GAUGE THEORIES OF WEAK INTERACTIONS

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1. Introduction

The purpose of these notes is to produce an introduction to the presently popular standard model of weak and electromagnetic interactions. This model has a number of ingredients, leptons, quarks, vector bosons, photons and finally also Higgs scalars, and of these only the leptons and photons are directly observed particles. The evidence for quarks comes from another domain of physics, namely strong interactions supposedly described by quantum chromodynamics. The massive vector bosons as well as the Higgs particles are theoretical inventions, and while few people doubt the existence of vector bosons, many are sceptical with respect to the Higgs system.

Meanwhile, theorists have gone beyond the standard model, and speculations on possible grand unified schemes have emerged. Also there are speculations on the possible compositeness of quarks and leptons, or Higgs particles. In these notes we will not discuss grand unified theories, and the question of physics beyond 1 TeV (where compositeness may show up) will be touched upon only sketchily.

Our metric is such that $k^2 < 0$ if k is a timelike vector.

2. The Four-Fermion Interaction

In 1957, Marshak and Sudarshan, and Feynman and Gell-Mann[1] proposed the V-A theory. These latter authors then also introduced the current-current theory. In this theory the interaction between fermions is written as the product of two currents that carry a charge. If we restrict ourselves to muons, electrons and their neutrinos then the interaction Lagrangian is:

$$\mathscr{L}_{cc} = G_F j^+_\alpha j^-_\alpha \tag{2.1}$$

$$\begin{aligned} j^-_\alpha &= i(\bar{\nu}_e\gamma^\alpha(1+\gamma^5)e) + i(\bar{\nu}_\mu\gamma^\alpha(1+\gamma^5)\mu) \\ j^+_\alpha &= i(\bar{e}\gamma_\alpha(1+\gamma^5)\nu_e) + i(\bar{\mu}\gamma^\alpha(1+\gamma^5)\nu_\mu) \ . \end{aligned} \tag{2.2}$$

Thus:

$$\begin{aligned} \mathscr{L}_{cc} = -G_F[& (\bar{\nu}_e\gamma^\alpha(1+\gamma^5)e)(\bar{e}\gamma^\alpha(1+\gamma^5)\nu_e) \\ &+ (\bar{\nu}_e\gamma^\alpha(1+\gamma^5)e)(\bar{\mu}\gamma^\alpha(1+\gamma^5)\nu_\mu) \\ &+ (\bar{\nu}_\mu\gamma^\alpha(1+\gamma^5)\mu)(\bar{e}\gamma^\alpha(1+\gamma^5)\nu_e) \\ &+ (\bar{\nu}_\mu\gamma^\alpha(1+\gamma^5)\mu)(\bar{\mu}\gamma^\alpha(1+\gamma^5)\nu_\mu)] \ . \end{aligned} \tag{2.3}$$

We thus have four basic interactions, and they all go with the same coupling constant (see Fig. 1).

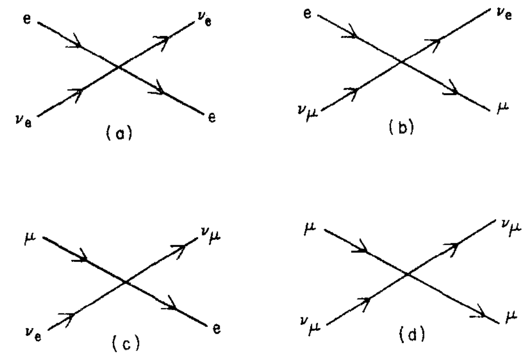

Figure 1

Vertices b and c are relevant for μ^+ and μ^- decay respectively.

Using the expression corresponding to vertex c we may compute the muon lifetime as a function of the fermi coupling constant G_F.

$$\frac{1}{\tau_\mu} = \frac{m_\mu^5}{96\pi^3} G_F^2 \ . \tag{2.4}$$

Comparing this with the experimentally known lifetime we find

$$G_F = 8.2297\times10^{-12}\ \mathrm{MeV}^{-2} = \frac{1.0246}{m_p^2\sqrt{2}} \times 10^{-5} \tag{2.5}$$

where m_p is the proton mass. To good approximation $G_F \sim 10^{-5}/m_p^2\sqrt{2}$. In the literature one often writes $G_F/\sqrt{2}$ in front of the interaction rather than G_F, as we did, and consequently values quoted may differ by $\sqrt{2}$ from the value that we give here.

At this level the current-current theory produces a prediction for ν_e-e scattering. The lowest order amplitude is:

$$- (2\pi)^4 i G_F (\bar{e}\gamma^\mu (1+\gamma^5)\nu_e)(\bar{\nu}_e \gamma^\mu (1+\gamma^5) e) \tag{2.6}$$

which, after a Fiertz transformation is equivalent to

$$(2\pi)^4 i G_F (\bar{\nu}_e \gamma^\mu (1+\gamma^5)\nu_e)(\bar{e}\gamma^\mu (1+\gamma^5) e) \tag{2.7}$$

This leads to the total cross section:

$$\sigma^{tot}_{\nu_e e} = \frac{16}{\pi} G_F^2 \frac{m^3 E_\ell^3 (E_\ell + \frac{1}{2}m)}{E^4 E_\nu} \tag{2.8}$$

m = electron mass,

E_ν = neutrino energy,

E_ℓ = total lab energy = $E_\nu + m$

E = c.m. energy = $\sqrt{2mE_\ell + m^2}$.

If E_ℓ and E large with respect to m we may approximate $E_\ell = E_\nu$ and $E^2 = 2mE_\ell$. The result is then:

$$\sigma^{tot}_{\nu_e e} \simeq \frac{4}{\pi} G_F^2 \, mE_\ell \simeq \frac{2}{\pi} G_F^2 \, E^2 \, . \tag{2.9}$$

We will not analyze here to what extent this prediction agrees with the data obtained from low energy reactor experiments. As a matter of fact, it seems that there is no conflict with the measurements. However, the above equation shows a strong dependence on energy; the cross section grows indefinitely with increasing energy. This is not acceptable; at some point one gets into conflict with conservation of probability. Stated differently, above a certain energy the formula for the cross section violates unitarity, and thus must be wrong. In the following section we will analyze this problem.

3. The Unitarity Limit

The fact that Eq. (2.9) for the total cross section violates unitarity actually implies that we have not made a correct calculation. We must include radiative corrections, and they will restore unitarity. From the fact that at high energy the cross section (2.9) violates unitarity by an arbitrarily large amount we deduce that the radiative corrections become very large at high energies. Since we have no data at high energy, that cannot be excluded. However, it may well be that some of these radiative corrections are also large at low energies, in which case they are of immediate interest.

To study this we must introduce a cut-off in the theory. Instead of the Fermi constant G_F we will use the expression

$$\frac{g^2}{q^2 + \Lambda^2} \tag{3.1}$$

where q is the four-momentum transfer between the currents. Thus in the process under consideration it is the momentum as indicated in Fig. 2. In our metric q^2 is positive for spacelike q. If we choose g and Λ such that $g^2/\Lambda^2 = G_F$ then at low energies there will be no difference with the original interaction.

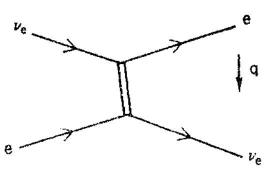

Figure 2

The lowest order radiative corrections to the above process correspond to a number of diagrams, and the relevant diagrams are shown in Fig. 3.

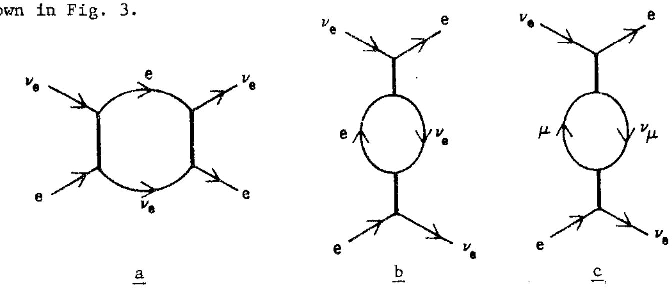

Figure 3

All of these diagrams give large contributions at high energies. At low energies all these diagrams lead to effects that can be eliminated by redefining g and Λ.

However, there is also μ-decay. The relevant diagrams to this decay are shown in Fig. 4.

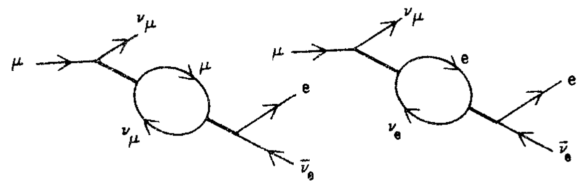

Figure 4

Again, the above diagrams can be eliminated by redefining g and Λ. However, this redefinition evidently differs from that needed for ν_e-e scattering, as there is no equivalent of diagram a of Fig. 3 in the case of μ-decay. This diagram will therefore induce a difference between the rate for μ-decay and the ν_e-e scattering cross section relative to the lowest order predictions.

