Dissociation and Exemplarity

Proposing a Mathematical Alternative to Heideggerian Questioning

Master's thesis

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To be frank, I am not particularly gifted when it comes to public displays of emotions of any kind—be it gratitude, joy, or anger, whether in the flesh or through the written word. I would much rather write a story. But skipping something like this will not do.

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Abstract

It remains a very good example, we just don’t know of what.

— Anne Carson, *The Albertine Workout*

Within the philosophy of Martin Heidegger, the role of modern mathematics and modern natural science is largely limited to that of “producing results” (rather than “thinking”), because, he argues, science is always ontologically founded on a being—for example, the transcendental subject—which itself remains unelucidated as to its ontological bearings. At the same time, however, the usefulness of Heidegger's alternative to scientific inquiry, philosophical questioning, is seemingly limited to the vague “pondering of essences.” I argue that the work of Albert Lautman, a French philosopher of mathematics active for only a few years on the eve of the Second World War, offers a way out of this impasse. His conception of the dissociative movement by which mathematics progresses can be understood as a mode of Heideggerian questioning, and his notion of exemplarity, by which he interprets the relation of being and beings, provides mathematics with a foundation that escapes Heidegger’s conception of modern science. This means that the activity of mathematics thus understood is not reducible to thoughtless result-production, but is in fact deeply philosophical—even in Heidegger's sense.

Introduction

The question put forward in this thesis is the following: *Can Albert Lautman's conception of mathematics provide an alternative to Martin Heidegger's mode of philosophical questioning?* Thus posed, the question requires some elucidation. What is Albert Lautman's conception of mathematics? What is meant by Heidegger's mode of philosophical questioning? Why would it be desirable to look for alternative modes, and what would an alternative mode entail? These questions will be dealt with throughout the thesis. By way of introduction, the structure of the thesis can here be given in outline.

The first chapter will deal with the questions concerning Heidegger. Throughout his work, Heidegger can be said to become progressively critical of the sciences. While he will never univocally argue that science is evil, bad, or has taken a wrong turn, the distance between science and philosophy will, in his conception, only increase. At first, he will still conceive of philosophy itself as being scientific, while in his later work he will 'locate the essence of science in the essence of technology' and will conceive of philosophy and the arts as providing a force opposing the rising danger posed by this essence of technology (Glazebrook 2000, 5; Heidegger 1977c, 33-35). The precise way in which they are opposed remains rather abstract, and Heidegger himself, for the most part, turns to readings of poems (Lysaker 2010, 204-206). What is clear is that there is an opposition between philosophy, which 'questions' or 'reflects', and science and technology, which produce results. Precisely the fact that apart from this, the notion of questioning remains rather unelucidated, is one of the reasons why one might want to look for alternatives. Another is that Heidegger, at several points, seems to associate this mode of questioning with national socialism, and argues that it must give the German people their "world."

I refer to Heidegger's *mode* of questioning because Heidegger himself, at one point, uses this term when he relates this practice to that of taking the transcendental viewpoint as in Kant (Heidegger 1967, 179). One of the questions which must be answered is the question what an alternative mode of questioning would entail, and what it would mean to provide an alternative mode. The question is, in any case, to provide an alternative mode *of* questioning, which is the same as providing an alternative *to* Heidegger's mode of questioning. Both phrases are used since it cannot from the start be clear what the alternative entails, and only a reflection on a proposed alternative can make clear to what extent it *supplements* the original mode of questioning, and to what extent it *replaces* it.

In the first chapter, the condition which an alternative should satisfy is determined. Since the thesis will be proposing a *mathematical* alternative, first Heidegger's own conception of mathematics is determined by first looking at his general stance on science, and then by analyzing his relation to Kant, whose conception of mathematics, I will argue, he mostly takes over—be it with the remark that Kant's conception of thinghood is too narrow, and Kant's own view of mathematics must thus be amended by an analysis of what, in Kant's work, has remained hidden. In contrast to his view of science, Heidegger's conception of *philosophical questioning* can be put forward. This is only done after determining his conception of science because, to a large extent, his conception of questioning is negatively defined through its relation to scientific 'calculation.' This conception of philosophical questioning is then asked after the effect it would supposedly have on mathematics, in order to show both why an alternative might be desirable, and the pitfalls a different conception of mathematics should try to escape. At the end of the first chapter, a condition can then be formulated which a (mathematical) alternative mode of questioning should satisfy.

The alternative, it will be argued here, might be provided by Albert Lautman's philosophy of mathematics, or mathematical philosophy—it has been called both. This is already suggested by Simon B. Duffy, the translator of Lautman's work to English, who writes that 'Lautman's claim for the utility of mathematical philosophy for metaphysics … runs counter to the aesthetic move that Heidegger eventually makes against the risks posed by mathematics toward the fine arts', thus challenging 'Heidegger's turn away from mathematics' (Duffy 2018, 89). Before Lautman's thought can be put forward as providing an alternative, however, several problems must be dealt with—some of them due, perhaps, to the short time in which he has been active (almost all of his work was published between 1937 and 1939, excluding one posthumous publication in 1946). One of the biggest open questions is precisely Lautman's relationship to Heidegger, since he refers to Heidegger succinctly—but at rather crucial moments.

The second chapter, then, will first try to establish Lautman's philosophy of mathematics by looking at it generally, from the point of view of philosophy, from the point of view of mathematics, and by looking at interpretations and criticisms in the (modern) secondary literature. When Lautman's philosophy has been established, it can be related to the results given in the first chapter—the condition which an alternative mode of questioning would have to satisfy, and the conception of mathematics held by Heidegger. Lautman will be shown to conceive of the foundation of mathematics rather differently from his contemporaries: rather than providing the foundation of certain basic entities from which the rest of mathematics might be derived, Lautman wants to provide a notion of existence that permeates all of mathematics equally; to this end, he conceives a rather eccentric version of Platonism. This conception can, because it is not ontologically dependent on the axiomatic structure which nevertheless characterizes mathematics, already be shown to diverge from that of Heidegger. At the end of the second chapter, it will be shown that the question whether or not Lautman can be seen to provide an alternative hinges on the sense in which one takes his notion of *exemplarity*, which is his interpretation of Plato's *participatory* relation of entities in the realm of Ideas.

One of the apparent problems will be that the notion of exemplarity can be said to be somewhat improper—in the words of Pierre Cassou-Noguès, it seems to 'short-circuit'. Either the Ideas can't be said to exist at all, it seems, and entities cannot be said to be examples of them, or the entities must be said to *precede* the Ideas, in which case the notion of example seems wrong, since, usually, an example does not precede that of which it is said to be an example, while in Lautman's case the examples seem to precede, or almost precede, that of which they are examples even *ontologically*. In the third chapter, I will argue that the apparent circularity in Lautman's notion of exemplarity is not a fault or error to be amended, but that it is precisely this "improper" notion of exemplarity by which Lautman can be said to provide a mode of questioning. To this end, I turn to the work of Jacques Derrida, and his analyses of quasi-metaphoricity in both Plato and Heidegger. Both Heidegger and Plato, it will be shown, have to use "improper" metaphors precisely because they are dealing with the very foundations of their thought, in order to provide a philosophy which does not rule out the possibility of a truth in general (in contrast to, for example, Kant's notion of truth, which according to Heidegger must remain subjective).

At the point where Lautman's conception of mathematics is shown to be a possible alternative, it will also be cleared up to what extent this is an alternative mode *of* questioning or an alternative *to* Heidegger's mode of questioning—that is, it will be cleared up to what extent Lautman's conception of mathematics can be said to *supplement* and to what extent it can be said to *replace* Heideggerian questioning.

At this point, then, the main question of the thesis is answered in theory. The third chapter is, in other words, the focal point of this thesis, both in terms of the argument made, and the addition the thesis makes to existing scholarly literature. While the first two chapters are mostly exegetical, and try to establish the connection between the philosophies of Heidegger and Lautman, the third chapter provides an original contribution to the interpretation—and the possible *use*—of both.

The fourth chapter consists of a series of examples by which I will try to show what Lautman's conception of mathematics entails *in practice*. That is, several mathematical examples are used in order to show, or begin to show, how mathematics can be read in a Lautmanian fashion, and what aspects of it become important in light of it.

This thesis can be seen to participate in several discussions. Since it puts forward a mathematical alternative mode of questioning, it can be seen as one attempt to bridge the gap which has emerged between continental philosophy and scientific practice. The large influence which Heidegger has had on perhaps all major continental philosophers has also caused them to sometimes somewhat repeat his turn away from science, or at least remain quite indifferent to questions of scientific practice. Given that Heidegger, at one point, explicitly links his thought of questioning to Nazi ideology, both the mathematical alternative proposed and the analysis what such an alternative must entail is a fruitful addition to the various debates concerning the relation of Heidegger's thought to his life and politics. Furthermore, by putting forward Lautman's philosophy as providing such an alternative, Charles Alunni's statement that his 'reference to Martin Heidegger' should have caused 'professional philosophers' to take a closer look to his work, which has instead been met with an 'oppressive silence', is taken up (Alunni 2006, 67).

On the side of the philosophy of mathematics, the reverse happens. Lautman's philosophy, almost all commentators agree, is unique within both the context of his close colleagues and of the first half of the twentieth century more generally. His philosophy can be seen to unite David Hilbert's formalism with León Brunschvicg's idea that the truth of mathematics must be seen in light of its history—in other words, Lautman's philosophy tries to account for mathematical existence *both* from the point of view of axiomatic and deductive rigor, and from the point of view of mathematics as it emerges *in practice*. Precisely concerning his notion of existence, however, thus the *foundational* relevance of his work, problems arise. Paul Bernays already in 1940 finds many aspects of Lautman's work interesting, but argues that he 'does not really give an account of what mathematical existence means' (Bernays 1940a, 21); modern commentators emphasize that, to understand Lautman's philosophy and its relevance to the foundation of mathematics, the relation of his Platonism to the philosophy of Heidegger must be cleared up. By doing precisely that, this thesis can be said to establish more univocally the foundational importance of Lautman's philosophy for mathematics.

Answering the questions left open through Lautman's short period of activity by turning to Jacques Derrida's analyses of quasi-metaphoricity is, I believe, an original and fruitful addition to the literature on Lautman's philosophy, which only recently has begun to receive more attention, and to the research on Heidegger, whose turn away from science and mathematics leaves a lot to be desired.

Chapter 1: Heidegger

Mathematics and Philosophical Questioning

The question to be answered in this chapter is that concerning the condition or conditions which an alternative mode of Heidegger's notion of questioning should satisfy, and why such an alternative might be desirable. This will be done in several steps. First, Heidegger's general stance on science—and whether there is one—will be determined (§1, §2). Followingly, Heidegger's conception of mathematics specifically (§3). Then, Heidegger's thought of philosophical questioning and its relation to scientific inquiry will be determined (§4). Finally, after the effect of philosophical questioning as Heidegger conceives it on mathematics is assessed (§5), a condition which an alternative to this mode of questioning—in the case of this thesis, Lautman's mathematical alternative—must satisfy will be put forward (§6).

§1. Heidegger's Conception of Science: Context

Heidegger's general stance on science cannot be determined without answering the question whether or not he has such a stance at all, and in this case, that claim has not gone uncontested. Joseph J. Kockelmans argues that 'Heidegger has never developed a systematic philosophy of science', even if he admits that Heidegger's philosophy has a bearing on 'several issues which are of vital importance for a philosophy of science' (Kockelmans 1970b, 184). This would imply that asking after Heidegger's general stance on science is, if not illegitimate, at least poorly put. Trish Glazebrook, however, argues that Heidegger certainly *is* a philosopher of science in one respect, namely when it comes to the question of 'what constitutes science' (Glazebrook 2000, 1). One might argue that Kockelmans does not deny that Heidegger can be seen as a philosopher of science *in some respect*, but Glazebrook argues more strongly that the way in which Heidegger is concerned with the constitution of science remains consistent throughout his entire work; he can indeed be seen, then as putting forward a 'systematic philosophy of science' when it comes to this question.

Glazebrook, who has written the most sustained account on Heidegger as a philosopher of science, points out that 'over several decades, he explores the thesis that science is the mathematical projection of nature. … This conception of science binds together his thinking of the question of science over sixty years' (Glazebrook 2000, 1). The various reformulations of this stance are not so much signs of previous failure, she argues, but symptomatic of Heidegger's insight 'that the relation between thinking and science is not what he has previously taken it to be' (Glazebrook 2000, 2-3). That is, if Heidegger is fundamentally concerned with the constitution of science, and if he's interested in the relation between science and philosophy, then progressive insights into the nature of philosophical thought will automatically result in reformulations of the relation of thought to science.

Glazebrook distinguishes three phases of Heidegger's philosophy of science: an early view characterized by the fact that 'he held that philosophy is itself scientific', a transitional phase 'in which he turns to questions of scientific practice and away from problems of philosophy', and a final phase 'in which he locates the essence of science in the essence of technology.' Throughout all of these, the thesis itself—that science is "the mathematical projection of nature"—remains (Glazebrook 2000, 5).

In order to determine Heidegger's general stance on science from the point of view of philosophy, then, one might look at two moments in his course of thought. Firstly, the moment where the first phase flows into the second: before Heidegger turns away from problems of philosophy, but at the point at which he does no longer consider philosophy itself a science. That would be the moment where Heidegger's conception of science is most developed in its own right, and which thus gives us the most mature conception of the thesis that science is the mathematical projection of nature—which will thus help make clear what this thesis means at all. Secondly, a moment in the last phase, where 'the essence of science is [located] in the essence of technology.' There it would be a question of situating Heidegger's eventual conception of science, determined by looking at the first moment, within his eventual reflections on technology. By approaching the question in such a way, one will be able to establish Heidegger's conception of science in its own right, without neglecting his eventual turn towards questions of technology

Only in conclusion will the conception of science determined in this chapter be situated, somewhat, within Heidegger's reflections on technology. Most of this chapter, however, will deal with the first moment. To this end, the focus will lie on Heidegger's work *What is a Thing?* (also translated as *The Question Concerning the Thing*), because this is a work in which Heidegger has abandoned the view that philosophy is a science, but is still concerned with the constitution of science more than questions of scientific practice (Glazebrook 2000, 17).

To supply the reading of *What is a Thing* some context, one can first look at relevant passages from his early major work, *Being and Time*, and from the 'transitional' *Contributions to Philosophy (Of the Event)*, which was written shortly after the lecture course of which *What is a Thing?* is the text was given—and both of which deal with questions of science and its constitution.

In *Being and Time*, as Glazebrook argued, the relation of philosophy to the sciences is still one of a specific science to the other sciences. This can be seen as a particularly Husserlian influence (Glazebrook 2000, 3; Kisiel 1970, 168). If Heidegger, as Glazebrook argues, at some point conceives of this relation as radically different and perhaps even oppositional, it can be ascribed to Heidegger only later on.

Heidegger's view of science in *Being and Time* is set forth as early as the third section, after introducing the question of the meaning of being and elaborating on its formal structure and urgency. Heidegger considers this question ontologically prior to the positive sciences. He defends the question against accusations that being is (simply) the most universal concept, that it is indefinable, and that we (nevertheless) supposedly already understand what we mean by it. Even if it were the most universal concept, he argues, this would only emphasize its obscurity, not its clarity; and even if it were indefinable, this would not dispose of the question, but force it upon us even more strongly. That we already know what we mean by it, even if we cannot define or clarify it, lastly, only shows that 'an enigma lies *a priori* in every relation and being towards beings as beings' (Heidegger 2010, 1-3).

In other words, Heidegger does not so much disagree with these accusations as much as that he argues that they pose no argument *against* but *for* the priority of the question of the meaning of being. The last accusation, that we somehow already know what we mean by being—our pre-ontological understanding of being, as he calls it—indeed occupies an important place in his thought.

'Scientific research,' Heidegger now writes, 'demarcates and first establishes [various] areas of knowledge in a rough and ready fashion.' They do so through 'pre-scientific experience' and 'interpretation of the domain of being to which the area of knowledge is itself confined', which results in fundamental concepts. The true progress of science, Heidegger argues, lies not so much in collecting results as it does in 'being forced to ask questions about the basic constitution of each area' (Heidegger 2010, 8).

His early view on the progress of science he puts like this:

> The real "movement" of the sciences takes place in the revision of these basic concepts, a revision which is more or less radical and lucid with regard to itself. A science's level of development is determined by the extent to which it is *capable* of a crisis in its basic concepts. In these immanent crises of the sciences the relation of positive questioning to the matter in question becomes unstable (Heidegger 2010, 9).

He then sums up a few contemporary examples, among which is the debate in the philosophy of mathematics between formalism and intuitionism, which according to him 'centers on obtaining and securing primary access to what should be the object of this science' (Heidegger 2010, 9).

The 'preliminary research that creates the fundamental concepts', which might in a crises undergo revision, Heidegger describes as a 'leap ahead' 'into a particular realm of being' which 'discloses it for the first time' and 'makes the acquired structures available to the positive sciences' (Heidegger 2010, 9-10). This preliminary, interpretative inquiry which constitutes (and revises) the positive sciences is thus *ontological* inquiry. One might interpret the difference between the ontic inquiry of the positive sciences and such ontological inquiry as being the difference between mathematics and the philosophy of mathematics, or physics and the philosophy of physics.

The ontological inquiry will however 'remain naïve' if it '[leaves] the meaning of being in general undiscussed' (Heidegger 2010, 10). This is a more specific formulation of what Heidegger already argues in defense against the claims that that the meaning of being is supposedly already known to us: as long as we do not clarify what we mean by being, or do not even know how to clarify it, relying on what we think we already know—and even if this is true—we cannot grasp the basic constitution of various fields of knowledge with clarity, and the *meaning* of our scientific research and its results will remain obscure to us.

To sum up, in *Being and Time* Heidegger conceives of a three-part structure in which the various positive sciences are grounded by their various ontologies, and ontological inquiry itself is guided by investigations into the meaning of being, into 'the *a priori* enigma' preceding every relation toward beings as beings—what Heidegger here still calls fundamental ontology.

Some ten years later, much of this will have changed. Indeed, by the time of writing *Contributions to Philosophy*, Heidegger has a rather oppositional view of philosophy and science, where science is by its very nature 'specialized'—there is no such thing as science in general, of which Heidegger still wrote in *Being and Time*—while philosophy is *always* philosophy in general. Furthermore, according to Heidegger, there is *only* positive science—the sciences make use of general concepts which never undergo radical questioning in the sense envisioned in *Being and Time*—and philosophy itself is something radically different (Heidegger 2012, 114).

Heidegger thus still conceives of the sciences as researching something 'pre-given' which the sciences themselves can never grasp as such; there is still a kind of basic constitution. 'Beings, *as a region*, are something available for science; they are a *positum*, and every science (even mathematics) is in itself *"positive" science*' (Heidegger 2012, 114). That the sciences themselves can *never* grasp this basic constitution makes one wonder about what Heidegger earlier pointed out as being the true movement of the sciences, namely, the questioning of this basic constitution. And indeed, despite all the examples given in *Being and Time*, Heidegger now writes that 'it must be admitted … that talk of a "crisis" of science was in fact mere babble.' The essence of science drives forward 'unchanged' and 'not self-alterable' toward its ‘extreme end-state’, and this excludes ‘any possibility of a “crisis” of science or, in other terms, any essential transformation of knowledge and truth’ (Heidegger 2012, 117).

