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Horizons

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Abstract

The gravitational force harbours a fundamental instability against collapse. In standard General Relativity without Quantum Mechanics, this implies the existence of black holes as natural, stable solutions of Einstein's equations. If one attempts to quantize the gravitational force, one should also consider the question how Quantum Mechanics affects the behaviour of black holes. In this lecture, we concentrate on the horizon. One would have expected that its properties could be derived from general coordinate transformations out of a vacuum state. In contrast, it appears that much new physics is needed. Much of that is still poorly understood, but one may speculate on the way information is organized at a horizon, and how refined versions of Quantum Theory may lead to answers.

1. Introduction: Black Holes as Inevitable Features of General Relativity

The fact that the gravitational force acts directly upon the inertial mass of an object, makes this force unique in Nature, and allows for an unambiguous description of the classical (*i.e.* unquantized) case, called "General Relativity". However, unlike the situation in electromagnetism, the gravitational force produces attraction rather than repulsion between like charges. An inevitable consequence of this is a fundamental instability: masses attract to form bigger masses, which attract one another even more strongly. Eventually, gigantic implosions of large accumulated quantities of mass may result. There is no obvious limit here, so one cannot avoid that the gravitational potential might cross an important threshold, where the escape velocity exceeds that of light.

Indeed, as soon as one is ready to accept the validity of General Relativity for classical systems, one can easily calculate what will happen. The final state that one then reaches is called a "black hole". In astronomy, the formation of a black hole out of one or several stars depends on the circumstances, among which is the equation of state of the material that the stars are made of. Because of this, the physics of black hole formation is sometimes challenged, and conjectures are uttered that black holes "are probably nothing else but commercially viable figments of the imagination"[1]

It is however easy to see that such a position is untenable. To demonstrate this, let me here show how to construct a black hole out of ordinary objects, obeying non-exotic equations of state. These objects could, for example, be television sets, acting on batteries. During the process of black hole formation, these objects will each continue to be in perfect working order. We begin with placing these in the following configuration: let them form a shell of matter, of thickness d and radius R. If d is kept modest, say a few kilometers, then R has to be taken vary large, say a million light years. The television sets may stay at convenient distances away from each other, say a meter. The initial velocities are taken to be small; certainly objects close to each other must have very small relative velocities so that collisions have no harmful effects.

The objects attract one another. They don't feel it because, locally, they are under weightless conditions, but they do start to accelerate. So, the sphere shrinks. After thousands of years, the configuration is still spherical, the relative velocities for close-by objects are still small, the density is still low, the televisions are still in working order, but they pass the magical surface called "horizon". What happens is, that light emitted by the objects can no longer reach the outside world. The calculation is straightforward and robust, which means that small perturbations will not affect the main result: no light can be seen from these objects; they form a black hole.

What happens next, is that the sphere of objects continue to contract, and at some point, long after the horizon has been past, the objects crush, television sets will cease to function, for a while the Standard model still applies to them, but eventually the matter density will

