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A complex network approach to urban growth

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Abstract. **Economic geography can be viewed as a large and growing network of interacting activities. This fundamental network structure and the large size of such systems makes the complex network approach an attractive model for growth dynamics modeling. In this paper the authors propose the use of complex networks for geographical modeling and demonstrate how such an application can be combined with a cellular model to produce output that is consistent with large-scale regularities such as power laws and fractality. Complex networks can provide a stringent framework for growth dynamic modeling where concepts from, for example, spatial interaction models and multiplicative growth models, can be combined with the flexible representation of land and behaviour found in cellular automata and agent-based models. In addition, there exists a large body of theory for the analysis of complex networks that have direct applications in urban geographic problems. The intended use of such models is twofold: (1) to address the problem of how the empirically observed hierarchical structure of settlements can be explained as a stationary property of a stochastic evolutionary process rather than as equilibrium points in a dynamic process, and, (2) to improve the predictive quality of applied urban modeling.**

1 Introduction

Economic geography today is often considered to be less about the 'space of places', where the success of places is attributed to characteristics of locations, and more about the 'the space of flows' where places derive their success from their position in networks (Castells, 2000; Hohenberg and Lees, 1995). Networks consist of interacting nodes, and interactions between geographical locations are fundamental to our present understanding of geographical systems of human origin (see Isard, 1960). However, networks in the economy and society are typically hard to analyse just by observing them. They are very large, involving thousands or millions of nodes and connections and, rather than representing some schematic structure, they represent arbitrary outcomes of a growth dynamics. Furthermore, these systems generally change slowly and irreversibly over time, which means that they do not lend themselves well to equilibrium analysis.

Complex networks—an area of study that stems from graph theory and statistical physics—provides a new paradigm for the study of complex evolving systems (Barabási and Albert, 1999). This approach has been extensively explored over the past decade for studying large growing networks in a wide range of areas, including the World Wide Web (Barabási et al, 2000), the Internet (Cohen et al, 2000; 2001; Pastor-Satorras et al, 2001), human sexual contacts (Liljeros et al, 2001), ecological networks (Camacho et al, 2002), cellular networks (Jeong et al, 2000; Ravasz et al, 2002), social networks

(Girvan and Newman, 2002; Holme, 2003; Jin et al, 2001), phone calls (Abello et al, 1999; Aiello et al, 2000), citations (i Cancho and Solé, 2001; Redner, 1998), and protein folding (Scala et al, 2001). Owing to the present availability of large geographical datasets and the inherent characteristics of economic geography as a large growing network, complex networks appear to be an attractive baseline model. The creation of urban growth models that are consistent with global regularities, which at the same time are open-ended for the addition of the wealth of details necessary for application (for example, scenario prediction), must be considered an important research direction. This paper represents an attempt to work in this direction through the application of the complex network approach to urban growth models.

Most urban theorists agree that the best explanations for large-scale regularities in urban systems are still provided by simple multiplicative growth models based on the so-called Gibrat's law, which states that the relative growth rates of cities are constant, and notably by the models of Gabaix (1999) and Simon (1955). Although crucial as criteria for the admissability of models, the importance of large-scale regularities in urban systems should not, however, be overstated (Gabaix, 1999; Pumain, 2004). In complex systems, such as the urban system, macroscopic states of this kind do not exhaust the properties of the system nearly as well as what is often the case for physical systems. First of all, it is not at all certain that any specific simple distribution describes these states particularly well. For example, there is no general consensus on the value and applicability of the rank-size rule: empirical observations are simply not convincing enough, and too many exceptions exist. What can be asserted with more confidence, however, is that there is clear stationary hierarchisation in urban systems. In the sense that multiplicative growth models generate stationary hierarchies they are attractive as models for this type of phenomenon. Such simple models do not serve as final models of observed phenomena but rather as kernels of models that can in principle be made arbitrarily complex.

Models formulated on the macrolevel also hide a lot of potentially important internal dynamics of the objects and relations in a model, and this has some implications. For example, it is hard to relax assumptions on local growth rates in a realistic way in a macroscopic model in which growth rates apply to entire cities, cities being objects that lack a proper unambiguous definition *casu quo* delineation (Pumain, 2004). Such models do not allow us properly to understand when, why, and in which ways urban systems conform to or deviate from these regularities. To do this, it appears reasonable that microdynamics need to be considered explicitly. For instance, spatial interaction and a variety of often cited 'forces' (for example, agglomeration economies, economies of scale, and immobile inputs) act on levels below that of entire cities. Another implication is that macroscopic models cannot be used to address equally important issues of internal structure of urban systems—something that, apart from being of theoretical interest, is also of critical importance for forecasting and scenario exploration models.

To be able to use the hypothesis that stationary power laws stem from multiplicative growth in more detailed models we must be able to incorporate more detailed growth dynamics. By using a complex network model mapped onto a cellular space we may use multiplicative growth on a microlevel and incorporate, in a flexible and open-ended manner, any number of submodels that capture forces such as transportation costs, external economies, economies of scale, and agglomeration economies. Doing this we may investigate whether, in which way, and under what circumstances such forces affect global regularities. The representation of land that we use here is very similar to that found in a cellular automaton, and many of the attractive properties that are associated with cellular models also apply here. However, the mathematical toolbox that applies to complex networks, and at the same time pertains to relevant issues for geographic modeling, is considerably more general. Furthermore, the central importance of interactions in economic geography is reflected in the fundamental structure of complex networks. In that sense, complex networks models retain and extend the functional advantages of previous models and, additionally, bring a substantial battery of analytical and statistical methods to bear on the problem.

We will adhere to the following structure. First, we discuss some properties of economic geography as a complex dynamic system and arrive at a rationale for using complex networks to address some important questions pertaining to the generation of macrostructure from microscopic mechanisms. We then introduce a baseline model of complex evolving networks. Thereafter, we proceed to define a model of the proposed type for the purpose of simulating the geographical evolution of land values, clusters, and patterns. We also present some results and interpretations of this model and discuss some of the problems and benefits of this formulation. Finally, we discuss the application of complex networks to economic geography in a wider context. The basic assumptions underlying our model have also been used in previous papers (Andersson et al, 2003; 2005). In this contribution, we elaborate on the model by introducing the concept of node fitness and by discussing the methodology, model setup, and model properties in the context of economic geography more widely.

2 A conceptual model that can be extended to more topical incarnations

The term 'model' bears somewhat different connotations for different readers. Whereas 'model' to a planner might indicate a tool for the prediction of specific scenarios, a 'model' to a theoretically minded person might suggest a more general and conceptual tool for understanding the connection between mechanisms and phenomena. We will refer to these two ends of the spectrum of models as the *topical* and *conceptual* types of models, respectively. For example, Simon's model would be a more conceptual model whereas a calibrated cellular automaton would be a more topical model. This distinction is important because models along the entire spectrum are being developed and explored in the context of urban growth, and this might influence the expectations of individual readers.

Conceptual models are constructed to address fundamental questions about systems. In the case of a dynamic system such as an urban system we might, for example, pose questions about possible types of behavior and about the way in which the system responds to changes in global parameters. Such models are not created with the primary intention of predicting any urban system in particular but to explore basic system behaviour such as catastrophes (in the mathematical sense), bifurcations, fractality, agglomerations, and power laws. Examples of conceptual models in urban dynamics would include multiplicative growth models, models of correlated percolation, diffusion-limited aggregation, agent-based evolutionary models, and neoclassical economic models (for example, Alonso, 1964; 1972; Arthur, 1987; Axtell and Florida, 2001; Batty, 1991; Batty and Longley, 1994; Berry and Garrison, 1958; Dendrinos and Rosser, 1992; Fujita et al, 1999; Gabaix, 1999; Henderson, 1974; Lane, 2002; Makse et al, 1998; Manrubia et al, 1999; Marsili and Zhang, 1998; Reed, 2002; Rosser, 1998; Simon, 1955). A drawback of such models has been that the lessons that have been learned have not easily been carried over to applied models of the more topical type.

