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WEAK INTERACTIONS AND HIGGS MECHANISMS

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1. Introduction

Quantum field theory,as a logical extension of quantum mechanics was first presented by Heisenberg and Pauli[1] in 1929. The history of the subject is long and troubled, but again and again field theory in one way or another has proven to be of relevance in the description of elementary particle physics. The main technical limit nowadays is that the only way to compute things is by means of perturbation theory ; but this already allows very extensive application to weak and e.m. interactions. Slowly, other than perturbative methods are coming up : asymptotic freedom, solitons are present day symptoms of this. Hopefully this will lead eventually to a better understanding of mythical objects such as quarks and bags or strong interactions in general.

On the other hand, the recent developments in super symmetry[2] may lead to further insight in the basic theory of this world : perhaps it is possible to integrate gravitation into the existing scheme . Here again perturbation theory may prove relevant.

In these notes we will aim at a discussion of the Higgs sector of renormalizable models of weak and e.m. interactions. Our point of view is that it is now a matter of time before neutral currents , charm, charm spectroscopy are also experimentally established in a detailed manner. The next question, inevitably, is to the experimental investigation of the Higgs system, supposedly responsible for the masses of vector bosons, leptons, and indeed, quarks. It is the aim of these lectures to focus attention on this system, as has been done in the literature already by Ellis, Gaillard and Nanopoulos[3]. Our main attitude in this is somewhat ambivalent ; we do not really believe the Higgs mechanism as formulated in renormalizable gauge theories, but on the other hand use such theories to infer information on the Higgs system. Briefly, the idea is that the usual Higgs system in a perturbation theory context is a highly simplified representant of a much more complex system. And in this sense study of the theoretical models may lead us to the formulation of good experimental questions.

As a form of presentation we follow more or less the lines of development shown in the history of the subject. In this way it is possible to identify the various ingredients in the theory, and to emphasize the ideas that have governed the construction. Since the theory has met with considerable success we deduce from that support for these ideas. But we must be careful in understanding which ideas are really supported by the data.

2. Photon degrees of freedom

Wigner's work on the representations of the Lorentz group[4] has shown that we must distinguish massive and massless particles. The number of degrees of freedom for massless particles of spin 1/2 and higher is always two ; left and right handed neutrinos, two states of polarization for photons (spin 1) or gravitons (spin 2). For massive particles the number of degrees of freedom is $2j+1$, where j is the spin of the particle in question.

However, in the Lagrangian, or the equations of motion for the fields it is not so obvious what happens. Basically, the difference between the massive and massless cases is that in the massless case a symmetry appears to be valid, and this symmetry has as a consequence that one degree of freedom becomes unobservable.

In its most elementary form this may be seen for the photon. The Lagrangian for a photon with mass M is :

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \frac{1}{2} M^2 A_\mu A_\mu \qquad (2.1)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad .$$

The equations of motion resulting from this Lagrangian are :

$$\partial_\beta(\partial_\beta A_\alpha - \partial_\alpha A_\beta) - M^2 A_\alpha = 0 \ . \qquad (2.2)$$

Applying ∂_α to this we obtain : $\quad -M^2 \partial_\alpha A_\alpha = 0$

which is therefore a consequence of the equations of motion. Because of this the four component vector field has only three degrees of freedom. If, in classical theory, we take for A_α a plane wave with polarization vector e_α :

$$A_\alpha = e_\alpha e^{ikx} \quad , \quad kx = \vec{k}\vec{x} + k_4 x_4 = \vec{k}\vec{x} - k_o x_o$$

then the equation of motion is

$$-k^2 e_\alpha e^{ikx} + k_\alpha (k,e) e^{ikx} - M^2 e_\alpha e^{ikx} = 0$$

where $(k,e) = k_\beta e_\beta$ is the scalar product of k and e. Multiplication with k_α gives:

$$-k^2 (k,e)\, e^{ikx} + k^2 (k,2)\, e^{ikx} - M^2 (k,e)\, e^{ikx} = 0$$

or

$$(k,e)\, e^{ikx} = 0 \rightarrow (k,e) = 0 \ .$$

Thus the scalar product $(k,e) = k_\alpha e_\alpha$ must be zero. In the center of mass where k takes the form $(0,0,0,ik_o)$ we have as allowed polarization vectors

$$e^1 = (1,0,0,0) \ , \quad e^2 = (0,1,0,0) \ \text{and} \ e^3 = (0,0,1,0)$$

indeed three degrees of freedom. Using $(e,k) = 0$, the equation of motion reduce

$$-(k^2 + M^2)\, e_\alpha e^{ikx} = 0$$

or

$$k^2 + M^2 = 0 \ .$$

Only solutions with $k_o = \sqrt{\vec{k}^2 + M^2}$ are allowed : we are dealing with plane waves representing particles with mass M.

