

Quantum information and information loss in General Relativity ‡

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ABSTRACT

When it comes to performing thought experiments with black holes, Einstein-Bohr like discussions have to be re-opened. For instance one can ask what happens to the quantum state of a black hole when the wave function of a single ingoing particle is replaced by an other one that is orthogonal to the first, while keeping the total energy and momentum unaffected. Observers at $t \rightarrow \infty$ will not notice any difference, or so it seems in certain calculational schemes.

If one argues that this cannot be correct for the complete theory because a black hole should behave in accordance with conventional quantum mechanics, implying a unitary evolution, one is forced to believe that local quantum field theory near the black hole horizon is very different from what had hitherto been accepted. This would give us very valuable information concerning physics in the Planck length region, notably a mathematical structure very close to that of super string theory, but it does lead to conceptual difficulties.

An approach that is somewhat related to this is to suspect a breakdown of General Relativity for quantum mechanical systems. It is to some extent unavoidable that Hilbert space is not invariant under general coordinate transformations because such transformations add and remove some states. Finally the cosmological constant problem also suggests that flat space-time has some special significance in a quantum theory. We suggest that a new causality principle could lead to further clues on how to handle this problem.

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1. INTRODUCTION

General relativity and Quantum mechanics are two disciplines in theoretical physics that have been abundantly tested both experimentally and from logical mathematical viewpoints. Yet it seems to be all but impossible to combine the two theories into one. For a short time it was believed that (super) string theory holds the promise to answer this need, but one cannot avoid the impression that the conceptually most difficult issues are not properly assessed by string theory and that it rather adds its own peculiar interpretation difficulties to what we already have. As we will briefly explain at a later stage of this lecture (beginning of Sect. 4), string theory is more likely to represent some sort of long distance limit, or continuum limit, of a more detailed theory, as yet understood. In such a detailed theory space and time may well be discrete in nature, and the information contained in it must play a central role.

Just because of this discreteness of space and time it is likely that it will not be possible to make any kind of approximation, or simplification, so that solvable models would be obtained. Consequently it may well be that the only way to formulate a correct theory is by guessing all at once the correct procedure. To make such a guess is beyond our present capacities. The only alternative as I see it is to try to argue, as accurately as we can, what the dynamical equations are likely to be close to the Planck length scale and to try to deduce from that a picture that is as clear as possible.

This picture should be free from contradictions under all conceivable circumstances, and this is why it is of importance to consider in particular the most extreme situations one can imagine. We will therefore concentrate on the highest possible energy concentrations and the strongest possible gravitational fields. Quite generally one expects that the answers one might come forward with will have a bearing as well on all other related issues in quantum gravity. Indeed, one answer that seems to emerge from our considerations requires a reconsideration of the meaning of quantum theory as a theory describing reality. More than in previous enterprises where we struggled to find theories that are appropriately in harmony with quantum mechanics, for instance the construction of the Standard Model for elementary particles, the discussion about the interpretation of quantum mechanics may become relevant¹. In fact, the same can be said about General Relativity². The extent to which the axioms of this theory will survive in any ultimate theoretical structure will also have to be discussed, as we will see in Sects. 5 and 6.

2. BLACK HOLES, HAWKING RADIATION, AND COUNTING STATES

As for extreme configurations, our obvious choice is the black hole. The black hole is a logically inevitable outcome of the (unquantized) theory of general relativity.

If we take a sufficiently large quantity of matter and give it a spherically symmetric initial configuration, the collapse into a black hole can easily be seen to be unavoidable: imagine that the total mass would be that of an entire galaxy. In that case one can deduce that at the moment that the matter particles cross the horizon (the point of no return near the black hole) the local density would still be less than that of water, and the laws of nature needed to describe what happens at that moment are in no way more exotic than the laws describing water under terrestrial circumstances[†]. At first sight therefore one would argue that a correct treatment of physics at the horizon should not lead to any controversy, and that the real difficulty should be the question what happens at the origin $r = 0$ of the black hole, where a genuine, physical singularity develops. Curiously, quite the opposite is true. The singularity at the origin of coordinate space is of no direct concern since it is safely hidden from our observation, and the physical relevance of questions about that region is hard to defend. For all practical purposes one may ignore the singularity. It is the horizon that gives rise to problems.