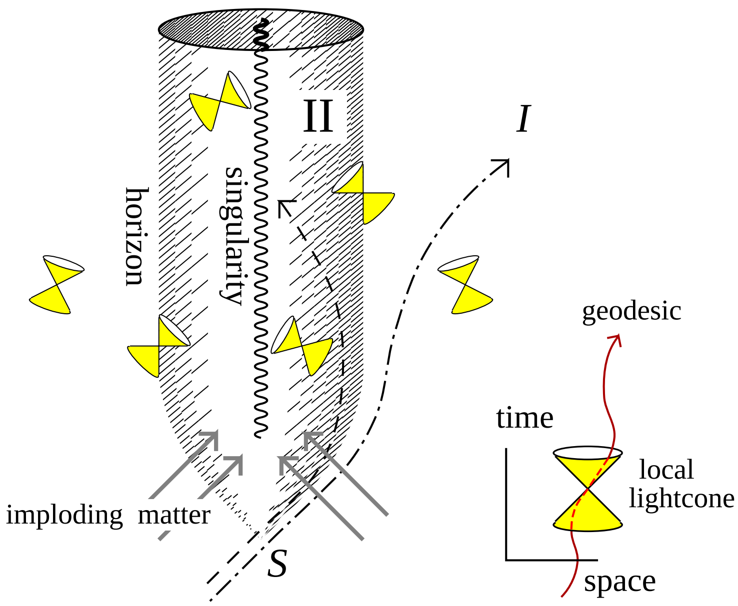

Figure 1: The space-time of a black hole

exceed all bounds, and a true singularity is encountered[1]. It is here, at this singularity, where the laws of physics as we know them no longer apply, but whatever happens there is totally irrelevant for the phenomenology of a black hole; whatever an outside observer sees is determined by known laws of physics. the horizon acts as a "cosmic sensor", preventing us from observing the singularity. Whether all singularities in all solutions to the equations are always screened by this cosmic sensor is still being debated, but we do see this happen in all practical solutions known.

In Fig. 1, it is sketched what happens to space-time. The solution to Einstein's equations in General Relativity dictates that, locally, the light cones are tilted. The shaded surface, the horizon, is formed by constructing the tangent of these local light cones. Inside this surface, all local lightcones are pointed inwards, towards the central singularity. The radius of this horizon is found to be

$$R = 2G_N M/c^2 \ , \tag{1.1}$$

where M is the total mass-energy of the system.

Note that all signals seen by an outside observer in region I, when observing a black hole, originate at the point S. If only a finite amount of light is emitted from there, this

[1]Small perturbations from spherical symmetry do affect the singularity in a complicated way, but this is not relevant for the nature of the horizon.

light is spread over an infinite amount of time, and therefore infinitely red-shifted. Hence, one expects no signal at all; the black hole is black. It can be formed out of ordinary matter.

2. Black holes in particle physics

In elementary particle physics, the gravitational force is extremely weak, and can normally be ignored. It is, however, understood that there must be regions of particle physics where this force *must* play a decisive role. this is when the energy per particle tends to exceed the Planck scale. The Planck scale is set by considering the three fundamental constants of nature:

$$\begin{array}{lrcll} \text{The velocity of light,} & c & = & 2.9979\times 10^{8}\ \ \text{m/sec}\ , & \\ \text{Planck's constant,} & h/2\pi=\hbar & = & 1.0546\times 10^{-34}\ \text{kg m}^2/\text{sec}\ , & \text{and} \\ \text{Newton's constant,} & G_N & = & 6.672\times 10^{-11}\ \text{m}^3\ \text{kg}^{-1}\,\text{sec}^{-2}\ . & \end{array} \tag{2.1}$$

Out of these, one finds the following fundamental units:

$$\begin{aligned} L^{\text{Planck}} &= \sqrt{\hbar\, G_N/c^3} = 1.616\times 10^{-33}\ \text{cm}\ , \\ T^{\text{Planck}} &= \sqrt{\hbar\, G_N/c^5} = 5.39\times 10^{-44}\ \text{sec}\ , \\ M^{\text{Planck}} &= \sqrt{\hbar\, c/G_N} = 21.8\ \mu\text{g}\ . \end{aligned} \tag{2.2}$$

If particles collide with the enormous c.o.m. energy of $M^{\text{Planck}}c^2$, gravitational effects must be important. If many particles with such energies accumulate in a region of size L^{Planck}, gravitational implosion must take place. Black holes must play an important role there.

It was S. Hawking's fundamental discovery [2], that, when applying the laws of Quantum Field Theory (QFT), black holes are no longer truly black. Particles are emitted at a fundamental temperature, given by

$$k\,T^{\text{Hawking}} = \frac{\hbar\ c^3}{8\pi\, G_N\, M_{\text{BH}}} = \frac{\hbar\ c}{4\pi R_{\text{BH}}}\ . \tag{2.3}$$

For astronomical black holes, this temperature is far too low to give observable effects, but in particle physics the Hawking effect is essential. For a further discussion of this phenomenon by the present author, see [3].