Dynamic models of urban evolution toward the topical end of the spectrum in urban growth modeling include primarily cellular automata, or, at least, models that include a cellular automaton as an important submodel, and agent-based models. Models of this type are designed typically for predicting the future of specific urban systems and are often very complex, involving numerous submodels. The use of cellular automata for urban growth modeling dates back over two decades to work by Tobler (1979) and was further developed by Couclelis, White, Clarke, Itami, and others (for example, Andersson et al, 2002; Clarke and Gaydos, 1998; Clarke et al, 1997; Couclelis, 1985; Engelen et al, 1997; Itami, 1988; Torrens and O'Sullivan, 2001; White et al, 1997; Xie, 1996). The cellular automata framework has been useful because of its flexibility: it is easy to incorporate any number of states and transition rules, and cellular models can easily be layered hierarchically with a mix of micro and macro models. The drawback from a theoretical point of view is that the mapping between urban systems and cellular automata does not in itself translate into much in terms of pertinent theory: not much may be learned about urban systems from the fact that they may be viewed as cellular automata, and the framework is in this respect too flexible. The agent-based approach, such as the SIMPOP model (Sanders et al, 1997), shares not only the implementation flexibility of a cellular automaton but also the theoretical shortcomings. In addition, other approaches, related more to time-series prediction in general than to urban evolution processes, have been used. Examples of such methods include artificial neural networks and logistic regression (for example, Pijanowski et al, 2002; Yeh and Li, 2002), and Markov models (for example, López et al, 2001). A class of other models that have seen extensive topical use consists of spatial interaction models; such models, however, generally are not aimed at reproducing urban evolution.

Under the distinction between conceptual and topical models used here, the model presented below is to be viewed as a conceptual model that is constructed with the aim of being openended for elaboration into topical models. Some steps toward making the model more topical to an urban growth context are taken: we embed the basic network model into a cellular space and we make a distinction between different types of nearest-neighbour neighbourhood types. The method of model construction that we have used and that we argue can be applied further involves the use of a conceptual model as a kernel with a step-by-step approach towards a more topical model through the addition of more and more mechanisms. For each new set of additions we validate the behaviour of the model. Additions should not destroy the desirable behaviour of the simpler parent models, but should add new meaningful behaviour in agreement with empirical observations. In summary, the intention is to formulate a model that (1) is reasonably stringent, (2) is formulated at the microlevel, (3) is geographic (incorporating spatial interaction), (4) captures growth dynamics, (5) is reasonably openended for additions, and (6) is consistent with the large-scale statistical properties of urban systems.

3 The core model of a complex evolving network

Our point of departure is the graph-theoretical model of Barabási and Albert (1999), which has become a baseline model to explain uneven distributions as an outcome of growth dynamics within a network space. Essentially, the Barabási–Albert model elaborates on the principle of preferential attachment of the stochastic growth model of Simon (1955) in the context of networks. A network is a collection of nodes and connections between nodes, and the term 'complex networks' is commonly used for networks that have a structure that evolves over time as a result of the application of some mechanism by which new nodes are created and existing nodes are selected as endpoints for new connections. Depending on the nature of this update mechanism and the properties of the network (for example, whether it is directed or undirected) such networks come to have various properties as they grow. As will be outlined, one such property is the stationary power-law distribution of node degrees found in the Barabási–Albert model. In this model it is assumed that an increase in the connectivity (or degree) of a node in a network is a function of its current connectivity. More precisely, the probability $\Pi_i(t)$ that a new node connects to an existing node i at iteration t is linearly proportional to the degree $x_i(t)$ of node i at iteration t; that is,

$$\Pi_i(t) \propto x_i(t) . \tag{1}$$

Several variations and extensions of the initial model have been proposed, as reviewed by Barabási et al (2002), Dorogovtsev and Mendes (2002), and Newman (2003). A simple, but fairly general, version can be defined formally as follows. Consider a growing, enumerated set of nodes, $\{1, 2, ..., N\}$, connected by undirected edges. As the network develops, new nodes and edges are added according to a stochastic model. There are no restrictions on multiple edges between two nodes, and a node is allowed to have any number of connections to itself. The degree (number of connections) of node i is denoted by x_i.

The network is initialised by connecting n_0 nodes and at each iteration t the network is updated by the addition of one edge between two nodes that are chosen independently according to the following rules.

Rule 1: with probability q_1 the node is chosen uniformly between existing nodes. The probability that node i will be selected, Π_i^{u}, is given by

$$\Pi_i^{\mathrm{u}} = \frac{1}{N} . \tag{2}$$

Rule 2: with probability q_2 the node is chosen preferentially, which corresponds to the uniform selection of an edge endpoint in the system and the subsequent location of its node. The probability that node i will be selected from among nodes j, Π_i^{p}, is given by

$$\Pi_i^{\mathrm{p}} = \frac{x_i}{\sum_j x_j} . \tag{3}$$

This means that the probability is linearly proportional to the node degree.

Rule 3: with probability q_3 a new node is added to the network. This node will have a degree of 1.

The parameters q_1, q_2, and q_3 fulfill the condition $q_1 + q_2 + q_3 = 1$, and are assumed to be constant during the evolution of the network.

When the growth rules are formulated, a number of questions can be posed about the properties of the network after a large number of iterations. The node degree distribution is a property that can be analysed rather easily. It is possible to find an exact expression with use of a master equation approach (Dorogovtsev et al, 2000) but for our purposes an approximate solution is sufficient. Such a solution can be obtained by using a continuum formulation of the model. For node i the time evolution of the expected node degree is as follows:

$$x_i(t+1) = x_i(t) + 2q_1 \frac{1}{N(t)} + 2q_2 \frac{x_i(t)}{\sum_j x_j(t)} , \tag{4}$$

where $N(t)$, equal to $2q_3 t$, is the expected number of nodes developed after t iterations. By using the continuous time method introduced by Albert et al (1999) we find that, after a sufficiently long time, the degree distribution approaches the form

$$P[x_i = x] \approx (x+B)^{-\gamma} , \tag{5}$$

where

$$B = \frac{q_1}{q_2 q_3}, \tag{6}$$

and

$$\gamma = 1 + \frac{1}{q_2}. \tag{7}$$

The reason for including uniform growth is to allow pure (additive) random growth that is not correlated to the present amount of activity in the particular land lot where the growth occurs. For example, the development of new land is surely additive because there is no present activity in the lot to multiply: areas that are in connection with the transportation network will have a market potential and be eligible for development. This corresponds to the addition of new cities in, for example, Simon's model but here it also serves the crucial role of edge growth of urban clusters. In the context of urban growth, the argument for multiplicative growth (also referred to as Gibrat's law) can be formulated in a number of ways, with perhaps the most robust being based on a lack of information. It is fair to assume that growth often is generated as a direct result of existing activities. Now, if we do not know anything about the activity in two cities of size X and $2X$ then we must assume that the next growth event will occur in the first city with probability $\frac{1}{3}$ and in the second with probability $\frac{2}{3}$ This argument, which has been used for cities most notably in models derived from that of Simon, should also be applicable to other areas within which activity is located. The use of constant growth rates is, of course, an approximation and it is perhaps more realistic instead to use constant moments for a distribution from which growth rates are taken. This has been done by Gabaix at the city level (with constant mean and variance) and will be introduced into the present model in future. In network terms, which holds greater generality than in this application, the model can be described as a linear combination between a random network (Erdős – Rényi network) and a scale-free network (Barabási – Albert network). In a random network the node selection mechanism is uniform per growing unit (in this paper, per land lot) whereas in the scale-free network the selection mechanism is uniform per unit size of the growing unit (in this paper, per unit activity in the land lot) and corresponds to Gibrat's law and appears to be a realistic base model in many real-world situations, including the above-cited systems that have been analysed as scale-free networks.

From equation (5), this simple stochastic model can be used to reproduce the power law distribution of degrees of nodes observed in many networks. Importantly, as shown by Barabási et al, both the assumption of growth and the assumption of preferential attachment are needed to reproduce the stationary power laws observed in empirical data on networks. If one assumes only uniform, and no preferential, attachment ($q_2 = 0$) the scale-free distribution is not reproduced. Also if one assumes a given nongrowing ($q_3 = 0$) set of nodes, preferential attachment first produces a power law, yet, ultimately, the network evolves into a state in which all nodes are connected and the degree distribution tends to a Gaussian distribution. It should be noted, though, that there are other network models that generate power-law degree distributions without any of these mechanisms [see Newman (2003) and the references therein].

The Barabási – Albert model, in a simple and reasonably robust manner, thus reproduces the scale-free degree distribution observed empirically in many complex networks. In the context of economic geography, the next step is to argue for the mapping between model and reality and to introduce the additional 'forces' involved in urban dynamics.

