GRAVITATIONAL COLLAPSE AND PARTICLE PHYSICS

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ABSTRACT

1. Introduction

In a world described without quantum mechanics gravitational collapse can occur whenever a too large amount of matter is accumulated in a too small volume. The laws of general relativity tell us that this is inevitable regardless the equation of state for the material considered. Astrophysicists[1)] know that the threshold for spontaneous gravitational collapse is at a few solar masses when this material is sufficiently cooled so that the occupied volume is reduced. Quantum mechanical effects may then safely be ignored because the mass scales and distance scales are large.

In principle however, one could consider smaller systems. Quantum field theories do not seem to deny the possibility of having a certain amount of very heavy particles packed together in a volume as small as

we please. Just because the possibility is not denied, we should realize that apparently quantum field theories have "channels" that allow (sub-) systems to undergo gravitational collapse. As soon as the masses ans sizes involved are so small that quantum mechanical effects may not be ignored we must conclude that the collapsed state should represent a new set of vectors in Hilbert space, to be taken into account for instance in considerations of unitarity and the ensuing dispersion relations. What would the Schrödinger equation then be like?

At first sight one might expect that existing approximative theories of quantum gravity, regularized for instance by using terms that are quadratic in the Riemann curvature, should answer the question. In any case, for systems in which the mass- and distance scales are macroscopic these theories should at least in principle give straightforward answers. And for systems at the Planck length scale, superstring theories, claimed to be finite, could be expected to be powerful enough to allow us to calculate what happens.

Quite alarmingly, this turns out not to be the case. There are fundamental problems, even for large collapsing systems. A key ingredient in ordinary quantum field theories, but also in general relativity, is locality: of any pair of points in space-time, regardless how close together, one can tell which is earlier and which is later. But now we run into a problem separating two closeby points: if a beam of heavy particles would be used then their power of resolution will always be limited, if not by the wavelengths, then by the finiteness of the Schwarzschild radii.

And in string theories one nowadays assumes the string constant T

to be of the order of the Planck length. The mass-energy of a string with total length L is then of the order of

$$M \gtrsim TL \quad ; \quad T = \mathcal{O}(1) \ , \tag{1.1}$$

so that the Schwarzschild radius of a string,

$$R = 2M \tag{1.2}$$

is of the same order as the length L. In other words, strings should collapse easily, a fact entirely ignored in existing interpretations of the string concept.

The most logical way out of this dilemma would be to identify strings with black holes. Only then could the inevitable existence of collapsed matter configurations be reconciled with the hypothesis of highly excited string modes. Such an idea is not as absurd as it looks. The dynamical properties of the horizon of a black hole are very reminiscent of strings. In this lecture I will give our preliminary findings pointing towards a unified theory. The promises of such a theory are great: the highly excited black hole/string modes may one day be found using low energy field theory only.

2. The black hole

We write

$$M = Gm \ , \tag{2.1}$$

and the black hole metric as[2)]

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2 d\Omega^2 \ . \tag{2.2}$$

The Kruskal coordinates x and y, defined by

$$xy = \left(1 - \frac{r}{2M}\right)e^{r/2M} \ ,$$

$$x/y = -\, e^{t/2M} \tag{2.3}$$

can replace r and t, after which we can extend analytically [3] to the entire space $(x,y \mid xy < 1)$.

This metric may arise naturally when a large amount of matter is accumulated. The physical interpretation of the various regions in x-y space has been discussed at various places[3,4)].

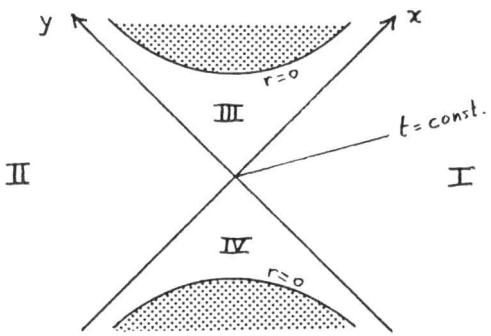

Fig. 1. Black hole in Kruskal coordinates.