In fact, the two changes pointed out here—that philosophy is no longer regarded as a science grounding the various other sciences, and that a crisis of science is no longer conceived of as possible—are intimately connected. The three-part structure in which the various positive sciences were grounded by various regional ontologies, and the regional ontologies themselves by fundamental ontology, the inquiry of the meaning of being, has lost its middle part. Philosophy is one, and the sciences are multiple, and they can only indirectly affect each other—there are no regional ontologies properly asking after the meaning of the being of the beings which a particular science studies, questions which themselves might be guided by fundamental ontology.

It must be noted here that Heidegger is commenting on science ‘in its current actual constitution’ (Heidegger 2012, 113). That is, he is speaking about science in its *current* actual constitution—the contemporary talk of a crisis was mere babble—but he is also speaking about science in its current *actual constitution*, the sciences *are* currently constituted in such a way so as to exclude even the possibility of crisis. The ambiguity inherent in this statement—what it would take for the sciences to be constituted differently—is something that cannot be directly treated here, although the results of this thesis will have some bearing upon the question.

Interestingly, Heidegger feels the need to emphasize that *even* mathematics is a positive science, to which the region of beings is pre-given. Indeed, later on he emphasizes that ‘even mathematics’ requires ‘experience’ or ‘the simple cognizance of its simplest objects and of their determinations in axioms’ (Heidegger 2012, 117). Heidegger then still conceives of the sciences, including mathematics, as being founded on ‘basic concepts’, and by now he adds that this foundation is an *axiomatic* foundation. But even if Heidegger tries not to make an exception for mathematics, by such insistencies he at least makes of mathematics an exceptional case. Apparently, mathematics marks something of a limit (*even* mathematics is subject to the general condition of the sciences). The relation of mathematics to (other) exact sciences for now remains an open question.

To sum up, Heidegger first conceives of philosophy and science in a three-part structure, where the various positive sciences, producing results, are grounded by their respective regional ontologies, which ground them by providing basic concepts and which measure their development by the capability of a crisis in these concepts. These regional ontologies are themselves guided by investigation into the meaning of being, fundamental ontology. Later, Heidegger disposes of the possibility of a crisis in such a sense, and argues that the sciences (in their 'current actual constitution') are fully incapable of asking after their basic concepts, everything that might have seemed like it, was in fact 'mere babble.' The basic concepts of all the sciences, drawn from experience, are according to Heidegger established in fundamental axioms, from which all scientific activity then derives.

This last fact constitutes the major difference between Heidegger's early view in *Being and Time*—where philosophy was still a science among sciences—and his later view. The various regional ontologies are, in a sense, replaced by 'the simple cognizance of [science's] simplest objects and … their determinations in axioms.' Throughout *Contributions to Philosophy*, Heidegger does not elaborate on these 'axioms' replacing the work of the regional ontologies from *Being and Time*, only hinting, at some point, that they are related to the role of the "I" in Descartes's *cogito ergo sum*, and the sense of certainty presupposed in establishing the subject there (Heidegger 2012, 116). Given that *Contributions to Philosophy* was written shortly after delivering *What is a Thing?*, the question as to the nature of these axioms replacing regional ontology might serve as something of a guideline in the next section.

§2. Heidegger's Conception of Science: In *What is a Thing?*

Having thus sketched the early progress of Heidegger's conception of science—from a three-part view of positive, ontic sciences, their regional ontologies and fundamental ontology to a two-part view where positive sciences are founded upon axioms which determine their simplest objects, and are essentially opposed to philosophy and inquiry into the meaning of being—the question becomes what gives rise to this change in view. The lectures in *What is a Thing*? trace Heidegger's conception of science as mathematical projection—as Glazebrook puts it—to 'the grounding function of transcendental subjectivity in Cartesian and Kantian idealism' (Glazebrook 2013, 339). What this entails, precisely, remains to be seen. In any case, while in *Being and Time* Heidegger envisioned preliminary investigations as disclosing various fields of scientific knowledge ahead of research, in *What is a Thing* Heidegger will actually define 'the mathematical' precisely 'as what is a priori rather than being found in experience'—the sciences, thus, will be delimited by 'the imaginative projections of transcendental subjectivity' as described by Kant (Glazebrook 2013, 339). *What is a Thing?,* then, can be used to more rigorously determine Heidegger's conception of science in general and mathematics specifically.

Seeing as how *What is a Thing?* originated as a lecture course on Kant's *Critique of Pure Reason*, its contents must be situated with regard to Heidegger's own thought—that is, it must be clear in which measure the work is merely exegetical of Kant's position. Glazebrook argues that Heidegger considers Kant's project a failure by falling pray to idealism, and writes that Heidegger's analytic of Dasein 'is an attempt to achieve the aim of the first *Critique*' (Glazebrook 2000, 42). Heidegger's relation to Kant would then be one of rejection (Glazebrook 2000, 8). Various other scholars, however, do not so much speak of a relation of rejection but of delimitation. 'Heidegger delimits the mathematical project through an existential analysis of it', writes Michael Roubach (1997, 200)—where the mathematical project is thus the delimitation of science by 'the imaginative projections of transcendental subjectivity', as Glazebrook wrote. Heidegger's analytic would thus not *replace* but *delimit* the *Critique*. David Farrell Krell too writes that *What is a Thing?* 'accomplishes essential steps in the destruction of the history of ontology' announced but unfinished in *Being and Time* (Heidegger 1997a, 246)—where destruction must be understood, as Heidegger wrote back then, as the 'dissolution of the concealments produced by' the philosophical tradition (Heidegger 2010, 21). In other words, Krell too sees the work not so much as rejecting Kant, but as elucidating what remains 'concealed' in his work. Frank Schalow, finally, quotes a passage in which it becomes clear that Heidegger's criticism of Kant is one of *narrowness*, not of failure (Schalow 1992, 333). Kant, according to Heidegger, 'immediately fixes on the thing as an object of mathematical-physical science', and this is where he conceals the possible question after the thingness of things more generally (Heidegger 1967, 128).

Especially this last fact suggests that when it comes to Kant's views concerning mathematics itself, Heidegger does not so much reject them as much as that he finds they require further explication of their foundation. Kant does not ask the *wrong* question, but he *fixes* upon one narrow possibility of answering it. Of course, from the point of view of a philosophy of science—which is what Glazebrook is trying to retrieve from Heidegger—this might be argued to be a rejection: Kant's foundation of the sciences is not enough of a foundation for Heidegger, it is itself in need of further grounding.

Before reading *What is a Thing?*, already from this discussion of Heidegger's relationship to Kant one might anticipate the conclusion that Heidegger does not have any view of mathematics other than that which he ascribes to Kant, since Heidegger does not reject Kant, nor his views, but tries to situate them within a more fundamental analysis. Reading the work might thus be guided by two questions: what precisely is it that remains concealed within Kant's work, and how does the elucidation of these concealments affect the conception of mathematics put forward by Kant himself? The answers to these two questions would together result in something of a 'conception of mathematics' of Heidegger himself; if Heidegger's relation to Kant is indeed not one of absolute rejection but of delimitation, Kant's conception amended by a discussion of what, in Kant's conception, remains unthought, would in effect be Heidegger's conception of mathematics.

What is a Thing? is divided into two large parts, the first of which is an introduction to the main question of the title, and the second of which deals with Kant's *Critique*. It is useful to work through both parts, because the first part will provide the most comprehensive account Heidegger gives of science as 'mathematical projection', and will thus clear up this notion, and the second will elucidate Heidegger's relationship to Kant.

Heidegger deems the introduction to the question necessary because, he says, 'philosophy is that thinking with which we can begin to do nothing immediately', and the introduction, in sum, provides a preliminary answer to the question "What is a thing?", namely, that 'a thing is the bearer of properties, and the corresponding truth has its seat in the assertion, the proposition, which is a connection of subject and predicate' (Heidegger 1967, 39). He continues to say that, although this answer and the reasons for it are 'quite natural', or 'self-evident', it has not always been so, thus, this natural and self-evident answer is in fact historical. This does not merely mean, for Heidegger, that the question and its answer have a past, but also that in questioning, something of the past is 'still happening', and in questioning we must, he argues, 'remain equal to this happening' so that we can truly develop the question we are asking (Heidegger 1967, 39, 44). To question historically means to 'set free and into motion the happening which is quiescent and bound in the question', 'setting into motion the original inner happening of this question according to its simplest characteristic moves, which have been arrested in a quiescence'—it is not a matter of correcting a previous answer, then, but undoing the historical sedimentations that make the answer look like something natural, which is 'an indifferent falsification' (Heidegger 1967, 48-49). Here again, we see that Heidegger's relation to this conception of thinghood and truth is not a matter of rejection or of differing opinions, but of situating or delimiting a certain determination.

This will not change the "natural" answer to the question—the determination of the thing as something present-at-hand (*Vorhanden*) and the corresponding truth as proposition, as he had already shown in *Being and Time*, which has, according to Heidegger, an 'unshattered preeminence'—but it unsettles the manner in which the question was posed, because instead of looking for a simple answer, it prepares a renewed determination of the thing (Heidegger 1967, 48, 52). How so? Because the question "What is a thing?"

> is not a proposition but a transformed basic position or, better still and more cautiously, the initial transformation of the hitherto existing position toward things, a change of questioning and evaluating, of seeing

> and deciding; in short, of the Being-there (*Da-sein*) in the midst of what is (Heidegger 1967, 50).

That is, if we understand this question to require a simple propositional answer, we in fact fall pray to a certain circularity. By answering the question concerning what something *is* (in this case a thing) with an answer in the form of a proposition can precisely only be done on the basis of the determination of the thing which was just said to be historical rather than "natural"—the thing as bearer of properties and the corresponding truth as proposition connecting a subject and a predicate. What Heidegger wanted to do, however, was to ask the question anew, instead of falling back on this supposedly "natural" determination. This is why Heidegger tells us that, instead of looking for a simple answer, the question is posed here so as to prepare a renewed determination of the thing. The question put "historical" in Heidegger's sense must precede the determination of thinghood—and precisely therefore a merely propositional answer cannot suffice from the start, and we cannot expect the answer to take such a form.

Here, then, there begins to arise a difference between "historical" or philosophical questioning and scientific inquiry. The problem of trying to escape the nowadays natural determination of thinghood and its corresponding truth results in a more opposed relation between philosophy and science. If the scientific "axioms" by way of which the basic objects of science are determined are caught up in this "natural" determination, then the questioning enacted by philosophy, 'with which we can begin to do nothing', and which, it seems, only *prepares* a renewed decision as to the thinghood of things, is something radically different. Indeed, according to Heidegger, 'what most holds us captive and makes us unfree in the experience and determination of the things' is 'modern natural science, insofar as it has become a universal way of thinking along certain basic lines.' It must thus become clear which 'basic lines' these are, according to Heidegger, in order to elucidate the difference between science and philosophy.

It is important to note, here—since the question of the relation between science and mathematics remained somewhat open—that Heidegger excludes mathematical entities like numbers, and mathematical symbols like

‘<’, from the “natural” determination of thinghood to begin with. Heidegger does not deny that these are *something*, but they do not belong, according to him, to the dominant sense of thinghood (Heidegger 1967, 6-7).

Summing up this introduction, Heidegger thus emphasizes that the question concerning the thing is tied up with the history of that question, and of questioning itself; asking it in the way Heidegger envisions requires one to expect something other than an answer in the form of the proposition or assertion. Modern natural science is an obstacle in renewing the question in such a way, but it is not a matter of “correcting” it—the conception of truth as correspondence to a thing, a correspondence which can be true or false and might merely be corrected, is itself part of modern natural science as Heidegger conceives of it.

A few things remain unclear. The relation of mathematics to science is vague, for on the one hand, Heidegger is keen on emphasizing that it is a science among others, in need of basic objects given to it beforehand, on the other hand, it does not concern itself, apparently, with ‘things’ in the same way that modern natural science supposedly does. Heidegger will indeed later write that ‘mathematics is as little a natural science as philosophy is one of the humanities’, ‘philosophy in its essence belongs as little in the philosophical faculty as mathematics belongs to natural science’ (Heidegger 1967, 69). Two relations are thus of importance if the conditions which an mathematical alternative mode of philosophical questioning would have to satisfy are to become clear: the distinction between philosophical questioning and science, and the relation between science and mathematics. Concerning the first relation, it seems that philosophy is different from science insofar science is founded upon a determination of thinghood which isn’t as natural or self-evident as it would seem, while philosophy is concerned with posing the question of thinghood anew—and the propositional nature of answering questions seems to be a key axis. The second relation Heidegger has not yet cleared up: on the one hand, mathematics is regarded as a science like others, on the other, its notion of thinghood seems to be different. Seeing as how this difference is precisely crucial to the first relation, both relations must be further elucidated if we are to retrieve Heidegger’s conception of mathematics.

The second part of *What is a Thing?* is itself divided into two parts, the first assessing the *Critique of Pure Reason* in light of its historical situation, the second actually reading and working through the *Critique* itself.

Two questions arose concerning Heidegger's relation to Kant. After discussing the secondary literature, it seemed plausible that Heidegger does not so much reject or accept Kant as much as that he takes up Kant in order to find what has remained hidden in his work. In other words, when reading *What is a Thing?* it was important to keep wondering to what measure it is merely exegetical of Kant's work. The two guiding questions concerning the matter were *what* remained hidden in Kant's work, and how its elucidation would affect the conception of mathematics present in this work as Heidegger sees it. Heidegger would seemingly not have any other comment than that this determination of thinghood is too narrow, and the determination of what remains concealed would be Heidegger's own "addition" to Kant.

At the beginning of the first part, Heidegger writes that 'henceforth, only Kant shall speak.' From that it must not be concluded, however, that the whole work is merely exegetical, for Heidegger writes that he will turn 'our question'—the question concerning the thing—into Kant's question and vice-versa (Heidegger 1967, 56). In an overview of the history of Kant's work, he argues that both the German Idealists and the Neo-Kantians have failed to engage with what he calls Kant's 'basic position' (*Grundstellung*), while this is precisely what must be done. Kant cannot be rejected, followed or surpassed, according to Heidegger, only passed by (Heidegger 1967, 60-61). Heidegger then quickly sketches a development from Greek *logos*—saying something about something—through medieval *ratio* to German *Vernunft*—reason—in order to link Kant's *Critique of Pure Reason* to the question "what is a thing?" (Heidegger 1967, 62-65). This is because he wants to show that the necessity and possibility of Kant's *Critique* is a crucial moment in the history of that question.

It does seem correct, then, to argue that Heidegger's relation to Kant and Kant's *Critique* is not one of acceptance, rejection or surpassing, seeing as how he himself deems such a relation naïve and in fact impossible.

In the large section which follows, Heidegger will try to clarify the 'basic feature' of modern natural science with the intention of understanding 'the possibility and necessity of something like Kant's *Critique of Pure Reason*' (Heidegger 1967, 66). This basic feature is that 'modern science is *mathematical*' (Heidegger 1967, 68). He does not mean, by this, that it makes use of mathematics. According to Heidegger, mathematics is 'only a particular formation of the mathematical' (Heidegger 1967, 68-69). We must, then, keep distinguishing between mathematics and the mathematical. When Heidegger tells us that modern natural science is not mathematical because it is numerical, but numbers are something mathematical, he means mathematical in this broader sense which he is trying to pin down (Heidegger 1967, 70). The mathematical, he writes, is 'a taking where he who takes only takes what he actually already has', and this 'most difficult learning is to come to know all the way what we already know' (Heidegger 1967, 73). He tries to elucidate this with an example:

> The *mathemata*, the mathematical, is that "about" things which we really already know. Therefore we do not first get it out of things, but, in a certain way, we bring it already with us. From this we can now understand why, for instance, number is something mathematical. We see three chairs and say that there are three. What "three" is the three chairs do not tell us, nor three apples, three cats nor any other three things. Moreover, we can count three things only if we already know "three." In thus grasping the number three as such, we only expressly recognize something which, in some way, we already have. This recognition is genuine learning. The number is something in the proper sense learnable, a *mathema*, i.e., something mathematical. Things do not help us grasp "three" as such, i.e., threeness. "Three"—what exactly is it? It is the number in the natural series of numbers that stands in third place. In "third"? It is only the third number because it is the three. And "place"—where do places come from? "Three" is not the third number, but the first number. "One" isn't really the first number. For instance, we have before us one loaf of bread and one knife, this one and, in addition, another one. When we take both together we say, "both of these," the one and the other, but we do not say, "these two" or 1+1. Only when we add a cup to

> the bread and the knife do we say "all." Now we take them as a sum, i.e., as a whole and so and so many. Only when we perceive it from the third is the former one the first, the former other the second, so that one and two arise, and "and" becomes "plus," and there arises the possibility of places and of a series. What we now take cognizance of is not created from any of the things. We take what we ourselves somehow already have. What must be understood as mathematical is what we can learn in this way (Heidegger 1967, 74-75).

Heidegger, it seems, does not (yet) want to determine the mathematical more precisely—or at least he does not do so. One of the most clear sentences is near the end, 'what we *now* take cognizance of is not created from any of the things', where what had just been taken cognizance of is the "plus" that arose from the earlier "and", the "one" and "two" which arose from what had just been "both." These were not taken from the things, Heidegger argued, but we somehow already have them and are able to learn them—take them from ourselves—as soon as we took the bread, knife and cup as "all", as a sum, 'a whole and so and so many.' This taking-as-a-sum then is something mathematical which we somehow already have.

What can be determined at least, then, is that the mathematical symbols and entities first excluded form the discussion concerning the notion of thinghood prevalent throughout modern natural science *return* in the determination of modern natural science as 'mathematical' in this still somewhat vague sense. The argument, however, remains unclear. Michael Roubach has argued that three is the first number because it 'establishes the number series', and only after it has been established, 'we go back and characterize the second and first places in the series, rendering two and one as numeric characterizations' (Roubach 2008, 85). That is, 'one, as a characterization of an entity, does not yet make reference to a place in the number series, and two can be characterized as a pair, which necessitates neither reference to the number series, nor reference to an order within the pair' (Roubach 2008, 84). Elsewhere, he summarizes this by saying numbers 'are mathematical because we assume them in counting' (Roubach 1997, 201). This interpretation clears up Heidegger's argument somewhat, but does not yet quite satisfy. First of all, one can imagine arrangements of three elements which do not establish the number series—for example, of three elements in a circle. This would yield the notion of "betweenness," which is *some* kind of ordering, but not the ordering if *the* number series. On the other hand, the interpretation is not satisfying because even if we *can* characterize two without reference to the number series, this does not exclude the possibility of characterizing two *with* reference to the number series.

The insight that three, as a number, by necessity makes reference to an order—where a pair of two do not *necessarily* do so, even if they *can*—is perhaps a fruitful addition to Heidegger's example, but this does not tell us why the first number is not, for example, seventeen. Characterizing three as the first number because it is the first one *following* one and two necessarily referencing an order or number series seems like a circular argument. That we do not ordinarily say "these two" but "both" does not prevent us from doing so; we do not need the presence of cups to take bread and knife 'as a whole and so and so many.'

The only thing which is clear is that Heidegger wants to argue that precisely this taking something as a whole, as a sum—be it of three, seventeen, one or a million—is something we do without learning it from the things thereby counted. We do not learn numbers from things that are so and so many, rather, it is because we are capable of taking things as being so and so many that we can count them. *How* we are capable of this, where this ability to take sums (and thus "sums", "plus", "three") comes from, at this point is still unclear.