Imagine sending an observer into a black hole shortly after the collapsing material disappeared³. Looking at the solutions to the equations of motion one finds that this observer will be surrounded by vacuum. In the simplest, spherically symmetric, case the space-time he is in is characterized by the Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{1 - 2M/r} + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (2.1)$$

where M stands for $G_N m$ and G_N is Newton's constant, m is the total mass of the original configuration. One expects that in terms of his local coordinate frame nothing drastic happens the moment the observer passes through the horizon, which is at $r = 2M$. This is because in spite of appearances space-time is not singular at that spot. Since we can compute precisely what the observer will see, we can also deduce what the outside observer will observe, simply by performing the relevant coordinate transformation. The first non-trivial calculation involving quantum field theory here was done by Hawking⁴ in 1975. He found that the outside observer will experience thermal radiation at a temperature

$$kT = \frac{1}{8\pi M}, \quad (2.2)$$

in Planck units.

[†] In practice minute disturbances from the spherical symmetry of the initial configuration will cause material during its transition through the horizon to be heated to relativistic temperatures, which is why in reality black holes often radiate vast amounts of energy.

One may now imagine doing thermodynamical experiments with a black hole, injecting and extracting energy while monitoring the temperature. The temperature should always be given by Eq. (2.2). This way one derives^{4,5} the *entropy* S :

$$S = 4\pi M^2, \quad (2.3)$$

again in Planck units. This result seems to make a lot of sense physically. It suggests that the black hole can be compared to a macroscopic object such as a balloon filled with a gas that may escape through an opening. The emission spectrum is thermal as long as the balloon is macroscopic, but a microscopic description, in terms of a microcanonical ensemble, should be possible also. In such a system the entropy is directly related to the total number of quantum states that can describe the system.

One can also try to compute S directly by counting physical degrees of freedom near the horizon, and it is here that the first real difficulties show up. Any attempt to count the physical degrees of freedom near the horizon tends to yield infinity as an answer⁶. Certainly this is what happens if one applies perturbative field theory near the horizon, treating all Fourier modes of a field as mutually non-interacting. This is simple to see. Consider for instance a scalar field ϕ . Let us perform the coordinate replacement

$$r - 2M \Rightarrow e^\sigma, \quad (2.4)$$

near the horizon. Then in terms of the coordinates σ and t , we have the Euler-Lagrange field equation:

$$\ddot{\phi} - \frac{1}{(2M)^2} \phi_{\sigma\sigma} + e^\sigma \left(-\frac{1}{(2M)^2} \partial_\Omega^2 \phi + m^2 \phi \right) = 0, \quad (2.5)$$

where ∂_Ω^2 stands for the angular Laplacian. Closer to the horizon the term with e^σ may be ignored altogether. Thus we obtain simple plane wave solutions, but since the boundary is now at $\sigma = -\infty$ these plane waves continue their journey towards the horizon without ever returning – of course, since this material will vanish into the hole. This implies that there is a continuum of Fourier modes there, and the heat capacity of the vacuum here should be strictly infinite, which is not at all in agreement with the finite expression (2.3) for the entropy.

This infinity problem for the entropy is directly related to the so-called quantum information problem: one may compare two one-particle states sent into the black hole, one described by a wave packet that is orthogonal to the other. These wave packets will continue their journeys into the hole forever, following the line $\sigma = -t/2M$, yet it is difficult to see how they can then affect the outgoing waves, which will continue to form a thermal spectrum of Hawking particles. The two in-states could be chosen to be

orthogonal to each other, whereas the out-states are indistinguishable. This appears to violate unitarity.

The entropy infinity problem is closely related to the (one-loop) non-renormalizability of quantum gravity⁷; the low-energy contributions to the entropy can be accommodated for by renormalization of Newton's constant, but this observation does not solve the problem as it requires an infinite bare Newton constant at a finite (Planckian) distance scale. See also Ref⁸.

3. INTERACTIONS

It is tempting now to suggest that the reason for this apparent conflict is that we ignored interactions. Now it is true that if we approach the horizon closer than one Planck length unit (10^{-33} cm) the effective gravitational interactions become strong there, and omitting them seems to be a serious mistake. It is however far from easy to see how one can avoid the disaster just mentioned even when one does take interactions into account. The local observer sees only particles going in, nothing comes out. Why would these interact significantly, and how could this alter the result that the number of possible states near the horizon diverges to infinity? The particles a local observer might see coming out seem to be in no way related to the ingoing objects, and since one is free to choose the initial data for the ingoing waves during a near infinite amount of time it is hard to see how nevertheless only a finite amount of independent states would be possible. To be sure, if one tries to stick to what could be considered to be well-established rules for applying known laws of physics, one still does not obtain a finite density of states, even when one thinks that the interactions *have* been taken into account!

