VECTOR BOSONS AND THE HIGGS SYSTEM

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I. INTRODUCTION

The long predicted and searched for vector bosons of the weak interactions were finally demonstrated to exist in early 1983 at the proton/anti-proton collider at CERN [1]. The masses were found to be those predicted by the standard model to within 3%, including radiative corrections of about 3%. At the same time, ironically, a number of events not foreseen by the standard model were recorded. At this time it is not known whether these events (the $e^+e^-\gamma$ events at 90 GeV) are a statistical fluke or some real new physics; meanwhile, subsequent announcements of other remarkable and for the time being un-explainable events at still higher energies seem to confirm the onset of new phenomena [2].

While clearly the data are too scarce to build on, we will nevertheless focus on these events in these lectures. That will lead us automatically to an analysis of the standard model, its proven points, and its open questions.

II. THE VECTOR-BOSON MASSES

In this section we shall concentrate on the values of the vector boson masses within the standard model. The masses are supposedly generated by the Higgs mechanism, and we must first review that.

In setting up the Higgs sector, one must first make a choice concerning the Higgs particles to be used, that is, a choice concerning the weak isospin properties of the Higgs particles. At least a complex iso-doublet must be used (much like the system of K-mesons); but, a priori, higher representations cannot be excluded. A convenient way to parametrize this choice for confrontation with experiment is the ρ-parameter, introduced by Ross, et al. [3] in 1975. At that time the parameter was called β; in a later paper by Hung and Sakurai [4] this parameter was re-introduced in a somewhat different context and called the ρ-parameter, a name that has become standard despite the fact that Michel used the same name for a parameter in mu-decay.

The ρ-parameter is defined as

$$\rho = M^2/c^2M_o^2 \ .$$

Here $c = \cos\theta_W$ [5], M is the mass of the charged W, and M_o is the mass of the neutral W (often called Z_o, but called W^o in these lectures). Disregarding radiative corrections for the moment, the result

$$\rho \geqslant \frac{1}{(2I)}$$

was proved in Ref. 3, where I is the highest weak isospin found among the Higgs particles. In the case of the simplest Higgs system as introduced by Weinberg [6], I = 1/2 and ρ = 1.

To compare theory and experiment, at least four measurements are needed. The standard model contains four free parameters (not counting the fermion masses), namely, the coupling constant g, the weak mixing angle θ_W, and the masses of the charged and neutral vector bosons. The Higgs mass(es) is also a free parameter but the measurements relevant for us depend very weakly or not at all on that parameter. Two measurements are readily available: the fine-structure constant α and the muon decay rate. Two other measurements are needed to determine the ρ-parameter. For this, one may use the mass values found by the CERN groups (the upper value is UA1, the other, UA2):

$$M = \begin{matrix} 80.9 \pm 1.5 \pm 2.4 \text{ GeV} \ , \\ 83.1 \pm 1.9 \pm 1.3 \text{ GeV} \ , \end{matrix} \quad \text{(first error is statistical)}$$

$$M = \begin{matrix} 95.6 \pm 1.4 \pm 2.9 \text{ GeV} \ , \\ 92.7 \pm 1.7 \pm 1.4 \text{ GeV} \ . \end{matrix}$$

Most radiative corrections are of the order of 1% or less, with the exception of radiative corrections to the vector boson masses due to low-mass fermion insertions, which are +3 and +3.3 GeV for M and M_o, respectively [7]. Since the data are clearly not better than 1% it is pointless to be more precise at this point. We thus take the average of the published data, subtract the corrections mentioned, and determine the parameters from this as well as from the values for the lowest order muon lifetime and the fine structure constant α:

$$\frac{1}{\tau_\mu} = \frac{m_\mu^5}{192\pi^3}\,\frac{g^4}{32M^4} = \frac{1}{2.197\text{x}10^{-6}}\ \text{sec}^{-1}\ ,$$

$$\alpha = \frac{e^2}{4\pi} = \frac{g^2 s^2}{4\pi} = \frac{1}{137.04}\ .$$

To be precise, we have used M = 79000 MeV and M = 90850 MeV. From this we determine s^2 = 0.22 and ρ = 0.97. Below, for comparison, we also do the simplified method for the UA2 results alone, showing perfect agreement with the numbers quoted by UA2.

It should be noted here that there is confusion concerning what one calls the weak mixing angle. For example, UA2 in comparing their results with the standard model quotes two different values for s^2, differing by 10%. This is due to the otherwise excellent work of the authors of Ref. 8. In that reference one defines the cos of the weak mixing angle as the ratio of charged to neutral vector boson mass. The reason for this will be discussed below. However, this would make ρ = 1 by definition. At that time, the authors were probably not aware of the importance of the ρ-parameter, since they started from the minimal Higgs sector, period. They resolve the dilemma by defining

$$\rho = (1 - A^2/M^2)\cdot M^2/M_0^2\ ,\quad A = 38.65\ \text{GeV}\ ,$$

as quoted by UA2. This stubbornness really serves no purpose in this case, and here we would like to advocate the simple procedure given above. At some point, if the measurements become more precise than 1%, it will be necessary to take all one-loop radiative corrections into account. That presents some difficulties, because they depend on the process considered. For example, we may expect that in the future the really precise measurements come from angular distributions in W_0 decay seen in electron machines. Also the mass M_0 will be measured to within a fraction of a percent. It would seem a proper division of labor if the experimental groups would present their results using the standard model and simply ignoring radiative corrections except the above mentioned 3% corrections. Then there would be no disagreement up to the 1% level. Later, theorists could fight out the details. For instance, if the masses were really very precisely known, then one might compute the radiative corrections to the above procedure, and conclude that the above derived ρ-parameter should be 1+δ for the simplest Higgs system, where δ is the total of radiative corrections to mu-decay, α, and the vector-boson masses, each of which is well known (and infinite, although the combination δ is and must be finite).

