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POWER CORRECTIONS IN EIKONAL CROSS SECTIONS

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We discuss power corrections associated with the infrared behavior of the perturbative running coupling in the eikonal approximation to Drell-Yan and other annihilation cross sections in hadron-hadron scattering. General properties of the eikonal approximation imply that only even powers of the energy scale are necessary.

1 Introduction

Power corrections [1,2] are phenomenologically significant in many QCD hard-scattering cross sections for which the operator product expansion is not directly available. Examples that have received considerable attention include event shapes in electron-positron annihilation and transverse momentum distributions in Drell-Yan cross sections. In each of these cases, a perturbative description of the cross section leads to integrals of the form $I_p \equiv Q^{-p} \int_0^Q d\mu\, \mu^{p-1}\, \alpha_s(\mu)$ with Q the hard scale and $p \geq 1$. In perturbation theory with a fixed coupling, I_p is just a number, but when the coupling runs, the integral becomes ill-defined at its lower limit. This observation requires us to introduce a minimal set of power corrections of the form λ_p/Q^p, one for each ambiguous I_p that we encounter [3,4]. The perturbative expression is cut off, or otherwise regularized to make it finite without changing the set of exponents p. The values of the coefficients λ_p are then to be found by comparison with experiment; they depend on the nature of the perturbative regularization that is employed. In any case, it is only the sum of regularized perturbation theory and power corrections that has physical meaning.

The first step in this process is to show that in some self-consistent approximation the cross section at hand may be written in terms of integrals like the I_p above. In many cases, this step involves the resummation of logarithms associated with soft gluon emission, for which the eikonal approximation is useful. In this talk[a], we discuss an expression for the eikonal approximation in hadronic collisions, where the analysis of power corrections through the running coupling is particularly transparent.

2 The Eikonal Cross Section

To be specific, we discuss the eikonal approximation as it appears when partons a and b combine through an electroweak current, such as the Drell-Yan annihilation of quark with antiquark to a lepton pair or gluon fusion to a Higgs boson,

$$\sigma_{ab}^{(\mathrm{eik})}(q) = \int d^4x \; \mathrm{e}^{iq\cdot x} \langle 0|W_{ab}^{\dagger}(x) W_{ab}(-0)|0\rangle . \tag{1}$$

The operators W_{ab} are defined by

$$W_{ab}(0) \equiv \Phi^{\dagger}_{\beta'}(0)\Phi_{\beta}(0)\,, \tag{2}$$

in terms of nonabelian phase operators for a and b, $\Phi_\beta(0) = P\; \mathrm{e}^{-ig\int_0^\infty d\lambda \beta\cdot A(\lambda\beta)}$, with lightlike velocities β and β', $\beta\cdot\beta' = 1$.

The eikonal cross sections reproduce the logarithms, as singular as $(Q/q_0)\alpha_s^n \ln^{2n-1}(q_0/Q)$ and $(Q/q_T)\alpha_s^n \ln^{2n-1}(q_T/Q)$, that characterize the edges of partonic phase space at which the energy of radiation, q_0, or its total transverse momentum, q_T, vanish. The resummation of these logarithms is most convenient in terms of transforms,

$$\tilde\sigma_{ab}^{(\mathrm{eik})}(N,\mathbf{b}) = \int d^4q \; \mathrm{e}^{-Nq_0 - i\mathbf{b}\cdot\mathbf{q}}\, \sigma_{ab}^{(\mathrm{eik})}(q)\,. \tag{3}$$

In the transformed functions we find logarithms at each order up to $\alpha_s^n \ln^{2n} N$ and $\alpha_s^n \ln^{2n}(bQ)$, which exponentiate. The exponentiation of energy logarithms is known as threshold resummation [5,6,7], of transverse momentum logarithms as k_T resummation [7,8].

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3 Exponentiation

Transforms of the eikonal cross section may be written in exponential form on the basis of algebraic considerations that have been known for a long time,

$$\tilde\sigma_{ab}^{(\mathrm{eik})}(N,\mathbf{b}) = \exp\left[E_{ab}^{(\mathrm{eik})}(N,\mathbf{b}Q,\epsilon)\right]\,, \tag{4}$$

where the exponent is an integral over functions w_{ab}, sometimes called "webs" [9], which are defined by a modified set of diagrammatic rules,

$$\begin{aligned} E_{ab}^{(\mathrm{eik})} = 2\int^{Q} &\frac{d^{4-2\epsilon}k}{\Omega_{1-2\epsilon}} \\ &\times\, w_{ab}\left(k^2, \frac{k\cdot\beta k\cdot\beta'}{\beta\cdot\beta'}, \mu^2, \alpha_s(\mu^2),\epsilon\right) \\ &\times \left(\mathrm{e}^{-N(k_0/Q)+i\mathbf{k}\cdot\mathbf{b}} - 1\right)\,. \end{aligned} \tag{5}$$

