

Condensed matter physics with trapped atomic Fermi gases

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Summary. — We present an overview of the various phase transitions that we anticipate to occur in trapped fermionic alkali gases. We also discuss the prospects of observing these transitions in (doubly) spin-polarized ${}^6\text{Li}$ and ${}^{40}\text{K}$ gases, which are now actively being studied by various experimental groups around the world.

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1. – Introduction

After the formidable achievement of Bose-Einstein condensation in spin-polarized alkali gases [1-6], the next challenge that experimentalists have already set themselves is to realize quantum degenerate conditions also in fermionic alkali vapors. One particular motivation in this respect is the prediction that a gas of spin-polarized atomic ${}^6\text{Li}$ becomes superfluid at densities and temperatures comparable with those at which the Bose-Einstein experiments are performed [7]. As a result of this experimental interest, the first theoretical studies of an ideal Fermi gas trapped in a harmonic external potential have recently appeared [8,9]. Moreover, the effects of an interatomic interaction have also been considered [10-12]. It is interesting to note that Oliva's calculations for atomic

deuterium were already performed a decade ago, even though magnetically trapped deuterium had not been observed at that time. In fact, it has still not been observed, because the loading of the trap cannot be accomplished in the same way as for atomic hydrogen [13,14]. This is presumably caused by the fact that deuterium binds more strongly to a superfluid helium film, that the surface recombination rate is much larger, and that the sample is contaminated with atomic hydrogen [15]. Fortunately, such problems do not arise for experiments with alkali gases and both ^6Li and ^{40}K have indeed been trapped already [16,17].

The most important qualitative feature of a trapped Fermi gas is that its density profile ‘freezes’ at low temperatures. This is a result of the Pauli exclusion principle and can be easily understood by considering an ideal gas in the trapping potential $V^{\text{trap}}(\mathbf{x}) = m\omega^2 \mathbf{x}^2/2$. In general the extent of the gas cloud is determined by the classical turning point of the most energetic particles. If the gas is fully classical, i.e., it obeys Maxwell-Boltzmann statistics, these particles have an energy of a few $k_B T$ and the size of the cloud R_{TF} follows from $m\omega^2 R_{TF}^2/2 \simeq k_B T$, implying that $R_{TF} \simeq (2k_B T/m\omega^2)^{1/2}$. We see that, as we lower the temperature, the size of the cloud shrinks. Moreover, if we keep the number of particles N fixed, the density increases. This process gradually continues until we reach zero temperature and the density profile becomes equal to $n(\mathbf{x}) = N\delta(\mathbf{x})$. If the gas obeys Fermi-Dirac statistics, however, the most energetic particles have in the degenerate (nonclassical) regime an energy that is equal to the Fermi energy ϵ_F and the size of the gas cloud is always given by $R_{TF} \simeq (2\epsilon_F/m\omega^2)^{1/2}$ for temperatures $T \ll \epsilon_F/k_B$. Comparing this also with the density profile for an ideal Bose gas, which in the degenerate regime consists of a large and narrow condensate peak with a width of about $(\hbar/m\omega)^{1/2}$ on top of a broad thermal background of size $R_{TF} \simeq (2k_B T/m\omega^2)^{1/2}$, we conclude that the density profile of a degenerate Fermi gas is indeed ‘frozen’. In contrast to the ideal Bose gas, the ideal Fermi gas also does not have a phase transition. From a point of view of condensed matter physics, an atomic Fermi gas thus appears much less interesting than a Bose gas. The main objective of this contribution is to argue that this is no longer true if there are interactions between the atoms.

2. – Interactions

For atomic alkali gases the most important interatomic interaction is the so-called central interaction $V^c(r)$, which consists of a sum of the usual singlet and triplet interactions. More precisely, we have that

$$(1) \quad V^c(r) = V_S(r)\mathcal{P}^{(S)} + V_T(r)\mathcal{P}^{(T)} ,$$

where $\mathcal{P}^{(S)}$ and $\mathcal{P}^{(T)}$ denote the projection operators on the subspace of singlet and triplet states, respectively. Besides this interaction that is the net result of the Coulomb attractions and repulsions between the electrons and nuclei of the atoms, we also have to consider the weak magnetic dipole-dipole interactions. Of these, the electron-electron

magnetic dipole interaction is most important and obeys [18]

$$(2) \quad V^d(\mathbf{r}) = \frac{\mu_0 \mu_e^2}{4\pi} \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - 3(\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}})(\boldsymbol{\sigma}_2 \cdot \hat{\mathbf{r}})}{r^3} \equiv -\frac{\mu_0 \mu_e^2}{4\pi r^3} \sqrt{\frac{4\pi}{5}} \sum_{\mu} (-1)^{\mu} Y_{2\mu}(\hat{\mathbf{r}}) \Sigma_{2,-\mu}^{ee} .$$

Here μ_e is the electron magnetic moment and the tensor operator $\Sigma_{2,-\mu}^{ee}$ is obtained from coupling the Pauli spin matrices $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ of the two valence electrons to a tensor of rank 2.

It should be noted that both these interactions do not commute with the electron spin operators $\hbar \boldsymbol{\sigma}_i / 2$ and therefore also do not commute with the atomic hamiltonian, which in a magnetic field contains both a hyperfine and a Zeeman term. As a consequence the central and dipole-dipole interactions are not fully diagonal in the basis in which the atoms are in definite hyperfine states. This is important in principle, because it implies that two atoms can also collide inelastically, i.e., their hyperfine states can change during the collision. Together with three-body recombination events, these inelastic collisions in fact always seriously limit the lifetime of a trapped alkali gas. Nevertheless, we will in the following mostly neglect the nondiagonal parts of the interatomic interaction by restricting ourselves to (doubly) spin-polarized gases for which the ‘good’ elastic collisions dominate the ‘bad’ inelastic ones. Clearly such a restriction is a minimum requirement for our discussion to be also of some experimental interest.