Now after considering changes from Aristotle, through Galileo, to Newton and the physics of Kant's day, Heidegger sums up his determination of the mathematical. This summary does not give us a view of actual modern science, but is merely 'the fundamental outline' along which 'the entire richness of posing [scientific] questions and experiments, establishing of laws and disclosing of new districts of what is [unfolds]' (Heidegger 1967, 94)—these are thus the 'basic lines' along which one might begin to distinguish philosophy and science:

1. The mathematical is a 'project of thingness' which 'skips over things' and 'first opens up a domain (*Spielraum*) where things … show themselves' (Heidegger 1967, 92).

2. This projection posits 'that which things are taken as' in 'anticipating determinations and assertions', in fundamental propositions or axioms (Heidegger 1967, 92).

3./4. The axiomatic nature of the mathematical projection anticipates the essence of things. It is both the blueprint for things and their relations amongst themselves, as the measure for 'the realm, which, in the feature, will encompass all things of that sort.' More specifically, nature has become 'the realm of the uniform space-time context of motion', Heidegger writes (Heidegger 1967, 92).

5. Things are only 'what they *show* themselves as, within this projected realm.' This allows one to pose questions to which nature must answer 'in one way or another', i.e., to perform experiments (Heidegger 1967, 93).

6. Mathematics in the narrow sense, 'a particular kind of mathematics,' first became possible and, above all, necessary, on the grounds of the basically mathematical character of thinking', and not vice-versa (Heidegger 1967, 93-4).

The first five points, together, seem to coincide with a conception of science already suggested by *Being and Time* and especially *Contributions to Philosophy*. Science has been given basic concepts and objects through this projection which has 'leapt ahead,' and has secured them in the form of axioms. Only on the basis of this projection are experiments possible, not vice-versa.

The sixth point can help elucidate the strange position of mathematics in the narrow sense within all this. It is not a natural science. The fundamental trait of modern natural science is that it is *mathematical*. Modern mathematics is a particular formation of the mathematical which became possible and necessary because of the projection which establishes modern natural science. In any case, this implies that other kinds of mathematics are or were possible, have existed or could perhaps exist—i.e., there is room within this scheme for something like Ancient mathematics as differing from modern mathematics. It must become clear, now, why modern mathematics as a 'particular formation of the mathematical', this 'particular kind of mathematics', becomes possible and necessary only on the basis of the project which leaps ahead and discloses things ahead of time when mathematics does not seem to concern itself with such things. That is, if the relation between science and philosophy is now somewhat clear, the relation of mathematics to science must be cleared up.

Their complicity has to do with the axiomatic character that Heidegger ascribes to the modern mathematical project. This axiomatic character replaced the earlier conception of science as being grounded by various regional ontologies. It will take some time to clear this up.

Heidegger turns to Descartes in order to do so. Descartes did not become a doubter, Heidegger writes, 'because he was a skeptic', but 'because he posits the mathematical as the absolute ground' (Heidegger 1967, 103). Beginning to philosophize through doubt, as Descartes does, presupposes a certain conception of certainty and truth. 'If mathematics, in the sense of a *mathesis universalis*, is to ground and form the whole of knowledge, then it requires the formulation of special axioms', Heidegger writes (Heidegger 1967, 102). That is, the notion of *certainty* and truth implicit in the method of radical doubt determines in advance the establishment of truths in axioms—propositions regarded as self-evident or accepted, whose truth is indubitable. This, according to Heidegger, also entails a change in the conception of what propositions are. 'Up till now,' he writes, 'the proposition had been taken only as what offered itself, as it were, of itself. The simple proposition about the simply present things contains and retains what the things are' (Heidegger 1967, 103). Within the mathematical project, these *things* themselves cannot be presupposed. One cannot begin with a proposition that 'offers itself' retaining things which are 'simply present.'

That is why, says Heidegger, the beginning must be *positing* itself. The proposition which would not depend on anything else is the proposition 'I posit', and the determination of 'thinking' as always being an "*I* think." The thingness of things in the mathematical project will be determined 'out of the "I am" as the certainty of the positing' (Heidegger 1967, 104). Before Descartes, anything 'present-at-hand for itself' was a subject, but Descartes raises the I to be the 'special subject' 'with regard to which all the remaining things first determine themselves as such', i.e., become objects (Heidegger 1967, 105). Only from Descartes on does the guideline for reason become 'the subjectivity of the subject', and every assertion 'must always posit what lies in the *subjectum*.' From this, Heidegger writes, also flows forth the principle of non-contradiction: what is thought must be able to be thought in the unity of the subject (Heidegger 1967, 107). This is for Heidegger the basis on which Kant's project becomes possible and necessary: 'the principles of mere reason are the axioms of pure reason', and 'the question about the thing is now anchored in pure reason, i.e., the mathematical unfolding of its principles' (Heidegger 1967, 107-108).

Heidegger already in *Being and Time* wrote that 'Kant dogmatically adopted Descartes' position' concerning this matter, and what prevented Kant from 'insight into the problem of temporality', with which Heidegger was there concerned, was besides 'the neglect of the question of being in general' specifically 'the lack of a thematic ontology of Dasein or, in Kantian terms, the lack of a preliminary ontological analytic of the subjectivity of the subject' (Heidegger 2010, 23).

Heidegger's delimitation of Kant in *What is a Thing?*—the emphasis that Kant 'immediately fixes' 'on the thing as an object of mathematical-physical science' (Heidegger 1967, 128)—is then virtually the same as that in *Being and Time*. Not posing the question concerning the thingness of things is the other side of the coin of not inquiring into the subjectivity of the subject. This, then, answers one of the two questions concerning Heidegger's relation to Kant: what remains hidden in Kant's *Critique of Pure Reason* is this lacking 'preliminary ontological analytic of the subjectivity of the subject.'

This also explains why Heidegger could not be more clear than writing that the mathematical is something we somehow already have. This something is precisely 'what lies in the *subjectum*', and what this *is*, precisely, is what has not been properly brought to light.

Furthermore, this gives a way to determine the relation of mathematics with regard to the mathematical. The particular formation of mathematics which becomes possible and necessary on the basis of the mathematical projection is that formation of mathematics which builds upon the same implicit notion of *certainty* and thus of truth; it is the axiomatic structure of proceeding from first propositions which establish self-evident truths. The main difference between modern mathematics and ancient mathematics is that the final ground of such self-evidence has become the subject, the "I think" at the heart of thought. It is the role of this subject which thus must be clear if we are to understand what an alternative conception of mathematics would have to look like.

§3. Heidegger's Conception of Mathematics

With the execution of the critique of pure reason, writes Heidegger, 'the "mathematical" in the fundamental sense first comes to its unfolding and, at the same time, to its being lifted up (*Aufhebung*), i.e., to its own limit' (Heidegger 1967, 121). That is, the *Critique* both positively founds and delimits reason; we do have 'a priori knowledge'—the sensible world 'necessarily conforms to certain fundamental laws'—precisely because 'the human mind constructs it according to those laws', and this is knowledge of 'any possible human experience', but this also means, Kant writes, that 'we can cognize of things a priori only what we ourselves have put into them', and thus 'we cannot have a priori knowledge about things whose existence and nature are entirely independent of the human mind' (Rohlf 2018, §2.2). A critique of pure reason thereby delimits the 'determination of the being of what is' 'from out of pure reason', Heidegger says, and thereby already the mathematical character of modern metaphysics is retained (Heidegger 1967, 122). Indeed, Kant precisely writes that we can 'cognize only' of things what we 'have put into them' ourselves, and this was how Heidegger characterized the *mathema* as something 'properly learnable.' The mathematical, in Heidegger's sense, is basically the Kantian project of founding and limiting the possibility of *a priori* knowledge on the transcendental subject.

Heidegger's focus in reading the *Critique* is on the second book of the Transcendental Analytic, and more specifically on the second chapter, the 'System of all principles of pure understanding.' He gives various reasons for considering this the center of the *Critique*, and argues that it was likewise at the center of it for Kant himself (Heidegger 1967, 124-127). One of Heidegger's preliminary comments is, as was said before, that 'Kant does not pose the thingness of the things that surrounds us' and that his view 'immediately fixes itself on the thing as an object of mathematical-physical science' (Heidegger 1967, 128). Heidegger emphasizes that this narrow focus cannot simply be amended by adding, so to speak, the missing things. Since ‘the definition of the thing and the way it is set up include fundamental presuppositions which extend over the whole of being and to the meaning of being in general’, Kant’s narrowness of view results from a narrow interpretation of the meaning of being. To amend his project would require, as Heidegger wrote already in *Being and Time*, reflection on the meaning of being in general. All of this more or less repeats the criticism Heidegger has repeated again and again, that Kant founds the existence of objects on a subject whose own existence remains unelucidated.

Heidegger writes that Kant’s use of the word ‘object’ in both a proper narrow sense and an ‘improper wider sense’ already indicates ‘that Kant has broached and decided the question of human knowledge and its truth only in a certain respect’:

> Kant has disregarded what is manifest (*das Offenbare*). He does not inquire into and determine in its own essence that which encounters us prior to an objectification (*Vergegenständlichung*) into an object of experience (Heidegger 1967, 141).

The section Heidegger focusses upon is the section in which the thingness of things is in fact determined through the highest principle: the conditions of the possibility of experience in general are at the same time the conditions of the possibility of the objects of experience’ (Kant 1998, A154/B197). Or, in the words of Yirmiyahu Yovel: ‘the a priori *knowledge* conditions under which alone we are able to unite representations in one consciousness and create a scientific, objective world picture are the same as the ontological *existence* conditions under which the objects in that world can *be* what they are, and what our knowledge says they are’ (Yovel 2018, 71). Kant’s narrow focus on one kind of object—the mathematical-physical object of nature—and the highest principle of all synthetic judgments, which asserts that the conditions of knowledge of objects are the conditions of existence of things at all, together determine the thingness of things in advance in such a way as to leave no residue.

This leaves the matter of mathematics in the narrow sense, what one might call pure mathematics, somewhat undecided still. Indeed, Kant focusses on the mathematical-*physical* object. As was already noted, Heidegger excluded mathematical entities like numbers, and mathematical symbolism, from the “natural” determination of thinghood which he set out to show was in fact historical. The strange position of pure mathematics within Heidegger’s conception of the mathematical projection of nature which resulted from his analyses came down to the fact that although mathematics does not itself deal with mathematical-physical things, or objects, it is, according to Heidegger, in its current actual constitution, axiomatically founded upon a subject whose “I posit” serves as a first principle. The conception of *truth* in both physics and mathematics comes to them from the implicit notion of *certainty* in the thought of so positing the subject. What can be said, then, is that mathematics does not deal with cognizing what we put in nature, but consists of what is there to ‘put in’ at all; as *a priori* science it does not deal with the physical world, but it is itself, within Heidegger’s analysis, still founded upon a subject the existence of which Kant has failed to elucidate.

Kant has disregarded what is manifest (*das Offenbare*), Heidegger writes. He has, however, through his work hit upon the open (*der Offene*). The proofs surrounding his highest principle have a strange circularity, in that the principle must make possible the experience by which it must be proven, and so what it makes possible must itself in proving it be presupposed. The nature of experience made possible by the principle is thus ‘not a thing present-at-hand, to which we return and upon which we then simply stand’:

> Experience is in itself a circular happening through which what lies within the circle becomes exposed (*eröffnet*). This open (*Offene*), however, is nothing other than the between (*Zwischen*)—between us and the thing (Heidegger 1967, 242)

This notion of the “between” is thus quite crucial when it comes to the question what remains hidden in Kant’s work; what he hits upon while disregarding it himself. Heidegger will return to this notion of the *between* at the end of *What is a Thing?*. For the current research, the passage now important is that where Heidegger treats ‘the axioms of intuition’, since these in particular bear on the question concerning the nature of entities of mathematics in the narrow sense.

The axioms of intuition ensure 'the passage from pure mathematics to an applied "mathematics of nature" to which all natural phenomena must submit' (Yovel 2018, 71). The measurement of magnitudes in nature through mathematics is possible since all intuitions are given in space or time which are 'equally the origins of mathematics', and which are themselves 'extending systems' (Yovel 2018, 71). Kant stresses, in the words of Yovel, 'that the content of the mathematical axioms (such as Euclid's) and the primary modes of demonstration cannot be known by the understanding but only by pure intuition':

> The a priori understanding states only that there can be no object in nature that is not subject to the possibility of mathematization (or, for space, geometrization). But what are the axioms of the geometrical system to which the object is subject—this cannot be discovered by the understanding, but must be derived from pure intuition. (Thereby, perhaps, Kant's theory leaves an opening for using non-Euclidean geometry in physics, even though Kant himself did not come up with such an idea, which to him would have looked outlandish.) (Yovel 2018, 72).

The possibility which Yovel here spots is quite crucial, since if Kant's conception of mathematics leaves open—without specifically meaning to—the possibility of other kinds of geometry, then what remains unelucidated in Kant's subject is not merely something Kant himself *fails* to do, but this is precisely *also* the work of future mathematics. To respond to Heidegger, then, one might say that Kant indeed fixes upon the mathematical-physical object of nature, but *without* restricting these objects to the mathematics of his day, precisely. The determination of thinghood of the things of nature would then be prone to new insights because of the progress of mathematics. It would not, however, free the whole enterprise from the positing character which Heidegger recognizes at the heart of it, which he also wishes to delimit.

What does Heidegger have to say about all this? Predictably, he doesn't go into questions of Euclidean or non-Euclidean geometries, as he has never done. He does, however, go into the question what makes synthetic a priori judgments—such as the one Kant argues exist in mathematics—possible.

First of all, he identifies the a priori with ‘what belongs to the subjectivity of the subject’ (Heidegger 1967, 166). The a priori is thus Kant’s incarnation of the mathematical which we somehow already have, the projection of the thingness of things, as was shown. Through the grounding of the a priori in principles, Kant’s recourse to *axioms* for determining the thingness of things, Kant remains within the mathematical tradition, even if the way in which he does so—by hitting upon the open without himself making it into a theme—‘brings about a revolution’ (Heidegger 1967, 184).

After working through the forms of pure intuitions, space and time, in a rather common way, emphasizing their anteriority to everything that can be intuited, Heidegger shortly comments upon the being of space itself. Since space is not itself in space, what is it? It cannot be something present at hand, for what is present at hand is determined by it. Therefore, writes Heidegger, explaining Kant, ‘‘being-intuited is the granting (*einräumende*) beings-space of space’ (Heidegger 1984, 203; own translation).

> The being-space of space consists in granting (*einräumt*) that which shows itself (*dem sich Zeigenden*) the possibility of showing itself in its extendedness (*Ausbreitung*) (Heidegger 1984, 203; own translation).

Heidegger now makes a rare comment on ‘the difficulties of the Kantian interpretation of space.’ He writes that these do not lie ‘where most people like to find them’, they lie not in the formulation of the being of space itself, but ‘in attributing space as pure intuition to a human subject, whose being is insufficiently defined’ (Heidegger 1967, 200).

In other words, Heidegger does not disagree with the ‘room-making’ or granting nature of space, but with its being founded upon a subject after whose subjectivity Kant does not further ask. That is, Heidegger’s main problem with Kant’s *Critique* as a whole is also his specific problem with the foundation of mathematics in the narrow sense. It can be concluded that Heidegger does not disagree with the *character* of the being of space, and in a sense thus does not disagree with the idea that it grants spatial being to spatial beings—this precisely being a determination *of its own being*—but that this idea still leaves something to be desired as to its clarification. It is then indeed only the attribution of the forms of intuition to a subject which constitutes Heidegger's problem with mathematics in the narrow sense. Thereby, in a way, the whole progress of mathematics, and thus the different possibilities of thinghood opened up by its application to nature, leave Heidegger indifferent, it is the attribution to a subject which remains firmly in place throughout which he takes aim at.

And indeed, apart from this comment on the character of space, Heidegger only comments upon the reasons Kant gives for the possible application of mathematics to nature. Regarding what one might call Heidegger's own conception of mathematics—pure mathematics, or mathematics in the narrow sense—it would be wasted time to consider all of them.

What can be said, then, of the relation of Heidegger's thought to mathematics, is the following. We can understand his early comments upon "three"—which is not taken from things, but is something we somehow already have. It is made possible by the forms of intuition in Kant, by the a priori structures of the subject, and is what is there to "put in" nature so that we might cognize it *from* nature. Furthermore, although Kant, with his highest principle of synthetic judgments, has stumbled upon the "open" or "between," he has himself disregarded this manifestness and focused right away on the object of mathematical physics. Although Kant does not delimit mathematics, and this object might thus be further determined as to its possible thinghood, it remains dependent on a subject whose subjectivity, for Heidegger, remains unquestioned. The sense given to judgments remains entirely dependent on an "I" which is undisputed, while the "between"—which is something between this I and its objects, which for Heidegger is of decisive importance—is never as such made a theme.

§4. Philosophical Questioning

Heidegger at the end of *What is a Thing?*, comments upon this "between." We must 'move in the between', he writes, 'between man and thing', in a between which 'exists only while we move in it', and which is an 'anticipation' (Heidegger 1967, 243). He has, in passing, said that animals cannot do so, the animal 'cannot bring itself into a stand-point as that against which an objective other could stand', it cannot ever 'say "I."' (Heidegger 1967, 221).

This moving within the between which only exists in moving, similar to how the highest principle grounds that which becomes the ground for their own proof, can be seen as an elaboration upon what Heidegger has spoken of as the 'leap ahead' which first gives science its things at all, the counterpart to which was the inexhaustible self-questioning of philosophy.

Heidegger identifies what he has called questioning with what Kant calls transcendental, that is, considering objects as they are a priori—as they belong to the subjectivity of the subject (Heidegger 1967, 177-178):

> Whenever, within a science, we reflect in some way upon that science itself, we take the step into the line of vision and onto the plane of transcendental reflection. Mostly we are unaware of this. Therefore our deliberations in this respect are often accidental or confused. But, just as we cannot take one reasonable or fruitful step in any science without being familiar with its objects and procedures, so also we cannot take a step in reflecting on the science without the right experience and practice in the transcendental point of view.
>
> When, in this lecture, we constantly ask about the thingness of the thing and endeavor to place ourselves into the realm of this question, it is nothing else than the exercise of this transcendental viewpoint and mode of questioning. It is the exercise of that way of viewing, in which all reflection on the sciences necessarily moves. The securing of this realm, the acknowledged and knowing, taking possession of it, being able to walk and to stand in its dimensions, is the fundamental presupposition of every scientific *Dasein* which wants to comprehend its historical position and task (Heidegger 1967, 179).

Kant, eventually, could go only so far in this direction. Heidegger wrote in *Being and Time* that he was *unable* to go further because of the neglect of 'the question of being in general' and precisely the lack 'of a preliminary ontological analytic of the subjectivity of the subject' (Heidegger 2010, 23). In any case, only by constantly questioning its own presuppositions can a knowledge 'preserve things in their inexhaustibility, i.e., without distortion' (Heidegger 1967, 65). Philosophical questioning is something constant and inexhaustible, it is perpetual self-questioning of presuppositions. It is necessary to conceive of this questioning as pursuing something other than an answer, Heidegger would argue—and given that Kant developed his *Critique* in the historical situation he was in, whereby the notion of thinghood he would properly found was in fact already becoming the "natural" notion, and thus the question "what is a thing?" was already predetermined, he could not do so.

