SEMINAR 2

THE UNIFICATION OF BLACK HOLES WITH ORDINARY MATTER

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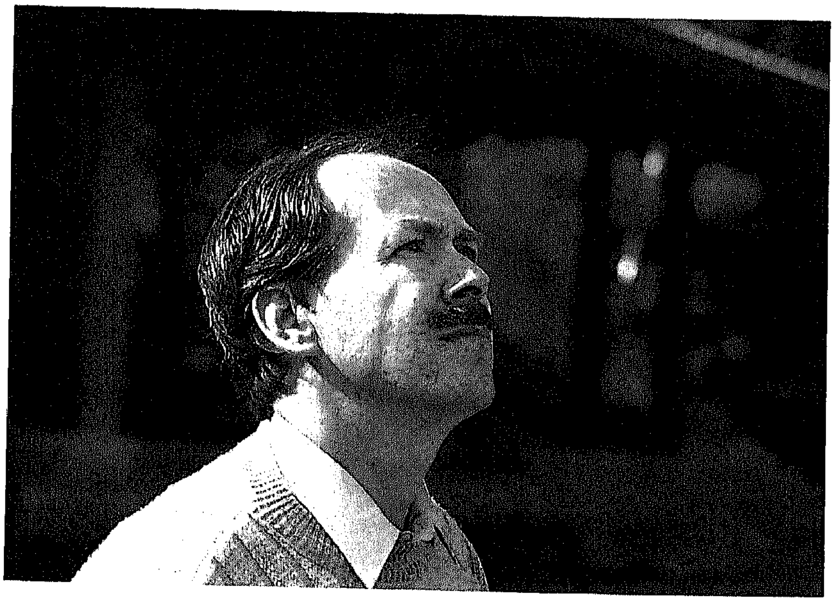
Gravitation and Quantizations



The problem of reconciling the theory of general relativity with the principles of quantum mechanics is one of the deepest and fundamental problems of theoretical physics and it continues to mystify many of us. Now, the procedure of replacing a "classical" theory by a corresponding "quantum mechanical" one is straightforward in many cases, in particular when we are dealing with relatively tiny interaction strengths or a small number of degrees of freedom. Indeed, when we consider circumstances where the gravitational force is weak and therefore accessible to a perturbative treatment, we know fairly precisely how to perform this so-called "quantization procedure". The resulting theory, perturbative quantum gravity, turns out to be similar to any other gauge theory, except that when increasing accuracies are required, new, undetermined physical parameters are required: subtraction constants associated with unrenormalizable interactions. This complication, though of course a fundamental one, is relatively mild compared to the obstacles one encounters when a "non-perturbative" formalism is required. One then notices that any attempt even at giving a sensible frame for a description of what might happen will falter at distance scales smaller than the Planck length. A fundamentally new approach is needed.

One reason why any attempt based on the classical description of gravity must break down is a basic instability of the gravitational force: the possibility of gravitational collapse. As soon as too much energy is concentrated within one tiny volume element, a black hole – sometimes with considerable size – emerges, while, on the other hand, a quantum theory would not suggest any such constraint on a quantity such as the Hamiltonian density. This brings us quite naturally to the consideration that indeed black holes are the prototype testing facilities for any quantum gravity theory, because they carry the strongest possible gravitational fields in any given volume. A proper incorporation of black holes in any theory of quantized gravity must be absolutely essential, since they form the natural upper boundary of the energy spectrum.

Most standard theories of gravity do not incorporate black holes properly. In a proper theory black holes, or at least objects that would behave like black holes in the limit of large mass and size, should occupy a natural position in Hilbert space, be included in the unitarity conditions of the S-matrix, and so on.

Instead, what is usually done is that black holes are treated in the so-called background formalism. One specifies the metric as if it were a classical one, and then performs quantum field theory with respect to this background. At first sight one would expect that this were a correct procedure, comparable to, for instance, the treatment of magnetic monopoles in a gauge theory for elementary particles. But the outcome is drastically, and catastrophically, different [1]. It is found that, when viewed this way, quantum black holes extract and destroy "quantum information" [2]. In terms of pure quantum states this means that when we start with two states that are orthogonal to each other in Hilbert space, for instance because they differ by the presence of one extra particle moving into the black hole, these states become indistinguishable after a while, and hence cannot continue to be orthogonal to each other; if they did, the number of possible states inside a black hole would rapidly exceed the total number of possible states in the universe. In a slightly different interpretation of the same mental exercise one would say that a quantum mechanically pure state evolves naturally into a quantum mechanically mixed state*.