We can introduce a source term, a current, in the Lagrangian representing s(system that emits waves. If we add to $\mathscr{L}$ a term $-A_\mu j_\mu$

$$\mathscr{L} \rightarrow \mathscr{L} - A_\mu j_\mu$$

then the equations of motion (2.2) become

$$\partial_\beta(\partial_\beta A_\alpha - \partial_\alpha A_\beta) - M^2 A_\alpha = j_\alpha \tag{2.3}$$

Remember that $\partial_\beta = \partial/\partial x_\beta$ is differentiation with respect to x. This equation ca\ be solved

$$A_\alpha(x) = -\frac{1}{(2\pi)^4} \int d_4 y \int d_4 k \; e^{ik(x-y)} \frac{\delta_{\alpha\mu} + k_\alpha k_\mu / M^2}{k^2 + M^2} \, j_\mu(y) \tag{2.4}$$

Indeed, introducing this in the left hand side of the equation gives

$$-\frac{1}{(2\pi)^4} \int d_4 y \int d_4 k \; \frac{e^{ik(x-y)}}{k^2 + M^2} \left\{ (-k^2 - M^2)(\delta_{\alpha\mu} + k_\alpha k_\mu / M^2) \right.$$

$$\left. + k_\alpha k_\beta \, (\delta_{\beta\mu} + k_\beta k_\mu / M^2) \right\} j_\mu(y)$$

$$= -\frac{1}{(2\pi)^4}\int d_4y\int d_4k\ \frac{e^{ik(x-y)}}{k^2+M^2}\left\{-(k^2+M^2)\ \delta_{\alpha\mu}\right\}j_\mu(y)$$

$$= \frac{1}{(2\pi)^4}\int d_4y\int d_4k\ \ e^{ik(x-y)}\ j_\alpha(y) \quad .$$

Now

$$\delta_4(x-y) = \frac{1}{(2\pi)^4}\int d_4k\ e^{ik(x-y)}$$

and we get

$$\int d_4y\ \ \delta_4(x-y)\ j_\alpha(y) = j_\alpha(x) \quad .$$

The quantity

$$\frac{\delta_{\alpha\mu} + k_\alpha k_\mu/M^2}{k^2+M^2} \tag{2.5}$$

governs the propagation of the field A_μ generated by the source current j_α .

Physical particles are created or absorbed by sources satisfying a divergence equation

$$\partial_\alpha j_\alpha = 0 \quad . \tag{2.6}$$

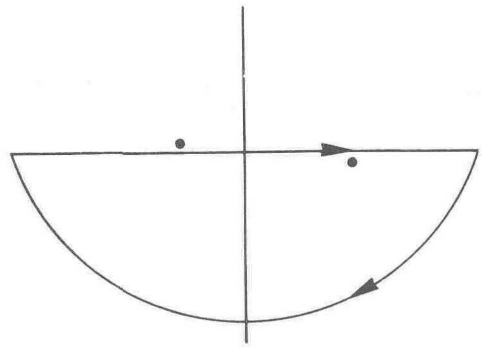
To see this suppose that $j_\mu(y)$ is different from zero only if $y = 0$ (creation or absorption of a particle at the origin, $\vec{y} = 0$, and time $y_o = 0$).For large distances A_α is non zero only if $k^2 + M^2 \sim 0$, that is the pole part of the above solution. A discussion of this requires defining precisely the zero of the denominator. We write $k^2 + M^2 - i\varepsilon$, that is we are at the pole if $k^2 + M^2$ is very small positive imaginary. Other choices are possible and correspond to other situations. Now the denominator has a zero if

$$k^2 + M^2 = i\varepsilon \quad \text{or} \quad k_o = \pm\sqrt{\vec{k}^2 + M^2 - i\varepsilon} = \pm\sqrt{\vec{k}^2 + M^2} \mp i\varepsilon$$

Consider

$$\int e^{ikx}\ \frac{d_4k}{k^2+M^2-i\varepsilon} = \int d_3k\ e^{i\vec{k}\vec{x}}\ \ dk_o\ \frac{e^{-ik_ox_o}}{(k_o-\sqrt{\ })(k_o+\sqrt{\ })}$$

with $x_o > 0$, thus at a time after the source was active. The integral over k_o can be done ; in the complex k_o plane we have two poles, and for $k_o \to -i\infty$ the exponential becomes very small because $x_o > 0$. Completing the contour of integration with a big semi-circle in the lower half plane (which gives no contribution due to the exponential as argued above) we have that the integral is equal to $2\pi i$ times the residue of the role at $k_o = \sqrt{\vec{k}^2 + M^2} - i\varepsilon$. The result is :



$$\int e^{ikx} \frac{d_4 k}{k^2 + M^2 - i\varepsilon} = \int d_3 k\, e^{i\vec{k}\vec{x}}.\, 2\pi i.\, \frac{e^{-ik_o x_o}}{2k_o}$$

with now $k_o = +\sqrt{\vec{k}^2 + M^2}$. This shows that due to the pole we get plane wave solutions for positive times. If there is no pole there is no long distance solution for positive times ; for instance, trivially

$$\int d_4 k\, e^{ikx} = (2\pi)^4\, \delta_4(x)$$

which is non zero only at the origin at time 0. All this is to show that M represents indeed a mass, and that sources indeed emit waves (particles) with $k^2 = -M^2$.