To be very specific, we will do this in detail for the case at hand, finding the ρ-parameter from the mass values 82 and

94.15 GeV, not neglecting radiative corrections. Including radiative corrections, the experimental quantities α, the mu-decay rate, and the W masses are given by:

$$\alpha = \frac{1}{137.04} = \frac{g^2 s^2}{16\pi^2}\left(1 + \frac{1}{2}\,\delta_\ell\right) ,$$

$$\frac{1}{\tau_\mu} = \frac{m_\mu^5}{192\pi^2}\,\frac{g^4}{32M^2}\left(1 + \delta_\mu\right) ,$$

$$M^2_{\mathrm{exp}} = M^2\left(1 - S_+(k^2)/M^2\right) ,$$

$$M^2_{o,\mathrm{exp}} = M_o^2\left(1 - S_o(k^2)/M_o^2\right) ,$$

where the radiative corrections δ_μ, δ_ℓ, S_+, and S_o are given in Ref. 9. S_+ must be evaluated for $k^2=-M^2$ and S_o for $k^2=-M_o^2$, that is, for the W on mass shell. It is easy to check that the ρ-parameter becomes:

$$\rho = \frac{M^2_{\mathrm{exp}}}{M^2_{o,\mathrm{exp}}\,c_o^2}\left[1 + \frac{c_o^2 - s_o^2}{c_o^2}\,\frac{S_+}{M^2} - \frac{S_o}{M_o^2} + \frac{s_o^2}{c_o^2}\left(\frac{1}{2}\,\delta_\mu - \frac{1}{2}\,\delta_\ell\right)\right] .$$

The various quantities are by themselves infinite, but the combination occurring in this equation for the ρ-parameter is finite, as it should be. For the calculation of all these quantities one uses the values for the parameters found in zero'th order, that is, solving the above equations with δ_μ, etc., set to zero. The results are denoted by s_o, c_o, M and M_o=M/c_o etc. Here we find, for example, s_o^2 = 0.2072, and the lowest order value for the ρ-parameter is 0.9567. Including now the radiative corrections as indicated above leads to correction of the ρ-parameter by a factor (1 + 0.0174), which gives the corrected value 0.9734. This is in fact the same as found with the simplified procedure outlined above.

The correction to $\sin^2\theta_W = s^2$ is not finite and is also gauge dependent, which makes this parameter unsuitable for comparison with the data. However, one may determine s^2 from any given data point and compare s^2 for different data points. Or one may use one data point, and predict what should be the result from a measurement of another data point. This is what the authors of Ref. 8 probably had in mind: Compute for any given experiment what prediction that would give for the ratio M^2/M_o^2 and call that $\cos^2\theta_W$. Indeed, there is something to be said for that, except for two facts: this particular choice of standard reference point is unsuitable because one would rather reserve it for the ρ-parameter; and secondly it introduces big errors in the values quoted for s^2

if the W masses are taken as input parameters, because a 1% error in $\cos\theta_W$ corresponds to a 4% error in $\sin\theta_W$. That is why UA2 obtains such different values for s^2. Alternatively one could use as reference point, for example, neutrino-electron scattering at low energy; such a usage would produce values of s^2 very close to the values obtained by the simplified procedure, and also very close to what one would find in any low energy process by simply ignoring radiative corrections. The latter is usually what experimentalists quote. We are not about to propose here one way versus another, because after all there is more in life than just s^2. Also, as long as the vector boson masses are not input parameters, then the differences between the methods are beyond today's precision.

Doing the same work for the UA2 mass values 83.1 and 92.7 GeV leads to the result $\rho = 1.024$, including radiative corrections. With the simplified procedure, we first correct the masses by subtracting 3 and 3.3 GeV, giving 80.1 and 89.4 GeV. Computing s^2 and ρ from that using the simple uncorrected lowest order equations gives $s^2 = 0.217$ and $\rho = 1.026$, which is not significantly different from the precise result, as well as the values quoted by UA2.

A similar analysis of the UA1 results gives $s^2 = 0.23$ and $\rho = 0.92$.

The above considerations show that experimentally the ρ-parameter is found to be 1 with an error margin of about 10%. Using some other relatively low energy data as input instead of the W masses (for example, data concerning neutral currents) gives $\rho = 1$ with an error of 3%.

The world average for s^2 is 0.217 ± 0.014, which compares well with the above results. A more precise method for comparing the experimental values is to quote the values predicted for the W masses from the low energy data, including all radiative corrections. That is clearly totally unambiguous. Those predictions are:

$$M = 83^{+2.9}_{-2.7} \text{ GeV} , \qquad M_0 = 93.8^{+2.4}_{-2.2} \text{ GeV} ,$$

from the work of Ref. 8. The agreement with the results of UA1 and UA2 is impressive. Apparently, the low-energy data predict the 80-100 GeV data with an accuracy in the range of 3%.

III. THE LOW-ENERGY DATA

The vector-boson self-energy functions S_+ and S_0 are very interesting objects. The reason for this is two-fold: (i) they can be measured quite directly, both at high and low energy; and (ii) they are sensitive to the interactions of the vector-bosons with fermions and possibly other objects, and the effects of this are visible even if the masses of these objects are of the order or larger than 100 GeV. To demonstrate this we consider here the contribution of a fermion generation to these functions. One finds, calculating simple self-energy diagrams:

$$S_+(k^2) = \frac{g^2}{(2\pi)^4 i} \{(m_e^2 - m_\nu^2)\, B_1(k^2,m_\nu,m_e) - m_\nu^2\, B_0(k^2,m_\nu,m_e)$$

$$+ 6k^2[B_{21}(k^2,m_u,m_d) + B_1(k^2,m_u,m_d)]$$

$$+ 3(m_d^2 - m_u^2)\, B_1(k^2,m_u,m_d) - 3m_u^2\, B_0(k^2,m_u,m_d)\}$$

$$S_0(k^2) = \frac{g^2}{(2\pi)^4} \{\frac{k^2}{16c^2} [(4s^2 - 1)^2 + 1][8B_{21}(k^2,m_e,m_e)$$

$$- 4B_0(k^2,m_e,m_e)]$$

$$+ \frac{k^2}{8c^2} [8B_{21}(k^2,m_\nu,m_\nu) - 4B_0(k^2,m_\nu,m_\nu)]$$

$$+ \frac{3k^2}{16c^2} [(1 - \frac{8}{3} s^2)^2 + 1][8B_{21}(k^2,m_u,m_u) - 4B_0(k^2,m_u,m_u)]$$

$$+ \frac{3k^2}{16c^2} [(\frac{4}{3} s^2 - 1)^2 + 1][8B_{21}(k^2,m_d,m_d) - 4B_0(k^2,m_d,m_d)]$$

$$- \frac{1}{2c^2} [m_\nu^2\, B_0(k^2,m_\nu,m_\nu) + m_e^2\, B_0(k^2,m_e,m_e)$$

$$+ 3m_u^2\, B_0(k^2,m_n,m_n) + 3m_d^2\, B_0(k^2,m_d,m_d)]\}$$

The functions B appearing in these expressions are defined by:

$$B_0;\; k_\mu B_1;\; k_\mu k_\nu B_{21} + \delta_{\mu\nu} B_{22} = \int d_n q \, \frac{1;\; q_\mu;\; q_\mu q_\nu}{(q^2+m_1^2)[(q+k)^2 + m_2^2]} \,.$$

More details on these expressions can be found in Refs. 7 and 9.