3. – Cooper pairing

As we will see in detail below, the most common phase transition in a weakly-interacting Fermi gas is due to the formation of so-called Cooper pairs. The main idea behind the famous Bardeen-Cooper-Schrieffer theory for this phenomenon is in fact a Bose-Einstein condensation of these pairs [19]. To see how we can arrive at a mean-field theory for this phase transition, let us first recapitulate the mean-field (Hartree) theory for Bose-Einstein condensation in an atomic Bose gas. At zero temperature we can then use a variational many-body wave function in which all the atoms are in the same one-particle state $\phi(\mathbf{x})$, so

$$(3) \quad \Psi_N(\mathbf{x}_1, \dots, \mathbf{x}_N) = \prod_{i=1}^N \phi(\mathbf{x}_i) .$$

In the language of second quantization this essentially reads

$$(4) \quad |N\rangle = \frac{\left(\int d\mathbf{x} \phi(\mathbf{x}) \psi^\dagger(\mathbf{x}) \right)^N}{N!} |0\rangle ,$$

where $\psi^\dagger(\mathbf{x})$ is the creation operator for an atom at position \mathbf{x} and $|0\rangle$ is the ‘vacuum’ state in which there are no atoms present in the trap. Calculating the average of the

hamiltonian H in this variational wave function and minimizing with respect to $\phi(\mathbf{x})$ leads of course to the Gross-Pitaevskii equation [20] if $N \gg 1$.

The above variational wave function has a definite number of particles. For practical calculations, in particular if we want to consider also nonzero temperatures or corrections to the mean-field theory, it is much more convenient to consider a variational wave function in which not the number of particles N , but instead the phase ϑ is fixed. This state is given by

$$(5) \quad |\vartheta\rangle = \sum_N \frac{\left(\int d\mathbf{x} \phi(\mathbf{x}) e^{i\vartheta} \psi^\dagger(\mathbf{x})\right)^N}{N!} |0\rangle = \exp\left(\int d\mathbf{x} \phi(\mathbf{x}) e^{i\vartheta} \psi^\dagger(\mathbf{x})\right) |0\rangle ,$$

as can be seen from the fact that now the matrix element $\langle\vartheta|\psi(\mathbf{x})|\vartheta\rangle = \phi(\mathbf{x})e^{i\vartheta}$ has a definite phase in contrast to the matrix element $\langle N|\psi(\mathbf{x})|N\rangle = 0$.

It is not difficult to show that by minimizing the average energy in the state $|\vartheta\rangle$ we again recover the Gross-Pitaevskii equation. More important for our purposes is that we can easily show by Fourier analysis that

$$(6) \quad |N\rangle = \int_0^{2\pi} \frac{d\vartheta}{2\pi} e^{-iN\vartheta} |\vartheta\rangle ,$$

which shows that the number of particles N and the phase ϑ are conjugate variables and obey the commutation relation $[N, \vartheta] = i$. The Heisenberg equation of motion for the average phase thus becomes

$$(7) \quad i\hbar \frac{\partial\langle\vartheta\rangle}{\partial t} = \langle[\vartheta, H]\rangle = -i \left\langle \frac{\partial H}{\partial N} \right\rangle \equiv -i\mu ,$$

with μ the chemical potential of the gas. We therefore recover the important Josephson relation

$$(8) \quad \hbar \frac{\partial\langle\vartheta\rangle}{\partial t} = -\mu ,$$

which is also well known from the hydrodynamic formulation of the Gross-Pitaevskii equation [21]. Indeed, taking the gradient of this equation and using the definition of the superfluid velocity, i.e., $\mathbf{v}_s = \hbar \nabla \langle\vartheta\rangle / m$, we obtain the desired result that

$$(9) \quad \frac{\partial \mathbf{v}_s}{\partial t} = -\frac{1}{m} \nabla \mu .$$

Let us now return to Cooper-pair formation in degenerate Fermi gases. In analogy with Bose-Einstein condensation we can now use at zero temperature the variational

wave function

$$(10) \quad |N\rangle = \frac{\left(\int d\mathbf{x} \int d\mathbf{x}' \phi_{\alpha,\alpha'}(\mathbf{x}, \mathbf{x}') \psi_{\alpha'}^\dagger(\mathbf{x}') \psi_\alpha^\dagger(\mathbf{x}) \right)^{N/2}}{(N/2)!} |0\rangle.$$

A few remarks are in order. First, we have denoted the various hyperfine states of the atoms by $|\alpha\rangle$. In our previous discussion of Bose-Einstein condensation we should in principle also have indicated the hyperfine state of the atoms and used $\psi_\alpha^\dagger(\mathbf{x})$ instead of $\psi^\dagger(\mathbf{x})$ in the variational wave function. However, as long as all the atoms are in the same hyperfine state, the precise atomic state which is trapped is unimportant from a theoretical point of view and only influences the interatomic interaction, i.e., the particular value of the scattering length, that should be used in the Gross-Pitaevskii equation. For clarity we therefore suppressed the spin degrees of freedom in that case.

Second, the fermionic creation operators anticommute. As a result the Cooper-pair wave function must obey $\phi_{\alpha,\alpha'}(\mathbf{x}, \mathbf{x}') = -\phi_{\alpha',\alpha}(\mathbf{x}', \mathbf{x})$, reflecting the Pauli exclusion principle. There are essentially two ways to fulfill this antisymmetrization requirement. If all the atoms are in the same hyperfine state we have $\alpha' = \alpha$ and the orbital part of the Cooper-pair wave function is antisymmetric with respect to an exchange of the atoms. The relative angular momentum of the pairs must therefore be odd. In particular, we can have p -wave pairing. This situation occurs in doubly spin-polarized Fermi gases and also in liquid ^3He [22]. If we have an equal number of atoms in two hyperfine states, which is implicitly assumed in the above variational wave function, the spin part of the Cooper-pair wave function can already be antisymmetric and the relative angular momentum of the pairs must then be even. We now can have s -wave pairing as in ordinary superconductors. In principle, we can of course also have d -wave pairing as in the high-temperature superconductors, but this turns out to be extremely unlikely for dilute gases.

Third, in actual applications it is again more convenient to use a variational wave function with a well defined phase. In this case it is obtained by multiplying the Cooper-pair wave function in the right-hand side of Eq. (10) with $e^{2i\vartheta}$ and summing over all even values of N . We then find

$$(11) \quad |\vartheta\rangle = \exp \left(\int d\mathbf{x} \int d\mathbf{x}' \phi_{\alpha,\alpha'}(\mathbf{x}, \mathbf{x}') e^{2i\vartheta} \psi_{\alpha'}^\dagger(\mathbf{x}') \psi_\alpha^\dagger(\mathbf{x}) \right) |0\rangle$$

and exactly the same relation between the states $|N\rangle$ and $|\vartheta\rangle$ as for a condensate of single atoms. The Josephson relation given in Eq. (8) is therefore also valid in this case. Moreover, in the state $|\vartheta\rangle$ we have a nonvanishing expectation value

$$(12) \quad \langle \vartheta | \psi_\alpha(\mathbf{x}) \psi_{\alpha'}(\mathbf{x}') | \vartheta \rangle = \phi_{\alpha,\alpha'}(\mathbf{x}, \mathbf{x}') e^{2i\vartheta},$$