Take now a source for which $\partial_\alpha j_\alpha \neq 0$, i.e. $k_\alpha j_\alpha \neq 0$. Now the four vector j_α can always be decomposed in a four vector along k and a four vector perpendicular to k. Thus :

$$j_\alpha = k_\alpha\, j^a + j_\alpha^b$$

where $k_\alpha j_\alpha^b = 0$. Let us see the consequences of a source of the type j^a. The solution for the field becomes :

$$\int d_4 y \int d_4 k\; e^{ik(x-y)}\; \frac{\delta_{\alpha\beta} + k_\alpha k_\beta / M^2}{k^2 + M^2}\; k_\beta j^a$$

$$= \int d_4 y \int d_4 k\; e^{ik(x-y)}\; \frac{k_\alpha (1 + k^2/M^2)}{k^2 + M^2}\; j^a$$

$$= \frac{1}{M^2} \int d_4 y \int d_4 k\; e^{ik(x-y)}\; k_\alpha j^a$$

The pole has disappeared, and in fact

$$\int d_4k \quad e^{ik(x-y)} \, k_\alpha \;=\; \frac{1}{i}\frac{\partial}{\partial x_\alpha}\int d_4k \; e^{ik(x-y)}$$

$$= \frac{(2\pi)^4}{i} \; \frac{\partial}{\partial x_\alpha} \, \delta_4(x-y)$$

showing that no long range fields are created by this current.

In conclusion, consider a massive photon with four-momentum k, and $k^2 + M^2 = 0$. There are three degrees of freedom. They are created by sources satisfying $k_\alpha j_\alpha = 0$, and in the system where $k = (0,0,0,iM)$ we may take as independent sources quantities proportional to (1,0,0,0), (0,1,0,0) or (0,0,1,0).

Consider now the massless case. The Lagrangian and equations of motion in the presence of a source are

$$\mathcal{L} \;=\; -\frac{1}{4} F_{\mu\nu}F_{\mu\nu} - j_\alpha A_\alpha$$

$$\partial_\beta(\partial_\beta A_\alpha - \partial_\alpha A_\beta) \;=\; j_\alpha \qquad (2.7)$$

We now must take $\partial_\alpha j_\alpha = 0$, because else we get into straight contradictions ; applying ∂_α to the equation of motion leads to

$$0 \;=\; \partial_\alpha j_\alpha$$

This has as consequence that the Lagrangian is invariant under gauge transformations (apart from a term that is a total divergence of no relevance). Let us substitute

$$A_\mu \rightarrow A_\mu - \partial_\mu \Lambda \qquad (2.8)$$

where Λ is an arbitrary function of space-time. Then, starting from (2.7) :

$$\mathcal{L} \;\rightarrow\; \mathcal{L} + j_\alpha\partial_\alpha\Lambda \;=\; \mathcal{L} + \partial_\alpha(j_\alpha\cdot\Lambda) - \partial_\alpha j_\alpha\cdot\Lambda$$

$$= \mathcal{L} + \partial_\alpha(j_\alpha\Lambda)$$

Such total divergencies do not contribute to eqs. of motion or otherwise. Therefore we can conclude that physics is invariant to the addition of $\partial_\alpha\Lambda$ to the field

A_α . Now the Fourier transform of $\partial_\alpha \Lambda$ has the form

$$k_\alpha \int d^4x \; e^{ikx} \; \Lambda(x) \tag{2.9}$$

and the conclusion is that pieces in A_α with a polarization vector proportional to k_α are of no physical consequence.

Consider now the solutions of the eqs. of motion

$$A_\alpha = -\frac{1}{(2\pi)^4} \int d_4y \int d_4k \; e^{ik(x-y)} \; \frac{\delta_{\alpha\beta}}{k^2 - i\varepsilon} \; j_\beta \tag{2.10}$$

That this is a solution requires the validity of $k_\beta j_\beta = 0$. Let us for a given k see what source we can have. Suppose $\vec{k}$ is along the positive third axis :

$$k = (0,0,K,ik_o) \quad , \quad K > 0$$

The requirement $k_\alpha j_\alpha = 0$ leaves as possible sources expressions proportional to

$$(1,0,0,0)$$
$$(0,1,0,0)$$
$$(0,0,k_o,iK)$$

and $(0,0,k_o,-iK)$ is excluded. This still suggests three degrees of freedom. However, if the photon is on mass shell the last source becomes proportional to k itself, because $k^2 = 0$ implies $K = k_o$. And from (2.10) we see that we then get a field A_α of the form (2.9), i.e. of the form $\partial_\alpha \Lambda$ that is physically undetectable. And we are thus left with two physically relevant degrees of freedom.

It is clear from the above that there is something very special about the mass-less case. The situation with $M = 0$ differs radically from the case $M \neq 0$, no matter how small this mass M. This goes back to a simple fact in the theory of relativity : for a particle with finite mass, no matter how small, there is a rest frame where its momentum takes the form $(0,0,0,iM)$, independent of the magnitude of M. But no such thing holds for a particle of zero mass.

It is clear that gauge invariance is very essential for the photon, and is needed for very basic reasons. Mass zero particles of spin 1 or higher give nonsensical theories unless a certain gauge invariance holds. This is true also for gravitation (spin 2) where indeed such an invariance exists in Einstein's theory.

