Mass Scale of Weak Interactions.

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1. — Introduction.

The work of 't Hooft on the renormalization of gauge theories [1] has resulted in a fundamental change in theoretical elementary-particle physics. Gauge theories of weak and e.m. interactions [2] have become credible, and the standard model [3] including a colour gauge theory of strong interactions is now very popular. The observation of neutral currents [4] as well as the apparent experimental verification [5] of the prophetic paper of Gaillard, Lee and Eosner [6] concerning charm [7] is most encouraging.

In spite of these successes we must be careful to maintain an objective attitude. What can be concluded given that neutral currents and charm exist as required? Prom a phenomenological point of view can we say that the data fit a current x current type model, where the currents satisfy an algebra. Experimentally, this is not only tested through the measurement of neutral currents, but also the validity of the Adler-Weisberger relation and other similar low-energy theorems can be considered as evidence for a current algebra. But we have no direct evidence for the existence of vector bosons, and the Higgs mechanism is experimentally totally unverified. The vector-boson hypothesis may perhaps be verified with the new accelerators presently under construction. Even if the energy is insufficient to actually create the vector bosons, the energy dependence as implied by the propagator structure $(-s + M^2)^{-1}$ is perhaps observable.

The question is now: what is really experimentally established, and what experiments are needed in order to settle the various problems. Thus, first of all, we must review the existing data, which we will do in the form of a convenient model.

2. — The experimental facts.

In order to be able to present things in a convenient way, we will assume the quark model, more specifically the existence of 4 quarks (u, d, s and c).

The experimental data show a current x current structure:

$$\mathscr{L}_{\text{int}} = 2G j_\alpha^- j_\alpha^+ + 2G' j_\alpha^0 j_\alpha^0 ,$$

$$\begin{aligned}
j_\alpha^+ \quad &= \frac{1}{\sqrt{2}} (\bar{\nu}_\mu \gamma^\alpha (1+\gamma^5)\mu) + \frac{1}{\sqrt{2}} (\bar{\nu}_e \gamma^\alpha (1+\gamma^5) e) + \\
&\qquad + \frac{\cos\theta_C}{\sqrt{2}} (\bar{u}\gamma^\alpha(1+\gamma^5) d) + \frac{\sin\theta_C}{\sqrt{2}} (\bar{u}\gamma^\alpha(1+\gamma^5) s) - \\
&\qquad - \frac{\sin\theta_C}{\sqrt{2}} (\bar{c}\gamma^\alpha(1+\gamma^5) d) + \frac{\cos\theta_C}{\sqrt{2}} (\bar{c}\gamma^\alpha(1+\gamma^5) s) ,
\end{aligned}$$

$$\begin{aligned}
j_\alpha^0 \quad &= \frac{1}{2\cos\theta_w} (\bar{\nu}_\mu \gamma^\alpha (1+\gamma^5)\nu_\mu) + \frac{1}{2\cos\theta_w} (\bar{\nu}_e \gamma^\alpha (1+\gamma^5) \nu_e) + \\
&+ \frac{1}{2\cos\theta_w} \Bigg[(\bar{e}\gamma^\alpha(-1 + 4\sin^2\theta_w - \gamma^5) e) + (\bar{\mu}\gamma^\alpha(-1 + 4\sin^2\theta_w - \gamma^5)\mu) + \\
&+ \left(\bar{c}\gamma^\alpha\left(\frac{3 - 8\sin^2\theta_w}{3} + \gamma^5\right) c\right) + \left(\bar{u}\gamma^\alpha\left(\frac{3 - 8\sin^2\theta_w}{3} + \gamma^5\right) u\right) + \\
&\qquad + \left(\bar{s}\gamma^\alpha\left(\frac{-3 + 4\sin^2\theta_w}{3} - \gamma^5\right) s\right) + \left(\bar{d}\gamma^\alpha\left(\frac{-3 + 4\sin^2\theta_w}{3} - \gamma^5\right) d\right)\Bigg] ,
\end{aligned}$$

$$G \quad = \frac{1.02\cdot 10^{-5}}{m_p^2\sqrt{2}} , \qquad m_p = \text{proton mass} .$$

G' is not very well determined, but may be close to $G\cos^2\theta_w$. Further $\theta_C =$ = Cabibbo angle and θ_w = weak-mixing angle.

The above currents satisfy an algebra of currents which we will show in the case of the neutral muon current.

First, introduce the muon multiplet

$$l = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} .$$

With this notation

$$\begin{aligned}
j_\alpha^+ &= (\bar{l}\gamma^\alpha(1+\gamma^5)\tau^+ l) , \qquad & j_\alpha^- &= (\ldots\ \tau^-) , \\
\tau^+ &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad & \tau^- &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} .
\end{aligned}$$

The commutator is

$$[\tau^+, \tau^-] = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \equiv \tau^0 .$$

The algebra closes, $[\tau^+, \tau^0] = -\tau^+$.

The weak and e.m. currents are now supposed to be mixtures of τ^0 currents and currents that involYe matrices commuting with τ^+, τ^- and τ^0 (including the factor $1+\gamma^5$):

weak: $$\cos\theta_w(\bar{l}\gamma^\alpha(1+\gamma^5)\tau^0 l) - \sin\theta_w\{X\}\,,$$

e.m.: $$\sin\theta_w(\bar{l}\gamma^\alpha(1+\gamma^5)\tau^0 l) + \cos\theta_w\{X\}\,.$$

The general expression for X involves the identity combined with $1+\gamma^5$ and an arbitrary matrix combined with $1-\gamma^5$:

$$X = \lambda_1(\bar{l}\gamma^\alpha(1+\gamma^5)Il) + (\bar{l}\gamma^\alpha(1-\gamma^5)tl)$$

with 7 = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and t arbitrary 2x2 matrix. Now we require λ_1 and t to be such that the e.m. current contains only $(\bar{\mu}\gamma^\alpha\mu)$, thus no γ^5 part and no $(\bar{\nu}_\mu\nu_\mu)$ part. This gives

$$\lambda_1 = -\frac{\sin\theta_w}{2\cos\theta_w}\,, \qquad \mathbf{t} = \begin{pmatrix} 0 & 0 \\ 0 & -\dfrac{\sin\theta_w}{\cos\theta_w} \end{pmatrix}$$

By using this, the neutral weak current becomes

$$\left(\bar{l}\gamma^\alpha\left[\frac{1}{2}\begin{pmatrix} \cos\theta_w + \dfrac{\sin^2\theta_w}{\cos\theta_w} & 0 \\ 0 & -\cos\theta_w + \dfrac{\sin^2\theta_w}{\cos\theta_w} \end{pmatrix}(1+\gamma^5) - \right.\right.$$
$$\left.\left. - \begin{pmatrix} 0 & 0 \\ 0 & -\dfrac{\sin^2\theta_w}{\cos\theta_w} \end{pmatrix}(1-\gamma^5)\right] l\right) =$$
$$= \left(\bar{l}\gamma^\alpha\cdot\frac{1}{2\cos\theta_w}\begin{pmatrix} 1+\gamma^5 & 0 \\ 0 & -1+4\sin^2\theta_w - \gamma^5 \end{pmatrix} l\right)\,,$$

which is the current shown before. Similarly the e.m. current

$$\left(\bar{l}\gamma^\alpha\begin{pmatrix} 0 & 0 \\ 0 & -2\sin\theta_w \end{pmatrix} l\right)\,.$$

In the Lagrangian there is a factor $2G'$ in front of the neutral currents. The neutral-vector-boson mass M_0, as yet an arbitrary parameter, is defined by

$$2G'\cdot 4\sin^2\theta_w = \frac{e^2}{M_0^2}\,.$$

In other words, apart from the boson mass we also take the coefficients of the weak and e.m. interactions to be equal. From this one derives with $\sin^2\theta_w = \frac{1}{3}$ the value

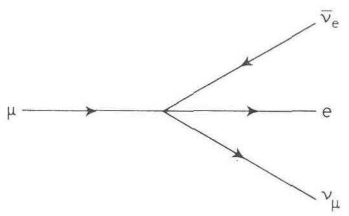
$$M_0^2 \simeq (90\, m_p)^2\,.$$

The above Lagrangian checks with experiment, insofar as has been tested (mainly ν-hadron scattering, and also $\nu_\mu e$ scattering) to about 15 %. Further, the fact that the hadron currents satisfy an algebra has also been tested through the Adler-Weisberger relation and other low-energy theorems.

3. — Higher-order corrections.

