Limits on the Perception of Local Shape from Shading

Roderik Erens, Astrid M. L. Kappers, and Jan J. Koenderink
Utrecht Biophysics Research Institute, The Netherlands

Abstract. Local shape is not determined uniquely by shading. This leads to a wrong interpretation of shape by the human visual system when only shading information is present. When a cast shadow indicates the direction of the illuminant, concave and convex shapes are distinguishable, but elliptic and hyperbolic shapes are not distinguishable.

Introduction

Artists are very skilled in creating depth in natural images with the use of only monocular cues. Shading is generally expected to be one of the most important cues for creating this depth impression. But extracting shape from shading is not a trivial task. The only input of the visual system is the luminance data, determined by the surface orientation, the direction of the illuminant and the surface properties. Therefore the shape from shading problem is vastly underconstrained. Simplifying assumptions have to be made by the human visual system to solve this problem.

Several psychophysical experiments have been performed on the perception of shape from shading (Mingolla & Todd, 1986; Pentland, 1989). Some experiments have treated the problem of distinguishing between concave and convex shapes (Berbaum, Bever, & Sup Chung, 1984; Ramachandran, 1988), but the question of distinguishing between hyperbolic and elliptic shapes by the use of shading information alone has never been attempted before.

We have conducted several experiments on the perception of quadric surfaces with shading as depth cue. Quadric surfaces describe the whole range of elliptic and hyperbolic shapes. We are interested in the performance in distinguishing different shapes. The contribution of a cast shadow, which reveals the direction of the illuminant, is also investigated.

Methods

Every solid shape can be described locally by patches of quadric surfaces. We used the shape index and curvedness introduced by Jan Koenderink (1990), to describe the quadric shape. The shape index scale represents continuously all possible quadric shapes, but can roughly be divided in a concave elliptic, a hyperbolic and a convex elliptic part (Figure 1).

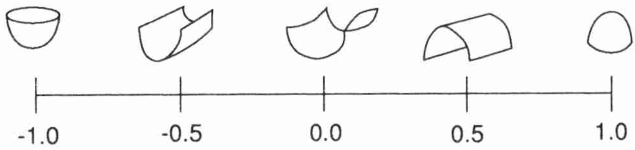

Figure 1. Local shapes presented on the shape index scale.

A quadric surface is described in tangent coordinates by the expression:

$$F(x,y) = \frac{1}{2}\left(K_1 x^2 + K_2 y^2\right) \quad (1)$$

where K_1 and K_2 are the principle curvatures. The shape index and curvedness are given by:

$$S = -\frac{2}{\pi}\arctan\frac{K_1 + K_2}{K_1 - K_2} \quad (2)$$

$$C = \sqrt{\left(K_1^2 + K_2^2\right)/2} \quad (3)$$

$S \in [-1,1]$, $C \in [0,\infty\rangle$

The shape index varies from -1 for a concave symmetric paraboloid up to 1 for a convex symmetric paraboloid. The curvedness varies from 0 for a flat surface up to infinity for an extremely curved surface.

The images are projected perspectively on a high resolution monitor and viewed monocularly from 1 m distance. The quadric surfaces are shaded diffusely, and the curvedness is 30 m^{-1} (think of a tennisbal).

The perspective projection gives every shape a specific outline (this is not an occluding contour!), by which it can be recognized easily. Therefore, the quadric shapes are covered with a random frayed grey mask. The quadric shapes are oriented randomly around the z-axis; otherwise the principle curvatures are always aligned along the x- and y-axis. The outline and orientation of the quadric surface would be a very strong cue for recognizing the image. The direction of the illuminant was chosen randomly from four directions: 45, 135, 225, or 315° with the positive x-axis. With our setup the only cue for perceiving the shape is shading.

In two experiments 3 series of 200 images are presented for 4 s with the shape index chosen randomly. Four subjects participated in the experiment, all familiar with the scale of the shape index and curvedness. In the experiments the shape index scale was divided into eight equal parts. Subjects answered on the shape index scale by picking a category.