3. Global and local invariance

The Lagrangian for free electrons of mass m is :

$$\mathscr{L} = - \bar{\psi}(\gamma^{\mu}\partial_{\mu} + m)\psi$$

from which the Dirac equation follows. By inspection we note the invariance under multiplication of ψ by a phase factor :

$$\psi \rightarrow e^{-iC} \psi$$
$$\bar{\psi} \rightarrow \bar{\psi} e^{iC}$$

with C a constant. This is called a global invariance, or gauge invariance of the first kind as introduced by Pauli[5].

If C depends on space time then the invariance breaks down. Let us take a function Λ instead of C. We have

$$\mathscr{L} \rightarrow - \bar{\psi} e^{i\Lambda} (\gamma^{\mu}\partial_{\mu} + m) e^{-i\Lambda} \psi$$

$$= - \bar{\psi} (\gamma^{\mu}\partial_{\mu} + m) \psi + i\partial_{\mu}\Lambda (\bar{\psi}\gamma^{\mu}\psi)$$

Thus $\mathscr{L}$ changes by an amount $\partial_{\mu}\Lambda(\bar{\psi}\gamma^{\mu}\psi)$. We now enlarge $\mathscr{L}$ with new terms, also not invariant but such that the whole is invariant. The new term is

$$+ ig A_{\mu}(\bar{\psi}\gamma^{\mu}\psi)$$

and we transform A_{μ} as follows :

$$A_{\mu} \rightarrow A_{\mu} - \frac{1}{g} \partial_{\mu}\Lambda$$

where g is an arbitrary constant (the electric charge). The transformation law for A_{μ} is precisely the gauge transformation of the previous section, and we may add the e.m. Lagrangian that is invariant by itself. The new Lagrangian becomes

$$\mathscr{L} = - \frac{1}{4} F_{\mu\nu}F_{\mu\nu} - \bar{\psi}(\gamma^{\mu}D_{\mu} + m)\psi$$

with

$$D_{\mu} = \partial_{\mu} - ig A_{\mu}$$

The global invariance has been enlarged to a local invariance (space-time dependent Λ) at the expense of introducing the vector field A_μ . Pauli called this gauge invariance of the second kind.

What about degrees of freedom ? Due to the fact that gauge invariance holds any part of the form $\partial_\alpha \Lambda$ in A_α can still be transformed away and is thus unobservable even if electrons are present.

The above Lagrangian defines the theory of quantum-electrodynamics. It is renormalizable, and this may be related to the absence of a term $k_\alpha k_\beta$ in the photon propagator (compare 2.4 and 2.10). And that is related to the gauge invariance of the theory.

4. Non-abelian global and local invariance.

In section 3 we have seen how global invariance can be enlarged to local invariance at the expense of introducing vector fields. Can this trick be generalized to more complex cases ? The essential step was taken by Yang and Mills[6], in 1954, on the example of isospin invariance.

Consider the Lagrangian for free protons and neutrons with rigourously identical masses. It is

$$\mathscr{L} = - \bar{P}(\gamma^\mu \partial_\mu + m)\, P - \bar{N}(\gamma^\mu \partial_\mu + m)\, N$$

$$= - \bar{N}(\gamma^\mu \partial_\mu + m)\, N$$

with N a two component quantity

$$N = \begin{pmatrix} P \\ N \end{pmatrix} \quad , \quad \bar{N} = \begin{pmatrix} \bar{P} \\ \bar{N} \end{pmatrix}$$

This Lagrangian is invariant for a rotation of the proton and neutron into each other

$$N \rightarrow e^{-iC^a \frac{\tau^a}{2}} N$$

There are three real coefficients C, and the usual three Pauli spin matrices. It follows

$$\bar{N} \rightarrow \bar{N}\, e^{iC^a \frac{\tau^a}{2}}$$

because $\tau = \tau^+$, that is $\tau^* = \tilde{\tau}$ (complex conjugation = reflection). It is actually not necessary to work to all orders in the coefficients C ; from now on we will consider only small coefficients C and neglect terms of order C^2. The transformation laws become to first order in C :

$$N \rightarrow N - iC^a \frac{\tau^a}{2} N \tag{4.1}$$

from which it follows :

$$\bar{N} \rightarrow \bar{N} + \bar{N}.\ iC^a \frac{\tau^a}{2}$$

Can this global invariance be enlarged to a local invariance ? Take now for the C^a space time dependent functions Λ^a. We get :

$$\mathscr{L} \rightarrow \mathscr{L} + i\partial_\mu \Lambda^a \ (\bar{N}\gamma^\mu \frac{\tau^a}{2} N) \quad .$$

At first sight it seems that this can be compensated if we introduce an extra term

$$i\ g\ W^b_\mu (\bar{N}\gamma^\mu \frac{\tau^b}{2} N)$$

and assign the transformation property

$$W^b_\mu \rightarrow W^b_\mu - \frac{1}{g}\ \partial_\mu \Lambda^b$$

However, the problem is that the newly introduced term itself is then not invariant. It changes by the amount :

$$\begin{aligned} (\bar{N}\gamma^\mu \frac{\tau^b}{2} N) \rightarrow\ & (\bar{N}\gamma^\mu \frac{\tau^b}{2} N) \\ & + i\Lambda^a \{\bar{N}\gamma^\mu (\frac{\tau^a}{2}\frac{\tau^b}{2} - \frac{\tau^b}{2}\frac{\tau^a}{2}) N\} \\ = \ & (\bar{N}\gamma^\mu \frac{\tau^b}{2} N) - \varepsilon_{abc}\Lambda^a (\bar{N}\gamma^\mu \frac{\tau^c}{2} N) \end{aligned}$$

where we used

$$[\frac{\tau^a}{2}, \frac{\tau^b}{2}] = i\varepsilon_{abc} \frac{\tau^c}{2} \tag{4.2}$$

