

Chapter 8

Integration of Mathematics and Didactics in Primary School Teacher Education in the Netherlands



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Abstract From the 1970s, curricula of primary mathematics teacher education in the Netherlands drastically changed. This occurred simultaneously with the changes in primary mathematics education. Teacher educators systematically discussed mathematics teacher education and implemented new content and new approaches in primary teacher education. This chapter provides a chronological overview of how Dutch primary mathematics teacher education developed from the 1970s until the present. We describe ideas about learning to teach mathematics and ideas about the relationship between the development of mathematical literacy and didactical proficiency of student teachers. Furthermore, the influence of national measures such as the introduction of nationwide tests for primary mathematics teacher education is discussed. The chapter ends with an impression of recent learning materials for student teachers and a reflection on new perspectives for integrating theory and practice, emphasising the continuous search for a well-balanced way to interconnect mathematics and didactics.

8.1 Introduction

In 1858 criticism on the education of teachers in the Netherlands led to the establishment of the first teacher education institutions by law (Van Essen, 2006). The curricula of these institutions were almost the same as those for the higher grades of secondary school, with pedagogy and half a day a week for teaching practice

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added. The mathematics curriculum included, for example, algebra, planimetry and stereometry (Van Beek & Van Heek, 1924, 1925, 1926).

This situation did not change fundamentally for decades. Even in 1952, when curricula changed as a result of the *Nieuwe Kweekschoolwet* (New Teacher Education Act), little changed in daily practice because most teacher educators were the same people as before the Act, and they saw no reason to change their teaching approaches.

In 1968 teacher education institutions were renamed into *Pedagogische Academies* (Pedagogical Academies). In 1985, after years of discussion (see *Innovatie Commissie Opleidingen Basisonderwijs*, 1980) kindergarten schools and primary schools, were integrated into primary schools for children from 4 to 12. The respective teacher education institutions were also integrated into the *Pedagogische Academies voor het Basisonderwijs* (Pedagogical Academies for Primary Education). After a few years, criticism from the *Onderwijsraad*, the Dutch national advisory council for education (Onderwijsraad, 1988) and OCW, the Ministry of Education (OCW, 1990), mainly on the insufficient academic level and the lack of a clear education concept at the teacher education institutions, again led to a revision of the teacher education curriculum. This time the curriculum had to be based on a well-defined education concept and should fit within the framework of a specific teacher education didactics. Problem-based learning and thematic education were adhered to, and teachers from all disciplines were expected to develop their own educational materials according to these two concepts (Goffree & Oonk, 1999).

From the beginning in 1991, external quality monitoring of teacher education institutions mostly considered general characteristics of the institutions' curriculum and hardly focused on school subjects. Over the years less and less attention was paid to the development and enforcement of mathematics and other subjects in teacher education, which resulted in a decrease in attention for subject-specific content knowledge in the teacher education institutions (Van Mulken, 2002; Onderwijsraad, 2005). However, from time to time the HBO-raad, the Council for Higher Professional Education, took measures to secure the mathematical proficiency of student teachers, mainly through including mandatory mathematics tests (Keijzer & Hendrikse, 2013).

In this chapter, we describe primary school teacher education in the Netherlands, taking the year 1971 as a starting point, which is the year that the IOWO¹ (the first predecessor of the Freudenthal Institute) was founded. In the final section of this chapter we provide a new perspective on learning to teach mathematics, the relationship between the development of mathematical literacy and didactical proficiency and recent influences on primary school mathematics teacher education.

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8.2 Mathematising and Didacticising

8.2.1 *The Influence of Freudenthal on Mathematics Teacher Education*

In 1971 Hans Freudenthal became the first director of the IOWO, where two years earlier the Wiskobas² project had started (see Freudenthal, 1978; Treffers, 1978; 1987). This Wiskobas project for mathematics in primary school was intended to develop a new approach to mathematics education together with teachers and student teachers, with the intention to teach them at the same time how to implement this new approach. A strong belief of Freudenthal and his team was that the curriculum for primary school mathematics teacher education should be developed in close connection with the primary school mathematics curriculum. Shortly after the first Wiskobas curriculum publications were published, an educational experiment was started at a teacher education institution in Gorinchem (Goffree, 1977; Goffree & Onk, 1999). Every week, Freudenthal and two members of the Wiskobas team went to this small town and attended the lectures given to the student teachers and visited the school where the student teachers acquired practical experience. Freudenthal worked with the children to show the student teachers how it is possible to initiate and observe mathematical learning processes. His observations and analyses were intended to convey to the student teachers the idea of the teacher as a researcher and give them the feeling that there was much that could be learned from the children themselves. Freudenthal also introduced a narrative element into the teaching with stories such as “Walking with Bastiaan” (Freudenthal, 1977). Materials for the student teachers were developed and tested. Freudenthal made theoretical contributions to these materials as evidenced, for example, by his work on the analysis of fractions and ratio as mathematical objects (Freudenthal, 1983). Freudenthal’s approach had a great impact on the teacher education institution in Gorinchem (see, e.g., Goffree, 1979).

8.2.2 *A Model for Learning to Teach Mathematics*

The experiences in Gorinchem were discussed regularly in a national group of mathematics teacher educators. Materials for student teachers were tried out nationwide and rewritten by the staff of the Wiskobas project. All this led to a model for learning to teach mathematics (Fig. 8.1).

The model shows, starting from the left, that mathematics education, both for student teachers and children, takes meaningful mathematical situations as its starting point. For children, it implies the activation of subjective, informal structures, which allows the mathematical learning process to start with children’s intuitive

²Wiskunde op de Basisschool (Mathematics in Primary School).