which suggest that $\langle \psi_\alpha(\mathbf{x}) \psi_{\alpha'}(\mathbf{x}') \rangle$ is the appropriate order parameter for the Bardeen-Cooper-Schrieffer transition, just like $\langle \psi_\alpha(\mathbf{x}) \rangle$ is the order parameter for Bose-Einstein

condensation. Although this identification of the order parameter is correct, it turns out that it is more convenient in practice to work with the so-called BCS gap parameter

$$(13) \quad \Delta(\mathbf{r}; \mathbf{R}) \equiv V_{\alpha, \alpha'}(\mathbf{r}) \langle \psi_{\alpha}(\mathbf{R} + \mathbf{r}/2) \psi_{\alpha'}(\mathbf{R} - \mathbf{r}/2) \rangle ,$$

where $V_{\alpha, \alpha'}(\mathbf{r}) = \langle \alpha, \alpha' | V^c(r) + V^d(\mathbf{r}) | \alpha, \alpha' \rangle$ is a shorthand notation for the elastic part of the interatomic interaction, and $\mathbf{r} = \mathbf{x} - \mathbf{x}'$ and $\mathbf{R} = (\mathbf{x} + \mathbf{x}')/2$ are the relative and center-of-mass coordinates of the Cooper pair, respectively.

Knowing the order parameter of the phase transition of interest, it is straightforward to obtain the corresponding mean-field theory. The main idea is first to write in the interaction part of the hamiltonian

$$(14) \quad H = \sum_{\alpha} \int d\mathbf{x} \psi_{\alpha}^{\dagger}(\mathbf{x}) \left(-\frac{\hbar^2 \nabla^2}{2m} + V^{\text{trap}}(\mathbf{x}) - \mu_{\alpha} \right) \psi_{\alpha}(\mathbf{x}) \\ + \frac{1}{2} \sum_{\alpha, \alpha'} \int d\mathbf{x} \int d\mathbf{x}' \psi_{\alpha'}^{\dagger}(\mathbf{x}') \psi_{\alpha}^{\dagger}(\mathbf{x}) V_{\alpha, \alpha'}(\mathbf{x} - \mathbf{x}') \psi_{\alpha}(\mathbf{x}) \psi_{\alpha'}(\mathbf{x}')$$

the operators $\psi_{\alpha}(\mathbf{x}) \psi_{\alpha'}(\mathbf{x}')$ and $\psi_{\alpha}^{\dagger}(\mathbf{x}) \psi_{\alpha'}^{\dagger}(\mathbf{x}')$ as a sum of a mean value and fluctuations, and then to neglect terms that are quadratic in the fluctuations. In this manner we arrive at the mean-field hamiltonian

$$(15) \quad H_{MF} = \sum_{\alpha} \int d\mathbf{x} \psi_{\alpha}^{\dagger}(\mathbf{x}) \left(-\frac{\hbar^2 \nabla^2}{2m} + V^{\text{trap}}(\mathbf{x}) - \mu'_{\alpha}(\mathbf{x}) \right) \psi_{\alpha}(\mathbf{x}) \\ + \frac{1}{1 + \delta_{\alpha, \alpha'}} \int d\mathbf{x} \int d\mathbf{x}' \left(\Delta(\mathbf{r}; \mathbf{R}) \psi_{\alpha'}^{\dagger}(\mathbf{x}') \psi_{\alpha}^{\dagger}(\mathbf{x}) + \Delta^*(\mathbf{r}; \mathbf{R}) \psi_{\alpha}(\mathbf{x}) \psi_{\alpha'}(\mathbf{x}') \right) ,$$

where the renormalized chemical potentials are in a good approximation (but see below) given by

$$(16) \quad \mu'_{\alpha}(\mathbf{x}) = \mu_{\alpha} - \sum_{\alpha' \neq \alpha} \int d\mathbf{x}' V_{\alpha, \alpha'}(\mathbf{x} - \mathbf{x}') n_{\alpha'}(\mathbf{x}')$$

and the density profile of atoms in spin state $|\alpha\rangle$ obeys $n_{\alpha}(\mathbf{x}) = \langle \psi_{\alpha}^{\dagger}(\mathbf{x}) \psi_{\alpha}(\mathbf{x}) \rangle$. It is important to note that in trapped alkali gases it is indeed appropriate to have a chemical potential for each spin state, because the time scale for relaxation towards equilibrium in spin space is always much larger than the equilibration time for the spatial degrees of freedom. This is for example quite dramatically demonstrated by the two overlapping condensate experiments by Myatt *et al.* [4].

To complete the mean-field theory we should now calculate the mean values of the operators $\psi_{\alpha}(\mathbf{x}) \psi_{\alpha'}(\mathbf{x}')$ and $\psi_{\alpha}^{\dagger}(\mathbf{x}) \psi_{\alpha'}^{\dagger}(\mathbf{x}')$ in a thermal ensemble with the hamiltonian H_{MF} . This clearly makes the theory selfconsistent. To perform the calculation we write

the annihilation operators at the positions \mathbf{x} and \mathbf{x}' as

$$(17) \quad \psi_\alpha(\mathbf{R} \pm \mathbf{r}/2) = \int \frac{d\mathbf{k}}{(2\pi)^3} \psi_\alpha(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{R} \pm \mathbf{r}/2)} ,$$

respectively. Substituting this in the mean-field hamiltonian and neglecting gradients in the center-of-mass coordinate \mathbf{R} , we find that

$$(18) \quad H_{MF} = \sum_\alpha \int \frac{d\mathbf{k}}{(2\pi)^3} \psi_\alpha^\dagger(\mathbf{k}) \left(-\frac{\hbar^2 \mathbf{k}^2}{2m} + V^{\text{trap}}(\mathbf{R}) - \mu'_\alpha(\mathbf{R}) \right) \psi_\alpha(\mathbf{k}) \\ + \frac{1}{1 + \delta_{\alpha, \alpha'}} \int \frac{d\mathbf{k}}{(2\pi)^3} \left(\Delta(\mathbf{k}; \mathbf{R}) \psi_{\alpha'}^\dagger(-\mathbf{k}) \psi_\alpha^\dagger(\mathbf{k}) + \Delta^*(\mathbf{k}; \mathbf{R}) \psi_\alpha(\mathbf{k}) \psi_{\alpha'}(-\mathbf{k}) \right) .$$