The required invariance can be obtained if we complicate the transformation property of W^a_μ :

$$W^a_\mu \rightarrow W^a_\mu + \varepsilon_{abc}\ \Lambda^b\ W^c_\mu - \frac{1}{g}\partial_\mu \Lambda^a \tag{4.3}$$

This finishes the nucleon part. But now the W-part. The expression

$$F^a_{\mu\nu}\ F^a_{\mu\nu} \ , \quad F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu$$

is no more invariant due to the new term in the transformation law. But it can be made invariant if we change $F_{\mu\nu}$. The final invariant Lagrangian is :

$$\mathcal{L} = -\frac{1}{4} g^a_{\mu\nu} g^a_{\mu\nu} - \bar{N}(\gamma^\mu D_\mu + m) N \qquad (4.4)$$

$$g^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\, \varepsilon_{abc} W^b_\mu W^c_\nu \qquad (4.4)$$

$$D_\mu = \partial_\mu - ig W^a_\mu \frac{\tau^a}{2}$$

The above Lagrangian differs from q.e.d. notably in the following respects :

i. there are three mass-less vector fields

ii. The fields interact with each other due to the terms trilinear and quadrilinear in the W-fields in the piece $g^a_{\mu\nu}\, g^a_{\mu\nu}$

iii. the gauge invariance is non-abelian, that is the result of two successive gauge transformations depends on the order in which the transformations are applied.

Since only one massless vector particle, the photon, is known it is not immediately clear that this Yang-Mills theory is relevant for the description of nature. Therefore not much attention was devoted to this theory. In fact, it took almost 10 years before Feynman and De Witt established Feynman rules for this theory. Renormalizability seemed to be allright, from these rules. However, the simultaneous occurrence of three massless particles introduces new difficulties, the infrared divergences. At this moment these infrared problems are being studied intensively by several groups of theorists.

5. Development of models for weak interactions.

The application of Yang-Mills type theories to weak interactions was advocated by Schwinger[7] even before the V-A theory was well established. The first major paper in this context was due to Bludman[8], who build a Yang-Mills theory for weak interactions. In fact, he took the Lagrangian (4.4) of the previous section, however replacing the nucleon doublet by a left handed lepton doublet :

$$N \to L_+ = \frac{(1+\gamma^5)}{2} \binom{\nu}{e}$$

There are some minor additional complications that we will not discuss here. The main point is that a local SU_2 symmetry was proposed for weak interactions. Bludman did not think through how to incorporate electromagnetism ; in fact he did not have much confidence in the existence of vector bosons and suggested that the theory should be understood in that currents are always as if they originated from a Yang-Mills scheme. This great idea, to abstract the properties of the currents from the theory, has been taken over and expanded by Gell-Mann to his well known algebra of currents.

After Bludman people tried to include e.m. First there were some attempts to identity the neutral W with the photon. This goes wrong because the coupling of W_μ^3 is ;

$$ig\, W_\mu^3\, (\bar{\ell}\, \gamma^\mu \frac{1+\gamma^5}{2} \frac{\tau^3}{2}\, \ell)$$

with

$$\ell = \binom{\nu}{e}$$

Now

$$\tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and W_μ^3 thus couples also to the neutrino, contrary to the fact that the neutrino is electrically neutral and not coupled to the photon. Further, the coupling to the electron is parity violating, which is also contrary to experiment.

The correct way out was indicated by Glashow[9] who added simply another vector boson and another symmetry. The new vector boson W_μ^o had couplings with the left handed neutrino and electron and with the right handed electron :

$$g\, g_1\, W_\mu^o\, (\bar{\ell}\, \gamma^\mu \frac{1+\gamma^5}{2}\, \ell)$$

$$+ \quad g\, g_2\, W_\mu^o\, (\bar{e}\, \gamma^\mu \frac{1-\gamma^5}{2}\, e)$$

with arbitrary g_1 and g_2. In this scheme the photon field is a linear combination of W_μ^3 and W_μ^o :

$$A_\mu = \sin\theta\, W_\mu^3 + \cos\theta\, W_\mu^o$$

For a given choice of θ the coupling constants g_1 and g_2 can be adjusted so that the photon decouples from the neutrino while furthermore its interaction with the electron is parity conserving. The electric charge turns then out to be related to g, the coupling constant of weak interactions ;

$$e = g \sin\theta$$

The free parameters are now

$$M_c,\ M_o,\ \sin\theta$$

where M_c and M_o are the charged and neutral vector masses. The relation (from μ-decay) :

$$\frac{g^2}{M_c^2} = 4\sqrt{2}\ .\ \frac{1.02\times 10^{-5}}{m_p^2}$$

with m_p = proton mass = 938 MeV may be rewritten in the from :

$$\left(\frac{M}{m_p}\right)^2 = \frac{\pi\alpha . 10^5}{1.02\ \sqrt{2}\ \sin^2\theta} = \frac{1589}{\sin^2\theta}$$

where $\alpha = e^2/4\pi = 1/137$. Experiments indicate $\sin^2\theta \sim 0.32$, and this leads to $M_c \sim 66$ GeV. Furthermore, also from neutral current data one finds

$$M_o^2 \sim \frac{M_c^2}{\cos^2\theta} = 80\ \text{GeV}$$

The extension of this model to include hadrons is best made in the quark model and leads to the Glashow-Iliopoulos-Maiani[10] structure including charm. But we will turn now our attention to other things and just assume that all this agrees well with experiment.