Suppose now the existence of a new as yet undiscovered fermion family. Such a family would also contribute to the vector boson self-energies as given by the above equations; all one has to do is substitute their masses for the electron, neutrino, and up and down quark masses. What would one observe? We will give the answer for a few different cases of interest. To begin with, the W masses shift upwards by about 60 MeV for M and 94 MeV for M_0. Assuming that 1% will be observable shortly, we see that 10 new families would produce an observable result. The present limit is around 30 families. Secondly, mass differences within the new family (lepton mass difference and quark mass difference) produce observable results at low energy. At low energy the expressions for S_+ and S_0 work out to ($k^2 = 0$):

$$S_+(0) = \frac{g^2}{(2\pi)^4 i}\left[(m_e^2 - m_\nu^2)\, B_1(0,m_\nu,m_e) - m_\nu^2\, B_0(0,m_\nu,m_e)\right.$$

$$\left. + 3(m_d^2 - m_u^2)\, B_1(0,m_u,m_d) - 3m_u^2\, B_0(0,m_u,m_d)\right] ,$$

$$S_0(0) = \frac{g^2}{(2\pi)^4 i}\,\frac{-1}{2c^2}\left[m_\nu^2\, B_0(0,m_\nu,m_\nu) + m_e^2\, B_0(0,m_e,m_e)\right.$$

$$\left. + 3m_u^2\, B_0(0,m_u,m_u) + 3m_d^2\, B_0(0,m_d,m_d)\right] .$$

The only way such a new family could contribute to low-energy processes is through vector-boson self-energy diagrams, and we need not worry about other radiative corrections. Now the input data are certain low energy cross sections, such as for example ν_μ-e or $\bar{\nu}_\mu$-e scattering. The total cross section for these processes is, in lowest order and keeping only leading terms in the limit of low energy:

$$\sigma_{tot} = \frac{g^4}{256\pi c^4 (M_0^2)^2}\; m_\ell E\,\left[(a+b)^2 + \frac{1}{3}(a-b)^2\right] ,$$

where $a = 4s^2 - 1$ and $b = -1$ for ν_μ-e and $b = 1$ for $\bar{\nu}_\mu$-e scattering. E is the laboratory energy. The radiative corrections due to a new family amount to the replacement $M_0^2 \rightarrow M_0^2 - S_0(0)$.

Actually there is also a contribution due to the fact that a new family gives rise to a W-photon transition, but that is zero for zero energy. In the other low-energy input expressions, only mu-decay changes; and in fact there the contribution due to a new family amounts to also replacing M^2 by $M^2 - S_+(0)$ in the expression for mu-decay. The experimentalists, unaware of the new generation, will essentially solve the equations not taking these effects into account; that means that they will find:

$$\rho = [M^2 - S_+(0)]/[M_0^2 - S_0(0)]c^2 = 1 - (S_+ - c^2 S_0)/M^2 ,$$

where we took $M_0 = M/c$. This works out to [16]:

$$\rho = 1 + \frac{g^2}{64M^2\pi^2} [f(m_\nu,m_e) + 3 f(m_u,m_d)] ,$$

with

$$f(m,M) = m^2 + M^2 + \frac{2m^2M^2}{m^2-M^2} \log M^2/m^2 .$$

The function f can be expanded in the mass difference; writing

$$M^2 = m_0^2(1+\delta) , \quad m^2 = m_0^2(1-\delta) , \quad \delta = (M^2-m^2)/(M^2+m^2) ,$$

one has

$$f(m,M) = 4m_0^2 \left[\frac{1}{3}\delta^2 + \frac{1}{15}\delta^4 + \frac{1}{35}\delta^6 + \ldots \frac{1}{(2n-1)(2n+1)}\delta^{2n} + \ldots\right] ,$$

which shows that f is always positive. That is also clear from the integral representation

$$f(m,M) = \frac{1}{i\pi^2} \int d_n q \frac{q^2(m^2-M^2)^2}{(q^2+m^2)^2(q^2+M^2)^2} .$$

Numerically, using that $g^2 = 8M^2G_f$, where $G_f = 1.02\times10^{-5}/\sqrt{2}m_p^2$, m_p = proton mass, one finds that significant corrections result if the mass differences among the leptons or quarks are of the order of the vector-boson mass, i.e., 100 GeV. Looking to the known families, in particular to the quark masses, one notes mass differences of a few MeV for the first family, 1.25 GeV for the second, and at least 15 GeV for the third. If this trend continues it appears that perhaps one more family, but certainly not two new families, can be accommodated without driving the ρ-parameter too far away from 1.

One may ask what happens if other types of fermion or boson multiplets exist. For example, one might speculate that perhaps in a supersymmetric theory bosonic contributions cancel against fermionic ones; but this turns out not to be true; in general they add up [10]. Actually, for a while it was suspected that the contributions to ρ would always be positive; that turns out not to be true, although it is true for a very large number of cases. There exists a conjecture concerning when that would be so [11], but up to now the subject has not been fully clarified.

It is clear from the above that the value 1 for the ρ-parameter is closely connected to the degree of iso-spin breaking as observed in the form of mass differences between the lepton doublets and the quark doublets. A more formal derivation can be given by rewriting the Higgs sector in the notation of the σ-model.

IV. THE σ-MODEL AS HIGGS SECTOR

Many of the results quoted in this section have been derived in the papers of Ref. 12.