This is why, initially, many researchers indeed found that the density of states should be infinite⁹. In thermodynamical language this might simply mean that the entropy is not given by Eq. (2.3), but by that with an *infinite* constant added to it. This now would have physically important consequences. It would make the black hole 'phase space' infinite, and simple arguments from quantum scattering theory then imply that the majority of these states should be absolutely stable against decay, they would form stable 'black hole remnants'. Remnants are very unlike any known physical objects. Since their Hilbert space would be strictly infinitely degenerate they would violate some of the rules of quantum field theory, such as a recognizable spin statistics connection, and it would be difficult to see how to avoid infinities in calculations of for instance gravitational pair production of remnants. Although we realize that none of these observations are absolute proofs that the remnant theory is deficient, it is generally considered to be unelegant. It seems to be more natural to search for a theory that avoids such unconventional behaviour.

It is certainly possible to find weaknesses in the ‘naive’ arguments that would point to the infinite black hole degeneracy. In terms of locally regular coordinates¹⁰ the horizon can be described as in Fig. 1. In terms of the degrees of freedom in a free field theory the majority of states we are interested in are elements of a product Hilbert space,

$$\mathcal{H} = \{|\text{in}\rangle\} \times \{|\text{out}\rangle\}, \quad (3.1)$$

where the ingoing states $|\text{in}\rangle$ include the entire past history and the outgoing states $|\text{out}\rangle$ include the entire future history. This means that there are infinite amounts of information squeezed onto the past horizon as well as the future horizon.

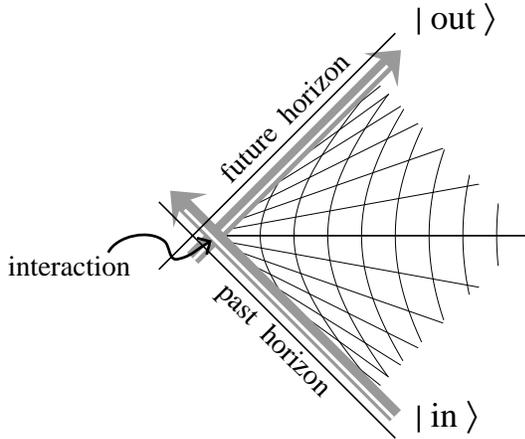


Fig. 1

This Hilbert space is larger than may be considered acceptable. In terms of the regular coordinates as shown in Fig. 1 the ingoing particles in the far past and the outgoing particles in the distant future pass each other near the origin with a center-of-mass energy far beyond the Planck mass, indeed exponentially increasing to beyond the total energy in the universe. Certainly the mutual gravitational interactions¹¹ may then not be ignored. The true Hilbert space is therefore probably not Eq. (3.1). The crucial question here is how to deduce what the correct Hilbert space is.

The approach advocated by us for some time now¹² is to start from the other end: let us *assume* that the black hole Hilbert space is not infinitely degenerate and that its evolution can be described by a unitary operator $U(t)$. One can then use conventional laws of physics to derive constraints that this evolution operator will have to obey. This assumption at first sight looks modest and reasonable but actually turns out to be quite restrictive, with far-reaching consequences, since this way we can obtain expressions for the actual form of $U(t)$. Just because our result is almost in contradiction with what one would be tempted to derive from standard laws of physics, some consequences of this assumption imply important revisions of what one would have thought to be reasonable physics at the Planck length. We refer to this approach as *the S-matrix Ansatz*.

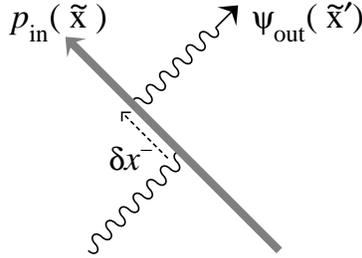
The S -matrix Anstaz only makes sense if we do take interactions between in- and outgoing matter into account. Most important of all these interactions is the gravitational one. As has been described in detail elsewhere, the main effect an ingoing particle has on an outgoing one (and *vice versa*) is that its trajectory is shifted¹¹. In terms of the regular local coordinate frame of Fig. 1, let the ingoing momentum distribution be given by a function $p_{\text{in}}(\tilde{x})$, where \tilde{x} stands for the transverse coordinates, typically the angles θ and φ on the horizon. p_{in} is actually the (integrated) energy momentum component $\int dx^+ T_{++}(\tilde{x}, x^+, x^-)$ where the continuity equation $\partial_\mu T_{\mu\nu} = 0$ removes nearly all x^- dependence (the spacelike components T_{+a} and T_{ab} ($a, b = 1$ or 2) of $T_{\mu\nu}$ are usually negligible). Then one finds that the geodesic of an *outgoing* particle at transverse position \tilde{x}' is shifted by an amount

$$\delta x^- = \int d^2 \tilde{x} f(\tilde{x} - \tilde{x}') p_{\text{in}}(\tilde{x}) \quad , \quad f(\tilde{z}) = -4G_N \log |\tilde{z}|. \quad (3.2)$$

The function $f(\tilde{z})$ can be seen to be a Green function on the transverse plane, obeying

$$\tilde{\partial}^2 f(\tilde{z}) = -8\pi G_N \delta^2(\tilde{z}). \quad (3.3)$$

The direction of the shift is in the positive time direction at close distances, as indicated in Fig. 2.