4 A complex network model of urban evolution

4.1 Definition of objects and interactions

From assumptions of statistical averages regarding behaviour and activity type we construct a model that fits closely statistical empirical data on relating to geographical land-value distribution concurrently at several levels of abstraction: for land value per unit area, for land value per city, and for the relation between cluster area and perimeters. The parameters are approximated from empirical data and the model reacts smoothly to changes in parameter values. The model explains urban power laws in relation to multiplicative growth that is sufficiently dominant with respect to other types of growth and in relation to the propensity for new land development to occur along the urban perimeter or along roads. It also isolates the geographically heterogeneous growth biases that give rise to cluster dynamics as a combination of transportation diseconomies and availability of infrastructure. These biases, which will be defined later in this section are not strong enough to cause appreciable deviations from the power law properties of the Barabási – Albert network but are sufficiently strong to cause geographic clustering.

Objects in a network model are represented as nodes, and interactions are represented as connections between nodes. Agents are modeled only implicitly and their behaviour is integrated in the selection of locations for growth. In the example used here we will use small fixed-sized nonoverlapping land lots as objects, and we will use trade streams between land lots as connections. Note that no explicit types of land use are modeled and that this amounts to assuming that any number of land uses and goods or services exist. Given a connection endpoint of an unknown type, each node is equally likely to have a demand or supply that makes it eligible for being the second endpoint. With respect to transportation, interaction probabilities or rates rather than explicit costs of transportation are used, with short connections being more common than long connections.

We note that profit (here considered before rent is paid) is generated as a consequence of trade (in a wide sense) between pairs of largely immobile specialised producers. That is, growth is the addition of a new connection, and this causes an increase in profit for each endpoint of the connection. We approximate the amount of profit in a lot simply by counting the number of connection endpoints in that lot. This gives the basic network structure of the geographic economic system. Interactions, defined as trade streams, can simply be added and would also constitute a valid definition at the city level. At the lot level, because lots here are by definition constant in area, the addition of a new connection does, however, have an additional important effect: it implies an increase in profit per unit area. If we assume that the market pricing of land is a fast process compared with the process of urban growth we can also invoke the 'leftover principle' from urban economics to say that an increase in profit per unit area implies an increase in land rent. A new connection represents a new means of earning money in this location and, because this behaviour can be copied by other agents, the landowner can generally charge rent for this. That is, if the present tenant does not want to pay more rent then other potential tenants might. Land value, in turn, can be approximated by the present value of all future income streams stemming from rent. This allows us to compare empirical and simulated land values by taking the node degree as proportional to the market land value. Obviously, it does not allow us to reconstruct an explicit network.

4.2 Model formulation

When we introduce explicit spatiality in the model, the choice of the nodes in a pair can no longer be independent. This situation is handled by selecting one of the nodes before the other. We call the first node the place of primary growth and the other node the place of secondary growth. The selection of these geographic locations is made either multiplicatively (proportional to the current node degree) or additively (independent of the current node degree).

Additive growth, then, represents growth processes that are not directly correlated with present activity. In the nonspatial model we treated this case separately, but now it turns out that we can handle node creation as additive growth on previously undeveloped sites. We therefore drop q_3 from the model and what was formerly node addition now becomes the transition of nodes from being unconnected to connected by additive growth. As nodes are taken to be land lots the number and identity of nodes remains the same throughout the simulation, only their connections evolve (see also figure 1).

If we assume constant fractions q_1 and q_2 for additive and multiplicative growth, respectively, with $q_1 + q_2 = 1$, the probability of selecting a node i multiplicatively in primary growth is

$$\Pi_i^{1,\mathrm{mul}} = q_2 \frac{x_i}{\sum_j x_j}, \tag{8}$$

which is similar to the nonspatial case.

The probability of selecting a node i additively as a primary effect must be dependent on the availability of basic infrastructure. To keep it simple, we divide the sites into three categories—developed sites, perimeter sites, and external sites. If we ascribe a weight a_i to each site (dependent on which category it belongs) and pose the condition that the sum of all probabilities for additive primary growth should be q_1, we get

$$\Pi_i^{1,\mathrm{add}} = q_1 \frac{a_i}{\sum_j a_j}. \tag{9}$$

The property a_i of nodes is determined by a simple local infrastructure availability model that can be viewed as a cellular incarnation of cluster growth and birth mechanisms in cluster-based multiplicative models. In contrast to cluster-based models, in which we differentiate only between cluster growth and cluster birth, here we also wish to capture the spatial patterns of clusters and the spatial interaction between cells. The developed sites are defined to be the base case, and $a_i = 1$ for these. Perimeter sites are all undeveloped sites adjacent to a developed site, and all these are assigned $a_i = b$, where b is a parameter. In most realistic cases $b < 1$, because there is, compared with a developed site, less probability of an average perimeter site having adequate infrastructure. Government growth control will also decrease b (see also figure 2).

External nodes are nodes that are neither developed nor on the urban perimeter. In this case, usually only sites with direct access to roads can be considered for development. We assume that all external sites have access to relevant roads and other infrastructure (thus effectively making them perimeter nodes) with some probability $0 < a_i < b$. This probability must surely grow with the system as more and more roads cross the hinterland, and we represent this by making it proportional to the ratio between the number of perimeter nodes, $n_i^{(\mathrm{P})}$, and the number of external nodes, $n_i^{(\mathrm{E})}$ (the t-indices denote that these variables change during the evolution of the system). We then get, for external sites, $a_i = b\varepsilon[n_t^{(\mathrm{P})}/n_i^{(\mathrm{E})}]$, where ε is a constant parameter describing the relative density of infrastructure. [For this to be reasonable, we must assume that the lattice is not too crowded; that is, $\varepsilon_t^{(\mathrm{P})} < n_t^{(\mathrm{E})}$.]

Now when all three categories of sites have been considered, equation (9) can be written as follows:

$$\Pi_i^{1,\text{add}} = q_1 \left[\delta_i^{(\mathrm{D})} + b\delta_i^{(\mathrm{P})} + b\varepsilon \frac{n_t^{(\mathrm{P})}}{n_t^{(\mathrm{E})}} \delta_i^{(\mathrm{E})} \right] \Big/ \sum_j \left[\delta_j^{(\mathrm{D})} + b\delta_j^{(\mathrm{P})} + b\varepsilon \frac{n_t^{(\mathrm{P})}}{n_t^{(\mathrm{E})}} \delta_j^{(\mathrm{E})} \right]$$

$$= \delta_i^{(\mathrm{D})} + b\delta_i^{(\mathrm{P})} + b\varepsilon \frac{n_t^{(\mathrm{P})}}{n_t^{(\mathrm{E})}} \delta_i^{(\mathrm{E})} \Big/ \left[n_t^{(\mathrm{D})} + b(1+\varepsilon) n_t^{(\mathrm{P})} \right], \qquad (10)$$

where $n_t^{(\mathrm{D})}$ is the number of developed nodes, $\delta_j^{(\mathrm{D})} = 1$ if node j is developed, and $\delta_j^{(\mathrm{D})} = 0$ otherwise. The meanings of $\delta_j^{(\mathrm{P})}$ and $\delta_j^{(\mathrm{E})}$ are analogous to that of $\delta_j^{(\mathrm{D})}$, with P

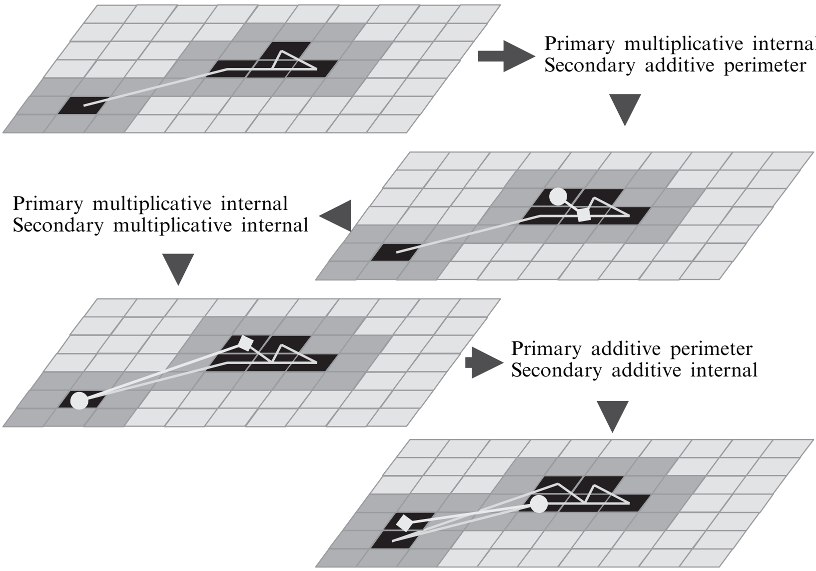

Figure 1. A sample spatial network configuration with external, perimeter, and internal nodes. The network is updated stochastically by the addition of new connections. In this figure we show how nodes (cells) are selected at random according to the increase in the specified types of activity. For example, in the first update the first endpoint (diamond) is selected according to the multiplicative primary effect scheme, and the second endpoint (circle) is selected with the secondary additive scheme.