The current X current theory is not renormalizable, and we must introduce a cut-off Λ if we want to compute radiative corrections. As Λ goes to infinity, the radiative corrections become infinite, and in principle the observed smallness of weak radiative corrections gives us an upper limit on Λ. For energies larger than Λ the current x current theory can no longer be valid. In fact, it turns out not to be easy to obtain a sensible limit on Λ (meaning a limit below 1000 GeV), due to the fact that the currents satisfy an algebra, more particularly also the existence of charm. Nevertheless, a limit can be obtained, and we will discuss this in detail. This will develop insight that will be helpful later as the Higgs system is discussed.

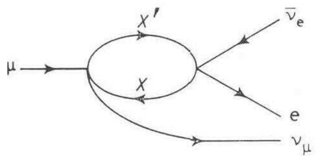
The most obvious candidate for such considerations is muon decay. The lowest-order amplitude is represented by one diagram:

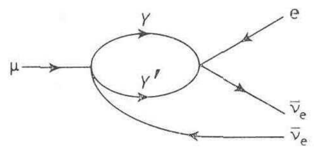


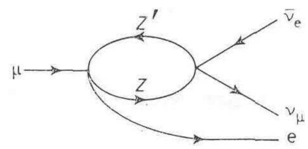
$$(2\pi)^4 iG\big(\bar\nu_\mu\gamma^\alpha(1+\gamma^5)\mu\big)\big(\bar e\gamma^\alpha(1+\gamma^5)\nu_e\big)\,.$$

In here $\bar\nu_\mu$, μ, $\bar e$ and ν_e stand for the various spinors of the particles in question. The above expression is usually employed to determine G from the μ lifetime.

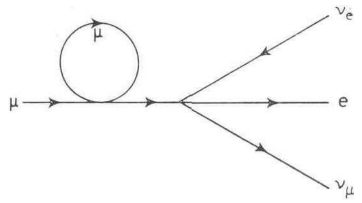
The lowest-order radiative corrections as generated by the Lagrangian of sect. 2 are







$$
\begin{aligned}
(X, X') &= (\bar{\nu}_e, e)\,,\ (\bar{\nu}_\mu, \mu)\,,\ (\bar{u}, d),\ \text{etc.}\,,\\
(Y, Y') &= (\nu_\mu, e)\,,\ (\mu, \nu_e)\,,\\
(Z, Z') &= (\nu_\mu, \bar{\nu}_e)\,,\ (\mu, \bar{e})\,,
\end{aligned}
$$



$+$ similar diagrams

A typical contribution (the first diagram with. $(X, X') = (\bar{u}, d)$) is

$$
G^2\big(\bar{\nu}_\mu\gamma^\alpha(1+\gamma^5)\mu\big)\big(\bar{e}\gamma^\beta(1+\gamma^5)\nu_e\big)\cdot\cos^2\theta_C\cdot I\,,
$$

$$
I=\int \mathrm{d}^4q\,\frac{\mathrm{Tr}\,\{\gamma^\alpha(1+\gamma^5)(-i\gamma q+m_u)\gamma^\beta(1+\gamma^5)(-i\gamma(q-p)+m_d)\}}{(q^2+m_u^2)((q-p)^2+m_d^2)}\,,
$$

where m_u and m_d are the masses of the up and down quarks, and further q is the four-momentum of the up quark *(Z′)* and p the total four-momentum of the electron and electron neutrino. In this expression we encounter the integral

$$
\int \mathrm{d}^4q\,\frac{q_\lambda\, q_k}{(q^2+m_u^2)((q-p)^2+m_d^2)}\,.
$$

This integral is quadratically divergent. We introduce the cut-off factor

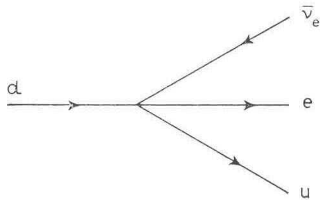
$$
\frac{\Lambda^4}{(q^2+\Lambda^2)^2}\,.
$$

For low q^2 this equals one; for high q^2 this behaves as $1/q^4$, thereby making the integral finite.

On the basis of purely dimensional considerations (the integral has the dimensions of a mass squared) we have

$$\int \mathrm{d}^4 q \frac{q_\lambda q_\varkappa \Lambda^4}{(q^2+m_\mathrm{u}^2)((q-p)^2+m_\mathrm{d}^2)(q^2+\Lambda^2)^2} =$$
$$= a_1 \delta_{\lambda\varkappa}\Lambda^2 + a_2 \delta_{\lambda\varkappa} m_\mathrm{d}^2 \ln \Lambda^2 + a_3 \delta_{\lambda\varkappa} m_\mathrm{u}^2 \ln \Lambda^2 + a_4 \delta_{\lambda\varkappa} p^2 \ln \Lambda^2 +$$
$$+ a_5 p_\lambda p_\varkappa \ln \Lambda^2 + a_6 \delta_{\lambda\varkappa} m_\mathrm{u}^2 + a_7 \delta_{\lambda\varkappa} m_\mathrm{d}^2 + \text{further finite terms}\,.$$

The coefficient a_1 is the most important to us; its value (to be computed later) is $i\pi^2/4$. However, the a_1-term is unobservable, because, if we add this term to the lowest-order amplitude, we merely obtain the same expression with G replaced by $G + \lambda G^2 \Lambda^2$ (and λ proportional to a_1). This simply leads to a redefinition of G in the Lagrangian. Only if we had another way of determining G, an effect would be observable. Luckily there is one other place where G can be seen reasonably clean, and that is in neutron decay. In this decay a d-quark decays into an u-quark, emitting an $\mathrm{e}\bar{\nu}_\mathrm{e}$ pair (recall N = ddu, P = duu). The relevant diagram is



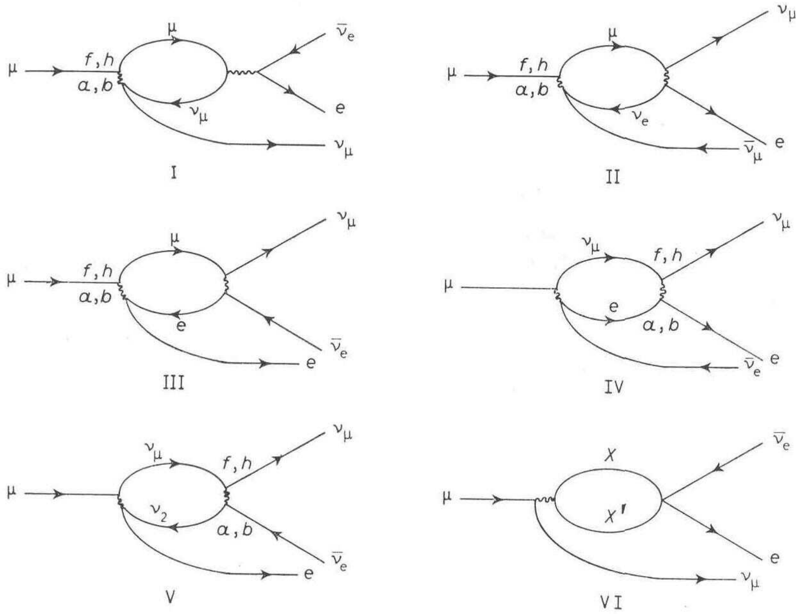
$$(2\pi)^4 iG \cos\theta_\mathrm{C} (\bar{u}\gamma^\alpha(1+\gamma^5)d)(\bar{e}\gamma^\alpha(1+\gamma^5)\nu_\mathrm{e})\,.$$

The diagrams corresponding to radiative corrections are similar to those shown before; however, due to the fact that the quark neutral currents are different from the lepton neutral currents, we get a different result for those radiative corrections. This difference leads to a deviation from 1 for the ratio of the experimentally observed vector coupling constant in neutron decay and the muon coupling constant. The CVC theorem ensures us that strong interactions do not influence this ratio, and we can make a comparison with experiment.

4. - Radiative corrections to G_β/G_μ.

We will now compute the radiative corrections proportional to Λ^2 to the ratio G_β/G_μ, the β-decay and μ-decay vector coupling constant ratio. Within the Weinberg model supplemented with some quark model the computation of these radiative corrections has been done, and we refer to an article by Eoos [8] for a comparison with the data as well as further references (in particular to SIRLIN [9], whose calculation has much in common with what follows below).

The radiative corrections to G_β/G_μ arise because of the differences between the quark neutral currents and the lepton neutral currents. There are 6 diagrams of which we need only the leading part in terms of the cut-off Λ, in addition to external-line renormalization diagrams that will be treated later:



The subscripts a, b and f, h indicate the structures $\gamma^\alpha(a + b\gamma^5)$ and $\gamma^\alpha(f + h\gamma^5)$, respectively. Similar diagrams can be written down for d-quark decay: d and u instead of μ and ν_μ in the above diagrams. In diagram VI X and X' stand for possible fermion pairs. Since these diagrams will be equal for μ and d decay, we need not consider them.