The effect this has on the wave functions $\psi_{\text{out}}(x)$ is now easy to compute. In Ref¹² it is shown that the shift (3.2) indeed can be made compatible with a unitary evolution operator provided that both the ingoing and the outgoing states are *exclusively* characterized by their momentum distributions $p_{\text{in}}(\tilde{x})$ and $p_{\text{out}}(\tilde{x})$. In terms of the Hilbert space elements $|\{p(\tilde{x})\}\rangle$ one finds the scattering matrix

$$\langle \{p_{\text{out}}(\tilde{x}')\} | \{p_{\text{in}}(\tilde{x})\} \rangle = \mathcal{N} e^{-i \int d^2 \tilde{x} \int d^2 \tilde{x}' p_{\text{in}}(\tilde{x}) f(\tilde{x} - \tilde{x}') p_{\text{out}}(\tilde{x}')}, \quad (3.4)$$

which can be rewritten as the functional integral

$$\langle \{p_{\text{out}}(\tilde{x}')\} | \{p_{\text{in}}(\tilde{x})\} \rangle = \mathcal{N}' \int \mathcal{D}u^\mu(\tilde{x}) e^{i \int d^2 \tilde{x} (p_\mu(\tilde{x}) u^\mu(\tilde{x}) + \frac{1}{8\pi G_N} u^+(\tilde{x}) \tilde{\partial}^2 u^-(\tilde{x}))}, \quad (3.5)$$

with

$$p_\mu(\tilde{x}) = (p_+, p_-, 0, 0) = (p_{\text{out}}, -p_{\text{in}}, 0, 0) \quad \text{and} \quad u^\mu = (u^+, u^-, 0, 0). \quad (3.6)$$

The quantities \mathcal{N} and \mathcal{N}' are normalization factors, to be adjusted such that the matrix (3.4), (3.5) be unitary.

The result (3.5) can be seen to be remarkably closely related to string theory amplitudes in spite of the fact that none of the usual premisses of string theory has been appealed to. The interpretation of (3.5) in terms of strings has been further explained in Ref¹².

4. COMMUTATION RULES

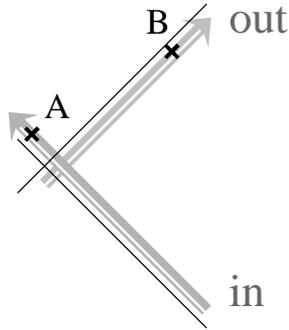
The expressions (3.4), (3.5) are still matrices in an infinite-dimensional Hilbert space, defined in terms of the continuous functions $p(\tilde{x})$. In reality there are reasons to expect only discrete degrees of freedom. It is important in this respect to emphasize that in our approach the expressions obtained so-far were evidently not more than approximations. We suspect that the real physical degrees of freedom near a black hole horizon must be discrete, corresponding to one bit of information per Planckian unit of surface area. Thus, string theory is probably the continuum limit of a discrete theory.

One should keep in mind that these expressions were derived assuming that the transverse momenta were small, otherwise a *sideways* gravitational displacement would have to be taken into account. This means that the expressions are only to be trusted on a transverse length scale large compared to the Planck length. One may hope that the introduction of more kinds of interactions may lead to corrections of this shortcoming, and also to the expected dimensionality as suggested by the finite entropy (2.3). Attempts in this direction were made with only partial success^{13, 14}

In this lecture I now wish to focus on the implications of expressions of this sort. What has been achieved is that ‘quantum information’, originally thought to be lost onto the black hole, is actually returned to us, transformed under the scattering matrix given in some approximation by (3.4), (3.5). However, there seems to be a price that has been paid. It is often argued that the result, a non-trivial unitary evolution law, cannot be correct. These objections were based on the commutation rules. We will now show why commutation rules in our scheme have to be handled with great care^{15, 16}.