The contribution of diagram a of Fig. 3 can be computed. It is given by:

$$\int d_4q \frac{g^4}{(q^2+\Lambda^2)^2 q^4} \cdot \{\bar{\nu}_e \gamma^\mu (1+\gamma^5)(-i\gamma q)\gamma^\nu (1+\gamma^5)\nu_e\}$$
$$\times \{\bar{e}\gamma^\mu (1+\gamma^5)(+i\gamma q)\gamma^\nu (1+\gamma^5) e\} \tag{3.2}$$

where we neglected the electron mass as well as the external momentum dependence. Averaging over directions amounts to the replacement $q_\alpha q_\beta \to \frac{1}{4} q^2 \delta_{\alpha\beta}$, and furthermore using the formula

$$\gamma^\mu \gamma^\alpha \gamma^\nu = \gamma^\mu \delta_{\alpha\nu} + \gamma^\nu \delta_{\mu\alpha} - \gamma^\alpha \delta_{\mu\nu} + \varepsilon_{\mu\alpha\nu\lambda}\gamma^5\gamma^\lambda$$

we obtain:

$$\begin{aligned} &\int d_4q \frac{12 g^4 q^2}{(q^2+\Lambda^2)^2 q^4} (\bar{\nu}_e \gamma^\mu (1+\gamma^5)\nu_e)(\bar{e}\gamma^\mu (1+\gamma^5) e) \\ &= 12 g^4 \cdot i\pi^2 \cdot \frac{1}{\Lambda^2} (\bar{\nu}_e \gamma^\mu (1+\gamma^5)\nu_e)(\bar{e}\gamma^\mu (1+\gamma^5) e) \\ &= 12 i\pi^2 \, G_F^2 \Lambda^2 (\bar{\nu}_e \gamma^\mu (1+\gamma^5)\nu_e)(\bar{e}\gamma^\mu (1+\gamma^5) e) \ . \end{aligned} \tag{3.3}$$

The addition of this to the lowest order amplitude Eq. (2.7) shows that we have a radiative correction to ν_e-e scattering (relative to μ-decay) given by a factor

$$1 + \frac{3}{4\pi^2} G_F \Lambda^2 = 1 + \frac{\Lambda^2}{(1365 \ m_p)^2} \tag{3.4}$$

to the amplitude.

Clearly, in the limit $\Lambda \to \infty$ the relation between μ-decay and ν_e-e scattering gets completely lost. Of course, one should also take into account still higher order corrections, and in fact they give again arbitrarily large corrections.

If we assume that the four-fermion theory remains valid up to some energy then we see that this energy must be less than 410 m_p if the correction to the amplitude is to be less than 9%.

The above example shows in full clarity the following fact: in a non-renormalizable theory relations between coupling constants may be strongly affected.

The model described above is incomplete in many respects: no quark currents have been included, and also current-current interaction involving neutral currents have not been considered. A more complete discussion[2] along these lines leads to a limit on the validity of the four-fermion theory of about 150-300 GeV. We refer to the literature for more extensive and/or alternative discussions relating to these matters.[3]

4. Gauge Theories

The discussion of the previous section resulted in the statement that the four-fermion theory of weak interactions cannot be true to all energies. The real question is then this: how to modify the four-fermion theory in such a way that the low energy results remain unaffected, and furthermore such that radiative corrections remain small. One of the basic ingredients in such a construction is the introduction of a symmetry. For instance, one introduces a symmetry that guarantees the equalness of the μ-decay and ν_e-e scattering coupling constants, and tries to arrange things such that most interactions respect this symmetry. Invariably such considerations lead to the same result, namely a gauge theory of weak interactions. We will give here the standard theory[4] of weak and electromagnetic interactions, and describe in detail the options available within the framework of that theory.

The first choice is the choice of the symmetry. Here this is $SU_2 \times U_1$. Thus there is a two-dimensional complex space and a one-dimensional complex space. SU_2 is the collection of 2×2 unitary matrices with determinant 1 defined in the two-dimensional space, and U_1 is the collection of 1×1 unitary matrices in the one-dimensional space. Alternatively one may see U_1 as matrices in n×n space differing from the unit matrix by a phase factor.

Having chosen the symmetry the vector particle content of the theory is fixed. There are as many vector particles as degrees of freedom in the symmetry. An SU_2 matrix may be written in the form

$$S = e^{-i\rho_a \frac{\tau^a}{2}} , \quad a = 1,2,3 \tag{4.1}$$

where the τ^a are the Pauli matrices and the ρ_a are three real numbers. For the U_1 matrices we may write

$$e^{-i\rho_o \frac{\tau^o}{2}} \tag{4.2}$$

involving one real number ρ_o and the unit matrix τ^o.

Next we define vector boson fields B^α_μ, $\alpha = 1,2,3$ and C^o_μ, and vector boson matrices:

$$b_\mu = -\frac{i\tau^\alpha}{2} B^\alpha_\mu ; \qquad c_\mu = -\frac{i\tau^o}{2} C^o_\mu . \tag{4.3}$$

With every SU_2 and U_1 transformation we will associate a transformation of the vector boson matrices. Let S be an SU_2 matrix of the form (4.1). With this matrix we associate a vector boson transformation involving an arbitrary constant g:

$$b'_\mu = S\, b_\mu S^\dagger + \frac{1}{g} S\, \partial_\mu S^\dagger . \tag{4.4}$$

This defines b'_μ. If we write b'_μ as a function of new vector boson fields B'_μ just like in Eq. (4.3) for b then, after some work, one finds the relation between the B' and B. Just for completeness we quote this relation:

$$B'^a_\mu = U_{ab} B^b_\mu - \frac{1}{g} \left(\frac{U-1}{\overline{\Lambda}}\right)_{ab} \partial_\mu \rho^b \tag{4.5}$$

where $\overline{\Lambda}$ and U are matrices:

$$\bar{\Lambda}_{ab} = -f_{cab}\rho^{c}$$

$$U = e^{\bar{\Lambda}} \tag{4.6}$$

and the f are structure constants:

$$\left[\frac{-i\tau^{a}}{2}, \frac{-i\tau^{b}}{2}\right] = f_{abc}\,\frac{-i\tau^{c}}{2} . \tag{4.7}$$

In the case at hand $f_{abc} = \varepsilon_{abc}$, the completely antisymmetrical tensor in three dimension.

A similar transformation holds for c_μ with respect to the U_1 transformations (4.2).

We now can write down the vector boson Lagrangian. Define the matrix $g_{\mu\nu}$:

$$g_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu + g[b_\mu, b_\nu] . \tag{4.8}$$

If we decompose

$$g_{\mu\nu} = -\frac{i\tau^{a}}{2}\,G^{a}_{\mu\nu} \tag{4.9}$$

Then we deduce:

$$G^{a}_{\mu\nu} = \partial_\mu B^{a}_\nu - \partial_\nu B^{a}_\mu + g f_{abc} B^{b}_\mu B^{c}_\nu . \tag{4.10}$$

The SU_2 vector boson Lagrangian is now:

$$\mathscr{L} = -\tfrac{1}{2}\,\mathrm{Tr}(g^{\dagger}_{\mu\nu} g_{\mu\nu}) = -\tfrac{1}{4}\,G^{a}_{\mu\nu} G^{a}_{\mu\nu} \tag{4.11}$$

where Tr stands for trace. To this we must add the U_1 vector boson Lagrangian constructed in the same way. However, there the commutator part of $g_{\mu\nu}$ is zero, thus no further coupling constants enter in the theory.

We will now show some important results, namely that two successive transformations with matrices S_1 and S_2 give the same result as one transformation with a matrix S_3, where $S_3 = S_2 \cdot S_1$. Secondly we show that the Lagrangian (4.11) is invariant under the transformation (4.4).

(i) Let

$$b''_\mu = S_2 b'_\mu S^{\dagger}_2 + \frac{1}{g}\,S_2 \partial_\mu S^{\dagger}_2$$

with b'_μ related to b_μ as shown in Eq. (4.4) with S_1 for S. We find:

$$\begin{aligned} b''_\mu &= S_2(S_1 b_\mu S^{\dagger}_1 + \frac{1}{g}\,S_1 \partial_\mu S^{\dagger}_1)S^{\dagger}_2 + \frac{1}{g}\,S_2 \partial_\mu S^{\dagger}_2 \\ &= S_3 b_\mu S^{\dagger}_3 + \frac{1}{g}\,S_3(\partial_\mu S^{\dagger}_1)S^{\dagger}_2 + \frac{1}{g}\,S_2 \partial_\mu S^{\dagger}_2 \\ &= S_3 b_\mu S^{\dagger}_3 + \frac{1}{g}\,S_3 \partial_\mu S^{\dagger}_3 \end{aligned}$$

where we used

$$\partial_\mu S^{\dagger}_3 = \partial_\mu(S^{\dagger}_1 S^{\dagger}_2) = (\partial_\mu S^{\dagger}_1)S^{\dagger}_2 + S^{\dagger}_1 \partial_\mu S^{\dagger}_2$$

and

$$S^{\dagger} = S^{-1} .$$

(ii) From (4.4) and (4.8) we deduce:

$$\begin{aligned} g'_{\mu\nu} &= \partial_\mu b'_\nu - \partial_\nu b'_\mu + g[b'_\mu, b'_\nu] \\ &= \partial_\mu S\ b_\nu S^{\dagger} + S\ b_\nu \partial_\mu S^{\dagger} + S\partial_\mu b_\nu S^{\dagger} - \partial_\nu S\ b_\mu S^{\dagger} - S\ b_\mu \partial_\nu S^{\dagger} - S\ \partial_\nu b_\mu S^{\dagger} \\ &\quad + \frac{1}{g}\,\partial_\mu(S\ \partial_\nu S^{\dagger}) - \frac{1}{g}\,\partial_\nu(S\ \partial_\mu S^{\dagger}) + \frac{1}{g}\,S[b_\mu, b_\nu]S^{\dagger} \\ &\quad + [S\ \partial_\mu S^{\dagger}, S\ b_\nu S^{\dagger}] + [S\ b_\mu S^{\dagger}, S\ \partial_\nu S^{\dagger}] + \frac{1}{g}\,[S\ \partial_\mu S^{\dagger}, S\ \partial_\nu S^{\dagger}] . \end{aligned}$$

With

$$\partial_\mu S = -S(\partial_\mu S^{-1})S = -S(\partial_\mu S^\dagger)S$$

one obtains

$$g'_{\mu\nu} = S(\partial_\mu b_\nu - \partial_\nu b_\mu + g[b_\mu, b_\nu])S^\dagger = S\ g_{\mu\nu}\ S^\dagger .$$

Now the Lagrangian (4.11) contains the trace of $g_{\mu\nu}g_{\mu\nu}$, and if we use the property Tr(ABCD) = Tr(BCDA) then we see that $\mathrm{Tr}(g'_{\mu\nu}g'_{\mu\nu}) = \mathrm{Tr}(g_{\mu\nu}g_{\mu\nu})$. This is precisely what we mean by invariance of the Lagrangian under the transformations (4.4).