The calculation is straightforward, and will be exemplified in the case of diagram II. We get

$$\mathrm{II} = \frac{2GG'}{4\cos^2\theta_w}\cdot\int \mathrm{d}^4q\, \frac{\left(\bar{e}\gamma^\alpha(1+\gamma^5)(+i\gamma(q-p))\gamma^\beta(a+b\gamma^5)\,\nu_e\right)}{((q-p)^2-i\varepsilon)(q^2+m_\mu^2-i\varepsilon)}\cdot$$
$$\cdot\left(\bar{\nu}_\mu\gamma^\alpha(1+\gamma^5)(-i\gamma q+m_\mu)\gamma^\beta(f+h\gamma^5)\mu\right).$$

Here p is the sum of the momenta of ν_μ and e. Setting p and m_μ to zero (only the leading divergence is required), we get

$$\mathrm{II} \simeq \frac{GG'}{2\cos^2\theta_w}(a+b)(f+h)\left(\bar{e}\gamma^\alpha\gamma^\lambda\gamma^\beta(1+\gamma^5)\,\nu_e\right)\left(\bar{\nu}_\mu\gamma^\alpha\gamma^{\lambda'}\gamma^\beta(1+\gamma^5)\mu\right)\cdot\int \mathrm{d}^4q\,\frac{q_\lambda q_{\lambda'}}{q^\varkappa}\,.$$

With

$$q_\lambda q_{\lambda'} \to \tfrac{1}{4} q^2 \delta_{\lambda\lambda'}$$

and

$$\gamma^\alpha \gamma^\lambda \gamma^\beta = \gamma^\alpha \delta_{\lambda\beta} + \gamma^\beta \delta_{\alpha\lambda} - \gamma^\lambda \delta_{\alpha\beta} - \varepsilon_{\alpha\lambda\beta\varkappa} \gamma^\varkappa \gamma^5$$

this expression reduces to

$$\mathrm{II} \simeq \frac{2GG'}{\cos^2\theta_\mathrm{w}} (a+b)(f+h)\big(\bar{e}\gamma^\lambda(1+\gamma^5)\nu_\mathrm{e}\big)\big(\bar{\nu}_\mu \gamma^\lambda(1+\gamma^5)\mu\big)\int \mathrm{d}^4 q\, \frac{1}{q^2}\,.$$

Now let us regulate the integral by a factor

$$\frac{\Lambda^4}{(q^2+\Lambda^2)^2}\,.$$

We obtain

$$\int \mathrm{d}^4 q\, \frac{1}{q^2} \to \int \mathrm{d}^4 q\, \frac{\Lambda^4}{(q^2+\Lambda^2)^2 q^2} = \int_0^1 \mathrm{d}x \int \mathrm{d}^4 q\, \frac{2x\Lambda^4}{(q^2+\Lambda^2 x)^3} = \int_0^1 \mathrm{d}x\, \frac{i\pi^2 \cdot 2x\Lambda^4}{2\Lambda^2 x} = i\pi^2 \Lambda^2\,.$$

This completes the computation of diagram II. In this way all diagrams may be calculated with the result

$$\mathrm{I} = -C\,, \quad \mathrm{II} = 4C\,, \quad \mathrm{III} = -C\,, \quad \mathrm{IV} = 4C\,, \quad \mathrm{V} = -C\,,$$

$$C = \frac{i\pi^2 GG'}{2\cos^2\theta_\mathrm{w}} (a+b)(f+h)\Lambda^2 \cdot A_0\,.$$

For μ-decay

$$A_0^\mu = \big(\bar{e}\gamma^\lambda(1+\gamma^5)\nu_\mathrm{e}\big)\big(\bar{\nu}_\mu \gamma^\lambda(1+\gamma^5)\mu\big)\,.$$

For d-decay

$$A_0^\mathrm{d} = \big(\bar{e}\gamma^\lambda(1+\gamma^5)\nu_\mathrm{e}\big)\big(\bar{u}\gamma^\lambda(1+\gamma^5)d\big)\,.$$

Next we must insert the explicit values for a, b and f, h for μ- and d-decay:

$$\mathrm{I}_\mu: \quad a = 1\,,\ b = 1\,,\ f = -1 + 4\sin^2\theta_\mathrm{w}\,,\ h = -1\,,$$
$$(a+b)(f+h) = -4 + 8\sin^2\theta_\mathrm{w}\,;$$

$$\mathrm{II}_\mu: \quad a = 1\,,\ b = 1\,,\ f = -1 + 4\sin^2\theta_\mathrm{w}\,,\ h = -1\,,$$
$$(a+b)(f+h) = -4 + 8\sin^2\theta_\mathrm{w}\,;$$

$$\mathrm{III}_\mu: \quad a=-1+4\sin^2\theta_w\,,\ b=-1\,,\ f=-1+4\sin^2\theta_w\,,\ h=-1\,,$$
$$(a+b)(f+h)=4-16\sin^2\theta_w+16\sin^4\theta_w\,;$$
$$\mathrm{IV}_\mu: \quad a=-1+4\sin^2\theta_w\,,\ b=-1\,,\ f=1\,,\ h=1\,,$$
$$(a+b)(f+h)=-4+8\sin^2\theta_w\,;$$
$$\mathrm{V}_\mu: \quad a=1\,,\ b=1\,,\ f=1\,,\ h=1\,,$$
$$(a+b)(f+h)=4\,.$$

For d-decay

$$\mathrm{I}_d: \quad a=1-\frac{8}{3}\sin^2\theta_w\,,\ b=1\,,\ f=-1+\frac{4}{3}\sin^2\theta_w\,,\ h=-1\,,$$
$$(a+b)(f+h)=-4+8\sin^2\theta_w-\frac{32}{9}\sin^4\theta_w\,;$$
$$\mathrm{II}_d: \quad a=1\,,\ b=1\,,\ f=-1+\frac{4}{3}\sin^2\theta_w\,,\ h=-1\,,$$
$$(a+b)(f+h)=-4+\frac{8}{3}\sin^2\theta_w\,;$$
$$\mathrm{III}_d: \quad a=-1+4\sin^2\theta_w\,,\ b=-1\,,\ f=-1+\frac{4}{3}\sin^2\theta_w\,,\ h=-1\,,$$
$$(a+b)(f+h)=4-\frac{32}{3}\sin^2\theta_w+\frac{16}{3}\sin^4\theta_w\,;$$
$$\mathrm{IV}_d: \quad a=-1+4\sin^2\theta_w\,,\ b=-1\,,\ f=1-\frac{8}{3}\sin^2\theta_w\,,\ h=-1\,,$$
$$(a+b)(f+h)=-4+\frac{40}{3}\sin^2\theta_w-\frac{32}{3}\sin^4\theta_w\,;$$
$$\mathrm{V}_d: \quad a=1\,,\ b=1\,,\ f=1-\frac{8}{3}\sin^2\theta_w\,,\ h=1\,,$$
$$(a+b)\,(f+h)=4-\frac{16}{3}\sin^2\theta_w\,.$$

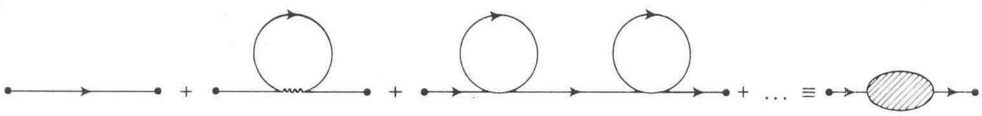
For μ-decay:

$$(\mathrm{I}\text{-}\mathrm{V})_\mu=\frac{i\pi^2GG'}{2\cos^2\theta_w}(-36+72\sin^2\theta_w-16\sin^4\theta_w)\Lambda^2A_0^\mu\,.$$

For d-decay:

$$(\mathrm{I}\text{-}\mathrm{V})_d=\frac{i\pi^2GG'}{2\cos^2\theta_w}\left(-36+72\sin^2\theta_w-\frac{400}{9}\sin^4\theta_w\right)\Lambda^2A_0^d\,.$$

In addition to these contributions there are contributions due to wave function renormalization. Let us first consider the μ-propagator correction due to a self-energy insertion. We have the series



Prom the self-energy diagram we need only the part proportional to $-i\gamma p$, and Ave denote this by $-\lambda i\gamma p$. The corrected propagator becomes

$$\frac{1}{(2\pi)^4 i}\,\frac{1}{i\gamma p+m_\mu}+\frac{1}{(2\pi)^4 i(i\gamma p+m_\mu)}\cdot(-\lambda i\gamma p)\,\frac{1}{(2\pi)^4 i(i\gamma p+m_\mu)}+\ldots=$$

$$=\frac{1}{(2\pi)^4 i}\,\frac{1}{i\gamma p+m_\mu}\cdot\frac{1}{1-(-\lambda i\gamma p/(2\pi)^4 i(i\gamma p+m_\mu))}=$$