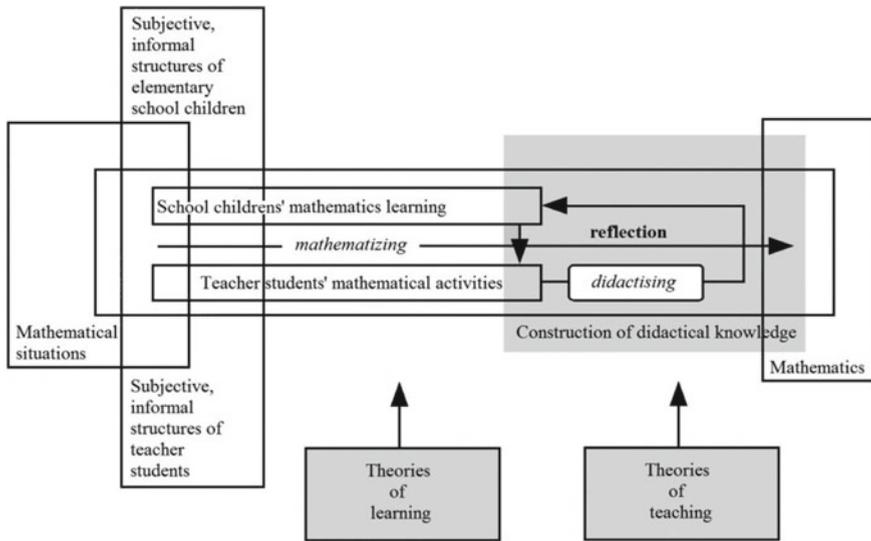


Fig. 8.1 Model for learning to teach mathematics (Goffree, 1979, p. 313; Goffree & Oonk, 1999, p. 209)

notions and informal procedures. Under the guidance of the teacher who knows the objective structures of mathematics (formal mathematics), they get the opportunity to rediscover mathematics (Freudenthal, 1983), experiencing a process of guided reinvention (Freudenthal, 1991).

For the student teacher, whose subjective, informal structures are affected by his or her earlier experiences with learning and teaching mathematics, learning to teach is considered a process of both mathematizing and didacticising (Freudenthal, 1991). Keijzer (1994, p. 4) expresses this as follows:

Reflecting on learning experiences as a starting point for learning to teach mathematics. In teacher education, student teachers' experiences with learning mathematics are discussed from time to time, especially shortly after student teachers enter teacher education. One teacher educator decides to use this focus in the very first meeting with first-year prospective teachers. One student teacher recalls learning the algorithm for long division and another tells how she masked her struggle with mental arithmetic by finger calculations hidden from the teacher. The teacher educator concludes: "Our talk on early experiences with learning mathematics showed that reflecting on one's own mathematical acting forms a fruitful starting point for exploring didactical content knowledge." (translated from Dutch by the authors)

The view on learning to teach mathematics as represented by the model in Fig. 8.1 is that learning to teach mathematics should start by student teachers carrying out mathematical activities at their own level. Reflections on children's learning processes combined with the student teachers' own experiences in learning mathematics contributes to the creation of an educational basis for teaching mathematics. Big ideas from general educational theory, rooted in either didactics or formal mathematics, can also contribute. It is assumed that in this way student teachers will get

into a cyclical process of solving mathematical problems, mathematisation, reflective problem solving, and mastering teaching approaches. Meanwhile, student teachers work with children and study their learning processes while continually referring to their own learning processes. While doing so the student teachers integrate their subject matter knowledge and didactical content knowledge,³ in other words, they coherently develop both mathematical and didactical knowledge.

8.3 New Developments in Primary School Mathematics Teacher Education

8.3.1 *Mathematics & Didactics as a New Subject for Student Teachers*

In the 1980s, the model described in Fig. 8.1 was elaborated into the new subject Mathematics & Didactics⁴ in primary school mathematics teacher education, based on the book series *Wiskunde & Didactiek* (Goffree, 1982/1994, 1983/1992, 1985, 1993/2005, 2000). This series of books was used in the 1980 s and 1990 s in more than eighty percent of the Dutch teacher education institutions. Goffree (1982, p. 7) formulated the approach to primary school mathematics teacher education as follows:

Learning to teach mathematics

We think that you will best learn to teach by first working on mathematical problems yourself. Of course, these problems have to refer to the subject matter you are going to teach. Therefore, most chapters of this book start with simple mathematics problems. Thinking through these problems together helps you to look back at your own and your peer students' solutions from a different point of view. We call this reflection: thinking deeply on finding new ways, using a sketch or material, getting another explanation, a state of still not understanding or suddenly grasping it [...]. We think this is important. It builds up the beginning of your didactical thinking, because you can expect similar situations if you are going to teach mathematics to children. (translated from Dutch by the authors)

³Although in international publications this is often referred to as 'pedagogical content knowledge', in the Netherlands, the term 'didactical content knowledge' is preferred. By using 'didactical', it is made clear that here knowledge is meant that is related to the teaching of mathematics and not knowledge about interacting with children and creating an environment at school that socially, psychologically and physically supports their development.

⁴The &-sign between mathematics and didactics symbolised the ambition of the author of the textbooks to optimize the student teachers' integration of their subject matter knowledge and didactical content knowledge.

8.3.2 *The Influence of Quality Monitoring*

In 1985 the Dutch Ministry of Education proposed a new system of quality monitoring in higher education, which, in the same year, was adopted by the Dutch parliament. In this system, the institutions for higher education themselves became responsible for quality monitoring, while the government would follow this process from a distance (OCW, 1985; Van Bommel, 2014). Following this advice, the HBO-raad⁵ (Council for higher professional education) arranged inspections in primary teacher education; the first one took place in 1991. Focal points in both this first inspection and in the second one included: curriculum, prospective teachers' level, assessment systems, and student teachers' educational process. These points potentially offered chances to evaluate the mathematics curriculum within teacher education. There was a need for such an evaluation, as primary student teachers' mathematics proficiency was a concern of many (Brandt, Feijs, Groen, & De Moor, 1987). Such an evaluation, however, did not happen. The inspection focused only on general aspects of teacher education and was not equipped to look at more domain specific issues (Keijzer, 1993).

