

DETERMINISM BENEATH QUANTUM MECHANICS¹.

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Abstract

Contrary to common belief, it is not difficult to construct deterministic models where stochastic behavior is correctly described by quantum mechanical amplitudes, in precise accordance with the Copenhagen-Bohr-Bohm doctrine. What is difficult however is to obtain a Hamiltonian that is bounded from below, and whose ground state is a vacuum that exhibits complicated vacuum fluctuations, as in the real world.

Beneath Quantum Mechanics, there may be a deterministic theory with (local) information loss. This may lead to a sufficiently complex vacuum state, and to an apparent non-locality in the relation between the deterministic (“ontological”) states and the quantum states, of the kind needed to explain away the Bell inequalities.

Theories of this kind would not only be appealing from a philosophical point of view, but may also be essential for understanding causality at Planckian distance scales.

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1. Motivation

The need for an improved understanding of what Quantum Mechanics really is, needs hardly be explained in this meeting. My primary concern is that Quantum Mechanics, in its present state, appears to be mysterious. It should always be the scientists' aim to take away the mystery of things. It is my suspicion that there should exist a quite logical explanation for the fact that we need to describe probabilities in this world quantum mechanically. This explanation presumably can be found in the fabric of the Laws of Physics at the Planck scale.

However, if our only problem with Quantum Mechanics were our desire to demystify it, then one could bring forward that, as it stands, Quantum Mechanics works impeccably. It predicts the outcome of any conceivable experiment, apart from some random ingredient. This randomness is perfect. There never has been any indication that there would be any way to predict where in its quantum probability curve an event will actually be detected. Why not be at peace with this situation?

One answer to this is Quantum Gravity. Attempts to reconcile General Relativity with Quantum Mechanics lead to a jungle of complexity that is difficult or impossible to interpret physically. In a combined theory, we no longer see “states” that evolve with “time”, we do not know how to identify the vacuum state, and so on. What we need instead is a unique theory that not only accounts for Quantum Mechanics together with General Relativity, but also explains for us how matter behaves. We should find indications pointing towards the correct unifying theory underlying the Standard Model, towards explanations of the presumed occurrence of supersymmetry, as well as the mechanism(s) that break it. We suspect that deeper insights in what and why Quantum Mechanics is, should help us further to understand these issues.

Related to the question of quantizing gravity is the problem of quantizing cosmology. Astrophysicists tell us that the Universe started with a “big bang”, but, at least at first sight, such a statement appears to be at odds with the notions of quantum mechanical uncertainty. In principle, we could know the state the Universe is in presently, and then one could solve the Schrödinger equation backwards in time, but this should yield a quantum superposition of many configurations, not just a Big Bang. Questions of this sort may seem of purely academic nature, but they become very concrete as soon as one attempts to construct some reasonable model for a “Quantum Universe”. The notion of a quantum state of the Universe appears to defy logic.

Attempts nevertheless to reconcile Quantum Mechanics with Cosmology were made. Whether the proposed schemes may be viewed as a satisfactory picture of our world, is difficult to discuss. To convince someone that they are flawed may be as difficult as changing someone's religious beliefs. Therefore, I shall refrain from trying to do this; instead, one further issue is as displayed in the next section.

2. Holography

Black holes are not only legitimate solutions of Einstein's field equations for the gravitational force, one can also show quite easily that black holes *inevitably* form under given favorable initial conditions of conventional matter configurations. Such 'conventional' black holes are very big, having a radius at least of the order of 10 km. Therefore, they are usually considered as classical, *i.e.* non-quantum mechanical, objects. But, at least in principle, they should also obey the laws of Quantum Mechanics. Elementary particles in the vicinity of a black hole should be described by Quantum Field theory, and the laws of General Relativity should dictate how to handle Quantum Field theory here. As was shown by S. Hawking, this exercise leads to the astonishing result that particles must emerge from a black hole.[1]

Mathematically, the explanation for this effect is that *time* is measured by freely falling observers in a coordinate frame that is fundamentally different from the coordinate frame used by the onlooking observer outside the black hole. Physically, one may explain the emission as a gravitational tunneling effect, comparable to the pair creation of charged particles in the presence of a strong electric field. The emission rate is precisely computable, and conventional theory gives a flux of particles corresponding to a temperature

$$T_{\text{H}} = \frac{\hbar c^3}{8\pi k G M_{\text{BH}}} , \quad (2.1)$$

where k is Boltzmann's constant, and M_{BH} is the mass of the black hole.

