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## Meson Production by Neutrinos Incident on Nucleons.

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**Summary.** — Production and exchange of nonstrange pseudoscalar and vector mesons are considered in the peripheral model. For nonstrange mesons a consequence of this model is that the corresponding  $\nu$  and  $\bar{\nu}$  cross-sections are equal, assuming that  $\nu$  and  $\bar{\nu}$  couple to the same isospin multiplet. The cross-section for production or  $\rho$  and  $\omega$  with  $\pi$  exchange, with and without simultaneous excitation of the  $\frac{3}{2}, \frac{3}{2}$  resonance, is found to be a fraction (with strong dependence on badly known form factors) of the « elastic » cross-section,  $\sigma(\nu+n \rightarrow p+\mu^-)$ , at a few GeV incident-neutrino energy. Production of  $\pi$  and  $\eta$  mesons with  $\rho$  and  $\omega$  exchange is computed also, and is found to be very small.

### 1. - Introduction.

It is conceivable that vector mesons play an important role in the inelastic pion production by neutrinos incident on nucleons as observed in the CERN neutrino experiment <sup>(1)</sup>. Production of single pions <sup>(2,3)</sup> and kaons <sup>(4,5)</sup> has

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<sup>(2)</sup> J. S. BELL and S. M. BERMAN: *Nuovo Cimento*, **25**, 404 (1962); P. DENNERY: *Phys. Rev.*, **127**, 664 (1962); N. DOMBEY: *Phys. Rev.*, **127**, 653 (1962); N. CABIBBO and G. DA PRATO: *Nuovo Cimento*, **25**, 611 (1962).

<sup>(3)</sup> S. M. BERMAN and M. VELTMAN: to be published.

<sup>(4)</sup> A. FUJII and E. CELEGHINI: *Nuovo Cimento*, **28**, 90 (1963).

<sup>(5)</sup> G. R. HENRY, J. LØVSETH and J. D. WALECKA: to be published.

been discussed by several authors. Here we consider peripheral meson production involving nonstrange vector and pseudoscalar mesons.

In Sect. 2 we compute cross-sections for events of the type  $\nu + \mathcal{N}^0 \rightarrow \mathcal{N}' + \mu^- + b$  where  $b$  is a  $\rho$  or  $\omega$ . In Sect. 3, cross-sections for events of the type  $\nu + \mathcal{N}^0 \rightarrow \mu^- + b + X$  are considered, where  $X$  represents the final state in the reaction  $\pi + \mathcal{N}^0 \rightarrow X$ . Production of  $\pi, \eta$  mesons by vector-meson exchange is computed in Sect. 4 and is found to be very small.

2. - Cross-sections for  $\rho$  and  $\omega$  production.

The cross-section for  $\rho$  and  $\omega$  production are computed according to the peripheral Feynman diagram in Fig. 1a. The four-momenta associated with

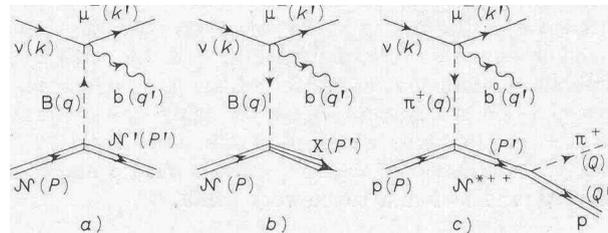
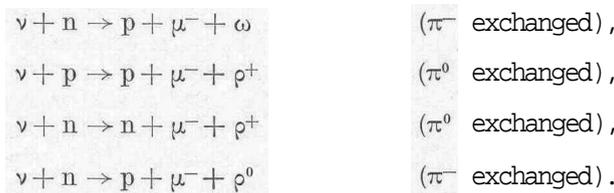


Fig. 1. - a) Peripheral Feynman diagram for  $\nu + \mathcal{N}^0 \rightarrow \mathcal{N}' + \mu^- + b$ . The four-momenta of the particles are indicated in parentheses; dashed lines correspond to pseudoscalar mesons, wavy lines to vector mesons. b) Peripheral diagram for  $\nu + \mathcal{N}^0 \rightarrow \mu^- + b + X$ . c) Diagram for  $\mathcal{N}^{*}$  production with vector meson production; expected to be a major contributor to diagram 1b.

each particle is indicated in parentheses. Considering only nonstrange particles, the possible reactions indicated in graph 1» are:



In these reactions the meson current coupled to the leptons transforms supposedly as the (+) component of an isovector. The antineutrino reactions are found by replacing  $\nu$  by  $\bar{\nu}$  and taking the isospin (—) part of the meson current; apart from very small effects of electromagnetic origin (e.g., n,p mass difference, etc.) the corresponding neutrino and antineutrino cross-sections are

equal. This is a situation peculiar to this model, and depends entirely on the assumption that  $\nu$  and  $\bar{\nu}$  couple to the same isospin multiplet, or equivalently (if time-reversal invariance holds), that the meson current is of the first kind under  $G$ -parity<sup>(6)</sup>. From the isospin properties we find, apart from small electromagnetic effects again,

$$\sigma(\nu + p \rightarrow p + \mu^- + \rho^+) = \sigma(\nu + n \rightarrow n + \mu^- + \rho^+) = \frac{1}{2}\sigma(\nu + n \rightarrow p + \mu^- + \rho^0).$$

Thus there are two independent cross-sections which we take to be  $\sigma(\nu + n \rightarrow p + \mu^- + \omega)$  and  $\sigma(\nu + p \rightarrow p + \mu^- + \rho^+)$ .

The lepton-meson vertex of graph. 1a is given by

$$(1) \quad i\bar{U}_\mu(k')\gamma_\alpha(1 + \gamma_5) U_\nu(k)\langle b | J_\alpha^V + J_\alpha^A | B \rangle.$$

The leptonic part is as usual, and  $J^V$  and  $J^A$  are the vector and axial vector components of the mesonic weak current (\*). Requiring now that  $GJ^VG^{-1} = J^V$  and  $GJ^AG^{-1} = -J^A$  (where  $G$  is the  $G$ -parity operator) implies that either  $\langle b | J^V | B \rangle$  or  $\langle b | J^A | B \rangle$  is absent, for under 6

$$(2) \quad \langle b | J^V + J^A | B \rangle = g_b g_B \langle b | J^V - J^A | B \rangle,$$

where  $g_b$  and  $g_B$  are the  $G$ -parities of  $b$  and  $B$  (equal to  $\pm 1$ ), and therefore

$$\langle \omega | J^A | \pi \rangle = \langle \rho | J^A | \eta \rangle = \langle \rho | J^V | \pi \rangle = 0.$$

Considering omega production by pion exchange ( $b = \omega$ ,  $B = \pi^-$ ), the coupling strength of the  $\pi$ - $\omega$  current to the lepton current for  $(k - k')^2 = 0$  may be found from the observed (purely  $\Delta I = 1$ )  $\omega \rightarrow \pi^0 + \gamma$  decay rate<sup>(7)</sup> and CVC theory<sup>(8)</sup>. The lepton-meson vertex is then given by (momenta denoted in Fig. 1a)

$$(3) \quad ig_\nu \bar{U}_\mu \gamma_\alpha (1 + \gamma_5) U_\nu \varepsilon_{\alpha\beta\sigma} S_\beta q'_\rho q_\sigma F_1([k - k']^2),$$

where  $F_1$  is a form factor with  $F_1(0) = 1$ . Furthermore,  $g_\nu \equiv \lambda G$  and  $\lambda^2$  is found<sup>(5)</sup> to be  $7.5 \text{ GeV}^{-2}$ ;  $G$  is the weak coupling constant  $GM_\nu^2 \approx 10^5$ , and  $S_\beta$  is the omega linear polarization vector with  $S_\beta q'_\beta = 0$ . Note that the meson current is conserved, i.e.,  $(q' - q)_\alpha \varepsilon_{\alpha\beta\sigma} S_\beta q'_\rho q_\sigma = 0$ . Thus, according to ADLER<sup>(9)</sup>,

<sup>(6)</sup> T. D. LEE: Lecture Notes, CERN 61-30 (1961); S. WEINBERG: *Phys. Rev.*, **112**, 1375 (1958).

(\*) We use the metric  $k'^2 = \mathbf{k}'^2 + k_4'^2 = \mathbf{k}'^2 - k_0'^2 = -m_\mu^2$ .

<sup>(7)</sup> M. ROOS: *Nucl. Phys.*, **52**, 1 (1964).

<sup>(8)</sup> R. P. FEYNMAN and M. GELL-MANN: *Phys. Rev.*, **109**, 193 (1958).