Of particular interest is the region close to the origin. Writing

$$\zeta^2 = \left(\frac{r}{2M} - 1\right)e^{r/2M} ,$$

$$\tau = t/4M , \tag{2.4}$$

we see that

$$\tfrac{1}{2}(x+y) = \zeta \sinh \tau ,$$

$$\tfrac{1}{2}(x-y) = \zeta \cosh \tau . \tag{2.5}$$

At the origin the metric is regular:

$$ds^2 \to -\frac{16M^2}{e}\, dxdy + 4M^2 d\Omega^2 . \tag{2.6}$$

Since we shall mainly concentrate on the origin of xy space we could just as well replace (2.6) by a flat Minkowski metric. Consider a flat Minkowski space with coordinates x_{tr},z,t, and

$$ds^2 = dx_{tr}^2 + dz^2 - dt^2 \tag{2.7}$$

(where x_{tr} is a two component vector replacing the angles θ and φ, and where t is not to be confused with the Schwarzschild time t in (2.2)). The Rindler coordinates ζ and τ are now defined by[5)]

$$z = \zeta \cosh \tau$$

$$t = \zeta \sinh \tau \ . \tag{2.8}$$

An observer for whom τ acts as a time coordinate feels a strong gravitational field, singular at $\zeta = 0$. Since his time-translations correspond to Lorentz transformations in the original Minkowski space, the observer in this new world, called Rindler space[5], experiences laws of nature that are constant in time.

Let the Minkowski Hamiltonian be the integral over a Hamilton density:

$$H_M = \int H(\vec{x}) d^{3}\vec{x} \ . \tag{2.9}$$

Of course, (2.9) is the generator of time-translations. The generator of Lorentz transformations in Minkowski space is the Rindler Hamiltonian:

$$H_R = \int d^{3}\vec{x} \left(H(\vec{x}) z - P_3(\vec{x}) t \right) , \tag{2.10}$$

where $P_3(\vec{x})$ is the momentum density. We can split H_R into two parts (taking $t = 0$):

$$H_1 = \int_{z>0} H(\vec{x}) z \, d^{3}\vec{x} \ ; \tag{2.11}$$

$$H_2 = \int_{z<o} H(\vec{x})\ |z|\ d^3\vec{x}\ ; \tag{2.12}$$

$$H_R = H_1 - H_2\ . \tag{2.13}$$

In most field theories one finds easily:

$$[H_1, H_2] = 0\ . \tag{2.14}$$

The physical interpretation of (2.14) is that no signals can be transmitted between the regions I ($z > |t|$) and II ($z < -|t|$).

Suppose we had a scalar field φ in Minkowski space satisfying the Euler-Lagrange equations generated by the Lagrangean

$$\mathcal{L} = -\tfrac{1}{2}(\partial_z\varphi)^2 + \tfrac{1}{2}(\partial_t\varphi)^2 - \tfrac{1}{2}(\partial_{tr}\varphi)^2 - \frac{m^2}{2}\varphi^2 \tag{2.15}$$

If we write

$$\zeta = e^{\sigma} \qquad (-\infty < \sigma < \infty) \tag{2.16}$$

then

$$\mathcal{L}\, dz\, dx_{tr}\, dt = \mathcal{l}\, d\sigma\, dx_{tr}\, d\tau\ , \tag{2.17}$$

with

$$\mathcal{l} = -\tfrac{1}{2}(\partial_\sigma\varphi)^2 + \tfrac{1}{2}(\partial_\tau\varphi)^2 + e^{2\sigma}\left(-\tfrac{1}{2}(\partial_{tr}\varphi)^2 - \frac{m^2}{2}\varphi^2\right)\ . \tag{2.18}$$

