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LETTER TO THE EDITOR

Monte Carlo simulation of carrier number noise spectra in the integral quantum Hall regime

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Abstract. We report Monte Carlo simulations of the spectral distribution of carrier number fluctuations due to transitions between localised and extended states in a Landau level. Surprisingly, it is found that the noise spectrum consists of a single Lorentzian, even though the density of states near the mobility edge is assumed to be continuous. The spectral features of the noise spectrum strongly resemble those observed in experiments on GaAs/AlGaAs heterostructures. We show that comparison of experiment and simulation yields an estimate of the energy separation between the mobility edges in the experimental system.

It is generally accepted that localised electronic states play an important role in the integral quantum Hall effect (IQHE) [1]. In a quasi-two-dimensional electron gas (2DEG) at low temperatures and under the influence of a strong magnetic field, the density of electron states consists of a series of equidistant Landau levels separated by the cyclotron energy. For a non-interacting 2DEG these Landau levels would be delta-functions. Scattering, however, causes broadening of the levels. Due to spatial fluctuations in the static potential experienced by the electrons, the states in the tails of the Landau levels are localised, whereas the states in the centre of a Landau level are extended. The latter states are responsible for electrical transport. When the Fermi level is located in a region of localised states and the thermal energy kT is small compared to the energy difference between the Fermi level and the energy of an extended state, scattering is suppressed, so the current flows virtually without dissipation. A model based on this picture can account for quantised plateaus in the Hall resistance and broad minima in the magnetoresistance. There are, however, other possible experimental manifestations of the presence of localised electronic states in the quantum Hall regime. For instance, at not too low temperatures, spontaneous fluctuations in the occupation of such localised states should lead to excess noise in the resistance of a 2DEG in the IQHE regime. Indeed, recent experiments on the electrical noise in the Hall resistance and in the magnetoresistance of GaAs/AlGaAs heterostructures at 4.2 K [2–5] provide evidence for the existence of carrier number noise. The noise power spectrum measured in these experiments consisted of a Lorentzian, the intensity of which was proportional to the square of the electric current, in addition to the (white) thermal noise that was independent of current. Intuitively, such a voltage-noise spectrum is just what one would expect if the noise were

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due to generation–recombination processes associated with transitions of electrons to and from localised states. However, the standard theory of generation–recombination noise [6] would predict a noise spectrum consisting of well defined Lorentzians only if the fluctuation processes take place between just a few energy levels, or possibly energy bands. In the latter case the rate at which intra-band electronic transitions take place should be much higher than the frequency of inter-band transitions, such that the different bands effectively behave as reservoirs, each at local thermodynamic equilibrium.

It is, however, not at all clear that such a description of generation–recombination noise is compatible with the picture of localised and extended Landau states sketched above. After all, a broadened Landau level is expected to consist of a continuous distribution of electronic states, not just a few well separated levels. Moreover, there is no *a priori* reason why the rate of electronic jumps between localised and extended states (i.e. across the mobility edge) should be lower than the rate of either localised–localised or extended–extended transitions.

In this Letter we present a numerical (Monte Carlo) study of generation–recombination noise in a broadened Landau level with localised states in the tails and extended states in the centre. At first sight, one would expect such a distribution of localised and extended states to give rise to a generation–recombination noise spectrum with a continuous distribution of relaxation times. Depending on the precise nature of this distribution one may obtain different kinds of 'coloured' noise. A typical example is a power-law spectrum $S(f) = c/f^{\alpha}$, with α of order 1. In contrast, the noise spectra reported in [3] consist of one or, occasionally, two Lorentzians.

The primary aim of the present simulations is to test whether the simple picture of a Landau level with localised states in the tails and extended states in the centre can account for the qualitative features of the Lorentzian noise spectra observed experimentally. If so, our calculations should provide us with an estimate of physically interesting parameters of the experimental systems, such as, for instance, the position of the mobility edge.