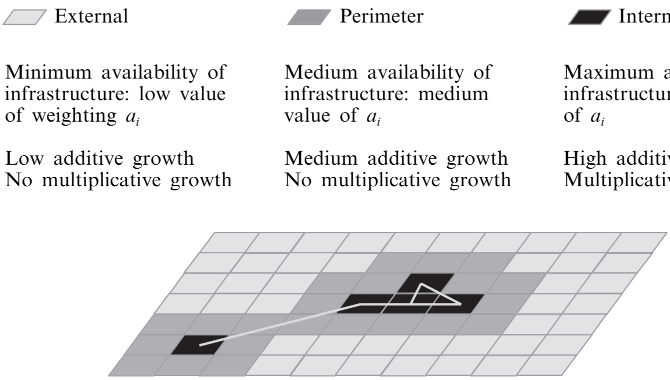

Figure 2. A simple local infrastructure availability model: a sample spatial network configuration with external, perimeter, and internal nodes; connections between nodes are also shown.

referring to perimeter sites, and E referring to external sites. A site must belong to one, and only one, of these categories at a time, which means that for each j one and only one of $\delta_j^{(D)}$, $\delta_j^{(P)}$, or $\delta^{(E)}$ is equal to 1, with the other two equal to 0.

Now, after the selection of the primary site (by means of one of the two mechanisms described above) the secondary site should be selected with a probability decreasing with distance from the primary one. To accomplish this, we define D_{ij} to be the strength of spatial interaction between sites i and j. We have the restrictions $D_{ij} \leqslant 1$, and $D_{ij} = D_{ji}$.

The probability of secondary preferential growth at site i as a consequence of primary growth at site j is

$$\Pi_{ij}^{2,\mathrm{mul}} = q_2 \frac{D_{ij} x_i}{\sum\limits_k D_{kj} x_k}, \tag{11}$$

and, by analogy, for secondary uniform growth, it is

$$\Pi_{ij}^{2,\mathrm{add}} = q_1 \frac{D_{ij} a_i}{\sum\limits_k D_{kj} a_k}; \tag{12}$$

with the same site categories as for primary growth, $\Pi_{ij}^{2,\mathrm{add}}$ can be written as follows:

$$\Pi_{ij}^{2,\mathrm{add}} = q_1 \left[D_{ij} \left(\delta_i^{(D)} + b\delta_i^{(P)} + b\varepsilon \frac{n_t^{(P)}}{n_t^{(E)}} \delta_i^{(E)} \right) \right] \Bigg/ \sum_k D_{kj} \left[\delta_k^{(D)} + b\delta_k^{(P)} + b\varepsilon \frac{n_t^{(P)}}{n_t^{(E)}} \delta_k^{(E)} \right]. \tag{13}$$

Several choices for D_{ij} are possible, but it is clear that the interaction strength should decay with increasing transportation costs. We have, for the most part, used the following:

$$D_{ij} [1 + cd(i, j)]^{-\alpha}, \tag{14}$$

where $d(i, j)$ is the Euclidean distance between sites i and j. The nonnegative parameters c and α control the impact of spatiality.

4.3 Results

Agreement between model output and empirical data is important because failure to agree would allow us to disqualify the model or point to necessary modifications. However, empirical agreement does not validate the model in any stronger sense, because there generally exist many possible models that are capable of producing some sought-for behavior. Thus, the validation must be complemented with an argument for the mapping of the mechanisms onto real-world mechanisms. The predictions made from the model, a comparison between these predictions and measurements on empirical data, and an argument for the choices and simplifications made in the model will be discussed in the following sections.

4.3.1 *Analytical results*

To simplify the analysis of the model it is useful to assume that the development of new land (the addition of an active node) takes place at a constant rate q_A (compared with other types of growth, not as a function of physical time). To explain the assumption, let us consider the growth of developed clusters and their perimeters. It is true in simulations of the model, and it can be verified empirically (see figure 3), that the cluster area distribution is close to a simple power law with density function f(A),

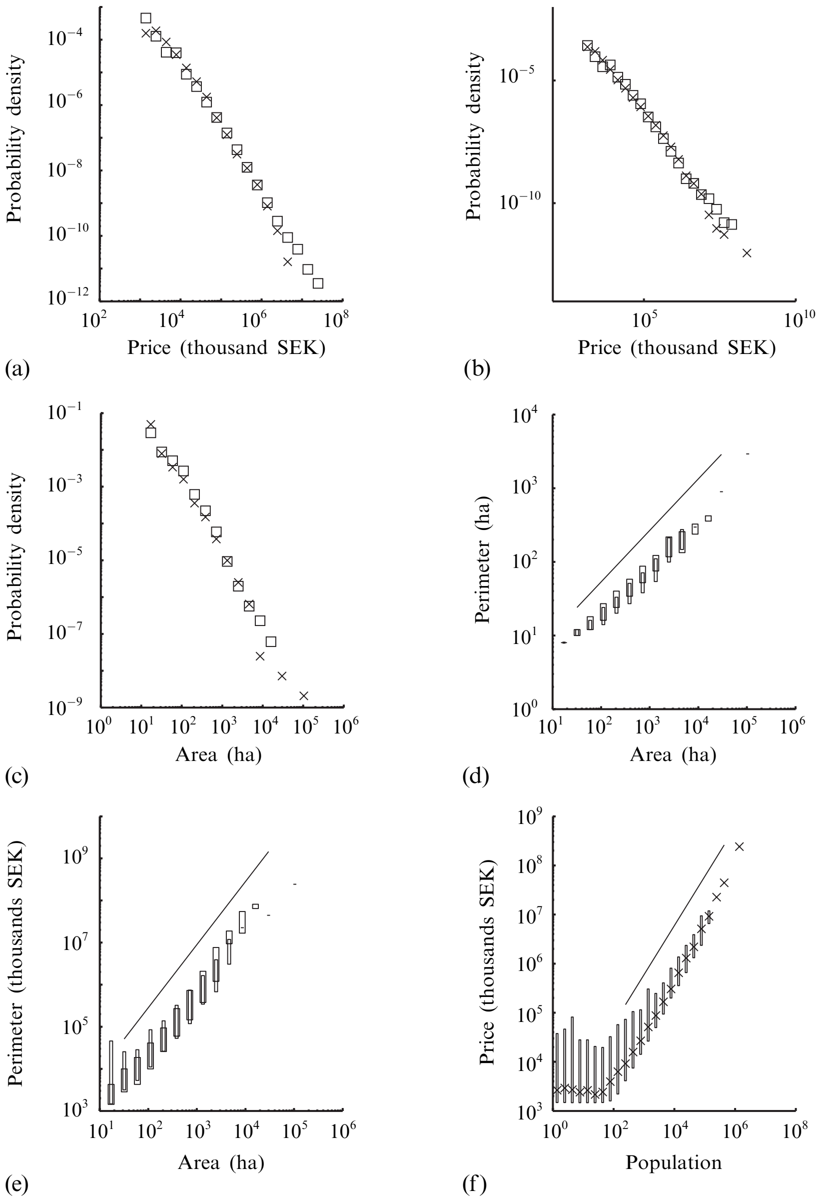

Figure 3. Parts (a), (b), and (c) show double logarithmic histograms with exponentially binned empirical (×) and simulated (□) observables: (a) land value per 400 m × 400 m cell; (b) aggregated cluster land value; and (c) cluster area. For empirical cluster measurements, land values were aggregated to 400 m × 400 m cells, and a threshold of SEK 1 425 000 per cell was applied. All contiguous (eight-cell neighbourhood) areas above this threshold were identified as clusters. In parts (d) and (e), empirical (broad boxes) and simulated (thin boxes) results for the cluster area are plotted against exponentially binned cluster perimeters [part (d)] and aggregated cluster land values [part (e)]. The vertical interval of the boxes contains 90% of the observations in the corresponding bins. The reference lines have slopes 0.7 in part (d), and 1.5 in part (e). Part (f) shows empirical cluster population plotted against exponentially binned empirical aggregated cluster land values. The vertical interval of the boxes contains 90% of the observations in the corresponding bins, and the crosses indicate the median cluster land values in the bins. The reference line has a slope of 1.0, which indicates that there is a near linear relationship between cluster price and population, for clusters with a population larger than 100.