<sup>(9)</sup> S. L. ADLER: *Phys. Rev.*, **135**, B 963 (1964).

if the lepton factor is proportional to  $(q' - q)$ , as happens if the  $\mu$  (with  $m_\mu$  taken to be 0) is in the forward direction, the cross-section will be zero. At the nucleon-pion vertex the usual pseudoscalar coupling is used, namely,

$$(4) \quad i\sqrt{2}g_{N\pi} \bar{U}_{N'} \gamma_5 U_N F_B(q^2),$$

where  $g_{N\pi}^2/4\pi \approx 15$ , and  $F_B$  is a form factor with  $F_B(0) = 1$ .

With these vertices the following differential cross-section is obtained for omega production (averaging over incident spins and summing over outgoing spins):

$$(5) \quad d_9\sigma = \frac{\lambda^2 G^2 g_{N\pi}^2 F_1^2([k - k']^2) F_B^2(q^2) (q^2 + \Delta M^2)}{(2\pi)^5 (-p \cdot k) (q^2 + m_B^2)^2} \delta_4(k' + q' + p' - p - k) \cdot \\ \cdot \{k \cdot q' [(q \cdot q')(k' \cdot q) - q^2 (q' \cdot k)] + k \cdot q [(q \cdot q')(k' \cdot q') + m_b^2 (q \cdot k')]\} \frac{d_3 \mathbf{q}'}{q'_0} \frac{d_3 \mathbf{k}'}{k'_0} \frac{d_3 \mathbf{p}'}{p'_0},$$

where momenta are again defined in Fig. 1a, and  $\Delta M = M_{N'} - M_N$ . The total cross-section may be trivially reduced to a four-dimensional integral, and it was then numerically integrated. All numerical integrations were done on the CERN 7090 computer.

The form factors were taken to be of the Clementel-Villi type<sup>(10)</sup>:

$$(6) \quad F_1(\{k - k'\}^2) = \left[1 + \frac{\{k - k'\}^2}{M_{F1}^2}\right]^{-1}, \quad F_B(q^2) = \left[1 + \frac{q^2}{M_{FB}^2}\right]^{-1}.$$

The total cross-section is very dependent on the choices of  $M_{F1}$  and  $M_{FB}$ . The pseudoscalar  $\pi-N$  coupling gives rise to the  $(q^2 + \Delta M^2)$  factor in eq. (5), partly cancelling the  $(q^2 + m_B^2)^{-2}$  of the propagator and making high- $q^2$  values more important (which is, in fact, contrary to the spirit of the peripheral model). The strong dependence on  $M_{F1}$  arises from the fact that  $\sigma$  is zero if the muon is in the forward direction, as noted above. Explicitly this may be verified by observing that the dot products in the  $\{ \}$  in eq. (5) may be reduced to

$$\{ \} = -\frac{1}{2}(k - k')^2 \left[ (k \cdot q')^2 + (k' \cdot q')^2 + \frac{m_b^2}{2} (k - k')^2 \right],$$

where terms in  $m_\mu^2$  are neglected. Thus  $F_1$  clearly plays an important role in the cross-section obtained from eq. (5). Table I lists the total production cross-section for several choices of form factors. We see that the cross-section is approximately proportional to the lepton form factor mass squared ( $\sigma \sim M_{F1}^2$ ).

(10) E. CLEMENTEL and C. VILLI: *Nuovo Cimento*, **4**, 1207 (1956).

Finally it is now immediately possible to estimate  $\rho$  production by  $\eta$  exchange on the basis of graph 1 $\llcorner$  (\*). Generalizing the Cabibbo weak coupling scheme <sup>(11)</sup> by requiring that the meson current be an octet (\*\*), the  $\eta$ ,  $\rho$  and  $\pi$ ,  $\omega$  couplings

TABLE I. -  $\sigma(\nu+n \rightarrow p+\mu^-+\omega)$  and  $\sigma(\nu+p \rightarrow p+\mu^-+\rho^+)$  in units of  $10^{-40}$  cm<sup>2</sup>.