Let there be given a set of four fields, σ^0 and ϕ^i, i=1,2,3. Consider the 2×2 matrix Φ:

$$\Phi = \sigma^0\tau^0 + i\phi^\alpha\tau^\alpha$$

$$= \begin{pmatrix} \sigma^0+i\phi^3 & i\phi^1+\phi^2 \\ i\phi^1-\phi^2 & \sigma^0-i\phi^3 \end{pmatrix} = i\sqrt{2}\begin{pmatrix} -\phi^\dagger & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix} ,$$

with

$$\phi^0 = -\frac{i}{\sqrt{2}}(\sigma^0-i\phi^3) , \quad \phi^\dagger = \frac{i}{\sqrt{2}}(\sigma^0+i\phi^3) .$$

With this matrix Φ a Lagrangian may be constructed:

$$\mathcal{L}_H = -\frac{1}{4}\mathrm{Tr}(\partial_\mu\Phi^\dagger\partial_\mu\Phi) - \frac{\lambda}{8}\left[\frac{1}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \frac{2\mu}{\lambda}\right]^2 .$$

This Lagrangian is invariant under two independent SU2 transformations called SU2(left) and SU2(right) and denoted by G and H respectively:

$$\Phi \to G\Phi , \quad G = e^{i\Lambda_L^\alpha\tau^\alpha} e^{i\Lambda_L^0\tau^0} ,$$

$$\Phi \to \Phi H , \quad H = e^{i\Lambda_R^\alpha\tau^\alpha} e^{i\Lambda_R^0\tau^0} .$$

We make SU2(left) into a local invariance by the replacement

$$\partial_\mu \to D_\mu = \partial_\mu - \frac{ig}{2}\tau^\beta B_\mu^\beta .$$

Also, e.m. must be included, for which we need a U1 symmetry. We may take any of the remaining Λ_L^o or Λ_R^α, Λ_R^o for this purpose. To establish what to take, we compute the vector-boson masses, given a certain vacuum expectation value for Φ:

$$\mathrm{Tr}\{[(-\frac{ig}{2}\tau^\beta B_\mu^\beta - \frac{i\lambda^o}{2}\tau^o C_\mu^o)\Phi - \Phi\,\frac{ig\,g^\alpha\tau^\alpha}{2}C_\mu]^\dagger \times [\,]\} \; .$$

The question is what to take for λ^o or the g^α. Writing

$$B_\mu^3 = c\,W_\mu^o + s\,A_\mu ,$$

$$C_\mu = -s\,W_\mu^o + c\,A_\mu ,$$

where W_μ^o and A_μ are the neutral vector-boson and e.m. fields, we require that: (i) there are no AW terms (whether W is the charged or neutral W field), and (ii) no AA terms (no mass for the photon). This is enough to fix everything and the result is

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}[(D_\mu\Phi)^\dagger D_\mu\Phi] - \frac{\lambda}{8}[\frac{1}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \frac{2\mu}{\lambda}]^2 \; ,$$

$$D_\mu\Phi = (\partial_\mu - \frac{ig}{2}T^\beta B_\mu^\beta)\Phi - \frac{igg'}{2}\Phi\tau^3 C_\mu \; ,$$

$$g' = -\frac{s}{c} \; .$$

The vacuum expectation value for Φ is proportional to the identity:

$$\Phi = \sqrt{-\frac{2\mu}{\lambda}}\, I \;\; (\equiv F\sqrt{2}\, I) \; ,$$

and the W masses are given by:

$$M^2 = \frac{g^2}{4}(-\frac{2\mu}{\lambda}) = \frac{1}{2}g^2F^2 \; ,$$

$$M_o^2 = \frac{g^2}{4}(-\frac{2\mu}{\lambda})\frac{1}{c_o^2} = \frac{1}{2}g^2\,F^2/c_o^2 \; .$$

Thus:

$$\rho = M^2/M_o^2c^2 = 1 \; .$$

In the limit c = 1 we have $M = M_0$. Moreover we have a residual global symmetry if we chose $G = H^\dagger = e^{i\Lambda^\alpha \tau^\alpha}$, i.e.,

$$\Phi \rightarrow G\Phi G^\dagger .$$

This symmetry is not broken by the vacuum expectation value of Φ, since that vacuum expectation value is a multiple of the unit tensor.

This residual symmetry, which is nothing but ordinary isospin, is clearly broken by the U1 gauge symmetry, i.e., the e.m. interactions, which breaking is proportional to s. The fact that the ρ-parameter is 1 is really a consequence of this symmetry, where one has explicitly corrected for W^0-A mixing.

V. THE NONLINEAR σ-MODEL

The above Lagrangian is the usual Higgs sector of the standard model. In this form it is recognized to be the linear σ-model. There is one free parameter in this linear σ-model, which may be taken to be the Higgs mass, i.e., the mass of σ^0 after substituting $\sigma^0 = \sigma^0 + F\sqrt{2}$, where $F\sqrt{2}$ is the vacuum expectation value. The Higgs mass, denoted by m in the following, is related to the original parameters by

$$m^2 = 2\lambda F^2 = 4\lambda M^2/g^2 ,$$

where M is as usual the charged W mass. With these notations, the Higgs Lagrangian, not including coupling to the vector bosons, is:

$$-\frac{1}{2}(\partial_\mu H)^2 - \frac{1}{2}(\partial_\mu \vec{\phi})^2 - \frac{1}{2}m^2H^2 - \frac{m^2}{4M}gH(H^2+\vec{\phi}^2) - \frac{m^2}{32M^2}g^2(H^2+\vec{\phi}^2)^2 .$$

Again, this is the linear σ-model. Since the Higgs has not been seen so far, one might be interested in the limit $m\rightarrow\infty$. This limit may be taken on the Lagrangian level. First select all terms that become large with large m:

$$-\frac{m^2}{8M^2}\{2MH + \frac{g}{2}(H^2+\vec{\phi}^2)\}^2 .$$

If $m\rightarrow\infty$, the equation of motion for the Higgs field H corresponds to the minimum of this part of the Lagrangian, which is

$$H = -2M(1 - \sqrt{1 - \frac{g^2\vec{\phi}^2}{4M^2}}) .$$

Substituting this value for H in the rest of the Lagrangian gives us the nonlinear σ-model. The Higgs field has disappeared. The Lagrangian contains only the massive vector fields and the photon field as physical fields, which seems just like the Yang-Mills theory with masses put in by hand (massive Yang-Mills theory). Indeed, with some work that will not be given here, the massive Y-M Lagrangian (Glashow's model [5]) can be worked over into a form which is precisely the above limit from the standard model.

That raises an interesting question. Is the limit of large Higgs mass the same as taking the massive Y-M theory, i.e., using the nonlinear σ-model instead of the linear σ-model? That is certainly true at the tree level, because then manipulations in the Lagrangian are legitimate. As it happens it is also true for one-loop diagrams [12], but discrepancies arise at the two-loop level [13]. This shows that there may be more than one way to get to a theory without a Higgs. At the moment, as the existence of the Higgs becomes more dubious than ever, one might be quite interested in this problem; but so far nothing more is known.

The vector-boson masses, and notably the ρ-parameter, and the 3-W and 4-W couplings, are the only objects that are sensitive to the limit of large Higgs mass at the one-loop level. Even here, the sensitivity is only logarithmic; for example, the radiative correction to the ρ-parameter in the limit of large Higgs mass is

$$\rho = 1 - \frac{3g^2}{64\pi^2} \frac{s^2}{c^2} \log \frac{m^2}{M^2} .$$

This rather weak sensitivity of the low-energy theory to the limit of large Higgs mass has been called the screening theorem [14]. It was also noted that for Higgs mass larger than 1000 GeV perturbation theory breaks down, and higher loop effects may contribute as much as one-loop effects.