One might suspect now that black holes therefore behave a bit more like ordinary matter. Not only can they be formed by high-energy collisions, but they can also decay. Apparently, QFT restores (some) time-reversal symmetry in processes involving black holes. Are black holes elementary particles? Are elementary particles black holes? Probably, particles and black holes become indistinguishable at the Planck scale. It is instructive to consider the entire formation and decay of a black hole as if described by quantum mechanical amplitudes.

3. Information in a black hole

The absorption cross section σ is roughly given by

$$\sigma = 2\pi R_{\rm BH}^2 = 8\pi M_{\rm BH}^2\ , \tag{3.1}$$

and the emission probability *for a single particle in a given quantum state*:

$$W{\rm d}t = \frac{\sigma(\mathbf{k})\, v}{V}\, e^{-E/kT}\ {\rm d}t\ , \tag{3.2}$$

where $\mathbf{k}$ is the wave number characterizing the quantum state of the particle emitted, and T is the Hawking temperature. E is the energy of the emitted particle. Now, *assume* that the process is also governed by a Schrödinger equation. this means that there are quantum mechanical transition amplitudes,

$$\mathcal{T}_{\rm in} \ = \ {}_{\rm BH}\langle M + E/c^2|\ |M\rangle_{\rm BH}|E\rangle_{\rm in}\ , \tag{3.3}$$

$$\mathcal{T}_{\rm out} \ = \ {}_{\rm BH}\langle M|_{\rm out}\langle E|\ |M + E/c^2\rangle_{\rm BH}\ , \tag{3.4}$$

where $|M\rangle_{\rm BH}$ is the black hole state without the absorbed particle, having mass M, and $|M{+}E/c^2\rangle$ is the slightly heavier black hole with the extra particle absorbed. The absorption cross section is then

$$\sigma = |\mathcal{T}_{\rm in}|^2 \varrho(M + E/c^2)/v\ , \tag{3.5}$$

where $\varrho(M + E/c^2)$ is the level density of the black hole in the final state. This is what we get when applying Fermi's Golden Rule. The same Golden Rule gives us for the emission process *for each quantum state of the emitted particle*:

$$W = |\mathcal{T}_{\rm out}|^2 \varrho(M)\frac{1}{V}\ . \tag{3.6}$$

Here, as before, v is the velocity of the emitted particle, and V is the volume, to be included as a consequence of the normalization of the quantum state.

We can now divide Eq. (3.1) by Eq. (3.2), and compare that with what we get when (3.5) is divided by (3.6). One finds:

$$\frac{\varrho(M + E/c^2)}{\varrho(M)} = e^{E/kT} = e^{8\pi G_N M\, E/\hbar\, c^3}\ . \tag{3.7}$$

One concludes that

$$\begin{aligned} \varrho(M) \ &= \ e^{S(M)}\ , \\ S(M + {\rm d}M) - S(M) \ &= \ 8\pi G_N M {\rm d}M/\hbar\, c\ ; \qquad (3.8) \\ S(M) \ &= \ \frac{4\pi\, G_N}{\hbar\, c} M^2 + C^{\rm nt}\ . \qquad (3.9) \end{aligned}$$

Thus, apart from an overall multiplicative constant, $e^{C^{\rm nt}}$, we find the *density of states* $\varrho(M)=e^{S(M)}$ for a black hole with mass M. It can also be written as

$$\varrho(M)=2^{A/A_0}\ , \tag{3.10}$$

where A is the area $4\pi R^2$ of the black hole, and A_0 is a fundamental unit of area,

$$A_0=0.724\times10^{-65}\ {\rm cm}^2\ . \tag{3.11}$$

Apparently, the states of a black hole are counted by the number of bits one can put on its horizon, one bit on every A_0.