What is the difference between this form of questioning, which would not result in an answer, and scientific inquiry? Practically all scholars who have written on Heidegger and science or on *What is a Thing?* have commented upon it. It is characterized as 'serious conflict' between 'the questioning proper to metaphysics, which *seeks to question itself*' and 'scientific investigation', which 'fosters an indifference towards ways of manifestness other than what occurs in thematizing [segmented areas of beings]' (Schalow 1992, 312); it has been remarked that 'sciences themselves cannot question [what may or may not be considered valid reality]', and that 'philosophy must be unscientific precisely to make possible thinking the question of the relation between being and beings' (Glazebrook 2013, 338-9); someone notes that scientists must recognize 'that an essentially different kind of thinking is needed in the self-reflection on [their] science', and that philosophy tries to recognize 'a higher form of knowledge hidden *in* every science' (Kisiel 1970, 169). 'Philosophy does not intend to speak against science', one writes, but takes it as its task to ask the questions to which science cannot itself turn—questions concerning 'the essence of its own field of study' (Kockelmans 1970a, 148). It becomes clear that scientific investigation proceeds within a certain given constitution and produces results, and that philosophical questioning is a kind of radical *self*-questioning which cannot take the form of the simple retrieving of answers, because it is even the form of questioning which is questioned. Science has an unshakable essence which it needs in order to proceed, and which it thus cannot question by its normal means; the means to do so are in fact philosophical. It must be noted that this description of the difference between scientific investigation and philosophical questioning becomes somewhat prescriptive. Theodore J. Kisiel writes that 'when a biologist does venture to address [what a living thing is, which cannot be decided by biology as biology], he speaks no longer as a biologist but as a metaphysician' (Kisiel 1970, 169). Indeed, Heidegger at one point notes that 'Niels Bohr and [Werner] Heisenberg think in a thoroughly philosophical way, and only therefore create new ways of posing questions and, above all, hold out in the questionable' (Heidegger 1967, 67). This tension of taking this particular distinction between philosophy and science as being either prescriptive or descriptive does not seem particularly harmful, but it does signal that one must remain attentive to the fact that it cannot simply be argued that because something is presented as being a scientific investigation, it does not include properly philosophical questioning. Whether or not some form of thought is philosophical or scientific cannot be determined from the context in which it occurs.

Questioning, as self-questioning, or 'holding out in the questionable', then, is precisely what Heidegger has called 'moving in the between.' Indeed, Heidegger wrote that the between only exists while one moves within it, and this is true precisely of the activity of radically and perpetually self-questioning, trying to preserve things in their inexhaustibility by not settling on a determination. Only by holding out in the questionable can scientists 'create new ways of posing questions', although, in doing so, they precisely *stop* holding out in the questionable insofar as they produce results.

In a way, this renews the possibility of a crisis of science of which Heidegger in *Being and Time* spoke, and which, by the time of *Contributions to Philosophy* he deemed impossible. In *Being and Time*, this was still the capacity of science to question its own basic constitution, the foundation of sciences by respective regional ontologies. Now, it is the possibility that a scientist, *besides* proceeding in scientific ways, thinks in 'a thoroughly philosophical manner' and thus 'creates new ways of posing questions'. A scientific crisis is thus not so much a moment in history wherein a science, for some amount of time, turns 'ontological'; rather, at every moment a scientist can hold a philosophical questioning stance with regard to his scientific practice. It is not a crisis occurring *in* science, then, thus a crisis *of* science in the sense of *Being and Time* indeed is not possible, but by holding out in the questionable *while* proceeding scientifically as one does is a possibility of renewing science.

The question remains, however, what this might result in if it "succeeds," and given that this radical self-questioning is perpetual, it is unclear how the creation of new questions becomes possible through it. It seems that philosophy and science are still split by a chasm; the new ways of posing questions become possible by holding out into the questionable, but acting according to such new ways itself *is not philosophical*. What is the positive result, if any, of the questioning stance itself? Is philosophy merely negative?

Two years before, during his Rectorial Address, Heidegger had already written on a way of questioning which would 'no longer [be] a preliminary step, to give way to the answer and thus to knowledge', but which would become 'itself the highest form of knowing'; that is, the 'completely unguarded exposure to the hidden and uncertain, i.e., the questionable' (Heidegger 1985, 474). This repeats everything already said about questioning and the between. One cannot refrain, however, from noting the link Heidegger sets up between this highest form of knowing and Nazism.

> If we will the essence of science understood as the *questioning, unguarded holding of one's ground in the midst of the uncertainty of the totality of what-is, this* will to essence will create for our people its world, a world of the innermost and most extreme danger, i.e., its truly *spiritual* world. (…) Spirit is the primordially attuned, knowing resoluteness toward the essence of Being. And the *spiritual world* of a people is not the superstructure of a culture, no more than it is an armory stuffed with useful facts and values; it is the power that most deeply preserves the people's strengths, which are tied to earth and blood; and as such it is the power that most deeply moves and most profoundly shakes its being (*Dasein*). Only a spiritual world gives the people the assurance of greatness (Heidegger 1985, 474-475).

One of the reasons that such a passage cannot be disregarded is that, in light of it, some parts of *What is a Thing?* take on a distinctive air. Not only the references to contemporary German scientists, of whom Heidegger writes that they hold out into the questionable (as they, it seems, ought to do); also the passage in which he insists on a non-German origin of the mathematical. Indeed, considering the beginning of modern philosophy, he firmly denies that it begins earlier than Descartes, explicitly denying that it could have anything to do with the (German) Meister Eckhart. ‘It is no accident’, he writes, ‘that the philosophical formation of the mathematical foundation of modern *Dasein* is primarily achieved in France, England and Holland’, from which, furthermore, Leibniz ‘received his decisive inspiration’—it was because he ‘passed through that world and truly appraised its greatness’ that he was able to ‘lay the first foundation for its overcoming’ (Heidegger 1967, 98). It is thus not only in the highly politically charged Rectorate that Heidegger links questioning to the German people and their innermost *Dasein*, the mathematical character of modern *Dasein* to which it is opposed is distinctly non-German. Heidegger furthermore tries to “save” the ‘Greek origin’, which only in a ‘changed way’ ‘also governs’ ‘what holds us most captive and makes us unfree in the experience and determination of things’—‘modern natural science, insofar as it has become a universal way of thinking along certain basic lines’ (Heidegger 1967, 51). Such comments begin to look like acts of purification. There is a distinct Greco-German character of questioning, which is the highest form of knowledge, vis-à-vis the modern mathematical determination of human existence, which perhaps has (had) its greatness, but which is or was to be overcome. In any case, it has its origin outside of Germany and Greece, in England, Holland and France. Even Kant at most ‘dogmatically’ adopted his fault from Descartes (Heidegger 2010, 23), and surely the way in which he remains in this tradition ‘brings about a revolution’ (Heidegger 1967, 184).

None of which is to say that a desire for rigorous self-questioning should simply be discarded. But neither should one fall for the trap of admiring all these descriptions because they seem so desirable; preserving things in their inexhaustibility, seeking the basis of principles perpetually, and so on. Indeed, the only positive result of such questioning Heidegger makes explicit is the creation of a ‘truly spiritual world’ for the German people which would preserve their strengths tied to earth and blood.

Thus there are now two important conclusions which can be drawn. First, an alternative to Heidegger’s conception of philosophical questioning is quite desirable. One can go along a long way with his analyses of Kant’s *Critique*, his only solution to the thought he delimits, however, remains very vague, and is related by himself to his Nazi sympathies. Second, Heidegger's conception of mathematics is marked above all by its being founded on a subject which remains unelucidated as to its own ontological foundation. This is Heidegger's 'problem' with the *Critique* in general, but also—given his remarks on the character of space—with mathematics in particular.

The question what a mathematical alternative mode of philosophical questioning would look like can be put after the question how these two—philosophical questioning and mathematics founded in the transcendental subject—are related, and how this affects mathematics.

§5. Questioning Mathematics

The question of the relation between philosophical questioning and mathematics can be made more explicit if we ask what would happen to mathematics if mathematicians were to rigorously question their science in the manner Heidegger envisions, and how this is related to his problem with the foundation of mathematics in a subject. What, thus, does successful questioning look like from the perspective of mathematics?

Heidegger wrote that a 'particular' kind of mathematics became 'possible and necessary' in modernity. This seemingly implies that another kind of mathematics was possible, and perhaps will be possible if one were to "escape" the mathematical projection of nature grounded in the "I think." It was the foundation of mathematics in this "I think" at the heart of the transcendental subject, and not the specific character of space, for example, which Heidegger found problematic. The character of space as making room (*einräumen*), as granting, is itself not something in need of change, and would thus, perhaps, not change as a result of the questioning stance. Since the *whole* of the mathematical project is axiomatically founded upon the subject, and the unity of this project is in the end the unity of its "I", which secures both the necessary *objectivity* of the objects of science and through its unity the law against contradiction, Heidegger is looking for a questioning of what it means to be man.

Heidegger wrote in *Being and Time* that the 'preliminary ontological analytic of the subjectivity of the subject' which Kant lacked was, in other words, a 'thematic ontology of Dasein' (Heidegger 2010, 23). Kockelmans has written that man is 'the "mediation" between beings and Being', insofar as he is Dasein (Kockelmans 1970b, 201). Schalow, too, has written that the 'attunement to the "between"' is 'a deeper appropriation of the ontological difference' (Schalow 1992, 311). Self-questioning, then, or moving in the "between," which man does or is insofar as he is Dasein, comes down to a 'deeper appropriation' of Heidegger's thought of ontological difference. One moves in the between, and self-questions, insofar as the question of the meaning of being remains up for decision, and man as the "mediator" between being and beings, remains himself questionable.

In a 1938 text, 'The Age of the World Picture', some of these matters are highlighted. Heidegger here calls questioning 'reflection' (*Besinnung*), which can be seen through the fact that he writes that reflection is 'the courage to make the truth of our own presuppositions and the realm of our own goals into the things that most deserve to be called in question' (Heidegger 1977b, 116). Indeed, 'reflection transports the man of the future into that "between" in which he belongs to Being and yet remains a stranger amid that which is', and by this power of reflection, which is 'creative questioning', man will 'know, i.e. carefully safeguard into [his] truth, that which is incalculable' (Heidegger 1977b, 136). Heidegger furthermore notes a distinction between modern science and what he calls Greek science: 'Greek science was never exact, precisely because, in keeping with its essence, it could not be exact and did not need to be exact', and furthermore, 'it is still more impossible to say that the modern understanding of whatever is, is more correct than that of the Greeks'—because, of course, *correctness* itself is part of the modern understanding of whatever is (Heidegger 1977b, 117). In *Contributions to Philosophy*, Heidegger specified that 'exact science' is ambiguous, but that it can be taken to mean 'calculated, measured, and determined numerically', and that a science '*must* be exact … if its subject area is determined in advance as a domain (the modern concept of "nature") accessible solely to quantitative measurement and calculation and only thus guaranteeing results' (Heidegger 2012, 117). In other words, exactness is to be taken again as being founded in the mathematical projection of nature, i.e. the preliminary determination of thinghood according to the mathematical-physical object—this only is the reason that Greek science would not be exact.

A decisive sentence is the one where Heidegger writes that '[modern observation] remains essentially different even when ancient and medieval observation also works with number and measure, and even when that observation makes use of specific apparatus and instruments' (Heidegger 1977b, 121). A particular kind of mathematics became possible and necessary, as was shown, *because* of the way modern observation differs from earlier modes. Greek *episteme* and medieval *doctrina* too observed, Heidegger writes, they too 'worked with number and measure', but they did not do so within the mathematical projection of nature which came to a limit in the work of Kant; they lack, in a way, the "Copernican revolution" of the *Critique*. 'Greek man', Heidegger writes, 'must gather (*legein*) and save (*sozein*), catch up and preserve, what opens itself in its openness, and he must remain exposed (*aletheuein*) to all its sundering confusions' (Heidegger 1977b, 131); by which he basically repeats what he had already written in *What is a Thing?*, that 'up till now, the proposition had been taken only as what offered itself, as it were, of itself. The simple proposition about the simply present thing contains and retains what the things are' (Heidegger 1967, 103). The decisive change is thus not the use of propositions, or of mathematics in the narrow sense, but their grounding in an axiomatic structure that does not take anything from things, but takes everything out of what we 'somehow already have', cognizing only what we 'put in' nature, as Kant wrote.

Even though this is a decisive difference between modern science and Greek thought, however, the fact that 'the beingness of whatever is, is defined for Plato as *eidos* is the presupposition, destined far in advance and long ruling indirectly in concealment, for the world's having to become picture' (Heidegger 1977b, 131)—i.e., the world as "image" of a subject. This, then, is perhaps the way in which the Greek origin 'also governed' the rise of modern scientific *Dasein*, as Heidegger wrote in *What is a Thing?*. In an appendix, Heidegger indeed writes that the thought of Plato and Aristotle 'has been able to pass for Greek thinking' and 'proves to be the end of Greek thought, an end that at the same time indirectly prepares the possibility of the modern age' (Heidegger 1977b, 143). This does not mean that Plato and Aristotle, too, are already subject to a thought of the mathematical project—the change that takes place in their thought 'always remains on the foundation of the Greek fundamental experience of what is' (Heidegger 1977b, 143). Only with Descartes does the "I" become *the* subject—which for the Greek experience it could never be—and only in that way does the mathematical project find firm foundations (Heidegger 1977b, 143-147). Nevertheless, the modern conception is "prepared."

All of these considerations for Heidegger imply that 'being subject as humanity has not always been the sole possibility belonging to the essence of historical man … nor will it always be.' And Heidegger asserts that 'truth as the certainty of subjectivity lays [a darkening] over a disclosing event [*Ereignis*] that … remains denied to subjectivity … to experience' (Heidegger 1977b, 153).

Two things can then be concluded. First of all, questioning will not entail, for mathematics, the disappearance of numbers, or mathematical symbolism, or geometrical entities. These themselves do not pose any problem. Secondly, however, questioning seems only able to *prepare* for an event that could not be understood from out of our current situation. It is, then, something that cannot be adequately described, it is only to be 'pondered'—this fact that the current conception of truth Is not the first and will not be the last.

A similar picture is sketched in a later text, 'Art and Space.' Heidegger there asks whether the space upon which the (plastic) arts work 'is … that homogenous expanse, not distinguished at any of its possible places, equivalent toward each direction, but not perceptible with the senses?'—the space of Galileo and Newton, of Kant (Heidegger 1973, 3-4). In other words, the question there is whether scientific space takes privilege over all other kinds of space. 'How can this be so', Heidegger writes, 'if the objectivity of the objective world-space remains, without question, the correlate of the subjectivity of a consciousness which was foreign to the epochs which preceded modern European times?'(Heidegger 1973, 4). This is the same argument as before: the modern conception of space is, for Heidegger, ascribed to a subject which only became the measure of truth in modernity.

Nevertheless, 'before space there is no retreat to something else', space seems to be a 'primal phenomenon' which we cannot overcome. The character of space must be understood 'from space itself' (Heidegger 1973, 4). Heidegger then defines the being of space from the word space [*Raum*] as 'clearing-away' [*Räumen*], 'the release of places' (Heidegger 1973, 5):

> How does clearing-away happen? Is it not making-room (*Einräumen*), and this again in a twofold manner as granting and arranging? First, making room admits something. It lets openness hold sway which, among other things, grants the appearance of things present to which human dwelling sees itself consigned. On the other hand, making-room prepares for things the possibility to belong to their relevant wither and, out of this, to each other (Heidegger 1973, 6).

Interestingly enough, the being of space as it was for Kant, granting or making room, is still the sense of space which Heidegger envisions as being more originary than Kant's conception of space. In his case, however, this making room prepares for things to have their place and 'belong to each other'—which sounds a lot like the simple proposition which gathers that which offers itself.

On the question which comes first—gathering places, or making-room—Heidegger answers that 'we would have to learn to recognize that things themselves are places and do not merely belong to a place' (Heidegger 1973, 6). Physical-technological space, however, 'unfolds itself only through the reigning of places of a region' (Heidegger 1973, 6). What, then, will happen to space through successful reflection?

> If it stands thus, what will become of the volume of the sculptured, place embodying structures? Presumably, volume will no longer demarcate spaces from one another, in which surfaces surrounds an inner opposed to an outer. What is named by the word "volume," the meaning of which is only as old as modern technological natural science, would have to lose its name. … The places seeking and place forming characteristics of sculptured embodiment would first remain nameless (Heidegger 1973, 7).

The suggestion is thus that successful questioning is the undoing of names. Whatever our concepts currently name must (first) remain nameless. This can perhaps be linked to the fact that Heidegger wrote that, for now, one can only ponder and anticipate the truth that is still to come, which would succeed the truth of the mathematical project. One might wonder how desirable this namelessness is—even if one agrees with the hierarchy sketched by Heidegger, and with his criticisms. Once more, an alternative to this mode of philosophical questioning seems desirable.

All of these conclusions return in Heidegger's 'The Question Concerning Technology', wherein Heidegger develops his thought of the mathematical projection into the idea that, in modernity, we are 'enframed' by the essence of technology so that man doesn't any longer 'encounter himself, i.e., his essence' (Heidegger 1977c, 26-27). It goes too far too completely summarize Heidegger's thoughts on technology here. It can be noted, however, that the 'challenging Enframing' which he there tries to analyse, which is akin to what in *What is a Thing?* is the mathematical projection, 'not only conceals a former way of revealing, bringing-forth [*poiesis*], but it conceals revealing itself and with it That wherein unconcealment, i.e., truth, comes to pass' (Heidegger 1977c, 27). The "between", which is crucial to Heidegger, and in which man would be able to encounter himself in his essence, would thus become inaccessible. This is a 'danger', according to Heidegger. What might 'foster the growth of the saving power' which 'may awaken and found anew our look into that which grants and our trust in it', Heidegger suggests, are 'the fine arts' (Heidegger 1977c, 35). Heidegger thus returns to this character of space, which he already coined earlier, as 'that which grants', or makes room, and wonders whether the fine arts might be able to preserve it, while the essence of technology progressively conceals not only other ways of 'revealing', i.e. determinations of thinghood, but also the open in which the question of thinghood might be perpetually posed. How the fine arts are to do so, however, remains completely obscure. Heidegger does not think of anything else than to 'ponder' the essence of technology so that 'the essence of art becomes' 'more mysterious' (Heidegger 1977c, 35).

What remains, then, is that which 'grants', or 'makes room', of which the problem within modernity is that it is ascribed to an obscure subject, and which, through the progress of technology, becomes obscure altogether. An alternative mode of questioning should, somehow, retain this space which ‘makes room’ which however precedes objective space, which thus isn’t ascribed to a subject as in Kant. This is the most specific condition which can be found which an alternative to Heidegger’s notion of questioning should satisfy, and there are seemingly no specific entities or methods which an alternative, ‘non-modern’ mathematics, would have to leave behind, as these specific entities are themselves not the problem, but their foundation upon a subject which founds that which grants is.