VI. THE UNITARITY LIMIT

Another approach to the large m limit is to consider the magnitude of tree diagrams as a function of energy [15]. It has been found that the WW scattering amplitude, calculated in the tree approximation and considered for energies very large with respect to the vector boson mass, becomes larger than permitted by unitarity if the Higgs mass m is larger than 1000 GeV. This result is actually not immediately identifiable with the last remark of the previous section, because of the limit of large energy. As long as one remains at low energies (energies below 1000 GeV) there is no breakdown of unitarity at the tree level. For that reason the unitarity argument becomes useful only if measurements above 1000 GeV become available. Then such measurements may be used to derive the Higgs mass, provided it is seen that the results follow perturbation theory, implying m less than 1000 GeV. If m is larger than 1000 GeV, then one can only conclude that above 1000 GeV perturbation theory is no longer valid.

The upshot of the above is that nothing can be said about the Higgs mass on the basis of present data. The Higgs may or may not exist, and if the Higgs exists, its mass may be anywhere from a few GeV upwards to infinity.

VII. THE ANOMALOUS EVENTS

Let us now return to the anomalous events seen at CERN. There are three such events at this time, with the following characteristics:

	Exp	Decay		$m(e^+e^-)$ (GeV)	E_γ (GeV)	Total mass
(a)	UA1	$e^+e^-\gamma$	$\theta_{e^-\gamma} = 14.4°$	42.7±2.4	38.8	98.7±5.0
(b)	UA2	$e^+e^-\gamma$	$\theta_{e^-\gamma} = 31.8°$	50.4±1.7	24.4	90.6±1.9
(c)	UA1	$\mu^+\mu^-\gamma$	$\theta_{\mu^+\gamma} = 7.9°$	70.9^{+37}_{-12}	28.3	88.4^{+46}_{-15}

Calling m_1, m_2 and m_3 the masses of the ee and the two $e\gamma$ systems (we will by definition take m_2 to be the smaller of the two $e\gamma$ masses) one finds:

(a) m_2 = 4.6 ±1.0 Gev

(b) m_2 = 9.1 ±0.3 Gev

(c) m_2 = 5.3 ±0.3 GeV

What is so peculiar about these events? In principle we might expect such events simply as bremsstrahlung of the normal $W^o \to ee$ decay. If this were so, then we should also see similar happenings with the charged W decay: $W \to e\nu\gamma$. UA2 has 7 events of the type $W^o \to ee$, and 1 $ee\gamma$ event. On the other hand there are 37 events $W \to e\nu$, and essentially no $W \to e\nu\gamma$. UA1 sees 52 events $W \to e\nu$, and 18 $W \to \mu\nu$, but no candidates for events with an extra photon. On the other hand there is 1 $ee\gamma$ and 1 $\mu\mu\gamma$ event compared to 3 $W^o \to ee$ and 4 $W^o \to \mu\mu$ events. This is the problem: the rate is far too large. Now that may always be a statistical fluke, but it really becomes difficult to ignore these numbers. Let us go on analyzing the situation.

First of all there is no reason whatsoever to say that the events are W^0 decay. The mass values quoted allow no such conclusion as one can see. In fact, they are hardly compatible with one mass value. However, for definiteness we will assume that all the events occur at some common mass, namely 95 GeV, and we will speak of the S particle. This S may or may not be the W, and may or may not be a particle with well defined quantum numbers.

Let us now consider the kinematics of this type of event in some detail. Assume momenta q, p_+, p_-, and k for the S, e^+, e^-, and γ. We neglect the fermion mass and go into the S rest system. Here the mass of S will be taken to be 1, as a reference mass.

$$m_1^2 = -(p_+ + p_-)^2 = -(q - k)^2 = 1 - 2k_0 ,$$

$$m_2^2 = -(p_- + k)^2 = -(q - p_+)^2 = -2\ p_-\cdot k \ ,$$

$$m_3^2 = -(p_+ + k)^2 = -(q - p_-)^2 = -2\ p_+\cdot k \ .$$

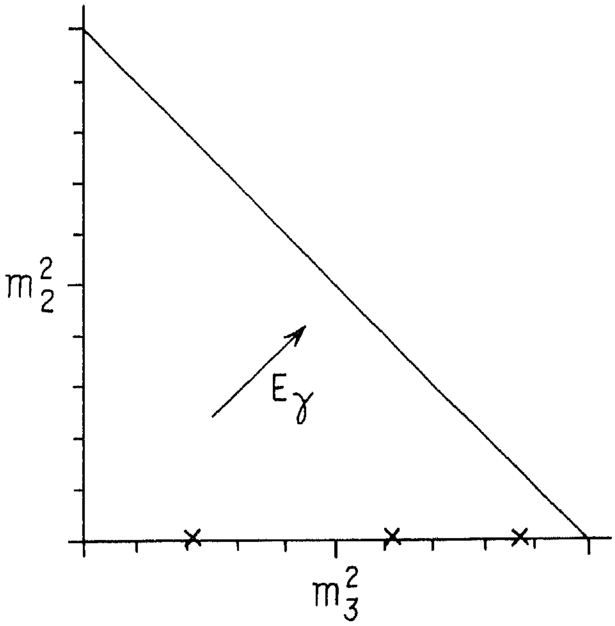
These variables are not independent:

$$m_1^2 + m_2^2 + m_3^2 = 1 \ .$$

Note that m_1^2 is essentially the photon energy. Fig. 1 shows the available phase space, and the three CERN events are indicated. All three events are placed practically on the horizontal axis. This looks a lot like bremsstrahlung, $W^0 \to e^+e^-\gamma$. The equation for bremsstrahlung is (Passarino, Ref. 17; we omit terms proportional to the lepton mass):

$$\frac{d^2\Gamma}{dm_1^2 dm_2^2} = \frac{\alpha}{\pi}\,\Gamma_0\ \left[2\,\frac{1+m_1^2}{1-m_1^2}\,\left(\frac{1}{m_2^2} + \frac{1}{m_3^2}\right) - 4\right] ,$$

where Γ_0 is the decay width without photon. That gives a distribution closely tied to either the horizontal or vertical axis and increasing going into the origin (Fig. 2). The trouble with this interpretation (i.e., bremsstrahlung) is that the CERN events are too much to the right, the photon energy is too large. Not only that, the values of m_2^2 are really too big also. Fig. 3 shows relative probabilities for fixed E_γ; these are diagonal lines across the plots of Figs. 1 and 2. The bremsstrahlung interpretation is difficult not only concerning rate, but also concerning distribution. However, any other type of distribution, like the decay of some scalar, is off even more (see Fig. 4).



	m_2^2	m_3^2
(a)	0.0023	0.87
(b)	0.0092	0.62
(c)	0.0031	0.20

Fig. 1: CERN eeγ events.

