PHYSICAL BASIS OF BALLISTOCARDIOGRAPHY. III

H. C. Burger, D.Sc., and A. Noordergraaf, B.Sc.

Utrecht, Netherlands

INTRODUCTION

By means of a ballistocardiograph (BCG) record is made of the displacement, the velocity, or the acceleration with which the center of gravity of the subject moves because of the action of the heart. The subject lies on a BCG, and the displacement, the velocity, or the acceleration of subject or BCG is recorded. From these curves something is to be detected about the occurrences within the body. The type of BCG that is made use of determines which quantity concerning the center of gravity is represented by the recorded curve. These relations were dealt with in preceding papers.[2,3] Summarized, these relations are:

1. The displacement of a *low-frequency* BCG, damped critically or somewhat less than critically (natural frequency about or less than 0.3 c/s), represents the displacement of the center of gravity of subject and BCG caused by the circulation.
2. The displacement of a *middle-frequency* BCG (natural frequency about 1 to 2 c/s) represents the velocity of the center of gravity, provided that the damping is much more than critical.
3. The displacement of a *high-frequency* BCG (natural frequency of the loaded BCG about 15 c/s) represents the acceleration of the center of gravity of subject and BCG reliably up to about 10 c/s. (The internal force is to be found by means of multiplication of the acceleration and the mass.)

In the deduction of these relations it was assumed that the binding between the body of the subject and the BCG were infinitely strong. We remarked that in reality this binding is far from this extreme case.[2,3] So the movement of the subject differs from the movement of the BCG and that difference is greater the higher the frequency that is to be represented.

Nickerson and Mathers[1] give formulas by which they calculate how the representation of the forces that cause the center of gravity to move are distorted if (a) the displacement of the high-frequency BCG or (b) the displacement of their "low-frequency" BCG is recorded. We call this type of BCG the middle-frequency BCG, because the natural frequency of this type is neither high nor low with respect to the frequency of the heart. The type used by us has a considerably lower natural frequency. We will call the latter the low-frequency BCG.

From the Department of Medical Physics, Physical Laboratory of the University of Utrecht.
Received for publication June 1, 1955.

THEORY

In its simplest form the problem of the relative movement is a special case of a more general problem: that of two coupled harmonic oscillators.

The problem in the form we have to face it can schematically be represented in the form of Fig. 1. The subject with the mass m_s is represented by a cart that is coupled to the BCG with mass m_b by a directive force and a frictional force. The system of subject and BCG is in the same way coupled to the surroundings.

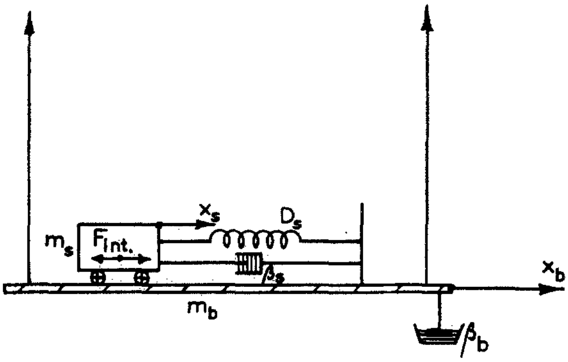

Fig. 1.—Schematic representation of the ballistocardiographic system in which the relative movement is taken into account. The subject with mass m_s is coupled to the BCG with mass m_b by a directive force and a frictional force. The whole system is coupled to the surroundings in an analogous way.

Nickerson and Mathers do not give the deduction of their formulas to calculate the distortion, but refer to a textbook.[4] From this we deduced that they reasoned as follows. Suppose that the movement of the BCG does not influence the movement of the subject. Then, the distortion caused by the vibration properties of the subject can be calculated from the differential equation:

$$m_s \ddot{x}_s + \beta_s \dot{x}_s + D_s x_s = F_{int} \qquad (1)$$

x_s, $\dot{x}_s$ and $\ddot{x}_s$ are the respective displacement, velocity, and acceleration of the body with mass m_s with respect to the fixed BCG. $D_s x_s$ is the directive force acting between subject and BCG that drives the subject to the zero position on the BCG; $\beta_s \dot{x}_s$ is the frictional force between subject and BCG. F_{int} is the internal force that causes the center of gravity to move.