Alejandro A. Vallega has written on ‘Art and Space’, where spatiality ‘has the performative character of letting beings be.’ It ‘appears as a figure similar to Timaeus’ *chora*, a kind beyond kind, a figure of the presencing of events of beings that remains outside determination in terms of objective and ideal presence’ (Vallega 2003, 179). The comparison with *chora* is fruitful, and this chapter will end by considering Heidegger’s thoughts on it, as this would be a starting point for proposing an alternative mode of questioning.

§6. *Chora* as the Condition of Any Alternative Questioning

Heidegger has remarked on *chora* twice, once explicitly referring to the connection between *chora* and space, in the *Introduction to Metaphysics*:

> The Greeks have no word for “space.” This is no accient, for they do not experience the spatial according to extension but instead according to place (*topos*) as *chora*, which means neither place nor space but what is taken up and occupied by what stands there. The place belongs to the thing itself. The various things each have their place. … But in order for this to be possible, “space” must be bare of all the modes of appearance, any modes that it may receive from anywhere (Heidegger 2014, 69).

Further on, he remarks that

> Platonic philosophy—that is, the interpretation of Being as *idea*—prepared the transfiguration of place (*topos*) and of *chora*, the essence of which we have barely grasped, into “space” as defined by extension. Might not *chora* mean: that which separates itself from every particular,

> that which withdraws, and in this way admits and “makes room [*Platz*]” precisely for something else? (Heidegger 2014, 70).

Heidegger here already claims, then, that Platonic philosophy anticipates the modern mathematical projection which would succeed Greek thought. And, furthermore, the *other* possibility which he sees indeed still retains the character which space will have in Kant, i.e. that of space as granting, making room, making place, with the additional remark that it does so by ‘withdrawing’, by ‘separating itself from every particular.’

The second reference to *chora* happens in the lecture series *What is Called Thinking*, written some two decades later. In the tenth lecture, Heidegger embarks upon a reflection on ontological difference by way of the notion of the *participle*—the two distinctive meanings of “being” as *a* being and as the *being* of this being; the nominal and verbal meanings. All participles in the end have their roots in this dual character of being, according to Heidegger, wherein ‘a being has its being in Being, and Being persists as the Being of a being’ (Heidegger 1968, 221). The term participle itself comes from the Greek *metoche*, ‘taking part of something in something’, as Heidegger translates;

> This word is fundamental to Plato’s thinking. It designates the participation of any given being in that through which it—say, this table—shows its face and form (in Greek, *idea* or *eidos*) as this being. In this appearance it is in present being, it *is*. According to Plato, the idea constitutes the Being of a being. … Now Plato designates the relation of a given being to its idea as *methexis,* participation. But this participation of the one, the being, in the other, the Being, already *presupposed* that the duality of being and of Being does exist (Heidegger 1968, 222).

This determination of the being of beings as participation becomes indicative of Western metaphysics, Heidegger writes. However, ‘the duality of individual beings and Being must first lie before us openly, be taken to heart and there kept safely, before it can be conceived and dealt with in the sense of the participation of the one, a particular being, in the other, Being’ (Heidegger 1968, 223). Heidegger writes that from Parmenides on, however, ‘*no* further inquiry and thought is given to the duality *itself*, of beings and

Being', 'Philosophy's procedure in the sphere of this duality is decisively shaped by the interpretation Plato gave to the duality. That the duality appears as participation does not at all go without saying' (Heidegger 1968, 224).

At the end of the lecture the reference to *chora* is made:

> When we say "Being," it means "Being of beings." When we say "beings," it means "being in respect of Being." We are always speaking *within* the duality. The duality is always a prior datum, for Parmenides as much as for Plato, Kant as much as Nietzsche. The duality has developed beforehand the sphere within which the relation of beings to Being becomes capable of being mentally represented. That relation can be interpreted and explained in various ways.
>
> An interpretation decisive for Western thought is that given by Plato. He says that between beings and Being there prevails the *chorismos*; the *chora* is the *locus,* the site, the place. Plato means to say: beings and Being are in different places. … To make the question of the *chorismos,* the *difference* in placement of beings and Being at all possible, the *distinction*—the duality between the two—must be given beforehand, in such a way that this duality itself does not as such receive specific attention (Heidegger 1968, 227).

What Heidegger has variously called ontological difference, the between, the duality, or the open can thus itself never be thematized, it cannot become an object or thing for us in any sense, for it makes possible taking objects *as* objects, beings *as* beings in the first place. What Heidegger calls questioning is thus eventually directed to the fact that this duality 'can be interpreted and explained in various ways'—while we cannot interpret it on the basis of what we know, for what we know is made possible on the basis of a certain interpretation.

An alternative to Heidegger's conception of philosophical questioning should thus, most crucially, take up the task of allowing for this duality, within which we always already speak, to remain open; the interpretation of this duality (for example, as participation) and interpretation the relation *between* the two terms of the duality (for example, as difference in place), must remain questionable, open to indetermination, so that the thinghood of things (and the subjectivity of the subject, or the essence of man) are not decided upon once and for all. An alternative must be attentive not only to the things with which we deal, to beings, but also to that which "grants" these beings their being, which "makes room" or "place" for them. In other words, the alternative should take up Heidegger's question in *Introduction to Philosophy* whether *chora* might not be taken as 'that which separates itself from every particular, that which withdraws, and in this way admits and "makes room" precisely for something else'.

Conclusions

The aim of this chapter was to determine the condition or conditions which Lautman's mathematical alternative to Heidegger's mode of questioning—opposed to scientific inquiry—should satisfy, and answer the question why such an alternative might be desirable. In order to do so, first Heidegger's conception of science in general and mathematics specifically had to be elucidated. It became apparent that Heidegger's conception of mathematics does not differ from Kant's. Since Heidegger's relation to Kant is not one of rejection or correction, but of situating or demarcating, the question became how Heidegger's relation to Kant would affect this conception of mathematics.

What remained hidden, in Kant's *Critique*, was what Heidegger variously called "the open," "the between," or the duality between being and beings—that is, his notion of ontological difference. This duality, according to Heidegger, must be preserved as a question. In Kant, the duality is covered by the grounding function of the transcendental subject. The determination of thinghood—which is another way of saying a determination of the duality between being and beings—is reduced to being the objects of a subject whose a priori structures condition all possibility of appearing. The being of this subject itself, however, thereby remains in the dark.

Science, for Heidegger, is precisely the progress of inquiring within such a determination of the being of all things. That is, modern science in Heidegger's view basically conforms to Kant's grounding of it. The relationship between philosophy and science was shown to be opposed:

science progresses by taking this basic constitution—the determination of the duality—and deriving results from it, philosophy tries to preserve the duality in such a way so as not to determine it once and for all.

The question which thereby became important is how mathematics functions within all of this. It is not a natural science like the others, and does not deal with the same kind of 'things'—Heidegger admits this. The way it is still, however, part of the modern constitution of science—what Heidegger calls the mathematical—is how its notion of truth is similarly founded upon the certainty of the subject. The specific character of mathematics, however, was thus not the problem. The 'granting' nature of space itself is not something which has to be discarded, indeed, Heidegger's own desire for an alternative takes over precisely this character of 'granting that which shows itself the possibility of showing itself' as what it is. The problem of space is its foundation, as form of intuition, in the transcendental subject.

The reason an alternative to Heidegger's mode of questioning is desirable was twofold. On the one hand, Heidegger explicitly links this mode of questioning to his Nazi sympathies, saying that it creates for the German people its spiritual world, tied to earth and blood. The only other positive result of questioning seemed to be turning concepts to namelessness. Otherwise, questioning was the pondering of essences and the deepening of mysteries.

The condition which an alternative to this mode of questioning should satisfy is thus the other interpretation of *chora* which Heidegger proposes in an offhand remark in the *Introduction to Philosophy*. The interpretation of being as *idea* or *eidos* in Plato, and the relation of beings to this interpretation of being as one of *participation*, whereby *chora* is interpreted as 'difference in place', being and beings as being in different places, anticipates the modern constitution of science. Another possible interpretation of *chora* would be 'that which separates itself from every particular, that which withdraws, and in this way admits and "makes room" precisely for something else'. It is a matter, then, of interpreting *chora* in such a way so as to not split being and beings. An alternative mode of Heideggerian questioning must interpret the 'participatory' relation of beings in being otherwise.

Chapter 2: Lautman

Mathematics and Dialectics

The aim of this chapter is to position Albert Lautman's philosophy of mathematics (or mathematical philosophy—it has been called both) over and against the results retrieved in the previous chapter. That is, given the results of the first chapter—that an alternative must think ontological difference otherwise than a split between being and beings—it must be assessed whether or not Lautman's conception of mathematics is a viable alternative mode of Heideggerian questioning. Charles Alunni's assertion that Lautman aligns his thought of 'dialectical Ideas and mathematical theories' to Heidegger's thought of the ontological difference between being and beings will be a guiding thread throughout, even if the question whether this analogy fits will, at least first and provisionally, be a question left open.

Albert Lautman was a philosopher of mathematics shortly active in the third decade of the twentieth century. Almost all of his work was published between 1937 and 1939, with one posthumous publication after the second world war, in which he was killed as a member of the French resistance. Lautman worked in the tradition of Brunschvicg, in close contact with both Gaston Bachelard and Jean Cavaillès (Colin 1987, 129).

Lautman's work has been met, according to Alunni, with an 'oppressive silence' from the side of 'professional philosophy', while 'there was at least one major occurrence in the context of contemporary French philosophy' within the work of Lautman 'which should have served as an injunction to 'professional philosophers' to take a closer look: Lautman's reference to Martin Heidegger' (Alunni 2006, 67). Lautman, according to Alunni, aligns his philosophy of dialectical Ideas and mathematical theories to Heidegger's thought of ontological difference 'by affirming that dialectical Ideas are to mathematical theories what Being and the sense of Being is to being and the existence of being', introducing into his philosophy of mathematics Heidegger's conception of truth as *unconcealment* (Alunni 2006, 72).

This, in any case, looks promising. In order to see whether Lautman's conception of mathematics provides an alternative mode of questioning, Lautman's philosophy will first be sketched in a fourfold way: first a general context will be established (§1), then a summarizing overview will be given from the point of view of philosophy (as opposed to mathematics) (§2), this philosophical summary will be supplied with commentary from secondary literature (§3), and finally Lautman's philosophy will be looked at from the point of view of mathematical practice (§4). Together, these four aspects will provide a conception of mathematics which can be analyzed over and against the results sketched in the first chapter (§5, §6), after which, finally, the question will be put forward how Lautman's philosophy of mathematics must be interpreted so as to satisfy the condition established—that it does not determine ontological difference as difference in place (§7).

§1. Lautman's Philosophy of Mathematics: Context

Since Heidegger eventually designated the debate between formalism and intuitionism, which he first identified as the site of crisis and development within mathematics, as 'mere babble', it is interesting to note that Lautman distances himself from either side. According to Lautman, even if both schools differ regarding the existence of entities—formalism arguing that an entity exists if it can be defined in a non-contradictory way, intuitionism insisting that it must be able to be 'effecitvely' constructed in a finite number of steps—there remains, he says, one shared characteristic: 'they still conceive of the relation of essence to existence as arising with regard to the same entity' (Lautman 2011, 28). Lautman, on the other hand, argues that one must deal with 'passages from the essence (structure) of something (e.g. a domain) to the existence of *other* things', which in 1940 Paul Bernays recognized as being Lautman's main thesis (Bernays 1940a, 21). Bernays, however, argues that while this seems 'intended to displace the previous foundational discussions of the "naïve period" [of the debates on the foundation of mathematics]', it 'does not really give an account of what mathematical existence means, but simply adopts in each case the existential assumptions of the theory in question' (Bernays 1940a, 21). Lautman, indeed, discussing the matter of essence and existence, asks us to '[c]onsider, for example, what mathematicians call existence theorems—that is to say, theorems that establish the existence of certain functions or certain solutions without actually constructing them', which 'establishes a link between the degree of completion of the internal structure of a certain mathematical being … and the existence of another mathematical being' (Cavaillès and Lautman, 1939, 9). For example, a proof which asserts the existence of an absolute maximum and an absolute minimum value on a closed interval for any continuous function does not in fact give the values for which the function returns these absolutes. In first instance, then, it might seem like Bernays is right, and Lautman does not give an account of mathematical existence beyond what is already *assumed* in mathematical practice. If this is the case, the attempt to use Lautman's work as an alternative to Heidegger's mode of questioning would fail from the start, since its notion of existence would be founded in mathematical theory, which itself would fall back on the foundations which Heidegger seeks to escape.

At the end of the text discussed by Bernays, however, Lautman writes that 'the nature of mathematical reality can be defined from four different points of view'—that of mathematical facts, entities, theories and the Ideas that according to Lautman 'govern these theories.' These 'fit naturally together: the facts consist in the discovery of new entities, these entities are organized in theories, and the movement of these theories incarnate the schema of connections of certain Ideas' (Lautman 2011, 183). 'The reality inherent in mathematical theories' thus 'comes to them from their participation in an ideal reality that is dominating with respect to mathematics, but that is only knowable through it' (Lautman 2011, 30). In other words, not only do the four viewpoints of mathematical reality fit together naturally, there is a certain hierarchy: the reality of theories comes from their participation in an ideal reality, and the existence of mathematical entities, being organized in theories, is equally dependent on this participation. Lautman elsewhere has indeed stressed that 'the properties of a mathematical being depend essentially upon the axioms of the theory within which that being appears' (Cavaillès and Lautman 1939, 8). As Alunni has also remarked, then, Lautman does not give a notion of existence for mathematical entities other than the notion of existence assumed in theory because 'scientific philosophy must take theories, not isolated concepts, as its object' (Alunni 2006, 68). Alunni sees in this another affinity with Heidegger's thought, where beings are not 'founded upon' being serving as a kind of final concept or explanation, but the relation of beings and being must be explained in terms of ontological difference (Alunni 2006, 70-71).

Lautman himself likens his own philosophy to that of Martin Heidegger on several occasions (cf. Cavailles and Lautman 1939, 10; Lautman 2011, 200). Given that Lautman is also a self-proclaimed Platonist—which also becomes apparent through his use of terms like 'ideal reality', 'participation' and 'incarnation'—one might wonder how this is possible. Indeed, as was shown in the first chapter, Heidegger precisely sees in the determination of being as *idea* the 'end of Greek thought' and the beginning of the modernity which he wants to delimit. Several commentators have argued that the relation between mathematical facts, beings and theories on the one hand, and the ideal reality on the other, is the most crucial (open) question in understanding Lautman's thought (Lebel 2010, 163-164; Duffy 2018, 79-80). Since it is precisely concerning this relation that Lautman turns to the thought of Heidegger, this question of the relation between Lautman's Platonism and his references to Heidegger should be the guiding question of the considerations here. This includes the question whether or not Lautman rightfully aligns his notions of mathematical theory and dialectical Ideas with Heidegger's use of beings and being.

§2. Lautman's Philosophy of Mathematics: Overview

'Lautman's work is based on the idea of a fundamental difference in kind between a problem and its solution,' writes Daniel W. Smith (Smith 2003, 428). This is the difference between what Lautman calls dialectics and mathematical theory itself. In an early, short talk, Lautman positions his thought over and against the reduction of mathematics to tautology:

> Mathematical philosophy tends often actually to be mistaken for the study of different logical formalisms. This attitude generally entails as a consequence the assertion of the tautological character of mathematics. The mathematical edifices that appear to the philosopher so hard to explore, so rich in results and so harmonious in their structures contain in fact no more reality than is contained in the principle of identity (Lautman 2011, 27).

Lautman believes the reality of mathematics to be far greater than the principle of identity, it is, 'as all reality', something which 'the mind encounters' as 'an objectivity that is imposed on it.' This reality of mathematics 'is not made in the act of the intellect that creates or understands, but it is in this act that it appears to us and it cannot be fully characterized independently of the mathematics that is it's indispensable support' (Lautman 2011, 28). The 'Platonic conclusion' which Lautman believes is necessary is that 'the reality inherent to mathematical theories comes to them from their participation in an ideal reality that is dominating with respect to mathematics, but that is only knowable through it' (Lautman 2011, 30). What is key, as was shown in the conclusion of the first chapter, is how Lautman interprets these notions of "participation" and "domination."

Lautman has emphasized that he 'recognize[s] the impossibility of … a conception of an immutable universe of ideal mathematical beings', that is, the understanding of Platonism as 'a theory of the 'in-itself' existence of mathematics' (Cavaillès and Lautman 1939, 8). Instead, 'the properties of a mathematical being depend essentially upon the axioms of the theory within which that being appears; and this dependency strips them of the immutability that supposedly characterizes an intelligible universe' (Cavaillès and Lautman 1939, 8). Mathematical beings are those described within mathematical theory, and mathematical theory is dependent on axioms. Lautman thus takes over Hilbert's structural, axiomatic conception of mathematics (Alunni 2006, 68), adding, however, the Platonic conclusion he deems necessary, since 'this objectivity of mathematical beings … only reveals its true meaning within a theory of the participation of mathematics in a higher and more hidden reality—a reality which, in my view, constitutes the true world of ideas' (Cavailles and Lautman 1939, 8).

Lautman thus doesn't believe in an immutable reality of mathematical entities which would only need to be 'discovered' and then 'described', emphasizing that mathematical beings *are* those defined through axioms and theory. Their meaning, however, only becomes truly clear when these beings are seen in light of their participation in an ideal reality. If the alignment of Lautman to Heidegger holds up, this would mean that Lautman asserts that beings are only understood as they *are* when they are taken in light of being, when the duality which divides both, and within which according to Heidegger we always already speak, is not covered over.

Lautman defines various terms. Dialectical *notions* are notions like whole and part, form and matter, essence and existence. These are 'not mathematical notions', he tells us, but 'it is toward them that the consideration of effective mathematical theories leads.' Dialectical *ideas* are 'the problem of the possible liaison between dialectical notions thus defined' (Cavaillès and Lautman 1939, 9). The relation of dialectics to mathematics, furthermore, lies in the fact 'that the problems of dialectics can very well be conceived of and formulated independently of mathematics, but that every sketching out of a proposed solution to these problems will necessarily rest upon some mathematical example designed as a concrete support for the dialectical liaison in question' (Cavaillès and Lautman 1939, 9). That is, one can envision the problem of a liaison between "whole" and "part", but every concrete description of this liaison will be actual mathematical theory—an example Lautman gives is the analysis of conditions which are sufficient to make a topological surface "whole" (Lautman 2011, 103). Dialectics is thus, as Smith already noted, 'a pure problematics, antithetic' and 'fundamental', and mathematical theory consists of 'mixtures' that are constituted through the composition of some concrete (and axiomatically founded) example, through which the dialectical notions which at first sight 'appear … opposed' are in fact 'composited together' (Cavaillès and Lautman 1939, 10).

Insofar as Lautman describes mathematical theory as *participating* in dialectics, he seems at first to adopt Plato's interpretation of the dualism of being and beings, which we saw Heidegger comment upon at the end of the first chapter. Nevertheless, it is precisely regarding this participatory relation—which he does not envision as a relation between historical entities participating in some immutable reality independent of them—that Lautman likens his own thought to that of Heidegger. 'The extension of the dialectic into mathematics corresponds', it seems to him, 'to what Heidegger calls the genesis of ontic reality from the ontological analysis of the idea. One thus introduces, at the level of Ideas, an order of before and after which is not that of time' (Cavaillès and Lautman 1939, 10). The anteriority of the dialectic is elsewhere defined as the anteriority of a question with regard to its response, and Lautman again calls this, after Heidegger, an 'ontological' anteriority (Lautman 2011, 204):

> Insofar as 'posed questions', [dialectical ideas] only constitute a problematic relative to the possible situations of entities. It then happens to be once again exactly as in Heidegger's analysis, that the Ideas that constitute this problematic are characterized by an essential insufficiency, and it is yet once again in this effort to complete the understanding of the Idea, that more concrete notions are seen to appear relative to the entity, that is, true mathematical theories (Lautman 2011, 204).