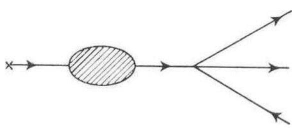
$$=\frac{1}{(2\pi)^4 i}\,\frac{1}{(1+\lambda/(2\pi)^4 i)\,i\gamma p+m_\mu}=\frac{1}{(2\pi)^4 i}\cdot\frac{1}{1+\lambda/(2\pi)^4 i}\cdot\frac{1}{i\gamma p+m'_\mu},$$

where m'_μ is a corrected mass. In order to get properly normalized amplitudes, we must give external μ lines the factor

$$\frac{1}{\sqrt{1+\lambda/(2\pi)^4 i}}\simeq 1-\frac{\lambda}{2(2\pi)^4 i}\,.$$

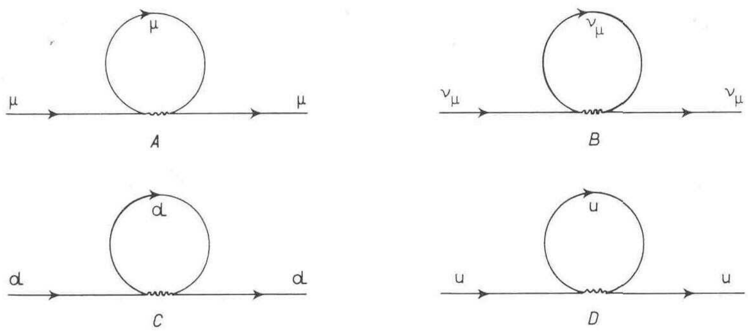
(In terms of sources this arises because sources must be given a factor $\sqrt{1+\lambda/\ldots}$ so that the two-muon-source diagram

remains as it was. For a process like μ-decay we have one μ source



which leaves a factor $1/\sqrt{\quad}$.)

The following diagrams must be taken into account:



All other diagrams are the same in both processes, or cancel. Example:

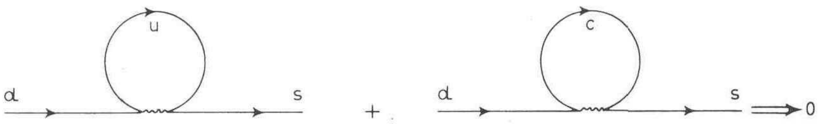


Diagram *A* gives rise to an expression of the form

$$\Lambda^4\int \mathrm{d}^4q\, \frac{\gamma^\alpha(a+b\gamma^5)\big(-i\gamma(q+p)\big)\gamma^\alpha(f+h\gamma^5)\cdot\frac{1}{2}(1+\gamma^5)}{(q+p)^2(q^2+\Lambda^2)^2}\,.$$

We have put a projection factor $\frac{1}{2}(1+\gamma^5)$ since that is the only relevant part in the processes under consideration. Only the $i\gamma p$ part is needed. This requires the computation of

$$\Lambda^4\int \mathrm{d}^4q\, \frac{i\gamma p}{(q+p)^2(q^2+\Lambda^2)^2}\,.$$

The *p* in the denominator can be set zero, and this gives

$$i\gamma p\cdot i\pi^2\Lambda^2\,.$$

Next we must compute

$$\int \mathrm{d}^4q\, \frac{\Lambda^4 q_\lambda}{(q+p)^2(q^2+\Lambda^2)^2} = \int_0^1 \mathrm{d}x\int \mathrm{d}^4q\, \frac{2xq_\lambda\Lambda^4}{(q^2+2pq(1-x)+p^2(1-x)+\Lambda^2x)^3} =$$

$$= \int \mathrm{d}x\int \mathrm{d}^4q\, \frac{2x\Lambda^4\big(q-p(1-x)\big)_\lambda}{(q^2+p^2x(1-x)+\Lambda^2x)^3} =$$

$$= -\int \mathrm{d}x\, \frac{i\pi^2p_\lambda x(1-x)\Lambda^4}{p^2x(1-x)+\Lambda^2x} \Rightarrow -\Lambda^4\int \mathrm{d}x\, \frac{i\pi^2p_\lambda(1-x)}{\Lambda^2} = -\frac{i\pi^2}{2}\Lambda^2p_\lambda\,.$$

Using this diagram A gives

$$-i\pi^2\Lambda^2\gamma^\alpha i\gamma p(-\tfrac{1}{2}+1)\gamma^\alpha(a+b)(f+h)\cdot\tfrac{1}{2}(1+\gamma^5)=$$
$$=+i\pi^2\Lambda^2(a+b)(f+h)\,i\gamma p\,.$$

The various diagrams can now be computed:

$$A:\quad -\frac{iG'\pi^2}{2\cos^2\theta_w}\cdot(a+b)(f+h)\Lambda^2\cdot(-i\gamma p)\,,$$
$$f+h=a+b=-2+4\sin^2\theta_w\,,$$
$$(a+b)(f+h)=4-16\sin^2\theta_w+16\sin^4\theta_w\,;$$
$$B:\quad (a+b)(f+h)=4\,.$$

If λ is the coefficient of $-i\gamma p$, we get the correction

$$1-\frac{1}{2}\frac{\lambda}{(2\pi)^4 i}=1-\frac{1}{2}\frac{i\pi^2G'\Lambda^2}{(2\pi)^4 i\cdot 2\cos\theta_w}(-8+16\sin^2\theta_w-16\sin^4\theta_w)\,.$$

For diagrams C and D, relevant for d-decay,

$$C:\quad (a+b)(f+h)=4-\frac{16}{3}\sin^2\theta_w+\frac{16}{9}\sin^4\theta_w\,,$$
$$D:\quad (a+b)(f+h)=4-\frac{32}{3}\sin^2\theta_w+\frac{64}{9}\sin^4\theta_w\,.$$

This then gives to d-decay the factor

$$1-\frac{1}{2}\frac{i\pi^2G'\Lambda^2}{(2\pi)^4 i\cdot 2\cos^2\theta_w}\left(-8+16\sin^2\theta_w-\frac{80}{9}\sin^4\theta_w\right).$$

All together we get for μ-decay and d-decay:

$$\mu:\quad (2\pi)^4 iGA_0^\mu\Big\{1+\frac{i\pi^2G'\Lambda^2}{(2\pi)^4 i\cdot 2\cos^2\theta_w}(-36+72\sin^2\theta_w-16\sin^4\theta_w+$$
$$+4-8\sin^2\theta_w+8\sin^4\theta_w)\Big\}\,,$$

$$\mathrm{d}:\quad (2\pi)^4 iGA_0^{\mathrm{d}}\Big\{1+\frac{i\pi^2G'\Lambda^2}{(2\pi)^4 i\cdot 2\cos^2\theta_w}\Big(-36+72\sin^2\theta_w-\frac{400}{9}\sin^4\theta_w+$$
$$+4-8\sin^2\theta_w+\frac{40}{9}\sin^4\theta_w\Big)\Big\}\,.$$

For the ratio of d- to μ-decay only factors $\sin^4\theta_w$ remain (this can also be seen directly) and we find

$$\frac{G_d}{G_\mu} = 1 + \frac{i\pi^2 G' \Lambda^2 \sin^4\theta_w}{(2\pi)^4 i \cdot 2 \cos^2\theta_w}\left(-\frac{360}{9}+8\right),$$

or

$$\frac{G_d}{G_\mu} = 1 - \frac{G'}{2}\frac{\sin^4\theta_w}{\cos^2\theta_w}\cdot\Lambda^2 .$$

What is experimentally the situation! In fact, a correction of about 2 % is observed (after Cabibbo-angle corrections etc.) However, there are also finite corrections, and ln Λ^2 corrections that can be quite large, and that we have not computed here. Within the Weinberg model (which means effectively using a cut-off of (60 $\div$ 80) GeV) the calculation has been done [8, 9], and the result is indeed a correction of about 2 %. This then we take as an estimate of the finite pieces. Consequently we may say that the above factor should not differ from its value at Λ = 80 GeV by more than, say, 1 %. Indeed from 0.224 $\leqslant \sin\theta_C \leqslant$ 0.252 (see ref. [8]) we obtain 0.968 $\leqslant \cos\theta_C \leqslant$ 0.975. Thus

$$\frac{G'}{\pi^2}\frac{\sin^4\theta_w}{\cos^2\theta_w}(\Lambda^2 - 80^2) < 0.01 .$$

Using the value $\sin^2\theta_w$ = 0.36 we obtain

$$\frac{1.02\cdot 10^{-5}}{\pi^2\sqrt{2}\,m_p^2}\cdot 0.13\cdot(\Lambda^2 - 80^2) < 0.01 ,$$

or

$$\Lambda^2 < 10^5 + 6400 \simeq 100\,000\, m_p^2 , \qquad \Lambda < 315\, m_p \simeq 300 \text{ GeV} .$$

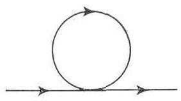
There are a few interesting observations to be made here. The above limit appears to be inescapable, and very likely can be improved considerably if a more careful evaluation is made. The main source of uncertainty is the Cabibbo angle. Further, note that G_d becomes smaller as Λ is increased. Now, if the d-quark is mixed with yet another, new, quark (the bottom quark), then this mixing must be very small simply because we cannot let G_d go down even more (by another factor $\cos\theta'_C$). Thus θ'_C must be very close to zero. This means that such a bottom quark, left-handed, would decay very slowly into a u-quark, since this would contain a factor sin θ'_C (just as the decay of the strange quark into an up quark). Other, as yet unknown forces would then perhaps become visible. Indeed, top and bottom quarks may be extremely interesting objects. An e^+e^- machine operating in the (10 $\div$ 15) GeV region may turn out to be a very exciting machine if the new discovery [10] of a resonance in $\mu^+\mu^-$ pairs at 9.4 GeV indeed relates to such quarks.