In Fig. 3. two points near the black hole horizon are indicated, A and B , where operators can be considered: $\mathcal{A}(x_A)$ and $\mathcal{B}(x_B)$. Now one may observe that these points are spacelike separated, and therefore one expects

$$[\mathcal{A}(x_A), \mathcal{B}(x_B)] \stackrel{?}{=} 0. \quad (4.1)$$



On the other hand however, operator \mathcal{A} covers all of the Hilbert space of the ingoing particles, and operator \mathcal{B} can act on all states of the Hilbert space of outgoing particles. From this one would deduce that they cannot always commute. Here we have an apparent paradox.

This is precisely where the S -matrix Ansatz deviates from what would be done in more conventional approaches. The Ansatz implies that the in- and out-Hilbert spaces are not independent but instead connected by equations of motion. In terms of the space of states that describe the Black hole completely and unambiguously the operators \mathcal{A} and \mathcal{B} do not commute, but in terms of the states in the corresponding flat space they do. The situation can be clarified further by regarding these operators in a Heisenberg picture (i.e. the scheme where operators are time dependent but states $|\psi\rangle$ are not) Suppose that the operator \mathcal{A} acts on particles that went into the hole in the far past. This means that these particles in the coordinate frame of Fig. 1 are Lorentz boosted towards the past horizon with boost parameter that diverges exponentially with time. Particles with this much energy cause a large gravitational shift among the particles going in the other direction, that is, the outgoing particles. Thus the operator \mathcal{A} causes a large deformation of the trajectories of the outgoing objects.

Now consider the operator \mathcal{B} . One could still maintain that its effect is independent of what \mathcal{A} did because it is spacelike separated. It should be clear however that operator \mathcal{B} has a similar effect on the trajectories of the ingoing objects. It is true that in order to see this effect we have to extrapolate back to the past, but this is totally legal in the Heisenberg picture. If we were to apply the conventional argument that \mathcal{A} and \mathcal{B} commute this would mean that we would be working in a Hilbert space where one can create independently from each other anomalously energetic leftmoving particles and rightmoving particles. In a Heisenberg picture these particles would collide with excessive center-of-mass energies near the origin, causing each other's trajectories to be wildly distorted by gravitational fields. Since the center-of-mass energy would quickly exceed the energy needed to form a black hole, we see that black holes, or rather, their time reverses, sometimes called 'white holes', would become dominant elements of this combined Hilbert space. It is the philosophy of the S -matrix Ansatz that these states

would be unphysical and therefore should be removed from the allowed states. As soon as one says this however, delicate questions will be raised about which states should be admitted in Hilbert space, and why. Should one admit white holes in the far past or not?

It is not unreasonable to speculate that constraints will have to be introduced to limit oneself to those elements of Hilbert space that describe in a more or less reasonable way histories of the universe as we are familiar with. This excludes the states dominated by white holes. If this is what should be done we can easily see that \mathcal{A} and \mathcal{B} will not commute. Since we are looking at operators corresponding to ultra-energetic particles, far outside the realm of conventional quantum field theory, and since we now do concentrate on *the Hilbert space describing the black hole, and not the Hilbert space of empty space-time* we think this is legitimate. The argument goes as follows.

Let us first consider the product $\mathcal{B}\mathcal{A}$. The operator \mathcal{A} is computed in the point x_A which is after the ‘observer’ crossed the horizon. One generally argues that the observer did not experience any special effect while crossing this horizon. In particular his geodesic was not seriously affected by the gravitational fields of any particle that might be near him on its way out of the black hole. After this one considers that action of operator \mathcal{B} which is at a point x_B on the future horizon. This future horizon is defined by examining the fate of the outgoing geodesics in the distant future. Therefore the geodesics of the particles B created by the operator \mathcal{B} will be distorted in the region of space-time *before* the trajectory of particle A that went in.

Now when we compute $\mathcal{A}\mathcal{B}$ we may argue that particle B is at the same spot as before, but what is the point x'_A where one should elaborate the operator $\mathcal{A}(x'_A)$? This operator must now be defined in a spacetime where all trajectories of *ingoing* particles A are severely distorted by a strongly gravitating beam of new B particles, that have been added to our state by operator \mathcal{B} . This now is in direct conflict with the ‘naive’ assertion that the ingoing observer feels no Hawking particles. The particle B created by the operator \mathcal{B} *will* be felt by him! Before continuing we should realize now that we have not yet defined what we exactly mean when we discuss the operator $\mathcal{A}(x_A)$: is x_A defined to be the one that is shifted by the gravitational effects of B , or should we take the point one would have obtained if the observer had indeed not noticed any shift? It would be reasonable to define $\mathcal{A}(x_A)$ to be the operator acting on the point x_A as it would have been if the ingoing observer had not experienced any gravitational effect from outgoing particles, since conventional arguments say that such effects will not be experienced. But if this is the definition we see that in the product $\mathcal{A}\mathcal{B}$ the real point x_A where \mathcal{A} is calculated is the shifted one. The most important effect of this shift is that other particles that crossed the horizon at slightly different transverse coordinates $\tilde{x} = (\theta, \varphi)$ will have landed at quite different locations, so that if \mathcal{A} depends on *different*