First experiment

In a first experiment we investigated the possibility to perceive difference between convex, concave or parabolic and hyperbolic shapes, with only shading.

The light intensity for a diffusely shaded image is given by:

$$I(x,y) = \rho\lambda(\mathbf{N} \cdot \mathbf{L}) \tag{4}$$

where ρ is the surface albedo, and λ is the intensity of the illuminant. For a arbitrary direction of the illuminant:

$$\mathbf{L} = \frac{(l_1, l_2, l_3)}{\sqrt{l_1^2 + l_2^2 + l_3^2}} \tag{5}$$

Then, the light intensity for a quadric surface is given by:

$$I(x,y) = \rho\lambda \frac{-K_1 x l_1 - K_2 y l_2 + l_3}{\sqrt{l_1^2 + l_2^2 + l_3^2}\sqrt{K_1^2 x^2 + K_1^2 y^2 + 1}} \tag{6}$$

It can be easily shown that we invert convex into concave by rotating the light source over 180°. This is the well known crater illusion. It is also possible to invert hyperbolic shapes into elliptic ones by rotating the light over 90° or 270°.

Second experiment

Information about the direction of the illuminant is needed. In real images there are several hints to knowing this direction by interreflections between objects, cast shadows, or the illuminance profile near occluding contours. In this second experiment we placed a small stick on the surface which generated a cast shadow. The direction of the illuminant (l_1 and l_2, formula 6) is visualized with the cast shadow. The influence of a cast shadow on the perception of convex and concave ellipsoids is often investigated (Berbaum, Bever, & Sup Chung, 1984; Ramachandran, 1988), but the influence of a cast shadow on the perception of elliptic and hyperbolic shapes is less clear and never investigated before.

The intensity distribution does not uniquely describe a certain shape. The intensity distribution will not vary so drastically for concave or convex elliptic patches. On a concave elliptic patch the intensity gradient will always point in the direction of the illuminant and on a convex elliptic patch in the opposite direction. A cast shadow will directly solve the concave-convex ambiguity.

It is always possible to get on hyperbolic patches intensity patterns like on elliptic patches, for specific directions of the illuminant. Therefore, hyperbolic shapes can be interpreted just as easily as being concave or convex elliptic shapes.

Results

Figure 2 shows the results of a subject for the first experiment with no cast shadow information. The eight graphs represent the eight categories. On the y-axis the score in % is plotted against the answered categories on the x-axis. Figure 2 clearly shows that shading information alone, is not enough to perceive the correct shape. Most shapes are perceived totally wrong and there is a small tendency to answer on the positive (convex) axes of the shape index scale. No difference between convex and concave elliptic and hyperbolic patches is found. Furthermore, the correlation between the answered shape index and the direction of the illuminant is checked. It seems that no specific direction of the illuminant is assumed.

The influence of the cast shadow on the perception of the shape is shown in Figure 3. All concave elliptic shapes are answered on the concave side of the shape index scale and also the convex elliptic shapes are reported as convex. But the hyperbolic shapes (Figure 3, categories 3 to 6) lead to random response on the whole scale of the shape index. Apparently, it is not possible to distinguish between elliptic and hyperbolic shapes. The knowledge of the direction of the illuminant cannot solve this ambiguity.

Solving the shape from shading problem is complicated and the information of the direction of the illuminant is not enough for a unique interpretation of the intensity distribution.

