SHELL-MODEL CALCULATION FOR NUCLEI OF MASSES 30 THROUGH 33 *

B. H. WILDENTHAL **, J. B. McGRORY and E. C. HALBERT
Oak Ridge National Laboratory, Oak Ridge, Tennessee, USA

and

P. W. M. GLAUDEMANS
Physisch Laboratorium, Utrecht, The Netherlands

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Level energies and spectroscopic factors for isotopes of P, Si, and S are calculated with a shell model which includes active particles in the $1d_{\frac{5}{2}}$, $2s_{\frac{1}{2}}$ and $1d_{\frac{3}{2}}$ shells.

This note presents some results of shell-model calculations for the low-lying even-parity states of nuclei with masses $30 \leq A \leq 33$. Several of these nuclei have been intensively studied experimentally [1-4], but until now there has been no comprehensive and detailed model offered for the large set of low-excitation energy data presently available.

In our calculations we restrict the shell-model basis to a subset of all the Pauli-allowed states formed by distributing $(A-16)$ particles in the orbits $1d_{\frac{5}{2}}$, $2s_{\frac{1}{2}}$, and $1d_{\frac{3}{2}}$ outside a closed $(1s)^4(1p)^{12}$ Core. Our restricted basis comprises all such states having at least ten $1d_{\frac{5}{2}}$ particles; in other words we allow no more than two holes in the $1d_{\frac{5}{2}}$ shell. Then for given A, T and J the number of many-particle basis states is typically between 100 and 300. (In the full s-d-shell vector space for these nuclei, typical dimensions would be 1000 to 3000.) Our hope is that the perturbative effects of further $1d_{\frac{5}{2}}$ holes and of 1p holes, 1f particles, etc. can be adequately treated by use of a suitable effective interaction. Indeed, inspection of the experimental data on parities of low-lying states does suggest that the neglect of explicit non-s-d-shell configurations is a better approximation for A = 30-33 than for nuclei near either end of the s-d shell.

The effective interaction for our model was obtained by varying its parameters so as to best fit the observed energy levels. We make the usual assumption of a (1+2)-body effective Hamiltonian. In our s-d shell basis, the most general (1+2)-body Hamiltonian is specified by 63 two-body matrix elements and 3 single-particle energies. However, it is not practical to vary all of these 66 parameters independently in a least-squares search. Instead, we restricted the two-body effective interaction to have the form of the modified surface delta interaction [5-7],

$$V_T(ij) = -4\pi A_T \delta(\boldsymbol{r} - \boldsymbol{r}_j) f_{ij} + B_T \ .$$

Here T indicates the isospin (0 or 1) of the two interacting particles; A_T and B_T are strengths depending only on T; and f_{ij} is an operator which has the effect of making

$$\iint r_i \, dr_i \, r_j \, dr_j \, R_a(r_i) R_b(r_j) \, \delta(r_i - r_j) f_{ij} \, R_c(r_i) \, R_d(r_j)$$

equal to unity (where R_a, R_b, R_c, R_d are the single-particle radial wave functions involved in $\langle abJT \,|\, V_T(ij) \,|\, cdJT \rangle$).

In this way the 63 two-body matrix elements are specified by the 4 parameters A_0, B_0, A_1, B_1 of the modified surface delta interaction. These 4 parameters, together with the 3 single-particle energies, were determined by adjusting them so as to give a least-squares fit to the experimental data for the 9 ground-state binding energies and for 44 excitation energies of ^{30}Si, ^{30}P, ^{31}Si, ^{31}P, ^{32}Si, ^{32}P, ^{32}S, ^{33}P and ^{33}S. The least-squares solution is quite stable against inclusion or exclusion of any particular level in the fitting criterion. (Indeed, a preliminary fit

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Table 1
Experimental and calculated energies for some even-parity states in $30 \leq A \leq 33$ nuclei.

Nucleus			First		Second		Third		Fourth	
A	T	J	E_{ex}	E_{th}	E_{ex}	E_{th}	E_{ex}	E_{th}	E_{ex}	E_{th}
30	0	1	156.22	156.31	0.71	0.93	3.02	3.34	-	3.75
30	0	2	1.45	1.65	2.72	2.34 [1]	4.34	4.75 [1]	-	5.05
30	0	3	1.97	2.19	2.54	2.95	-	3.39	-	4.51
30	0	4	-	4.18	-	5.01	-	6.08	-	6.44
30	0	5	-	3.74	-	5.11	-	6.01	-	6.92
31	$\frac{1}{2}$	$\frac{1}{2}$	168.50	168.67	3.14	3.56	*5.25	3.91	-	5.16
31	$\frac{1}{2}$	$\frac{3}{2}$	1.26	1.17	3.51	4.06	4.26	4.53	-	4.80
31	$\frac{1}{2}$	$\frac{5}{2}$	2.23	2.35	3.29	2.86	4.19	4.75	-	4.98
31	$\frac{1}{2}$	$\frac{7}{2}$	3.41	3.56	-	4.18	-	5.57	-	5.79
31	$\frac{1}{2}$	$\frac{9}{2}$	-	5.10	-	5.66	-	6.70	-	7.18
32	0	0	183.56	183.45	3.78	3.87	-	7.63	-	8.38
32	0	1	4.70	4.76	-	6.61	-	8.22	-	8.44
32	0	2	2.24	1.97	4.29	4.90	5.55	5.63	-	6.83
32	0	3	-	5.88	-	7.20	-	7.67	-	8.22
32	0	4	4.47	5.48 [14]	-	6.19	-	6.99	-	8.23

