

POLARISATION OF VECTOR BOSONS PRODUCED BY NEUTRINOS

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We have examined the polarisation of the hypothetical intermediate bosons W (of weak interactions) when produced by neutrinos incident on the Coulomb fields of nucleons and nuclei. We have found in computation that they are strongly circularly polarised in the same sense as the incident neutrinos. This greatly affects the distribution in energy and angle of their decay products.

Consider the reaction

$$\nu + T \rightarrow T + \mu + W,$$

where T may be a nucleon or nucleus initially at rest. Let q be the 3-momentum of the incident neutrino, k that of the boson, α the spin state of the latter, and let λ, ξ denote collectively all other momenta and spins needed to specify the initial and final states. Then we compute transition matrix elements

$$\langle \alpha k \xi | S | q \lambda \rangle.$$

The 3 X 3 Hermitean matrix

$$\rho_{\alpha\beta} = \sum_{\xi\lambda} \langle \alpha k \xi | S | q \lambda \rangle \langle \beta k \xi | S | q \lambda \rangle^*,$$

whose spur integrated over all k gives the total transition probability for the process, may be used as a density matrix specifying the polarisation of the bosons emerging with momentum k . We have used as the three basic spin states, $\alpha = 1, 2, 3$, states of linear polarisation along the direction $k \times q$, along $(k \times q) \times k$, and along k . There are nine complex numbers $\rho_{\alpha\beta}$ for any k , but they have certain symmetry properties. For instance, trivially

$$\rho_{\alpha\beta} = \rho_{\beta\alpha}^* \quad (1)$$

Moreover as we work only to lowest order (in weak and electromagnetic coupling constant) time reversibility gives 1)

$$\langle A | S | B \rangle = \langle A' | S | B' \rangle^*,$$

where prime denotes time reversal. It follows that

$$\langle A | S | B \rangle = \langle A'' | S | B'' \rangle^*,$$

where A'' and B'' are obtained from A and B by time reversal followed by rotation through π about the

axis $q \times k$ (made to restore the signs of the three momenta). The result of these operations may be to change the indices ξ or λ , which does not matter for the summation, and to reverse the sign of the polarisation state $\alpha = 1$. Thus $\rho_{\alpha\beta}$ is real unless one, and not both, of the α and β take the value 1. Using also (1) we have to compute only six real numbers

$$\rho_{11}, \rho_{22}, \rho_{33}, \rho_{23} = \rho_{32}, \rho_{12} = -\rho_{21}, \rho_{13} = -\rho_{31}$$

We have computed these numbers in a variety of cases 2). We find that in general to a good approximation

$$\rho_{11} \approx \rho_{22} \approx -\rho_{12} = \rho_{21},$$

while all other ρ are small compared to these. For instance for neutrino energy 6 GeV, M_W = mass of the proton, anomalous magnetic moment zero, coherent process, material Cu (Fermi charge distribution) we find when integrating over all k :

$$\begin{aligned} \rho_{11} &= 40.1, & \rho_{22} &= 37.7, & \rho_{12} &= 38.4i, \\ \rho_{33} &= 3.06, & \rho_{13} &= -5.84i, & \rho_{23} &= -5.97. \end{aligned}$$

All numbers are in 10^{-38} cm^2 per nucleus. This means in fact that the W is produced almost entirely in a state of left handed circular polarisation (with spin angular momentum projection - 1 in the direction of motion).

To check this result, and especially the sign of the polarisation, we have made the following argument. Consider for simplicity the coherent process. Taking over the notations of ref. 3) we rewrite the matrix element in lowest order (underlined part in formula (5.9) of ref. 3)):

$$\begin{aligned} \langle \alpha k \xi | S | q \lambda \rangle &\propto \frac{1}{Q^2} [\underline{l_\mu^t e_\mu^\alpha} \left(\frac{2\epsilon}{D_2} - \frac{2\omega}{D_1} \right) - l_\mu^t e_\nu^\alpha \frac{A_{\mu\nu}}{D_1} \\ &- \frac{i}{2D_2} (\bar{u}_t^m \gamma^4 \gamma^\nu \gamma^\mu (1 + \gamma^5) u^n) e_\mu^\alpha Q_\nu]. \quad (2) \end{aligned}$$

As argued before there is a strong cancellation in the first term in this matrix-element because $2\epsilon/D_2 \approx 2\omega/D_1$ for small angles. To see this we first rewrite $D_1 = M^2 - m^2 - 2q_\mu p_\mu$, $D_2 = m^2 - M^2 - 2q_\mu k_\mu$. If θ_1 and θ_1 are the angles be-

tween p and q , and k and q , respectively, one finds in lowest order in m^2/ϵ^2 , M^2/ω^2 , θ_1^2 and θ_2^2 :

$$\frac{\epsilon D_1 - \omega D_2}{\epsilon D_1} \approx \frac{E}{\epsilon M^2 + \omega m^2} \left\{ \frac{m^4}{4\epsilon^2} - \frac{M^4}{4\omega^2} + \epsilon p \theta_1^2 - \omega k \theta_2^2 \right\}$$

$$\approx -\frac{EM^2}{4\omega^2\epsilon} \quad \text{if } m \ll M, \theta_1 = \theta_2 = 0.$$

In general the terms proportional to θ_1^2 and θ_2^2 will be of the same order as the term proportional to $M^4/4\omega^2$, but we leave them aside for simplicity.