8.3.3 *Growing Attention to Student Teachers' Mathematical Literacy*

Alongside the development of a programme for primary school mathematics teacher education, there were growing concerns about the mathematical proficiency of student teachers (Jacobs, 1986; Brandt et al., 1987) As a reaction, the *Mathematics & Didactics* series was extended with a book especially aimed at the development of student teachers' mathematical proficiency (Goffree, Faes, & Oonk, 1988, 1994; Goffree & Oonk, 2004). This book contained mathematics problems selected from primary school mathematics textbooks, each problem was provided with reflective solutions at the level of student teachers. The view of the authors was that comparing one's own solutions with expected solutions of children and with expert solutions and discussing them, would raise the student teachers' mathematical literacy, and also strengthen their didactical proficiency.

Cooperating mathematics teacher educators also encouraged the Council for Higher Professional Education, to support the development of materials to tackle the problem of student teachers' low mathematics abilities. This led to the series *Wiskunde Verplicht* (Mathematics Required) (Faes & Oonk, 1989a, b, c, 1990) with which student teachers could refresh their mathematical skills. In addition, there was the series *Gecijferdheid* (Mathematical literacy) (Faes, Van den Bergh, & Olofsen, 1992) with example problems on mathematical literacy, which teacher educators could use to develop a test for student teachers, that could be administered parallel

⁵Currently 'Vereniging Hogescholen'.

- Calculate using an efficient strategy: $412 - 97.98 =$
- What scale should be used to fit a map of the Netherlands on an A4-sheet? Make sure the space on the sheet is maximally covered with the country. Write your answer as: $1 : \underline{\hspace{2cm}}$.
- Design a situation that leads to the number sentence ' $2.25 \div 0.75$ '.

Fig. 8.2 Problems from the 1992 Mathematical Literacy Test

to the other first-year test. The problems provided for the mathematical literacy test assessed, for example, student teachers' ability in using efficient strategies in tackling number problems or showing insight in measurement. Figure 8.2 shows some typical problems from this test.

From 1992 on, most teacher education institutions for primary education developed and used local adaptations of this test to assess their first-year student teachers' mathematics skills. In addition, as this test asked for mathematical knowledge of a specific nature, the first-year curriculum gradually incorporated this knowledge. This curriculum started to focus more on efficient strategies to solve number problems, various meanings of (rational) numbers and number relations, meaning in measurement (including personal references to measures), estimation, and geometry. However, although the tests used were inspired by a series of prototypical examples, the (adapted) tests differed significantly between institutions, with respect to the topic in the test, the difficulty level of the problems, and the pass mark.

8.4 Standards for Primary School Mathematics Teacher Education: Adapting the View on Learning to Teach Mathematics

8.4.1 Towards Standards for Primary School Mathematics Teacher Education

After discussions with all the experts involved, the new approach to primary mathematics education in the Netherlands that had been stimulated by Hans Freudenthal, and that was now known as Realistic Mathematics Education (RME), led to the *Proeve van een Nationaal Programma voor het Reken-wiskundeonderwijs op de Basisschool* (Treffers, De Moor, & Feijs, 1989), a first design for setting up a national programme for mathematics education in primary school. Following this programme and the standards for mathematics evaluation and teaching of the National Council of

Teachers of Mathematics (NCTM, 1989, 1992), a group named PUIK,⁶ consisting of ten mathematics educators, started in 1990 to develop standards for primary school mathematics teacher education. For example, on the student teachers' insight into children's learning processes these standards state the following:

- Student teachers acquire insight into children's learning processes in the area of mathematics.
- Student teachers analyse data from children's mathematical activities (written or oral data, or data on videotape) from various perspectives.
- Student teachers develop activities themselves to acquire insight into children's learning processes.
- The student teachers regularly talk with individual children (in clinical interviews) about specific problems and their solutions.
- Student teachers study material (such as from Kwantiwijzer⁷) about carrying out diagnostic interviews with children, and then hold interviews in accordance with it.
- Learning processes in the area of mathematics are a frequent topic of lectures, small group work and reading assignments.
- How to increase the level of understanding of both children and students is a topic of mathematical educational research.
- Children's own mathematical productions provide study material for small group work on mathematics education and also serve as illustrations of knowledge transfer (Goffree & Dolk, 1995, p. 74).⁸

8.4.2 *Constructive, Reflective, Narrative*

The philosophy of teacher education elaborated in the handbook of Goffree and Dolk (1995) was founded on three pillars: primary school mathematics teacher education should be constructive, reflective, and narrative. This approach to teacher education is an adaptation of the socio-constructivist vision of knowledge acquisition, reflection as the main driving force of the professionalisation of teachers, and the interpretation of practical knowledge as a way of narrative knowing. According to Oonk, Goffree, and Verloop (2004, p. 137), "Real teaching practice has to be the starting point of teacher education." In the attempt to have these pillars into new curriculum materials for primary school mathematics teacher education, the PUIK group faced essential questions: What represents 'real teaching practice'? How can curriculum designers give a learning environment a 'natural' aura? Moreover, what is meant by 'natural'? Fieldwork practice is natural by definition, but when student teachers discuss this

⁶Programming, Uitlijning, Invulling en Kwaliteit (Programming, outlining, filling-in and quality).

⁷The Kwantiwijzer project developed diagnostic instruments based on the ideas of Wiskobas (Van Eerde, 2005).