|                    |                 | $\sigma(\nu+n \rightarrow p+\mu^-+\omega)$ |       |      | $\sigma(\nu+p \rightarrow p+\mu^-+\rho^+)$ |       |      |
|--------------------|-----------------|--|-------|------|--|-------|------|
| Form factor        | $M_{\text{FI}}$ | 0.66                                       | 0.75  | 0.66 | 0.66                                       | 0.75  | 0.66 |
| masses, GeV        | $M_{\text{FB}}$ | 0.939                                      | 0.939 | 1.2  | 0.939                                      | 0.939 | 1.2  |
| $E_\nu$ (lab), GeV |                 |  |       |      |  |       |      |
|                    | 2               | 1.0  | 1.25  | 1.39 | 0.40                                       | 0.47  | 0.53 |
|                    | 3               | 3.76                                       | 4.93  | 5.34 | 1.15                                       | 1.39  | 1.57 |
|                    | 5               | 9.3  | 12.8  | 13.4 | 2.22                                       | 2.71  | 3.06 |
|                    | 7               | 14.0                                       | 19.7  | 20.4 | 2.90                                       | 3.54  | 4.02 |
|                    | 10              | 19.7                                       | 28.4  | 28.8 | 3.56                                       | 4.38  | 4.97 |

$\sigma(\nu+p \rightarrow p+\mu^-+\rho^+) = \sigma(\nu+n \rightarrow n+\mu^-+\rho^+) = \frac{1}{2} \sigma(\nu+n \rightarrow p+\mu^-+\rho^0)$ .

to the leptonic current are identical. Therefore eq. (5) is valid for  $p$  production by  $\eta$  exchange if we make the substitution  $2g_{\mathcal{N}\pi}^2 \rightarrow g_{\mathcal{N}\eta}^2$  and if we use the masses appropriate to the  $\rho$ ,  $\eta$  case. The coupling constant  $g_{\mathcal{N}\eta}^2$  has been estimated by DE SWART <sup>(13)</sup> on a unitary-symmetry model, and he finds  $g_{\mathcal{N}\eta}^2 \ll g_{\mathcal{N}\pi}^2$ . The high mass in the propagator also reduces production. Therefore,  $\rho$  production by  $\eta$  exchange is very much smaller than  $\omega$  production by  $\pi$  exchange.

Graph 1a may also be used to compute  $\rho$  production with  $\pi$  exchange. In this case the meson-lepton coupling is pure axial vector, and we take for the lepton vertex

$$(7) \quad ig_A \bar{U}_\mu \gamma_\alpha (1 + \gamma_5) U_\nu S_\alpha F_1(\{k - k'\}^2),$$

where  $S_\alpha$  is the linear  $\rho$  polarization vector ( $q'_\alpha S_\alpha = 0$ ) and derivative couplings

(\*) We can treat the  $\rho$  as a stable particle because:

- $I(\rho \rightarrow \pi + \pi)$  is a slowly varying function of  $m_\rho$  ( $I \sim m_\rho$ ),
- $[I(\rho \rightarrow \pi + \pi)/2m_\rho]^2 \ll 1$ , and

c)  $\sigma(\nu + \mathcal{N} \rightarrow \mathcal{N} + \mu^- + \rho^+)$  can be closely approximated by a linear function of the  $\rho$  mass in the interval  $0.75 \text{ GeV} - \frac{1}{2} \Gamma_\rho < \rho \text{ mass} < 0.75 \text{ GeV} + \frac{1}{2} \Gamma_\rho$ .

<sup>(11)</sup> N. CABIBBO: *Phys. Rev. Lett.*, **10**, 531 (1963).

(\*\*) We are indebted to Prof. V. TELEGGI for suggesting to us this generalization of the Cabibbo scheme.

<sup>(12)</sup> J. J. DE SWART: *Rev. Mod. Phys.*, **35**, 916 (1963).

are ignored. This interaction is equivalent to the usual  $\beta$ -decay or  $\mu$ -capture axial vector interaction in the sense that the quantum numbers of the exchanged systems are the same. In addition there will be a term like the induced pseudoscalar, presumably dominated by the one-pion pole. The coupling constant  $g_A$  may now be estimated by a Goldberger-Treiman<sup>(13)</sup> argument. In analogy with the  $\beta$ -decay case,

$$\langle \varrho | J_\alpha^A | \pi \rangle = g_A S_\alpha + \frac{S_\beta (q' - q)_\beta (q' - q)_\alpha g_{\pi\pi\rho} g_{\pi 1}}{(q' - q)^2 + m_\pi^2}$$

and

$$\langle \varrho | \frac{\partial J_\alpha^A}{\partial x_\alpha} | \pi \rangle = S_\alpha (q' - q)_\alpha \left\{ g_A + g_{\pi\pi\rho} g_{\pi 1} \left( 1 - \frac{m_\pi^2}{(q' - q)^2 + m_\pi^2} \right) \right\}.$$

