THE INFLUENCE OF BOSE-EINSTEIN CONDENSATION ON THE DECAY OF SPIN-POLARIZED ATOMIC HYDROGEN

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Introduction:

Wall-free confinement of double spin-polarized atomic hydrogen (H↑⇑) may lead to an ultra-cold gas in which the Bose-Einstein transition will take place in an almost ideal form. Furthermore, the critical density $n_c \propto T^{3/2}$ will be so low that only two-body dipolar-relaxation processes affect the decay of the gas. Here we present an expression for the rate constants of all dipolar transitions $dd \rightarrow \alpha\beta$ (a, b, c and d correspond to the hyperfine levels of the 1s ground state of atomic hydrogen in increasing order of energy), which applies to all temperatures and especially to temperatures below the critical temperature T_c. The deviation from the non-degenerate (high-temperature) limit opens up the possibility to study the approach to and achievement of the phase transition.

Method:

The calculation of the dipolar relaxation rates is based on Fermi's Golden Rule, treating the (electron-electron) dipole interaction as a first order perturbation. The special role played by the condensate in the initial and final states of the relaxation process is taken into account in a way similar to the method developed by Lee and Yang [1] and leads to a description of the gas as a assembly of non-interacting quasi-particles having a dispersion relation that differs from the free one and depends on temperature:

$$\epsilon_k = \frac{\hbar^2 k}{2m_H} \sqrt{k^2 + 16\pi a \bar{\xi} n}, \qquad (1)$$

where a is the (triplet) scattering length and $\bar{\xi}$ is the condensate fraction $N_{\vec{0}}(T)/N$.

Since this formalism is correct up to the lowest order of the parameters $(na^3)^{1/2}$ and $(na\Lambda^2)^{1/2}$, it is applicable to atomic hydrogen in which case these parameters are extremely small. ($\Lambda \propto 1/T^{1/2}$ is the thermal deBroglie wavelength.) In addition, we make use of the fact that the relevant thermal energies are small compared to the hyperfine splitting of hydrogen.

Results:

Our results are summarized in the following expression,

$$G_{dd\to\alpha\beta}(B,T) = \frac{1}{2}\, G^{MB}_{dd\to\alpha\beta}(B,T{=}0)\; \bar{\xi}^2 + \qquad (2)$$
$$+\, 2\, G^{MB}_{dd\to\alpha\beta}(B,T{=}0)\; \bar{\xi}(1-\bar{\xi}) \left\{ 1 - \frac{9}{16} \frac{g_{5/2}(\bar{\zeta})}{g_{3/2}(\bar{\zeta})} \frac{k_B T}{\Lambda_{\alpha\beta}} + \ldots \right\} +$$
$$+\, G^{MB}_{dd\to\alpha\beta}(B,T{=}0)\; (1-\bar{\xi})^2 \left\{ 1 - \frac{9}{4} \frac{g_{5/2}(\bar{\zeta})}{g_{3/2}(\bar{\zeta})} \frac{k_B T}{\Lambda_{\alpha\beta}} + \ldots \right\}.$$

Here $G^{MB}_{dd\to\alpha\beta}(B,T{=}0)$ denotes the T=0 result for the rate constant using a classical Maxwell-Boltzmann velocity distribution [2], and hence neglecting the importent role played by the condensate. $\Lambda_{\alpha\beta}$ is the (internal) energy released in the process $dd\to\alpha\beta$, $\bar{\zeta}$ is the fugacity which is related to the chemical potential μ by $\bar{\zeta}=\exp(\mu/k_BT)$ and $\bar{\xi}$ is the condensate fraction. By minimizing the free energy of the system one gets

$$\begin{cases} \bar{\xi} = 1-(T/T_c)^{3/2} \\ \bar{\xi} = 0 \end{cases} \quad \text{and} \quad \begin{matrix} \bar{\zeta} = 1 & \text{if } T \le T_c \\ g_{3/2}(\bar{\zeta}) = n\Lambda^3 & \text{if } T > T_c \end{matrix} \quad . \qquad (3)$$

Finally $g_n(z)$ are the well-known Bose-functions $\sum_{i=1}^{\infty} z^i/i^n$ [3].

Physically, the three terms in the above formula represent the transition $dd\to\alpha\beta$ involving two, one and no condensate particles, respectively, in the initial state. In the high temperature limit $T \gg T_c$ only the last term survives and we recover the result using Maxwell-Boltzmann statistics for the translational degrees of freedom since $\bar{\zeta}\to 0$ and $g_n(\bar{\zeta})\to\bar{\zeta}$ if $T\to\infty$.

In a magnetic trap the temperatures will ultimately be so low ($T\simeq 10$–$100\mu K$) that for the most important relaxation processes $k_B T/\Delta_{\alpha\beta} \ll 1$. In this case Eq. (2) reduces to

$$G_{dd\to\alpha\beta}(B,T) = \frac{1}{2}\, G^{MB}_{dd\to\alpha\beta}(B,T=0)\ \ (2-\bar{\xi}^2), \qquad (4)$$

a result which can also be found by extending a method due to Kagan [4] to the case of two-body relaxation processes. However near the bottom of the trap (where the condensate will be formed) the magnetic fields may be very small [5] and the condition $k_B T/\Delta_{\alpha\beta} \ll 1$ may not always be fulfilled. An example is the transition $dd\to cc$, because $\Delta_{cc}(B)\ \alpha\ B$ for small magnetic fields.

Conclusion:

The appearance of the condensate drastically affects the stability of double spin-polarized atomic hydrogen in magnetic traps: all relaxation rates are reduced by a factor of 2 (at $T\simeq 0$). The measurement of this reduction would be a clear signal for the achievement of Bose-Einstein condensation.

References

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