⁸See for all eighteen standards: <http://www.mtedu.utaipei.edu.tw/mathweb/opendata/%E8%8D%B7%E8%98%ADstandards.pdf>

practice, they often stick to a superficial interchange of ideas and opinions (Verloop, 2001; NCTM, 2000). Rarely do these discussions reach a level of theoretical reflections. How could the PUIK group solve this problem?

Oonk et al. (2004) mention three important issues that are central to the discussion about these problems. First, learning in practice is mostly a solo task because student teachers do rarely have the opportunity to discuss common experiences and observations, necessary to acquire deep rooted knowledge. Second, student teachers usually focus on fulfilling responsibilities and on survival issues, so their reflections on their profession are dominated by talking about actions. Third, as a result, student teachers do not acquire practical knowledge that can be generalised across situations or organise their narratives of teaching into a broader framework.

The PUIK group got a new perspective on these problems when they visited the School of Education of the University of Michigan where they were introduced to the *Student Learning Environment* (SLE) designed by Lampert and Loewenberg Ball (1998). The SLE became a source of inspiration for the making of the *Multimedia Interactive Learning Environment* (MILE) for primary mathematics student teachers in the Netherlands (Dolk, Faes, Goffree, Hermsen, & Oonk, 1996).

8.4.3 Mile

All Dutch teacher education institutions participated in the MILE project. The goal of MILE was to enable student teachers to investigate good practice in teaching primary mathematics. 'Good practice' here meant practice being in line with the Dutch standards for primary mathematics education and with those for primary school mathematics teacher education. Other characteristics of the good practice offered by MILE were:

- Showing authenticity of real practice in school.
- Representing the complexity of real teaching practice, exemplary for the programme of primary education.
- Taking into account learning strands and of students' learning processes: education in the vein of RME.
- Providing researchable reflective practice of expert teachers and some theoretical input by the designers (Oonk et al., 2004, p. 145).

The MILE database included materials on mathematics education from Kindergarten through Grade 6, involving recorded, connected lessons, discussions with teachers and supervisors, and textbooks and other materials. It was possible to study each lesson as a whole or in short fragments. Keyword searches of the fragment descriptions and lesson dialogues could be done using a search engine. Every lesson fragment reproduced a teaching instance and a short description that provided further clarification (Fig. 8.3).

Research showed that student teachers were often not only focused on the actual teaching of mathematics when watching the fragments in MILE, but also on general

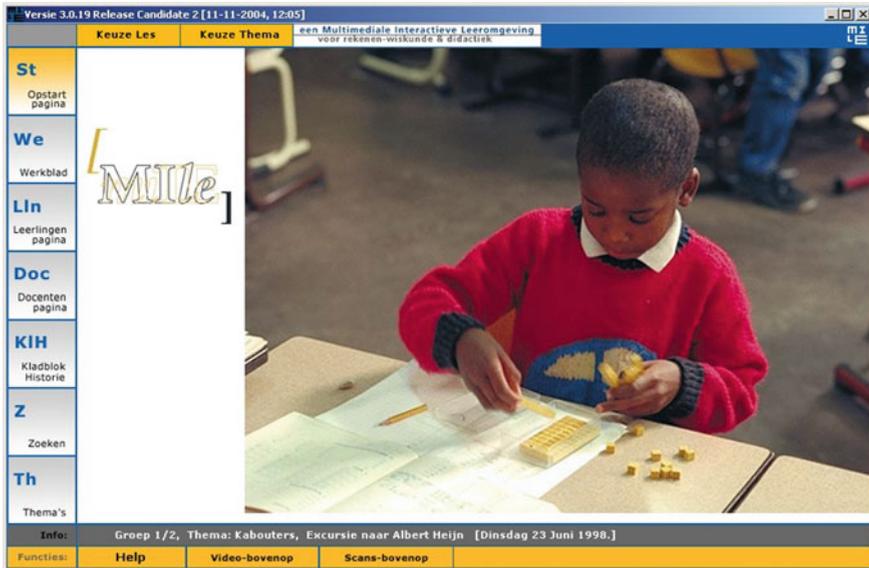


Fig. 8.3 The MILE start-up screen

didactical and pedagogical issues (Oonk, 2001, 2009; Goffree & Oonk, 2001; Oonk et al., 2004). MILE thus offered the possibility to use the school subject mathematics as an arena for theoretical reflections that connect with larger didactical and pedagogical ideas. Furthermore, working with MILE, four levels of student teachers' knowledge construction were distinguished (Oonk, 2009, pp. 74–75; Oonk et al., 2004, p. 152):

Four levels of student teachers' knowledge construction:

- Knowledge can be taken from the expert teachers in MILE; student teachers expand their own didactical repertoires through assimilation of the practice knowledge contained in MILE.
- Adaptation and accommodation of practice knowledge can modify the repertoires of the MILE teacher to suit student teachers' own purposes.
- Establishing (new) links between the events in MILE and events from student teachers own trainee practice and related theory; this is the level of integrating theory, in which they might (re)consider didactical insights and points of view.
- The level of theorising manifests itself when student teachers designed their own local theories; they built up ideas about causes and consequences through the observation and interpretation of fragments they themselves found in MILE.

The results of research on the activities of fifty second-year student teachers (Oonk, 2001) revealed that they used theory as a means to understand and explain practical situations. The majority of the student teachers themselves believed that working with MILE enabled them to apply and further explore the knowledge that they already had. The following transcript of a discussion shows how two second-year

student teachers working in MILE were searching for appropriate theory when they compared, faced and considered which material or model is (or is not) appropriate and why.

Denise and Marieke are watching and analysing a video clip about Fadoua, a Grade 2 student and her teacher at the instruction table. We see how the teacher identifies in a diagnostic discussion the way of thinking behind Fadoua's mistake ($18-6=11$). It appears that Fadoua counts backwards starting from 18 ('initial error') and while counting backwards also skips two numbers (12 and 14).