As usual, the divergence of the axial current is supposed to be dominated by the one-pion pole. Thus, we must have

$$g_A = -g_{\pi\pi\rho} g_{\pi 1},$$

where  $g_{\pi\pi\rho}$  and  $g_{\pi 1}$  are the coupling constants at the  $\rho\pi\pi$  vertex and the  $\pi\nu\mu$  vertex. These coupling constants are taken at their values for each particle on the mass shell:

$$\Gamma(\varrho \rightarrow \pi_1 + \pi_2) = \left( \frac{g_{\pi\pi\rho}^2}{4\pi} \right) \frac{1}{48m_\rho^2} [(m_\rho^2 - m_{\pi_1}^2 - m_{\pi_2}^2)^2 - 4m_{\pi_1}^2 m_{\pi_2}^2]^{\frac{3}{2}}$$

and

$$\Gamma(\pi^+ \rightarrow \mu^+ + \nu) = \left( \frac{g_{\pi 1}^2}{4\pi} \right) \left( \frac{m_\mu}{m_\pi} \right)^2 \frac{1}{m_\pi} \left[ 1 - \frac{m_\mu^2}{m_\pi^2} \right]^2.$$

Using the constants compiled by Eos<sup>(7)</sup>, we have

$$g_{\pi\pi\rho}^2 = 114, \quad g_{\pi 1}^2/m_\pi^2 = 2.21 \cdot 10^{-14}$$

and

$$(8) \quad g_A^2 = \lambda^2 G^2 \approx \lambda^2 \frac{10^{-10}}{M_D^4},$$

with  $\lambda^2 = 1.0 \text{ GeV}^{-2}$ .

<sup>(13)</sup> M. L. GOLDBERGER and S. B. TREIMAN: *Phys. Rev.*, **110**, 354, 1178 (1958).

With the lepton vertex of expression (7) we find for  $\sigma(\nu + p \rightarrow p + \mu^- + \rho^+)$

$$(9) \quad d_0 \sigma = \frac{g_{\mathcal{N}^* \pi}^2 g_A^2 F_B^2(q^2) F_1^2([q - q']^2) (\Delta M^2 + q^2)}{4(2\pi)^5 (-p \cdot k) (q^2 + m_B^2)^2} \delta_4(k' + q' + p' - p - k) \cdot \left\{ -k \cdot k' + \frac{2}{m_b^2} (k \cdot q') (k' \cdot q') \right\} \frac{d_3 p'}{p'_0} \frac{d_3 k'}{k'_0} \frac{d_3 q'}{q'_0},$$

where the notation of eq. (5) is followed. The total  $p$  production cross-section was also computed as a four-fold numerical integral. The results are listed in Table I. The dependence on form factors is still important, with  $\sigma \sim M_{F1}$ , roughly.

### 3. - Cross-sections for $\nu + \mathcal{N} \rightarrow \mu^- + b + X$ .

If in the peripheral graph 1a the outgoing nucleon is replaced by a state  $X$  (where  $B + \mathcal{N} \rightarrow X$ ) we arrive at a new graph (1b) which may also be important for  $\rho, \omega$  production. Since the most prominent feature of  $\pi, \mathcal{N}$  scattering is the  $\frac{3}{2} \frac{3}{2}$  resonance ( $\mathcal{N}^{*}$ ) we are led to consider the special case of graph 1c. This graph is evaluated using the Earita-Schwinger formalism<sup>(14)</sup>, and following closely the treatment of ref. (3) for the  $\mathcal{N}^{*}$ .