'Dialectic', he writes elsewhere, 'not being affirmative of any effective situation and being purely problematic, is necessarily extended into effective mathematical theories' (Cavaillès and Lautman 1939, 23).

Even though Lautman thus takes recourse to Platonism, and uses Platonic terms, it would seem that it remains possible to align his and Heidegger's thought. Lautman does not envision the duality of being and beings as being one of difference in *place*, as Plato according to Heidegger did. Dialectics and mathematics are not in different places: the first is characterized by an essential insufficiency, and is by necessity 'extended into' mathematical theory. Such essential insufficiency, furthermore, strengthens the affinity between Lautman's dialectics and Heidegger's conception of the inexhaustible questionability which characterizes his thought concerning being. It is the mathematical experience which 'should be the *sine qua non* of mathematical thought, this is certain', but 'we must find in experience something else and something more than experience; we must grasp, beyond the temporal circumstances of discovery, the ideal reality that alone is capable of giving its sense and its status to mathematical experience' (Cavailles and Lautman 1939, 23). There is a constant balancing, in Lautman's work, between the emphasis that this ideal reality is somehow *beyond* mere mathematical theory and its history, and the emphasis that it is *in* mathematical experience that we must find this 'something else and something more' than it.

Lautman furthermore argues that 'it is clear that it is only via an effort of regressive analysis that one gets back from the [mathematical] theory to the idea that it incarnates, but it is no less true that it is in the nature of a response to be a response to a logically anterior question, even if the consciousness of the question is posterior to the understanding of the response' (Cavaillès and Lautman 1939, 22). When reproached by Jean Hyppolite for using the term 'dialectic', Lautman again emphasizes that the dialectic cannot be 'self-sufficient, independently of mathematics', and writes that while 'Hyppolite says that positing a problem is not conceiving anything; I respond, after Heidegger, that it is to already delimit the field of the existent' (Cavaillès and Lautman 1939, 23). Thereby a further affinity between Lautman's position and Heidegger's insistence on perpetual self-questioning of assumptions is found.

Since any posed problem 'already delimits the field of the existent', by 'dialectic' Lautman thus seems to indicate *that which is thereby delimited*. The essential insufficiency of the dialectic would be, at the same time, its being inexhaustible by concrete examples.

Lautman would thereby indeed be quite close to Heidegger. Where Heidegger stresses the fact that only by thinking philosophically, to ask after the being of beings, is it possible to 'create new ways of posing questions' in science, Lautman too, while admitting that entities are defined completely by the axioms and theory in which they figure, emphasizes that these entities are only grapsed in their full meaning when taken as participating in an Ideal dialectic—which is at once foreign to mathematics and necessarily extended into it, a description akin to Heidegger's notion of ontological difference. Lautman deems it fruitful to remain attentive to this difference. 'It is not enough to posit the duality of the sensible and the intelligible', he writes, 'we must also explain the participation, that is to say, whatever we decide to call it, the deduction, the composition, or the genesis of the sensible from the intelligible' (Cavaillès and Lautman 1939, 9).

Finally, according to Lautman, it is precisely mathematics which 'in certain cases' gives 'remarkable examples' of determinations of this duality, determinations which might be studied—and this is the task of a philosophy of mathematics.

§3. Lautman's Philosophy of Mathematics: Reception and Criticism

Given that Lautman's philosophy of mathematics, after a first overview, at least does not seem *incompatible* with Heidegger's thought of ontological difference, it is fruitful to consider the various commentaries on it, so as to come to a more complete picture.

Several commentators emphasize the originality of Lautman's philosophy both within the context in which he worked, and within the twentieth century generally (Barot 2010b, 193; Catellana 2018, 45). Emmanuel Barot writes that the properly philosophical problems that Lautman's thought raises lie in 'Lautman's specific "dialectic" by which he partially reformulates Heidegger's distinction between Being and beings' in a way that, to him, seems akin to Hegel's *Science of Logic*, and he concludes that Lautman 'is a perfect heterodox who seizes, by any authority, theoretical motives that resonate with the mathematics in which he bathes', making him 'neither Heidegggerian, nor Hegelian, nor Platonist either, but rather [a promotor of] a metaphysics of a Platonist *spirit* which up to a certain point, but up to a certain point only, shows Hegelian traits' (Barot 2010b, 192).[1] Given that description, the affinity with Heidegger seems to disappear completely, and would merely be one of Lautman 'seizing' some motives. Mario Castellana too writes that Lautman "borrows categories" 'from Heidegger's contemporary philosophy' in order to escape 'the agnostic attitude prevalent in certain mathematical milieux' (Castellana 2018, 51). Simon B. Duffy, however, against Barot writes that Lautman's dialectic *is* in fact 'rather Platonic', and not merely a metaphysics in Platonist spirit (Duffy 2018, 84). This is because, he writes, Lautman regards hypotheses not as starting-points but 'as what they really are, things set down at the beginning of an inquiry to enable one to work one's way toward something else', which he regards as being distinctly Platonist:

> The practicing mathematician takes for granted the entities with which he works and gives no account of them, but treats them as starting points; his state or condition is thought/reasoning (Plato 5010C2-D3). … The dialectician, as distinguished from the practicing scientist or

[1] All English renderings of French sources are my own.

mathematician, sees things holistically, and leaves no assumption unexamined (Duffy 2018, 85).

Given such an interpretation of Lautman's Platonist dialectic, the leap towards Heidegger is also not very large as one might first assume, and Barot would in fact be wrong to leave Heidegger (and Plato) out of his characterization of Lautman; this difference between the practicing mathematician and the dialectician in fact reminds one strongly of Heidegger's distinction between the activity of science and of philosophy. Fernando Zalamea, too, writes that in it is between the notion of pre-ontological understanding and ontic existence that 'Lautman finds … an important echo of his own reflections' regarding structure and existence in mathematics (Zalamea 2012, 56-57).

Within the contemporary reception, then, despite the affinities shown in the second section, the status of the references to Heidegger, and the measure in which Lautman can be regarded a Heideggerian, so to speak, both remain rather unclear. Several scholars argue that the question of Lautman's relation to Heidegger is vital to current research, Duffy arguing that it is 'by clarifying Lautman's relation to the work of Plato and Heidegger that his account of the mathematical real and the dialectic operating in relation to it can best be understood' (Duffy 2018, 79-80). Lebel writes that '[it] may be hoped that returning to … Heideggerian descriptions will clarify what Lautman means by "ontological analysis," and will finally help to determine with greater certainty whether the transposition or analogy [of and with Heidegger's thought] proposed by Lautman … holds up well along the way' (Lebel 2010, 1640.

The question thus is whether Lautman's Heideggerrianism is reducible to the "borrowing of categories", and what we must understand when Zalamea writes that Lautman finds an "echo" of his reflections in Heidegger. Brendan Larvor, in any case, argues that Lautman has overestimated the value of Heidegger's "ontological difference" after using it to '[bolster] his Platonism', arguing that this difference 'collapses in mathematical practice (Larvor 2011, 185, 2020). Larvor, however, interprets ontological difference as the difference between phenomenology and science, an interpretation which perhaps is not precisely off the mark, but does reduce the difference to two practices, while Heidegger's does not characterize it in this way.

One aspect prevalent in the commentary would seem to agree, however, with Larvors argument that the distinction between dialectical notions and mathematical theory 'is neither clear nor stable' (Larvor 2011, 200). A host of scholars have commented upon the '"prophetic" capacity' of Lautman's work, 'in the sense that [his pairings of notions] seem to be relevant for the path and "philosophical concerns"' of the mathematics that was to emerge in the second half of the century (Catellana 2018, 49-50). Lautman's pairings of notions like whole and part, local and global, and his analysis of various fields of contemporary mathematics in terms of these notions, and in turn his linking of these fields with each other, would have prefigured several developments within mathematics, according to Zalamea, Mathieu Bélanger and David Corfield, with Corfield in particular concluding that 'the ideas he so brilliantly describes are immanent to mathematical practice, rather than belonging to "an ideal reality, superior to mathematics"', since these ideas can now be described *by* mathematics 'very thoroughly'—for example by category theory, sheaf theory or topos theory (Corfield 2010; Belanger 2010; Zalamea 2010).

What does this mean for the resemblance of Lautman's distinction between dialectics and mathematics to Heidegger's difference of being and beings? If the specific pairs of notions Lautman retrieved from his analysis of contemporary mathematics are not foreign to mathematics at all, this might mean that he has never, in fact, uncovered the duality which Heidegger deems so crucial. That mathematics can now describe 'very thoroughly' the concepts which Lautman suggested with his notions, however, does not mean that the dialectic and mathematical theory completely collapses. Larvor himself recognizes that aside from the difference between dialectical notions and mathematical theory, 'Lautman's distinction between dialectics and mathematics depends on the "essential insufficiency" of dialectical Ideas, that is, the fact that they cannot be understood except through the development of mathematical theory' (Larvor 2011, 200). That, indeed, seems to be the problem with the assessment that the dialectic would not be foreign to mathematics. Lautman, as we have seen, argues that the dialectic is *necessarily extended* into mathematical theory, every posing of a problem—*including that of a specific pair of notions like whole and part*, we might add—'already delimits the field of the existent', as he responded to Hyppolite. Lautman has never argued for the fixity of dialectical notions, quite the opposite, arguing that dialectical Ideas are intimately linked to the specific theories from which they are retrieved. That various mathematical theories analyzed in terms of the same pair of notions are now generalized in theories which encompass them both is not an argument against Lautman's conception of the difference between dialectics and mathematics, but *for* this conception. The point is, as Duffy correctly emphasizes, that the problem which dialectical pairs of notions for Lautman pose 'can comprise "an infinity of degrees"' (Duffy 2018, 85). And as Zalamea remarks, we can think of the continuum as being the saturation of the discrete, as in 'the Cantorian completion of the real line', but we can equally think the discrete as being detached from the continuum 'like [in] Brouwer's primordial continuum' (Zalamea 2012, 57). It is then not a question of which notions are (for all time to come) dialectical and which concepts are forever mathematical, but of realizing that any one mathematical concept might be *thought differently,* and that this is what characterizes the *essential insufficiency* of dialectical Ideas. The enigmas posed by dialectical Ideas are 'irreducible and unsolvable *as such*' (Barot 2010b, 193).

Given these assessments, the necessary conclusion seems to be that while Lautman uses specific notions of whole and part, local and global, these should not be the focus when it comes to interpreting the difference between dialectics and mathematics. In the end, dialectics remains anterior to mathematics the way a question is anterior to a response, and every question, for Lautman, by being posed is already limiting the field of the existent. The dialectic of Ideas is not composed of a finite group of specific problems summed up by the work of Lautman, dialectics instead is the irreducible possibility to pose such a problem. Thereby, the essential insufficiency which characterizes it is akin to Heidegger's inexhaustible possibility of questioning which characterizes any knowledge which wants to preserve things as they are.

Larvor has a second criticism, however. He recognizes a strange aspect of Lautman's interpretation of Plato's method of division. Lautman envisions dialectic Ideas as compromising the problem of possible liaison between *two* dialectical notions, Plato himself however never confines the method in such a way. Indeed, in all the dialogues in which the method of division is explained from the point of view of Socrates, 'all he insists is that the number of subclasses [into which a category may be divided] should be finite.' The dialogues displaying a method of division which results only in *pairs* first of all do not, or not always, divide a category into properly *opposed* concepts, and secondly, more importantly, sometimes divide a category into awkward and arbitrary pairs. Finally, they are written from the standpoint of the Eleatic philosopher, and not from the point of view of Socrates (Larvor 2011, 189-190).

The crucial aspect, according to Larvor, is that 'an object can participate in more than one Idea', and it is in this sense that 'a Platonic system of Ideas is somehow prior to and independent of the objects that participate in those Ideas' (Larvor 2011, 190). This seems to be a fair criticism, and might be seen as a minor correction to Lautman's thought: his kind of analysis might be diversified to include analyses of theory in more than two terms. One suspects, however, that the more terms one starts using to analyze a mathematical theory, the closer one comes to simply *practicing mathematics*, the more one tries to define them. What must be insisted upon, then, is that the most important aspect of Lautman's philosophy is regarding mathematical theory with an eye to the essential insufficiency of the dialectic which governs it—and the open question now becomes what this looks like in practice.

Summing up, what becomes clear is that the open problem within the reception of Lautman's thought is that of the relation of the dialectic to mathematical theory, i.e. the sense in which it "governs" or "dominates" this theory which "participates" within it. This relation is supposed to give us a notion of existence of mathematics which Bernays thought Lautman's philosophy lacked. Lautman, in a way, has reversed the foundational question, arguing that it is on the level of mathematical theory that the question of existence must be posed, while entities, being organized into theories, only receive their existence on account of the participation of theory in a dialectical Ideal reality.

There are, thus, two questions. One is what Lautman's philosophy looks like in practice, the other is how the relation between dialectics and mathematics must be envisioned. In order to make things more concrete, the first question will now be answered somewhat, by turning to a few examples—most of the answer, however, will be given in the fourth chapter, where the conclusions of this thesis will be regarded from the viewpoint of of mathematics. The second question will be dealt with after this section, by turning, as Duffy already pointed out one should do, to the question of Lautman's relation to both Plato and Heidegger.

§4. Lautman's Philosophy of Mathematics: Practice

Two aspects of the secondary discussion are important when it comes to the question what Lautman's philosophy of mathematics would look like in practice. First is Larvor's criticism of Lautman's interpretation of the Platonic method of division; Lautman for the most part sticking to (oppositional) pairs of terms, while in Plato's conception, this is not necessary at all. Second is the oft-noted fact that Lautman's specific choice of pairs, and specific analysis, often had a "prophetic" quality, anticipating many developments in mathematics that would take place the following decades. As several commentators remarked however, the fact that his notions now often have been given interpretations in mathematical theory obscures the fact that Lautman envisions the dialectic as a 'pure problematics'—*any* interpretation is already part of mathematical theory, and only as *problem* can terms be said to be 'foreign' to mathematics. An example was the notion of discrete and continuous: one can think the continuum as being the saturation of the discrete, as in 'the Cantorian completion of the real line', or think the discrete as being detached from the continuum 'like [in] Brouwer's primordial continuum'—and one might be able to envision a host of different 'answers' to the problem of this relation. That there *are* mathematical notions of the continuous and the discrete thus does not, in any way, reduce the 'problem' which the pair poses dialectically. The essential insufficiency of the dialectic must be understood as the inexhaustibility of taking several mathematical theories as answering the same problem.

Lautman, at one point, characterizes his thought as trying to reconcile 'the structural conception and the dynamic conception of mathematics', which at first seem to be opposed (Lautman 2011, 90). The structural conception referred to is Hilbert's, and Lautman's dialectical pairs of notions in a way serve to replace or extend Hilbert's notions of consistency and completion; they take up the 'dominant role of metamathematical notions' (Lautman 2011, 90). Lautman quotes the following passage from Hilbert's text on 'the logical foundations of mathematics':

> The axioms and provable theorems (i.e. the formulas that arise in this alternating game [namely formal deduction and the adjunction of new axioms]) are images of the thoughts that make up the usual procedure of traditional mathematics; but they are not themselves the truths in the absolute sense. Rather, the absolute truths are the insights (*Einsichten*) that my proof theory furnishes into the provability and the consistency of these formal systems (quoted in Lautman 2011, 90).

As in Hilbert, the truth of mathematical theories comes from their furnishing insight 'into the provability and the consistency' of these systems, so in Lautman, the truth of mathematical theories comes from their ability to provide an answer to a problem posed—'even if the idea of the question comes to mind only after having seen the response' (Lautman 2011, 204). Lautman remarks that 'the point of view of the logical notions of consistency and completion' from which theories might be examined only provide 'an ideal toward which the research is oriented', which in fact 'currently appears difficult to attain' (Lautman 2011, 90). He thus concludes that

> metamathematics can thus envisage the idea of certain perfect structures, possibly realizable by effective mathematical theories, and this independently of the fact of knowing whether theories making use of the properties in question exist, but then only the statement of a logical problem is possessed without any mathematical means to resolve it (Lautman 2011, 90)

It is then the ability to interpret mathematics as answering the problem of a more abstract relation, or meaning, which grants mathematics itself its meaning and consequently its reality, for Lautman, and as we have seen, it is precisely this difference between a problem posed and a mathematical solution which for Lautman proves fertile.

The question concerning the precise nature of the 'pure problematics', which is the dialectic governing mathematical theory, and the way in which it should grant mathematics its reality will be returned to in the next section. First, however, the other side of Lautman's work—the way in which he tries to reconcile these ideas with the 'dynamic conception of mathematics'—must also be elucidated.

This side is associated with the philosophy of Brunschvicg, who, according to Lautman, more than any philosopher of his day 'developed the idea that the objectivity of mathematics is the work of intelligence, in its efforts to overcome the resistance that is opposed to it by the matter on which it works' (Lautman 2011, 88). Thus where the structural view ascribes reality to theories which are viewed as finished and whole edifices, the dynamic view takes the conceptions of mathematics as 'never more than a provisional arrangement that allows the mind to go further forward' (Lautman 2011, 88). The opposition which the mind wants to overcome, the facts to be explained were throughout history

> the paradoxes that the progress of reflection rendered intelligible by a constant renewal of the meaning of essential notions. Irrational numbers, the infinitely small, continuous functions without derivatives, the transcendence of *e* and *P*, the transfinite had all been accepted by an incomprehensible necessity of fact before there was a deductive theory of them.

There are, then, two relations in Lautman which can be separated. Lautman envisioned the reality of mathematics on four levels: facts, entities, theories and Ideas, which were connected since 'the facts consist in the discovery of new entities, these entities are organized in theories, and the movement of these theories incarnate the schema of connections of certain Ideas' (2011, 183). The structural conception of mathematics is treated by the relation between mathematics—which encompasses the first three—and dialectics—which encompasses the Ideas. The dynamic conception must be described in terms of the movement of facts, entities and theories; i.e., by the rendering intelligible of essential notions which one is forced to do by entities 'accepted by an incomprehensible necessity of fact', of which Lautman thus cites several examples.

Given that Lautman adopts Hilbert's scheme of formal deduction following axioms and hypotheses, how does he describe this dynamic movement of mathematics? Axioms, he argues, are often viewed from a double perspective: they are seen both as 'a system of conceivable conditions independently of the mathematical entities that they realize' and simultaneously as 'defining the most extended class of entities likely to realize them' (Lautman 2011, 31). For example, the group axioms—closure, associativity, existence of an identity element and existence of an inverse element—can be seen as the conditions which *any* kind of group should satisfy, but also as defining the entity *group* in general. Lautman refers as an example to the axiomatic constitution of abstract spaces as envisioned by Maurice Frechet, wherein a D space—which satisfies axioms of distance—is more specific than an L space, 'wherein the convergence of sequences of elements can be defined' without use of the notion of distance, and V spaces, 'whose definition does not even appeal to the notion of convergence and relies solely on the notions of neighborhood and point of accumulation' (Lautman 2011, 31). Any D space is also an L space—since using distance, one can define convergence—and every L space is also a V space, for using the notion of convergence one can define the notions of neighborhood and point of accumulation; but vice versa not every V space is an L space, and not every L space is a D space. One can thus see that 'the axiomatic study of abstract spaces is able to be interpreted as a generalization' (Lautman 2011, 32).