nearby points \tilde{x} its effect on these states will be vastly different from what it did in the product $\mathcal{B}\mathcal{A}$. With this definition, \mathcal{A} and \mathcal{B} do not commute. Only if we use the other definition, where the point x_A is defined in a coordinate frame that had the curvatures due to the shift caused by B incorporated would \mathcal{A} and \mathcal{B} still commute. This however is a Hilbert space where from the start we allowed infinitely energetic particles leave the black hole, which in the far past of its history would be difficult to accept. Such particles really do not occur in the Hilbert space of the far past.

5. IS GENERAL RELATIVITY VIOLATED?

What was learned in the previous chapter is that one should limit oneself to ‘reasonable’ states in Hilbert space. But what does ‘reasonable’ mean? If we exclude elementary particles with energies beyond some large multiple of the Planck energy this may certainly be called ‘reasonable’, yet of course it violates Lorentz invariance. If we do not exclude such states we run the danger of allowing states that have white holes in their past, and this one would tend to call ‘unreasonable’. If one is ready to accept the idea of a unitary evolution of black hole states one must also accept that in the Rindler frame of Fig. 1, Hilbert space is only allowed to contain *either* all particle configurations arbitrarily Lorentz-boosted onto the past horizon (the Hilbert space of the in-states), *or* all particle configurations on the future horizon (the Hilbert space of all out-states), but not both sets simultaneously, since we expect an S -matrix to connect these two Hilbert spaces. For an observer in a local inertial frame this situation is unfamiliar. For him leftgoing particles and rightgoing particles should be allowed to run around independently, but on the other hand for him particles with energies much beyond the Planck mass (‘trans-Planckian particles’) are of little or no interest. We presently assume that all these Hilbert spaces are connected via quite non-trivial unitary matrices.

Let us consider again an ‘observer’ falling into the black hole. After he passed the horizon he might want to make some measurements at the point A of Fig. 3. Alternatively we might wish to study the effects these measurements have on states at the point B . The existence of a unitary evolution matrix should imply that the same measurements could indeed also have been carried out at B . This is reasonable from the point of view of the black hole physicist, but very strange as experienced by the observer in the inertial frame. Since A and B seem to be spacelike separated it looks as if we are dealing with the famous ‘quantum copying machine’ at the interaction point: the state at A is ‘duplicated’ at B .

Even though either the original or the duplicate is guaranteed to consist of trans-Planckian particles, so that one could maintain that no conflict has as yet arisen with conventional physics, there is nevertheless reason to worry. It does not seem unreason-

able after all that trans-Planckian particles are physically realizable. Why then should these have such odd properties? An answer to this may at first sight sound like a very radical one, since it will appear to imply that general relativity is violated. But hang on, I will explain my viewpoint about this afterwards.

The state at A is not duplicated at B , but rather transformed into it: there is no way of describing a Hilbert space that contains both all possible states in A and all possible states in B . Thus, we must *choose* which of these two representations of Hilbert space we wish to use. A simple model explains the situation best. Consider a quantum system described by a Hamiltonian,

$$\begin{aligned} \text{at } t \leq 0 : \quad & H(t) = H_0, \\ \text{at } t > 0 : \quad & H(t) = \begin{cases} H_1(t) \text{ in universe \#1,} \\ H_2(t) \text{ in universe \#2.} \end{cases} \end{aligned} \quad (5.1)$$

Thus at $t > 0$ we have two Hamiltonians to choose from. A unitary transformation relates the two branches, but the existence of such a transformation is a mere formality. For all we know the two ‘worlds’ at $t > 0$ are different. In a Heisenberg picture, operators in universe #1 at $t = +1$ sec do not commute with operators in universe #2 at $t = +1$ sec. Yet information cannot be readily transmitted from 1 to 2. Our two Hamiltonians could refer to the two cases: an observer does or does not pass through the horizon.