The two student teachers discuss the most appropriate way to assist Fadoua.

Denise I think solving the problem using 18 blocks (units) may help.

Marieke I don't think this will help, because it doesn't solve Fadoua's counting problem.

Denise Maybe the number line?... eh...

Marieke That will not help for the same reason.

Denise I suddenly think that the fives structure of the arithmetic rack with twenty beads can help Fadoua either by directly subtracting 6 or by splitting to yield 8-6 or 18-6. And that doesn't involve counting anymore.

Marieke I agree, I can well remember from earlier clips that Fadoua has most likely mastered splitting the numbers to ten (...). In this case we can probably indeed use the arithmetic rack teaching method. (Oonk, 2001, p. 21)

A number of the student teachers demonstrated a budding appreciation for theory. However, others lost their way in MILE and rarely reached beyond a superficial level of reflection. The frame of reference of these student teachers appeared somewhat diffuse and fragmented. An important side-product of the MILE project was the accompanying professional development for mathematics teacher educators at one-day conferences.

8.5 New Ideas About Learning to Teach Mathematics

After 2000, the earlier ideas about primary school mathematics teacher education remained as a kind of natural fundament within the community of mathematics teacher educators in the Netherlands. These included the ideas of RME, the pillars constructive, reflective and narrative, and the integration of mathematics and didactics.

However, important questions crossed the boundary of the centuries and remained to influence the discussions about the way to go: How could the integration of theory and practice for student teachers really be shaped? And how could student teachers' reflections on (their) teaching practice be brought to a higher level? For example, most teacher educators were convinced that video recordings served the purpose of reflection on real practice, but so far, the student teachers' experiences remained to be proved against this assumption. Developments at different levels of mathematics education in the Netherlands brought new perspectives to answer these questions.

First, the Freudenthal Institute, partly in cooperation with the Netherlands Institute for Curriculum Development (SLO) and the CED-Group,⁹ developed in the TAL¹⁰ project the so-called ‘Teaching-learning trajectories’ for most domains of primary mathematics education (Van den Heuvel-Panhuizen, 2008¹¹; Van den Heuvel-Panhuizen & Buys, 2008¹²; Van Galen et al., 2008¹³; Gravemeijer et al., 2016¹⁴). These extended descriptions of the learning pathways in mathematics provided teacher educators and authors of mathematics textbooks series with well thought out ideas about mathematical learning processes of primary school students.

Second, a large-scale research project for mathematics teacher education was set up. The purpose of this *Theorie In Praktijk* (TIP; theory into practice) project (Oonk, 2009) was to gain insight in the student teachers’ way of integrating theory and practice, and particularly to find out how they relate theory and practice and to what extent they are competent to use theoretical knowledge in multimedia educational situations. The study was performed at eleven teacher education institutions. A learning environment was designed to optimise the opportunity for theory use. Theory was recognisable in a multifunctional set of concepts,¹⁵ covering a local instruction theory (Gravemeijer, 2004). The set was multifunctional in the sense that it became manifest in expert reflections on video clips (Goffree, Markusse, Munk, & Olofsen, 2003), through a link that provided extra information, and in a list of concepts to use during the course. The latter functioned as a tool to check one’s own progress in understanding the concepts.

The TIP study showed among other things that the learning environment was a catalyst for the development of the student teachers’ so-called ‘theory-enriched practical knowledge’. The student teachers’ use of theory could be identified univocally and described at different levels. These levels turned out to have a positive correlation with the student teachers’ level of mathematical literacy and the level of their previous education. A rise in the level of theory use took place especially in interaction led by the teacher educator (Oonk, 2009).

A third development, that occurred simultaneously with the two previously described developments, is that the Panama Kerngroep (Panama core group)¹⁶ of

⁹The CED-Group trains and advises professionals in education and child care.

¹⁰Tussendoelen Annex Leerlijnen (Intermediate attainment targets and learning lines).

¹¹Published in Dutch in 1999 and 2001 (Treffers, Van den Heuvel-Panhuizen, & Buys, 1999; Van den Heuvel-Panhuizen, Buys, & Treffers, 2001) and in English in 2001 (Van den Heuvel-Panhuizen, 2001).

¹²Published in Dutch in 2004.

¹³Published in Dutch in 2005.

¹⁴Published in Dutch in 2007.

¹⁵The idea for such a set originated from a research project on MILE (Oonk, 2001), where a list of concepts was used (Bos, 1999) to inform the student teachers about keywords for the course at hand.

¹⁶Panama stands for: Pabo Nascholing Mathematische Activiteiten (Pedagogical Academy Training Mathematical Activities). Panama is the Dutch network of mathematics teacher educators for primary education. One of the activities of Panama is organising the annual Panama Conference.

mathematics teacher educators organised a discourse about the curriculum for primary school mathematics teacher education. This discourse inspired the members of this group to write a book about their experiences, considerations, and dilemmas when teaching primary mathematics student teachers (Van Zanten & Van Gool, 2007).

Reflections on the stories in this book were used to arrive at six quality landmarks of good practice in primary school mathematics teacher education:

1. Mathematics-specific coaching when student teachers practice their teaching in school
2. Enough opportunity to develop a mathematical and didactical repertoire
3. Including student teachers' mathematical literacy in the binding study advice
4. Developing student teachers' mathematical literacy
5. Opportunity for reflection and further professionalisation of mathematics teacher educators
6. Ample attention for mathematics specific development of student teachers, including mathematical attitude. (Van Zanten & Van Gool, 2007, pp. 111–115). (translated from Dutch by the authors)

Some of these landmarks came into being in the form of mandatory mathematics tests and the development of a knowledge base for mathematics for prospective teachers.