The variable k in this expression may be thought of as the momentum contributed by the web to the final state. The webs factor from each other under the transforms, and indeed in any symmetric integral over phase space. [11]

Webs have a number of restrictive properties. At fixed k, they are invariant under rescalings of the velocities in the eikonal phases, which corresponds to boost invariance under the axis defined by the two. In addition, at any fixed order, the web function has only one overall collinear and IR divergence, from $k_T \to 0$ and $k_0 \to 0$, respectively. Finally, the web functions have no overall renormalization:

$$\mu\frac{d}{d\mu}\, w_{ab}\left(k^2, \frac{k\cdot\beta k\cdot\beta'}{\beta\cdot\beta'}, \mu^2, \alpha_s(\mu^2),\epsilon\right) = 0\,. \tag{6}$$

Using boost invariance in the large-N limit, we find that the exponent takes the form

$$\begin{aligned} E_{ab}^{(\mathrm{eik})} = 2\int &\frac{d^{2-2\epsilon}k_T}{\Omega_{1-2\epsilon}} \\ &\times \int_0^{Q^2-k_T^2} dk^2\; w_{ab}\left(k^2, k_T^2 + k^2\right) \end{aligned} \tag{7}$$

$$\times \left[\mathrm{e}^{-i\mathbf{b}\cdot\mathbf{k}_T} \; K_0 \left(2N \sqrt{\frac{k_T^2 + k^2}{Q^2}} \right) - \ln \sqrt{\frac{Q^2}{k_T^2 + k^2}} \right] + \mathcal{O}\left(\mathrm{e}^{-N}\right) \, .$$

This expression, which requires dimensional regularization for its collinear divergences, is completely general for the eikonal cross section.

4 Factorization

The factorized hard-scattering function, $\hat{\sigma}_{ab}^{(\mathrm{eik})}$, for the eikonal cross section may be constructed in moment space to $\mathcal{O}(1/N)$ by dividing by moments of eikonal distributions, [10]

$$\tilde{\phi}_f^{(\mathrm{eik})}(N, \mu, \epsilon) = \exp\left[- \ln\left(N\mathrm{e}^{\gamma_E}\right) \int_0^{\mu^2} \frac{d\mu'^2}{\mu'^2} \; A_f\left(\alpha_s(\mu'^2)\right) \right] , \tag{8}$$

where A_a, with $A_a^{(1)} = C_a$, is the coefficient of $\ln N$ in the Nth moment of the $a \to a$ splitting function. Factorization theorems ensure the cancellation of collinear divergences in the resulting hard-scattering functions. Invoking this requirement, we find an explicit relation between the webs and the anomalous dimensions,

$$\int_0^{Q^2 - k_T^2} dk^2 \; w_{ab}\left(k^2, k_T^2 + k^2\right) = \frac{A_a\left(\alpha_s(k_T^2)\right) + A_b\left(\alpha_s(k_T^2)\right)}{(k_T^2)^{1-2\epsilon}} + \ldots . \tag{9}$$

In this fashion, we derive a general form for the eikonal approximation to the hard-scattering functions $\hat{\sigma}_{ab}$ of electroweak annihilation,

$$\hat{\sigma}_{ab}^{(\mathrm{eik})}(N, \mathbf{b}, Q, \mu) = \frac{\sigma_{ab}^{(\mathrm{eik})}(N, \mathbf{b}, Q)}{\tilde{\phi}_a(N,\mu)\,\phi_b(N,\mu)} = \exp\left[\hat{E}_{ab}^{(\mathrm{eik})}(N, b, Q) \right] , \tag{10}$$

where the collinear-finite exponent (here shown in a simplified form, accurate to NLL) is:

$$\hat{E}_{ab}^{(\mathrm{eik})} = \int_0^{Q^2} \frac{dk_T^2}{k_T^2} \sum_{i=a,b} A_i\left(\alpha_s(k_T^2)\right) \times \left[J_0\left(bk_T\right) \; K_0\left(\frac{2Nk_T}{Q}\right) + \ln\left(\frac{N\mathrm{e}^{\gamma_E} k_T}{Q}\right) \right] . \tag{11}$$

This result is the basis of the joint threshold-k_T resummation [12] developed in Ref. [13].

As described above, the resummation of logarithms as in (11) requires the inclusion of power corrections in both Q^{-1} as well as b, to compensate for the ill-defined behavior of the strong coupling at low scales. In [3] it was shown that for the Drell-Yan cross section only integer powers of Q^{-1} are necessary; in [14] models of the running coupling were invoked to suggest that power corrections begin at order Q^{-2}. Eq. (7) implies that only even powers of Q are present in all generality for the eikonal approximation. This is because, up to a single log, the expansion of the Bessel function $K_0(z)$ at small z involves only even powers of z. This conclusion includes, and requires, an expression which, like Eq. (7), is accurate to the level of "constant terms", $(\ln N)^0$. Other consequences of this approach have been discussed in Ref. [13].

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