The  $\pi \mathcal{N} \mathcal{N}^{*}$  interaction is taken to be of the form

$$(10) \quad (g_{\pi \mathcal{N} \mathcal{N}^*} / m_\pi) \mathcal{N}^* \left( \frac{\partial}{\partial x_\alpha} \pi \right) \mathcal{N},$$

where the form of the interaction and the coupling constant are chosen to yield the observed cross-section for  $\mathcal{N}^{*}$  production ( $g_{\pi \mathcal{N} \mathcal{N}^*} = 2.32 =$  renormalized  $\pi \mathcal{N} \mathcal{N}^{*}$  coupling constant). The  $\mathcal{N}^{*}$  propagator,  $D_{\mu\nu}(M')$ , is given by<sup>(3)</sup>

$$D_{\mu\nu}(M') = \left[ \delta_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{i}{3M'} (\gamma_\mu p'_\nu - p'_\mu \gamma_\nu) + \frac{2}{3M'^2} p'_\mu p'_\nu \right] \cdot (-i p'_\alpha \cdot \gamma_\alpha + M') (p'^2 + M'^2)^{-1},$$

where a prime refers to the  $\mathcal{N}^{*}$  (see Fig. 1c) and

$$M' = M_0 - \frac{i}{2} \Gamma [-p'^2]^{\frac{1}{2}} (-p'^2)^{\frac{1}{2}} / M_0, \quad M_0 = 1.238 \text{ GeV},$$

and

$$\Gamma(x) = \frac{g_{\pi \mathcal{N} \mathcal{N}^*}^2}{m_\pi^2} \frac{1}{6\pi} \frac{1}{(2x)^5} [(x + M_p)^2 - m_\pi^2] \cdot [(x^2 - M_p^2 - m_\pi^2)^2 - 4M_p^2 m_\pi^2]^{\frac{3}{2}}.$$

<sup>(14)</sup> W. RARITA and J. SCHWINGER: *Phys. Rev.*, **60**, 61 (1941).

Since  $\Gamma \ll M_0$  for the  $\mathcal{N}^{*}$ , we neglect all terms of order  $\Gamma/M_0$  or smaller everywhere except in the denominator of the  $\mathcal{N}^{*}$  propagator where the term of first order in  $\Gamma/M_0$  is kept. Graph 1c then gives the following cross-section:

$$(11) \quad d_6\sigma = \frac{2(g_{\pi\mathcal{N}\mathcal{N}^*}/m_\pi)^2 |l_\mu J_\mu|^2 k_0 k'_0 F_B^2(q^2) P_0' \Gamma}{3(p \cdot k)(2\pi)^6 (q^2 + m_B^2)^2 [(p'^2 + M_0^2)^2 - p'^2 \Gamma^2]} \cdot \left[ q^2 + \left( \frac{q \cdot p'}{M_0} \right)^2 \right] [M_0 M_{\mathcal{N}^*} - p \cdot p'] \frac{d_3 k'}{k'_0} \frac{d_3 q'}{q'_0},$$

where the momenta are defined in Fig. 1c, and  $l_\mu J_\mu$  represents the lepton-meson vertex (for  $\pi$  exchange,  $\omega$  production,  $l_\mu J_\mu$  is expression (3); for  $\pi$  exchange,  $\rho$  production  $l_\mu J_\mu$  is expression (7)).

Again there are only two independent cross-sections, taken as

$$\begin{aligned} \sigma(\nu + p \rightarrow \mu^- + \rho^0 + \mathcal{N}^{*++}) &= \frac{3}{2} \sigma(\nu + p \rightarrow \mu^- + \rho^+ + \mathcal{N}^{*+}) = \\ &= \frac{3}{2} \sigma(\nu + n \rightarrow \mu^- + \rho^+ + \mathcal{N}^{*0}) = 3 \sigma(\nu + n \rightarrow \mu^- + \rho^0 + \mathcal{N}^{*+}) \end{aligned}$$

and

$$\sigma(\nu + p \rightarrow \mu^- + \omega + \mathcal{N}^{*++}) = 3 \sigma(\nu + n \rightarrow \mu^- + \omega + \mathcal{N}^{*+}).$$

The corresponding antineutrino reactions are obtained as before.

From eq. (11) the total production cross-sections were computed as five-fold numerical integrals; they are listed in Table II.