Hawking's result can be used to estimate the *density of quantum states* of a black hole. Assuming a transition amplitude \mathcal{T}_{in} for the absorption process, we can write in two ways an estimate for the absorption cross section $\sigma(\vec{k})$ for an amount of matter δE with momentum \vec{k} by a black hole of mass M_{BH} :

$$\sigma \approx 2\pi r_+^2 = 8\pi M_{\text{BH}}^2 ; \quad (2.2)$$

$$\sigma = |\mathcal{T}_{\text{in}}|^2 \varrho(M_{\text{BH}} + \delta E)/v . \quad (2.3)$$

Here, r_+ is the radius of the outer event horizon, $\varrho(M)$ is the density of states of a black hole with mass M , and v is the velocity of the absorbed particle. The probability $W dt$ of a particle emission during a time interval dt can also be written in two ways:

$$W dt = |\mathcal{T}_{\text{out}}|^2 \varrho(M_{\text{BH}}) dt/V ; \quad (2.4)$$

$$W dt = \frac{\sigma(\vec{k})v}{V} e^{-\delta E/kT_{\text{H}}} dt . \quad (2.5)$$

Here, V is the volume of a box, in which the wave function of the emitted particle is normalized. Dividing Eqs. (2.2) — (2.5), we get, in Planck units,[2]

$$\frac{\varrho(M + \delta E)}{\varrho(M)} = \frac{|\mathcal{T}_{\text{out}}|^2}{|\mathcal{T}_{\text{in}}|^2} e^{\delta E/kT_{\text{H}}} = e^{8\pi M \delta E} . \quad (2.6)$$

We assumed here that $|\mathcal{T}_{\text{out}}| = |\mathcal{T}_{\text{in}}|$. All that is needed for this assumption is *PCT* invariance, since $\sigma(\vec{k})$ is symmetric under *P* and *C*. For all known field theories, *PCT*

is a perfect symmetry. Needless to say, we do not know this to be so for quantum gravity, but it would be a natural assumption.

Eq. (2.6) is to be seen as a differential equation that is easily integrated, to give:

$$\rho(M) = e^{4\pi M^2 + \mathcal{C}} = \mathcal{C}' 2^{A/A_0}, \quad (2.7)$$

where \mathcal{C} and \mathcal{C}' are integration constants, $A = 4\pi r_+^2$ is the black hole area, and $A_0 = 4 \ln 2$ in Planck units. One concludes that the density of quantum states of a black hole is that of an object with A/A_0 free Boolean parameters on its surface. The integration constant represents a fixed degree of freedom that all black holes have in common. The result (2.7) can also be derived using thermodynamics, but then one has to cope with the difficulty that black holes, embedded in a thermal environment, are unstable because of their negative specific heat (they cool off when energy is added to them).[3]

In one respect, this result appears to be quite interesting and acceptable. Apparently, the quantum states of a black hole form a discrete set, just as if the black hole were a fairly ordinary object, easily to become macroscopic, in the astronomical case. Black hole formation and evaporation can indeed be described in terms of quantum amplitudes, and if the black hole is very tiny, these amplitudes can be represented in Feynman diagrams.

On closer inspection, however, there are several problems with this result. We would have thought that a general coordinate transformation transforms *states* into *states*. An in-going observer describes what (s)he sees in terms of particle states superimposed on an approximately flat space-time environment, using regular coordinates. The outside observer uses the black hole coordinates featuring an horizon. The states observed by the outside observer are counted by discrete variables of one bit for every area unit A_0 . How can this mapping of discrete states onto the continuum of states for the in-going observer be unitary? How does the in-going observer count his/her states? We should have expected the number of these states to scale with the bulk volume of his environment, not with the area.