Diagonalizing the above hamiltonian by means of a Bogoliubov transformation, the spin-density profiles can then be calculated from

$$(19) \quad n_\alpha(\mathbf{R}) = \int \frac{d\mathbf{k}}{(2\pi)^3} \langle \psi_\alpha^\dagger(\mathbf{k}) \psi_\alpha(\mathbf{k}) \rangle$$

and, most importantly, the BCS gap parameter from

$$(20) \quad \Delta(\mathbf{k}; \mathbf{R}) = \int \frac{d\mathbf{k}'}{(2\pi)^3} V_{\alpha, \alpha'}(\mathbf{k} - \mathbf{k}') \langle \psi_\alpha(\mathbf{k}') \psi_{\alpha'}(-\mathbf{k}') \rangle .$$

Notice that the averages in the right-hand side of Eqs. (19) and (20) depend on the position \mathbf{R} , since the mean-field hamiltonian H_{MF} depends parametrically on this position. To arrive at such a simplified (Thomas-Fermi) description of the inhomogeneity of the gas we have to be able to neglect gradients of the densities $n_\alpha(\mathbf{R})$ and the BCS gap parameter $\Delta(\mathbf{k}; \mathbf{R})$. This is indeed true for present experiments with fermionic alkali gases, because the number of trapped atoms is so large that both the correlation length and the size of the Cooper pairs are small compared the typical length scale on which the trapping potential varies [11, 23].

We refer for the details of the diagonalization to our previous work [11]. For our present purposes it is however important to mention that in the optimal case of equal spin densities, the final result of Eq. (20) is the famous BCS gap equation [19, 22]

$$(21) \quad \Delta(\mathbf{k}; \mathbf{R}) = - \int \frac{d\mathbf{k}'}{(2\pi)^3} V_{\alpha, \alpha'}(\mathbf{k} - \mathbf{k}') \frac{\Delta(\mathbf{k}'; \mathbf{R})}{2\hbar\omega(\mathbf{k}'; \mathbf{R})} \tanh \left(\frac{\hbar\omega(\mathbf{k}'; \mathbf{R})}{2k_B T} \right) ,$$

where the so-called Bogliubov dispersion $\hbar\omega(\mathbf{k}; \mathbf{R})$ obeys

$$(22) \quad \hbar\omega(\mathbf{k}; \mathbf{R}) = \sqrt{\left(\frac{\hbar^2 \mathbf{k}^2}{2m} - \epsilon_F(\mathbf{R}) \right)^2 + |\Delta(\mathbf{k}; \mathbf{R})|^2} .$$

In principle, one can solve the BCS equation for any interatomic potential $V_{\alpha,\alpha'}(\mathbf{r})$. However, for alkali gases the complete central interaction is usually not very well known and we have only information on the two-body scattering length a . We therefore would like to reformulate the gap equation in such a way that only this scattering length enters. This is achieved by noting that the gap equation is very similar to the Lippmann-Schwinger equation for the two-body T(ransition) matrix. Indeed the latter reads [24]

$$(23) \quad T_{\alpha,\alpha'}^{2B}(\mathbf{k}, \mathbf{k}''; z) = V_{\alpha,\alpha'}(\mathbf{k} - \mathbf{k}'') + \int \frac{d\mathbf{k}'}{(2\pi)^3} V_{\alpha,\alpha'}(\mathbf{k} - \mathbf{k}') \frac{1}{z - \hbar^2 \mathbf{k}'^2/m} T_{\alpha,\alpha'}^{2B}(\mathbf{k}', \mathbf{k}''; z) .$$

Using the Lippman-Schwinger equation with $z = 2\epsilon_F(\mathbf{R}) + i0$, we can after some algebraic manipulation show that the gap equation is equivalent to

$$(24) \quad \Delta(\mathbf{k}; \mathbf{R}) = - \int \frac{d\mathbf{k}'}{(2\pi)^3} T_{\alpha,\alpha'}^{2B}(\mathbf{k}, \mathbf{k}'; 2\epsilon_F(\mathbf{R})) \left(\frac{1}{2\hbar\omega(\mathbf{k}'; \mathbf{R})} \tanh \left(\frac{\hbar\omega(\mathbf{k}'; \mathbf{R})}{2k_B T} \right) - \frac{1}{\hbar^2 \mathbf{k}'^2/m - 2\epsilon_F(\mathbf{R}) - i0} \right) \Delta(\mathbf{k}'; \mathbf{R}) .$$

This result serves our purposes since the two-body T matrix is directly related to the two-body scattering length as we will see next.

4. – Doubly spin-polarized Fermi gases

In doubly spin-polarized Fermi gases all the atoms are in the same hyperfine state. Although it is straightforward to generalize the following to an arbitrary state $|\alpha\rangle$, we will in first instance restrict ourselves to the fully stretched state $|\zeta\rangle$ in which both the electron as well as the nuclear spin have a maximal projection on the magnetic field axis. The relevant interaction matrix element is then equal to

$$(25) \quad V_{\zeta,\zeta}(\mathbf{r}) = V_T(r) + \frac{\mu_0 \mu_e^2}{4\pi r^3} (1 - 3 \cos^2(\theta_{\mathbf{r}})) ,$$

with $\theta_{\mathbf{r}}$ the angle between the interatomic separation \mathbf{r} and the magnetic field \mathbf{B} . For this potential we have to calculate the two-body T matrix. Treating the weak electron-electron magnetic dipole interaction in Born approximation [18] and making use of the fact that for the central interaction only p -waves contribute at low momenta, we find

$$(26) \quad T_{\zeta,\zeta}^{2B}(\mathbf{k}, \mathbf{k}'; z) = \frac{12\pi a^3 \hbar^2}{m} \mathbf{k} \cdot \mathbf{k}' + \frac{\mu_0 \mu_e^2}{8\pi} (\cos^2(\theta_{\mathbf{k}-\mathbf{k}'}) - \cos^2(\theta_{\mathbf{k}+\mathbf{k}'})) .$$

Here a is the triplet p -wave scattering length and $\theta_{\mathbf{k}\pm\mathbf{k}'}$ denotes the angle between the momentum transfer $\mathbf{k} \pm \mathbf{k}'$ and the magnetic field axis.