where $f(A) \approx A^{-\beta}$, and that the relation between cluster perimeter size P and cluster area A has the form $P \approx A^{\lambda}$, with $\lambda < 1$. From this we observe that for the entire system of clusters we have

$$\frac{n_t^{(\mathrm{P})}}{n_t^{(\mathrm{D})}} \approx \frac{\displaystyle\int_1^{\infty} A^{-\beta} A^{\lambda}\, \mathrm{d}A}{\displaystyle\int_1^{\infty} A^{-\beta} A\, \mathrm{d}A} = \frac{\beta - 2}{\beta - \lambda - 1}, \tag{15}$$

assuming that $\beta > 2$, and $\lambda < 1$. This means that it can be expected that the ratio between the total number of perimeter nodes and developed nodes is reasonably constant. We now define q_1' as the fraction of primary activity increments that occur on developed nodes under additive growth; from equation (10) we obtain

$$q_1' = \sum_i \delta_i^{(\mathrm{D})} \Pi_i^{1,\,\mathrm{uni}} = q_1 \left[1 + b(1+\varepsilon) \frac{n_t^{(\mathrm{P})}}{n_t^{(\mathrm{D})}} \right]^{-1}, \tag{16}$$

which, because of equation (15), is approximately constant. This means that the rate of primary-node activation, q_{A}, is given by $q_{\mathrm{A}} = q_1 - q_1'$, and can also be considered constant.

It can be argued that expected secondary growth behaves in a very similar way to primary growth, even when the impact of spatiality is strong (Andersson et al, 2003). This means that the time evolution of expected activity on a developed site i can be approximated by

$$x_i(t+1) = x_i(t) + 2q_1' \frac{1}{n_t^{(\mathrm{D})}} + 2q_2 \frac{x_i(t)}{\sum_j x_j(t)}, \tag{17}$$

which is similar to the nonspatial model described in equation (4). Thus, after a long time the degree distribution can be expected to approach a generalised power law, $P[x_i = x] \sim (x+B)^{-\gamma}$, with

$$B = \frac{q_1'}{q_2 q_{\mathrm{A}}} = \frac{q_1'}{q_2(1 - q_2 - q_1')}, \tag{18}$$

and

$$\gamma = 1 + \frac{1}{q_2}. \tag{19}$$

4.3.2 *Endogenous node fitness*

The basic growth mechanism in this model is stochastic multiplicative growth, which means that there is no need directly to invoke agglomeration economies, external economies, or transportation costs for explaining the presence of urban hierarchies: the urban hierarchy can essentially be viewed as a consequence of stochastic multiplicative growth (land value per unit area) and city edge growth (land value per city). Agglomeration economies, local increasing returns to scale, and transportation costs are important because we know that they exist in the real system and a model must be able to include such forces and remain valid. In other words, although the mentioned forces may not be responsible for observed hierarchies or agglomerations, they are important for explaining other urban phenomena, and their presence must be consistent with the existence of hierarchies. Hence, the question may be not whether transportation diseconomies and agglomeration economies cause skewed distributions but rather if the statistics of the simple model 'survives' the introduction of such forces. It will be discussed later, in sections 3.1 and 4, that local variation in growth rates may be crucial in a secondary way because it enables fast convergence from initial configurations that are not governed by a power law.

Knowing the criterion for scale-free node distributions in growing networks of the proposed kind to be that growth is asymptotically multiplicative we want to be able to analyse the complete node growth rates to investigate whether they still fulfill this criterion when some additional forces are included. To facilitate this sort of analysis it is useful to observe what we call here 'endogenous node fitness'. Fitness, in short, specifies the full growth rate of the nodes and allows us to observe in what way that particular growth rate deviates from a purely multiplicative process.

We can calculate the expected value of the growth, Δx_i, of any site by summing over the probabilities of all primary and secondary growth events at i,

$$\mathrm{E}[\Delta x_i] = \Pi_i^{1,\,\mathrm{uni}} + \Pi_i^{1,\,\mathrm{pref}} + \sum_j (\Pi_j^{1,\,\mathrm{uni}} + \Pi_j^{1,\mathrm{pref}}) + (\Pi_{ij}^{2,\,\mathrm{uni}} + \Pi_{ij}^{2,\mathrm{pref}}) \,. \tag{20}$$

By separating the terms containing a multiplication with x_i from those that do not, we can write the expression as a combination of multiplicative (preferential) and additive (uniform) growth:

$$\mathrm{E}[\Delta x_i] = \frac{\eta_i x_i}{\sum_k \eta_k x_k} + \zeta_i \,, \tag{21}$$

with

$$\frac{\eta_i x_i}{\sum_k \eta_k x_k} = \Pi_i^{1,\,\mathrm{pref}} + \sum_j (\Pi_j^{1,\,\mathrm{uni}} + \Pi_j^{1,\,\mathrm{pref}}) \Pi_{ij}^{2,\,\mathrm{pref}} \,, \tag{22}$$

and

$$\zeta_i = \Pi_i^{1,\,\mathrm{uni}} + \sum_j (\Pi_j^{1,\,\mathrm{uni}} + \Pi_j^{1,\,\mathrm{pref}}) \Pi_{ij}^{2,\,\mathrm{pref}} \,. \tag{23}$$

For large enough x_i, additive growth will always be negligible compared with multiplicative growth. This means that we have asymptotic preferential growth, which has been shown to be a sufficient criterion for obtaining a scale-free network (Krapivsky et al, 2000). Thus, the factor with most potential notably to affect the type of degree distribution is the site-dependent and time-dependent η_i. The structure of equation (21) shows that there is a strong resemblance between η_i and the concept of node fitness (Bianconi and Barabási, 2001; Ergün and Rodgers, 2002). This means variation in η_i might cause the degree distribution to be a sum of power laws with different exponents. But in all simulations of the model, η_i turns out to fall in an interval narrow enough for multiscaling to be negligible.

In the appendix it is shown that the multiplicative fitness can be written as

$$\eta_i = q_2 \left[1 + \sum_j \left(\sum_k x_k \Pi_j^{1,\,\mathrm{uni}} + q_2 x_j \right) \frac{D_{ij}}{\sum_k D_{jk} x_k} \right] . \tag{24}$$

It is important to note that η_i is not particularly dependent on the value of x_i.

From the above equations and figures we can learn a number of things about the model. First, as we can say that a model where growth is asymptotically multiplicative will yield scale-free statistics we also know the types of functional forms that we can allow η_i and ζ_i to have if we want to retain power-law statistics. Any content that is added, meaning forces that influence location decisions, will show up in these functions.

We may add any content that we wish and still maintain scale-free behaviour as long as node growth remains essentially multiplicative. Now, if we assume that real-life urban systems are in fact growing networks we may also analyse known deviances from scale-free behaviour by using the fitness concept.