This result is quite general. It also holds for black holes that carry electric charge or angular momentum or both. Usually, one expects the constant $C^{\rm nt}$ in Eq. (3.9) to be small, although its value is not known.

4. The Brick Wall

This result[4], obtained in the 1970's, is astounding. Black holes come in a denumerable set of states. These states seem to be situated on the horizon, and, as was stated in the Introduction, the physical properties of the horizon follow from simple coordinate transformation rules applied on the physical vacuum. We seem to have hit upon a novel property of the vacuum itself.

Naturally, we wish to learn more about these quantum states. It should be possible now to derive all their properties from General Relativity combined with Quantum Field Theory. However, when one tries to do these calculations, a deep and fundamental mystery emerges: direct application of QFT leads to an infinity of states, described by much more parameters than one bit of information per quantity A_0 of area. Writing the radial coordinate r and the external time coordinate t as

$$r=2M+e^{2\sigma}\ ;\qquad t=4M\tau\ , \tag{4.1}$$

in units where all Planckian quantities of Eq. (2.2) were put equal to one, it is quickly found that, at the horizon, in-going and out-going waves are plane waves in terms of σ and τ:

$$\psi(\sigma,\tau)\rightarrow\psi_{\rm in}(\sigma+\tau,\,\Omega)+\psi_{\rm out}(\sigma-\tau,\,\Omega)\ , \tag{4.2}$$

where Ω stands short for the angular coordinates θ and φ on the horizon. Since σ runs to $-\infty$, an infinite amount of information can be stored in these waves.

By way of exercise, one can now compute how much information will be stored in these waves if

- the particle contents will be as dictated by the Boltzmann distribution corresponding to the Hawking temperature (2.3), and

- a *brick wall* is placed at some postion $r_w = 2M + h$, where some boundary condition is imposed on the fields. One could impose a Neumann or Dirichlet boundary condition for the fields there, or something more sophisticated[2].

In a theory with N scalar fields, in the limit of small h one finds[3, 5] for the total energy of the particles:

$$U = \frac{2\pi^3}{15h}\left(\frac{2M}{\beta}\right)^4 N \ , \tag{4.3}$$

and for the total entropy:

$$S = \frac{16\pi^3 M}{45h}\left(\frac{2M}{\beta}\right)^3 N \ . \tag{4.4}$$

We can place the wall in such a way that the entropy matches Eq. (3.9):

$$h = \frac{N}{720\pi M} \ . \tag{4.5}$$

The total energy of the particles then makes up for $\frac{3}{8}$ of the black hole mass.

Only with this brick wall in place, a black hole would exactly live up to our intuitive expectations. Infalling waves would bounce back, so that an unambiguous S-matrix can be derived, and the entropy S would correspond to the total number of physical states. Although the wall position may seem to depend on the total mass M of the black hole, one finds that the *covariant* distance between wall and horizon is M independent:

$$\int_{r-2M}^{r=2M+h} \mathrm{d}s = \sqrt{\frac{N}{90\pi}} \ . \tag{4.6}$$

But what would be the physical interpretation of this result? Surely, an infalling observer would not notice the presence of such a wall. For some reason, a quantum field cannot have physical degrees of freedom between the wall and the horizon, but why?

One obvious observation is that this is a region comparable to the Planck size (or even somewhat smaller). Surely, one is not allowed to ignore the intense gravitational self interactions of particles confined to such a small region, so that perturbative quantum field theory probably does not apply there. However, one could concentrate *only* on either the in-going or the out-going particles. They are just Lorentz transforms of regular states. Why should their degrees of freedom no longer count?

A more subtle suggestion is that, although we do have fields between the wall and the horizon, which do carry degrees of freedom, these degrees of freedom are not physical. They could emerge as a kind of *local gauge degrees of freedom*, undetectable by any observer. Such a suggestion ties in with what will be discussed later (Section 7).