We now have two cases to consider. Generically we expect the p -wave scattering length to be of the order of the range of the triplet potential and therefore $k_F(\mathbf{R})|a|$ to be much smaller than one. In that case the contribution of the triplet potential to the two-body

T matrix is negligible and the effective interaction between the atoms is dominated by the long-range dipole-dipole interaction. Due to the complicated angular dependence of this interaction it is not possible to solve the BCS gap equation analytically. However, we can nevertheless make progress by noting that the magnetic dipole-dipole interaction is most attractive when \mathbf{r} is directed along the magnetic field. We thus expect that if Cooper pairs are formed their wavefunction $\phi_{\zeta,\zeta}(\mathbf{r};\mathbf{R})$ is most likely proportional to $Y_{1,0}(\hat{\mathbf{r}}) = \sqrt{3/4\pi} \cos(\theta_{\mathbf{r}})$, which implies that $\Delta(\mathbf{k};\mathbf{R}) = Y_{1,0}(\hat{\mathbf{k}})\Delta(\mathbf{R})$. Since this gap is anisotropic in the relative wave vector \mathbf{k} , the gas is below the critical temperature an anisotropic superfluid, just like ^3He in the so-called A phases [22].

To obtain an estimate for the critical temperature we explicitly consider only the p -wave part of the dipole-dipole interaction, which results in the approximation

$$(27) \quad T_{\zeta,\zeta}^{2B}(\mathbf{k},\mathbf{k}';z) \simeq -\frac{4}{15}\pi\mu_0\mu_e^2 \sum_m (-1)^m (1 + \delta_{m,0}) Y_{1,m}(\hat{\mathbf{k}}) Y_{1,m}^*(\hat{\mathbf{k}}') .$$

This explicitly confirms that the dipole-dipole interaction is only attractive in the channel $m = 0$. Substituting the above two-body T matrix into the BCS gap equation and linearizing with respect to $\Delta(\mathbf{0})$ to obtain an equation for the critical temperature, we find

$$(28) \quad \frac{2}{15\mu_0\mu_e^2} = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\mathcal{P}}{\epsilon_F(\mathbf{0}) - \hbar^2\mathbf{k}^2/2m} N(\mathbf{k};\mathbf{0}) .$$

Here we used the notations \mathcal{P} for the Cauchy principle value part of the integral and $N(\mathbf{k};\mathbf{R})$ for the Fermi distribution function $N(\epsilon) = (e^{\beta\epsilon} + 1)^{-1}$ evaluated at $\hbar^2\mathbf{k}^2/2m - \epsilon_F(\mathbf{R})$. Note that we have also used that the BCS gap equation will have a nontrivial solution in the center of the trap first, since the density of the gas is highest there. Introducing a ‘scattering length’ a^d for the electron-electron magnetic dipole interaction by means of $4\pi a^d \hbar^2/m \equiv -15\mu_0\mu_e^2/2$, i.e., $a^d = -15\mu_0\mu_e^2 m/8\pi\hbar^2 \simeq -0.2(m/m_H) a_0$ with m_H the mass of a hydrogen atom and a_0 its Bohr radius, the equation for the critical temperature becomes identical to the linearized BCS gap equation that has been studied previously in the context of s -wave superconductors by Sá de Melo *et al.* [25]. Using their result, we have for the critical temperature

$$(29) \quad T_c = \frac{8\epsilon_F(\mathbf{0})e^{\gamma-2}}{k_B\pi} \exp \left\{ -\frac{\pi}{2k_F(\mathbf{0})|a^d|} \right\} ,$$

with $\gamma \simeq 0.5772$ Euler’s constant. Unfortunately, the BCS transition to an anisotropic superfluid thus occurs at extremely low temperatures in this case and appears to be out of reach experimentally. For example for ^6Li at a density of $n \simeq 1 \times 10^{12} \text{ cm}^{-3}$, we have $\epsilon_F/k_B \simeq 600 \text{ nK}$ and $k_F|a^d| \simeq 2 \times 10^{-4}$.

The second case to consider appears to be more promising. As mentioned before, the triplet p -wave scattering length is in general too small to be able to dominate over the dipole-dipole interaction. For ^6Li it is only $-35 a_0$, for instance. However, we can imagine

that it is possible, in the same way as in the recent experiment by Inouye *et al.* [26], to (optically) trap a hyperfine state that has a p -wave Feshbach resonance [27] and tune the external bias magnetic field such that the p -wave scattering length becomes large and negative. The two-body T matrix is then well approximated by

$$(30) \quad T_{\alpha,\alpha}^{2B}(\mathbf{k}, \mathbf{k}'; z) \simeq \frac{12\pi a^3 \hbar^2}{m} \mathbf{k} \cdot \mathbf{k}' = \frac{(4\pi)^2 a^3 \hbar^2}{m} k k' \sum_m Y_{1,m}(\hat{\mathbf{k}}) Y_{1,m}^*(\hat{\mathbf{k}}') .$$

Substituting this into the BCS gap equation, it is not difficult to show that it is solved by the *ansatz*

$$(31) \quad \Delta(\mathbf{k}; \mathbf{R}) = k \Delta(\mathbf{R}) \sum_m d_m^* Y_{1,m}(\hat{\mathbf{k}}) = \sqrt{\frac{3}{4\pi}} \Delta(\mathbf{R}) \mathbf{d} \cdot \mathbf{k} ,$$

where d_m are the spherical components of a vector \mathbf{d} that is normalized as $\sum_m |d_m|^2 = \mathbf{d} \cdot \mathbf{d} = 1$. Furthermore, linearizing the BCS gap equation with respect to $\Delta(\mathbf{0})$, we find that the critical temperature is now determined by

$$(32) \quad -\frac{m}{4\pi a^3 \hbar^2} = \int \frac{d\mathbf{k}}{(2\pi)^3} \mathbf{k}^2 \frac{\mathcal{P}}{\epsilon_F(\mathbf{0}) - \hbar^2 \mathbf{k}^2 / 2m} N(\mathbf{k}; \mathbf{0}) .$$

The solution can be obtained by the same methods as before and reads

$$(33) \quad T_c = \frac{8\epsilon_F(\mathbf{0}) e^{\gamma-8/3}}{k_B \pi} \exp \left\{ -\frac{\pi}{2(k_F(\mathbf{0})|a|)^3} \right\} .$$

It is important to realize that up to this point the direction of the vector \mathbf{d} is arbitrary. This reflects the rotational symmetry of the problem. However, as we have seen, the magnetic dipole-dipole interaction breaks this symmetry and will cause \mathbf{d} to lie either parallel or perpendicular to the magnetic field, depending on precisely which hyperfine state $|\alpha\rangle$ is trapped. Because we are dealing with an attractive interaction we also have to make sure that the gas is mechanically stable. This leads to the restriction that $n(\mathbf{0})|a|^3 \leq 5/96\pi$, or equivalently $(k_F(\mathbf{0})|a|)^3 \leq 5/576\pi^3$. Unfortunately, the latter again severely limits the feasibility of experimentally observing the in principle interesting possibility of a transition to an anisotropic superfluid.