6. Almost renormalizable models.

In the model developed by Bludman, Glashow, Iliopoulos and Maiani the vector boson masses are simply put in "by hand". Such mass terms break the local gauge invariance, but the idea is that symmetry breaking shall be done through mass terms only. The important question is now : is such a theory renormalizable ? The answer is "almost", and we will specify what we mean by this.

In perturbation theory infinities appear due to integrals that do not converge for high energies. Crudely cutting off the integrations at some energy makes the theory finite, but as this cut-off point is made higher and higher then higher order effects become larger and larger.

One can now take the following point of view. A non-renormalizable theory is a theory that needs additional physics in order to become a correct theory, and this new physics sets in at high energies and modifies the theory such that integrals become convergent. We will call a theory "almost renormalizable" if the energy where modifications are needed is well beyond present experimental energies. In the case of the model mentioned this energy is somewhere in the range 150-1000 GeV. That is, somewhere in this range the models cannot correspond precisely with nature else one would have observed the radiative corrections.

How should the theory look like at very, very high energies ? Well, above the point mentioned above the theory must look like a renormalizable theory, that is there should be no convergence problems. And we must make an inventory of the existing renormalizable theories and devise experiments to investigate what alternative has possibly been chosen by nature.

The idea that the theory at low energies should not exactely look like a renormalizable theory is in itself not very shocking. For instance, the electromagnetic interactions at low energies between atoms (the van der Waals forces) look very different from the simple forces assumed in quantum electrodynamics. At high energies all bound state systems dissolve in their components, and what was complicated becomes much simpler.

The fact that the above mentioned models, with symmetry breaking only through masses, are "almost" renormalizable is due to the circumstance that the lowest order radiative corrections (the one loop corrections) contain at most logarithmic divergencies in observable quantities. And these logarithms are then accompanied by factors m/M, where m and M are lepton (i.e. electron or muon) and vector boson masses. Only at the two loop level, where another factor $\alpha = 1/137$ is present, are the effects more significant.

The analysis of the infinities of Yang-Mills theories was started[11] in 1968 in response to the success of Gell-Mann's current algebra (such as the Adler-Weisberger relation). First the case with hand-inserted masses was investigated with the above mentioned result, Only after that the differences with the massless theory were understood[12] further progress was possible and resulted finally in the proof that mass generation through spontaneous breakdown leads to a renormalizable theory[13]. This mechanism, the so-called Higgs mechanism[14] had been proposed already for Glashow's model by Weinberg, and the resulting model is now called the Weinberg model[15]. It is special in the sense that the most simple Higg's mechanism is chosen, which leads to the mass relation $M_o^2 = M_c^2/\cos^2\theta$. More involved

Higgs structures are possible[16], and then the neutral vector boson mass becomes essentially a free parameter. Experiment is rather close to the value quoted for the simplest Higgs mechanism as used originally by Weinberg.

7. Renormalizable models.

A distinction must be made between two classes of models, namely

i. Massless Yang-Mills theories

ii. Yang-Mills theories with spontaneous symmetry breakdown.

Both classes differ at the high and low energy sides. The second class may be made to look at the low energy side precisely like the "almost" renormalizable models of the previous section that we take to agree well with experiment. It must be realized that also the first class models can look very different from what one would infer at first sight if bound states are involved. In this we simply wish to take a neutral attitude. After all, also in the second class models there are new things needed, namely the Higgs particles.

As all these things have their parallel already in quantum electrodynamics we will consider that case.

Class i is simply the usual quantum-electrodynamics, with the Lagrangian given before

$$\begin{aligned} \mathscr{L} &= -\frac{1}{4} F_{\mu\nu}F_{\mu\nu} - \bar{\psi}(\gamma^{\mu} D_{\mu} + m)\psi \\ F_{\mu\nu} &= \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} \end{aligned} \tag{7.1}$$

In here ψ is the electron field. This theory is renormalizable, there are some infrared problems that can be dealt with. Bound states such as positronium occur, but do not complicate the structure of the theory.

The theory of class ii corresponding to this is described by :

$$\begin{aligned} \mathscr{L} = &-\frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \bar{\psi}(\gamma^{\mu}D_{\mu} + m) \psi \\ &- (D_{\mu}\phi)^{*}(D_{\mu}\phi) - \mu\, \phi^{*}\phi - \frac{\lambda}{2} (\phi^{*}\phi)^{2} \end{aligned} \tag{7.2}$$

In here ϕ is a complex scalar field, coupled to photons with charge e, and also coupled to itself. In total we have 2 degrees of freedom more than in the previous case, because a complex scalar field has two degrees of freedom.