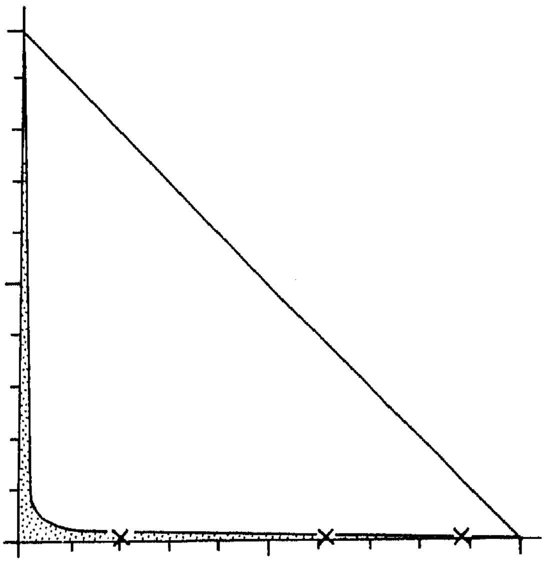


Fig. 2: Bremsstrahlung

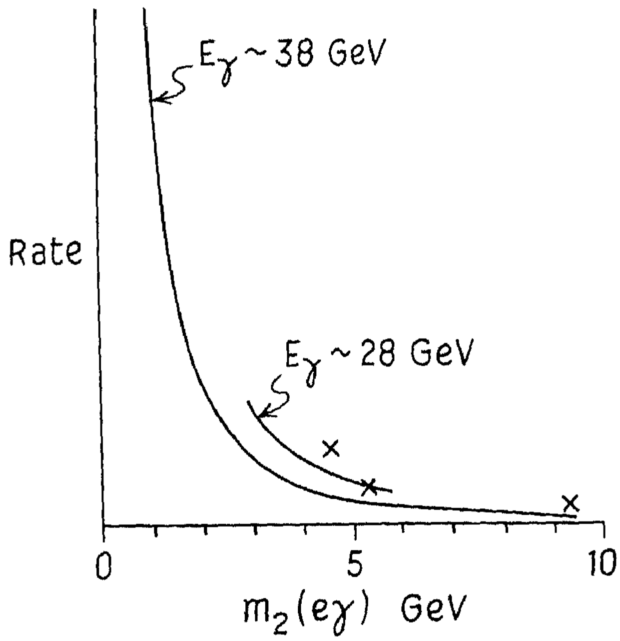


Fig. 3: Bremsstrahlung, fixed E_γ.

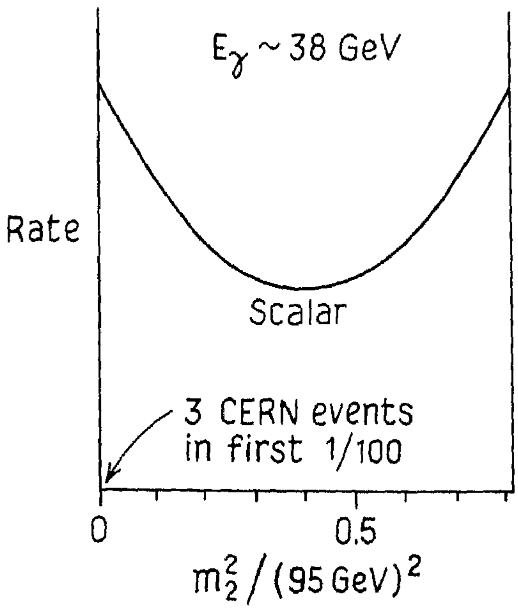


Fig. 4: Scalar decay into $ee\gamma$.

One conclusion can certainly be drawn: if this is not bremsstrahlung, then it must at least have something in common with it. Somehow the photon and e^- (or e^+) have some interaction, or something in common, because there is clearly a strong correlation.

How do the various proposed models compare to this? None comes even near. Roughly speaking, the following types have been proposed*:

1) New narrow pseudoscalar or vector resonance [18-20]:

$$W^o \to \gamma + P \to \gamma + e^+ + e^- .$$

2) Scalar boson with derivative couplings to fermions [21].

3) Composite vector bosons [20,22]:

$$W^o \to Y(\to\gamma) + W^o(\to e^+e^-) \ , \ Y = \text{isoscalar} \ .$$

4) Excited electron e*, with mass e* about 75 Gev [23]:

$$W^o \to e^*e \to e\gamma e \ .$$

Possibilities 1 and 4 require definite m_1^2 or m_3^2, corresponding to the lines of Fig. 5. The distributions along the line are not particularly peaked. Possibility 2 gives a distribution that is down by a factor of 2 in the middle compared to the edges. As for possibility 3, Ref. 22 gives an equation for the distribution:

$$d\Gamma \propto 1 + \frac{1}{2}\,(m_1^2 - 2)[2m_2^4 - 2(1 - m_1^2)m_2^2 + 1 + m_1^4] \ .$$

This distribution actually peaks at $m_2^2 = \frac{1}{2}\,(1 - m_1^2)$. For example, with a photon energy of 38.3 Gev (or $m_1^2 = 0.187$) the rate at the maximum is 5.8 times larger than the rate at the endpoints.

It is clear that the explanation of the distribution is going to be difficult. Whatever S is, its decay is no simple matter. Some sort of final state interaction acting on the photon and the electron seems to be present. Right now we will leave this issue.

Let us now concentrate on the following question. Is $S = W^o$ or not? One thing is clear: S must have a structure, and probably a very complicated one. But we certainly do not believe that the W^o has a structure, as that seems totally at variance with the earlier analysis, highlighting the success of perturbation theory.

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* The following is a summary of an analysis by G. Passarino.