[2] Since one expects *all* continuous symmetries to be broken by the black hole, a *random* boundary condition could be preferred, but in practice the details of the boundary condition are not very important.

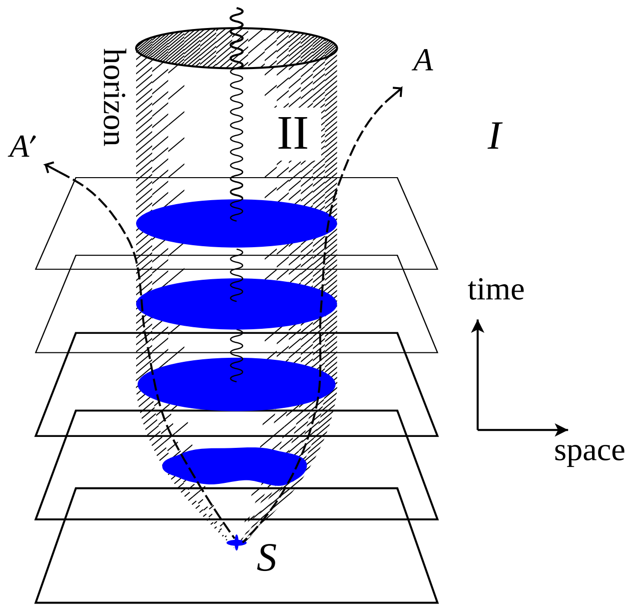

Figure 2: The black hole caustic

5. The black hole caustic

One can do a bit more than speculate. In Ref. [3], it is described how to take into account that in-going and out-going particles interact gravitationally. We know now that such interactions may not be ignored. What is found is that the position of the horizon depends on the mass distribution of matter falling in. In turn, this affects the quantum states of the particles moving out, so, unlike what one finds from linearized quantum field theory, there is a relation between in-going and out-going particles, and this relation can indeed be cast in the form of an S-matrix. The problem with this derivation is that one still does not find the correct density of distinct quantum states — there are too many states. It turns out that the quantum state of out-going matter appears to be described completely by the *geometry of the dynamic horizon.*

This can be understood in the following way. We define the horizon as the boundary between the region I of space-time from where signals can escape to infinity, and the region II from which no signals can be received. This means that the exact location of the horizon is obtained by following all light rays all the way to time $\rightarrow +\infty$. if we have obtained the horizon at some (large) value of time t, we can integrate the equations to locate the horizon at earlier times. The procedure is sketched in Fig. 2. If, finally, we reach the instant when the black hole is formed, the horizon shrinks, until the region II ends. The horizon opens up at the point S, but, from its definition, we see that, there, it is a caustic. The details of this caustic can be quite complex, in principle. Indeed, the quantum state of the out going particles depends on where on this caustic these particles originated. One might conclude that the quantum state of the out-going particles might be determined completely by the geometric features of this caustic.

As yet, however, it has not been possible to distill a Hilbert space of out-going particles from such an assumption. In Fig. 2, we see that a signal observed by observer A, may meet the signal seen by an observer A' at the caustic. A and A' need not be opposite to one another; there is a duality mapping from points A to points A' on the horizon. This mapping may be the one that determines the black hole's quantum state.

A particle seen by A and a particle seen by A' meet at S with tremendous c.o.m. energy. Let us consider their scattering in a time-reversed setting. Gravitational interactions cause both particles to undergo a large coordinate shift[6]. These shifts turn both particles into showers (backward in time). The quantum states of A and A' are determined by overlapping these showers with the vacuum fluctuations in the region below S.