5. – Spin-polarized Fermi gases

We now turn our attention to the case of fermionic gases that are a mixture of two hyperfine states. In such a gas we can have s -wave collisions between atoms in different hyperfine states, which has as an advantage that interacting effects are expected to be much more important. Moreover, it has an additional advantage that it is now in principle possible to use evaporative cooling to cool the gas to low temperatures. A fermionic gas consisting of three hyperfine states has recently been considered as well [28], but since it

does not lead to any qualitative different physics we restrict ourselves here to mixtures of only two spin states. We denote the two hyperfine states involved from now on by $|\uparrow\rangle$ and $|\downarrow\rangle$. Experimentally, there are two ways to realize such a system. We can either magnetically trap two low-field seeking states, or use an optical trap. In the latter case it is presumably necessary to load the trap by precooling the gas in a magnetic trap using sympathetic cooling.

5.1. Positive scattering length. – If the s -wave scattering length a between two atoms in different spin states is positive, a first many-body effect that we have to consider is the phase separation of the gas into two phases with opposite ‘magnetization’. Roughly speaking, this implies that instead of having overlapping spin densities $n_\uparrow(\mathbf{R})$ and $n_\downarrow(\mathbf{R})$, the gas prefers to separate into two phases in which (almost) all the atoms are either in the state $|\uparrow\rangle$ or in the state $|\downarrow\rangle$. The driving force behind this instability is that although the phase separation increases the kinetic energy of the gas, this increase is more than compensated by the decrease in interaction energy. More precisely, the gas is stable if the free-energy surface $F[n_\uparrow, n_\downarrow]$ has a positive curvature in all directions. Using that

$$(34) \quad T_{\uparrow,\downarrow}^{2B}(\mathbf{k}, \mathbf{k}'; z) = \frac{4\pi a \hbar^2}{m},$$

the free energy in the degenerate regime is given by

$$(35) \quad F[n_\uparrow, n_\downarrow] = \int d\mathbf{R} \left\{ (6\pi^2)^{2/3} \frac{3\hbar^2}{10m} \left(n_\uparrow^{5/3}(\mathbf{R}) + n_\downarrow^{5/3}(\mathbf{R}) \right) + \frac{4\pi a \hbar^2}{m} n_\uparrow(\mathbf{R}) n_\downarrow(\mathbf{R}) \right\}$$

and stability of the gas requires that the spin densities obey $n_\uparrow(\mathbf{0}) n_\downarrow(\mathbf{0}) a^6 \leq (\pi/48)^2$. In the particular case of equal spin densities this reduces to the condition that $k_F(\mathbf{0}) a \leq \pi/2$. Note that above we have used the T-matrix approximation to evaluate the average interaction energy. Because the chemical potential μ_α is equal to the derivative $\delta F / \delta n_\alpha(\mathbf{R})$, we must for consistency use the same approximation to determine the renormalized chemical potentials $\mu'_\alpha(\mathbf{R})$. Therefore, we use in the following always that

$$(36) \quad \mu'_\alpha(\mathbf{R}) = \mu_\alpha - \sum_{\alpha' \neq \alpha} \frac{4\pi a \hbar^2}{m} n_{\alpha'}(\mathbf{R})$$

instead of the less accurate (only Born approximation) relation given in Eq. (16).

Even though the interaction between the atoms is effectively repulsive, there can nevertheless occur a BCS pairing transition to a superfluid state due to the so-called Luttinger-Kohn instability [29]. Physically, the instability is a result of the fact that two atoms in the same hyperfine state can exchange a fluctuation (phonon) in the density of the other hyperfine state, leading to an effectively attractive interaction between the atoms involved. The p -wave transition associated with the Luttinger-Kohn effect has been studied by Baranov *et al.* [30]. These authors obtain for the critical temperature

the estimate

$$(37) \quad T_c \simeq \frac{\epsilon_F(\mathbf{0})}{k_B} \exp \left\{ -13 \left(\frac{\pi}{2k_F(\mathbf{0})a} \right)^2 \right\} ,$$

where notably a is the s -wave scattering length. It should, however, be kept in mind that to be able to observe the transition we must require that the gas is mechanically stable and does not phase separate. As we have already seen, this implies for equal spin densities that $k_F(\mathbf{0})a \leq \pi/2$. Moreover, for generic values of the scattering length we will even have that $k_F(\mathbf{0})a \ll 1$ at the low densities of interest. It therefore seems once again practically impossible to observe the above transition in a real atomic gas.

5.2. Negative scattering length. – Our last chance of achieving a superfluid phase in a fermionic gas thus appears to be a spin-polarized gas with a large and negative s -wave scattering length. Besides the possibility of using a Feshbach resonance to tailor the scattering length, we can also directly make use of the anomalously large ^6Li triplet scattering length of $-2160 a_0$ by trapping a spin-polarized ^6Li gas in a bias magnetic field of at least 0.05 T [31]. Substituting Eq. (34) into the BCS gap equation we see that the solution $\Delta(\mathbf{k}; \mathbf{R})$ is now independent of the wave vector \mathbf{k} and therefore equal to $\Delta(\mathbf{R})$. Furthermore, a linearization in $\Delta(\mathbf{R})$ gives

$$(38) \quad -\frac{m}{4\pi a \hbar^2} = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\mathcal{P}}{\epsilon_F(\mathbf{0}) - \hbar^2 \mathbf{k}^2 / 2m} N(\mathbf{k}; \mathbf{0}) ,$$

which results in the critical temperature

$$(39) \quad T_c = \frac{8\epsilon_F(\mathbf{0})\epsilon^{\gamma-2}}{k_B\pi} \exp \left\{ -\frac{\pi}{2k_F(\mathbf{0})|a|} \right\} .$$

Since the mechanical stability of the gas requires also for a negative scattering length only that $k_F(\mathbf{0})|a| \leq \pi/2$, this expression shows that the prospects of observing a BCS transition in this case are indeed most favorable.