The view wherein axiomatic systems provide definitions is inherited from Hilbert, who viewed his axiomatization of geometry—'perhaps together with propositions assigning names to concepts'—as defining the concepts of "point," "line" and "plane" (quoted in Fontanella 2019, 173). That is, in order to put a halt to the infinite regress of trying to provide a foundation by way of "classic" definitions, which try to define one unknown term by ways of known terms which however would then themselves have to be defined, axiomatization defines concepts ‘by describing their relations to one another through certain axioms’ (Fontanella 2019, 173). Given the impossibility of Hilberts program to, ultimately, found mathematics on ‘a unique axiomatic system and then prove its direct consistency’, Laura Fontanella writes that axioms should not be viewed as expressing absolute or self-evident truths, but ‘meaningful only insofar as it contributes with the other axioms of the system to the definition of a concept’ (Fontanella 2019, 175). This can be taken as coming close to Lautman’s view, where he adds—over Fontanella’s agnosticism when it comes to ‘whether mathematics is simply not a body of truths’, writing that ‘the axioms of a theory do not entail any ontological commitment to the schema of concepts [thereby] defined’ (Fontanella 2019, 175-176)—that the reality of these concepts comes to them through their participation in Ideal reality of the dialectic.

It is fruitful to note here in passing that such a view of axioms and axiomatic systems already departs from Heidegger’s conception of axioms within the mathematical project. Axioms do not, in this view, define basic concepts which mathematics then has secured for itself, and there might not be need to envision any of this as an infinite regress which would be put to a halt by an “I think.” The concepts are defined relationally by multiple axioms at a time, and it is the theory as a whole which requires an ontological foundation—such as Lautman tries to provide.

Fontanella has shown that it is possible to regard ‘axiomatic systems that do not have a background theory’ as definitions. That is, the group axioms define groups with the help of set theory: the group axioms condition the pair of a set and an operation, the “background theory.” Axiomatic systems without such background, according to Fontanella, ‘fix the very meaning of the non-logical symbols of the language of the theory, such as ϵ and = in the case of set theory’ (Fontanella 2019, 170). Thus when one writes that ‘the axioms of set theory define the concept of “set”, or the axioms of arithmetic define the concept of “number”’, what is meant is actually that they define ‘the symbols ϵ and =’ and ‘0, S, + and ×, and so on’ respectively (Fontanella 2019, 170). A “set” is thus not a collection of objects, but rather ‘the possibility of performing specific operations on such collections’, and she challenges the view wherein there should be one absolute set theory founding all of mathematics, set theory only plays a fundamental role 'providing a *conceptual* basis for mathematics by determining a concept of "set" as general as possible' (Fontanella 2019, 169). The role of such a theory is thus not the foundation of primitive truths, but as providing a rich, abstract concept (Fontanella 2019, 180). In any case, 'the natural outcome of our definitional perspective [of axiomatics] is pluralism', 'the view that there are many distinct and equally legitimate concepts of sets' (or indeed, of any concept). She mentions that some set theorists conceive of this as 'an extreme form of Platonism' where 'many universes exist as an independent reality'—given the present research here, it could be over and against this conception that Lautman's Platonism is placed (Fontanella 2019, 181).

Lautman deems it possible 'to give to axiomatic thought a completely different bearing' than simply interpreting it as aiming at generalization, and in order to show this turns to the work of Georges Bouligand and his notion of "causal proofs." (Lautman 2011, 32). The term "causal proof" is informed by the idea of interpreting 'conclusions as effects resulting from the choice of the premises and the hypotheses' (Bouligand 1971, 58). A causal proof is a proof wherein the 'relation between hypotheses and a conclusion is such that any reduction carried out in the statement of the hypotheses is likely to compromise the conclusion' (Lautman 2011, 32). The hypotheses are to be understood as the "requirements" in a proof—for example, the requirements, given some topology, that it is both Hausdorff and compact for some conclusion to hold. The notion of a causal proof is not exact—indeed, it might be impossible to know for sure if a given proof is *the* causal proof—but one *is* able to compare proofs with respect to their "causality," or improve a proof. (This, incidentally, is why a causal proof is formulated as being the proof in which changes in the requirements *likely* compromise the result.)

Bouligand links 'the concern of causality' to 'the revision of initial notions implicated by the axiomatic method and the search for greater generality'—and therefore, according to Lautman, this search for generality is in no way a concern in its own right, but 'presents itself rather as a consequence of the search for the *necessary connection*' between initial notions and results (Lautman 2011, 32).

Deductively, this necessary connection is in a sense meaningless: conclusions can be derived from propositions, one might use different propositions and still arrive at the same conclusions. One might, for example, use more specific requirements than is necessary, and the conclusion will still follow. A proof which concludes something for Frechet's L spaces will still work of one requires a D space, but the proof will no longer show as clearly *what* aspect of the space "causes" the conclusion. For a mathematician, the "necessary connection" supplied by a proof of greater causality is very insightful, for one has not merely some necessary conditions from which to obtain some result, but, as far as one knows, *the* necessary conditions for specifically *this* result. It is likely precisely *these* requirements from which the conclusion follows.

This notion of the necessary connection, which is made clear by the notion of (greater) causality in mathematical proofs, is precisely something between logical deduction and mathematical psychology, which Lautman is looking for, even if it is irreducible to either one: only within an axiomatic, deductive structure can there ever exist something as a necessary connection, but it is only necessary in a sense not really provided by this structure. The proof, so to speak, is indifferent from the choice of requirements. If its requirements are met, the conclusion holds. Another proof might produce the same conclusion. No matter.

Furthermore, the fact that a causal proof is defined as a proof where changing the requirements *likely* will compromise the result accounts for the fact that we might not ever be able to proof that we have arrived at *the* causal proof—it is a notion which describes something very much in the movement of mathematical practice.

Using these notions, Lautman will try to give a different bearing to axiomatic systems and generalization by way of the notion of dissociation. The aim is not one of 'subsuming the particular under the general', he writes, but 'carrying out the dissociations comparable to those that condition the process of physical knowledge.' He distinguishes two kinds of dissociation: the first where 'two properties are wrongly identified' as being one and the same, which becomes apparent in the discovery of an entity 'in which one is realized without the other', showing their difference. An example here is Weierstrass's discovery of a continuous function without derivatives, dissociating "continuity" and "derivability." The second kind of dissociation 'establishes difference between certain elements having use of a common property', for example, distinguishing between saddle-points and extrema of a function, both of which are critical points (Lautman 2011, 33).[2]

The example of Weierstrass's function is significant, because it is one of the examples which Lautman mentioned as being 'accepted by incomprehensible necessity of fact' before there was a deductive theory of it, one of the resistances to be overcome in the progress of mathematics. Weierstrass published his proof of the existence of a completely continuous function not derivable in any point in 1872, and at that moment himself writes that according to his knowledge 'even in the writings of Gauss, Cauchy and Dirichlet one cannot find a statement which makes it univocally clear that these mathematicians (…) were of any other opinion' than that continuous functions of real variables have a derivative in all points except perhaps some.[3] Weierstrass then constructs a continuous function which has no derivative at *any* point. If before, having a derivative was almost coextensive with being continuous, from now in it is clear that what one thinks by the name of these notions must be *wholly distinct*.

Such a dissociation, furthermore, precisely has bearing on the likeliness of something being a causal proof: if one does not dissociate completely these notions, one will not be able to generalize or specify requirements in such a way so as to show that a certain conclusion follows necessarily *only* from the fact of continuity *or* derivability. A proof which before was considered the most causal, then, might be refined further after the discovery of a continuous function without derivatives. So while it might seem, on the one hand, that such a discovery will not significantly change any earlier proof—dissociation can never render a previous proof false—it *does*

[2] Note that here, one might apply Larvor's criticism on a smaller scale than that of the Ideas: the critical points might also be distinguished into saddle-points, minima and maxima, all of which are critical points.
[3] It is easy enough to construct a continuous function which lacks a derivative at least in one point, for example the function $f(x) = |x|$, which has no derivative at $x = 0$.

change earlier proofs precisely with regard to the necessary connection between their requirements and their conclusion. From now on, it might be shown whether it is really the *continuity* or the *derivability* which is necessary for the conclusion to follow. It is only after the dissociation of these notions that a causal proof might be improved so as to show more clearly this necessary connection.

This, then, is the other side of Lautman's attempted reconciliation between Brunschvicg's dynamic conception of mathematics and Hilbert's structural conception. On the side of the structural conception, Lautman envisions a dialectic of Ideas, which are pure problems which can never be exhausted by an answer (for example, all the different possible conceptions of a "set" which are defined, in the way specified, by different axiomatic systems). The dialectics is thus beyond mathematics like metamathematics, and its Ideas will always remain orienting ideals. On the side of the dynamic conception, Lautman links the activity of generalization to dissociative activity in search of the necessary connection between certain premises and the results deduced from them—and it is thus precisely the attempt to grasp the *meaning* of certain proof which guides the resulting generalizations. Given the criticism of Larvor, one might append to Lautman's own conception the possibility of conceiving of Ideas on different levels of abstraction. Lautman's own investigations use quite general notions, which are usually quite distant from the theories which he thereby analyses—precisely in order to see the common motivations between radically different theories. Multiple theories, whose axiomatizations might hardly have anything to do with each other, might in practice be informed by *similar* guiding ideals. Besides Lautman's choices, however, we might also argue that single mathematical concepts, such as a "set", are guiding ideals—and thus an Idea in Lautman's sense—insofar as there are multiple ways of answering the problem posed by it. That is, given Lautman's understanding of axiomatic systems defining mathematical concepts, as Fontanelle already argued, 'pluralism of concepts' is inevitable (Fontanelle 2019, 182). It is this pluralism, and the corresponding inexhaustibility if the concept which can be plurally conceived, which constitutes the 'pure problematics' and 'essential insufficiency' characterizing Lautman's dialectic.

As Fontanelle also wrote, however, such a conception of axiomatic systems defining concepts does not yet inform any specific ontological commitment. This is where Lautman's Platonism comes in, whose specific sense was still an open question requiring the investigation of the relation between his Platonism, which was looked at already quite elaborately in previous sections, and his references to Heidegger. This relation will now be considered.

§5. Lautman and Heidegger

It was shown that Lautman admits that 'the properties of a mathematical being depend essentially upon the axioms of the theory within which that being appears', and that this was his argument for discarding all forms of Platonism which envision 'a theory of the 'in-itself' existence of mathematics.' For Lautman, the 'intelligible universe' which dominates mathematics is not immutable. This means that more modern conceptions of multi-universe Platonisms, as Fontanella referred to, must also be excluded. Lautman, instead, tries to account for the fact that various mathematical theories might "incarnate" *the same* Ideas, and only in light of its participation in this 'world of ideas' can the true meaning of mathematical entities and theories be revealed (Cavaillès and Lautman 1939, 8).

Lautman's Platonism might be positioned with respect to Heidegger's conception of it if one recalls Heidegger's characterization of Plato as a philosopher who determined the being of beings ad *idea*, and envisioning the relation of beings and being as one of *participation* of the former in the latter, explaining, through the notion of *chora*, that being and beings are 'in different places'. Since, obviously, if Lautman's Platonism coincides with Heidegger's conception of Platonism—from which Heidegger distances himself—the alignment of Lautman to Heidegger would become more problematic.

Furthermore, since for Lautman, the properties of mathematical beings depend essentially on the axiomatics within which these beings appear, it would at first seem that Lautman positions mathematical theory firmly within the range of what Heidegger called the mathematical—such as the Kantian *a priori* grounded in the transcendental subject.

Nevertheless, for Lautman the meaning of mathematical beings can only be revealed through reflection upon their participatory nature, even if the world of ideas is simultaneously dependent on mathematical theory. As Catherine Chevalley notes, Lautman contests the conception of Platonism which 'interprets science as a copy, a reproduction … in short a simple transposition of ideal elements, unchanged by this assimilation of their substance by the human intelligence. The "true Platonic sense" … *removes* the idea of an irreducible distances between the "eidos" and its representation' (Chevalley 1987, 61). Indeed, in a passage concerning the matter functioning as a receptacle for the Ideas–in other words, *chora*—Lautman writes that 'the cut between the dialectic and mathematics cannot in effect be envisaged', and that it is necessary rather 'to clarify a mode of emanation from the one to the other' which 'connects them closely and does not presuppose the contingent interposition of a Matter heterogenous to the Ideas' (Lautman 2011, 199-200). This would mark Lautman as at least a mathematician who thinks 'thoroughly philosophical', as Heidegger stated about for example Heisenberg—but perhaps it does even more.

Plato, for Heidegger, anticipates the Kantian notion of space through his notion of *chora*. As being and beings are, for Plato, in different places, so objects are 'in' a space which is ascribed to something wholly other (for Plato, the being of objects in space comes to them from the Ideas, for Kant, they are founded upon the transcendental subject). And even if Heidegger does not deny that it is 'correct' that objects are in space, that to which this space is ascribed, in Kant, remains for him unelucidated as to its being, and its temporality remains unquestioned.

It seems Lautman's Platonism cannot simply be reduced to this anticipation of Kant, or, to speak less anachronistically, it seems Lautman's Platonism cannot be reduced or analogous to Kantianism. He envisions the Ideas as mutable and historical—which would, in the most radical sense, be incompatible with the static Kantian transcendental subject. The anteriority of the dialectic over mathematics might not simply be the *a priori* conditioning of its objects. It seems, thus, that for Lautman the Ideas and mathematical entities—mathematical beings and their being—are not in different places. It is within mathematical experience that we must find something ‘more and beyond’ it, Lautman said, even if the mathematical experience is the ‘sine qua non of mathematical thought’ (Cavaillès and Lautman 1939, 22). The tension between these two assertions, which is a tension present everywhere in Lautman’s work, might be likened to the fact that Heidegger always emphasizes both the duality of being and beings—ontological difference—whilst equally eager to emphasize not only that ‘being is not a being’, but that such an expression precisely still speaks of being as if it were a being.

There is, then, a non-temporal anteriority of dialectics over mathematics in Lautman, or, as it slowly seems we are correct in saying, of being over (mathematical) beings. This substitution of terms is not to say that the analogy with Heidegger has already been conclusively proven, but to bear witness to the fact that Lautman himself argues that mathematics receives its ‘reality’ from the world of Ideas, and that this is an argument for the *foundational* relevance of his work. Lautman’s theory of participation is the part of his philosophy whose purpose is to supply the notion of reality and existence. Given that Lautman also does not envision these two—being and beings—as split, at least *some* proximity of his work to that of Heidegger must be admitted.

The anteriority of the dialectic shows itself, for example, through the *dissociations* that occur throughout the history of mathematics: in some sense, they change nothing, all previous proofs remain firmly in place and no proposition, it seems, is significantly affected from the point of view of their formal deduction. Looked at through the lens of causal proofs, however, dissociations precisely effect the measure in which the *necessary connection* between hypotheses and conclusions can be grasped. Given that, within the understanding of axioms as defining concepts which Lautman inherits from Hilbert, it is not *single axioms* which define *single concepts*, but it is rather the *system of axioms* which defines multiple concepts in their interrelatedness—indeed, these concepts *being nothing else than the ways in which they are related*—this notion of the necessary connection becomes crucial for understanding the *meaning* of mathematical beings. It is, Lautman can be said to argue, the necessary connection between various mathematical entities which allows us to grasp their true meaning.

It is, thus, not the proofs which significantly change—with respect to the various deductions preformed—but our understanding of the necessary connection of the requirements and conclusions, and thus of the connection between various mathematical entities within them. This is precisely the continuous questioning of starting-points which Duffy wrote was characteristic of the Platonic dialectician, vis-à-vis the assuming of starting points of a 'mere' mathematician—and this is thus also, as was already said, greatly resembles Heidegger's distinction between science and philosophy.

Lautman, at one point, responds to criticism that some "necessary connections" simply appear as a result of 'rigging' mathematical entities to obtain certain results, and that this is not in fact significant or surprising at all. One finds simply 'what one has already put there.' This would make mathematics sound, again, more like Heidegger's conception of it. Lautman, however, emphasizes that 'presenting things in this way … does not place sufficient emphasis on the fact that there exists two sorts of 'rigging' … *those which are fruitful and those which are not*' (Cavaillès and Lautman 1939, 23). Lautman thus does not deny that it is possible to define entities precisely in order to obtain something from them, perhaps defining them in all sorts of cumbersome ways, which would make any necessary connection seem contrived, but he notes that *some definitions "work" better than others*. That is, it seems that even if things can be rigged, so as to obtain results which are desired from the start, *what is there to want* can in some sense only be *given* through the measure of *possible* rigging. Being possible in this sense does not depend on our whim. It is precisely the 'fruitfulness of certain structural properties'—of some and not others—which for Lautman distinguishes '*within* the possibilities of axiomatic definition, creative conceptions from those which lead to nothing really new' (Cavaillès and Lautman 1939, 23).

Furthermore, the dissociation of mathematical notions does not happen through first dissociating a certain notion, and then embarking upon an exploration of which mathematical beings this dissociation calls into being. It is usually not a matter of simply inventing axioms and throwing together requirements and hoping for the best. In the history of mathematics, cases appear which *force* one to dissociate between certain notions. It is a being which must already be accepted to exist which makes possible the dissociation of meaning at all.

Axiomatics, then, is governed by dialectics in such a way so as to not so much threaten any of its deductive characteristics, but which might change *the meaning* of the beings axiomatically established. And this is precisely why Lautman emphasizes that the reality of mathematics must be placed between logical deduction and the psychology of mathematicians without being mistaken for either one. One might wonder if this does not precisely coincide with what Heidegger noted about the between, that it exists only as long as one moves within it, and that it is between man and thing. Similarly, at the moment a dissociation is made possible through the occurrence of a mathematical being which, more or less, forces one to recognize the difference in meanings produced, it is neither the mathematician nor the thing which takes precedence: the beings will change as to their meaning, but this change is not a 'choice' founded within a subject which simply decides on their meaning. Which is not to say that Lautman is simply a Heideggerian—a major difference seems to be that Heidegger calls for the scientist to be philosophical, while Lautman sees a history of mathematics *forcing* scientists to be philosophical. The coincidence, however, does explain why Lautman sees his conception of Platonism reflected within Heidegger's work.

The question remaining is what becomes of the "matter", of *chora*, within Lautman's conception of Platonism. He was already quoted as saying that the cut between dialectics and mathematics cannot be envisioned, and no "Matter" 'heterogenous' to the Ideas can be 'contingently interposed' between them. How then must this participatory relation be understood?