For completeness we write explicitly some equations for the U_1 field c_μ. Under a transformation (4.2) the transformation law (4.4) simplifies for the 1×1 matrix c_μ to

$$c'_\mu = c_\mu + \frac{1}{g}\, i\, \frac{\tau^o}{2}\, \partial_\mu \rho_0 \tag{4.12}$$

and Eq. (4.5) becomes:

$$C'^o_\mu = C^o_\mu - \frac{1}{g}\, \partial_\mu \rho_0 \tag{4.13}$$

For convenience we also give some equations that are useful for the derivation underlying the connection between Eqs. (4.4) and (4.5). A useful trick is to write:

$$\frac{U-1}{\overline{\Lambda}} = \int_0^1 dy\ e^{y\overline{\Lambda}}\ , \quad U = e^{\overline{\Lambda}}\ . \tag{4.14}$$

Other useful equations are (A and B are matrices):

$$e^{-A}\, B\, e^{A} = B + [B,A] + \frac{1}{2!}\,[[B,A],A] + \ldots \tag{4.15}$$

If A is a matrix depending on x:

$$e^{-A} \frac{d}{dx} e^{A} = A' + \frac{1}{2!}\,[A',A] + \frac{1}{3!}\,[[A',A],A] + \ldots$$

where A' is the matrix whose elements are the derivative of the elements of A. Equations like these are proven by substituting A = yA and subsequently working out d^n/dy^n of the left-hand side to obtain a power series in y. For instance:

$$\frac{d}{dy}\left\{e^{-yA}\, B\, e^{yA}\right\} = e^{-yA}[B,A]\, e^{yA}\ .$$

For y = 0 the expression on the right is the coefficient of y in the series expansion. So we get the second term in Eq. (4.15).

5. Strong Interactions

In the previous section we introduced vector-boson matrices b and c. They may be combined elegantly into one matrix $\underline{b}$:

$$\underline{b}_\mu = b_\mu + c_\mu = -\frac{i}{2}\begin{pmatrix} C^o_\mu + B^3_\mu & B^1_\mu - iB^2_\mu \\ B^1_\mu + iB^2_\mu & C^o_\mu - B^3_\mu \end{pmatrix} .$$

The Lagrangian is now simply $-\mathrm{Tr}(\underline{g}_{\mu\nu}\underline{g}_{\mu\nu})/2$ where $\underline{g}_{\mu\nu}$ is defined analogous to Eq. (4.8).

The identification with physical particles goes by establishing the mass eigenstates, and also by charge (or other quantum number) eigenstates. Such things become well defined only after coupling of the vector-bosons to fermions and Higgs fields, where then the latter generate masses for the vector bosons. Things will be arranged such that off-diagonal fields represent quantum eigenstates. Along the diagonal things may be more complicated. There the mass-eigenstates may be combinations of the fields found along the diagonal, but that can be established only after mass generation by the Higgs system is worked out.

The physical fields in the $SU_2 \times U_1$ standard model will be denoted by $W^\pm_\mu$, W^o_μ and A_μ, where W^o_μ is the neutral vector boson (often called the Z^o) and A_μ is the photon field. The fields B^3_μ and C^o_μ are linear combinations of W^o and A:

$$B^3_\mu = c_\theta W^o_\mu + s_\theta A_\mu$$

$$C^o_\mu = -s_\theta W^o_\mu + c_\theta A_\mu$$

$$c_\theta = \cos\theta \qquad s_\theta = \sin\theta .$$

The as yet unspecified angle θ is the weak mixing angle. In terms of the physical fields the vector boson matrix is:

$$\underline{b} = -\frac{i}{2}\begin{pmatrix} (c_\theta - s_\theta)W^o_\mu + (c_\theta + s_\theta)A_\mu & \sqrt{2}\,W^+_\mu \\ \sqrt{2}\,W^-_\mu & -(c_\theta + s_\theta)W^o_\mu + (c_\theta - s_\theta)A_\mu \end{pmatrix} .$$

If $\mathscr{L}$ was properly normalized with respect to the B and C then this will also be the case for the W and A.

Quantum chromodynamics is presumably described by an SU_3 symmetry. This amounts to writing 3×3 matrices λ^a instead of the 2×2 matrices τ^a. The vector bosons of QCD are called gluons, and may also be grouped in a matrix. A gluon is characterized by two colors (or one color and one anticolor), and in terms of these color states the gluon matrix is

$$g_\mu = -\frac{i}{2}\begin{pmatrix} g^w + \frac{1}{\sqrt{3}}g'^w & \sqrt{2}\,g^r_g & \sqrt{2}\,g^r_b \\ \sqrt{2}\,g^g_r & -g^w + \frac{1}{\sqrt{3}}g'^w & \sqrt{2}\,g^g_b \\ \sqrt{2}\,g^b_r & \sqrt{2}\,g^b_g & -\frac{2}{\sqrt{3}}g'^w \end{pmatrix} .$$

The subscripts r,g,b refer to the colors red, green and blue, and the superscript to their anticolors. There are two "white" gluons that may be considered also as combinations of red-antired, green-antigreen and blue-antiblue.

The Lagrangian now simply gets an extra term involving the gluon fields completely analogous to the b fields as described before. One may combine the weak, electromagnetic and gluon vector bosons by introducing a 5×5 matrix made up from the 2×2 b matrix and the 3×3 g matrix:

$$\begin{pmatrix} b & b & 0 & 0 & 0 \\ b & b & 0 & 0 & 0 \\ 0 & 0 & g & g & g \\ 0 & 0 & g & g & g \\ 0 & 0 & g & g & g \end{pmatrix} .$$

The Grand Unified Theory of Georgi and Glashow is obtained by assuming the existence of 6 new vector bosons at the locations with zeroes in the above matrix. Moreover, the U_1 part of $SU_2 \times U_1$, represented by C^o, is redefined as a multiple of the matrix with 1,1,-2/3,-2/3,-2/3 along the diagonal. It is no longer a U_1 symmetry, but part of the SU_5 symmetry.

On the face of it this SU_5 symmetry is quite far from the reality of physics. The new vector bosons must be very heavy, or else they would have been observed either directly or indirectly. Furthermore, in a Lagrangian defined with such a matrix as in Eq. (4.11) we have only one free parameter, the coupling constant g as shown in Eqs. (4.8) and (4.10), while at low energies we observe 3 different coupling constants for electromagnetic, weak and strong interactions. However, it may be shown that the apparant experimental differences arise from radiative corrections. The calculation of these corrections involves one new parameter, inherent to the renormalization procedure. It turns out that there exists one choice for this parameter such that the radiative corrections, starting from one coupling constant, give rise to about the experimentally observed coupling constants. Stated differently, two parameters may be chosen such that three quantities fit the data. In these notes we will not discuss SU_5 any further, we will focus attention on other areas of interest.

6. The Fermions

In constructing the fermion Lagrangian, including the coupling to vector bosons, one must assign SU_2 and U_1 transformation properties to these fermions. With respect to SU_2 the popular view is that the left-handed fermions are SU_2 doublets while the right-handed fermions are singlets under SU_2. The transformation properties with respect to U_1, always given by phase factors, are fixed such that the proper coupling to the photon results.

Consider a left-handed fermion doublet, denoted by f_i^+, i = 1,2. Under an SU_2 transformation X_{ji}, with X of the form (4.1), the f_i transform as

$$f_i'^+ = X_{ij} f_j^+ \quad . \tag{6.1}$$

Once the fermions are chosen to be doublets there is no freedom in this transformation assignment. The requirement is that the matrix X corresponding to an SU_2 transformation as in Eq. (4.1) must be of the form

$$e^{-i\rho_a \frac{t^a}{2}} \quad , \quad a = 1,2,3 \tag{6.2}$$

where the t^a have the same commutation rules as the τ^a, and apart from trivialities this has only the solutions $t^a = \tau^a$ or $t^a = 0$. The situation is quite different with respect to the U_1 symmetry. Corresponding to the U_1 transformation (4.2) we have a transformation

$$e^{-i\rho_o \frac{t^o}{2}} \tag{6.3}$$

and the requirement that t^o commutes as τ^o gives only that t^o must be a multiple of the identity. In other words, the U_1 transformation takes the form

$$e^{-ig_1\rho_o \frac{\tau^o}{2}} \qquad (6.4)$$

with g_1 as arbitrary constant.

Formally, the right-handed counterparts of the left-handed doublet may also be written as a doublet, ℓ_i^-, $i = 1,2$. These right-handed fermions are singlets under SU_2, i.e., the transformation matrices X for the f^- are given by an expression of the form (6.2) with $t^a = 0$, which is the unit matrix. With respect to t^o we now may take any hermitean 2×2 matrix, and denote that by T^o.