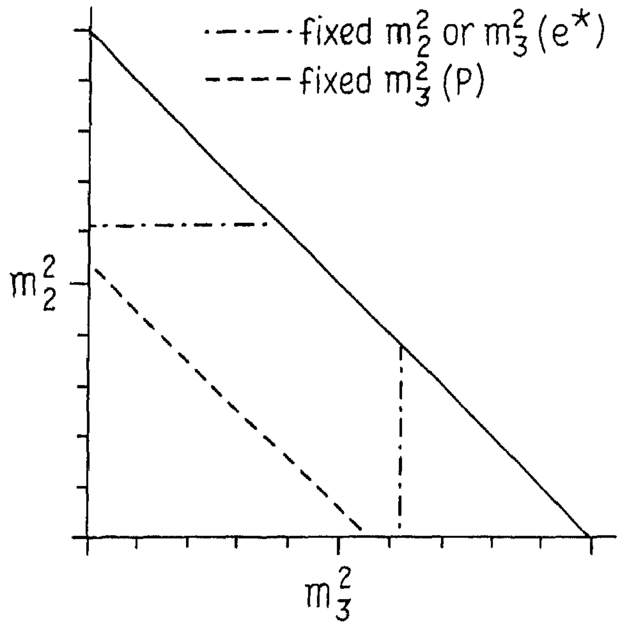


Fig. 5: e^* and P=(ee) hypothesis.

We will elaborate this point even further, to show the general effects to be expected.

Assume that there is a scalar particle R that couples only to the W, strongly enough that it gives the W a structure. To make the effects of R otherwise minimal, we assume R to be a scalar, and also isoscalar, without any couplings to fermions.

In any case, R must have an interaction with the W^o. We assume an interaction term

$$h\, R\, W^o_\mu W^o_\mu \,,$$

where h is a coupling constant (with the dimension of a mass).

We now must take care that gauge invariance is not broken. A term of the form WW is like a mass term; and we know how to write a gauge invariant Lagrangian that generates such a term, namely, simply the Higgs Lagrangian. Thus we arrive at the interaction:

$$h\, R\, (D_\mu\Phi)^\dagger D_\mu\Phi \,.$$

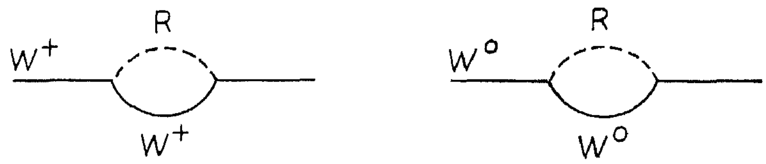
One may or may not also couple R to the other terms of the Higgs

Lagrangian, but this is the minimum needed. After the Higgs develops its vacuum expectation value, we get the term mentioned above, but also a term where R interacts with the charged vector bosons, in a proportion just like the W mass terms. The terms involving three fields are:

$$h\ R\ [-\frac{1}{2}(\partial_\mu H)^2 - \partial_\mu\phi^+\partial_\mu\phi^- + M(W_\mu^+\partial_\mu\phi^- + W_\mu^-\partial_\mu\phi^+)$$

$$+\frac{M}{c}W_\mu^o\partial_\mu\phi^o - M^2W_\mu^+W_\mu^- - \frac{1}{2}\frac{M^2}{c^2}W_\mu^oW_\mu^o]\ .$$

The interactions shown give rise to certain W self-energy diagrams:



These W^o self-energy diagrams will have a factor c^{-4} compared to the W^+ diagrams, and there will be a contribution to the ρ-parameter (which involves the combination $S_+ + c^2\ S_0$). In fact, this contribution is infinite, and is not cancelled out by other diagrams.

Here are some questions that must be answered. Since we have only a doubled Higgs, why is $\rho \neq 1$ by an infinite amount? The reason is that the theory is non-renormalizable; and to make the theory finite one must also introduce certain terms involving more than 4 fields into the Lagrangian, having a more complicated behavior with respect to isospin than the usual terms.

From the arguments given above we are strongly tempted to assume $S \neq W^o$. Ultimately, however, the answer must come from experiment. How can one decide this point? Very simply. If the $e^+e^-\gamma$ events are just some other decay mode of the W^o, then the ratio

$$\frac{W^o \to e^+e^-\gamma}{W^o \to e^+e^-}$$

must be independent of the production mechanism. If $S \neq W^o$, then in general the relative production rate will depend on the production mechanism, and is not likely to be constant over the whole experimental range. Thus one may, for example, divide events in two groups, based on the configuration or the total visible energy. If the above branching ratio is different for those two groups, then one can conclude that $S \neq W^0$.

The next question concerns the leptons. While we know already at this stage that something very remarkable is going on, in view of the distribution seen, we nevertheless would like to develop here some criterion relating to the way that the leptons occur in the process. Let us first ignore the question of the mass distribution seen in the $ee\gamma$ events. We now ask the following question: Is the interaction giving rise to the e^+e^- pair a new interaction, or is it basically the well-known interaction found in the standard model? If the latter is the case then the e^+e^- pair originates somewhere from a virtual W^0 or γ, or some linear combination of the two. Let us for definiteness assume that it is purely a W^0. Since a W^0 also couples to neutrino and quark pairs, we expect in such a case also to see events of the type $\nu\bar{\nu}\gamma$ and $\bar{q}q\gamma$, in the same proportion as that with which they couple to the W^0. In general, allowing for any W^0-γ mixture, we have a one-parameter situation, which was worked out in Ref. 24. One finds:

$$R_{e\nu} = \frac{\Gamma(\nu_e) + \Gamma(\nu_\mu) + \Gamma(\nu_\tau)}{\Gamma(e)} = 3\,\frac{1 - 2x + x^2}{1 + 2x + 5x^2}\ ,$$

where $\Gamma(a)$ is the decay rate of $S \to \bar{a}a\gamma$. Similarly

$$R_{eq} = \frac{3\Gamma(d) + 3\Gamma(u)}{\Gamma(e)} = \frac{18 + 22x^2}{1 + 2x + 5x^2}\ .$$

Events of the type $\nu\bar{\nu}\gamma$ show one γ and missing energy totaling the S mass. The $\bar{q}q\gamma$ events may be more difficult to detect, especially if the photon also has here a tendency to align with one of the two jets.

This brings us again to the question of the decay distribution. The fact that the photon and one of the particles align implies either an interaction between the photon and that particle (such as bremsstrahlung), or else that some mechanism forces both electron and photon in the same direction. Since we have opted for not believing bremsstrahlung we must conclude for the latter case. At this point we dare not speculate about what goes on in S-decay. But we might assume that this mechanism has no major influence on the total decay rate. Under that assumption the branching ratios quoted above remain valid.

If the branching ratios mentioned do not agree with experiment, then the possibility that there are new forces coupling to electrons must be taken seriously. For the time being we consider that unlikely.