In view of this encouraging situation we have studied in more detail the behavior of the spin density profile $n_\alpha(\mathbf{R})$ and the BCS gap parameter $\Delta(\mathbf{R})$ for a spin-polarized ^6Li gas in the same magnetic trap that has been used for the Bose-Einstein condensation experiments with ^7Li [2,5]. The results for equal spin densities are shown in Fig. 1 and lead to three important conclusions. First, we see explicitly that the density profile of a degenerate Fermi gas is indeed completely ‘frozen’. Clearly, even the BCS transition to a superfluid has essentially no effect on the density profile. From an experimental point of view this is somewhat unfortunate, because it implies that the appearance of a condensate of Cooper pairs cannot be observed by the same methods that have been so successful in the Bose-Einstein condensation experiments. We come back to this problem shortly. Second, for a total density of $n \simeq 4 \times 10^{12} \text{ cm}^{-3}$ the critical temperature is about 37 nK. In view of the achievements with bosonic alkali gases, this appears to be a density-temperature combination which is certainly within reach experimentally. Finally, we

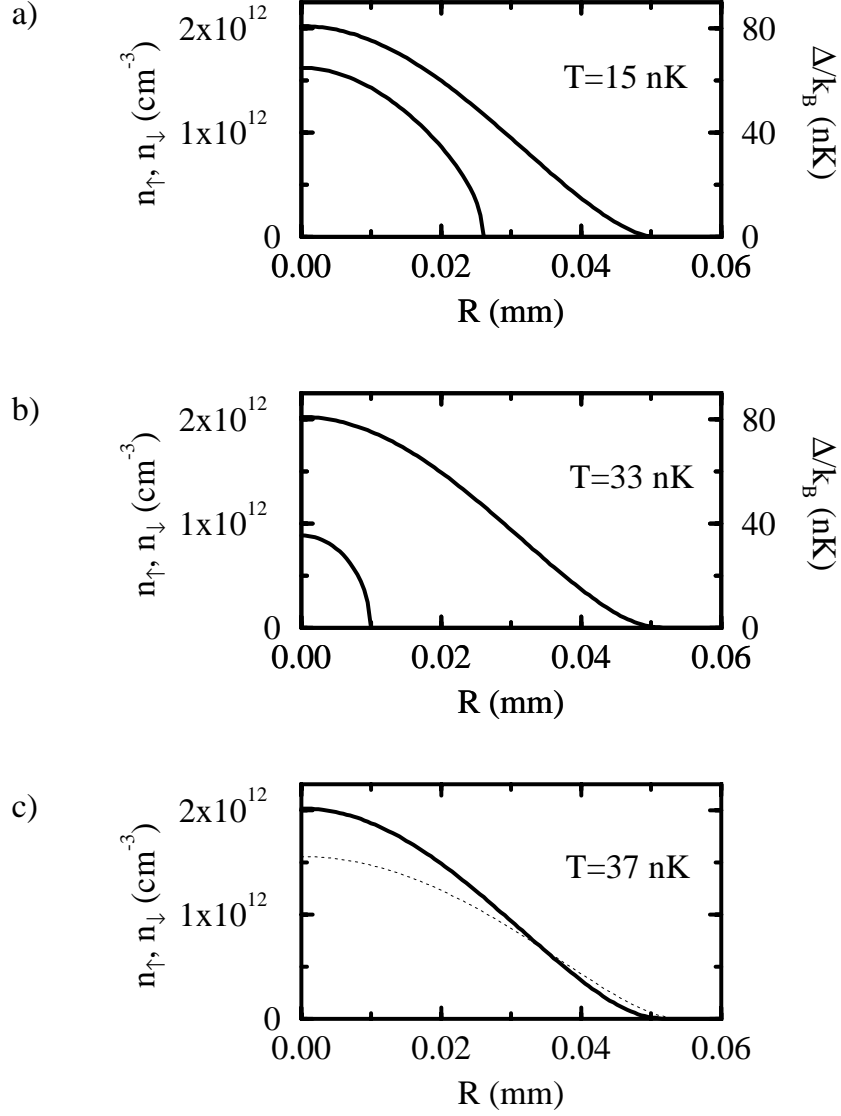


Fig. 1. – Density distribution $n_{\uparrow}(\mathbf{R}) = n_{\downarrow}(\mathbf{R})$ and energy gap $\Delta(\mathbf{R})$ for a ${}^6\text{Li}$ atomic gas consisting of 2.865×10^5 atoms in each spin state a) at $T = 15 \text{ nK}$, b) at $T = 33 \text{ nK}$, slightly below T_c , and c) at $T = T_c = 37 \text{ nK}$. The left scale of each plot refers to the density and the right scale to the energy gap. The dotted line in c) shows the density distribution for an ideal Fermi gas with the same number of particles and at the same temperature.

have also compared the density profile of ^6Li with that of a noninteracting Fermi gas with the same number of particles. The strong attractive interaction between the ^6Li atoms evidently results in a substantial increase of the density in the center of the trap. Experimentally, this is a favorable effect because for a fixed number of atoms in the trap it enhances the critical temperature as is shown quantitatively in Fig. 2. For future

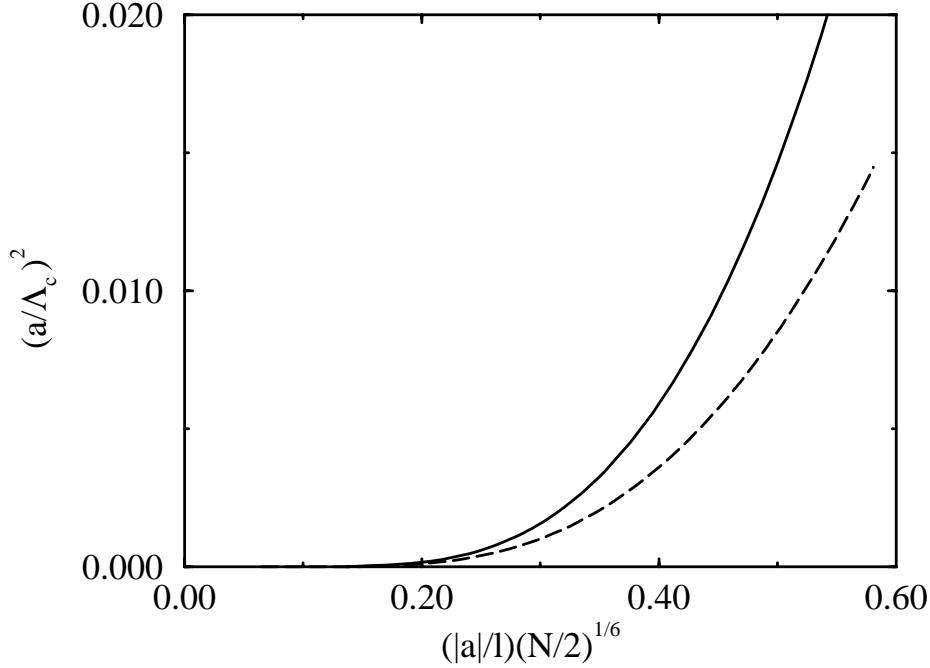


Fig. 2. – Critical temperature as a function of the number of particles (solid line) when there are $N/2$ particles present in both spin states. The dashed line represents the critical temperature for a gas whose density distribution is not altered by mean-field interactions.

convenience we mention that in a good approximation the critical temperature obeys

$$(40) \quad \left(\frac{a}{\Lambda_c}\right)^2 \simeq 0.037 \exp \left\{ -1.36 \frac{\ell}{|a|N^{1/6}} + 2.66 \frac{|a|N^{1/6}}{\ell} \right\} ,$$

with $\Lambda = (2\pi\hbar^2/mk_B T)^{1/2}$ the thermal de Broglie wavelength and $\ell = (\hbar/m\omega)^{1/2}$ the size of the harmonic oscillator ground state. It must be kept in mind that the latter formula can only be used for values of $N^{1/6}|a|/\ell$ that are less than about 0.74, since for larger values the gas is mechanically unstable and undergoes a spinodal decomposition first.