Equation (11), as can be seen from graph 1b, entails the cross-section  $B + \mathcal{N} \rightarrow X$  (i.e., here  $\pi + \mathcal{N} \rightarrow \mathcal{N}^{*}$ ) where, however, B is virtual. The fact

TABLE II. -  $\sigma(\nu + p \rightarrow \mathcal{N}^{*++} + \mu^- + \omega)$  and  $\sigma(\nu + p \rightarrow \mathcal{N}^{*++} + \mu^- + \rho^0)$  in units of  $10^{-40} \text{ cm}^2$ .

|                         |                 | $\sigma(\nu + p \rightarrow \mathcal{N}^{*++} + \mu^- + \omega)$ |       |      | $\sigma(\nu + p \rightarrow \mathcal{N}^{*++} + \mu^- + \rho^0)$ |       |       |
|-------------------------|-----------------|--|-------|------|--|-------|-------|
| Form factor masses, GeV | $M_{\text{FB}}$ | 0.66   | 0.75  | 0.66 | 0.66   | 0.75  | 0.66  |
|                         | $M_{\text{F1}}$ | 0.939  | 0.939 | 1.2  | 0.939  | 0.939 | 1.2   |
| $E_\nu$ (lab), GeV      |                 |  |       |      |  |       |       |
| 3                       |                 | 1.1  | 1.37  | 1.71 | 0.58   | 0.697 | 0.876 |
| 5                       |                 | 5.5  | 7.48  | 9.26 | 2.43   | 2.96  | 3.81  |
| 7                       |                 | 10.8   | 15.1  | 18.7 | 4.34   | 5.29  | 6.89  |
| 10                      |                 | 19.1   | 27.2  | 33.8 | 7.21   | 8.75  | 11.5  |

$$\sigma(\nu + p \rightarrow \mathcal{N}^{*++} + \mu^- + \omega) = 3 \sigma(\nu + n \rightarrow \mathcal{N}^{*+} + \mu^- + \omega),$$

$$\begin{aligned} \sigma(\nu + p \rightarrow \mathcal{N}^{*++} + \mu^- + \rho^0) &= \frac{3}{2} \sigma(\nu + p \rightarrow \mathcal{N}^{*+} + \mu^- + \rho^+) = \\ &= \frac{3}{2} \sigma(\nu + n \rightarrow \mathcal{N}^{*0} + \mu^- + \rho^+) = 3 \sigma(\nu + n \rightarrow \mathcal{N}^{*+} + \mu^- + \rho^0). \end{aligned}$$

that the  $\pi$  is virtual necessitates the use of an explicit formula for the cross-section rather than directly the experimental  $\pi, \mathcal{N}$  cross-section since in the latter case one neglects important kinematical effects. For a given pion energy (in the  $\mathcal{N}^{*}$  rest system) a virtual pion may have as compared to a real pion considerably higher three-momentum and it therefore penetrates the angular momentum barrier more easily and has relatively a higher cross-section than a physical pion. Using the experimental  $\pi, \mathcal{N}$  cross-section with the same form factors as before, and without allowance for the angular momentum barrier, leads to an underestimate of graph 1c by roughly a factor of 4 for neutrino energies of a few GeV.

#### 4. - Production of $\pi, \eta$ mesons by $\rho, \omega$ exchange.

Production of  $\pi, \eta$  mesons by  $\rho, \omega$  exchange has been computed on the basis of the graph in Fig. 2. All of these cross-sections are small because of the weak coupling of the vector mesons to nucleons and the high masses in the propagators. The three independent cross-sections are:

$$\begin{aligned} & \sigma(\nu + p \rightarrow p + \mu^- + \pi^+) = & (\omega \text{ exchanged}), \\ = & \sigma(\nu + n \rightarrow n + \mu^- + \pi^+) & (\omega \text{ exchanged}), \\ & \sigma(\nu + n \rightarrow p + \mu^- + \eta) & (\rho^- \text{ exchanged}), \end{aligned}$$

and

$$\begin{aligned} & \sigma(\nu + p \rightarrow p + \mu^- + \pi^+) = & (\rho^0 \text{ exchanged}), \\ = & \sigma(\nu + n \rightarrow n + \mu^- + \pi^+) = & (\rho^0 \text{ exchanged}), \\ = & \frac{1}{2} \sigma(\nu + n \rightarrow p + \mu^- + \pi^0) . & (\rho^- \text{ exchanged}). \end{aligned}$$

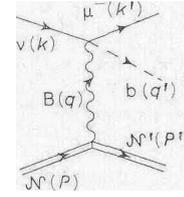


Fig. 2. - Diagram for pseudo-scalar meson production by vector-meson exchange.