4.3.3 *Model validation*

Most of the parameters in the model can be roughly estimated from empirical data. For instance, because ε controls the ratio of external development to perimeter development, and external growth for the most part gives a remaining new cluster, we have the approximate relationship

$$\frac{\varepsilon}{1+\varepsilon} \simeq \frac{n_t^{(\mathrm{C})}}{n_t^{(\mathrm{D})}}, \tag{25}$$

where n_t^{C} is the number of clusters. The exponent γ in the distribution of land prices gives information about the size of q_2 (and q_1) via equation (19). The perimeter parameter b controls the ratio of perimeter development and internal additive growth, and it can then be determined by

$$q_{\mathrm{A}} \simeq \frac{n_t^{(\mathrm{D})}}{\sum_i x_i(t)}, \tag{26}$$

and, with the use of equation (16), we get

$$b \simeq q_{\mathrm{A}} \left[(q_1 - q_{\mathrm{A}})(1+\varepsilon) \frac{n_t^{(\mathrm{P})}}{n_t^{(\mathrm{D})}} \right]^{-1} . \tag{27}$$

The fact that the current state of the system constrains the value of these parameters does not mean that they are purely empirical in nature. They are aggregated measures of fundamental properties of the current economic system (and region) of interest, and in the general case they may change with time. If the model were to be used for prediction, instead of explanation as is the case here, then more elaborate ways of estimating current (and future) parameters would be needed. The small number of parameters that are used in the present model appear to be sufficient for reproducing some large-scale regularities of the system but, quite naturally, not for prediction at any finer resolution.

The spatial parameters c and α in equation (14) reflect the statistics of transport characteristics of the economic configuration. Their effects on the statistical properties of the system are somewhat subtle and they are thus not as easily estimated from data as the parameters mentioned above. In any case, the results are robust to their exact values and the functional form of D_{ij}. A variety of parameter values as well as linear, exponential, and constant functional forms have been used and alterations appear to impact primarily the internal structure of the network but not to a large degree the statistical properties investigated here. This is also indicated by the properties of equation (24).

The data used for all empirical results are based on a database delivered by Sweden Statistics that covers estimations of the market value of all land in Sweden (2.9 million data points). The database is constructed from street addresses of taxing units, mapped to coordinates. The estimated land values of all units with coordinates within a 100 m $\times$ 100 m cell are summed to give the reported land value of the given lot. In figure 4, the distributions of these land prices (1.18 million lots) are shown. It is evident that a large part of Swedish land values is power law distributed, an observation also reported for Japan (Kaizoji, 2003). To connect the empirical land price data to our model, we need to choose a threshold for what we consider a developed site. We choose

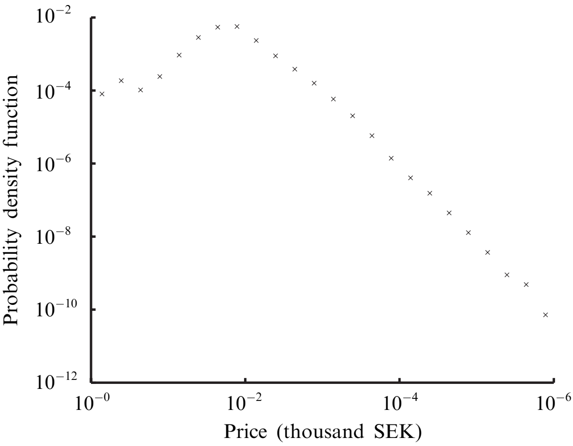

Figure 4. A double logarithmic histogram with exponentially binned Swedish land values. The prices are per hectare and are based on 2.9 million taxing units, which give information on land values for a total of 1.18 million ha. A rough estimate for the power-law exponent γ of the right hand side of the distribution is 2.1. We have chosen the price at the transition from the left to the right-hand side of the distribution (approximately SEK 75 000 per hectare) as the threshold for what is to be considered developed urban land.

this threshold to be SEK 75 000 per ha (SEK 10 $\approx$ US $1), which lies at the transition between the two regions in the empirical histogram. Some taxing units are larger than one hectare, and roads and water separate developed sites from each other; there are effects that do not appear in the model. To get around this, and to make the dataset somewhat more manageable in size, we aggregate the land values to 400 m $\times$ 400 m cells. A new threshold for developed land is chosen to be SEK 1425 000 per 16 ha, to achieve the same ratio between developed and total land area as in the 100 m $\times$ 100 m case (2.0%).

Cluster measurements were obtained by identifying clusters with use of this threshold for developed sites. To identify clusters in simulated and empirical data we used a computer program that masks all data points below a threshold value and then treats all contiguous (eight-cell neighbourhood) areas as clusters. With this procedure, 7747 empirical clusters were identified.

Simulation of network evolution produces a network configuration in which the nodes are characterised by their location on the lattice and the connections that they have to other nodes. Node degrees (the number of connections to a node) in such configurations are taken to correspond to land value by the mapping that was explained in section 4.1. From the mentioned properties, additional output is computed, such as spatial clusters of high node degrees, which correspond to urban places. The theoretical properties of the model suggest that it can reproduce the observed power-law distribution of urban land values to the extent that this mapping is correct. To further validate the model, some statistical measures that faithfully capture the spatial configuration of the node degrees are needed. Probability distributions of cluster observables, such as areas, perimeters, and aggregated prices, are such measures. By using the model presented in this paper we obtained agreement between simulated and empirical statistics for a range of these higher-order structures in successive orders of upward causation (cells to clusters; see figure 3). All figures reflect data from the same simulation run of the model. To reproduce the power-law exponent in the empirical distribution of land values, a value of $q_1 = 0.2$ was used. The parameters b and ε were then estimated from empirical cluster data as described above in this section, to give $b = 0.34$, and $\varepsilon = 0.18$. The spatial parameters were $c = 0.1$ and $\alpha = 2$. Investigation of sensitivity shows that exponents and proportions change slowly and smoothly with all parameters. We investigated this by carrying out simulations for a wide range of parameter values. Furthermore, the values of the measured quantities, being stationary properties of the network under growth, reappear robustly regardless of the random seed used. As will be discussed in more depth later, in section 4, the present model with constant multiplicative rates of node growth converges at a power-law node degree distribution only when it is grown from a small initial configuration or, obviously, from a configuration that already is power-law distributed. For convergence to take place from a wider range of seed configurations, variations in local growth rates need to be introduced in a fashion analogous to that of the Gabaix model (Gabaix, 1999). A square grid of 1600×1600 cells (each cell representing a 16 ha square) was used to match the land area of Sweden (41 093 400 ha). The number of iterations was 267 956, to give the final simulated configuration the same total price as the aggregated empirical price of developed sites (SEK 763 675 011 000). In comparisons with empirical land values, the model land values were taken as SEK 1425 000 x_i (degree 1 then corresponds to the threshold land value for developed sites).

The results are not trivial consequences of the node degree (land-value) distribution; it is perfectly possible to arrange the developed cells into any system of clusters. The same is true of the relation between cluster area and perimeter. It can also be noted that as the growth model is based on random multiplicative growth the model does not explain the presence of a stationary hierarchical organisation as being caused by increasing returns or agglomeration economies. However, such effects can be incorporated to allow other properties of urban systems to be studied. However, in agreement with other multiplicative growth models, the results indicate that these concepts are not central to explaining skewed distributions in urban systems.

In the tails of the distributions there are some deviations between empirical and model data. In figure 3 there is more very-high-priced land in the model than in the empirical data, and it is evident that the largest empirical clusters have larger areas than in the model. None of these results is very surprising. Regarding the high land values, there are two reasons—first, there is great uncertainty in the estimations and reporting of market value for the most expensive central urban land, and, second, there might be congestion factors, not present in the model, that prevent too high a degree of activity concentration, which should show up as a cutoff in the empirical land-price distribution. With regard to the deviations of large cluster areas, there are also several reasons. One reason is that the largest cluster (Stockholm) is something of an outlier in the distribution, with an aggregated price 5.5 times and an area 3.1 times that of the second largest cluster. Another reason is that coastal areas with high land value around the large cities extend their areas in a way not captured by the present model. Also, the positive feedback from the availability of transportation networks around large cities is not fully present in the model because D_{ij} is taken as constant, which, of course, is a simplification. Such forces are significant among the previously discussed deviations (section 4.2) from pure multiplicative growth and, because such forces are known to exist, we must also expect that real-life distributions deviate from simple power laws to some extent.

In figure 3 we demonstrate that, for Sweden, aggregated cluster population and aggregated cluster land value are linearly related. This could be interpreted as an indication that population growth might be understood as a response to the growth of the economic network or simply that it grows obeying similar rules.