The coupling of the vector bosons to the fermions is dictated by the fact that the covariant derivative D_μ must involve the vector bosons in such a way that $D_\mu \ell$ transforms precisely as ℓ under $SU_2 \times U_1$. Consider f^+. Under an SU_2 transformation S we have:

$$f'^+ = Sf^+ \qquad (6.5)$$

and

$$\partial_\mu f'^+ = (\partial_\mu S)f + S\partial_\mu f \ .$$

We define (compare Eq. (4.4))

$$D_\mu = \partial_\mu + gb_\mu \ .$$

Note that D_μ is a 2×2 matrix. The ∂_μ part is ∂_μ times the unit matrix. We have:

$$\begin{aligned} D'_\mu f'^+ &= (\partial_\mu + gb'_\mu)f'^+ \\ &= (\partial_\mu + gSb_\mu S^\dagger + S\partial_\mu S^\dagger)Sf^+ \\ &= S(\partial_\mu + gb_\mu)f^+ + (\partial_\mu S)f^+ + S(\partial_\mu S^\dagger)Sf \ . \end{aligned}$$

Since

$$\partial_\mu S^\dagger = - S^\dagger(\partial_\mu S)S^\dagger$$

we see that

$$D'_\mu f'^+ = SD_\mu f^+ \ .$$

With respect to the U_1 transformations we must use t^o instead of τ^o in the use of the vector boson matrix. In other words, let

$$\bar{c}_\mu = - \frac{i}{2} t^o C_\mu = t^o c_\mu$$

(compare Eq. (4.3)). Then, including also the SU_2 part of the correct definition for D is:

$$D_\mu = \partial_\mu + gb_\mu + g\,\bar{c}_\mu \ .$$

For the right-handed SU_2 singlet doublet we have the definition

$$D_\mu = \partial_\mu + g\,\bar{\bar{c}}_\mu \ , \qquad \bar{\bar{c}}_\mu = T^o c_\mu$$

where now in the definition of $\bar{\bar{c}}$ the T^o occurring in the transformation rule of f^- is to be used.

The above shows how the SU_2 fields b and the U_1 field c appear in several different combinations. In a truly unified theory this would not happen.

The Lagrangian for the fermion fields can now be written:

$$\mathcal{L}_f = - \overline{f^+}\gamma^\mu D_\mu f^+ - \overline{f^-}\gamma^\mu D_\mu f^- \ .$$

D_μ involves t^o in the f^+ term and T^o in the f^- term. Several arbitrary parameters (relating to the behaviour under U_1) remain to be fixed. This is done requiring that the electromagnetic field has couplings as required. This amounts to the following

(i) Leptons. Consider the ν_e e system. The left- and right-handed doublets are:

$$f^{\pm} + \ell^{\pm} = \tfrac{1}{2}\,(1 \pm \gamma^5)\begin{pmatrix}\nu_e \\ e\end{pmatrix} \quad .$$

For t^o and T^o

$$\text{leptons:} \quad t^o = -\frac{s_\theta}{c_\theta}\,\tau^o, \qquad T^o = -2\,\frac{s_\theta}{c_\theta}\begin{pmatrix}0 & 0 \\ 0 & 1\end{pmatrix}$$

must be used. The construction decouples the neutrino from the photon, and the interaction between photon and electron is parity conserving (equal for left- and right-handed electrons).

(ii) Quarks. Consider the u-d system. The left- and right-handed doublets are

$$f^{\pm} + q^{\pm} = \tfrac{1}{2}\,(1 \pm \gamma^5)\begin{pmatrix}u \\ d\end{pmatrix}.$$

Now one must take

$$\text{quarks:} \quad t^o = \frac{1}{3}\frac{s_\theta}{c_\theta}\,\tau^o \;, \quad T^o = \frac{s_\theta}{c_\theta}\begin{pmatrix}4/3 & 0 \\ 0 & -2/3\end{pmatrix}.$$

7. Higgs Couplings

In order to get a realistic model we must have mass terms for the vector bosons and the fermions. This is to be generated using the Higgs mechanism. This amounts to introducing scalar particles that must transform according to some $SU_2 \times U_1$ representation. The simplest non-trivial choice is a doublet, much like the left-handed fermion doublet. Thus we assume a complex doublet K:

$$K = \begin{pmatrix}K_1 \\ K_2\end{pmatrix} \equiv \begin{pmatrix}\sigma_0 + i\phi^3 \\ -\phi^2 + i\phi^1\end{pmatrix} \quad . \tag{7.1}$$

Now σ_0, ϕ^1, ϕ^2 and ϕ^3 are real fields. We have written things in this way because now (7.1) can also be written as:

$$K \equiv M\binom{1}{0} = \begin{pmatrix}\sigma_0 + i\phi^3 & \phi_2 + i\phi_1 \\ -\phi_2 + i\phi_1 & \sigma_0 - i\phi^3\end{pmatrix}\begin{pmatrix}1 \\ 0\end{pmatrix} \tag{7.2}$$
$$= \left\{\sigma_0\tau^o + i\phi^a\tau^a\right\}\binom{1}{0} .$$

This notation is sometimes advantageous. We now establish the $SU_2 \times U_1$ properties of K. Under SU_2, $K' = SK$, with S an SU_2 transformation as before, and under U_1 we have a transformation involving a t_0 that must be a multiple of the identity τ_0. All this is precisely like for the fermion doublet. The Lagrangian for the Higgs system is now:

$$\mathcal{L}_H = -\tfrac{1}{2}\,(D_\mu K)^+(D_\mu K) - \frac{\mu}{2}\,(K^+K) - \frac{\lambda}{8}\,(K^+K)^2 \tag{7.3}$$

where we have added two terms that are $SU_2\times U_1$ invariant and furthermore of a renormalizable type. Two arbitrary parameters μ and λ are involved. Further

$$D_\mu = \partial_\mu + gb_\mu + g\underline{c}_\mu \tag{7.4}$$

with $\underline{c}_\mu = -\frac{i}{2} g_1 \tau^o C^o_\mu$. Also g_1 is a new parameter, to be fixed by requiring the proper behaviour for electromagnetic interactions.

The state of lowest potential energy corresponding to the Lagrangian (7.3) is not necessarily the state with $K = 0$. In fact we may write:

$$\frac{\mu}{2}(K^+K) + \frac{1}{8}\lambda (K^+K)^2 = \frac{\lambda}{8}\left\{(K^+K) + \frac{2\mu}{\lambda}\right\}^2 - \tfrac{1}{2}\frac{\mu^2}{\lambda}. \tag{7.5}$$

This potential will have a minimum if $\lambda > 0$ (remember that $\mathscr{L}$ = kinetic-potential energy). If $\mu > 0$ then the minimum is for $K = 0$, which is not what we want. If $\mu < 0$ then we have a minimum for $(K^+K) = -2\mu/\lambda$. This happens for some K. By performing an $SU_2\times U_1$ transformation this K may be turned such that it is of the form:

$$\begin{pmatrix} F\sqrt{2} \\ 0 \end{pmatrix}.$$

Let us now rewrite the potential substituting $\sigma_0(x) = F\sqrt{2} + H(x)$ in the expression for K. Writing

$$K = \begin{pmatrix} F\sqrt{2} \\ 0 \end{pmatrix} + \underline{K}, \qquad \underline{K} = \begin{pmatrix} H + i\phi^3 \\ -\phi^2 + i\phi^1 \end{pmatrix} \tag{7.6}$$

we find for the potential (7.5)

$$\frac{\lambda}{8}\left\{(\underline{K}^+\underline{K}) + 2HF\sqrt{2} + 2F^2 + 2\frac{\mu}{\lambda}\right\}^2 - \tfrac{1}{2}\frac{\mu^2}{\lambda}.$$

In the following we will ignore the term $-\mu^2/2\lambda$. Introducing the new parameters

$$M = \frac{g}{\sqrt{2}}F, \quad m_H^2 = 2\lambda F^2, \quad \beta = \mu + \lambda F^2, \quad \alpha = \frac{m_H^2}{4M^2} \tag{7.7}$$

we get:

$$\frac{g^2 m_H^2}{32M^2}\left\{(\underline{K}^+\underline{K}) + \frac{4M}{g}H + \frac{8\beta M^2}{g^2 m_H^2}\right\}^2. \tag{7.8}$$

Choosing $F^2 = -\mu/\lambda$ corresponds to $\beta = 0$. For $\beta = 0$ we have only one quadratic term, namely

$$\tfrac{1}{2} m_H^2 H^2. \tag{7.9}$$

What happened to the $SU_2\times U_1$ invariance? We used this invariance to write things in a convenient way. But the symmetry is of course unaffected.

At this point one may fix the U_1 constant g_1 involved in the Lagrangian (7.3) through D_μ as given in (7.4). Writing out this Lagrangian the constant part in K gives rise to vector boson masses, and g_1 must now be chosen such that the photon remains massless. This gives $g_1 = -s_\theta/c_\theta$.

It should be noted that we have turned things upside down. What happens is that the Higgs system involves a certain constant g_1. Then one rewrites the vector boson fields, introducing an angle θ such that one of the fields remains massless. This θ is then found by solving $s_\theta/c_\theta = -g_1$. In other words, the coupling constant g_1 of the Higgs system defines the weak mixing angle.

The Higgs Lagrangian can now be worked out. In doing so we will use the charged fields $\phi^\pm$ instead of ϕ^1, ϕ^2 and ϕ^o instead of ϕ^3:

$$\phi^\pm = \frac{1}{\sqrt{2}}(\phi^1 \mp i\phi^2), \quad \phi^0 = \phi^3.$$

8. Higgs-Fermion Couplings

Up to now also the fermions are massless, and we now use the Higgs fields to also generate fermion masses. This amounts to constructing $SU_2 \times U_1$ invariant Higgs-fermion couplings. Here we must be particularly careful with respect to the U_1 behaviour. This is most easily done by considering the right-handed fermions separately, thus no longer artificially written as doublets. Under a U_1 transformation $\exp(i\rho^o)$ any fermion field requires a factor $\exp(i\kappa\rho^o)$ with κ varying from field to field. For a given fermion field the associated value of κ will be called the U_1 hypercharge of that field. For a U_1 invariant term the sum of the U_1 hypercharges of the fields in that term must be zero.

In addition we will also consider strange, charmed, bottom and top quarks. As a matter of notation we introduce a generation index a. Thus there exist three quarks u_α with charge 2/3 and three quarks d_α with charge -1/3. In addition there is still the three-fold color degeneracy, but that is a trivial complication here. At this moment we will not introduce the names, up, charm, top, etc., since there is a matter of mixing involved. Similarly there are three lepton families, denoted by ν_α and ℓ_α.