The experimental conditions under which the quantum Hall effect is commonly observed are such that the separation between successive Landau levels is much larger than kT. For this reason, we consider in our simulations only transitions of electrons within a single Landau level. Furthermore we assume that all electrons have the same spin (i.e., we assume that the Zeeman splitting of the Landau levels is large enough to lift the spin degeneracy of these levels). Both assumptions have been made for computational convenience, and neither is essential. We assume that the Landau level has a Gaussian shape. Such an assumption is reasonable, in view of the available experimental evidence [7, 8]. We consider a relatively small system containing 1000 electronic states. All states are assumed to be non-degenerate, and the spacing between successive energy levels has been chosen such that an approximately Gaussian distribution of the density of states is reproduced. It is assumed that the states located in an energy range around the centre of the Landau level and bounded by two sharp, symmetrically placed mobility edges, are extended, while all other states are localised. Calculations were carried out for several different choices of the energy separation of the mobility edges. The filling of the Landau levels (and thereby the location of the Fermi energy) was varied by changing the number of electrons in the system while keeping the total number of available states constant. Alternatively, the filling of the Landau levels could have been varied by changing the degeneracy of the Landau levels at constant number of electrons (as is usually done experimentally by varying the applied

magnetic field). However, the dependence of the noise intensity on the position of the Fermi level is expected to be qualitatively similar in both cases.

Once a particular filling factor of the Landau level has been selected, the electrons are distributed over the available levels according to a Fermi–Dirac distribution. To simulate fluctuations in the number of occupied extended states, transitions are attempted between randomly selected pairs of states. A constant number of such trial moves is attempted per unit of time. A trial move is accepted if two conditions are satisfied: (i) the final state obeys the exclusion principle, and (ii) the Boltzmann factor $\exp(-\Delta E/kT)$ is larger than R, where R is a random number between 0 and 1, and ΔE is the energy difference between final and initial states. This set of rules guarantees that the Fermi–Dirac distribution is maintained during the Monte Carlo simulation. Note that, according to the present set of rules the *a priori* probability for a trial move from a localised state to an extended state is equal to that of a transition from one extended state to another. Clearly, this assumption is the most 'random' that we can make, and it is obviously an over simplification. It represents something like a 'worst possible' situation to observe a Lorentzian noise spectrum. Therefore, if we can reproduce a Lorentzian fluctuation spectrum with these transition probabilities, then we have demonstrated the main point of this Letter, namely that generation–recombination noise with a well defined relaxation time can take place even in a system where the energy levels are continuously distributed and not separated by a gap in energy or transition rate.

In our simulations we compute the fluctuation in the total number of extended states as a function of time. Only transitions in which one localised and one extended state are involved can change the number of occupied extended states. In practice we carry out a fixed number of trial moves per unit of time. The number of occupied extended states is sampled every time unit. The fluctuating number of electrons in extended states is converted to a function of frequency using fast Fourier transformation. The corresponding power spectrum of number fluctuations is obtained by taking the mean-square value of these Fourier components. Of course, in order to relate the 'frequency' in the simulation to the frequency scale of fluctuations in an experiment we have to know the actual value for the average transition matrix element for a typical scattering event in which an electron jumps from one energy state to another within the same Landau level. Conversely, by comparing simulated and experimental spectra we can arrive at an estimate for these matrix elements, that are in general not known.

Note that in the Monte Carlo simulations we are calculating the fluctuation spectrum of the *number* of extended electrons. The latter quantity is not necessarily proportional to the power spectrum of fluctuations in the Hall voltage. Hence, we make the additional assumption that all extended states contribute equally to the current. Once again, this is in a sense the most random and therefore the least favourable assumption that we can make. It should be stressed that this assumption is not in contradiction with the theoretical picture that explains how, in the quantum Hall regime at 0 K, the increase in the electric current carried by extended states exactly balances the loss of conductivity due to the presence of localised states [9]. In contrast, we are at present considering a finite-temperature fluctuation in the number of mobile *electrons*, while the number of extended *states* is constant.

