COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review.** *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on "Relativistic cluster dynamics of nucleons and mesons. II. Formalism and examples"

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In a recent paper [Phys. Rev. C **49**, 2142 (1994)], Haberzettl presented cluster N-body equations for arbitrarily large systems of nucleons and mesons. Application to the three-nucleon system is claimed to yield a new kind of three-nucleon force. We demonstrate that these three-nucleon equations contain double counting.

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Recently, Haberzettl proposed a relativistic formalism for the nuclear N-body problem that is based on the description of clusters rather than of individual particles [1,2]. In Ref. [2], this formalism, which he calls relativistic cluster dynamics (RCD), is illustrated by several examples, among them the three-nucleon system with explicit pion and Δ degrees of freedom in addition to the usual nucleonic ones. It is claimed that the RCD description yields, among others, a certain one-pion exchange three-nucleon force that has not been included in any three-nucleon calculation so far [2,3]. In this Comment, we evaluate the RCD graphs of this one-pion exchange three-nucleon force using the "RCD rules" for the calculation of cluster graphs as they are specified in the Appendix of [2]. We may compare it to a graph that results from the iteration of the so-called nucleon-exchange diagram which describes three-nucleon scattering solely in terms of the two-nucleon interaction. We find that the contributions of the one-pion exchange three-nucleon force are identical to the part of the iterated nucleon-exchange diagram that is due to the pionic component of the two-nucleon interaction. As a consequence, including the one-pion exchange three-nucleon force amounts to double counting.

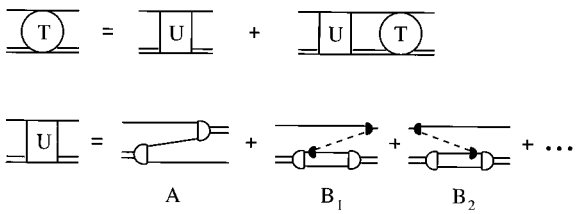

FIG. 1. Lippmann-Schwinger–type equation for three-body scattering. In all figures, solid lines are nucleons, dashed lines are pions. Open semicircles are two-nucleon form factors, hashed semicircles are pion production and absorption vertices.

The basic equation of RCD is an effective two-body Lippmann-Schwinger equation of the form

$$T^{\sigma\rho}=U^{\sigma\rho}+\sum_{\tau} U^{\sigma\tau}G_0^{\tau}T^{\tau\rho}, \tag{1}$$

where T is the total scattering amplitude for the transition from a two-cluster partition ρ to a two-cluster partition σ; G_0^{τ} describes the free propagation of two clusters, corresponding to the partition τ, that do not interact with each other but are internally fully interacting. The driving term U contains all diagrams that are irreducible with respect to G_0^{τ}. Figure 1 displays Eq. (1) in graphical form together with the three diagrams of U that we will focus on.

To demonstrate most clearly the double counting in these equations, let us consider the lowest order connected field-theoretical Feynman graph shown in Fig. 2. As a Feynman diagram, it sums all possible time orderings of pion exchanges, in particular also all processes where at least two pions are exchanged in overlapping time intervals. Within the RCD approach two classes of contributions can be identified with this Feynman graph. The diagrams B_1 and B_2 in

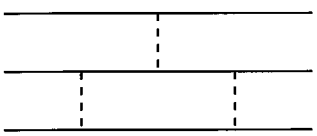

FIG. 2. Lowest-order connected pion exchange Feynman graph contribution to the three-nucleon t-matrix.

the driving force in Fig. 1 belong to one type, whereas there is also a contribution from iterating the Lippmann-Schwinger equation once.

Let us first consider the class of diagrams as occurring in the driving force in Fig. 1. In particular, we confine ourself to the graphs B_1 and B_2, which have been argued in Refs. [2,3] as leading to the new three-nucleon force contributions. If diagram B_1 of Fig. 1 is evaluated following the RCD rules of Ref. [2], we get (ignoring the three-momentum dependence and phase-space factors)

$$B_1 \propto \frac{i}{2\pi}\int dk_0[g_f(k_0+e-e',E-e')]^\dagger t_{\beta'}(k_0+e-e'-E_{\beta'}+i0)[f_{\delta\beta,\beta'}(e-e',k_0+e-e')]^\dagger t_\beta(k_0-E_\beta+i0)$$
$$\times t_\alpha(E-e-k_0-E_\alpha+i0)\tau_\delta(e-e'-\omega_\delta+i0)f_{\delta\gamma',\gamma}(e-e',e)g_i(k_0,E-e), \tag{2}$$

where t and τ are the nucleon and pion RCD propagators, g and f the two-nucleon and pion-nucleon vertex functions (or ''form factors'' in the terminology of separable interactions), and E_α and ω_δ are the on-shell energies of the nucleon α and the pion δ, respectively. Similarly, diagram B_2 yields

$$B_2 \propto \frac{i}{2\pi}\int dk_0[g_f(k_0+e-e',E-e')]^\dagger t_{\beta'}(k_0+e-e'-E_{\beta'}+i0)[f_{\delta\gamma,\gamma'}(e'-e,e')]^\dagger t_\beta(k_0-E_\beta+i0)$$
$$\times t_\alpha(E-e-k_0-E_\alpha+i0)\tau_\delta(e'-e-\omega_\delta+i0)f_{\delta\beta',\beta}(e'-e,k_0)g_i(k_0,E-e). \tag{3}$$