5. — General cut-off effects.

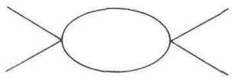
The above considerations show a general rule: for big effects one must look at relations that hold by virtue of a symmetry. In any nonrenormalizable theory the dependence on a cut-off goes for any diagram as :

$$\Lambda^m + k^2\,\Lambda^{m-2} + k^4\,\Lambda^{m-4} + \dots ,$$

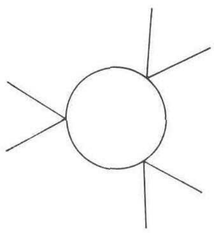
where k^2 is either a mass or an external momentum squared. Furthermore, the power m goes down when increasing external lines



Λ^3 ,

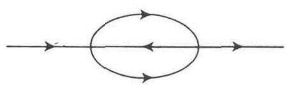


Λ^2 ,

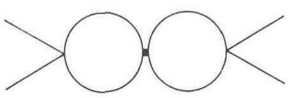


Λ .

Further, the power increases by two for every extra closed loop



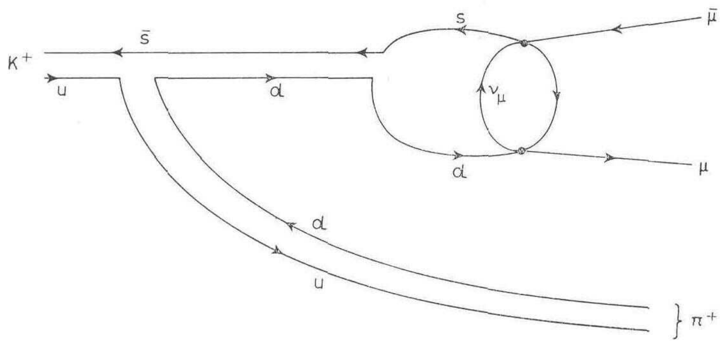
Λ^5 ,



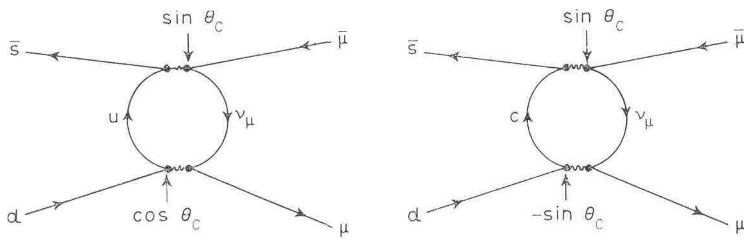
Λ^4 .

However, there are additional coupling constants.

It follows that we should look first at mass corrections (two external lines). However, such corrections are usually unobservable unless these corrections give rise to a breaking of some mass relationship. Similarly for coupling constants (three or more external lines, or through wave function renormalization from diagrams with two external lines with an explicit momentum dependence). This situation was encountered in the previous section. We will try to find other cases of this kind, but without success. The reason is that in current x current theories, where the currents satisfy an algebra, there is some sort of conspiracy to eliminate large effects. As an example we may quote weak neutral strangeness-violating decays. In lowest order it is not there (there is no d-s current). In fact, charm was introduced just to avoid this current while maintaining a current algebra. But in higher order such d-s currents will be generated, and in principle this could be an order-Λ^2 effect. Stated differently, the symmetry requires some coupling constant to be zero, and higher orders may produce a deviation of this relation. Unfortunately, such effects are strongly suppressed. Consider the decay $K^+ \to \pi^+\mu^+\mu^-$. In terms of quarks we may picture this as follows:

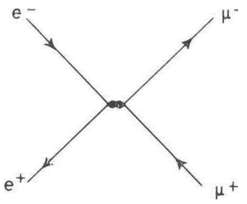


However, there are two diagrams that tend to cancel each other



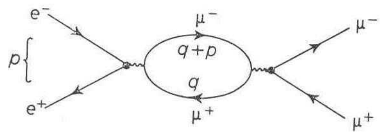
The cancellation is exact if the masses of the u- and the c-quark are equal. Therefore, the leading divergence Λ^2, which is independent of masses and momenta, is zero. The first effect is of the form $(m_c^2 - m_u^2)\ln \Lambda^2$. Even this could be used since the experimental limits on $K \to \pi\mu\mu$ (and also other similar decays) are quite strong. In fact, GAILLARD *et al.* [6] have used this to estimate the mass of the charmed quark, anticipating in this way quite correctly the subsequent experimental developments. However, we cannot use this to say something about Λ; going from Λ = 80 GeV to Λ •= 300 GeV gives too little change.

From the above we may nevertheless still draw an important conclusion: even if the leading term Λ^m cannot be observed, we can observe the next leading term provided some external momentum squared becomes very big. As an example we may consider $e^-e^+ \to \mu^-\mu^+$. The weak process is (we set the neutral current to $1+\gamma^5$)



$$(2\pi)^4 iG\big(\gamma^\alpha(1+\gamma^5)\big)_e\big(\gamma^\alpha(1+\gamma^5)\big)_\mu .$$

One of the lowest-order radiative corrections is



The corresponding expression is

$$-G^2\big(\gamma^\alpha(1+\gamma^5)\big)_e\big(\gamma^\beta(1+\gamma^5)\big)_\mu \cdot$$
$$\cdot \int d^4q \, \frac{\operatorname{Tr}\{\gamma^\beta(1+\gamma^5)(-i\gamma(q+p)+m_\mu)\gamma^\alpha(1+\gamma^5)(-i\gamma q+m_\mu)\}}{(q^2+m_\mu^2)((q+p)^2+m_\mu^2)} \Rightarrow$$
$$\Rightarrow -G^2\big(\gamma^\alpha(1+\gamma^5)\big)_e\big(\gamma^\alpha(1+\gamma^5)\big)_\mu \cdot 4i\pi^2 p^2 \left\{1-\ln\frac{\Lambda^2}{X^2}\right\},$$
$$X^2 = p^2+4m_\mu^2 .$$

This correction amounts to the replacement

$$G \to G\left\{1-\frac{4i\pi^2}{(2\pi)^4 i}\, Gp^2\left(1-\ln\frac{\Lambda^2}{X^2}\right)\right\}.$$

It becomes relevant if

$$\frac{1}{4\pi^2}\, Gp^2 \sim 0.01 ,$$

which is for $\sqrt{-p^2} \approx 250$ GeV.

This example reflects the so-called unitarity limit: at sufficiently high energy the higher-order corrections become arbitrarily large. The above calculation is indicative: one must go to some 200 GeV to be sure to see effects of the new physics that must set in at the energy Λ. In fact, even if we cannot go to the energy Λ, we can at least determine Λ from experiment, using the above formula. In other words, even if we cannot see the new physics directly, we can determine the threshold for this new physics from the observation of the radiative corrections at very high energy. This feature will remain true in the following, with respect to the Higgs particles.

6. — Gauge theories.

We have seen that at low energies the weak interactions are well described by a four-fermion current x current theory. However, this theory, being unrenormalizable, cannot be the complete field theory. What can we possibly guess in this respect? The first evident step is to introduce vector bosons. But further study reveals that this is not much of an improvement, that such a theory is also not renormalizable, at least not without additional particles.