6. Strings from black holes; white holes

How gravitational interactions lead to string-like amplitudes for the entire process of black hole formation and evaporation, has been described in refs. [3] and [7]. It is not exactly string theory what one gets, but rather a string with a purely imaginary string constant. Since the horizon itself acts as the string world sheet, this string may be some dual of the conventional string approach to black holes[8]. One can picture the scattering events as follows. The black hole is formed by a large number of particles contracting. Each of these particles is pictured as a closed string. Since the horizon acts as the string world sheet, our closed strings widen as they approach the horizon, and they scan the entire horizon as they do so. The strings recombine to form new closed strings, which then separate from the horizon acting as Hawking particles. A regular space-time with an expanding cloud of tiny closed strings forms the final state.

A peculiar by-product of this analysis is the resolution of an old problem in black hole physics: what is the time-reverse of a black hole? In the literature it is sometimes known as the "white hole": a shrinking black hole emitting classical objects and eventually disappearing. It may have been formed by a cloud of "time-reversed Hawking particles".

In our analysis the answer is as follows. By assuming that the out-state is controlled by the in-state through gravitational interactions, it is found that the amplitude automatically respects time-reversal invariance, basically because the gravitational forces respect Newton's law: action = reaction. It is found that the *positions* of the out-going particles are determined by the *momenta* of the in-going ones, and *vice-versa*. Quantum mechanically, the particles in the momentum representation are superpositions of the particles in the position representation. Therefore, one finds that *white holes are quantum superpositions of all possible black hole states* (in the same mass region), *and vice-versa*.

7. Information loss

Much of the investigations described above pertains to an apparent incongruity in any quantum picture of black holes. Classically, one sees that objects falling in cannot imprint all information contained in them on the out-going states. They are described by quantum waves that require an infinite amount of time to enter the black hole. In contrast, the out-going particles were there already at the very beginning, waiting close to the horizon, at σ in the far negative region, until it is their turn to leave. Our quantum picture requires that these objects are nevertheless closely related. The analysis sketched in the previous sections might suggest that we have come close to resolving this problem: all one has to do is switch on the gravitational forces between in-going and out-going objects. String theory[8] also suggests that this problem can be cured.

However, it should be possible to identify these quantum states in terms of features of the vacuum in relation to general coordinate transformations. In particular, this should be possible for the horizon in the large mass limit. The space-time one then describes is known as Rindler space-time[9]. What seems to be missing is the identification of the quantum states in Rindler space-time and their relation to the quantum states characterizing the vacuum in a flat world. This flat world appears to allow for an unlimited amount of information to disappear across the horizon. To see this, all one has to do is subject ordinary particles to unlimited Lorentz boost transformations. In spite of all that has been said, this problem has not been solved in a satisfactory manner.

Since we are dealing here with quantum phenomena in an extremely alien world of highly curved coordinate frames, it is natural to ask the following question: *Why should these issues* not *be related to the question of the foundation of quantum mechanics?* There are more than just instinctive reasons to ask this question. As soon as one allows space and time to be curved, one has to confront the possibility that they form a closed, finite universe. Of course, quantum cosmology must be a legitimate domain of quantum gravity. But the formulation of the quantum axioms for closed universes leads to new difficulties. One of these is the fact that there is no external time coordinate, which means that one will not have transition amplitudes or S-matrices. One then encounters the difficulty of interpretation: if the universe is finite, one cannot repeat an experiment infinitely many times at far separated places, so, if a quantum calculation leads to the evaluation of a "probability", how then can one verify this? In this universe, something happens or it does not, but probabilistic predictions then amount to imprecise predictions. Must we accept an imprecise theory? This difficulty shows up quite sharply in simple "model universes", such as the one formed by gravitating particles in 2 space-, 1 time dimension. This is a beautiful model with only a finite number of physical degrees of freedom[10], so quantization should be straightforward; unfortunately, it is not, and the fore-mentioned difficulties are the reason.

Should we return to the old attempts at constructing "hidden variable theories" for quantum mechanics?[11] Usually, such endeavor is greeted with skepticism, for very good reasons. Under quite general assumptions, it has been demonstrated that: "hidden variables cannot be reconciled with locality and causality".