The danger of theories of this sort is also clearly exhibited in this model. There needs not be any relation between $H_1(t)$ and $H_2(t)$. Indeed the same situation would occur if the theory of General Relativity that normally relates the two universes were completely violated. If our model would refer to a black hole one could interpret it as follows: *one* Hamiltonian describes the black hole for *all* observers, the other would describe *flat space only*. “Reality” is only described by *one* Hamiltonian. This is why our model is equivalent to a theory with explicit violation of General Relativity.

We therefore have to deal with the problem how to explain the accurate validity of General Relativity for weak gravitational fields. It must be possible to deduce the efficiency of General Relativity for the ordinary world, from some symmetry principle. This symmetry principle should also mean something for Rindler space. We believe that even though H_1 and H_2 need not be the same, there should still exist a well-defined transformation between them. This transformation law could however be much more complicated than the ones usually employed in General Relativity. In short: a symmetry principle that implies General Relativity in the classical limit may still exist, but Hilbert space itself cannot be completely invariant under coordinate reparametrizations. Our big challenge is now to identify the correct symmetries and transformation rules.

6. CAUSALITY[†]

If one would be ready to relax the demands of General Relativity in our description of the black hole horizon, other fundamental principles might have to be called for. A very fundamental principle in physics, often rudely disregarded, is causality. It is true that this demand has become more questionable, and it has become a fashion to reject it altogether. As the space-time metric $g_{\mu\nu}(x)$ has become a quantum variable itself it seems to be impossible to keep the speed of light as an upper bound of the velocity of information transport. Often it is suspected that there are contributions in the functional integrand where $g_{\mu\nu}$ has lost its Lorentzian signature¹⁷. Worse even, topologically non-trivial excitations such as wormholes may produce space-times harboring closed timelike loops¹⁸. Such proposals completely ignore the demand that there should be a strict separation between cause and effect, not only in a classical system but also in the Schrödinger equation.

My proposal is to restore causality, and it may even prove to be useful to restore the speed of light as an upper bound. First let me explain that in classical general relativity the *coordinate speed of light* is indeed an upper bound for information transport. More precisely, one has a theorem of the following sort:

In any gravitating system surrounded by asymptotically flat space, with a $T_{\mu\nu}$ satisfying a positive mass-energy condition, a coordinate frame $\{x^\mu\}$ can be found such that:

- 1) x^μ approaches the asymptotically flat coordinates x_0^μ sufficiently rapidly, and
- 2) for all infinitesimal dx^μ one has

$$g_{\mu\nu}dx^\mu dx^\nu \geq \eta_{\mu\nu}dx^\mu dx^\nu, \quad (6.1)$$

with

$$\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1). \quad (6.2)$$

This ensures that the dynamical lightcone resides inside the coordinate lightcone. The proof of the general theorem is as yet in a stage of a conjecture, but it is easy to verify for the Schwarzschild, Reissner-Nordstrom, Kerr and Kerr-Newman solutions. The theorem is illustrated by the well-known Shapiro delay of radar signals passing close by the Sun. It would not hold for negative Schwarzschild masses. The boundary condition on the coordinate frame has to be formulated with sufficient care. One may for instance demand that x^μ rapidly approaches the usual Schwarzschild coordinate frame.

[†] This chapter regrettably could not be discussed at the Conference for lack of time.

One may postulate this theorem now also to hold for the quantum case. This means that one is forced to use a particular coordinate frame. Transition towards any other frame will not be forbidden, but ensues in the admission of new states in Hilbert space (ones allowing values of the quantum variable $g_{\mu\nu}(x)$ that had been forbidden in the previous frame), while it will force the removal of others. This will be in accordance to the theme of the previous section where we describe how the Rindler space transformation adds and removes some elements in Hilbert space. Recognising that flat space-time at the boundary plays an essential role in formulating the quantum theory may well lead to an additional bonus: the resolution of the well-known cosmological constant problem may be impossible without the use of flat space-time as a special reference system where the energy density must be tuned to zero.

My causality postulate may perhaps not be welcomed by investigators who speculate on the occurrence of phenomena such as topology change, but the apparent validity of our theorem for classical gravity may provide some new thoughts.

