Complexity in Quantum Gravity

The key problem in reconciling the gravitational force with quantum mechanics is the question what the physical degrees of freedom are

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There are two difficulties with quantum gravity. One is that the theory has to be quantum mechanical, and the other is that it has to be invariant under general coordinate transformations. Presumably the resolution of these difficulties will lie in certain delicate revisions that will have to be considered for both disciplines.

Often, the fact that *perturbative* quantum gravity is in excellent shape, is not sufficiently appreciated. The theory is admittedly nonrenormalizable, which means that at every order in the perturbative expansion, new uncalculable "constants of nature" emerge. The number of uncalculable constants at every order is quite small however, certainly in comparison with the amount of information that the calculations could provide. There are two reasons why this theory is usually completely dismissed. One is its tremendous complexity, as already at low orders the number of algebraical manipulations needed in the calculations is gigantic. Secondly, of course, the emergence of uncomputable numbers (nonrenormalizability) renders the theory useless at the Planck scale. It is clear that a nonperturbative formulation of quantum gravity will have to be entirely different, but it is important to observe that most of the proposed alternatives to perturbative quantum gravity are actually much less predictive than the simple perturbation theory.

The most natural way to search for a resolution to the deficiencies in our present understanding is to attempt a logically consistent description of the *strongest possible gravitational fields*.

(Super)string theory is making rather vociferous claims for the status of "only consistent theory of quantum gravity," but this theory, too, is formulated perturbatively; the expansion, here, is one in topological complexity of string diagrams, and this expansion is as hopelessly divergent as the ordinary perturbative theory. However, the fact that, at each given order, there are no unknown counter terms strongly suggests a more powerful nonperturbative underlying system waiting to be uncovered. Yet there is a danger in such expectations, which can be illustrated by observing that there are many ordinary quantum field theories that have unique perturbative expensions but show completely new physics at the nonperturbative level.[1] Therefore, what is needed is a fundamentally nonperturbative formulation of a theory.[2]

The most natural way to search for a resolution to the deficiencies in our present understanding is to attempt a logically consistent description of the *strongest possible gravitational fields*. If the situation there can be brought under control, one may hope to achieve a completely coherent picture of all processes involving quantized gravitational fields. The strongest possible gravitational fields emerge when gravitational collapse occurs, so we must study black holes.

Large black holes can be described by contemplating familiar astrophysical processes [1] in the evolution of heavy stars or agglomerations of stars. There is little controversy over the fact that black holes are legitimate solutions. Their physi-

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cal properties can be understood and compared with observations. The essential theoretical ingredient is the complete equivalence of any sets of curved coordinate systems (general coordinate invariance). It is due to this principle that general relativity can make powerful predictions, which are being corroborated by observations.

The application of the laws of quantum mechanics to the environment of a black hole leads to highly interesting results. It was discovered by Hawking [2] that black holes will radiate elementary particles of all types, at a temperature

$$kT = \frac{\hbar}{8\pi c^3 G\,M}, \tag{1}$$

but the detailed quantum mechanical evolution of a black hole does not follow from applying general coordinate invariance. At first sight, unitarity seems to be violated because objects entering a black hole cannot reemerge or even transmit information to the outside world.

If we want to describe the laws of physics at tiny distance scales in such a way that quantum mechanics on the one hand, and invariance under general coordinate transformations on the other, are implicated, then we have a problem. The real difficulty appears to be that of bookkeeping. What is needed is an unambiguous and exhaustive description of *all dynamical degrees of freedom* [4]. A simple counting argument reveals that these degrees of freedom are not distributed over space and time in the way usually assumed in ordinary quantum field theories, but rather at the *boundaries* of a given physical system [5]. Not even string theory can reproduce this situation, although it is claimed that, in an indirect way, state counting leads to correct orders of magnitude in objects that one could call black holes [6]. The problem with these indirect arguments is that they do not seem to apply to large black holes with nondegenerated macroscopic horizons. It is precisely these objects to which one wishes to apply the laws of general coordinate invariance. Thus, our problem is to find a way to reformulate such laws in the presence of quantum mechanics.

What is needed is an unambiguous and exhaustive description of *all dynamical degrees of freedom*.

Since general invariance appears to be a universally valid law of nature, it seems to be inescapable that an extension of this law should exist for quantum mechanical black holes. Since this law should then reflect a basic property of space and time themselves, it is the very structure of space and time that we are confronting here. This is why this problem is of extreme importance to physics, and we do not even understand how to formulate the bookkeeping.

Quantum mechanics and statistics are clearly linked in this problem. It is only natural to suspect that the interpretation of the quantum mechanical laws should be reconsidered in this context. Could it be that determinism can be restored by combining quantum mechanics with gravitation? Such ideas are speculated upon, but not much progress is made. The author is cherishing the suspicion that some deterministic set of laws is producing semi-chaotic behavior at Planckian distance scales, which at the much larger distance scale of atoms and molecules, may lead to statistical behavior that require the Schrödinger equation for their description. This would be an extreme example of complexity in the submicroscopic world [7]. Often, the Bell [8] inequalities are cited as an obstacle against deterministic theories of quantum mechanics, but I suspect that such models are still permitted if one assumes long distance correlations in the vacuum oscillations, of a kind that physically are not unrealistic. But we do not have much to stand on; even simple models exhibiting some of the expected features could not be produced. Another avenue is the admission of a mild violation of general coordinate invariance. Since gravitational fields are generated by material objects, a coordinate frame which refers to a particular gravitational field may refer to a Hilbert space that contains states different from the states in other coordinate frames. Thus, in quantum mechanics one might suspect that general coordinate transformations fail to be unitary when applied within a given Hilbert space.

NOTES

1. Quantum electrodynamics, for example, is unique perturbatively, but requires new physics such as non-Abelian unifying fields, as substitutes for its Landau ghosts.
2. Or perhaps a scheme in which the perturbative expansion is used in an intermediate step, such as in quantum chromodynamics (QCD), which is a theory with *asymptotic freedom*, implying a domain where the expansion parameter can be made arbitrarily small. It is usually believed that this makes QCD uniquely defined, although rigorous proofs of this statement are lacking.
3. Thermal radiation had been predicted on more general grounds by Bekenstein [3].

REFERENCES

1. S. Chandrasekhar: The mathematical theory of black holes. Clarendon Press, Oxford University Press.
2. S. W. Hawking: Commun. Math. Phys. 43: p. 199, 1975; J.B. Hartle and S.W. Hawking: Phys. Rev. D13 p. 2188, 1976.
3. J. D. Bekenstein: Nuovo Cim. Lett. 4: p. 737, 1972; Phys. Rev. D7 (1973) p. 2333; D9: p. 3292, 1974.
4. G. 't Hooft, Nucl. Phys. B256: p. 727, 1985.
5. G. 't Hooft: J. Mod. Phys. A11: p. 4623, 1996; gr-qc/9607022.
6. A. Strominger and V. Vafa: hep-th/9601029
7. G. 't Hooft: Nucl. Phys. (Proc. Suppl.) B43: p. 1 , 1995.
8. J. S. Bell: Physica 1: p. 195, 1964.