To estimate the relative importance of the various terms inside square brackets in (2) we take as our standard of comparison $2\epsilon/D_2$. As seen from the above the combination $2\epsilon/D_2 - 2\omega/D_1$ is order $EM^2/2\omega^2\epsilon$. With the help of the relation $(k+Q)_\mu l_\mu^t = -p_\mu l_\mu^t = -iml^t$ where $l^t = (u^j m, u^n)$ one observes that the $A_{\mu\nu}$ term (due to relativistic terms in the equation of motion for the W) contains terms of order mQ/M^2 and Q/ω . The third term in square brackets (due to relativistic terms in the muon equation of motion) is of order Q/ϵ . As will be clear by methods to be described in the following, the factor $l_\mu^t e_\mu^\alpha$ is of order Q/ϵ with respect to all other spinor and polarisation vector combinations. This reduces the contribution of the $\epsilon/D_2 - \omega/D_1$ term to an insignificant fraction of that of the last term. Further the contribution of the $A_{\mu\nu}$ term may be neglected if $\epsilon \ll \omega$ (which is true in the region of small momentum transfer) and if $\epsilon \ll M^2/m$. In the region of small momentum transfer we have $\epsilon \approx mE/(M+m)$, and we conclude that the last term in square brackets of (2) will dominate provided $E \ll M^3/m^2$.

We will show now that the effect of the last term is to produce strongly polarised W. To this purpose we introduce explicit spinor and γ -matrix representations, as in ref. 4), where we take the direction of q to be the positive third (z) axis. We also take the vector k , whose magnitude will be of the order of the neutrino-energy if $m \ll M$, to be parallel to q . As a consequence the first two components of $Q = q - p - k$ are minus those of p . One easily finds:

$$I^\alpha \equiv (u^{m*} \gamma^\nu \gamma^\mu u^n) e_\mu^\alpha Q_\nu$$

$$= (u^{m*} u^n) a_j^\alpha Q_j + i(u^{m*} \sigma_{ij} u^n) a_j^\alpha Q_i,$$

where i, j take the values 1, 2, 3 only and $2i\sigma_{ij} = \gamma^i \gamma^j - \gamma^j \gamma^i$. Further with \hat{q}_j a unit vector in the q direction we have $a_j^\alpha = e_j^\alpha - \hat{q}_j e_4^\alpha / q_4$. We take the muon and neutrino to be in the state with spins down, the contribution of the other state being an order of magnitude smaller for small angles (i.e., small components p_1 and p_2) and relativistic ener-

gies. Working out now the spinor products one finds:

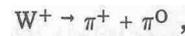
$$I^\alpha = \frac{1}{2} \sqrt{\frac{m+\epsilon}{\epsilon}} \left(1 + \frac{p_3}{m+\epsilon}\right) \{Q \cdot a^\alpha + iQ \times q \cdot a^\alpha\} + I' \cdot a^\alpha \times Q$$

where I' is zero if $p_1 = p_2 = 0$. Neglecting then the I' term we find

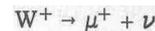
$$I^1 \propto Q_1 + iQ_2, \quad I^2 \propto Q_2 - iQ_1, \quad I^3 \propto \frac{M}{2\omega} Q_3,$$

which gives the aforementioned relative values to the ρ_{ij} provided that on the average Q_1 and/or Q_2 are of the same order of magnitude as Q_3 , and not an order M/ω smaller (of course this reasoning is correct only provided $Q^2 \ll \epsilon^2$, thus $Q_3 \ll \epsilon$, otherwise the approximation $p_1, p_2 \ll p_3$ would be contradictory). Thus, for the muon outgoing in directions such that $p_1, p_2 \ll p_3$, but $p_1, p_2 \approx Q_3 \approx M^2/2E$ we find in lowest order in p_1, p_2 the polarisation effect. More careful consideration of I^α for the case that both W and muon come out in the forward direction ($Q_1 = Q_2 = 0$) shows that then all ρ 's are small and furthermore all of the same order of magnitude ($\rho_{11} \approx \rho_{22} \approx \rho_{33} \approx -i\rho_{12}$), thus even then we have a slightly polarised W. The numerical computation shows, however, that the most important contributions come from processes where the muon comes out at a small but non zero angle. Numerical investigation also fully confirms the above reasoning, i.e., the cancellation mentioned and the relative unimportance of the $A_{\mu\nu}$ term.

For pure circular polarisation the angular distribution of pions in the decay



with W at rest is proportional to $\sin^2\theta$, where θ is the angle between either of the pions and the polarisation axis. In the decay



the angular distribution of the muon, neglecting its mass, is $(1 - \cos\theta)^2$, where $\theta = \pi$ is the direction of the W spin. Thus for the decay of fast W the muons are slower and have more angular spread than in the absence of polarisation.

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