The precise meaning of the occurring energy variables and cluster indices is defined in Fig. 3. Since energy (and momentum) is conserved at each vertex, it is sufficient to label the form factors by two arguments only, the choice of which is just a matter of taste. We choose our notation such that it is easy to follow the ''flow of energy'' through the diagrams along the path that includes the pion line. While the πNN vertices carry subscripts for all connecting clusters, we characterize the NN form factors only as initial (i) and final (f). Assuming pointlike pion-nucleon vertices, we get

$$B_1+B_2 \propto \frac{i}{2\pi}\Delta|f|^2\int dk_0[g_f(k_0+e-e',E-e')]^\dagger t_{\beta'}(k_0+e-e'-E_{\beta'}+i0)t_\beta(k_0-E_\beta+i0)$$
$$\times t_\alpha(E-e-k_0-E_\alpha+i0)g_i(k_0,E-e), \tag{4}$$

where Δ is the pion propagator,

$$\Delta=\tau_\delta(e-e'-\omega_\delta+i0)+\tau_\delta(e'-e-\omega_\delta+i0). \tag{5}$$

In order to establish the relation with the Feynman graph of Fig. 2, we note that the two-nucleon form factors g_i and g_f are directly related to the one-meson exchange graphs through the separable expansion of the two-nucleon t matrix. In view of this, Eq. (4) can readily be identified as corresponding to the Feynman graph of Fig. 2, with only the positive energy states kept in the nucleon propagators and pointlike pion-nucleon vertices taken.

We now turn to the other class of diagrams as obtained from iterating the integral equation. If we iterate diagram A of Fig. 1 once, we get the diagram depicted in Fig. 4. Part of the iterated graph is a full two-nucleon t matrix that can be determined at the hierarchically lower two-body level. It is an essential feature of the RCD strategy to build up cluster amplitudes at the N-body level from those of the (N-1)–body level in a consistent fashion. The kernel of the two-nucleon cluster equations from which the two-nucleon t-matrix is calculated is shown in Fig. 17 of Ref. [2]. At this point we are only interested in the one-pion exchange contributions to the two-nucleon interaction. The two-nucleon t-matrix, as illustrated in Fig. 5, consists of single pion-exchange processes plus all iterations (for this discussion, any other contributions than those of pions are irrelevant and not considered for simplicity).

If we keep only the lowest-order pion-exchange contribution to the two-nucleon t-matrix, the iterated one-nucleon exchange contribution reduces to the graphs of Fig. 6. We may note that the only difference between the two diagrams

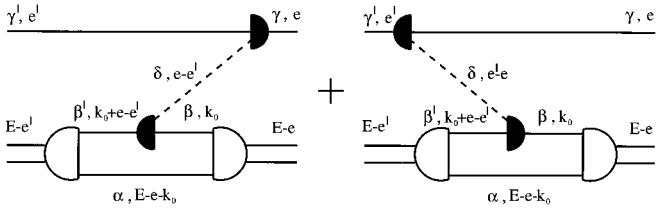

FIG. 3. Energy and cluster variables for the evaluation of diagrams B_1 and B_2 of Fig. 1.

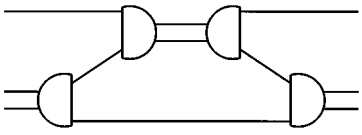

FIG. 4. Iterated one-nucleon exchange graph A of Fig. 1.

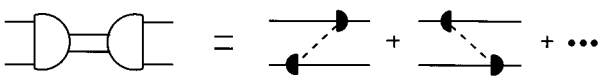

FIG. 5. Lowest-order pionic contributions to the two-nucleon t-matrix.

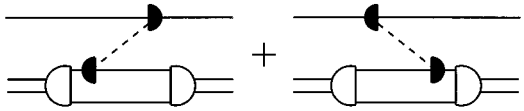

FIG. 6. Lowest-order pionic contributions to the iterated one-nucleon exchange term of Fig. 4.

of Fig. 6 and diagrams B_1 and B_2 of Fig. 1 consists in a different arrangement of the pion production and absorption vertices, respectively. At first sight, this suggests that different physical processes are involved. However, the cluster diagrams of RCD should not be confused with diagrams of ordinary time-ordered perturbation theory, nor with Feynman diagrams. They are ''time'' -ordered in the sense that vertices can ''open'' or ''close'' as one follows the lines representing clusters in a given diagram (from right to left, according to the convention of Ref. [2]). In other terms, a cluster can break up into two new clusters, and two clusters can combine to a single one, respectively. Since the two processes are not equivalent, the vertices induce an ordering that resembles a ''time'' ordering. This is different in a Feynman diagram where a vertex describes absorption and emission simultaneously. The rules of RCD as given in the Appendix of Ref. [2] assign vertex functions and propagators to each diagram; their energy and momentum dependence is completely determined by energy and momentum conservation at each vertex. It is therefore irrelevant whether, e.g., the pion production vertex of diagram B_1 is drawn on the right or left side of the two-nucleon form factor g_i. Hence, the expressions for the graphs of Fig. 6 are exactly the same as the ones for B_1 and B_2 of Fig. 1, namely Eqs. (2)–(4). Of course, since we are dealing with the same expressions as for B_1 and B_2, it follows immediately that identifying the two-nucleon vertex functions through Fig. 5 as corresponding to the one-pion exchange between the two nucleons leads again to the Feynman graph of Fig. 2 with only the positive energy parts of the nucleon propagators kept.

Hence, the Feynman graph contribution of Fig. 2 is counted twice in the RCD equations. Unlike in a nonrelativistic theory, iterating the two-nucleon interaction automatically includes all time orderings of particle exchanges, and time-overlapping processes need not be added by hand. From the above considerations it is obvious, that the RCD equations contain in general double counting of a certain class of field-theoretical graphs. The basic reason for this double counting is that the structure of the vertex functions is in principle dependent on the underlying interaction. As a result, certain contributions from the field-theoretical graphs will appear at different places in the RCD approach, leading to double counting of graphs.

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