The U_1 and SU_2 properties of the fermion multiplets are:

$\ell^+ = (\nu,e)_+$	$\kappa = -t_\theta$	SU_2 doublet
ν^-	0	singlet
e^-	$-2t_\theta$	singlet
$q^+ = (u,d)^+$	$1/3\ t_\theta$	doublet
u^-	$4/3\ t_\theta$	singlet
d^-	$-2/3\ t_\theta$	singlet
K	$-t_\theta$	doublet

In this table $t_\theta = \tan_\theta = s_\theta/c_\theta$. Note that for antiparticles the U_1 properties are simply the opposite of the ones given above. Furthermore, if the particle doublets transform under SU_2 with S then the antiparticles transform with $S^\dagger$, for example

$$K' = SK \ , \qquad K'^\dagger = K^\dagger S^\dagger \quad .$$

Interaction terms that are invariant under $SU_2 \times U_1$ may now be written down. First we do this assuming one generation only

$$\mathcal{L}_{fH} = \mu_1(\bar{\ell}^+\nu^- K) + \mu_2(\bar{\ell}^+ e^- K^\dagger) + \lambda_1(\bar{q}^+ u^- K) + \lambda_2(\bar{q}^+ d^- K^\dagger) + \text{herm. conj.}$$

The SU_2 properties have not yet been considered. Here we need the completely antisymmetrical tensor in two dimensions, ε_{ij}. This tensor is invariant under SU_2:

$$S_{ki}S_{\ell j}\varepsilon_{ij} = \varepsilon_{k\ell}\cdot\det(S) = \varepsilon_{k\ell} \quad .$$

Writing explicitly SU_2 indices we then have

$$\mathcal{L}_{fH} = \mu_1(\bar{\ell}_i^+\nu^-)K_i + \mu_2(\bar{\ell}_i^+ e^-)K_j^\dagger\,\varepsilon_{ij}$$

$$+ \lambda_1(\bar{q}_i^+ u^-)K_i + \lambda_2(\bar{q}_i^+ d^-)K_j^\dagger\,\varepsilon_{ij} + \text{herm. conj.}$$

Remember that the superscripts $\pm$ refer to factors $\frac{1}{2}(1 \pm \gamma^5)$ in front.

In the first instance we are interested in the mass terms only. Then only the constant part of K, i.e., $K = (F\sqrt{2},0)$ needs to be considered. Thus only $K_1 \neq 0 = 2M/g$. The result is

$$\mathscr{L}_{fH} = \frac{2M\mu_1}{g}(\bar{\nu}^+\nu^-) - \frac{2M\mu_2}{g}(\bar{e}^+e^-)$$

$$+ \frac{2M\lambda_1}{g}(\bar{u}^+u^-) - \frac{2M\lambda_2}{g}(\bar{d}^+d^-) + \text{herm. conj.}$$

Noting that $\nu^- = \frac{1}{2}(1+\gamma^5)\nu$, $\bar{\nu}^+ = \frac{1}{2}\bar{\nu}(1-\gamma^5)$, etc., we get:

$$\mathscr{L}_{fH} = \frac{M}{g}\left\{\bar{\nu}(\mu_1 + \mu_1^* + (\mu_1-\mu_1^*)\gamma^5)\nu\right\}$$

$$-\frac{M}{g}\left\{\bar{e}(\mu_2 + \mu_2^* + (\mu_2-\mu_2^*)\gamma^5)e\right\}$$

+ similar quark terms.

Actually, $\mu_1 \ldots \lambda_2$ may be taken to be real. This is because a phase factor in these parameters may be turned away by multiplying ν^-, e^-, etc., with a constant phase factor. Such a space-time independent phase factor leaves $\bar{\nu}^- D_\mu \nu^-$, etc., invariant.

Having done that we see that we can give an arbitrary mass to neutrino and electron. Assuming zero neutrino mass we thus take:

$$\mu_1 = 0\ , \qquad \mu_2 = \frac{gm_e}{2M}$$

$$\lambda_1 = \frac{gm_u}{2M}, \qquad \lambda_2 = \frac{gm_d}{2M}\ .$$

Things get more complicated if we consider three families. We will **explicitly treat** the quark sector.

Since all quark generations have the same behaviour under $SU_2 \times U_1$ we may use an arbitrary superposition of generations everywhere. In all generality the quark-Higgs couplings are then:

$$(\bar{q}^+_{i\alpha}\Lambda_1^{\alpha\beta}u^-_\beta)K_i + (\bar{q}^+_{i\alpha}\Lambda_2^{\alpha\beta}d^-_\beta)K_j^\dagger\varepsilon_{ij} + \text{herm. conj.}$$

In here Λ_1 and Λ_2 are arbitrary complex 3×3 matrices.

The Quark -W part of the Lagrangian is diagonal with respect to the generation index:

$$-(\bar{q}^+_\alpha \not{D} q^+_\alpha) - (\bar{u}^-_\alpha \not{D} u^-_\alpha) - (\bar{d}^-_\alpha \not{D} d^-_\alpha)\ .$$

Any unitary transformation U in generation space of q^+, u^- or d^- leaves this invariant, provided U is constant as a function of space time. For instance, if $q' = Uq$, with $U^+ = U^{-1}$ then

$$(\bar{q}'_\beta \not{D} q'_\beta) = (\bar{q}_\alpha U^+_{\alpha\beta}\not{D}U_{\beta\gamma}q_\gamma) = (\bar{q}_\alpha U^+_{\alpha\beta}U_{\beta\gamma}\not{D}q_\gamma) = (\bar{q}_\alpha \not{D} q_\alpha)\ .$$

Only the generation structure is shown.

We may use this freedom to simplify the Λ-matrices. First we perform a unitary transformation on u^- and another one on d^- so that Λ_1 and Λ_2 become hermitean. Thus we use the property that for arbitrary complex matrix Λ there exists a unitary matrix T such that ΛT is hermitean. Proof: consider the hermitean matrix $\Lambda^+\Lambda$. A suitable unitary transformation diagonalizes $\Lambda^+\Lambda$, thus $(e_i U^+\Lambda^+\Lambda U e_j) = \mu_i\delta_{ij}$, where the μ_i are the positive real eigenvalues and the e_i the normalized basis vector. Now define the vectors $v_i = Ue_i$. The last equation reads then $(v_j v_i) = \mu_i \delta_{ij}$. Thus the v_i are an orthogonal (but not normalized) system. There exists a unitary transformation V that turns the orthogonal v_i along the e_i, thus $Vv_i = \sqrt{\mu_i}\, e_i$, or $V\Lambda Ue_i = \sqrt{\mu_i}\, e_i$. Thus there exist unitary matrices V and U such that $V\Lambda U$ is diagonal and real positive, and evidently hermitean. But then also $V^{\dagger}V\Lambda UV = \Lambda UV$ is hermitean, and UV is the desired matrix T.

Thus Λ_1 and Λ_2 may be taken to be hermitean. Next we transform q^+, u^- and d^- all with the same unitary matrix U such that Λ_1 diagonalizes with positive real eigenvalues. The matrix Λ_2 remains hermitean.

Finally we perform a unitary transformation of the form

$$U(\phi) = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix}$$

to all fields. This leaves the diagonal matrix Λ_1 unchanged. The hermitean matrix Λ_2 changes; the off diagonal elements get phase factors of the type $e^{i(\phi_a - \phi_b)}$. There are 2 independent differences, $\phi_1 - \phi_2$ and $\phi_2 - \phi_3$, and thus two of the three arbitrary phases in Λ_2 may be turned away. After all this work Λ_2 is a hermitean matrix with only one complex phase. We may write

$$\Lambda_2 = CM^dC^{-1}$$

where M^d is diagonal, and C is of the form

$$C = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix}$$

where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$, and $\theta_1 \ldots \theta_3$ and δ are undetermined angles. They must be found from experiment.

If we substitute everywhere for the $d_\alpha^{\pm}$ the combination $C_{\alpha\beta}d_\beta^{\pm}$ then the d mass term becomes diagonal in the generation index. This is then what we call down, strange, and bottom quarks. But at various places we will see the generalized Cabibbo matrix C, in particular in the interaction of the charged vector bosons with u-d pairs.

9. Field Theory

The Lagrangian constructed so far is invariant under $SU_2 \times U_1$ transformations. Before one arrives at Feynman rules. that permit the calculation of radiative corrections, some further pieces must be added. Here we will not discuss this at length, but will just give the prescription.

First a gauge fixing part must be defined. To this purpose one must introduce 4 functions F_0, F_1, F_2, F_3 that are not invariant under the $SU_2 \times U_1$ transformations. Then $\mathscr{L}_{gf}$ is given by:

$$\mathscr{L}_{gf} = -\tfrac{1}{2}(F^a)^2 , \qquad a = 0,\dots,3 .$$

We will choose:

$$F^o = -\partial_\mu A_\mu$$
$$F^i = -\partial_\mu W^i_\mu + M\phi^i \qquad i = 1,2$$
$$F^3 = -\partial_\mu W^o_\mu + \frac{M}{c_\theta}\phi^3 .$$

Next the Faddeev-Popov ghost Lagrangian is to be constructed. It is obtained by subjecting the F to an infinitesimal $SU_2 \times U_1$ transformation generated by infinitesimal parameters ρ_0, ρ_1, ρ_2 and ρ_3. To first approximation one will have:

$$F^a \rightarrow F^a + M^{ab}\rho_b .$$

The Faddeev-Popov Lagrangian is then

$$\mathscr{L}_{FP} = \overline{X}^a M^{ab} X^b$$

where $X^o \dots X^3$ are four ghost fields with the wrong statistics, meaning that a minus sign must be given for every closed X loop in a Feynman diagram.