This would indeed be a good reason to abandon such attempts. But, how general is this result? In Ref. [12], some very simple models are constructed that could be viewed as counter examples of the general theorem. We hasten to add that these model are not at all free from problems. One might suspect, however, that the well-known no-go theorems for hidden variables do rely on some assumptions, which seem to be so natural that one tends to forget about them. Here, we list some of the small-print that may have gone into the derivation of the argument:

- It was assumed that an observer at all times is free to choose from a set of non-commuting operators, which of these (s)he wishes to measure.
- Rotations and other continuous symmetry operations can be performed locally, without disturbing any of the quantum states elsewhere.
- The vacuum is a single, unique state.

Assumptions of this kind may actually not be valid at the Planck scale. Indeed, in Ref. [12] it is assumed that only one class of operators can truly be observed at the Planck scale, and they all commute. They were called 'beables' there.

The most important problem of the ones just alluded to is that deterministic evolution seems to be difficult to reconcile with a Hamiltonian that is *bounded from below*. It is absolutely essential for Quantum mechanics to have a lowest energy state, *i.e.*, a vacuum state. Now the most likely way this problem can perhaps be addressed is to assume not only deterministic evolution, but also *local information loss*. As stated, information loss is difficult to avoid in black holes, in particular when they are classical. it now seems that this indeed may turn up to be an essential ingredient for understanding the quantum nature of this world.

Simple examples of universes with information loss can be modeled on a computer as cellular automata[13]. An example is 'Conway's game of life'[14].

Information loss may indeed already play a role in the Standard Model! Here, *local gauge degrees of freedom* are pieces of information that do play a role in formulating the dynamical rules, but they are physically unobservable. An unorthodox interpretation of this situation is that these degrees of freedom are unobservable *because* their information contents get lost, much like information drained by a black hole.

We stated earlier that the fields between the horizon and the brick wall could be local gauge degrees of freedom. Now we can add to that that probably they represent lost information.

8. Freezing the horizon

String theory has produced some intriguing insights in the nature of the black hole microstates. Unfortunately, the results reported apply to exotic versions of black holes, and the ordinary Schwarzschild black hole is conspicuously absent. Yet it is the Schwarzschild black hole that we hold here as the prototype. What went wrong?

The black holes handled in string theory are all *extreme black holes* or close-to-extreme black holes. What is an extreme black hole?

The prototype of that can be obtained from the Reissner-Nordstrøm black hole, a black hole with a residual electric charge. Due to the stress-energy tensor of the electric field, the metric is modified into

$$\mathrm{d}s^2 = -\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-\frac{2M}{r}+\frac{Q^2}{r^r}} + r^2\mathrm{d}\Omega^2 \ , \tag{8.1}$$

where M is the mass, as before, and now Q is the electric charge (in Planck units). As long as $Q < M$, the quantity $1-\frac{2M}{r}+\frac{Q^2}{r^2}$ has two zeros,

$$r_\pm = M \pm \sqrt{M^2-Q^2} \ . \tag{8.2}$$

The largest of these is the location of the horizon. The smaller value represents a second horizon, which is hidden behind the first. The physical interpretation of these horizons is nicely exhibited in Ref. [15], but does not concern us here very much. The *extreme case* is when Q approaches M. Then, the quadratic expression becomes $(1-\frac{M}{r})^2$, and it has two coinciding horizons at $r = r_+ = r_- = M$. It is this kind of horizon that can be addressed by string theories.