The state of lowest energy may be the vacuum, or it may be a state where the field ϕ has a certain value in all of space time. That depends on μ and λ. Let us see if the potential has a minimum. The potential is :

$$V(\phi^*,\phi) = \mu(\phi^*\phi) + \frac{\lambda}{2}(\phi^*\phi)^2 \tag{7.3}$$

This potential is positive for positive λ if ϕ gets sufficiently large because of the $(\phi^*\phi)^2$ term. For $\phi = \phi^* = 0$ we have zero. Let us see if there is an extremum (a minimum) somewhere. Changing ϕ by a small amount δ (and ϕ^* by δ^*) makes V change according to

$$V(\phi^* + \delta^*,\phi + \delta) = V(\phi^*,\phi) + \mu(\phi^*\delta + \delta^*\phi)$$

$$+ \lambda\,\{\phi^*(\phi^*\phi)\,\delta + \phi(\phi^*\phi)\delta^*\} + O(\delta^2) \quad .$$

The variation due to δ is zero if

$$\mu + \lambda(\phi^*\phi) = 0 \tag{7.4}$$

or

$$\phi = F\,e^{i\alpha} \qquad\qquad F = \sqrt{-\frac{\mu}{\lambda}} \tag{7.5}$$

where α is an arbitrary constant . This freedom manifest in the parameter α is due to the gauge invariance of the theory, in particular the invariance of the Lagrangian (7.2) for the global transformation

$$\phi \to e^{i\alpha}\phi \quad , \quad \phi^* \to e^{-i\alpha}\phi^* \quad . \tag{7.6}$$

Obviously, if a certain $\phi = F$ gives a minimum to the potential then also $\phi = e^{i\alpha}F$ is a minimum.

In the extremum found the value of the potential is :

$$\mu\,F^2 + \frac{\lambda}{2}F^4 = -\frac{\mu^2}{\lambda} + \frac{1}{2}\frac{\mu^2}{\lambda} = -\frac{1}{2}\frac{\mu^2}{\lambda} \; . \tag{7.7}$$

It must be noted that (7.4) has no solution if μ and λ have the same signs. Now λ must be positive, else the energy can be arbitrarily negative and no vacuum (by definition the state with lowest energy) would exist. Thus in order to have a minimum we must have $\mu < 0$. In that case we have a true minimum with a value for the potential given by (7.7).

We now substitute

$$\begin{aligned} \phi &\to \phi + F \\ \phi^* &\to \phi^* + F \end{aligned} \tag{7.8}$$

because we take it that the vacuum is the state where ϕ and ϕ^* have the value F (a possible phase can be rotated away by a global transformation). The Lagrangian becomes :

$$\begin{aligned} \mathscr{L} = & - \frac{1}{4} F_{\mu\nu}F_{\mu\nu} - \bar{\psi}(\gamma^\mu\partial_\mu + m_e)\,\psi \\ & - \partial_\mu\phi^*\partial_\mu\phi + ieA_\mu(\partial_\mu\phi^*\,\phi - \phi^*\partial_\mu\phi) - e^2A_\mu^2\,\phi^*\phi \\ & + i\,e\,F\,A_\mu(\partial_\mu\phi^* - \partial_\mu\phi) - e^2F\,A_\mu^2(\phi^* + \phi) - e^2F^2A_\mu^2 \\ & - \mu(\phi^*\phi) - \frac{\lambda}{2}(\phi^*\phi)^2 - \mu F(\phi^* + \phi) - \mu F^2 - \frac{\lambda}{2}\,F^4 \\ & - \lambda F(\phi^* + \phi)(\phi^*\phi) - \frac{1}{2}\,\lambda F^2(\phi^2 + 4\phi^*\phi + \phi^{*2}) - \lambda F^3(\phi^* + \phi). \end{aligned}$$

This becomes simpler if we substitute

$$\begin{aligned} \phi &= \frac{1}{\sqrt{2}}\,(B + iC) \\ \phi^* &= \frac{1}{\sqrt{2}}\,(B - iC)\ . \end{aligned} \tag{7.9}$$

The Lagrangian becomes

$$\begin{aligned} \mathscr{L} = & - \frac{1}{4}\,F_{\mu\nu}F_{\mu\nu} - \bar{\psi}(\gamma^\mu\partial_\mu + m)\,\psi \\ & - \frac{1}{2}(\partial_\mu B)^2 - \frac{1}{2}(\partial_\mu C)^2 - \frac{1}{2}\,m_H^2B^2 \\ & - e^2\,\frac{m_H^2}{4M^2}\,(B^2 + C^2)^2 - e\,\frac{m_H^2}{2M}\,B(B^2 + C^2) \\ & + e\,A_\mu(B\,\partial_\mu C - C\,\partial_\mu B) - \frac{1}{2}\,e^2A_\mu^2\,(B^2 + C^2) \\ & + M\,A_\mu\partial_\mu C - e\,M\,A_\mu^2B - \frac{1}{2}\,M^2A_\mu^2\ , \end{aligned} \tag{7.10}$$

where we introduced some new parameters :