Close to the horizon ($\sigma \to -\infty$), this Lagrangean generates plane waves:

$$(\partial_\tau^2 - \partial_\sigma^2)\varphi \to 0 \ . \tag{2.19}$$

The left-going waves will never reach the horizon itself, whereas the right-going ones are infinitely old. Therefore at first sight no boundary condition at $\sigma = -\infty$ (or $\zeta = 0$) seems to be necessary. Nevertheless, we expect "Hawking radiation". This is a thermal flux of particles arriving from $\sigma = -\infty$. This follows if one makes one extra assumption: total energy in the original Minkowski space must be finite. Since this inevitably involves linear combinations of different eigenstates of H_1 and H_2, one ends up with a quantummechanically mixed state in terms of H_1 alone, hence the thermal nature of the radiation .

Any "eigenstate" $|E_1,E_2\rangle$ with

$$H_1|E_1,E_2\rangle = E_1|E_1,E_2\rangle \ ; \qquad H_2|E_1,E_2\rangle = E_2|E_1,E_2\rangle \ , \tag{2.20}$$

being different from the Minkowski vacuum yet Lorentz-invariant, must have infinite energy in Minkowski space.

The infinite spectrum of equation (2.19) in the space (2.16), not bounded for small σ values, causes a nasty divergence at the horizon. Also, in Rindler space, but also for finite size black holes, no correlation whatsoever is expected between ingoing and outgoing particles. In a free theory all these states are orthogonal to each

other, and nothing can affect the truly stochastic nature of the thermal radiation.

As stated in the introduction, this situation seems to be unacceptable if we were to consider black holes as just some sort of elementary particles. What one expects there is a scattering matrix S:

$$|\psi_{out}\rangle = S|\psi_{in}\rangle \ . \tag{2.21}$$

The reason why (2.21) may perhaps not be incompatible with Rindler space dynamics is that the assumption of having only free fields in Minkowski space is clearly wrong. At sufficiently far negative values of the coordinate σ, sooner or later, gravitational forces (which are extremely non-linear), will dominate. If we would build such a combination of states (2.20) that the total energy in Minkowski space would become large, then gravitational interactions will cause severe curvature of space-time. Could the particles that went in, not be knocked out again by this curved space-time? Perhaps values of ζ smaller than the Planck length are forbidden. Are particles bounced back somehow?

3. The shifting horizon

Shifts in the parameter τ of transformation (2.8) correspond to Lorentz transformations in Minkowski space. Even the lightest particles become exceedingly energetic after a certain amount of "time" τ. It is therefore particularly important to study the gravitational effect of such energetic particles on space-time[6,7].

The metric of a very light particle at rest can be approximated

by

$$g_{\mu\nu} \simeq \delta_{\mu\nu}\left(1 + \frac{2m_o}{r}\right) + \frac{4m_o}{r}\, u_\mu u_\nu \ , \tag{3.1}$$

with

$$r^2 = x^2 + (x.u)^2 \ , \tag{3.2}$$

$$u^2 = -\ 1 \ . \tag{3.3}$$

u_μ is the 4-velocity of the particle. Since (3.1) - (3.3) have been written in a Lorentz-invariant way, they remain true also when the particle is boosted to tremendous energies. We let $m_o \to 0$, $u_\mu \to \infty$, with

$$p_\mu = m_o u_\mu \simeq (0,0,p,ip) \tag{3.4}$$

fixed. If $(x.u) \neq 0$ we have

$$r \to |x.u| = |x.p|/m_o \ , \tag{3.5}$$

so in most of space-time, r is large and indeed $|m_o/r| \ll 1$. Now consider the new coordinates $y^\mu_\pm$ with

$$y^\mu_\pm = x^\mu \pm 2m_o u^\mu \log r \ . \tag{3.6}$$

At

$$(x.u) > 0 : \quad ds^2 \to dy_+^2 , \tag{3.7}$$

And at

$$(x.u) < 0: \quad ds^2 \to dy_-^2 . \tag{3.8}$$

So space-time is flat at points x_μ with $(x.p) \neq 0$. At $(x.p) = 0$ we must make the transition from the y_+ to the y_- coordinates: At $(x.p) = 0$:

$$y_+^\mu = y_-^\mu + 2p^\mu \log y_{tr}^2 . \tag{3.9}$$

The second derivative of this shift with respect to y_{tr} produces real curvature across the seam. Notice that the shift, Δy^μ, obeys the two-dimensional Laplace equation

$$\partial_{tr}^2(\Delta y^\mu) = 8\pi\ p^\mu\ \delta^2(y_{tr}) . \tag{3.10}$$

Thus, spacetime can still be represented by flat coordinates y^μ, provided that we draw the geodesics that cross the plane $(y.p) = 0$ in a special way (Fig. 2)

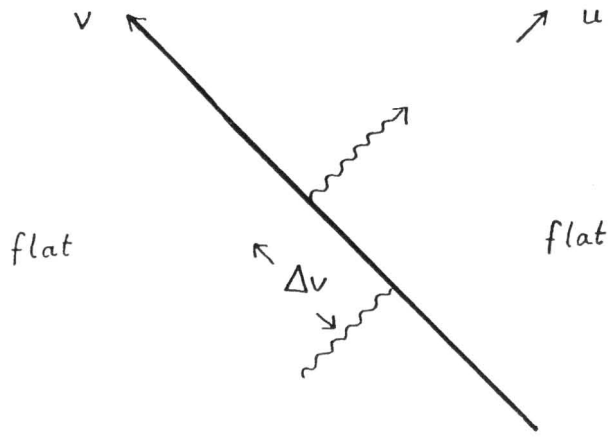

Fig. 2. A fast particle moving in the plane u = 0 causes a drag Δv in all geodesics that cross the plane u = 0, whose value depends on the transverse distance y_{tr} from the particle according to eq. (4.10).

In a finite-size black hole a particle approaching the horizon causes a similar drag. The laplacian ∂^2_{tr} is then replaced by an angular laplacian plus a constant[7], see Fig. 3.

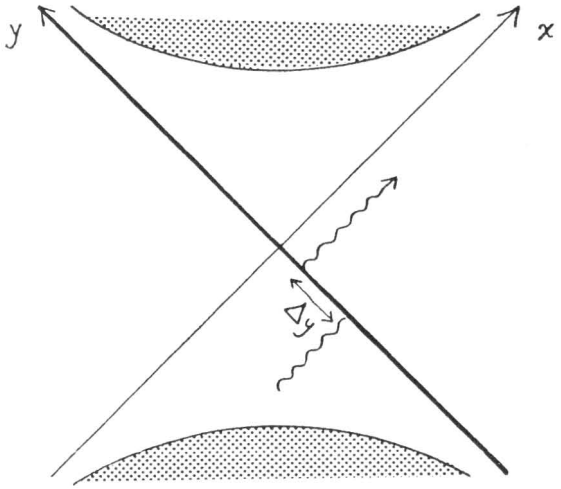

Fig. 3. A particle that moved into a black hole a long time ago causes a shift Δy in the trajectories of outgoing particles.

If we have very fast particles both on the u- and on the v-axis, complications arise. Clearly the two sets of particles disturb each other's motion, and the surrounding space-time will no longer be flat. As a first approximation however one may assume that the shifts due to the u-particles and the v-particles can be added linearly.

4.A link with string theory

Consider now a black hole that has been formed in the distant past. A freely falling observer detects very few particles. It seems to be very well possible to consider the analytic extension of his space-time to the entire Kruskal frame ($xy < 1$). Of course we realize that this analytic extension went beyond the line $t = -\infty$, which is before the black hole was actually made. In reality a large amount of collapsing matter had accumulated on that line and the analytic extension the mathematical model one gets was invalid. Nevertheless the mathematical model one gets is important. For example, if an outgoing particle is actually detected one enters into an element of Hilbert space which contains the past singularity of Kruskal space, a feature further discussed in ref. 8.