What can we guess about the theory of these vector bosons! The main fact that presents itself is that the currents to which these bosons are coupled satisfy an algebra. Thus we must look for theories that have precisely this feature. As was pointed out by BELL [11], one obtains automatically currents with this property if one requires local gauge invariance. To be precise, current algebra can also be presented in the form of divergence conditions on the weak hadronic currents [12], and such conditions were shown by BELL to follow from gauge invariance. This fundamental remark leads to the idea that the weak interactions are a gauge theory and, if we believe that Nature employs only renormalizable theories, then it follows that gauge theories must be renormalizable [13]. It is this line of reasoning which ultimately led to the proof of renormalizability of gauge theories by 'T HOOFT [1]. And seen in this way the theoretical confirmation of this idea can be seen as some sort of evidence that this reasoning is correct.

Thus let us now assume that the weak interactions (together with the e.m. interactions) constitute a gauge theory. It is our first task to devise means to test this assumption.

There are many different kinds of gauge theories that can be constructed in this sense. The Weinberg model involves the $SU_2 \times U_1$ symmetry and gives rise to the currents given before, at least for the lepton sector. It can be enlarged to contain also quarks, including charm, as done by HAEA, BJORKEN and GLASHOW, ILIOPOULOS and MAIANI [7]. This gives the quark currents as written down in the beginning. Whether there are other currents or possibly other or larger symmetries is right now an open question. We will limit ourselves to this enlarged Weinberg model, because the features that will be discussed here will certainly arise in other models as well.

The basic ingredients added to transform the current x current theory into a gauge theory are the following:

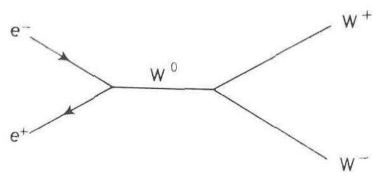
i) charged and neutral vector bosons $W^{\pm}$ and W^0 with masses M_c and M_0 coupled to the currents in the required gauge fashion with a coupling constant *g;*

ii) couplings among these vector bosons themselves according to the gauge prescription, involving the same coupling constant *g;*

iii) introduction of one or more scalar particles coupled to the vector bosons and fermions in such a way that the theory becomes renormalizable.

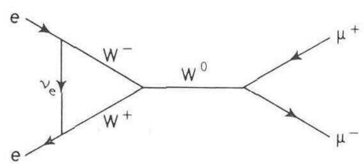
These facts must be investigated experimentally. The first two points are fairly straightforward: we must first observe the vector bosons. In the Weinberg model $M_c \sim 60$ GeV and $M_0 = 80$ GeV. In any case, the masses must be below 300 GeV, as argued before. The quoted mass values seem quite reasonable, but we can of course not be sure. Perhaps we will already get an indication at PETEA or PEP experiments that will investigate $e^+e^- \to \mu^+\mu^-$. A possible weak structure in this process may be visible at $(30 \div 40)$ GeV. The strength of the weak contribution at these energies is of the order of 30 % relative to the e.m. process, and it will at least be experimentally accessible. Thus perhaps point i) will be answered within a few years.

The situation with point ii) is much more difficult. The most straightforward method would be to observe the process



This involves precisely one of the vertices mentioned in point ii). However, an energy of at least 120 GeV is required (in the Weinberg model), and preferably one should be comfortably above this threshold, like 200 GeV.

In principle one could imagine this coupling to be observable indirectly through radiative corrections. Example:



This has not been worked out at this time. The calculation is complicated, and requires calculation of all corrections of this order.

Point iii) is much more difficult, and will be discussed in the following section.

7. — The Higgs system of the Weinberg model.

The complete Lagrangian of the Weinberg model is

$$
\begin{aligned}
\mathscr{L} = {} & \mathscr{L}_{\mathrm{YM}}(B_\mu) - \frac{1}{2}(\partial_\mu B^\alpha_\mu)^2 - \frac{1}{2} M^2 (\underline{B}^a_\mu)^2 - \bar{e}\gamma\partial e - \bar{\nu}\gamma\partial\nu - m_e(\bar{e}e) + \\
& + \frac{ig}{2}\begin{pmatrix}\bar{\nu}\\ \bar{e}\end{pmatrix}\begin{pmatrix}\underline{B}^3_\mu & B'_\mu - iB^2_\mu \\ B^1_\mu + iB^2_\mu & -\underline{\underline{B}}^3_\mu\end{pmatrix}\gamma^\mu\left(\frac{1+\gamma^5}{2}\right)\begin{pmatrix}\nu\\ e\end{pmatrix} + \frac{ig}{2}(\underline{B}^3_\mu - \underline{\underline{B}}^3_\mu)\left(\bar{e}\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)e\right) - \\
& - \frac{1}{2}(\partial_\mu Z)^2 - \frac{1}{2} m^2 Z^2 - \frac{1}{2}(\partial_\mu \varphi_a)^2 - \frac{1}{2} M^2(\varphi_1^2 + \varphi_2^2) - \frac{M^2}{2c^2}\varphi_3^2 - \\
& - \frac{gm_e}{2M}\{Z(\bar{e}e) + i\varphi_3(\bar{e}\gamma^5 e)\} + \frac{gm_e}{4M} i(\varphi_1 + i\varphi_2)(\bar{e}(1+\gamma^5)\nu) - \\
& - \frac{gm_e}{4M} i(\varphi_1 - i\varphi_2)(\bar{\nu}(1-\gamma^5)e) - \frac{1}{2} gMZ\underline{B}^2_\mu - \frac{1}{8} g^2 Z^2 \underline{B}^2_\mu - \\
& - \frac{1}{2} g\varepsilon_{abc}\varphi_a\partial_\mu\varphi_b \underline{\underline{B}}^c_\mu + \frac{1}{2} g\underline{B}^a_\mu(Z\partial_\mu\varphi_a - \varphi_a\partial_\mu Z) - \frac{1}{8} g^2\varphi_a^2 \underline{\underline{B}}^2_\mu - \\
& - \frac{g^2}{4} Z\varepsilon_{abc}\underline{B}^a_\mu\varphi_b \underline{\underline{B}}^c_\mu - \frac{g}{2} M\varepsilon_{abc}\underline{B}^a_\mu\varphi_b\underline{\underline{B}}^a_\mu - \alpha MgZ(\varphi^2 + Z^2) - \\
& - \frac{1}{8}\alpha g^2(\varphi^2 + Z^2)^2 - \beta\left\{\frac{1}{2}(Z^2 + \varphi^2) + \frac{2M}{g}Z\right\} + \frac{1}{2} g^2 \frac{s}{c}\varphi_3 B^0_\mu(B^a_\mu\varphi^a) - \\
& - \bar{\chi}^\alpha\partial^2\chi_\alpha - M^2\underline{\bar{\chi}}^a\underline{\chi}_a + g\varepsilon_{abc}\partial_\mu\bar{\chi}^a\chi_b B^c_\mu - \frac{1}{2} MgZ(\underline{\bar{\chi}}^a\underline{\chi}_a) + \frac{gM}{2}\varepsilon_{abc}\underline{\bar{\chi}}^a\underline{\underline{\chi}}_b\varphi_c\,, \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \alpha = 0, 1, 2, 3\,,\ a = 1, 2, 3\,,
\end{aligned}
$$

$$
\begin{aligned}
\underline{B}^3_\mu \quad & = B^3_\mu - \frac{s}{c}B^0_\mu = \frac{1}{c}W^0_\mu\,, \qquad \underline{\underline{B}}^3_\mu = B^3_\mu + \frac{s}{c}B^0_\mu = \frac{c^2 - s^2}{c}W^0_\mu + 2sA_\mu\,, \\
\underline{\chi}_a \quad & = \chi_a - \frac{s}{c}\delta_{a3}\chi_0\,, \qquad \underline{\underline{\chi}}_a = \chi_a + \frac{s}{c}\delta_{a3}\chi_0\,, \qquad s, c = \sin\theta_w, \cos\theta_w\,, \\
W^+_\mu \quad & = \frac{1}{\sqrt{2}}(B^1_\mu - iB^2_\mu)\,, \quad W^-_\mu = \frac{1}{\sqrt{2}}(B^1_\mu + iB^2_\mu)\,, \quad \alpha = \frac{m^2}{4M^2}\,, \quad \varphi^2 = \varphi_a\varphi_a\,, \\
g^2 \quad & = 8M^2G = 8M^2\cdot 1.02\cdot 10^{-5}/\sqrt{2}\,m_p^2\,, \qquad g = e/\sin\theta_w\,, \\
M \quad & = \mathrm{W}^\pm \text{ mass}\,, \quad m_e = \text{electron mass}\,, \quad m = \text{Higgs mass (Z-mass)}, \\
\varphi_a \quad & = \text{Higgs ghost}\,, \qquad \chi = \text{F-P ghost}\,, \qquad \text{in lowest order } \beta = 0\,, \\
\mathscr{L}_{\mathrm{YM}}(B_\mu) & = \text{Yang-Mills Lagrangian} = -\tfrac{1}{4}(G^a_{\mu\nu})^2 - \tfrac{1}{4}(G^0_{\mu\nu})^2\,, \\
G^a_{\mu\nu} \quad & = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g\varepsilon_{abc}B^b_\mu B^c_\nu\,, \\
G^0_{\mu\nu} \quad & = \partial_\mu B^0_\nu - \partial_\nu B^0_\mu\,.
\end{aligned}
$$

The parameter β must be chosen such that Z-tadpoles cancel out. It is a series in the coupling constant g and starts with a term proportional to g^2.