REFERENCES

- 1 G. 't Hooft, *J. Geometry and Physics* **1** (1984) 45-52; see also G. 't Hooft, *Black Holes and the Foundations of Quantum Mechanics*, in "Niels Bohr: Physics and the World", ed. H. Feshbach et al (Harwood Acad. Publ., London, Paris, New York, Melbourne, 1988), p. 171.
- 2 G. 't Hooft, *J. Stat. Phys.* **53**, (1988) 323.
- 3 S.W. Hawking and G.F.R. Ellis, *The Large Scale Structure of Space-time*, Cambridge Univ. Press (Cambridge, 1973).
- 4 R.M. Wald, *Commun. Math. Phys.* **45** (1975) 9; S.W. Hawking, *Comm. Math. Phys.* **43** (1975) 199; J.B. Hartle and S.W. Hawking, *Phys.Rev.* **D13** (1976) 2188; W.G. Unruh, *Phys. Rev.* **D14** (1976) 870.
- 5 R.M. Wald, *Phys. Rev.* **D20** (1979) 1271; J.D. Bekenstein, *Nuovo Cim. Lett.* **4** (1972) 737, *Phys. Rev.* **D5** (1972) 1239, 2403; **D7** (1973) 2333; **D9** (1974) 3292; *Nuovo Cim. Lett.* **4** (1972) 737.
- 6 G. 't Hooft, *Nucl. Phys.* **B256** (1985) 727.
- 7 J.L.F. Barbón and R. Emparan, *On quantum black hole entropy and Newton constant renormalization*, Princeton and Bilbao prepr. PUPT-1529 / EHU-FT 95/5, hep-th/9502155.
- 8 S. Carlip, *Statistical Mechanics and Black Hole Entropy* UCD-95-30, gr-qc/9509024, September 1995.
- 9 C. Callan, S. Giddings, J. Harvey and A. Strominger, *Phys. Rev.* **D45** (1992) 1005.

- 10 W. Rindler, *Am.J. Phys.* **34** (1966) 1174.
- 11 P.C. Aichelburg and R.U. Sexl, *Gen.Rel. and Gravitation* **2** (1971) 303; T. Dray) and G. 't Hooft, *Nucl. Phys.* **B253** (1985) 173; *Comm. Math. Phys.* **99** (1985) 613.
- 12 G. 't Hooft, *Phys. Scripta* **T15** (1987) 143; *Nucl. Phys.* **B335** (1990) 138; *Acta Physica Polonica* **B19** (1988) 187; *Strings from gravity* , In: *Unification of Fundamental Interactions*. Proceedings of Nobel Symposium 67, Marstrand, Sweden, June 2-7, 1986. Eds. L. Brink et al.; *Physica Scripta*, **T15** (1987) 143; *Scattering matrix for a quantized black hole* , In book: *Black Hole Physics*, ed. V. De Sabbata and Z. Zhang . 1992 (Kluwer Academic Publishers, The Netherlands), p. 381.
- 13 G. 't Hooft, *Nucl. Phys.* **B43** (Proc. Suppl.) (1995) 1; *Physica Scripta* **T36** (1991) 247.
- 14 G. 't Hooft, *More on the black hole S-matrix* , In: Proceedings of the Fifth Seminar *Quantum Gravity*, Moscow, USSR, 28 May-1 June 1990, ed. M.A. Markov, V.A. Berezin and V.P. Frolov. World Scientific, Singapore (1991) p. 251; *S-Matrix theory for black holes* , in Proceedings of a NATO Advanced Study Institute on New Symmetry Principles in Quantum Field Theory, held July 16-27, 1991 in Cargèse, France. Eds. J. Fröhlich, G. 't Hooft, A. Jaffe, G. Mack, P.K. Mitter and R. Stora NATO ASI Series (1992 Plenum Press, New York) p. 275.
- 15 C.R. Stephens, G. 't Hooft and B.F. Whiting, *Class. Quantum Grav.* **11** (1994) 621; E. Verlinde, *Black Hole Evaporation and Complementarity* , to appear in the proceedings of the 1993 Trieste Spring School, hep-th/9503120.
- 16 L. Susskind, L. Thorlacius and J. Uglum, *Phys. Rev.* **D48** (1993) 3743 (hep-th 9306069); D.A. Lowe *et al*, *Black Hole Complementarity vs. Locality* , prepr. NSF-ITP-95-47 / SU-ITP-95-13 ; *Phys. Rev.* **D48** (1993) 3743, *ibid.* **D49** (1994) 966; D. A. Lowe, L. Susskind, and J. Uglum, *Phys. Rev.* **B327** (1994) 226; L. Susskind, *The World as a Hologram*, preprint SU-ITP-94-33, hep-th/9409089.
- 17 J.B. Hartle, *J. Math. Phys.* **30** (1989) 452.
- 18 J.R. Gott, *Phys. Rev. Lett.* **66** (1991) 1126; S.W. Hawking, *Phys. Rev.* **D46** (1992) 603; C.J. Fewster and C.G. Wells, *Unitarity of Quantum Theory and Closed Timelike Curves* , Cambridge prepr. DAMTP-R94/35, hep-th/9409156.