One could try to maintain, as indeed is often done, that black holes must therefore be radically different from elementary particles, including solitons such as magnetic monopoles. But here is my problem: if black holes would indeed be ubiquitous in the high-energy regime of a quantized theory, then I find it virtually impossible to understand how such a theory could still behave entirely as if no such information drain ever took place. It is much more natural to assume that the background approach to black holes should merely provide for an approximate description, being quite accurate in a statistical sense, but failing when a more precise interpretation is required, even if that were a "quantum mechanical" interpretation.

Indeed, the background formulation of a black hole *is* at best approximate. An important complication was ignored: all interactions, in particular the gravitational ones, between the incoming and outgoing particles. Now, under normal situations this would not have been a great disaster. In quantum field theories one can easily correct for this by adding a series of successive, tiny perturbative corrections. But the gravitational interactions are not normal in this respect. If we want to know how the out-states react upon any variation of the incoming particles at an earlier epoch, we find a disturbing divergence: the strength of this mutual gravitational interaction diverges *exponentially* with the time difference. Hence any perturbative approach is out of the question whenever we wish to follow the evolution of some configuration over any appreciable time interval.

In these lectures, of which the present notes are just a summary [3], it is indicated

* A similar phenomenon seems to occur in theories of multiply connected universes. Here an uncertainty in the fundamental interactions arises on top and above the familiar quantum uncertainties. Pure states evolve into mixed states due to this uncertainty, but here this is clearly seen as a shortcoming in our information concerning the effective interactions. The uncertainty in question could be resolved for instance by performing accurate measurements.

how in principle a superior formalism could be constructed. We first observe that the nature of the gravitational interactions between incoming and outgoing particles can very easily be characterized. Incoming particles produce a *horizon shift*. This horizon shift may be very tiny, but its effects upon the outgoing particles grow exponentially with time. They are also readily computable [4]. The wavefunctions of all outgoing particles are simply shifted, by an amount that depends on the angular location on the horizon*.

But then one must ask what to do with this effect, whose importance is obvious, but which entirely disappears when, for instance, statistical correlations between the outgoing particles are computed. Indeed, this phenomenon has no effect at all upon the thermal spectrum of the Hawking radiation. We do find, however, that there is an effect on the nature of the quantum state of the outgoing objects. The quantum state is shifted, and hence the outgoing wavefunctions are all multiplied by factors $\exp(ip\,\delta y)$, where p is the momentum in Kruskal coordinates and δy the horizon shift, a function depending explicitly upon the angular coordinates θ and φ. The effect of this operation would be a harmless multiplication if the outgoing particles were in a Kruskal momentum eigenstate, but, of course, in more relevant circumstances they are not in such eigenstates. This way we conclude that any alteration of the form

$$|\psi\rangle_{\mathrm{in}} \to |\psi + \delta\psi\rangle_{\mathrm{in}},$$

where $\delta\psi$ carries a given momentum $p_{\mathrm{in}}(\theta,\varphi)$, affects the outgoing state by the above-mentioned operation.

This observation allows us to continue along the following essential speculation [3], the S-matrix ansatz:

> *The relation between incoming and outgoing states near a black hole must be described by an S-matrix such that, on the one hand, the mutual gravitational interactions are incorporated correctly, whereas, on the other hand, unitarity is maintained.*

Actually, unitarity will be restored, not just maintained, by this interaction. But we are not ready yet. A strange new problem arises. One may indeed insist that the resulting scattering matrix is unitary, but it will be so only in a very unconventional Hilbert space. Two states that have exactly the same momentum distribution for the incoming – or outgoing – particles, cannot be distinguished in any other way and therefore must be identical. This implies that the Fock space of elementary

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* This angular dependence is crucial for our arguments, since without such an angular dependence one could transform (practically) all its effects away. This is why one must be very careful in interpreting some popular two-dimensional toy models of black holes [5].

particles will eventually look very different from what it used to be in elementary-particle physics.