Instead of $X^o \dots X^3$ we will use X^+, X^-, Y^o and Y^a:

$$X^3 = c_\theta Y^o + s_\theta Y^A$$
$$X^o = -s_\theta Y^o + c_\theta Y^A$$
$$X^1 = \frac{1}{\sqrt{2}}(X^+ + X^-)$$
$$X^2 = \frac{i}{\sqrt{2}}(X^+ - X^-) .$$

10. The Complete Lagrangian

The complete Lagrangian is the sum of all previously described parts.

$$\mathscr{L}_{tot} = \mathscr{L}_W + \mathscr{L}_H + \mathscr{L}_{gf} + \mathscr{L}_f + \mathscr{L}_{fH} + \mathscr{L}_{FP}$$

$\mathscr{L}_W$, $\mathscr{L}_H$ and $\mathscr{L}_{gf}$ were discussed in Sections 4, 7 and 9. (n.b., $\alpha = \frac{m_H^2}{4M^2}$)

$$\begin{aligned}
\mathscr{L}_W + \mathscr{L}_H + \mathscr{L}_{gf} = & -\partial_\nu W_\mu^+ \partial_\nu W_\mu^- - M^2 W_\mu^+ W_\mu^- - \tfrac{1}{2}(\partial_\nu W_\mu^o)^2 - \tfrac{1}{2}\frac{M^2}{c_\theta^2} W_\mu^o W_\mu^o - \tfrac{1}{2}(\partial_\nu A_\mu)^2 \\
& -\partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \tfrac{1}{2}(\partial_\mu \phi^o)^2 - \tfrac{1}{2}\frac{M^2}{c_\theta^2}\phi^o\phi^o \\
& - \tfrac{1}{2}(\partial_\mu H)^2 - \tfrac{1}{2} m_H^2 H^2 \\
& -\beta\left\{\frac{2M^2}{g^2} + \frac{2M}{g} H + \tfrac{1}{2}(H^2 + (\phi^o)^2 + 2\phi^+\phi^-)\right\} + \frac{2M^4\alpha}{g^2} \\
& -igc_\theta[\partial_\nu W_\mu^o (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+) - W_\nu^o (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) \\
& \qquad + W_\mu^o (W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)] \\
& -igs_\theta[\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+) - A_\nu (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) \\
& \qquad + A_\mu (W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)] \\
& - \tfrac{1}{2} g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \tfrac{1}{2} g^2 W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- \\
& + g^2 c_\theta^2 (W_\mu^o W_\mu^+ W_\nu^o W_\nu^- - W_\mu^o W_\mu^o W_\nu^+ W_\nu^-) \\
& + g^2 s_\theta^2 (A_\mu W_\mu^+ A_\nu W_\nu^- - A_\mu A_\mu W_\nu^+ W_\nu^-) \\
& + g^2 s_\theta c_\theta \left\{A_\mu W_\nu^o (W_\mu^+ W_\nu^- + W_\nu^+ W_\mu^-) - 2A_\mu W_\mu^o W_\nu^+ W_\nu^-\right\} \\
& - \alpha g M\left\{H^3 + H(\phi^o)^2 + 2H\phi^+\phi^-\right\} \\
& - \frac{1}{8} g^2 \alpha \left\{H^4 + (\phi^o)^4 + 4\phi^+\phi^-\phi^+\phi^- + 4(\phi^o)^2\phi^+\phi^- \right. \\
& \qquad \left. + 4H^2\phi^+\phi^- + 2(\phi^o)^2 H^2\right\} \\
& - gMW_\mu^+ W_\mu^- H - \tfrac{1}{2} g \frac{M}{c_\theta^2} W_\mu^o W_\mu^o H \\
& - \tfrac{1}{2} ig\left\{W_\mu^+(\phi^o\partial_\mu\phi^- - \phi^-\partial_\mu\phi^o) - W_\mu^-(\phi^o\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^o)\right\} \\
& + \tfrac{1}{2} g\left\{W_\mu^+(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W_\mu^-(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right\} \\
& + \tfrac{1}{2} g \frac{1}{c_\theta} W_\mu^o (H\partial_\mu\phi^o - \phi^o\partial_\mu H) \\
& - ig\frac{s_\theta^2}{c_\theta} MW_\mu^o (W_\mu^+\phi^- - W_\mu^-\phi^+) + igs_\theta MA_\mu (W_\mu^+\phi^- - W_\mu^-\phi^+) \\
& - ig\frac{1-2c_\theta^2}{2c_\theta} W_\mu^o (\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_\theta A_\mu (\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) \\
& - \tfrac{1}{4} g^2 W_\mu^+ W_\mu^- \left\{H^2 + (\phi^o)^2 + 2\phi^+\phi^-\right\} \\
& - \frac{1}{8} g^2 \frac{1}{c_\theta^2} W_\mu^o W_\mu^o \left\{H^2 + (\phi^o)^2 + 2(2s_\theta^2-1)^2\phi^+\phi^-\right\} \\
& - g^2 s_\theta^2 A_\mu A_\mu \phi^+\phi^- \\
& - \tfrac{1}{2} g^2 \frac{s_\theta^2}{c_\theta} W_\mu^o \phi^o (W_\mu^+\phi^- + W_\mu^-\phi^+) \\
& - \tfrac{1}{2} ig^2 \frac{s_\theta^2}{c_\theta} W_\mu^o H (W_\mu^+\phi^- - W_\mu^-\phi^+) \\
& + \tfrac{1}{2} g^2 s_\theta A_\mu \phi^o (W_\mu^+\phi^- + W_\mu^-\phi^+) \\
& + \tfrac{1}{2} ig^2 s_\theta A_\mu H (W_\mu^+\phi^- - W_\mu^-\phi^+) \\
& - g^2 \frac{s_\theta}{c_\theta}(2c_\theta^2-1) W_\mu^o A_\mu \phi^+\phi^- \quad .
\end{aligned}$$

The fermion Lagrangian contains 3 lepton generations (e, μ, τ) and three quark generations. We assume zero neutrino mass, and label generations with an index α, β..., and colors with the index i:

$$
\begin{aligned}
\mathscr{L}_f = & -\bar{e}^\alpha(\partial\!\!\!/ + m_e^\alpha)e - \bar{\nu}^\alpha \partial\!\!\!/ \nu^\alpha \\
& - \bar{u}_i^\alpha(\partial\!\!\!/ + m_u^\alpha)u_i^\alpha - \bar{d}_i^\alpha(\partial\!\!\!/ + m_d^\alpha)d_i^\alpha \\
& + igs_\theta A_\mu \left\{-(\bar{e}^\alpha\gamma^\mu e^\alpha) + \frac{2}{3}(\bar{u}_i^\alpha\gamma^\mu u_i^\alpha) - \frac{1}{3}(\bar{d}_i^\alpha\gamma^\mu d_i^\alpha)\right\} \\
& + \frac{ig}{4c_\theta} W_\mu^o \left\{(\bar{\nu}^\alpha\gamma^\mu(1+\gamma^5)\nu^\alpha) + (\bar{e}^\alpha\gamma^\mu(4s_\theta^2-1-\gamma^5)e^\alpha)\right. \\
& \qquad \left. + (\bar{d}_i^\alpha\gamma^\mu(\frac{4}{3}s_\theta^2-1-\gamma^5)d_i^\alpha) + (\bar{u}_i^\alpha\gamma^\mu(1-\frac{8}{3}s_\theta^2+\gamma^5)u_i^\alpha)\right\} \\
& + \frac{ig}{2\sqrt{2}} W_\mu^+ \left\{(\bar{\nu}^\alpha\gamma^\mu(1+\gamma^5)e^\alpha) + (\bar{u}_i^\alpha\gamma^\mu(1+\gamma^5)C_{\alpha\beta}d_i^\beta)\right\} \\
& + \frac{ig}{2\sqrt{2}} W_\mu^- \left\{(\bar{e}^\alpha\gamma^\mu(1+\gamma^5)\nu^\alpha) + (\bar{d}_i^\alpha C_{\alpha\beta}^{-1}\gamma^\mu(1+\gamma^5)u_i^\beta)\right\}
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{L}_{fH} = & -\frac{i}{2\sqrt{2}} g \frac{m_e^\alpha}{M} \phi^+(\bar{\nu}^\alpha(1-\gamma^5)e^\alpha) \\
& + \frac{i}{2\sqrt{2}} g \frac{m_e^\alpha}{M} \phi^-(\bar{e}^\alpha(1+\gamma^5)\nu^\alpha) \\
& - \tfrac{1}{2} g \frac{m_e^\alpha}{M}(\bar{e}^\alpha e^\alpha)H - \frac{i}{2} g \frac{m_e^\alpha}{M}(\bar{e}^\alpha\gamma^5 e^\alpha)\phi^o \\
& - \frac{i}{2\sqrt{2}} g \frac{m_d^\beta}{M} \phi^+(\bar{u}_i^\alpha C_{\alpha\beta}(1-\gamma^5)d_i^\beta) \\
& + \frac{i}{2\sqrt{2}} g \frac{m_u^\alpha}{M} \phi^+(\bar{u}_i^\alpha C_{\alpha\beta}(1+\gamma^5)d_i^\beta) \\
& + \frac{i}{2\sqrt{2}} g \frac{m_d^\alpha}{M} \phi^-(\bar{d}_i^\alpha C_{\alpha\beta}^\dagger(1+\gamma^5)u_i^\beta) \\
& - \frac{i}{2\sqrt{2}} g \frac{m_u^\beta}{M} \phi^-(\bar{d}_i^\alpha C_{\alpha\beta}^+(1-\gamma^5)u_i^\beta) \\
& - \tfrac{1}{2} g \frac{m_u^\alpha}{M}(\bar{u}_i^\alpha u_i^\alpha)H - \tfrac{1}{2} g \frac{m_d^\alpha}{M}(\bar{d}_i^\alpha d_i^\alpha)H \\
& - \frac{i}{2} g \frac{m_d^\alpha}{M}(\bar{d}_i^\alpha\gamma^5 d_i^\alpha)\phi^o + \frac{i}{2} g \frac{m_u^\alpha}{M}(\bar{u}_i^\alpha\gamma^5 u_i^\alpha)\phi^o
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{L}_{FP} = & \bar{X}^+\partial^2 X^- - M^2\bar{X}^+X^- + \bar{X}^-\partial^2X^- - M^2\bar{X}^-X^- + \bar{Y}^o\partial^2Y^o - \frac{M^2}{c_\theta^2}\bar{Y}^oY^o + \bar{Y}^A\partial^2Y^A \\
& -igs_\theta W_\mu^+(\partial_\mu\bar{X}^+Y^A - \partial_\mu\bar{Y}^AX^-) + igc_\theta W_\mu^+(\partial_\mu\bar{Y}^oX^- - \partial_\mu\bar{X}^+Y^o) \\
& -igs_\theta W_\mu^-(\partial_\mu\bar{Y}^AX^+ - \partial_\mu\bar{X}^-Y^A) + igc_\theta W_\mu^-(\partial_\mu\bar{X}^-Y^o - \partial_\mu\bar{Y}^oX^+) \\
& -igs_\theta A_\mu(\partial_\mu\bar{X}^-X^- - \partial_\mu\bar{X}^+X^+) - igc_\theta W_\mu^o(\partial_\mu\bar{X}^-X^- - \partial_\mu\bar{X}^+X^+) \\
& - \tfrac{1}{2} gM(\bar{X}^+X^+H + \bar{X}^-X^-H + \frac{1}{c_\theta^2}\bar{Y}^oY^oH) \\
& + ig\frac{(1-2c_\theta^2)}{2c_\theta}M(\bar{X}^+Y^o\phi^+ - \bar{X}^-Y^o\phi^-) + ig\frac{M}{2c_\theta}(\bar{Y}^oX^-\phi^+ - \bar{Y}^oX^+\phi^-) \\
& -igs_\theta M(\bar{X}^+Y^A\phi^+ - \bar{X}^-Y^A\phi^-) - ig\frac{M}{2}(\bar{X}^-X^-\phi^o - \bar{X}^+X^+\phi^o) .
\end{aligned}
$$