Hawking's calculation gives no clues here. To the contrary, it appears to tell us that, even if the initial state of an imploding object would be a quantum mechanically pure state, the radiating black hole that emerges after some time would nevertheless be in a quantum mechanically mixed state.[4] Such a transition cannot be described by any Schrödinger equation. Does the black hole, viewed as an isolated object, disobey the quantum code? This is what was concluded initially, but most of us now agree that such a conclusion must be premature.[2]

If, however, on the other hand, information is conserved in unitary evolution equations, how is it that the information in the in-going particles is transmitted to the out-going ones?

A first clue towards answering this question was provided by taken into account the fact that ingoing particles interact with the outgoing ones when they pass each other. The gravitational interaction here diverges. Early ingoing particles meet late outgoing ones in an entirely different local Lorentz frame, so that the relative energy, that is, the energy in the center of mass frame, is large, and this number diverges exponentially with the

time difference of the two particles.[5] Taking this into account, one does find a unitary scattering matrix relating outgoing particles to the ingoing ones, but the spectrum of states does not seem to be bounded by the horizon area. Such a bound presumably has to come from the transverse components of the gravitational interactions, which is much harder to calculate.[6]

Requiring that the number of states in some region of space, described by a theory, is bounded by the surface area of this region, seems to be paradoxical. This paradox seems to be as deep and fundamental as the one that lead M. Planck to his postulates of Quantum Mechanics, or, in other words, we expect that its resolution requires a paradigm shift. How can we have locality in three-space, but numbers bound by two-space?

In certain versions of string theories, these apparently conflicting demands are met to some extent,[7] except that the concept of locality appears to be ignored. The amount of ‘magic’ required for these ideas to work is still not acceptable. It is this author’s belief that the true reason for the mysterious nature of this problem is our insistence to stick to the language of Quantum Mechanics. It seems to be only natural to see a link between the mysteries of string theory and those of the correct interpretation of Quantum Mechanics.

3. Harmonic oscillators.

It is instructive to ask how a deterministic system can be addressed using the mathematics of Quantum Mechanics. Our starting point is that we may have simple autonomous dynamical systems, where later we will decide how they should be coupled. Thus, we start with a deterministic system consisting of a set of N states,[8]

$$(0), (1), \dots, (N - 1)\}$$

on a circle. Time is discrete, the unit time steps having length τ (the continuum limit is left for later). The evolution law is:

$$t \rightarrow t + \tau \quad : \quad (\nu) \rightarrow (\nu + 1 \text{ mod } N). \quad (3.1)$$

Introducing a basis for a Hilbert space spanned by the states (ν) , the evolution operator can be written as

$$U(\Delta t = \tau) = e^{-iH\tau} = e^{-\frac{\pi i}{N}} \begin{pmatrix} 0 & & & & 1 \\ 1 & 0 & & & \\ & 1 & 0 & & \\ & & \ddots & \ddots & \\ & & & & 1 & 0 \end{pmatrix}. \quad (3.2)$$

The phase factor in front of the matrix is of little importance; it is there for future convenience. Its eigenstates are denoted as $|n\rangle$, $n = 0, \dots, N - 1$. They are found to be

$$|n\rangle = \frac{1}{\sqrt{N}} \sum_{\nu=1}^N e^{\frac{2\pi i n \nu}{N}} (\nu), \quad n = 0, \dots, N - 1. \quad (3.3)$$

This law can be represented by a Hamiltonian using the notation of quantum physics:

$$H|n\rangle = \frac{2\pi(n + \frac{1}{2})}{N\tau}|n\rangle. \quad (3.4)$$

The $\frac{1}{2}$ comes from the aforementioned phase factor. Next, we apply the algebra of the $SU(2)$ generators L_x , L_y and L_z , so we write

$$N \stackrel{\text{def}}{=} 2\ell + 1 \quad , \quad n \stackrel{\text{def}}{=} m + \ell \quad , \quad m = -\ell, \dots, \ell. \quad (3.5)$$

Using the quantum numbers m rather than n to denote the eigenstates, we have

$$H|m\rangle = \frac{2\pi(m + \ell + \frac{1}{2})}{(2\ell + 1)\tau}|m\rangle \quad \text{or} \quad H = \frac{2\pi}{(2\ell + 1)\tau} (L_z + \ell + \frac{1}{2}). \quad (3.6)$$