Binding energies (corrected for Coulomb energies as in ref. 9 are listed for the ground states along with excitation energies for the excited states. Levels marked by asterisks were not included in the least-squares search. Except where noted, experimental energies are taken from ref. 4.

was made to energies for $A = 30$ and 31 only. The resulting Hamiltonian, and the calculated energy levels up through $A = 33$, were essentially identical to those presented here [8].) The optimized values of the modified surface delta interaction strengths obtained from the 30-33 fit are $A_0 = = 0.892$ MeV, $A_1 = 0.873$ MeV, $B_0 = -1.071$ MeV and $B_1 = 0.708$ MeV; and the optimized binding energies of the $1d_{\frac{5}{2}}$, $2s_{\frac{1}{2}}$, and $1d_{\frac{3}{2}}$ nucleons are -7.55, -5.73, and -3.32 MeV respectively.

Table 1 shows some of the spectra from these calculations, together with the experimentally determined energy levels. The deviations between calculated and experimental energies in this table are typical of all the results. The r.m.s. deviation for the 53 pieces of data in the fit was 0.34 MeV, and the average absolute deviation was 0.27 MeV. In view of the simple and rigid nature of the interaction, we find the energy level agreement quite pleasing. Particularly significant are the several instances in which the observed energy spacings of three or four levels of one spin in a given nucleus are reproduced in the model spectra. It is not yet clear whether the few substantial deviations that do occur are to be traced to deficiencies in the model space or to the modified surface delta interaction assumption.

A test of some important features of the model wave functions obtained from these calculations is provided by comparing calculated and experimentally observed [11-15] values for single-nucleon spectroscopic factors. This comparison is shown in table 2. Again the agreement between theory and experiment is good. In particular, the relative strengths observed for $2s_{\frac{1}{2}}$ and $1d_{\frac{3}{2}}$ transfer are well reproduced. This indicates that the theoretical wave functions give the correct relative occupation probabilities for these two orbits. The absence of experimentally observed strong $1d_{\frac{5}{2}}$ stripping transitions, which implies that the $1d_{\frac{5}{2}}$ shell is effectively full in the ground states, is also correctly reproduced by the model. The main discrepancy between the model and experiment seems to be the lack of enough $1d_{\frac{5}{2}}$ single-hole strength in the second $\frac{5}{2}^+$ level calculated for $A = 31$, $T = \frac{1}{2}$ (^{31}P - ^{31}S).

This work is being extended to consider the $A = 34$ and 35 nuclei, electromagnetic transition rates, and alternative forms of the effective interaction. The computational methods used in this work are described in refs. 15 and 16.

Table 2
Experimental and calculated spectroscopic factors.

Transition	Final state (J^π, E_x(MeV))		S_x	S_{th}
$^{30}Si(^3He, d)^{31}P$ [10] *	$\frac{1}{2}^+$,	0.00	0.68	0.95
	$\frac{3}{2}^+$,	1.26	0.67	0.74
	$\frac{5}{2}^+$,	2.23	0.065	0.06
	$\frac{1}{2}^+$,	3.14	0.02	0.07
	$\frac{5}{2}^+$,	3.29	0.003	0.01
	$\frac{3}{2}^+$,	3.51	0.004	0.14
$^{30}Si(d, p)^{31}Si$ [11]	$\frac{3}{2}^+$,	0.00	0.86	0.70
	$\frac{1}{2}^+$,	0.75	0.27	0.17
	$\frac{5}{2}^+$,	1.70	0.02	0.002
	$\frac{3}{2}^+$,	2.32	0.06	0.01
	$\frac{5}{2}^+$,	2.79	0.04	0.01
$^{32}S(h,\alpha)^{31}S$ [12] **	$\frac{1}{2}^+$,	0.00	1.9	2.12
$^{32}S(p, d)^{31}S$ [13]	$\frac{3}{2}^+$,	1.24	2.0	1.66
	$\frac{5}{2}^+$,	2.23	5.5	4.52
	$\frac{1}{2}^+$,	3.08	weak	0.02
	$\frac{5}{2}^+$,	3.29	1.5	0.02
	$\frac{3}{2}^+$,	3.43	weak	0.01
$^{32}S(d, p)^{33}S$ [14]	$\frac{3}{2}^+$,	0.00	0.69	0.61
	$\frac{1}{2}^+$,	0.84	0.27	0.28
	$\frac{5}{2}^+$,	1.97	weak	0.002
	$\frac{3}{2}^+$,	2.31	0.05	0.08

* With arbitrary normalization.
** The values S_x listed here are the averages from refs. 12 and 13.

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