11. Discussion of the Lagrangian

The Lagrangian written down represents a large amount of experimental data. If we take the mass of the vector boson as energy scale, then all of the data are at very low energy.

Further, in constructing the Lagrangian, certain choices have been made. For instance, not having any direct information on the Higgs system, we have employed the simplest choice that covers our needs, namely an SU_2 doublet. This could be used to generate both vector boson and fermion masses. Other choices for the Higgs system lead to differences that are usually not observable at low energies, with one exception, namely the ratio of charged and neutral vector boson mass. In the quoted Lagrangian this ratio is equal to $\cos\theta_W$. If one allows also a Higgs triplet then the ratio depends on the choice of parameters in the Higgs sector, and may vary from 0 to $\cos\theta_W/\sqrt{2}$. This ratio may be changed by complicating this Higgs sector, but this is at the price of new hypothetical scalar particles.[5]

Experimentally one observes that the ratio is very close to $\cos\theta_W$. We will elevate this observation to an assumption: for calculations at low energy one may assume the simplest Higgs system, and we call this the Higgs $\Delta I = \frac{1}{2}$ rule.

It is a very difficult question to establish what is and what is not tested in this model. To a large extent this depends on one's cynicism with respect to the theory. For instance, at this time no top quark has been observed, and we do not know if the b quark is part of an SU_2 doublet. Also, it is quite possible to modify the vector boson structure without changing the low energy behaviour. We will know for sure only as soon as the vector bosons have been observed.

The situation with respect to the Higgs sector is even more obscure. The difference is that even in this simplest model the Higgs mass remains a free parameter. Moreover, there are theoretical suspicions with respect to the Higgs system.

Now it must be understood that the standard model is a system that, unlike the four-fermion theory, needs no cut-off. It is conceivable that this model describes physics up to 10^{19} GeV, the threshold where gravitation becomes important. In this sense it is a much better theory than the four-fermion theory. This fact is a consequence of the renormalizablility of the standard model.

There is however another aspect of the present situation. While the standard model is internally consistent, it nevertheless leaves many things unexplained. For instance, we may remember that the U_1 coupling for the various multiplets entered as a completely free parameter. Why should this coupling arrange itself such that the neutrino has zero charge? and so that the electromagnetic interactions conserve parity? In other words, the unification of weak and electromagnetic interactions is really a very superficial one in this model. All the time one must adapt parameters to the observed properties. In this respect grand unified theories represent progress, although a certain amount of arbitrariness remains.

The question of the observed fermion spectrum remains largely unanswered. Why are there three, or more, generations? Why do they have the particular $SU_2 \times U_1$ multiplet assignment as observed? Are the masses arbitrary parameters?, etc., etc.

In the remainder of these notes we will study some of these questions. The tools that we use are the radiative corrections in the standard model. The following results can be obtained

(i) There is a limit on new, as yet not observed generations.

(ii) The screening theorem: even if the Higgs mass is very large, which implies that the Higgs has strong interactions, no significant effects can be seen at low energies.

The screening theorem opens the door for all kinds of speculations on the Higgs system. To this author it is very plausible that the Higgs particle is really not a fundamental particle, but may be a bound state, or may be not even that but a system of new strong interactions. In itself, the idea that the Higgs is composite was advanced already some time ago[6]; through the screening theorem this idea gains credibility. We want to go further, and allow for the possibility that there is not even a well defined particle corresponding to the physical Higgs particle in the standard model.

12. Calculation of Radiative Corrections

In view of the complexity of the Lagrangian it is in general a considerable task to compute radiative corrections. However, for the two points cited the effort needed is rather small. Mainly one must first have a clear understanding of the renormalization procedure, after which the calculations are simple.

As in quantum electrodynamics, also in the standard model, infinities appear in the process of calculating radiative corrections. Therefore one must have a regularization scheme, that is a calculational scheme where everything is finite and where the actual model can be obtained by taking some limit. Thus we will introduce a parameter Δ, and the standard model obtains in the limit $\Delta \to \infty$. It is a property of renormalizable theories that the physical results are independent of Δ, and consequently the limit $\Delta \to \infty$ becomes trivial, at least insofar as experimentally observable results are concerned.

The standard model contains a number of parameters, and one must first have a number of data points such that the parameters can be fixed. Given the parameters new results can be computed and compared with experiment.

In the Lagrangian for the standard model we have as free parameters

- the coupling constant g
- the weak mixing angle θ_W
- the mass of the charged vector boson M
- the mass of the Higgs particle m_H

- the three angles θ_1, θ_2 and θ_3 in the generalized Cabibbo matrix as well as a phase angle δ
- the masses of the quarks and leptons.

Not all data points will be equally sensitive to all these parameters. For example, without neutral current experiments it would be very hard to determine θ_W to any precision. For the moment we will concentrate on the first four parameters.

Thus let g, θ_W, M and m_H denote the four parameters occurring in the Lagrangian. Comparison with the data points (four numbers, $d_1,\ldots,d_4$) fixes these parameters, for a given choice of Δ. Thus, for example:

$$g = g(d_1, d_2, d_3, d_4, \Delta) \ .$$

Having established g, θ_W, etc., we can now compute other experimentally observable effects. In these computations one must use the same Δ. But the final result will be independent of Δ provided the same Δ is used in fixing the parameters and in producing the new results.

A very simple example, with interesting physics consequences, will now be considered. Consider the standard model, as given before. This includes the Higgs $\Delta I = \frac{1}{2}$ assumption. Let us assume that all one-loop corrections have been computed. Let us furthermore assume that the following data points are used in determining the parameters g, $\sin^2\theta$ and M:

- μ decay
- the electric charge (e-μ scattering at zero momentum transfer)
- the ratio of $\bar{\nu}_\mu e$ to $\nu_\mu e$ total cross sections.

Of course, also the masses of electron, muon, etc., are used as input.

As experimentally testable consequence we will take the $\nu_\mu e$ total cross section.

Imagine the calculation has been done (actually, it has been done, see Ref. 7). We now pose the following question: imagine that there exists a new generation of quarks and leptons, with large masses, except for a **massless** neutrino. What would be the consequence of this new generation to the $\nu_\mu e$ prediction?

To find the result we must compute the influence of this new generation on all four processes mentioned above. Now it happens that the only way that such new fermions can contribute is through self-energy insertions in the vector boson propagators. Roughly speaking such diagrams produce radiative corrections to the vector boson masses, and these corrections violate the rule $M_0 = M/c_\theta$. In lowest order the three data points mentioned are independent of the mass of the neutral vector boson. The computed number (the $\nu_\mu e$ cross section) depends on this mass M_0, and the measurement is a test for the rule mentioned. And the new leptons and quarks produce deviations of this rule, so that experimental results give us information on these new particles.

The calculational work involved amounts to computing the contribution of vector boson self-energy graphs involving these new particles. Let us restrict ourselves to the new lepton doublet, ℓ and ν_ℓ. The following graphs need to be computed (see Fig. 5).

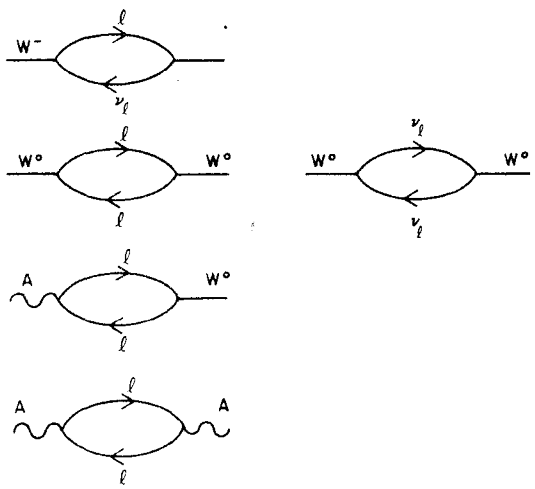

Figure 5

The calculation simplifies even further if we realize that we need the results only at low energy. In most cases, with the exception of photon self-energy diagrams, one can straight away set the external momentum to zero.