All these considerations show that the S is very likely a complex object. We may ask: what are the building blocks of S and what keeps them together? If it is true that the e^+e^- configuration originates in a W^o then we might conclude that at least some W's seem to be present. Also, since S is created in $q\bar{q}$ collisions, the building blocks must couple to quarks. One might think of colored objects; in that case, one wonders why S would decay into $e^+e^-\gamma$.

In Ref. 25 it is suggested that S is a compound state of vector bosons. It is also suggested that the force responsible for this state is the short range, strong part of the normal Y-M force between massive vector bosons. In the standard model, that force is compensated by the exchange of a Higgs particle, and becomes visible by taking the limit of large Higgs mass. If that is the case, we are surely in for big complications. To begin with, this strong force has a range of about 1 TeV^{-1}, which is very short range. In order to get a state of W's at 90 GeV, we very likely need a chain of compound states (a compound state with forces due to an exchange of compound states, like the pion being mainly responsible for the Yukawa forces in a nucleus); moreover, this force is strong only between vector bosons moving with high velocity with respect to each other, and with their spins aligned along the relative velocity. That precludes simple, approximately spherical systems. No wonder that attempts at a dynamical understanding met with great difficulties (Lee et. al., Ref. 15)

For the time being, we cannot hope to get a dynamical understanding, even if the basic assumptions mentioned above are correct. How can we hope to get insight into this? There are several consequences of the model that can be tested:

- branching ratios into leptons, neutrinos and quarks as in standard model;
- reasonable weak isospin conservation;
- not well defined W,γ number, since there are WWW and WWγ vertices in the standard model;
- V,A structure of the lepton currents.

Furthermore, if the above concepts are right, then there will surely be many other compound states. Mass permitting, such states would preferentially decay into vector bosons; this is the right moment to turn to the recent publications by UA1 and UA2 concerning new remarkable events other than the eeγ events. Reference 25 was written without knowledge of these new events.

Essentially, UA1 and UA2 publish data on events characterized by large missing momentum. Such missing momentum is presumably carried away by neutrinos or new, as yet unknown, non-interacting particles.

First the UA1 events. To begin with, there are two events containing one single photonic shower. That may either be a single photon, or a π^0, etc. Assuming for definiteness that these showers are single photons, then the transverse energy of the photons is 54 and 44 GeV. The events (G and H in the UA1 publication) fit precisely what one would expect for a decay $S \to \nu\bar{\nu}\gamma$. In fact, taking into account the missing transverse energy presumably carried away by the neutrinos, the total mass is 93±5 and 84±6 GeV, which is in the right range. Unfortunately, it is impossible to say anything about distributions; at best, at some time in the future we might learn something about the photon energy distribution, or angular distribution with respect to the initial beam.

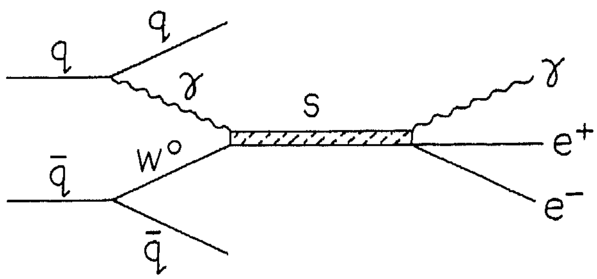
The other published UA1 events are hard to judge. In event A, for example, there seems to be a very fast muon, a "photon" jet, and a hadron jet, all more or less in the same direction, plus some hadron activity in the opposite direction.

The UA2 events all contain an electron in addition to hadron jets. Assuming that the missing energy is carried away by a neutrino, the first three events (A, B, and C) allow the interpretation that the $e\nu$ pair is from a real charged W decay. This interpretation also fixes the total mass at about 170 GeV. The hadron jets can perhaps be understood as a virtual W^0. Event D shows a considerably lower mass for the $e\nu$ system, and the interpretation of a real charged $W \to e\nu$ is difficult. Instead, the two jets make a mass of 86±7 GeV (this information is not in the paper in Ref. 1, but given in some transparancy shown at some conference); we might well have a real W^0 and a virtual charged $W \to e\nu$ in this event. This event can then also be seen as having a total mass of 170 GeV. Note that this is below the sum of the W^+ and W^0 mass, which is 176±3 GeV. We tentatively call the 170 GeV events a V particle, and while S is presumably an iso-scalar, V is perhaps an iso-vector.

If V is indeed a system with definite mass, then a crucial test is to measure the amount of decay of V into hadrons only. If V is a system made up from colored objects then we expect V to decay largely into quarks. If, however, V is a W compound, then the quark content should be limited to a level determined by W^0 and W^+ branching ratios into quarks.

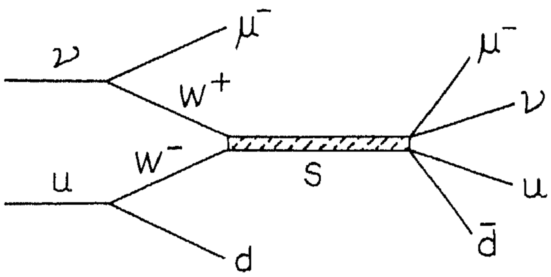
VIII. PRODUCTION MECHANISM

Given our ignorance about the S and V systems (the 90 and 170 GeV anomalous events) it is hard to say anything on the production of these particles. Within the picture of the compound vector boson model, we can at least guess a production mechanism for the decays observed. Thus S, decaying into γ and virtual W, might be produced by a near real γ and virtual W, as shown in the diagram below.



As it happens, the production kinematics is precisely sucn that we would expect the largest amplitudes from the decay configuration (photon and quarks aligning). That makes a relatively large production cross section plausible. The amplitude is significant only in a small kinematic domain, but that is precisely what is being excited in this kind of process.

If this view is correct, then we might speculate that S-like objects (after all, we do not know if there exist other such objects at even lower masses, since we really have no experimental evidence on that point*) might be produced in other processes. The production cross sections are presumable weak, and neutrino experiments might be the place to look. A typical event is shown in the diagram below; the S would probably be virtual.



* F. Sculli, private communication.

Such an event would show two negative muons; events of this type are called like-sign di-leptons, and they have been observed for some time [26] at a rate exceeding any conventional explanation.* Similarly wrong-sign leptons (μ^+ production in a ν beam for example) might be produced this way. Since we expect a substantial production rate increase with increasing neutrino energy, this is a process that could become open for better study in the near future at Fermilab. Similar remarks and speculations can be made for e^+e^- machines. We will not indulge in that any further here.

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