We computed the noise spectra of the fluctuations in the number of mobile electrons as a function of the location of the Fermi level ε_F. For convenience, the centre of the Landau level was chosen as energy zero. In addition, we varied the position of the mobility edges, $\pm\varepsilon_m$, and the temperature T. As our unit of energy we used the width of the Landau level. Both kT and ε_m, as well as ε_F, were expressed in this unit. Simulations

were carried out for $0.15 \leqslant \varepsilon_m \leqslant 2.4$ and $0.15 \leqslant kT \leqslant 2.0$. These values of kT correspond to typical temperatures in quantum Hall experiments ($T < 10$ K), if we assume that the width of the Landau level is of the order of 1 meV, as suggested on both theoretical [10] and experimental grounds [8]. In contrast, the estimates of ε_m (or more precisely, of the fraction of localised states) in the literature vary appreciably [11, 12].

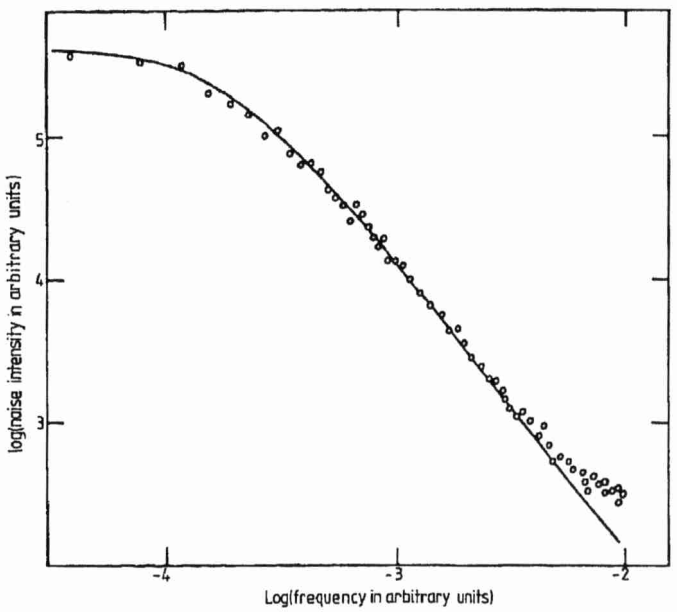

Figure 1. A typical example of a simulated spectrum of the fluctuating number of extended electrons in one Landau level. The open circles are the numerical data, the full curve is a Lorentzian fit to the data. The temperature $kT = 1.0$, the distance of the mobility edge from the centre of the Landau level $\varepsilon_m = 1.0$ and the Fermi energy $\varepsilon_F = 2.4$ (also with respect to the band centre). All energies are expressed in units of the width of the Landau level.

Figure 1 shows a typical example of a simulated noise spectrum. In general, we found that all simulated spectra could be fitted to one Lorentzian: $S(\omega) = S(0)/(1 + \omega^2\tau^2)$ with $\omega = 2\pi f$, $S(0)$ the spectral noise intensity at zero frequency and τ a relaxation time. We investigated the dependence of the two parameters $S(0)$ and τ on the location of the Fermi level ε_F, for various choices of the temperature T and the position of the mobility edge ε_m. In figure 2, $S(0)/\tau$ is plotted versus ε_F for three different values of ε_m/kT. For $\varepsilon_m/kT > 1$, the broad structure at the top of figure 2 gradually develops into two peaks which become more pronounced as the ratio ε_m/kT is increased. In fact, for $\varepsilon_m/kT \gg 1$, the peaks sharpen and the dip between them approaches zero. The width of the peaks, where present, is found to be equal to approximately $2kT$. The peak positions vary from ε_m at low temperatures to approximately $2\varepsilon_m$ at high temperatures. In fact, we observed similar behaviour for many different choices of ε_m and kT. The shape of the spectrum appeared to depend exclusively on the ratio ε_m/kT. However, the absolute intensity of the spectrum does depend on T and ε_m. $S(0)$ decreases as ε_m and T are increased simultaneously, such that ε_m/kT is constant.