This Hamiltonian resembles the harmonic oscillator Hamiltonian when $\omega = 2\pi/(2\ell + 1)\tau$, except for the fact that there is an upper bound for the energy. This upper bound disappears in the continuum limit, if $\ell \rightarrow \infty$, $\tau \downarrow 0$. Using L_x and L_y , we can make the correspondence more explicit. Write

$$L_{\pm}|m\rangle \stackrel{\text{def}}{=} \sqrt{\ell(\ell + 1) - m(m \pm 1)} |m \pm 1\rangle; \quad (3.7)$$

$$L_{\pm} \stackrel{\text{def}}{=} L_x \pm iL_y \quad ; \quad [L_i, L_j] = i\varepsilon_{ijk}L_k, \quad (3.8)$$

and define

$$\hat{x} \stackrel{\text{def}}{=} \alpha L_x \quad , \quad \hat{p} \stackrel{\text{def}}{=} \beta L_y \quad ; \quad \alpha \stackrel{\text{def}}{=} \sqrt{\frac{\tau}{\pi}} \quad , \quad \beta \stackrel{\text{def}}{=} \frac{-2}{2\ell + 1} \sqrt{\frac{\pi}{\tau}}. \quad (3.9)$$

The commutation rules are

$$[\hat{x}, \hat{p}] = \alpha\beta iL_z = i(1 - \frac{\tau}{\pi}H), \quad (3.10)$$

and since

$$L_x^2 + L_y^2 + L_z^2 = \ell(\ell + 1), \quad (3.11)$$

we have

$$H = \frac{1}{2}\omega^2\hat{x}^2 + \frac{1}{2}\hat{p}^2 + \frac{\tau}{2\pi} \left(\frac{\omega^2}{4} + H^2 \right). \quad (3.12)$$

Now consider the continuum limit, $\tau \downarrow 0$, with $\omega = 2\pi/(2\ell + 1)\tau$ fixed, for those states for which the energy stays limited. We see that the commutation rule (3.10) for \hat{x} and \hat{p} becomes the conventional one, and the Hamiltonian becomes that of the conventional harmonic oscillator:

$$[\hat{x}, \hat{p}] \rightarrow i; \quad H \rightarrow \frac{1}{2}\omega^2\hat{x}^2 + \frac{1}{2}\hat{p}^2. \quad (3.13)$$

There are no other states than the legal ones, and their energies are bounded, as can be seen not only from (3.12) but rather from the original definition (3.6). Note that, in the continuum limit, both \hat{x} and \hat{p} become continuous operators, since both α and β tend to zero.

The way in which these operators act on the ‘primordial’ or ‘ontological’ states (ν) of Eq. (3.1) can be derived from (3.7) and (3.9), if we realize that the states $|m\rangle$ are just the discrete Fourier transforms of the states (ν), see Eq. (3.3). This way, also the relation between the eigenstates of \hat{x} and \hat{p} and the states (ν) can be determined. Only in a fairly crude way, \hat{x} and \hat{p} give information on where on the circle our ontological object is; both \hat{x} and \hat{p} narrow down the value of ν of our states (ν).

The most important conclusion from this section is that there is a close relationship between the quantum harmonic oscillator and the classical particle moving along a circle. The period of the oscillator is equal to the period of the trajectory along the circle. We started our considerations by having time discrete, and only a finite number of states. This is because the continuum limit is a rather delicate one. One cannot directly start with the continuum because then the Hamiltonian does not seem to be bounded from below.

The price we pay for a properly bounded Hamiltonian is the square root in Eq. (3.7); it may cause complications when we attempt to introduce interactions, a problem that is not yet properly worked out.

Starting from this description of harmonic oscillators in terms of deterministic models, one may attempt to construct deterministic theories describing, for instance, free bosonic particles, see Ref.[9]

Strings can also be seen as collections of harmonic oscillators. A first attempt to write string theory in deterministic terms failed because conformal invariance could not be built in.[11] Apparently, further new ideas are needed here.