In above the $W^{\pm}$, W^0 and A_μ are the physical fields, corresponding to the usual intermediate vector bosons. Because $\underline{B}^3_\mu = (1/c)W^0_\mu$ we see that $M^2(B^a_\mu)^2$ implies a mass If for the charged W's and a mass $M/\cos\theta_w$ for the neutral W. This is specific for the Weinberg model where only the simplest Higgs system is used with only one Higgs scalar boson Z.

In the above only the electron and its neutrino have been written down. The muon terms are identical, with m_μ instead of m_e. For the quarks the situation is more complicated, they involve different combinations of W^0_μ and A_μ, and also the quark-Higgs coupling is somewhat different. It still contains a factor m_q/M; couplings between the Z and fermions are always proportional to the fermion mass (divided by the vector-boson mass), if left- and right-handed fermions are in different multiplets, *i.e.* if parity is violated.

To be specific, the quark Lagrangian is

$$\frac{ig}{2}\begin{pmatrix}\bar{u}\\ \bar{d}'\end{pmatrix}\begin{pmatrix}\frac{3-4s^2}{3c}W^0_\mu+\frac{4}{3}sA_\mu & \sqrt{2}\,W^+_\mu\\ \sqrt{2}\,W^-_\mu & \frac{-3+2s^2}{3c}W^0_\mu-\frac{2}{3}sA_\mu\end{pmatrix}\gamma^\mu\frac{1+\gamma^5}{2}\begin{pmatrix}u\\ d'\end{pmatrix}+$$

$$+\frac{ig}{2}\begin{pmatrix}\bar{u}\\ \bar{d}'\end{pmatrix}\begin{pmatrix}-\frac{4s^2}{3c}W^0_\mu+\frac{4}{3}sA_\mu & 0\\ 0 & \frac{2s^2}{3c}W^0_\mu-\frac{2}{3}sA_\mu\end{pmatrix}\gamma^\mu\frac{1-\gamma^5}{2}\begin{pmatrix}u\\ d'\end{pmatrix}+$$

$$+\text{ same with } u,\bar{u}\to c,\bar{c} \text{ and } \bar{d}', d'\to\bar{s}', s'+$$

$$(d'=d\cos\theta_C+s\sin\theta_C,\; s'=-d\sin\theta_C+s\cos\theta_C)$$

$$+\frac{m_d}{F}\bar{d}\varepsilon^{ij}\{\cos\theta_C\cdot(K_iQ^u_{+j})-\sin\theta_C(K_iQ^c_{+j})\}+\text{Hermitian conjugate}+$$

$$+\frac{m_s}{F}\bar{s}\varepsilon^{ij}\{\sin\theta_C\cdot(K_iQ^u_{+j})+\cos\theta_C(K_iQ^c_{+j})\}+\text{Hermitian conjugate}+$$

$$+\frac{m_u}{F}\bar{u}(K^{\dagger j}Q^u_{+i})+\frac{m_c}{F}\bar{c}(K^{\dagger j}Q^c_{+j})\,.$$

m_d, m_s, m_u and m_c are the respective quark masses. Further

$$K=\frac{1}{\sqrt{2}}\begin{pmatrix}F\sqrt{2}+Z+i\varphi_3\\ i\varphi_1-\varphi_2\end{pmatrix},\qquad K^\dagger=\frac{1}{\sqrt{2}}\begin{pmatrix}F\sqrt{2}+Z-i\varphi_3\\ -i\varphi_1+\varphi_2\end{pmatrix},$$

$$Q^u_+=\frac{1+\gamma^5}{2}\begin{pmatrix}u\\ d'\end{pmatrix},\qquad Q^c_+=\frac{1+\gamma^5}{2}\begin{pmatrix}c\\ s'\end{pmatrix},$$

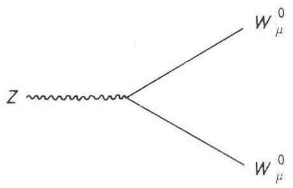
$$F=M\frac{\sqrt{2}}{g}\,.$$

We must now evaluate the leading radiative corrections as a function of the Z mass m. As stated before, they can occur only at a few places. First, in fermion-fermion scattering. Since all couplings of the Z to the fermions involve a factor $m_e/M \sim 10^{-5}$, these effects become only important for outrageously high values of m. Moreover, only logarithmic dependence is expected.

Secondly we must look to mass relations and coupling constant relations, and compute how such relations are possibly broken by m-dependent terms, and that become big if m becomes large. The first obvious candidate is the relation between the neutral and charged vector boson masses. Let us, therefore, write down vector-boson self-energy diagrams involving one (or more) Z internally. As an example, consider the interaction term

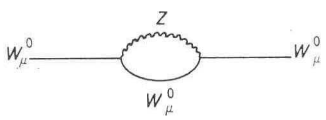
$$\tfrac{1}{2} gMZ\underline{B}^2_\mu = \tfrac{1}{2} gMZ\{(B^1_\mu)^2 + (B^2_\mu)^2 + (1/c^2)(W^0_\mu)^2\}\ .$$

For example, this implies the vertex



$$g\,\frac{M}{c^2}\,, \qquad c = \cos\theta_w\ .$$

A self-energy diagram arising from this is



This diagram involves a factor $1/c^4$. A similar diagram for $W^\pm_\mu$ involves no such factor. Thus we get a correction proportional to $1/c^4$ to the neutral-vector-boson mass squared:

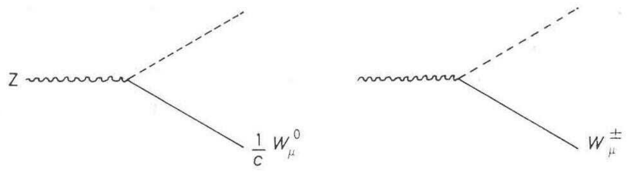
$$M_0^2 \to M_0^2 + \frac{1}{c^4}\,\delta = \frac{M^2}{c^2} + \frac{1}{c^4}\,\delta = \frac{M^2}{c^2}\left(1 + \frac{\delta}{c^2 M^2}\right).$$

For the charged mass this is

$$M^2 \to M^2 + \delta = M^2\left(1 + \frac{\delta}{M^2}\right),$$

and all depends now on the magnitude of δ. Unfortunately δ depends only logarithmically on m, the Higgs mass, as will be made clear in sect. 8.

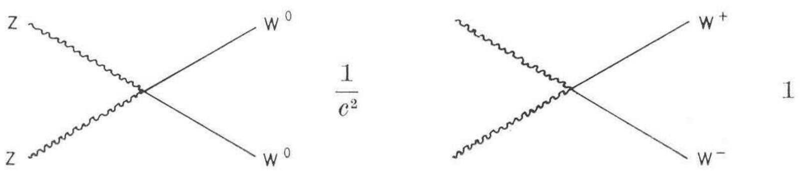
As another example consider the vertex $\underline{B}^a_\mu(Z\partial_\mu\varphi_a - \varphi_a\partial_\mu Z)$. It gives the vertices



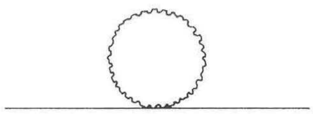
There is a factor 1/c. The self-energy diagrams get a factor $1/c^2$ for W^0 relative to $W^\pm$

As a consequence no deviation of the relation $M_0^2 = M^2/c^2$ is produced.

As a final example we consider the vertex $Z^2\underline{B}^2_\mu$



It gives rise to the self-energy diagram



which is quadratic in the Z mass *m*. But again, the W° correction has a factor $1/c^2$ and no deviation is produced.

There are other vertices containing both B_μ and Z that can produce effects. The conclusion is that in the Weinberg model there are no significant deviations from the rule $M_0^2 = M^2/\cos^2\theta_w$ for large Higgs mass *m*. This is very unfortunate because we would have had a direct possibility to find a limit on the Higgs mass. As we will see, we have no way, at this moment, to determine an upper limit. The best that we can do is to produce plausibility arguments and inspired speculations.

8. — Behaviour as a function of the Higgs mass.