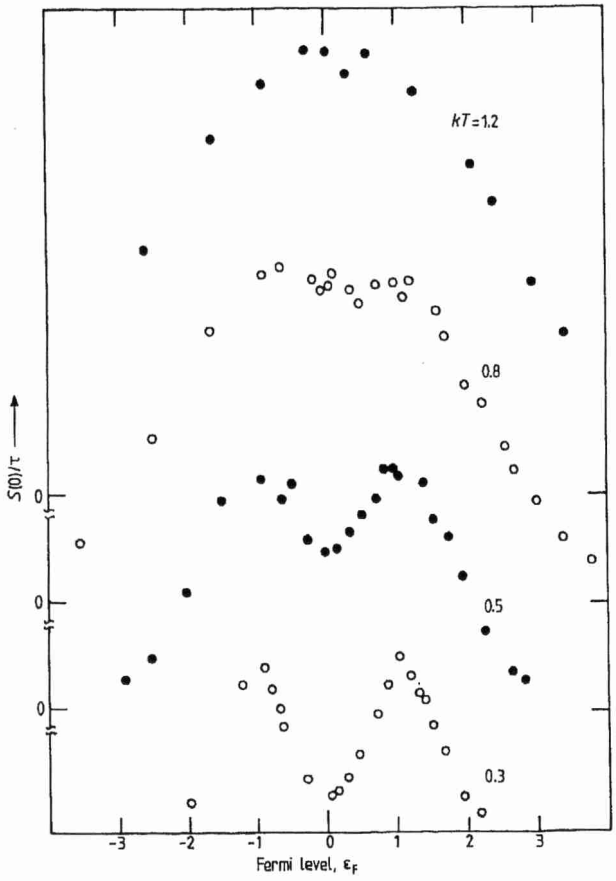

Figure 2. $S(0)/\tau$, obtained from the simulated Lorentzian noise spectra, plotted versus the position of the Fermi level for four different temperatures. For all simulations $\varepsilon_m = 1.0$. Units as in figure 1.

Figure 3 shows an example of the dependence of $S(0)/\tau$ and τ on B (and thereby on ε_F) as observed experimentally [3]. Clearly, the intensity modulation shown in figure 3 strongly resembles the simulation results shown in the lowest set of data points in figure 2. Apparently, the very simple model that we are considering in this Letter can account both for the fact that a Lorentzian spectrum is observed experimentally and for the dependence of the spectral intensity on the location of the Fermi level. In the latter case, however, a direct comparison is complicated by the fact that the experimental data are given as a function of B, rather than of ε_F. Moreover, the experimental data are centred around the middle of the mobility gap between two Landau levels, whereas the simulation results are centred around the middle of a Landau level.

Qualitatively we can understand the dependence of the noise spectrum on the position of the Fermi level as follows. The noise will be maximal if the Fermi level ε_F is close to either mobility edge, because then transitions between localised and extended states are most probable. A minimum in the noise intensity will occur if kT is too small