4.3.4 *Discussion*

The simulations we carry out are extrapolations from a near-empty initial configuration. We do not aim to reproduce the configuration of economic activities in Sweden but instead a system that is statistically similar to that configuration in some important observables. We are at this point interested only in large-scale regularities such as power-law exponents, and, needless to say, it can hardly be expected that a model with only a handful of parameters can make any detailed predictions about a real system. In order to predict a specific configuration (such as extrapolating the case of Sweden from the year 2000 to 2005) one would have to bring historically path-dependent phenomena into focus and to use a considerably more detailed model incorporating coast lines, roads, additional forces, and so on. Such a model, though well conceivable, is beyond the scope of this paper. According to the logic previously argued for reconstructing a system that is consistent with large-scale statistical regularities such as those studied here logically precede the implementation of a detailed model for scenario prediction. A level of detail needed for scenario prediction would serve only to confuse the model at this stage of the analysis.

In fact, it might turn out that the forces mentioned as affecting location choice indeed have an important role to play in the generation of hierarchies, although in a perhaps somewhat surprising and indirect way—they might conceivably be the 'impurities' in simple multiplicative growth that are necessary for rapid convergence (Gabaix, 1999). For variation of the required type to be endogenous to the model, the model needs to be extended in ways that remain to be determined. We leave a thorough investigation of this important aspect to subsequent papers. Another possible source of error when comparing the results obtained were with an empirical dataset relates to the fact that the real-world system is, in fact, only a part of a larger system. It might, for example, be such that Sweden lends itself fairly well to such a comparison because the largest region (Stockholm) and the second-largest region (Gothenburg) are located in a fairly central position in the dataset. Furthermore, the land borders of Sweden, which are mainly towards Norway, run through areas of comparably low development. If, however, some continental European country had been analysed, the fact that its borders would in all likelihood run through more densely developed areas might have caused problems. Still, it is clear that many connections in the real-world trade network of Sweden have their endpoints outside of Sweden. Furthermore, it must be suspected that such connections are not evenly distributed over the activities in the system: large cities with ports and airports may have a larger proportion of international trade compared with smaller cities.

The present Euclidean model of distance is, needless to say, another simplification that potentially could introduce systematic errors to the statistical regularities studied here. Basically, the present transportation model amounts to Euclidean distance with modifiers for transportation availability in external, internal, and perimeter areas. There are two main reasons why we have selected a very simple characterisation of transportation. First, the statistical behaviour of the system, in the properties we have observed, is not very sensitive to the distance decay of interaction rate. Second, a more complex representation of transportation, although not 'buying' much in terms of prediction, would mean that additional and potentially risky assumptions must be made. By the principle of parsimony, we choose at this stage the simplest possible model that captures the phenomena we wish to address. A better transportation model could, however, shed more light on, for example, the internal structure of clusters, so this is definitely still an interesting way in which to elaborate the model in future (for a qualitative illustration of the impact of using a simplistic transportation network model, see figure 5 (over); it may be noted that in figure 5 the most prominent visual difference

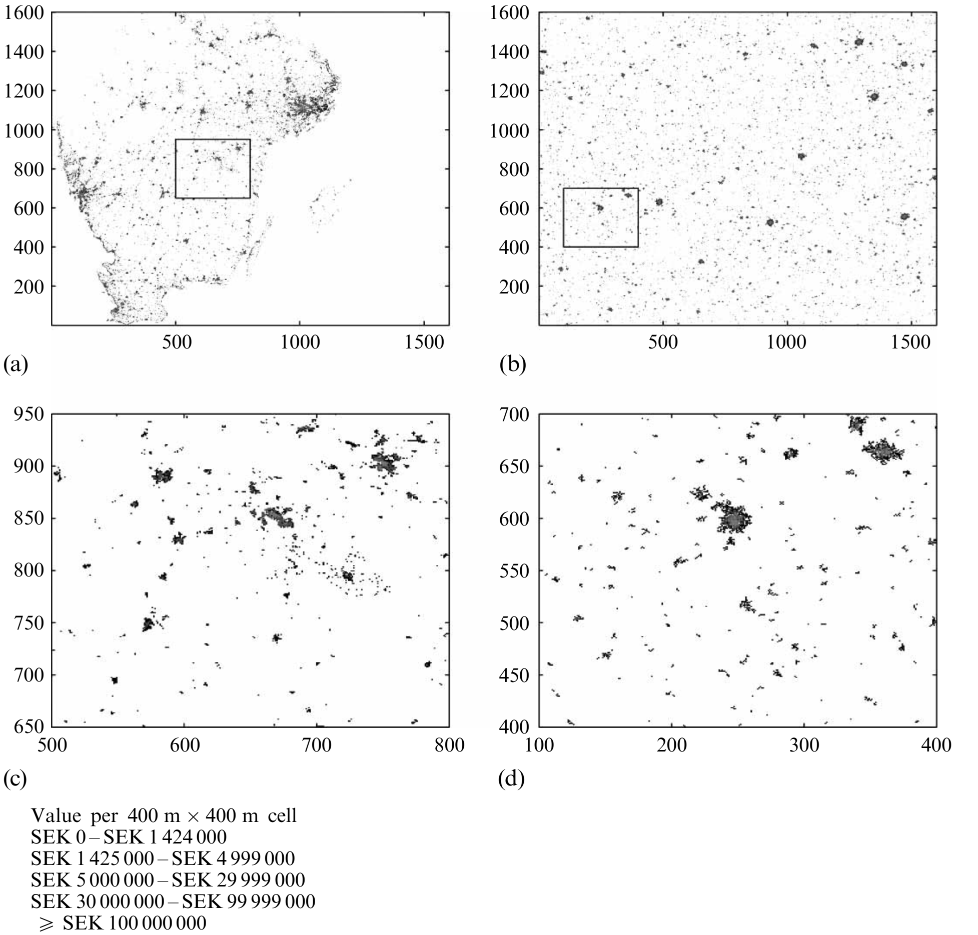

Figure 5. Empirical [parts (a) and (c)] and simulated (d) [parts (b) and (d)] geographical configurations of land values are shown at two scales. It can be seen clearly that the empirical configuration is arranged around natural and anthropogenic structures such as roads, rivers, and coastlines. These being absent in the simulation, the clusters become more idealised but still exhibit a similarity in both intraurban and interurban shape and composition. As is verified statistically in text, the statistical properties of the simulated and the real system under the observables that we have used are highly similar. Also, as pointed out in the text, no difference is made between intraurban and interurban dynamics in the model.

between simulation and reality is that real urban clusters are arranged largely in streaks, following the extent of major roads).

Finally, it must be noted that complex networks provide a solution to the problem of delineation in urban studies. In models an important aspect of selecting objects resides in their properties as 'individuals'. This subject has been discussed, for example, in biology in relation to models of selection based on objects on a higher and lower level than organisms (Ghiselin, 1974; Gould, 2002). To be a proper individual an object should have a definable temporal beginning and end and, in between, it should remain identifiable and integral. That is, it should not fuse with other objects or split up other than in very predictable ways (such as a biological cell division) or dissolve so that its identity becomes hard to define, neither should it pass in or out of existence. The reason for this is that we must be able to define mechanisms that govern the dynamics of the system and we must be able to maintain properly defined measures. This is a problem that exists for multiplicative growth models formulated on the level of cities, but it is circumvented by the present definition of nodes and fixed-size land lots. Cells, or nodes, remain exactly the same throughout the entire simulation.

The cellular formulation also allows us to tackle the problem of defining and measuring the quantities under study efficiently. Because cells stay the same throughout the simulation time we may also, in a strict way, study their properties: the number of connections per unit area, the number of cells per cluster, and so on. Furthermore, as the dataset with which we compare the model output is also in a cellular format we can use exactly the same analysis algorithms for both sets.

5 Complex networks in economic geography in a wider perspective

Models of urban systems have been developed along a variety of tracks and have progressed almost in isolation from each other. Neoclassical models are generally hard to apply and often lack realism because they are constructed with mathematical tractability as their primary design parameter. Models in geography, economic geography, and other areas not in the mainstream of economics are generally constructed with realism in the foreground, but this comes at the cost of a common analytic framework. Whereas neoclassical models rely on strong rationality assumptions and equilibrium analysis to achieve mathematical tractability, urban models from other disciplines generally rely on less restrictive assumptions to achieve the same thing. To understand the strengths and weaknesses of complex network models in this spectrum, we discuss a number of urban models from various disciplines.