4. Continuous degrees of freedom.

In the previous section, a discrete, periodic system was considered, and we took the continuum limit in the end. Could one not have started with a continuous model right from the beginning?

Take a Newtonian equation,

$$\frac{d}{dt}q^i(t) = f^i(\vec{q}). \quad (4.1)$$

We can write the *quantum* Hamiltonian,

$$H = \sum_i p_i f^i(\vec{q}), \quad p_i = \frac{\hbar}{i} \frac{\partial}{\partial q^i}. \quad (4.2)$$

This is quantum language for a classical, deterministic system. It works because the Hamiltonian is linear, not quadratic, in the momenta p_i . The difficulty linking this with

real Quantum Mechanics is that this Hamiltonian cannot possibly be bounded from below, so that there is no ground state.

5. Massless, noninteracting fermions.

Massless, non-interacting fermions are entirely deterministic. this can be demonstrated by identifying the ‘beables’ for this system. Beables are a complete set of observables $\mathcal{O}_i(t)$ that commute at all times:

$$[\mathcal{O}_i(t), \mathcal{O}_j(t')] = 0, \quad \forall t, t'. \quad (5.1)$$

First, consider only first-quantized, chiral fermions. They have a two-component complex wave function obeying the Hamilton equation for

$$H = \vec{\sigma} \cdot \vec{p}; \quad \sigma_i \sigma_j = \delta_{ij} \mathbb{1} + i \varepsilon_{ijk} \sigma_k, \quad (5.2)$$

σ_i being the Pauli matrices. Consider the set

$$\mathcal{O}_i(t) = \{\hat{p}, \hat{p} \cdot \vec{\sigma}, \hat{p} \cdot \vec{x}\}, \quad (5.3)$$

where

$$\hat{p}_i \equiv \pm \frac{p_i}{|p|}, \quad \hat{p}_x > 0. \quad (5.4)$$

These operators obey closed time evolution equations:

$$\frac{d}{dt} \vec{x} = \vec{\sigma}(t), \quad \frac{d}{dt} (\hat{p} \cdot \vec{x}(t)) = \hat{p} \cdot \vec{\sigma}, \quad \frac{d}{dt} (\hat{p} \cdot \vec{\sigma}) = 0. \quad (5.5)$$

$$\hat{p}(t) = \hat{p}(0), \quad \hat{p} \cdot \vec{\sigma}(t) = \hat{p} \cdot \vec{\sigma}(0), \quad \hat{p} \cdot \vec{x}(t) = \hat{p} \cdot \vec{x}(0) + \hat{p} \cdot \sigma(0)t. \quad (5.6)$$

The fact that all operators in Eq.(5.3) commute with one another is easy to establish, with the possible exception of $[\hat{p}_i, (\hat{p} \cdot \vec{x})]$. The fact that the latter vanishes is most easily established in momentum space, realizing that $\vec{p} \cdot \vec{x}$ is the dilatation operator, while \hat{p} keeps the same length 1 under dilatations:

$$[(\hat{p} \cdot \vec{x}), \hat{p}_i] = i \left(\hat{p} \cdot \frac{\partial}{\partial \vec{p}} \right) \hat{p}_i = 0. \quad (5.7)$$

The *physical* interpretation of this result is that the dynamical behaviour of a massless, chiral, non-interacting fermion is exactly like that of an *infinite, flat, oriented sheet*, moving with the speed of light in a direction orthogonal to the sheet. $\pm \hat{p}$ gives the direction of the sheet, $\hat{p} \cdot \vec{\sigma}$ gives the sign of its orientation and $\hat{p} \cdot \vec{x}$ its distance from the origin.

As before, we encounter the difficulty that, in this deterministic system, the Hamiltonian is not bounded below, again because it is linear in the momenta p_i . Thus, there exists no ground state. In this case, however, P.A.M. Dirac told us what to do: *second*

quantization. Assume an indefinite number of particles with hamiltonian (5.2). Consider the range of energies they can have. Take the state where all negative energy states are occupied by a particle, all positive energy states are empty. That is the state with lowest possible total energy, the vacuum state. It is standard procedure, but it does require our particles to obey Fermi statistics, or, in other words, no two particles are allowed in the same state, and interchanging two particles does not change a state into a different one.