From equation (1) follows the relation between the amplitudes of the internal force $|F_{int}|$ and the amplitude of the displacement of the subject $|x_s|$:

$$\frac{|x_s|}{|F_{int}|} = \frac{1}{[(m_s\omega^2 - D_s)^2 + \beta_s^2\omega^2]^{\frac{1}{2}}} \qquad (2)$$

($\omega = 2\pi\nu$, ν the frequency of a term of the Fourier expansion, in which the phenomena can be developed). By the movement of the subject a force F_{tr} is exerted on the BCG:

$$F_{tr} = D_s x_s + \beta_s \dot{x}_s \qquad (3)$$

with the amplitude $|F_{tr}|$

$$|F_{tr}| = |x_s|[D_s^2 + \beta_s^2\omega^2]^{\frac{1}{2}}. \qquad (4)$$

Equation (2) substituted in equation (4) gives the relation between the amplitudes of the force exerted on the BCG $|F_{tr}|$ and of the internal force $|F_{int}|$ thus gives the distortion of $|F_{int}|$ by the vibration properties of the subject:

$$\frac{|F_{tr}|}{|F_{int}|} = \left[\frac{D_s^2 + \beta_s^2\omega^2}{(m_s\omega^2 - D_s)^2 + \beta_s^2\omega^2}\right]^{\frac{1}{2}}. \tag{5}$$

Equation (5) can be written in another form

$$\frac{|F_{tr}|}{|F_{int}|} = \left[\frac{1 + 4\delta_s^2 \dfrac{\nu^2}{\nu_s^2}}{\left(1 - \dfrac{\nu^2}{\nu_s^2}\right)^2 + 4\delta_s^2 \dfrac{\nu^2}{\nu_s^2}}\right]^{\frac{1}{2}}. \tag{5a}$$

ν_s is the natural frequency of the subject on the fixed BCG; it is the frequency with which the subject would oscillate if there were no damping ($\beta_s = 0$). δ_s is the ratio of the damping β_s and the critical damping of the subject on the fixed BCG. So $\delta_s = 1$, if the damping of the subject with respect to the BCG is critical. The form on the right side of equation (5a) is called Ta by Nickerson and Mathers. So, in their notation:

$$|F_{tr}| = \text{Ta}\ |F_{int}|. \tag{5a}$$

The distortion caused by the vibration properties of the BCG can, according to Nickerson and Mathers, be calculated from a differential equation analogous to equation (1):

$$m_b\ddot{x}_b + \beta_b\dot{x}_b + D_bx_b = F_{tr} \tag{6}$$

In this equation x_b, $\dot{x}_b$ and $\ddot{x}_b$ are the respective displacement, velocity, and acceleration of the BCG with respect to the surroundings. The mass of the BCG itself is m_b. The frictional force with respect to the surroundings is $\beta_b\dot{x}_b$ and the directive force is D_bx_b.

If the amplitude $|x_b|$ of the displacement x_b is recorded, the distortion of $|F_{tr}|$ is:

$$\frac{|x_b|}{|F_{tr}|} = \frac{1}{[(m_b\omega^2 - D_b)^2 + \beta_b^2\omega^2]^{\frac{1}{2}}}. \tag{7}$$

Equation (7) can be written in the following form:

$$\frac{|x_b|}{|F_{tr}|} = \frac{1}{m_b\omega_b^2}\ \frac{1}{\left[\left(1 - \dfrac{\nu^2}{\nu_b^2}\right)^2 + 4\delta_b^2 \dfrac{\nu^2}{\nu_b^2}\right]^{\frac{1}{2}}}. \tag{7a}$$

$m_b\omega_b^2$ is a constant, so this distortion is given by

$$\text{Ra} = \frac{1}{\left[\left(1 - \dfrac{\nu^2}{\nu_b^2}\right)^2 + 4\delta_b^2 \dfrac{\nu^2}{\nu_b^2}\right]^{\frac{1}{2}}}. \tag{7b}$$

The symbol Ra is used by Nickerson and Mathers.

In formula (7b) ν_b means the natural frequency and δ_b the ratio of the damping β_b and the critical damping of the *unloaded* BCG. The formulas (7a) and (7b) do not hold good in case the BCG is suspended on wires as usually happens with the low-frequency BCG. This follows from the fact that D_b in formula (7) means the external force necessary to give the loaded BCG a deflection of 1 cm. In this case D_b can be calculated from

$$2\pi\nu_b' = \left[\frac{D_b}{m_s + m_b}\right]^{\frac{1}{2}}. \quad (8)$$

ν_b' is the natural frequency of the loaded BCG and is usually with good approximation independent of the load, just as with a pendulum.

If the BCG is mounted on springs, the same D_b can be found from

$$2\pi\nu_b = \left[\frac{D_b}{m_b}\right]^{\frac{1}{2}} \quad (9)$$

in which ν_b is the natural frequency of the unloaded BCG (because of the fact that ν_b depends on the mass m_s).