Urban economics stems from Ricardian rent theory via the monocentric-city model of von Thünen to its later extension for urban land uses by Alonso as well as influences from other schools of thought such as central-place theory. Since Henderson (1974), most contributors have modelled city size in terms of the effects of centrifugal and centripetal forces, that is, of economies and diseconomies of agglomeration and urbanisation. Furthermore, we can argue that agglomeration economies are often sector specific, so there is a tendency of production specialisation at the city scale. The ultimate size of a city represents an equilibrium between centripetal and centrifugal forces, and the point at which this equilibrium occurs is determined by the type of production in which the city specialises. The Henderson model, like many other urban economic models, identifies the forces that are at play but, because system states are viewed as points of equilibrium, it does not say much about growth dynamics. Furthermore, the Henderson model is not fully geographic: although there is cross-city migration there is no trade between cities, and the geographical location of cities therefore becomes meaningless.

The new economic geography models attributable to Krugman and others is a successful attempt to extend the neoclassical framework to a geographical setting. This is done with the expressed intent to provide a stronger theoretical basis to regional science. The core model of the new economic geography (Krugman, 1991) combines mobile firms and workers in a model with transportation costs and increasing returns to scale within firms. From this, it can be derived that, with transportation costs sufficiently low, firms and workers will cluster in cities to minimise the cost of production and transportation. The model has been shown to be sufficiently rich to allow for other extensions (Brakman et al, 2001; Fujita et al, 1999). The new economic geography models achieve analytic tractability, but this does come at a rather severe cost to realism. Because of this, these models have not been greeted warmly by all geographers. What is more, the new economic geography models do not address urban growth but are better understood as an evolutionary game in which economic actors (firms and consumers) decide rationally on a location by taking into account the expected decisions of all other agents. Given this reasoning, the resulting equilibrium is technically an evolutionary stable Nash equilibrium (Brakman and Garretsen, 2003). Systems in which states represent equilibria are typically characterised by the concurrent action of fast dynamics and slow dynamics or exogenous parameters: the slow dynamics or static exogenous parameters determine the location and types of equilibria, and the fast dynamics cause the system to relax to an equilibrium state. It is questionable, however, whether the forces that affect the evolution of urban systems typically change more slowly than the state of the system. If the system indeed does not have time to relax, then equilibrium analysis appears to be a poor choice for explaining geographical distributions of land use.

Complex network models, in the present application or any other application to economic or social geographic systems, simulate the formation of interactions in a specific or an abstract sense. Because of this, there exists a connection between complex network models and spatial interaction modeling. For example, it can be noted that the distance bias function that is used [equation (14)] bears a strong resemblance to a gravity function with the exponent as a parameter, as introduced by Huff (1963). The probabilities of persons from region i shopping in some other region j in Huff's retail model is also almost identical to the calculation of multiplicative secondary effects in the present model [see equation (11)]. Though spatial interaction models in dynamic incarnations are primarily equilibrium models and thus are not aimed at explaining questions about the evolution of urban systems the connection is nevertheless important. First, as complex network models simulate the formation of connections it is also possible to observe the quantities typically used in spatial interaction models, such as connection strength or number of trips between regions and amount of activity in regions. Second, a wide range of probabilities and relative densities can be observed in relation to cells and regions simulated with a complex network, and we are often interested in the relation between macrostates and microstates. This also means that entropy measures that are commonly used in spatial interaction modeling could also find use in relation to complex networks (Wilson, 1967; 1970). This may not only be the case for urban systems but also be the case from a cross-scientific perspective for other systems that are analysed using complex networks. Entropy has also been used by Curry (1964) to explain the rank-size rule, although he viewed the urban hierarchy as an equilibrium state of maximised entropy.

Another set of models to which the complex network approach is related is a family of 'complexity' approaches that explains urban growth and urban patterns by using conceptual models such as diffusion-limited aggregation (DLA), correlated percolation, reaction diffusion models, and multiplicative cluster growth models. The elementary model of Simon (1955) is truly foundational in this respect and reproduces the rank-size rule for cities yet is not geographically formulated and does not address morphological aspects of urban systems. DLA models attributable to Batty address the fractal properties of urban patterns but not other statistical properties of the urban system such as the rank-size rule (Batty and Longley, 1994). Also, the microformulation based on random walk appears not to be very realistic, and the site of the urban core must be specified at the beginning of the simulation. The correlated percolation model of Makse et al (1998) reproduces a number of power laws and appears to be more realistic in its micro formulation. Also, here the simulation starts from a configuration with an existing gradient centred on the site of the central business district. Zanette and Manrubia (1998) introduced a reaction diffusion model of population density evolution. This model combines predictions of characteristics of clusters and geographical patterns at the same time. The model captures some fundamental properties of urban growth such as population migration and multiplicative growth and has some statistical similarities to the present model. However, no secondary growth effects are used (growth events do not occur in spatially correlated pairs), no long-range interactions take place, and the micromechanisms are very abstract.

5.1 Concluding remarks

Attempting to link microscopic mechanisms to macroscopic structure is a defining theme in modern complexity-related research. For social and economic systems, the importance of this task is self-evident. Policies must be directed at making small changes that incrementally form the evolutionary economic system over time, as changes to the macrostructure would be exceedingly expensive and uncertain. At the same time, it is the macroscopic state of the system (for example, the geographical distributions of various measures) that most greatly affect agents' welfare and motivate policy programmes. Thus, an exploration of how the actions of agents, and policies directed at regulating them, map onto macroscopic states is central to the task of both research and policymaking.

The crucial asset of complex networks in this respect is that they can be applied to a wide spectrum of uses: from abstract models with the primary objective of paving the way for theoretical models, to arbitrarily complex models used for scenario prediction and exploration. It is not hard to see how multiple commodities and services can be used and how correlations between such types of commodities and services may be introduced by specifying types of activities that take a specified input and produce a specified output. Also, many of the global properties of complex networks such as those measured in this paper and related papers are the result of basic properties in the evolution mechanisms, and there is no reason to believe that these would not carry over well to models with greater complexity in these mechanisms. For example, the requirement of a Barabási – Albert scale-free network is that growth is asymptotically multiplicative. This is not a very constraining requirement and it leaves much room for elaboration regarding the specifics of the growth events, this includes the incorporation of other important forces in urban dynamics, such as increasing returns. This means that models can be constructed along a direction from the general to the special without the special case losing the desired properties arising in the general model. With respect to the application to land-use patterns, the complex network approach is compatible with cellular automaton models of urban development. It must be noted that a spatial network model is, in effect, a network model embedded in a cellular space. Thus, the appeal of cellular automata in geographic modeling—that of combining form and function in a common framework—must also be considered to apply in this case.

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Appendix

Because of the normalisation in equation (21), the exact expression for η_i can be chosen in different ways (that is, it can be varied by any multiplicative constant). To make individual η_i independent of the size of the system, we choose $\sum_i \eta_i x_i \sim \sum_i x_i$:

$$\eta_i = q_2 + \sum_j \left(\sum_k x_k \Pi_j^{1,\mathrm{uni}} + q_2 x_j \right) q_2 \frac{D_{ij}}{\sum_k D_{jk} x_k} = q_2 (1 + A_i + B_i) , \tag{A1}$$

with

$$A_i = \sum_k x_k \sum_j \frac{D_{ij} \Pi_j^{1,\mathrm{uni}}}{\sum_k D_{jk} x_k} , \tag{A2}$$

and

$$B_i = q_2 \sum_j \frac{D_{ij} x_j}{\sum_k D_{jk} x_k} . \tag{A3}$$

By using the symmetry $D_{ij} = D_{ji}$ and observing

$$\sum_i A_i x_i = \sum_i x_i \sum_k x_k \sum_j \frac{D_{ij} \Pi_j^{1,\mathrm{uni}}}{\sum_k D_{jk} x_k} = \sum_k x_k \sum_j \frac{\Pi_j^{1,\mathrm{uni}}}{\sum_k D_{jk} x_k} \sum_i D_{ij} x_i$$
$$= \sum_k x_k \sum_j \Pi_j^{1,\mathrm{uni}} = q_1 \sum_k x_k , \tag{A4}$$

and

$$\sum_i B_i x_i = q_2 \sum_i x_i \sum_j \frac{D_{ij} x_j}{\sum_k D_{jk} x_k} = q_2 \sum_j x_j \frac{D_{ij} x_j}{\sum_k D_{jk} x_k} = q_2 \sum_j x_j , \tag{A5}$$

it can be verified that

$$\sum_i \eta_i x_i = q_2 (1 + q_1 + q_2) \sum_i x_i . \tag{A6}$$