Discussion

The concave-convex ambiguity is rather familiar and is often investigated in psychophysics (Berbaum, Bever, & Sup Chung, 1984; Ramachandran, 1988). But an arbitrary solid shape is described by hyperbolic and elliptic patches. Strangely enough, in most shape from shading experiments only

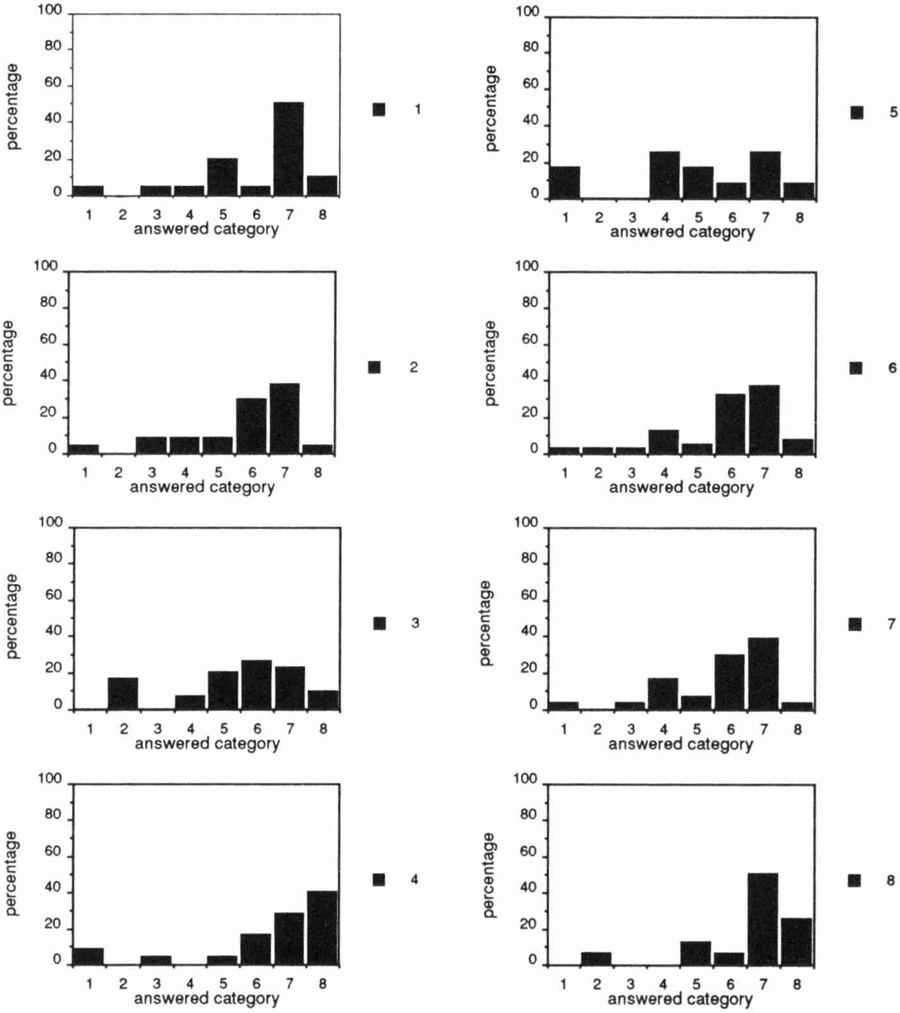

Figure 2. The results of the first experiment with no cast shadow. The score (%) is plotted against the answered categories. Each graph represents a different category.