8.6 A Mathematics Entrance Test for Student Teachers

Teacher education institutions adapted the 1992 Mathematical Literacy Test in various ways. As a result, this test did not secure a fixed mathematics ability level for the prospective primary school teachers (Straetmans & Eggen, 2005). The inspection of teacher education institutions already brought to the fore in 2002 that this situation was problematic, and also found that not all institutions took the level of mathematical proficiency into consideration in the binding study advice that was given to the students at the end of their first year. In school year 2006–2007, this led to the nationwide introduction of a mandatory entrance test, the Wiscat Test. From this year on, all prospective teachers needed to pass this test in the first year of their study. This meant that the third landmark of quality that the Panama Core Group formulated already had become reality, though not in the way they intended or had wished. The Wiscat Test was designed in such a way that the scores of the student teachers could be compared with the mathematics proficiency of primary school students. Student teachers had to show a better mathematics proficiency than eighty percent of students at the end of primary school. So, it can be concluded that the pass mark of the Wiscat Test is low. This pass mark requires less mathematical ability than was originally intended in the Mathematical Literacy Test, and for several teacher education institutions it meant a lowering of the required level of mathematical competence in the first year of teacher education (Van Zanten & Van den Brom-Snijders, 2007). Therefore, they decided to set additional requirements for student teachers.

Fig. 8.4 Two problems from the 2006 Wiscat Test

2.5 % of an amount is €9.-
 What is the full amount?
 €[]:[] (blanks should be filled in)

On a map with scale 1 : 12,500,000
 the distance between Den Bosch and Prague is 7 cm.
 The original distance is?
 [] km (blank should be filled in)

However, a significant number of teacher education institutions chose not to do so (Keijzer, 2010). Figure 8.4 shows typical problems from the Wiscat Test.

The Wiscat Test is a computer-based test and asks student teachers to provide only an answer. Open questions, as in the 1992 Mathematical Literacy Test, are not included. In a sense, this approach is also reflected in the curriculum for primary school mathematics teacher education. Teaching for mathematical literacy was replaced by teaching how teachers and students could produce answers. Several teacher education institutions offered many hours of support for student teachers who needed practise for the test (Keijzer, 2010). Student teachers in their turn often developed or maintained an instrumental way of learning and practicing mathematics, as this appeared to be appropriate to pass the test. Moreover, many institutions chose to not assess mathematics skills, other than those in the Wiscat Test (Van Zanten & Van den Brom-Snijders, 2007). Consequently, many prospective teachers considered the entrance level of this test as a sufficient end level of mathematical proficiency for teaching mathematics in primary education.

8.7 The Knowledge Base for Primary Mathematics Teacher Education

8.7.1 Background

Mathematics teacher educators kept articulating their concerns about the mathematical proficiency of their students (Van Zanten, 2006) and the inability of the Wiscat Test to address this problem. In the first decade of this century, there were also growing concerns about the declining amount of attention that teacher education institutions paid to mathematics (as well as to other subjects) (Onderwijsinspectie, 2008; Onderwijsraad, 2005).

After 2005, primary school mathematics teacher education in the Netherlands became one of the issues in a debate that followed the somewhat disappointing results for Dutch students in TIMSS and PISA (OECD, 2004; KNAW, 2009; Mullis,

Martin, Foy, & Arora, 2012). Especially, the study load in teacher education was discussed. In 2008, over the four years in teacher education, student teachers spent on average about 350 hours on studying mathematics, which many experts in the field consider a very low study load to cover the whole range of mathematical and didactical content knowledge. Moreover, there were enormous differences in the mathematics study load between teacher education institutions. In some institutions, the study load did not even exceed the amount of 120 hours in four years, i.e., an average of 30 hours per year (Keijzer, 2010).

The concerns about the proficiency of primary school teachers led to the decision to make a knowledge base for mathematics and language (HBO-raad, 2008; OCW, 2008). The HBO-raad assigned the development of a knowledge base for mathematics to a group of mathematics teacher educators under the name of ELWIE¹⁷/Panama. This resulted in the publication of *Kennisbasis Rekenen-wiskunde Voor de Lerarenopleiding Basisonderwijs* (Van Zanten, Barth, Faarts, Van Gool, & Keijzer, 2009), or in short, the Knowledge Base. The ELWIE/Panama group developed the Knowledge Base in close collaboration with mathematics teacher educators from all Dutch teacher education institutions (Van Zanten, 2010).

8.7.2 *Defining Professional Mathematics Literacy*

The Knowledge Base was meant to provide a description of mathematical knowledge for teaching. Therefore, it had to include both subject matter knowledge and didactical content knowledge. In line with the idea that mathematising and didacticising are interconnected, the developers of the Knowledge Base saw these types of knowledge as two sides of the same coin. Their basic assumption was that teachers' own mathematical literacy provides the foundation of their didactical repertoire. Mathematical content knowledge and knowledge of didactics for mathematics were seen as highly interrelated and inextricably connected. Following an earlier study on mathematical literacy for student teachers (Oonk, Van Zanten, & Keijzer, 2007), the ELWIE/Panama group defined professional mathematical literacy as a subject-specific competency for student teachers including: having a sufficient level of mathematical literacy of one's own, being able to give meaning to mathematics for students, being able to stimulate calculation methods and raise students' competency, and being able to stimulate students' mathematical reasoning (Van Zanten et al., 2009).

¹⁷Expertisecentrum Lerarenopleidingen Wiskunde en Rekenen (Expertise Centre Teacher Training Mathematics and Arithmetic).