From equations (5) and (7) the total amplitude distortion, according to Nickerson and Mathers, follows:

$$\frac{|x_b|}{|F_{int}|} = \left[\frac{D_s^2 + \beta_s^2\omega^2}{\{(m_s\omega^2 - D_s)^2 + \beta_s^2\omega^2\} \ \{(m_b\omega^2 - D_b)^2 + \beta_b^2\omega^2\}}\right]^{\frac{1}{2}}. \quad (10)$$

Written in their notation:

$$\frac{|x_b|}{|F_{int}|} = \frac{1}{m_b\omega_b^2}\,\text{Ta. Ra.} \quad (10a)$$

Schematizing as has been described already (Fig. 1) we ourselves calculated among other things the distortion in the representation of the amplitude of the internal force if the displacement of the BCG is recorded. In this calculation the influence of the movement of the BCG on the movement of the subject is taken into account.[5] We found:

$$\frac{|x_b|}{|F_{int}|} = \left[\frac{D_s^2 + \beta_s^2\omega^2}{p^2 + q^2}\right]^{\frac{1}{2}}. \quad (11)$$

From this calculation follow p and q:

$$p = m_s m_b\omega^4 - \omega^2(m_s D_b + m_b D_s + m_s D_s + \beta_s\beta_b) + D_s D_b$$
$$q = -\omega^3(m_s\beta_b + m_b\beta_s + m_s\beta_s) + \omega(\beta_s D_b + \beta_b D_s).$$

Nickerson and Mathers[1] give a formula for the phase shift analogous to formula (10a). We[5] calculated the phase shift analogously to the calculation of formula (11). This problem will not be discussed here.

COMPARISON OF THE RESULTS OBTAINED WITH THE CORRECT CALCULATION AND THE APPROXIMATION OF NICKERSON AND MATHERS

The curves in Figs. 2 and 3 represent the distortion according to formula (10a) of Nickerson and Mathers, interpreted in the correct manner (especially concerning ν_b and δ_b, see above), and those according to our formula (11).

To obtain Figs. 2 and 3, we substituted in formula (10), which is identical with formula (10a), and in the formula (11), the following data:

m_s = 60 kg.
$m_b \doteq$ 15 kg., according to data given by Starr (Nickerson and Mathers give no data about m_b).
D_s = 38.10^6 g.cm^{-2}, calculated with the aid of

$$2\pi\nu_s = [D_s/m_s]^{\frac{1}{2}}.$$

In this formula ν_s = 4.0 c/s, according to the measurements of Nickerson and Mathers[1] and ours.[5]

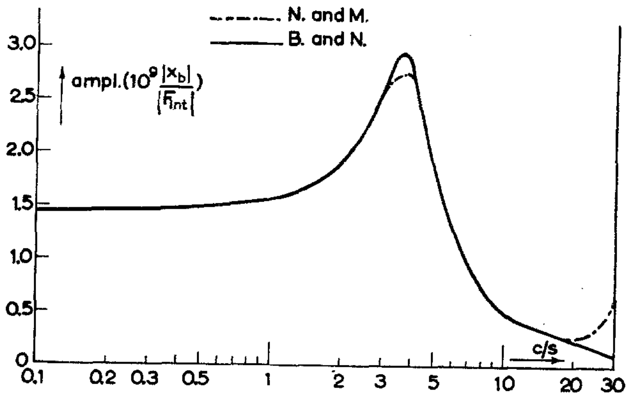

Fig. 2.

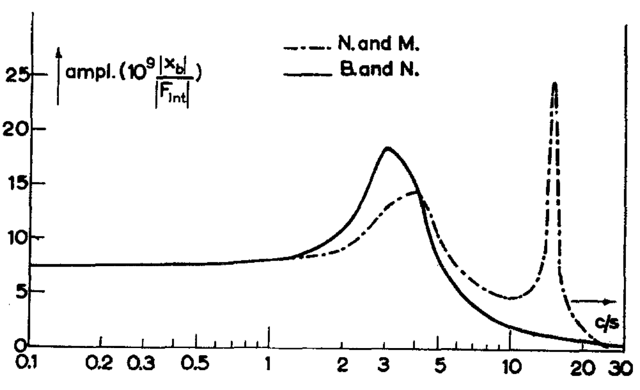

Fig. 3.

Fig. 2.—Amplitude characteristics of a high-frequency BCG (natural frequency of the *loaded* BCG 15 c/s) calculated according to the method of Nickerson and Mathers (broken line) and according to the authors' method (solid line).