Unconventional as this result may seem, there is one way in which it looks familiar. The functional integrals one ends up with contain Green functions defined on a two-dimensional surface (the intersection of future and past horizons), the external momenta are inserted in exponential form, and, if incoming and outgoing states are introduced in the form of wave packets, one is required to perform integrations over these wave packets. All this is exactly as in (super)string theory. Indeed, our results can best be characterized as reproducing string theory, although, remarkably, the string tension constant turns out to be purely imaginary*. Our interpretation of this observation is that the black-hole horizon can in some respects be regarded as the world sheet of a virtual closed string. The external particles are inserted there as vertex insertions in the usual sense.

As for the other interactions, besides the gravitational ones one may consider electromagnetic interactions between incoming and outgoing states. Their effect upon the S-matrix is very similar to the effect of the gravitational interactions; indeed, one may regard electromagnetism as gravitation in an internal, fifth dimension, à la Kaluza–Klein. But to incorporate other interactions is somewhat more difficult, and partly an open question. Quite generally, however, the following picture emerges:

Given one's favorite version of the standard model (augmented with perturbative quantum gravity) as a starting point, one can *construct* the S-matrix for a black hole. Limiting factor here is that since we only know the basic interactions up to a certain resolution in distances (distances larger than, say, $1\,\mathrm{TeV}^{-1}$), we can only specify the properties of the S-matrix with the same distance resolution. This S-matrix then appears to be generated by functional-integral expressions in two dimensions, as if we were dealing with a new field theory in two dimensions. This two-dimensional field theory is in some sense a projection of the four-dimensional theory. An unambiguous mapping seems to exist that provides us with a two-dimensional theory, given a four-dimensional one. Special features of this mapping are:

– The *gauge transformation generators* of the four-dimensional theory correspond to the *dynamical variables* in the two-dimensional one. Therefore the spins of the physical degrees of freedom in two dimensions are one less than the corresponding ones in four dimensions.

– Scalar and Dirac-spinor fields seem not to generate anything in two dimensions. An exception to this is the occurrence of spontaneous symmetry breaking: if in four dimensions a symmetry is broken spontaneously, the corresponding sym-

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* The fact that the string constant comes out imaginary should not be seen as a departure from unitarity, as was asserted by one author, but rather as a *consequence* of unitarity as required in our formalism.

metry in two dimensions is *explicitly* broken: the scalar field in four dimensions maps into a "spurion" field in two dimensions (spurions were used in the 60s to describe explicit symmetry breaking interactions). Indeed one may view the value of the scalar fields at the horizon intersection point as being the spurion parameter.

– A dual transformation in four dimensions corresponds to a similar dual transformation in two dimensions. Thus, magnetic monopoles entering the black holes generate a topological kink in the two-dimensional system; furthermore, quark confinement in four dimensions can be seen to correspond to an explicit symmetry breaking in terms of the scalar disorder parameter in two dimensions.

– Proceeding along these lines it is natural to suspect that a *gravitino* in four dimensions corresponds to a Dirac spinor in the two-dimensional theory. What we have not understood at present, however, is how to incorporate effects of Dirac spinors in four dimensions in the two-dimensional theory; they seem to leave no trace.

Ultimately, our goal is two-fold: we wish to understand quantum gravity and black holes better than we do at present. But we also would like to see in what way an eventual theory would pose constraints upon the more conventional particle theories. It is quite conceivable that not all versions of a standard model will be allowed. In particular, we cannot admit the presence of absolutely additively conserved global charges. They would render the black-hole Hilbert space strictly infinite-dimensional, whereas entropy arguments give us a rather precise piece of information concerning the finiteness of the black-hole Hilbert space. Most important of all is that we wish to clear up the near paradoxical difficulties of our theoretical understanding of quantum gravity and black holes. It is our belief that resolving these apparent paradoxes will bring us further toward new and exciting theories, just as this has happened in the past.

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