8.7.3 *Content of the Knowledge Base*

The final version of the Knowledge Base consists of two parts: general theory and domain descriptions. The first part broadly describes general information about mathematics education. This includes several kinds of goals of mathematics, varying from the legally established core goals to underlying values of mathematics education, such as preparing students for participation in society. Second, the general theory describes the varying learning processes that occur, for example mathematical reasoning, verbalising solution processes and memorisation, to name a few. In connection with this, didactical insights are given, with a prominent place for the notions of RME.

The second part includes descriptions of five mathematical domains: whole numbers; proportions, percentages, fractions and decimal numbers; measurement; geometry; and relations. These descriptions consist for the largest part of descriptions of domain specific mathematical and didactical content knowledge. Furthermore, appearances and relevance of the domain in reality are specified, as well as the intertwinement and connectedness with other domains and with other school subjects.

8.8 The Knowledge Base Test

8.8.1 *Content of the Knowledge Base Test*

After the Knowledge Base for mathematics was established, a test was developed for assessing student teachers' knowledge. This new nationwide Knowledge Base Test for third-year student teachers had to guarantee that the prospective teachers master the knowledge described in the Knowledge Base. However, this test only includes the mathematical content knowledge, and not the didactical content knowledge described in the Knowledge Base (Keijzer, Garssen, & Peijnenburg, 2012).

With the introduction of the Knowledge Base Test in 2013, prospective teachers' mathematics proficiency is not only assessed at a basic level in an entrance test (through the Wiscat Test) but also at a far higher level in the third year. This situation demanded specific investments in mathematics content matter knowledge in primary school mathematics teacher education (Keijzer, 2015a).

The Knowledge Base Test is computer-based. Student teachers' knowledge is assessed in all five domains described in the Knowledge Base. The test items are related to what Ball, Thames, and Phelps (2008) refer to as mathematics on the horizon, common content knowledge, and specialised content knowledge. Figure 8.5 shows some typical problems from this test.

- Estimate the number of minutes you have lived the day you celebrate your 18th birthday.
- Which numeral is in the ten-position in the answer of 877651×76523 ?
- What is the decimal number 25 written as a binary number?

Fig. 8.5 Problems from the 2013 Knowledge Base Test

8.8.2 Influence of the Knowledge Base Test on the Curriculum for Primary School Mathematics Teacher Education

Between 2009 and 2015 the average study load for primary school mathematics teacher education rose from about 350 hours to about 485 hours, again with huge differences between the teacher education institutions. The emphasis on the curriculum for primary school mathematics teacher education shifts from didactical content knowledge to all aspects of mathematical knowledge for teaching, including mathematics content matter knowledge. Furthermore, to support student teachers, there was a shift from preparing prospective teachers for the 2006 Wiscat Test to preparing them for the Knowledge Base Test (Keijzer, 2015b). But, when doing so, teacher educators struggled a bit with the intentions of the Knowledge Base Test. The Knowledge Base was introduced to guarantee mathematical knowledge for teaching as a connected body of knowledge for prospective teachers. Teacher educators expressed concerns that prospective teachers might be unable to pass the Knowledge Base Test and will drop out of teacher education, while shortly before these prospective teachers would have graduated and functioned well in teaching practice (Lit, 2011). Another concern was that teacher educators with insufficient background in mathematics foresaw that they might have difficulties when preparing their student teachers' for the Knowledge Base Test (Keijzer et al., 2012). Moreover, the nature of this test might force many mathematics teacher educators to spend a significant portion of teaching time to test preparation at the expense of paying attention to connecting the mathematical content to didactical content knowledge.

8.9 Recent Learning Materials for Student Teachers

Previous developments led to new learning materials for primary school mathematics teacher education. Currently, the market for this educational domain in the Netherlands is mainly determined by the book series *Reken-wiskundedidactiek* (Didactics

of mathematics)¹⁸ and *Rekenen-wiskunde in de Praktijk* (Mathematics in practice).¹⁹ These two book series that cover the complete curriculum for primary mathematics teacher education show many similarities in how they combine the implementation of the Knowledge Base and the Knowledge Base Test with the approach of integrating mathematical and didactical content knowledge. Both series encompass both the mathematical and didactical content knowledge described in the Knowledge Base and explicitly connect these two types of knowledge with each other. Both provide an overview of mathematical and didactical concepts in doing so. Also, the contents of both series are inspired by RME, and in both series the pillars constructive, reflective and narrative are recognisable. Each of the series pay attention to raising the mathematical literacy of student teachers towards the level required for the Knowledge Base Test. Further, both series stress student teachers' competences in learning to teach mathematics meaningfully, in stimulating students' mathematical reasoning. Other similarities include the attention paid to carrying out assignments, and the attention paid to differences between student teachers' in learning mathematics. However, the two series differ in their approach to student teachers learning to teach. The structure of *Reken-wiskundededidactiek* follows the mathematical content of the primary school curriculum. For example, the first book about whole numbers, starts with a chapter about the history and the properties of our number system, followed by a chapter about the teaching-learning trajectory of counting and number sense. The structure of the series *Rekenen-wiskunde in de praktijk* is built up along themes and big ideas of practice. For example, the first book of this series starts with an orientation on teaching practice in kindergarten followed by a chapter with a self-assessment to let student teachers figure out their knowledge of meaningful concepts.

In addition to these book series, there are also two books that focus on one part of the curriculum. *Rekenen met Hele Getallen op de Basisschool* (Veltman & Van den Heuvel-Panhuizen, 2010/2015), is a teacher education version of the teaching-learning trajectory for calculation with whole numbers developed in the TAL project (Van den Heuvel-Panhuizen, 2008). The book describes learning trajectories and intermediate attainment targets, and provides examples of activities for student teachers selected from primary mathematics textbooks. All eight chapters of the book start with a practical activity at the student teachers' own level followed by reflections. Then, each chapter discusses the relevant learning trajectory in detail, including examples of activities described in primary mathematics textbooks. In between, there are didactical tasks and ideas for student teachers' activities in school practice. In summary, it can be said that this book is in the first place a book about the didactics of whole numbers.