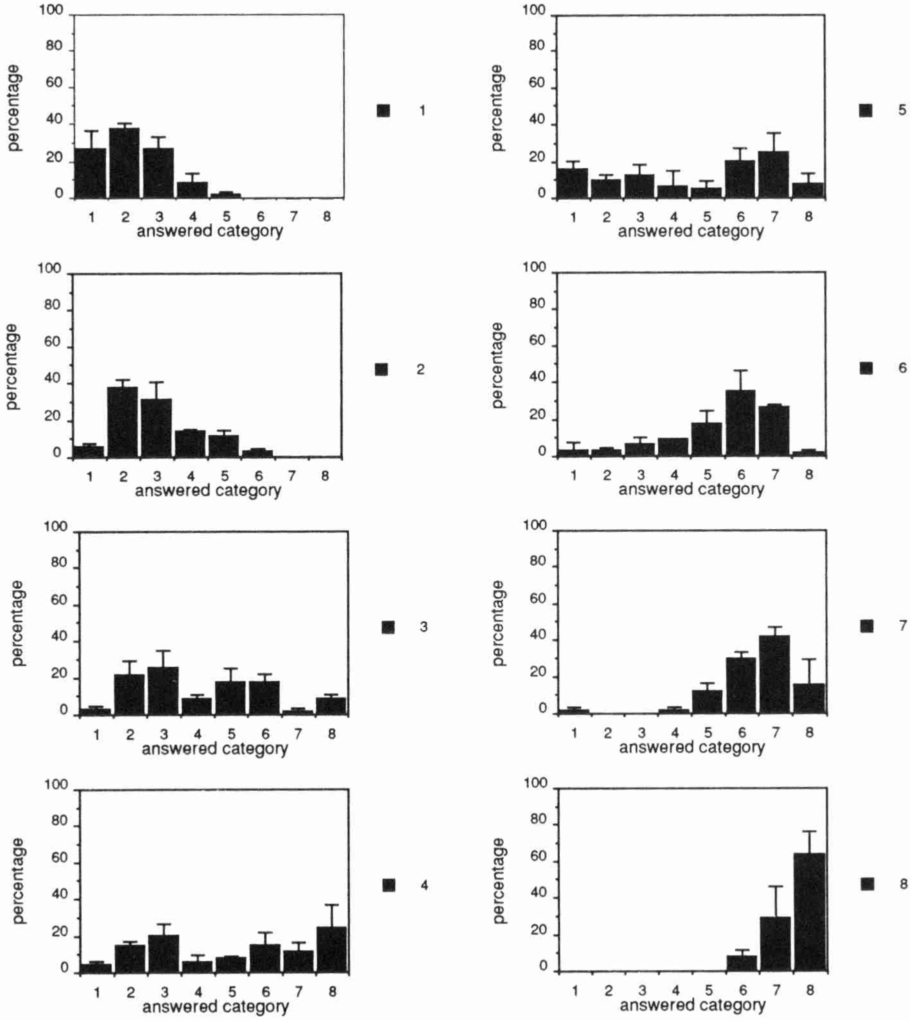

Figure 3. The results of the experiment with a cast shadow. The score (%) is plotted against the answered categories. Each graph represents a different category.

elliptic shapes are used. Therefore, the impossibility to distinguish between elliptic and hyperbolic patches with shading information only has never been noticed by others. This type of ambiguity is even stronger than the concave-convex ambiguity. With a cast shadow it is directly possible to distinguish between concave and convex shapes, but it is still impossible to distinguish between elliptic and hyperbolic patches.

Finding the local shape of a diffusely shaded object seems to be complicated. Even with knowledge of the direction of the illuminant it is not possible to distinguish between elliptic and hyperbolic shapes. However, in a natural image we get a strong 3D impression. In these images there are usually much more cues to get a 3D impression. For example the occluding contour, the interreflections between surfaces, the projection of a cast shadow on a surface, and the whole morphology of hyperbolic and elliptic regions on a complicated object (Koenderink & van Doorn, 1981). However, it is never investigated extensively what the contribution of each of these cues is to the perception of 3D shape.

Acknowledgment. This research is supported by the SPIN project '3D-Computer Vision' of the Dutch Ministry of Economic Affairs.

References

Berbaum, K., Bever, T., & Sup Chung, C. (1984). Extending the perception of shape from known to unknown shading. *Perception, 13*, 479-488.

Ramachandran, V. S. (1988). Perceiving shape from shading. *Scientific American, 259(2)*, 58-65.

Mingolla, E., & Todd, J. T. (1986). Perception of solid shape from shading. *Biological Cybernetics, 53*, 137-151.

Pentland, A. (1989). Shape information from shading: A theory about human perception. *Spatial Vision, 4*, 165-182.

Koenderink, J. J. (1990). *Solid shape.* Cambridge, MA: MIT Press.

Koenderink, J. J., & van Doorn, A. J. (1981). Photometric invariants related to solid shape. *Optica Acta, 27*, 981-996.