The matrix element for graph 2 may be written

$$\mathcal{M} = \frac{-i(2\pi)^4}{(2q_0')^{1/2}} (q^2 + m_B^2)^{-1} \bar{U}_\mu \gamma_\alpha (1 + \gamma_5) U_\nu F_1([k - k']^2) F_B(q^2) \cdot \bar{U}_{\mathcal{N}'} f_V \left( T_{\alpha\beta} [\gamma_\beta + \lambda_m \sigma_{\beta\rho} q_\rho] + \frac{ig_A}{m_B} \Delta M q_\alpha \right) U_{\mathcal{N}},$$

where  $T_{\alpha\beta} \equiv g_A \delta_{\alpha\beta} + g_V \varepsilon_{\alpha\beta\sigma\rho} q'_\sigma q_\rho$ , with  $g_A$  and  $g_V$  as defined above (for a given reaction, of course, either  $g_A$  or  $g_V \equiv 0$ , according to eq. (2)),  $\sigma_{\alpha\beta} \equiv i/2 \cdot (\gamma_\alpha \gamma_\beta - \gamma_\beta \gamma_\alpha)$ , and  $f_V$  and  $\lambda_m$  are coupling constants for the  $B\mathcal{N}\mathcal{N}'$  vertex.

These coupling constants have been estimated by Liu and Singer <sup>(15)</sup>, namely:

| Vertex        | $f_V^2/4\pi$ | $\lambda_m$             |
|---------------|--------------|-------------------------|
| $\omega N, N$ | 1.0          | 0                       |
| $\rho^- n, p$ | 0.2          | $-1.5 \text{ GeV}^{-1}$ |
| $\rho^0 N, N$ | 0.1          | $-1.5 \text{ GeV}^{-1}$ |

in units where  $g_{N\pi}^2/4\pi \approx 15$ .

The production cross-section is now given by

$$d_9\sigma = \frac{1}{(2\pi)^3} (\text{Flux})^{-1} \delta_4(p' + k' + q' - p - k) d_3 p' d_3 q' d_3 k' |\mathcal{M}|^2$$

where  $|\mathcal{M}|^2$  is averaged over initial spins and summed over final spins. The traces in  $|\mathcal{M}|^2$  are easily evaluated, but the tensor inner products lead to a

TABLE III. - Cross-sections for pseudoscalar meson production by vector meson exchange in units of  $10^{-40} \text{ cm}^2$ .

| Reaction           | $\omega$ exchange                       | $\rho$ exchange                         |  |
|--------------------|---|---|--|
|                    | $\nu + p \rightarrow p + \mu^- + \pi^+$ | $\nu + p \rightarrow p + \mu^- + \pi^+$ | $\nu + n \rightarrow p + \mu^- + \eta$ |
| $E_\nu$ (lab), GeV |   |   |  |
| 3                  | 0.422                                   | 0.267                                   | 0.243                                  |
| 5                  | 0.95                                    | 0.415                                   | 0.664                                  |
| 7                  | 1.47                                    | 0.543                                   | 1.11                                   |
| 10                 | 2.15                                    | 0.711                                   | 1.74                                   |

Form factor masses:  $M_{F1} = 0.66 \text{ GeV}$ ,  $M_{FB} = 0.939 \text{ GeV}$ .

large number of terms. In Table III are listed the computed cross-sections for pseudoscalar meson production with vector-meson exchange.

<sup>(15)</sup> L. S. LIU and P. SINGER: *Phys. Rev.*, **135**, B 1017 (1964).

\* \* \*

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## RIASSUNTO (\*)

Si studia, col modello periferico, la produzione e lo scambio di mesoni pseudoscalari e vettoriali non strani. Per i mesoni non strani una conseguenza di questo modello è che le corrispondenti sezioni d'urto dei  $\nu$  e  $\bar{\nu}$  sono uguali, nell'ipotesi che  $\nu$  e  $\bar{\nu}$  si accoppino allo stesso multipletto di isospin. Si trova che la sezione d'urto per la produzione di  $\rho$  e  $\omega$  con scambio di  $\pi$ , con e senza eccitazione simultanea della risonanza  $\frac{3}{2}^+$ ,  $\frac{3}{2}^-$ , è una frazione (con forte dipendenza da fattori di forma poco noti) della sezione d'urto « elastica »,  $\sigma(\nu+n \rightarrow p+\mu^-)$ , a pochi GeV di energia del neutrino incidente. Si calcola anche la produzione di mesoni  $\pi$  e  $\eta$  con scambio di  $\rho$  e  $\omega$  e si trova che è molto piccola.

(\*) Traduzione a cura della Redazione.