The actual calculation has been written down in great detail elsewhere.[8] The result is rather plausible. Denote the contribution of the W^+ and W^o self-energy diagrams by A^+ and A^o. Then the prediction for the $\nu_\mu e$ total cross section is modified by a factor

$$\left(\frac{1}{1-8G_F\delta}\right)^2 \qquad \delta = \frac{1}{(2\pi)^4 i g^2}\left(A^o c_\theta^2 - A^+\right) \quad . \tag{12.1}$$

Thus insofar as the correction follows the Higgs $\Delta I = \frac{1}{2}$ rule it is invisible. To give an idea of the quantities A^+ and A^o we quote the expression for A^+. Consider the expression:

$$g^2 \int d_4q \, \frac{\mathrm{Tr}\{\gamma^\mu(1+\gamma^5)(-i\gamma q+m)\gamma^\nu(1+\gamma^5)(-i\gamma q)\}}{(q^2+m)q^2} = g^2 \int d_4q \, \frac{2\{-8q_\mu q_\nu + 4\delta_{\mu\nu}q^2\}}{(q^2+m^2)q^2}$$

$$= g^2\left\{-16B_{22}(0,m,0) + 8A(m)\right\}\delta_{\mu\nu} \; .$$

The coefficient of $\delta_{\mu\nu}$ is the desired function A^+. We introduced the standard functions B_{22} and A defined by:

$$\int d_4q \, \frac{1}{q^2+m^2} = A(m^2) \tag{12.2}$$

$$\int d_4q \, \frac{q_\mu q_\nu}{(q^2+m_1^2)((q+k)^2+m_2^2)} = B_{21}(k,m_1,m_2)k_\mu k_\nu + B_{22}(k,m_1,m_2)\delta_{\mu\nu} \; .$$

Working out the expressions for A^+ and A^o one finds for δ:

$$\delta = \frac{m^2}{64\pi^2} \quad . \tag{12.3}$$

The correction is:

$$\left(\frac{1}{1-8G_F\delta}\right)^2 \simeq 1 + G_F\frac{m^2}{4\pi^2} \quad .$$

Thus the correction grows like the square of the lepton mass. Comparison with the experimental data leads to the constraint $m < 300$ GeV.

It seems that the pattern of new generations cannot continue much further, or else we would have seen this in neutral current cross sections as deviations from the standard model.

For completeness we note that in the general case the observed correction grows proportionally with the square of the mass differences in the new SU_2 multiplets. Thus new generations in which all masses would be about the same (including the neutrino mass) are not constrained by the present data.

13. The Screening Theorem[9]

We now focus attention on the Higgs system. The first question that arises is this: do we have any objective indication that it is there? Or stated differently: is there an upper limit to the Higgs mass from experimental data?

To a large extent this question is the same one as discussed in the 4-fermion theory, Section 2. From experiment an upper limit on Λ, the cut-off, could be established from low energy data (the equalness of the coupling constants of μ decay and neutron decay).

Here we have the following. Without the Higgs boson the theory is non-renormalizable. That is like the 4-fermion theory. The Higgs boson with finite mass is the equivalent of our Λ cut-off with finite Λ in the 4-fermion theory.

Thus in the limit of large Higgs mass there is a unitarity limit. This limit is actually around 1 TeV. Thus if there exists no Higgs then the theory becomes a strong interaction theory above 1 TeV. This is another way of stating that the radiative corrections are of the same order of magnitude as the lowest order results for energies above 1 TeV.

However, nothing is known at 1 TeV. For all we know there is no Higgs boson, but as an alternative strong interactions. If there is a Higgs boson with mass below 1 TeV then there is no unitarity problem, and no strong interactions are indicated.

The great problem is this: if there is no Higgs, would we note anything at low energies? We thus are led to study radiative corrections at low energies as a function of the Higgs mass. The question is if there are big corrections in the limit $m_H \to \infty$. We will consider here the most likely candidate for such corrections, namely the mass ratio $M/M_0 = c_\theta$.

We must go again through the whole procedure outlined before, and establish Higgs dependence in the relation between data points and parameters in the Lagrangian. After that we can establish Higgs dependence in some experimentally verifiable number.

In inspecting the Lagrangian of the standard model we observe that the Higgs H is not coupled to neutrinos, and that the coupling to electrons and muons is suppressed by a factor m_e/M or m_μ/M, which are of order 10^{-5} resp. 10^{-3}. We can safely ignore the Higgs-fermion couplings. So now we are back to a situation similar to that of the previous section: we need to consider only vector boson self-energy diagrams. Given that the external lines may be either a W^- or W^o the relevant diagrams are given in Fig. 6.

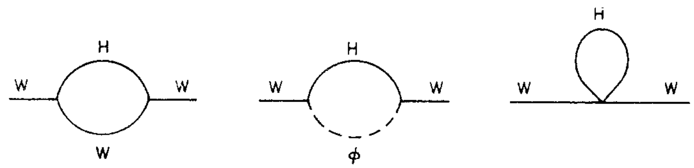

Figure 6

Besides, Higgs mass dependent contributions could come through tadpole type diagrams, Fig. 7.

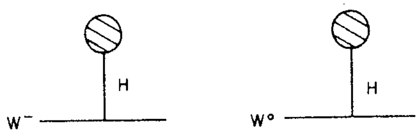

Figure 7

Some example of tadpole diagrams are given in Fig. 8.

Figure 8

The diagrams of Fig. 8 derive from HHW^o and H^3 terms in the Lagrangian. The contribution of these tadpoles to the masses derives from the W^-W^+H and W^oW^oH terms. But if we look to these latter terms we see that they have coefficients that are in the same ratio as the mass terms:

Mass terms: $$-M^2 W^+_\mu W^-_\mu - \tfrac{1}{2} \frac{M^2}{c_\theta^2} W^o_\mu W^o_\mu$$

WWH terms: $$-gMW^+_\mu W^-_\mu H - \tfrac{1}{2} g \frac{M}{c_\theta^2} W^o_\mu W^o_\mu H \quad .$$

For this reason the tadpole terms do not lead to any deviation from the rule $M/M_0 = c_\theta$.

Another contribution is the third diagram of Fig. 6. This derives from the terms of the type W^+W^-HH and W^oW^oHH. These are:

$$-\frac{1}{4}\, g^2 W^+_\mu W^-_\mu H^2 - \frac{1}{8}\, g^2 \frac{1}{c_\theta^2}\, W^o_\mu W^o_\mu H^2 \quad .$$

Again no deviation from the mass rule results. The contributions of the other diagrams must be computed, but that is an easy matter. Again, there is a contribution δ to an observable effect in $\sigma_{\nu e}$ scattering (see Eq. (12.1)), where now A^o and A^+ refer to contributions of the two first diagrams of Fig. 6. Now one finds:

$$A^o = g^2 \frac{M^2}{c_\theta^4} B_0(0,m_H,M_0) + g^2 \frac{1}{c_\theta^2} B_{22}(0,M_0,m_H)$$

$$A^+ = g^2 M^2 B_0(0,m_H,M) + g^2 B_{22}(0,M,m_H) \quad .$$

Working this out one finds the contribution to δ:

$$\delta \rightarrow \delta - \frac{3}{64\pi^2}\, M^2 \frac{s_\theta^2}{c_\theta^2}\, \ell n\, m_H^2 \quad .$$

All m_H independent terms, or terms becoming small for large Higgs mass, have been dropped.

The important point is that there is no contribution to δ proportional to m_H^2. The dependence is only logarithmic, and even for $m_H = 3$ TeV the effect is only 0.8% on $\sigma_{\nu e}$. For an effect of 2% a mass of about 700 TeV is required. These calculations are relative to $m_H^2 = M^2$.

This insensitivity to the Higgs mass is called the screening theorem. The effect persists also in other situations: the dependence on the Higgs mass is very weak and not visible at low energies even for very large Higgs mass.

The conclusion is this: if the Higgs mass becomes very large there will be new strong interactions in the TeV region. This follows because of the unitarity limit. But there is no substantial effect at low energies. In other words, because of the screening theorem it is quite possible that there are new strong interactions at the TeV level without this being visible at low energies.

Various authors, in particular Susskind and collaborators, have made models and conjectures concerning these possible new interactions. In these models an analogy with the usual strong interactions, quantum chromodynamics, is often made, and the new strong interactions are termed technicolor. The subject is quite open, and will not be discussed any further here.

14. Other Radiative Corrections

Various other radiative corrections have been computed, and the most interesting of these is the calculation of the corrections to the masses of the neutral and charged vector boson. In the spirit of the previous discussion the mass of the neutral vector boson is defined as the location of the resonance in e^+e^- annihilation. This must not be identified with the value of the parameter M, or $M_0 = M/c_\theta$ in the Lagrangian. It is simply another observable number.

With $s_\theta^2 = 0.238$ the lowest order calculation gives the values 76.5 and 87.6 GeV for M and M_0. Including radiative corrections the values shift by + 2.0 and 2.5 GeV.[10,11] These values differ from those quoted in Ref. 10 due to a calculational error in Ref. 7.* The results of Ref. 7, being part of the total, and quoted as 120 and 140 MeV, are in fact -940 and -690 MeV.

References to other calculations are given in Ref. 7 and 10. Similar calculations have also been done by Sirlin and Marciano, see Ref. 12.

* There is a sign error in the equation for F_1, Section 5. Instead of $-\frac{1}{2}k^2$ one must have $+\frac{1}{2}k^2$.

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