In this section we will write down some simple formulae that will give us insight in how diagrams depend on the Higgs mass. In a theory with Higgs system all infinities are renormalizable and need not to be cut off at some value Λ. We, therefore, will work in the framework of dimensional regularization.

Consider the integral

$$\int \mathrm{d}^n q \, \frac{1}{q^2 + m^2} = \frac{i\pi^{n/2}}{(m^2)^{-n/2}} \cdot \frac{\Gamma(1 - n/2)}{\Gamma(1)} .$$

Now

$$\Gamma\left(1 - \frac{n}{2}\right) = \frac{2}{2 - n} \Gamma\left(2 - \frac{n}{2}\right) = \frac{2}{2 - n}\left(\frac{2}{4 - n} + \gamma \, ...\right).$$

This contains a pole at $n = 4$ and finite terms

$$\Gamma\left(1 - \frac{n}{2}\right) = \frac{2}{n - 4} + C + O(n - 4) .$$

Further

$$(m^2)^{n/2} = m^2 \cdot (m^2)^{(n-4)/2} = m^2 \exp\left[\frac{n-4}{2} \ln m^2\right] = m^2 \left\{1 + \frac{n-4}{2} \ln m^2 + ...\right\}.$$

In this way we obtain for n around 4

$$\int \mathrm{d}^n q \, \frac{1}{q^2 + m^2} = \frac{2i\pi^2}{n - 4} m^2 + i\pi^2 m^2 \ln m^2 + ... ,$$

where ... stands for terms proportional to C and $\ln \pi^2$. They also contain a factor m^2, but they are never relevant and in fact can be renormalized away together with the pole term. In any case, we see that the finite part is proportional to m^2. This of course we could have guessed immediately because the integral has the dimensions of a mass squared.

This type of reasoning works generally. Thus

$$\int \mathrm{d}^4 q \, \frac{q_\alpha q_\beta}{((q + p)^2 + M^2)(q^2 + m^2)} ,$$

which has the dimensions of a mass squared, contains indeed terms proportional to m^2. And

$$\int \mathrm{d}^4 q \, \frac{1}{((q + p)^2 + M^2)(q^2 + m^2)}$$

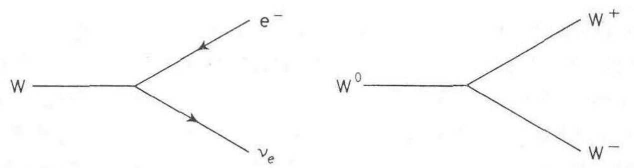
is dimensionless and contains at best $\ln m^2$ (which it does). It can be found easily, by setting both p and M to zero, that . .

$$\int \mathrm{d}^n q \, \frac{1}{q^2(q^2+m^2)} = \int_0^1 \mathrm{d}x \int \mathrm{d}^n q \, \frac{1}{(q^2+m^2x)^2} \Rightarrow i \int \mathrm{d}x \left(-\frac{2\pi^2}{n-4} + C - \pi^2 \ln m^2 \right) =$$
$$= -i\,\frac{2\pi^2}{n-4} + iC - i\pi^2 \ln m^2 \,.$$

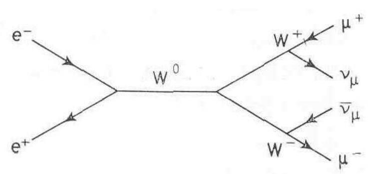
Obviously, m^2-dependence arises only from quadratically divergent integrals, $\ln m^2$ from logarithmically divergent integrals etc. This limits our search for m-dependent effects to those places where we have divergences, *i.e.* at mass and coupling constant type radiative corrections. This is of course precisely in accordance with the remarks made before. (Actually, the afore-mentioned divergence in fermion-fermion scattering seems to have vanished; we have not investigated this in detail.) It must be noted that the situation becomes more complicated if external Z lines occur. Then there are vertices in the Lagrangian containing a factor $\alpha = m^2/4M$ that may become involved.

9. — Coupling constants.

To find effects of a large Higgs mass, we must thus concentrate upon coupling constants. Experimentally, the coupling constant g occurs in lowest order in two places, namely in the $\mathrm{W}(e\nu)$ vertex, and in the WWW vertex:



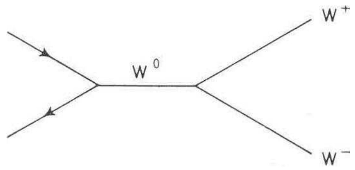
Eadiative corrections may give rise to observable differences proportional to $\ln m^2$. This indeed happens to be the case. In a slightly simplified model, namely the Weinberg model with e.m. interactions removed (this is the limit $\sin\theta_{\mathrm{w}} \to 0$, e = electric charge $\to 0$), this has been calculated, and we refer to ref. [14] for more details. The result is that the ampliude for the process



obtains a correction $1 - \frac{1}{2}\varepsilon$ in amplitude with

$$\varepsilon = \frac{g^2}{192\pi^2} \ln m^2 \sim 10^{-4} \ln m^2 .$$

This is really very small and not very practical. However, the calculation has been done by assuming that all quantities are small with respect to the Higgs mass. There are reasons to believe that in a W^+W^- production process



radiative corrections may become big if both energy and Higgs mass are large with respect to the vector-boson mass. Details have not yet been worked out.

Another area of interest is the Higgs-fermion coupling. This vertex obtains corrections proportional to m^2. In fact, the coupling constant

$$g\,\frac{m_e}{M}$$

obtains the correction

$$1 + \frac{g^2 m^2}{32\pi^2 M^2}\left(\frac{13}{8} - \frac{\pi\sqrt{3}}{4}\right).$$

That will not very likely be able to beat the factor $m_e/M \sim 10^{-5}$, nor even in the muon case the factor $m_\mu/M \sim 1/600$. But it sets us in some interesting direction of speculation. Imagine that $g^2 m^2/\pi^2 M^2$ is large. Then the whole perturbation series breaks down, and we may get very large corrections. If there exist very heavy fermions (quarks!) with a mass large with respect to the vector-boson mass *M*, then we may get a really strong Yukawa force between those fermions and moreover, for large Higgs mass, with a nonperturbative system attached to it. That is very much going in the direction of strong interactions. Perhaps our strong interactions are nothing else but some interplay between Higgs particles and heavy fermions. It is extremely difficult to envisage the details of such a configuration, and there are certainly many aspects that cannot be understood simply. In any case, nothing keeps us from speculating that perhaps hadrons contain a lot of Higgs-like bosons. The question is then: is there any way to detect this? This is very difficult because electrons and muons couple only very weakly to Higgs particles; they would be excellent probes, because electrons and muons are differently coupled to the Higgs scalars. Any difference in behaviour between electrons and muons moving through hadronic matter would, therefore, be very interesting. Up to now nothing significant has been seen; nevertheless it seems worth-while to investigate this point in more precise (experimental) detail [15].

We may close these lecture notes by mentioning another result, related to the methods explained above. As we have seen, mass relations are sensitive to radiative corrections due to high-mass systems. As noted, unfortunately, the Higgs mass cannot be estimated from corrections to the ratio of charged and neutral vector-boson masses. But it happens that very heavy fermions split indeed M_c and M_0, in a way depending on the mass differences within the fermion multiplet. In view of the fact that experimentally the mass relation holds Avell within some 15 % one can conclude that mass differences within any multiplet must be less than 800 GeV [16].

10. — Conclusions.

In summary we can say the following:

i) From the near equality of G_β and G_μ (vector coupling constants of β-decay and μ-decay) we conclude that new physics, presumably vector bosons, must set in before 300 GeV. New means here deviation of the current x current theory.

ii) If the Higgs boson is very heavy, it may be extremely difficult to detect. The only chance that radiative corrections are sizable is in W^+W^- production by e^+e^- collisions at high energy, say 300 GeV.

iii) Heavy leptons or heavy quarks (heavier than 300 GeV) coupled to the Higgs system just like the electron and its neutrino have a strong interaction with the Higgs boson.

iv) A strong interaction for such heavy fermions can be avoided if left- and right-handed multiplets are in the same representation. This implies pure vector coupling to the vector bosons.

v) If the Higgs bosons are very heavy also strong interactions arise.

vi) Breakdown of μ-e universality may be a probe for Higgs bosons inside hadrons.

vii) Lepton-neutrino pairs, left-handed, coupled just like e-ν_e, cannot have mass differences larger than 800 GeV if the mass relation $M_0^2 = M_c^2/\cos^2\theta_w$ is correct, and only one Higgs exist.

All in all, in the big unknown territory above 300 GeV there may be Higgs particles and fermions, but, unless these fermions have pure vector coupling, they will have strong Yukawa-type interactions. And if the Higgs bosons are very heavy, they also have strong interactions. It could be quite a mess, unless parity violation disappears for these high energies.

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