6. – Discussion and conclusions

We have argued that for condensed matter physics in trapped atomic Fermi gases, a spin-polarized gas with a large and negative s -wave scattering length between the atoms in the two different hyperfine states appears to be most promising. In view of the large uncertainties in the interatomic interaction potential of ^{40}K , the most likely candidate for the achievement of a gaseous BCS superfluid is at present ^6Li . However, before successful experiments with atomic ^6Li can be performed, some experimental problems need to be resolved. One problem is that in spin-polarized atomic ^6Li not only the s -wave scattering length but also the exchange and dipolar decay rates are anomalously large. As a result the gas has usually a very short lifetime. To enhance the lifetime to about 1 s at the densities of interest, we can either apply a bias magnetic field of about 5 T or use optical methods to trap two high-field seeking states. The latter solution seems to be most practical and is actively being pursued at the moment.

Assuming that we are able to achieve the necessary conditions for the BCS transition, the next problem that arises is the detection of the Cooper pair condensate. As we have seen, the density profile shows essentially no sign of the phase transition. Time-of-flight measurements, that were the ‘smoking gun’ for the Bose-Einstein condensation experiments, are therefore not appropriate here. Another possible signature that comes to mind are the frequencies of the collective modes. Because of the large s -wave scattering length required for relatively high critical temperatures, the collective modes are always in the hydrodynamical regime. They are therefore described by the (local) conservation laws and the Josephson relation. In the case of equal spin densities we thus obtain the following set of equations. The continuity equation for a superfluid is

$$(41) \quad \frac{\partial n}{\partial t} = -\nabla \cdot \mathbf{j} ,$$

with the total density $n = n_n + n_s$ and the total current density $\mathbf{j} = n_n \mathbf{v}_n + n_s \mathbf{v}_s$ consisting of a normal and superfluid contribution. In addition, Newton’s law gives

$$(42) \quad \frac{\partial \mathbf{j}}{\partial t} = -\frac{1}{m} (\nabla p + n \nabla V^{\text{trap}}) ,$$

where p denotes the pressure in the gas. Finally, we have also the Josephson relation, which leads to (cf. Eq. (9))

$$(43) \quad \frac{\partial \mathbf{v}_s}{\partial t} = -\frac{1}{m} \nabla \mu .$$

In principle, we must also take into account the conservation of energy. However, for a degenerate Fermi gas the specific heats at constant pressure and volume are almost equal and the continuity equation for the total energy density essentially decouples from the previous ones. This has important consequences, because for density fluctuations the Josephson relation just copies Newton’s law and we must conclude that the first sound

modes are not affected by the BCS transition. Of course, a measurement of second sound modes would be a clear signature of the transition, but this is presumably quite hard experimentally because second sound is primarily a temperature wave due to the fact that the energy fluctuations are almost decoupled. In analogy with sound attenuation in superconductors, it has been suggested by Fetter [32] that the damping of the first sound modes might be strongly influenced by the appearance of a Cooper pair condensate, but more work is needed to make sure whether this interesting suggestion would work.

A property of the gas that is certainly influenced by the BCS transition is the decay of the gas. Qualitatively this can be easily understood from the correlator method devised by Kagan *et al.* [33]. In this approach we can relate the decay rate constant $G(\mathbf{R}; T)$ for two-body decay to the rate constant in the normal phase $G(\mathbf{R}; T_c)$ by

$$(44) \quad G(\mathbf{R}; T) = G(\mathbf{R}; T_c) \frac{1}{n_{\uparrow}(\mathbf{R})n_{\downarrow}(\mathbf{R})} \left\langle \psi_{\uparrow}^{\dagger}(\mathbf{R})\psi_{\downarrow}^{\dagger}(\mathbf{R})\psi_{\downarrow}(\mathbf{R})\psi_{\uparrow}(\mathbf{R}) \right\rangle \\ = G(\mathbf{R}; T_c) \left(1 + \frac{|\phi_{\uparrow,\downarrow}(\mathbf{R}, \mathbf{R})|^2}{n_{\uparrow}(\mathbf{R})n_{\downarrow}(\mathbf{R})} \right) .$$

Hence, the Cooper pair condensate enhances the decay of the gas. A quantitative estimate of the effect is somewhat complicated by the fact that we cannot use a pseudopotential to calculate the Cooper pair wavefunction $\phi_{\uparrow,\downarrow}(\mathbf{R}, \mathbf{R})$ from our knowledge of the gap parameter $\Delta(\mathbf{R})$. In any case, the increase in the two-body decay rate of the gas can only be used as a detection method if the gas is trapped in a magnetic trap, since in an optical trap the two-body decay will essentially be eliminated and the lifetime of the gas is determined by the rate at which photons scatter off the atoms in the gas. Because this is a one-atom problem, it is also not affected by the BCS transition.

A final detection method that we would like to mention is the scattering of a beam of ^6Li atoms from the gas cloud. Since such an experiment is quite similar to a tunneling experiment, it is directly sensitive to the existence of the gap parameter $\Delta(\mathbf{R})$. Therefore, a measurement of the angular distribution of the scattered atoms appears to be a promising way to get detailed information about the condensate of Cooper pairs. Of course, to be most sensitive we need a very cold beam. However, this is clearly not an impossible requirement, because the first results of such experiments with a condensate of ^{87}Rb atoms have recently been reported [34]. From a theoretical point of view, we are at present performing a study of the above scattering process to see how large the effects of the BCS transition are. It is interesting to note that it is also possible to detect vortices in this manner, because the atomic beam (in contrast to a laser beam) does not only see the core of the vortex but its complete velocity profile. This is in fact also true for a Bose condensate. An experiment of this sort may therefore also be of interest in the context of ongoing research on the properties of trapped atomic Bose gases.

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