Fig. 3.—Amplitude characteristics of a high-frequency BCG (natural frequency of the *unloaded* BCG 15 c/s) calculated according to the method of Nickerson and Mathers (broken line) and according to the authors' method (solid line).

Nickerson and Mathers measured δ_s = 0.2. This accords with part of our measurements done under special conditions. We will take a somewhat different value of δ_s, namely, δ_s = 0.3, from which follows β_s = 10^6 g.sec^{-1}, a number near the mean of all measurements. From a paper of Bouhuys[6] it follows that β_b = 85.10^3 g.sec^{-1}. This means δ_b = 14.10^{-3}. We have chosen this number, which is much lower than the data used by Nickerson and Mathers, because the high-frequency BCG usually is, as far as we know, damped to this extent. (The damping of a BCG can be calculated directly from a curve representing the movement of the BCG carrying a load that is fixed to the BCG if the BCG is released without initial velocity from a position that is not the zero position. If the BCG carries a load that is not fixed to the BCG (e.g., a subject), then the damping of the BCG differs from the above-mentioned case.[6] The coefficients β_b and D_b from the formulas (10) and (11) mean those from the fixed load).

To complete the necessary data we have chosen different values of the natural frequency of the BCG. In Fig. 2 we have used a value of 15 c/s for the natural frequency of the *loaded* BCG. From this it follows with equation (8) that $D_b = 68.10^7$ g.sec^{-2}. In Fig. 3 we assumed the natural frequency of the *unloaded* BCG to be 15 c/s. So, $D_b = 136.10^6$ g.sec^{-2}.

From Fig. 2 it appears that the amplitude characteristics do not agree so badly for higher frequencies. However, if the natural frequency of the BCG approaches the natural frequency of the subject on the fixed BCG, the agreement is worse, also for lower frequencies (Fig. 3). Then the amplitude characteristic obtained by the method of calculation of Nickerson and Mathers is unreliable.

THE CURVES CONCERNING THE MIDDLE-FREQUENCY BALLISTOCARDIOGRAPH

Nickerson and Mathers also calculate the distortion appearing when a middle-frequency BCG is used (natural frequency about 1 to 2 c/s).

To calculate the amplitude distortion they use formula (10a). In other words, they calculate how the amplitude of the internal force is distorted if one records the amplitude of a middle-frequency BCG. Apart from the influence of the relative movement, the middle-frequency BCG has only a suitable amplitude characteristic and phase characteristic if its amplitude is regarded as a measure of the velocity with which the center of gravity moves.[2,3] But then, the loaded BCG must be damped far more than critical. Nickerson and Mathers damp their BCG less than critical and make calculations concerning the acceleration of the center of gravity instead of its velocity. For these two reasons their characteristics of the middle-frequency BCG are difficult to interpret.

Nickerson and Mathers give two curves regarding the middle-frequency BCG (their Fig. 1). In one of these cases the calculation is not correct, for they substitute for ν_b and δ_b of formula (10a) the data belonging to the *loaded* BCG (curve E).

SUMMARY

Two methods for calculating the amplitude characteristic of the high-frequency BCG, the difference in movement of subject and BCG taken into account, are compared and discussed.

Some remarks are made upon the calculation of the amplitude characteristic of the middle-frequency BCG according to Nickerson.

SUMMARIO IN INTERLINGUA

Es comparate e discutite duo methodos de calcular le amplitude characteristic del ballistocardiographo a alte frequentia, con consideration del differentia del movimento de subjecto e ballistocardiographo.

Es facite alicun remarcas in re le calculation del amplitude characteristic del ballistocardiographo a frequentia median secundo le methodo de Nickerson.

REFERENCES

1. Nickerson, J. L., and Mathers, J. A. L.: A Study of the Physical Properties of the Ballistocardiograph, AM. HEART J. **47**:1, 1954.
2. Burger, H. C., Noordergraaf, A., and Verhagen, A. M. W.: Physical Basis of the Low-Frequency Ballistocardiograph, AM. HEART J. **46**:71, 1953.
3. Burger, H. C., and Noordergraaf, A.: Physical Basis of Ballistocardiography. II. AM. HEART J. **51**:127, 1956.
4. Thomson, W. T.: Mechanical Vibrations, New York, 1948, Prentiss Hall, Inc.
5. Burger, H. C., Noordergraaf, A., Korsten, J. J. M., and Ullersma, P.: Physical Basis of Ballistocardiography. IV. In preparation.
6. Bouhuys, A.: High-Frequency Ballistocardiography, Proc. Roy. Netherlands Acad. Sc., Series C, **55**:126, 1952.