The latter condition is easily met, but to forbid two particles to be in the same state requires some sort of repulsion. The easiest procedure is to have at each value for the unit orientation vector \hat{p} a grid with some finite spacing a . No two sheets are allowed on the same lattice point. Then we can count the states exactly as in a fermion theory and the second quantization procedure works. The limit $a \downarrow 0$ can be taken without any difficulty.

6. Locality.

Let us focus a bit more on the ontological states for massless fermions. They are characterized by an orientation \hat{k} (obeying $|\hat{k}| = 1$), and a distance scalar z . Furthermore, we need the quantum operator conjugated to z , which we call $q \equiv -i\partial/\partial z$. We define η_3 to be the sign of q , and \hat{k} is the orientation of the sheet with its sign chosen such that it moves in the positive \hat{k} direction:

$$(\hat{p} \cdot \vec{\sigma})\hat{p} \equiv \hat{k} \equiv \eta_3 \frac{\vec{p}}{|\vec{p}|} ; \quad \eta_3 \equiv \frac{(\vec{p} \cdot \vec{\sigma})}{|\vec{p}|} = \pm 1 . \quad (6.1)$$

$z = (\hat{k} \cdot \vec{x})$ is the distance of the sheet from the origin, apart from its sign, which denotes whether the sheet moves away from or towards the origin. To define the original components of the vector \vec{p} , we first have to find its length $|\vec{p}|$. This we take to be the operator

$$|\vec{p}| \equiv |q| \equiv -i\eta_3 \frac{\partial}{\partial z} , \quad \text{so} \quad \vec{p} = -i\hat{k} \frac{\partial}{\partial z} . \quad (6.2)$$

\hat{k} and z are the ontological variables, or beables, whereas q and η_3 are changeables. We have $H = q$, so the dynamical equations are now simply

$$\dot{z} = 1 ; \quad \dot{\hat{k}} = 0 , \quad (6.3)$$

which are the equations of a sheet moving in a fixed direction. Since Eq. (6.2) defines the momenta, and their canonically conjugated operators the positions, we should now be in a position to compute the conventional wave function $\psi(\vec{x}, \sigma_3)$, $\sigma_3 = \pm 1$, if we have some wave function $\psi(\hat{k}, z)$. A fairly delicate calculation gives

$$\psi(\vec{x}, \pm) = \frac{1}{2\pi} \vec{\partial}_x^2 \int \sin \theta \, d\theta \, d\varphi \left(\begin{array}{c} \cos \frac{1}{2}\theta \\ e^{-i\varphi} \sin \frac{1}{2}\theta \end{array} \right) |\hat{k}, z\rangle , \quad (6.4)$$

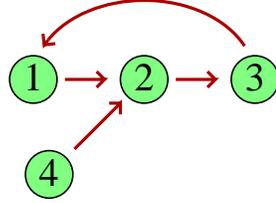


Figure 1: Mini-universe with information loss. The arrows show the evolution law.

where we used the notation

$$z = (\hat{k} \cdot \vec{x}) ; \quad \hat{k} = \begin{Bmatrix} \sin \theta \cos \varphi \\ \sin \theta \sin \varphi \\ \cos \theta \end{Bmatrix} . \quad (6.5)$$

Thus, apart from the Laplacian, all sheets contributing to $\psi(\vec{x}, \sigma_3)$ are going through the point \vec{x} .

7. Information loss

The reasons why *information loss* may be an essential ingredient in deterministic hidden variable models of the sort pioneered above, has been extensively discussed in Ref.[8],[10]. A prototype microcosmos with information loss is the model of Fig. 1. Following the arrows, one would conclude that the evolution matrix is

$$U = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} . \quad (7.1)$$

This, of course, is not a unitary matrix. One way to restore unitarity would be to remove state # 4. The problem with that is that, in universes with tremendously many allowed states, it would be very difficult to determine which of the states are like state number 4, that is, they have no state at all in their (distant) past.