Actually, the situation is a little bit deceptive. One could argue that, in the extreme limit (which can probably be approached physically, but never be reached exactly), the two horizons do not coincide at all. This is because, if we follow a $t =$constant path from r_+ to r_-, the metric distance between the horizons becomes

$$\int_{r_-}^{r^+} \sqrt{-g_{11}}\mathrm{d}r \to M\pi \ , \tag{8.3}$$

and this does not tend to zero in the limit (and it is a time-like distance, not space-like). Moreover, the distance between the r_+ horizon and any point in the regular region I of the surrounding universe (say the point $r = 2M$), tends to infinity:

$$\int_{r_+}^{2M} \sqrt{g_{11}} \to \infty \ . \tag{8.4}$$

In the extreme limit, the horizon is also infinitely red-shifted, the gravitational field κ there tends to zero, and so does the Hawking temperature. In all respects, the extreme horizon is a *frozen horizon.* Its surface area is still $4\pi r_-^2 = 4\pi M^2 \neq 0$. Accordingly, the entropy $S = \pi M^2 \neq 0$. However, sometimes it is argued that extreme black holes should have vanishing entropy. What happened?

The entropy for the Reissner-Nordstrøm black hole is

$$S = \pi r_+^2 = \pi\Big(M + \sqrt{M^2 + Q^2}\Big)^2\ . \tag{8.5}$$

Inverting this gives the mass-energy M as a function of Q and S:

$$M = \frac{Q}{2}\left(\sqrt{\frac{\pi Q}{S}} + \sqrt{\frac{S}{\pi Q}}\right) \quad , \quad T = \frac{\partial M}{\partial S}\bigg|_Q\ . \tag{8.6}$$

This curve, at a fixed value for Q, is sketched in Fig. 3 It begins at $S = \pi Q$ since, as we see from (8.5), $S \geq \pi Q$. At the extreme point, the temperature T is zero.

Our physical intuition, however, tells us that perhaps states with more order in them also exist, so that one can indeed lower S. The temperature will not become less than zero, so one simply expects a straight horizontal line from $S = 0$ to $S = \pi Q$ (dotted line in Fig. 3). One might suspect that, in a superior quantum theory, tiny deviations from the straight line may occur.

Now what happens if we take one of these ordered states, with $S \ll \pi Q$ (lowest cross in Fig. 3), and cause a minor disturbance, for instance by throwing in a light neutrino? The energy rises slightly (cross top left), and the hole will no longer be extreme . However, the correct solution is now the position on the curve at the right. Complete disorder must take place (arrow). Apparently, the slight perturbation from the neutrino now caused complete disorder. This can be understood in simple models. Since the horizon is no longer extreme, it is also no longer frozen. Dynamical evolution sets in, and this causes disorder. The situation can again be modeled in simple cellular automata.

9. Conclusion

With some physical intuition, one can view the horizon of a black hole as an intriguing physical object. Its microstates as yet cannot be linked to local properties of the vacuum configuration out of which the horizon is transformed, but string theory has made progress in picturing frozen or slowly evolving horizons. In principle, what has been discussed here should also apply to horizons in different settings, such as cosmological horizons. Considerable caution is then asked for, however, since quantum mechanics might not apply to an entire cosmos.

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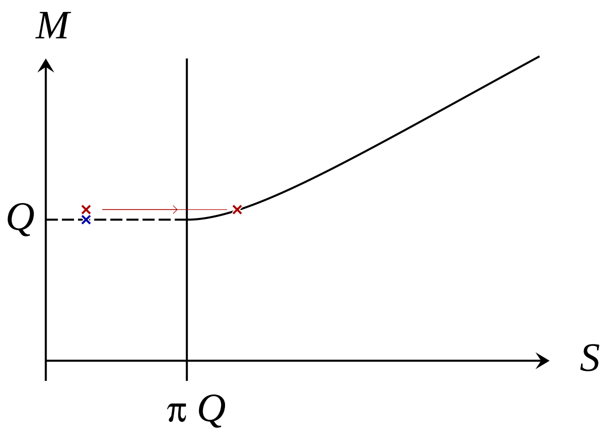

Figure 3: The energy plotted against entropy at fixed Q. The dotted horizontal line does not follow from Eq. (8.6), but from physical considerations.

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