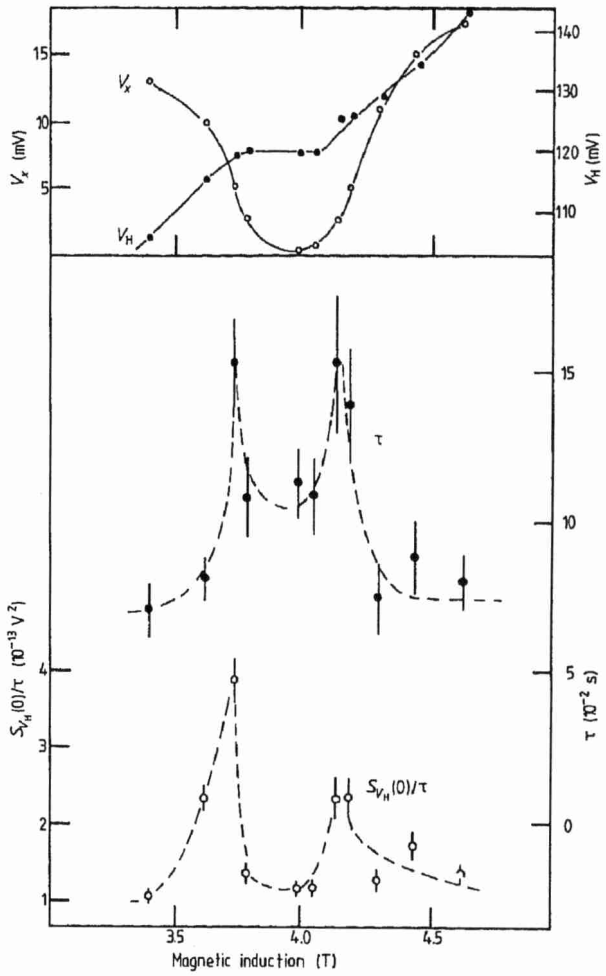

Figure 3. $S(0)/\tau$ and τ of the experimentally observed Lorentzian in the spectral noise intensity of the Hall voltage of a GaAs/AlGaAs heterostructure at 4.2 K. To indicate the position of the Hall plateau and DC Hall voltage v_H and the (longitudinal) Shubnikov–de Haas voltage v_{SdH} are also plotted. The broken curves are a guide to the eye.

to promote a particle from a localised to an extended state or vice versa. This can happen if ε_F lies either far in the tails of the Landau level or half way between the mobility edges. The latter minimum can obviously not be resolved if $kT > \varepsilon_m$, since then transitions across both mobility edges contribute substantially to the total noise intensity. Such behaviour is indeed observed in the Monte Carlo simulations where the two peaks start to overlap for $\varepsilon_m/kT < 1$.

The dependence of the characteristic time τ on ε_F is similar to that of $S(0)$, but less pronounced. At large values of ε_m/kT, τ peaks at approximately the same values of the Fermi energy as $S(0)$. A tentative explanation for this behaviour is discussed in [3]. For small values of ε_m/kT, τ depends very little, if at all, on ε_F.

It should be noted that our Monte Carlo simulations yield a noise spectrum consisting of a single Lorentzian. However, the standard theory of generation–recombination noise [13] would predict the number fluctuations involving three reservoirs (i.e. the extended states,the low-energy localised states and the high-energy localised states) should yield two distinct Lorentzians. One possible reason why we observe only one Lorentzian is the following. Calculations on an equivalent three-reservoir system indicate that the width of both Lorentzians is nearly the same, unless the intensity of the Lorentzian is much larger than the other. In either case one should expect that the two Lorentzian are hard to resolve in a numerical simulation, due to the limited statistical accuracy.

In this Letter we have shown that transitions between localised and extended states in a Landau level may give rise to generation–recombination noise characterised by a single Lorentzian. At first sight it seems surprising that one single characteristic relaxation time dominates the decay of number fluctuations. The probable explanation is that, due to the sharpness of the Fermi–Dirac distribution at low temperatures, only electrons from a narrow range of energy levels can contribute effectively to the generation–recombination process. We note that the relation between the spectral intensity $S(0)$ and the location of the Fermi energy is determined largely by the ratio ε_m/kT. Comparison of the experimental data presented in figure 3 and the simulation results in figure 2 suggest that in the GaAs/AlGaAs system studied in [3], the energy separation between the upper and lower mobility edges in the Landau level studied in that experiment is about 2 meV. Note however that the present simulation results do not allow us to arrive at a better estimate of the width of the Landau level itself, and thereby of the fraction of extended electronic states.

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