¹⁸This book series consists of the following publications: (Van den Bergh, Van den Brom-Snijders, Hutten, & Van Zanten, 2005/2012; Van den Brom-Snijders, Van den Bergh, Hutten, & Van Zanten, 2007/2014; Van Zanten, Van den Bergh, Van den Brom-Snijders, & Hutten, 2006/2008/2014; Hutten, Van den Bergh, Van den Brom-Snijders, & Van Zanten, 2010/2014).

¹⁹This book series consists of the following publications: (Oonk, Keijzer, Lit, & Barth, 2013; Oonk et al., 2011/2015; Oonk, Keijzer, Lit, & Figueiredo, 2016).

Rekenen+Wiskunde Uitgelegd (Ale & Van Schaik, 2011/2014) explains the primary mathematics subject matter at the level of the student teachers. These explanations are exemplified with contexts and models (e.g., bar, ratio table and number line). The text contains tips for student teachers, for example the suggestion to use special strategies for operations and examples for activities with students. In summary, it can be said that this book may be characterised as a book to strengthen student teachers' mathematical literacy before taking the Knowledge Base Test.

Although it might seem that these latter two books mean a break with the approach in which the development of mathematical proficiency and didactical proficiency are integrated, in practice it does not work out in this way. Teacher education institutions that use one of these books combine them with the book series mentioned previously.

8.10 Perspective: Searching for a Balance

Looking back over the past decades, we can observe a continuous search of in the field for a well-balanced way to interconnect the didactical education of student teachers and their development of mathematical literacy. That was—and is—not an easy enterprise. It occurred more than once that student teachers' mathematical proficiency was judged insufficient. As a consequence, many teacher educators struggled to adapt the curricula in the sense that less time could be spend on the didactical development of the student teachers in favour of more time for supporting student teachers' preparation for nationwide obliged tests.

Actually, two external forces influenced the development of curricula for primary school mathematics teacher education simultaneously. On the one hand, quality monitoring focused on the full curriculum in primary teacher education and did not signal the need for investments in the mathematics curriculum. On the other hand, concerns about primary student teachers' mathematics proficiency, also elicited by the TIMSS and PISA reports showing that mathematics results in the Netherlands went down in comparison to other countries, gave reason for introducing nationwide tests, namely two entrances tests (the Mathematical Literacy Test in 1992 and the Wiscat Test in 2006) and a third-year (the Knowledge Base Test in 2013). These nationwide tests did influence the mathematics curriculum in a sense that more focus was put on student teachers' development of mathematical literacy and less on their development of didactical proficiency. Teacher education institutions now offer a relatively large number of hours for preparing student teachers for the Knowledge Base Test (Keijzer, 2015b). Nevertheless, there are still big differences between curricula, which could be caused, in combination with the effects of the external forces, by the freedom that teacher education institutions have to organise their own curriculum.

Although teacher educators are concerned about the previously sketched situation, the majority keeps searching for new possibilities to maintain and increase the achievements of student teachers, especially their competence to integrate their mathematical and didactical knowledge.

First, the ELWleR group of primary mathematics teacher educators have focussed their practice-based research on improving mathematics teacher education. Questions addressed in this group include:

- What study load do primary teacher education institutions reserve for mathematics and how does this develop over time? (Keijzer, 2015b)
- How could horizontal content knowledge be included in primary mathematics teacher education? (Duman, 2015)
- How can high performers in mathematics be adequately supported in primary teacher education? (Kool & Keijzer, 2015)
- What are characteristics of low performers in mathematics in primary teacher education? (Keijzer & Boersma, 2017)
- How can strategies for connecting subject matter knowledge and didactics be schematised? (Keijzer & De Goeij, 2014; Keijzer, 2013).

Second, researchers on primary school mathematics teacher education presented a new approach of integrating theory and practice, called ‘enriching practical knowledge’,²⁰ as well as a way to assess and describe this knowledge in a systematic manner (Oonk, Verloop, & Gravemeijer, 2015). By analysing and discussing real teaching practice and describing their own reflections on that practice, student teachers show they are using theoretical ideas and terminology of mathematics and of teaching mathematics in a meaningful manner. In this way, practical knowledge can develop in ‘theory-enriched practical knowledge’. This approach is less or more recognisable in the current two mainly used book series for primary school mathematics teacher education.

Last but not least, there is an increasing interest in problem-oriented education in Dutch primary education (Platform Onderwijs 2032, 2016) as well in primary school mathematics teacher education. In line with this, the study group *Wiskunde voor Morgen* (Mathematics for tomorrow) (Gravemeijer, 2015) consisting of twenty experts in the area of mathematics teaching from primary to higher vocational education, is searching for an answer to the question how to adapt mathematics teaching to prepare the present generation of students for tomorrow’s society. The discussions about the development of 21st century skills in this group may also influence the goals, the content and the instructional formats of primary mathematics teacher education, not least because of the changing role of computers as an extension of human possibilities, and the increasing role of mathematics in other sciences. The latter argument reminds us of Freudenthal’s statement that mathematics (in the future—Freudenthal was talking about the year 2000) should not be taught as a separate subject, but as

²⁰The word knowledge in practical knowledge is used as an overarching, inclusive concept that summarises a variety of cognitions from conscious and well-balanced opinions to subconscious and unreflected intuitions (Verloop, Van Driel, & Meijer, 2001). Mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008) is considered as the core of practical knowledge for mathematics teaching.

a part of integrated education (Freudenthal, 1976). It requires an adapted view on learning to teach mathematics! We like to accept this challenge.

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