The future horizon is defined to be the boundary of that region of space-time from which escape out of the black hole is still possible. The past horizon is the boundary of the region of Kruskal space that can be reached from outside the black hole starting at $t = -\infty$. The analytic extension discussed above is realistic exactly up to the past horizon. The space-like horizon is defined to be the intersection of future and past horizon.

Suppose we take that part of the world that is accessible to us and add to it some ingoing particles with momenta p_{in}^{-}, and some outgoing particles with momenta p_{out}^{+}. How do these particles affect the shape of the space-like horizon?

As long as p_{in}^{-} and p_{out}^{+} are not too large we may assume their effects to be additive (as will turn out later, the products of the interesting values for p_{in}^{-} and p_{out}^{+} will be negligibly small). From

the previous section, eq. (4.10), we deduce for the coordinates $x^{\pm}(\theta,\varphi)$ of the space-like horizon:

$$\partial^2_{tr} x^- = 8\pi G p^-_{in}\, \delta^2(\Omega-\Omega_{in}) \; ; \tag{4.1}$$

$$\partial^2_{tr} x^+ = -\, 8\pi G p^+_{out}\, \delta^2(\Omega-\Omega_{out}) \; , \tag{4.2}$$

where Ω_{in}, Ω_{out} are the angles (θ,φ) at which the particles crossed the horizon. ∂^2_{tr} is the laplacian with respect to θ and φ. The constant term is neglected.

Writing the 4-vector

$$p^{\mu}_{ex} = \left(p^-_{in}, p^+_{out}\right) \tag{(4.3}$$

and replacing θ and φ by any set σ,τ via conformal transformations, we see that

$$\partial^2_{tr} x^{\mu} = 8\pi G \rho^{\mu}_{ex} \tag{4.4}$$

where ρ^{μ}_{ex} is the distribution of external momenta on the θ,φ (or $\sigma\tau$) plane.

Now in the presence of external sources with momenta

$$p^{\mu} = \int \rho^{\mu}\, d\sigma d\tau \; , \tag{4.5}$$

the coordinates $x^{\mu}(\sigma,\tau)$ of a string satisfy the classical equation

$$(\partial_\sigma^2+\partial_\tau^2)x^\mu = -i\rho^\mu/T \qquad (4.6)$$

where T is the string constant. We notice the similarity between (4.4) and (4.6). Could it describe the same physical world? The factor -i at first sight seems to be very troublesome, but may find a quite natural explanation: a factor -1 may be needed if, instead of the coordinates $x^\mu(\sigma,\tau)$ we consider the image of an ideal surface as seen through the "gravitataional lens" caused by the in- and outgoing particles, and the factor i may be seen as resulting from transformations of the sort

$$x^\pm \to \pm i\, x^\pm \ , \qquad (4.7)$$

which are Lorentz transformations with complex arguments. Also one must bear in mind that string theory is usually defined in a $\sigma\tau$ space with metric (+,-), in order to obtain timelike surfaces.

A complete formalism that explains string theory from black hole dynamics is still lacking. The above however suggests that these concepts are closely related. If indeed the factor $\pm i$ can be accounted for we expect the string constant T to be

$$T = 1/8\pi G \ , \qquad (4.8)$$

simply by comparing (4.4) with (4.6).

5. Conclusion

Even without understanding all technical details of the suggested link between string theory and black hole dynamics, we can consider its topological aspects. A conventional particle being a point in 3-space traverses a trajectory in 4-space. A virtual particle, giving rise to an instantaneous interaction between two points (such as a photon producing the Coulomb force), is a one-dimensional subspace of 3-space, but instantaneous in time (Fig. 4).

