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UNSTABLE OSCILLATION OF THE CRYOGENIC H MASER

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We show that the Maxwell-Bloch equations, describing the dynamics of the sub-Kelvin H maser, predict unstable and chaotic oscillations in a regime which should be easily accessible experimentally.

The room-temperature hydrogen maser (1) is the most stable existing frequency standard for a wide range of measuring times. As such it has been used very successfully for tests of general relativity, for Very Long Baseline Interferometry (VLBI) and for interplanetary navigation (Voyager 2 mission). Since some years work is going on to build a sub-Kelvin H maser for improving the frequency stability by one or more orders of magnitude (2).

A central result governing the operation of the (cryogenic) H maser is the so-called oscillation condition (3)

$$\frac{P}{P_c} = -2q^2\left[\frac{I}{I_{th}}\right]^2 + (1 - Cq)\frac{I}{I_{th}} - 1 > 0, \qquad (1)$$

expressing that the total radiated power P is to be positive (see Fig. 1). In Eq. (1) P_c is the critical power, I is the surplus flux of atoms entering the storage bulb in the upper (c) hyperfine level, I_{th} is its threshold value assuming only density-independent relaxation, q is the maser quality factor and $C = 2(T_1^0/T_2^0)^{1/2} + (T_2^0/T_1^0)^{1/2}$ with T_1^0 and T_2^0 being the density-independent longitudinal and transverse relaxation times.

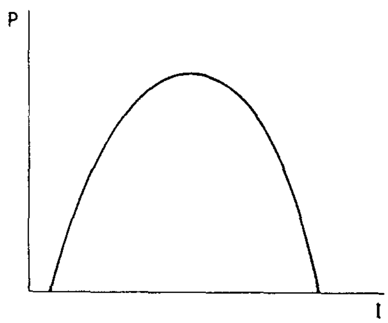

FIGURE 1: Power versus atomic flux

It has not been recognized until now that the condition (1) is only necessary but by no means sufficient. Reformulating the maser dynamics in the form of Maxwell-Bloch equations, it turns out that (1) guarantees only the existence of a steady solution, which is not necessarily stable, however. For stability a second condition $P < P_H$ has to be satisfied, where P_H is a threshold power beyond which unstable behavior begins. Restricting ourselves for definiteness to the case of small detuning $\delta \equiv T_2(\omega_{at} - \omega_m) = T_c(\omega_m - \omega_c)$ this condition has the form

$$\frac{T_1}{T_t}\frac{P - P_H}{P_H} = \left[-2q + \frac{T_c}{T_t}\frac{1 - 3\delta^2}{(1 + \delta^2)^2}\right]\frac{I}{I_{th}} - \frac{T_2^0}{T_t} < 0, \qquad (2)$$

with $1/T_c = \omega_c/2Q_c$ being the field loss rate and $T_t = (T_1^0 T_2^0)^{1/2}$. We have calculated the power P according to Eq. (1) as well as P_H, both on resonance, for the parameters of the University of British Columbia cryogenic maser (4). In particular, the cavity quality factor Q_c is equal to 1700. We find that $P < P_H$ for all fluxes satisfying the oscillation condition. Increasing Q_c to the (still modest) value 5×10^5, it is seen that the steady oscillation is unstable except for the very small fluxes (see Fig. 2).

An evaluation of the two conditions has also been carried out for the parameters of the Harvard-Smithsonian cryogenic maser (5). Again, the unstable regime seems easily accessible: Q_c has to be increased by a factor of 20 (see Fig. 3). From a similar comparison it follows that this regime is much more difficult to realize for a room-temperature H maser, mainly due to the much lower maximum atomic densities in the storage bulb allowed by the oscillation condi-

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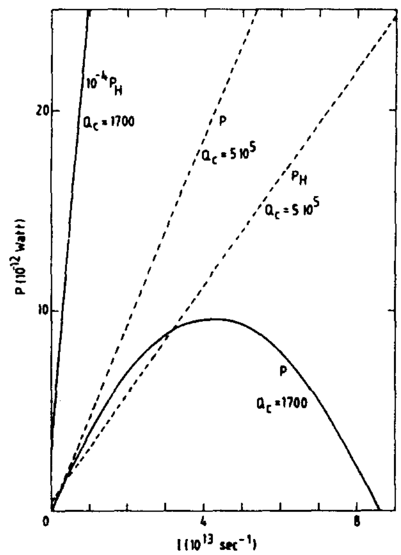

FIGURE 2: **Unstable regime for University of British Columbia maser**

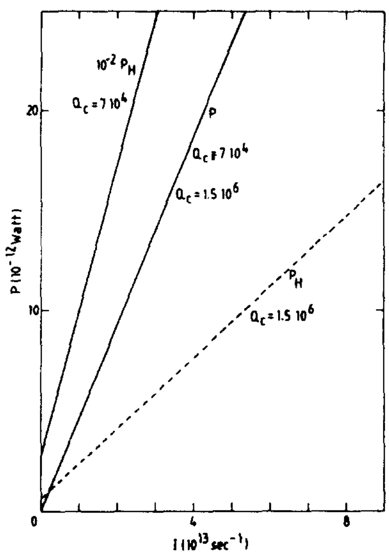

FIGURE 3: **Unstable regime for Harvard-Smithsonian maser**

tion due to the faster collisional relaxation rates.

The observation of instability may have important applications for obtaining information on the maser which would be very difficult to obtain otherwise. It is a priori to be expected that the nonsteady regime will offer much more information than the quantities frequency and amplitude, obtainable from stationary operation. This is especially welcome in view of the overwhelming number of experimental parameters such as hyperfine populations which determine the maser operation and are notoriously difficult to diagnose.

Apart from the prospect to get insight into the operation of the hydrogen maser, there is an intrinsic interest associated with the possibility to observe deterministic chaos in the cryogenic hydrogen maser. Not only does the derivation of the Maxwell-Bloch equations require far less simplifying assumptions than in the case of a laser, a system well-known to display chaos, but the ratios of the time constants entering the equations are that much different for both systems that the kind of nonsteady behavior to be expected in the H maser will differ largely from that of a laser. Although a detailed study has still to be carried out, it is already clear now at this stage that the operation of the H maser in the nonsteady domain will be characterized by a pulsed output power.

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