A preferred way to proceed is therefore to introduce *equivalence classes* of states. Two states are equivalent iff, some time in the near future, they evolve into one and the same state². In Fig. 1, states ## 1 and 4 are equivalent, so they form one class. By construction then, equivalence classes evolve uniquely into equivalence classes.

It should be emphasized that, at the Planck scale, information loss is not a small effect but a very large effect. Large numbers of ‘ontological’ states are in the same equivalence class, and the equivalence classes form a much smaller set than the class of all states. This is how it can happen that the total number of distinguishable quantum states (= the number of equivalence classes) may only grow exponentially with the *surface* of a

²It could also happen that two states merge into the same state in the *distant* future, but in many models such events become increasingly unlikely as time goes on.

system, whereas the total number of ontological states may rise exponentially with the volume. This seems to be demanded by black hole physics, when we confront the laws of quantum mechanics with those of black holes.

Information loss at the level of the underlying deterministic theory, may also explain the apparent lack of causality in the usual attempts to understand quantum mechanics in terms of hidden variables. The definition of an equivalence class refers to the future evolution of a system, and therefore it should not be surprising that in many hidden variable models, causality seems to be violated. One has to check how a system will evolve, which requires advance knowledge of the future.

Information loss at the Planck scale may also shed further light on the origin of gauge theories. It could be that, at the level of the ontological degrees of freedom at the Planck scale, there is no local gauge symmetry at all, but in order to describe a physical state, that is, an equivalence class, we need to describe a particular member of this class, a single state. Its relation to the other members of the same equivalence classes could be what is presently called a ‘gauge transformation’.

There is another aspect to be considered in theories with information loss. Theories with continuous degrees of freedom would have an infinity of possible states if there were no information loss. With information loss, there may be a discrete set of limit cycles, meaning that the equivalence classes may still form discrete sets. Discreteness, one of the prime characters of quantum physics, could thus be ascribed to information loss.

8. Conclusions.

Our view towards the quantum mechanical nature of our world can be summarized as follows.

- Nature’s fundamental laws are defined at the Planck scale. At that scale, all we have is *bits of information*.
- A large fraction of this information gets lost very quickly, but it is being replenished by information entering from the boundaries.
- A quantum state is defined to be an equivalence class of states which all have the same distant future. This definition is non-local and acausal, which implies that, if we would attempt to describe everything that happens purely in conventional quantum mechanical terms, such as what is done in superstring theories, locality and even causality will seem to be absent at the Planck scale. Only in terms of a deterministic theory this demand of internal logic can be met.
- These equivalence classes are described by observables that we call ‘beables’. In quantum terminology, beables are a complete set of operators that commute at all times, see Eq.(5.1). A beable describes what a Planckian observer would be able to register about a system — information that did not get lost.

- All other quantum operators are ‘changeables’, operators that do not commute with all beables.
- The wave function ψ has the usual Copenhagen/Bohr/Bohm interpretation, but:
- Many or all of the familiar symmetries of Nature, such as translation, rotation, Lorentz and isospin symmetry, must be symmetries that relate beables to changeables. This means that the ‘ontological’ theory behind Quantum Mechanics does not have these symmetries in a conventional form.

When we go from the Planck scale to the Standard Model scale,

- Our only way to obtain effective laws of physics at the larger distance scales is by applying the renormalization group procedure.
- Beables and changeables are then mixed up to the extent that it is impossible to identify them; they obey the same laws of physics.
- When we perform a typical quantum experiment, we therefore do not know in advance whether an operator we are working with is a beable or a changeable. Due to the symmetries mentioned above, beables and changeables may obey the same laws of physics. Only when we measure something, and not before, do we know that what we are looking at is a beable. In this way, we believe, apparent clashes with Bell’s inequalities may be avoided.
- The classical observables in the classical (macroscopic) limit, commute with the beables. They are beables as well.

There are numerous difficulties left. Most urgent is the need for a viable model, demonstrating how the mechanism works that we believe to be responsible for the conspicuous quantum mechanical nature of the world that we live in. It continues to be difficult to produce a non-trivial model, one showing particles that interact, for instance, such that its Hamiltonian is bounded from below.

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