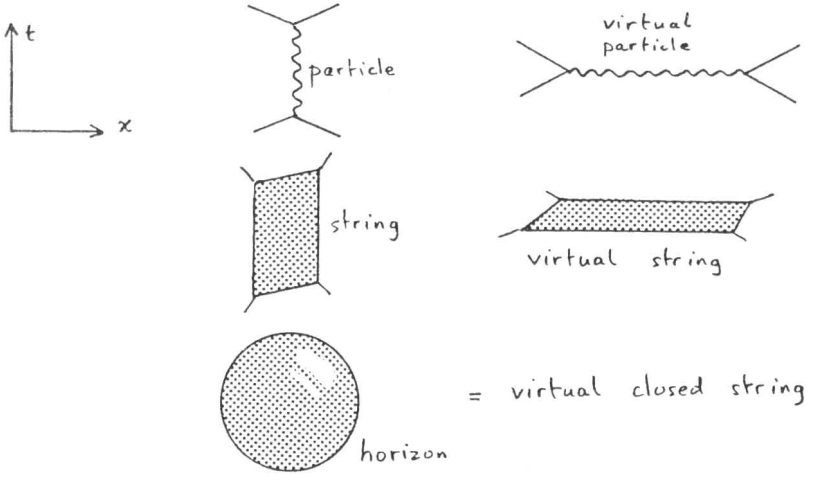

Fig. 4. A virtual string is sheet-like in 3-space and instantaneous in time. The space-like horizon can be a virtual closed string.

A virtual string can therefore be a sheet in 3-space. This is why the space-like horizon, being an S_2 sphere, has the same topology as a virtual closed string.

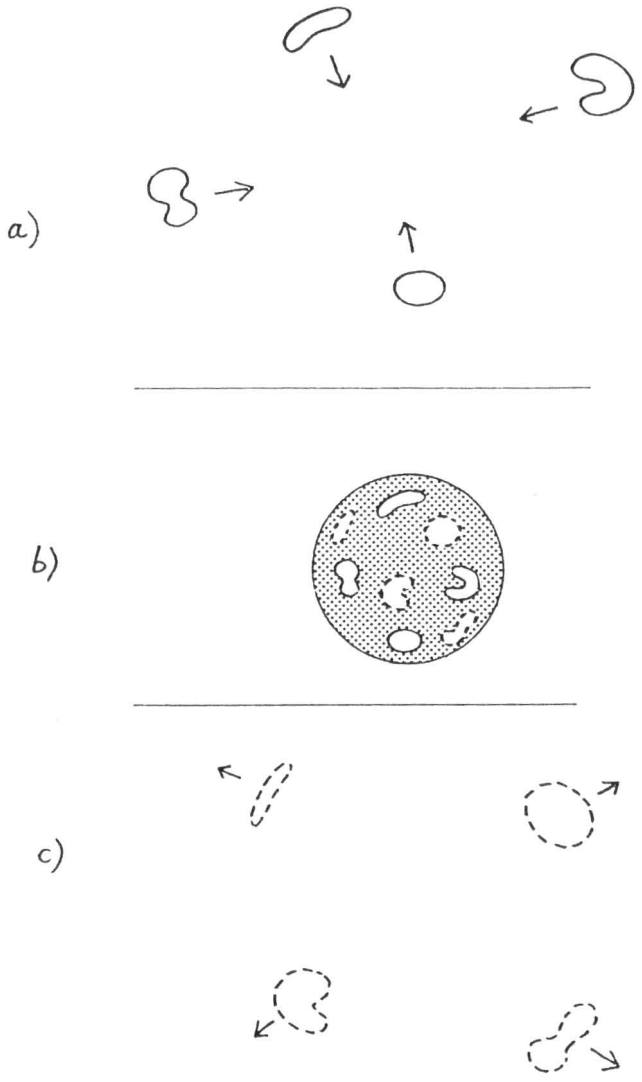

Fig. 5. Imploding particles (a), arrive at the horizon (b) and produce Hawking radiation (c).

In Fig. 5 we show how a black hole can perhaps be seen as a virtual closed string. Particles in the form of little closed strings come

together, exchange a virtual string that leaves holes; these holes connect to the outgoing Hawking particles, again closed strings. Our aim was to bring black hole physics and string theory together. What we see is that in some sense the functional integral for a string may actually describe the oscillations of a black hole horizon.

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