$$M^2 = 2e^2F^2 \qquad m_H^2 = -2\mu . \tag{7.11}$$

Note that

$$\lambda = -\frac{\mu}{F^2} = e^2\frac{m_H^2}{M^2} . \tag{7.12}$$

What has happened ? The photon has obtained a mass M (see the term $M^2A_\mu^2$). There are two fields, B and C, and the C field has zero mass. It is in fact unobservable, because it can be transformed away. The original gauge behaviour of the ϕ was :

$$\phi \to \phi - i\Lambda\phi$$

$$\phi^* \to \phi^* + i\Lambda\phi^*$$

which is the first order part of $\phi \to e^{-i\Lambda}\phi$. Substituting $\phi + F$ and $\phi^* + F$ for ϕ and ϕ^* gives for the new fields :

$$\phi + F \to \phi + F - i\Lambda\phi - i\Lambda F$$

or $$\phi \to \phi - i\Lambda\phi - i\Lambda F$$

and similarly :

$$\phi^* \to \phi^* + i\Lambda\phi^* + i\Lambda F$$

With

$$B = \frac{1}{\sqrt{2}}(\phi + \phi^*) \qquad C = -\frac{i}{\sqrt{2}}(\phi - \phi^*)$$

we get

$$B \to B + \Lambda C$$

$$C \to C - \Lambda B - \frac{M}{e}\cdot\Lambda \tag{7.13}$$

where we used $F = M/e\sqrt{2}$. The gauge transformation mixes B and C, but adds also an arbitrary amount to C, and by a suitable choice of Λ we can transform C away. Thus C has become an unphysical, or rather unobservable field.

The situation may be summarized as follows ;

If $\mu > 0$ then the state of lowest energy is the usual vacuum where all fields have zero value. The degrees of freedom are : 2 in the photon field and 2 in the

ϕ-field.

If $\mu < 0$ then the state of lowest energy is a state where the field ϕ has a certain space time independent value. This will then be what we observe as the vacuum. The photon has now a mass M. There are 3 degrees of freedom for the photon field and 1 for the field B. The field C is of no physical relevance.

Thus, in going from positive to negative μ a degree of freedom flips over from the ϕ sector to the photon.

The fact that the C-field is massless is formalized in the Goldstone theorem : spontaneous symmetry breaking implies a massless particle. This may be seen as follows. If the potential (7.3) has a minimum then it has a whole line of minima, because the potential is invariant for the symmetry (7.6). Therefore there is a direction in which the first derivative remains zero, and thus the second derivative is zero in that direction. But the second derivatives give the coefficients of the terms quadratic in the fields, thus some mass is going to be zero. The beautiful thing in the above scheme is that the massless field is unobservable.

The theory described by the Lagrangian (7.10) is renormalizable.

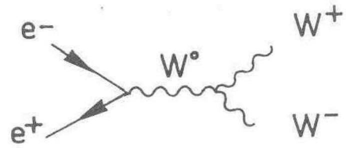
8. Large Higgs mass.

In the model with massive photons the Higgs mass is an arbitrary parameter. However, if it becomes very big then, as can be seen in the Lagrangian (7.10) certain self interactions of the physical Higgs particle B become also very big, and we see a strong interaction developing.

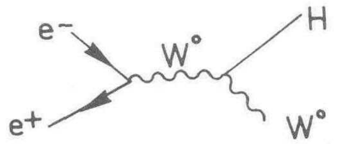
Furthermore, in general, in the limit of large Higgs mass the renormalizable model becomes an "almost" renormalizable model as discussed in section 6. By pure accident the case where the photon has a mass put in by hand is also renormalizable, and that is where the example of section 7 becomes useless. Nevertheless, in general we can conclude that certain amplitudes must blow up if the Higgs mass is taken to infinity. In fact, the Higgs system can be seen as the "new physics" discussed in section 6, it is the additional physics that provides for a cut-off on the almost renormalizable theories. As such the Higgs mass should not exceed, say, 600 GeV. Else couplings become of order 1, and in fact perturbation theory becomes useless. Very likely, such strong couplings can also be excluded experimentally.

We thus know that for high energies (say above 100 GeV) the almost renormalizable theory as we know it now experimentally should somehow change over into a massless theory or in a spontaneously broken theory. In the latter case Higgs particles should be observed.

If the theory smoothly turns over in a massless theory then we mean the following. It is not so much the mass itself that is important here, but rather the third degree of freedom of the vector boson. Will it gradually decouple and fade away at high energies, or will it remain ? The obvious place would be to look in very high energy $e^+e^- \to e^+e^-$, and to study the contribution of the neutral vector boson here. Unfortunately, the coupling $W^o e^+ e^-$ excites practically only the two transversal degrees of freedom, and effects would be very small. A more promising possibility is production of a W-pair through one intermediate W^o :



The threshold is at 120 GeV. Perhaps something can be observed here in the $W^{\pm}$ states. The fact that longitudinal polarizations do play a role comes also out through the fact that in the case of a spontaneously broken theory the process



may be important.

The investigation of this whole matter has not been completed, in fact is barely started. It seems however reasonable to conclude that e^+e^- experiments at an energy of 150 GeV or higher are essential to obtain further insight.

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