Larvor has written that Lautman has, in fact, passed over the fact that Plato himself is—according to him—unable 'to say what 'dialectical priority' means'. Lautman has 'stumble[d] over' this inability of Plato's precisely at the moment of being unable to determine the "matter" into which the Ideas are incarnated (Larvor 2011, 192). Regarding Lautman's apparent proximity to Heidegger, one might already get the idea that this inability is not precisely a shortcoming of Plato, nor for that matter of Lautman, and this question whether Plato indeed simply fails to determine this matter, or whether it is characteristic of this matter that it cannot be determined (like how the meaning of being is not a question which can simply be answered) will be returned to in the third chapter. Before that, Lautman's own conception of this matter, of *chora*, must be made clear as much as possible. Lautman's refusal of an contingently interposed Matter heterogenous to the Ideas can be seen in analogy with Heidegger's insistence that being is not a being, and that beings are not 'related' to being as to some other thing. The question is what thus becomes of the participatory relation which, in Heidegger's conception of Plato, is made possible by the interpretation of *chora* as a difference in place.

§6. The Stakes of the Question Concerning *Chora*

Heidegger did not only reference *chora* when he interpreted Plato's philosophy, but also wondered whether *chora*, instead of anticipating Kantian space, might not mean something else: 'that which separates itself forom every particular, that which withdraws, and in this way admits and "makes room" precisely for something else?' (Heidegger 2014, 70). And the main difference, shown in the first chapter, between this other possibility and Kantian space (which Heidegger equally described as making room) was the ascription of Kantian space to a transcendental subject. It is precisely the interpretation of *chora* which would settle the question regarding Lautman's proximity to Heidegger, for, as was shown in the previous section, Lautman's Platonism in any case does not correspond with the Platonism from which Heidegger distances himself.

It was already shown that Lautman's Platonism is very close to Heidegger's thought. Being, in terms of the Ideas incarnated in mathematical theory, is not something immutable, but is necessarily founded upon historical beings. It is 'governing' with respect to them, and they only appear truly when taken in respect of being and questioned with regard to their participation. Simultaneously, this ideal reality itself exists only inasmuch as it is incarnated. The only major difference was that Lautman envisions the history of science as sometimes forcing scientists to think philosophically, while for Heidegger it seemed that scientists might, individually, take on a philosophical stance. This difference might in fact be what makes Lautman's alternative—if it proves to be one—desirable.

Lautman has commented upon the matter of *chora*, referring to it through the common renderings of "place," "receptacle" and the Aristotelian interpretation "matter." The Ideas, he writes at one point, 'though not being in the time of the created world, are produced no less according to an order of the anterior and the posterior':

> [Léon] Robin shows how the constitution of bodies in the *Timaeus* assumes a matter which, before the existence of the world, has already been the receptacle of a geometric qualification. "There is therefore a generation and becoming anterior to the generation and the becoming of the world" (Lautman 2011, 190).

Conversely, as was already quoted, Lautman does not wish to assent to the conception of this matter as a contingent interposition, 'heterogenous to the Ideas'—the relation of dialectics and mathematics cannot be envisioned as difference in place. It is the question of this matter which thus is crucial when it comes to the sense of 'anteriority' which the dialectics has over mathematics, and it is at moments considering this receptacle that Lautman turns to Heidegger. Given that Lautman does not wish to ground his scheme of dialectics and mathematics upon some heterogenous third, and thus, like Heidegger, wishes to think *chora* otherwise, this becomes a shared concern of both. Heidegger shows the stakes of the question to be the demarcation of modern metaphysics—as was shown in the previous chapter, this other interpretation of *chora* is the condition which any alternative mode of questioning must satisfy. It is then indeed the interpretation of *chora* on which the alignment of Lautman's philosophy with Heidegger's hinges.

§7. Lautman: *Chora* and Exemplarity

According to Pierre Cassou-Noguès, Lautman's Platonism cannot be understood without reference to the influence of Brunschvcig, and in his analysis, he situates Lautman in relation to Brunschvicg and compares this to the influence of Brunschvicg on Bachelard and Cavaillès. The question regarding the “matter” onto which the Ideas are inscribed is thereby, in the end, crucial. Cassou-Noguès’s analysis will therefore be followed closely.

Brunschvicg’s heritage in the work of Bachelard, Cavaillès and Lautman, he argues, can be summed up in three points: ‘a methodological postulate, which is to seek the objectivity of the sciences in their history; the distinction between two factors of objectivity, one which is, say, intellectual, and the other empirical; the problem of the unity of science’ (Cassou-Noguès 2010, 60-61).

Now although Lautman does not, like Cavaillès, take over Brunschvicg’s practice of looking at ‘long periods’—Brunschvicg starting from ancient mathematics, Cavaillès covering a century of analysis and set theory—Lautman nevertheless ‘works on mathematics’ in a Brunschvicgian manner: his ‘insistence … on the fact that the dialectical Ideas are only revealed in the “proper movement of a mathematical theory”’ bears witness to this fact, his method is still one ‘of descriptive analysis’, even if his story is ‘punctual or … done on the spot, so to speak’ (Cassou-Noguès 2010, 64). For Brunschvic, however, history had ‘the role of “matter” and “means” [of philosophical reflection]’ (Cassou-Noguès 2010, 56). It will emerge that this is somewhat different in the case of Lautman.

The second point of Brunschvicg’s heritage—distinguishing two factors of objectivity, one intellectual and the other empirical—refers to ‘the Brunschvicgian schema of a dialogue between experience and reason’, wherein the term objectivity has some ‘vagueness’ which allows it ‘to be attributed sometimes to experience, sometimes to reason’ (Cassou-Noguès 2010, 60). Cassou-Noguès, now, argues that Lautman replaces the intellectual factor with his ideal reality, removing them from reason and fixing them ‘in Ideas independent of the mind.’ Lautman’s following ‘hesitations’ are according to Cassou-Noguès ‘not only due to the difficulty of thinking the relation of these Ideas to mathematical theories, but also due to the difficulty of recovering from this schema the second factor, the empirical aspect, of the objectivity of mathematics’ (Cassou-Noguès 2010, 61). Without radically disagreeing with him, I wonder whether this description presents things most clearly. Indeed, right after referring to Brunschvicg in his second thesis, taking over from him ‘the idea that the objectivity of mathematics is the work of intelligence, in its effort to overcome the resistance that is opposed to it by the matter on which it works' (elsewhere arguing that this objectivity is indeed 'imposed' on the intelligence), Lautman writes the following:

> Between the psychology of the mathematician and logical deduction, there must be room for an intrinsic characterization of the real. It must partake both of the movement of the intelligence and of logical rigor, *without being mistaken for either one* (Lautman 2011, 28, 88-89; my emphasis).

Now Lautman himself presents this as being in line with Brunschvicg's scheme, but if in Brunschvicg, objectivity is indeed, as Cassou-Noguès wrote, sometimes attributed to experience (here, it seems, 'the psychology of the mathematician'), sometimes to reason ('logical deduction'), there seems to be a difference in Lautman's statement that the characterization of the *real* must not be reduced to *either.*

That Lautman has been influenced regarding the third point, that of the unity of science, is clear from the very title of his first thesis—"Essay on the Unity of the Mathematical Sciences in their Current Condition"—and from the fact that the dialectical Ideas are used precisely to argue that *different* mathematical theories incarnate *the same* problems, and that this in fact is their hidden agreement. One might note in passing that it is this point of influence which sets Lautman in opposition to Heidegger, who argued, as was shown in the first chapter, that there is no such thing as unified science, only specialized science, and philosophy, rather, is unified. Now since the unity is for Lautman situated in the dialectic, this opposition cannot be called absolute, but it does make it so that this philosophical unity precisely *is* the unity *of* the sciences.

Lautman, writes Cassou-Noguès, recognizes his heritage 'when he defines his position by associating Brunschvicg and Hilbert', after which the relation to both is nicely summarized:

> From Hilbert, he takes up the idea that mathematical theories are susceptible to analysis in a meta-discourse that highlights their logical properties. These logical properties must be understood in a broader

> sense than that of Hilbert's program, since, instead of properties such as non-contradiction, which either does or does not verify a theory, Lautman evokes oppositions susceptible to an "infinity of degrees." On the other hand, Lautman takes from Brunschvicg the thesis that the philosophy of mathematics is based on the analysis of history or, punctually, the "movement" of theories (Cassou-Noguès 2010, 65).

The crucial question for Cassou-Noguès, then, is the question regarding the 'second factor' of objectivity, experience, after the intellectual factor first played by reason has been replaced, so to speak, with Lautman's ideal reality.

Now Cassou-Noguès writes that on the one hand, the Brunschvicgian heritage is the source of originality of Lautman's Platonism, while conversely, his Platonism 'gives Lautman a singular place in this tradition' (Cassou-Noguès 2010, 65). The first is because Lautman's Platonism 'is a historical Platonism', and 'the *place* where Ideas are incarnated … is not that of mathematical theories understood as propositional systems: it is the activity, movement, or again mathematical "experience."' (Cassou-Noguès 2010, 65; my emphasis). On the other hand, Lautman precisely through his Platonism, according to Cassou-Noguès, 'admits an ideal reality situated out of time' (Cassou-Noguès 2010, 65). The passages to which he refers, however, might be read a bit differently. Lautman does not so much write that the 'ideal reality' is situated *out of time*, but 'beyond the temporal circumstances of discovery' (Cavaillès and Lautman 1939; 23-24). Because of Lautman's particular Platonism, one should take careful note of this. And indeed, the other sentence to which Cassou-Noguès refers—on the necessity 'to relate to the intrinsic nature of [ideal] reality the modalities of spiritual experience in which it allows itself to be apprehended'—is not quoted in full. Cassou-Noguès quotes Lautman saying that 'the reality of mathematics is not made in the act of the intelligence that creates or understands, but it is in this act that it appears to us', but does not seem to find the rest of that sentence—*'and it cannot be fully characterized independently of the mathematics that is it's indispensable support'*—crucial (Cassou-Noguès 2010, 65; Lautman 2011, 28).

Here again, it seems to be a matter of wondering whether Lautman truly places the ideal reality *beyond* the factors of Brunschvicg, or rather *between* these factors in a way that cannot be simply reduced to either. As was shown before, the question concerning *chora*, or the "matter" onto which the Ideas are inscribed, hinges on the question how one is to understand it if it cannot be thought of as being a heterogenous *third* term, so to speak, which would account at once for both the existence of the Ideal reality and mathematical theory. It was a question of clarifying 'a mode of emanation from one to the other.'

For Cassou-Noguès, there is a 'hesitation' in Lautman's work regarding the question of *objectivity*, and, in his view, the missing second factor with respect to the Brunschvicgian scheme. Lautman's hesitation, now more specifically, is 'the problem … to know what is the matter in which the Ideas incarnate and to what extent it contributes to the reality of mathematics' (Cassou-Noguès 2010, 70). For Brunschvicg, this matter was the historical, mathematical experience. For Lautman, this does not (simply) seem to be the case.

At one point, Lautman compares—like Cavaillès—mathematical "signs" to 'the body of the Idea', which, according to Cassou-Noguès, shows that 'in these first texts, the experience, the matter of the Idea comes to contribute to the solidity, the reality of mathematics' (Cassou-Noguès 2010, 70).

At a second point, however, Lautman 'refuses in general the position of such a "matter" or an experience heterogenous to the Ideas, in which these would come to incarnate', and 'the passage from the Idea to mathematics would then come only from the mechanism of thought, from the "effort of understanding", which implies being able to illustrate the Idea with the concreteness of an *example*' (Cassou-Noguès 2010, 70). The mathematical concepts themselves would then become 'the material in which the Ideas are embodied' (Cassou-Noguès 2010, 71).

Cassou-Noguès reads this illustration of the Idea through an example as an outright refusal of any 'matter', and wonders whether 'Lautman's attempt to get rid of [Brunschvicg's] duality, to get rid of the second factor of objectivity represented by experience … succeeds' (Cassou-Noguès 2010, 71). In the eyes of Cassou-Noguès, Lautman hesitates between these two positions, and this hesitation leaves open 'the problem of the "how" of realization of the Ideas' (Cassou-Nogues 2010, 73).

He describes the two positions as follows: on the one hand, the Idea seems to need to exist for mathematics to be an example of it. This suggests, according to him, 'that it is the mind that makes the transition from the dialectical level to that of mathematics: a mind that hears the question and proposes an answer in the form of a mathematical theory.' From the perspective of this position, '[Lautman] admits at the origin of mathematics a subjectivity, analogous to Brunschvicgian consciousness, which creates theories beginning with Ideas' (Cassou-Noguès 2010, 72-73). It is clear, however, that Lautman does not only take on this position—indeed, he constantly writes about the objectivity of the ideal reality being *imposed* on the mind.

So on the other hand, 'Lautman admits that the dialectical Idea is not at the origin of the creation of mathematical theories', and the anteriority, being like that of a question with regards to a response, 'does not reflect the order of the course of thought.' The Ideas exist as an *unlived* history, which the philosopher retrieves. But in conceiving of them so, writes Cassou-Noguès, 'Lautman seems to be short-circuiting his analysis of the passage from the Idea to mathematics and of the example as the matter of the Idea. This passage from Idea to mathematics, this incarnation of the dialectical Idea in a concrete example has in fact never *taken place*', and mathematical theories develop without reference to these Ideas, autonomously (Cassou-Noguès 2010, 72-73).

In my opinion, Lautman never truly writes that the Idea precedes mathematical theory *chronologically*. This does not reduce the fact that in Lautman there is indeed a tension between the anteriority or priority of the Idea over mathematical theory, and the analysis of it which is by necessity, Lautman wrote, *regressive*. The two positions between which Lautman hesitates are that of a 'beyond' and a 'between.' The ideal reality which must be grasped beyond 'the temporal circumstances of discovery' according to Lautman must be found 'in experience' as being 'something else and something more than experience.' That is the most concise summary of Lautman's hesitation: something else and something more. There must be a characterization of the real 'between the psychology of the mathematician and logical deduction', it must 'partake both in the movement of the intelligence and of logical rigor, without being mistaken for either one'—whereby it remains unclear if it not to be mistaken for either one because it is in fact partly *both* or because it is truly beyond and thus *neither*.

It cannot be denied, I think, that Lautman seems to hesitate. Neither is Cassou-Noguès, in my opinion, mistaken on the fact that Lautman himself does not clearly express himself concerning this matter. But I would like to propose that Lautman's hesitation itself is surprisingly consistent, and that this consistent hesitation is in fact not fortuitous, but necessary. In order to do so, I will look at three sentences concerning the mode of *exemplarity* of mathematics with respect to the Ideas, and the question of the "matter" within it, since, as Cassou-Noguès has already shown, it is the notion of *example* which seems to take on the role of matter within Lautman.

1. First is Lautman's most explicit refusal of the "matter":

> It is necessary … to clarify a mode of emanation from the one to the other [dialectics and mathematics], a kind of procession that connects them closely and does not presuppose the contingent interposition of a Matter heterogenous to the Ideas (Lautman 2011, 199-200).

While Cassou-Noguès, regarding this sentence, writes that Lautman 'refuses in general the position of such a "matter"', it seems rather that he specifically refuses a Matter—written here with a capital, which Lautman only does twice, once in a letter—which is *contingently* interposed between the Ideas and mathematics, and which is heterogenous to the Ideas. That is, the existence of mathematics, the characterization of the reciprocal reality of mathematics and the Ideas that govern it, cannot be accounted for by some third term beyond them, without reference to them.

2. Then a sentence from the text in which Lautman turns to Heidegger to explain himself:

> To think [the Ideas] fully, it is necessary then to rely on some example, perhaps foreign to their very nature, but that gives shape, at least for thought, to the necessary matter (Lautman 2011, 204-205).

I would like to propose some comments on Duffy's translation of this sentence. The original French gives us, for the second half of the sentence, 'mais qui prend ainsi, tout au moins pour la pensée, figure de matière nécessaire' (Lautman 2006, 242). The verb "prendre" is thus by Duffy translated as 'gives shape', while one might translate that the *example* which is perhaps foreign to the very nature of the Ideas "*assumes the figure,* at least for thought, of (the) necessary matter." That is, Lautman can be read as writing that mathematical concepts, examples of the Ideas, serve *like* Plato's necessary matter. They assume—at least for thought—the figure of a matter on which the Ideas by necessity must be inscribed. This is why Lautman can later write that 'it is not necessary that the examples which correspond to a dialectical structure are such and such' (Lautman 2011, 207; translation slightly changed), and why Lautman can remain insistent on the fact that there is no contingent interposition of some Matter acting as a autonomous third.

3. Finally a sentence taken from a letter to the mathematician Maurice Fréchet:

> I wrote that mathematics is an example of incarnation, in the sense in which mathematical concepts constitute for example a matter on which relations envisaged as possible by the dialectic are effectively drawn (Lautman 2011, 223).

The part of the sentence translated as 'mathematical concepts constitute for example a matter on which relations … are effectively drawn' is 'les concepts mathématiques constituent comme une matière sur laquelle se dessinent effictivement les relations' (Lautman 2006, 262). Given the previous passage, I would propose to translate, rather than that the concepts 'constitute for example a matter', that they "serve as a matter"—where the important difference is not the change of the English verb "constitute" to "serve," but the sense in which to take "comme"—which I propose to take as a word signifying that mathematical concepts act *like* a matter for the dialectical Ideas.

From all of these passages, there emerges a sense in which Ideas are inscribed on mathematical concepts by way of a notion of *exemplarity*. Mathematical concepts serve as an example for the Ideas, and as such example, are ‘perhaps foreign’ to the Ideas. The dialectics as a whole, however, is not incarnated by way of a heterogenous matter. Mathematics, then, acts *like* this matter, in the sense that mathematical concepts serve as that on which the relations must be inscribed—but insofar as mathematical concepts are only ever provisional, and prone to change throughout history, they are only *examples* of these Ideas. They are not *interposed* between mathematical theory and the Ideas, for they are mathematical theory itself.

It can, in short, be said that this notion of exemplarity is Lautman’s interpretation of the dualism between being and beings; that it is this notion of exemplarity which in Lautman’s work serves to replace (or interpret) Plato’s notion of *participation*. *Chora*—or “place”—is not taken as a third, something which *divides* beings and being, rather, beings serve *as* this place for being by way of being *examples* of being.

Lautman’s hestitation, his ‘something more and something else’ than experience, comes down to the fact whether the Ideas can be said to exist, which in fact comes down to the question whether a “matter” in which they are inscribed, and upon which they rely for their own existence, exists. Now the mathematical examples, which figure like a matter for the Ideas, are never by necessity such or such, Lautman writes (this is precisely why several theories can incarnate the *same* Ideas, and what assures mathematics of its unity). But the Ideas can also only ever be retrieved by a regressive analysis of mathematical theory. There is, then, something provisional about which Ideas are said to be incarnated in mathematical theory, and we might even argue that there is a case of *one* Idea being incarnated in *several* theories *only through the comparison of those theories by the mathematician*. Taken thusly, Lautman’s conception indeed comes close to Heidegger’s notion of the “between” which questioning creates, which only exists *while we move within it*.

The Idea is anterior in the sense that both theories, after the Idea has been regressively retrieved from them, constitute *examples* of it, but only in this sense is it anterior. It must be concluded, then, that chronologically *the Idea does not precede its own examples.*

This is why Cassou-Noguès correctly writes that Lautman 'seems to be short-circuiting his analysis'—because the notion of "example" employed by Lautman seems somewhat improper. If Lautman's conception of mathematics is to be put forward as an alternative mode of questioning in Heidegger's sense, the reasons for this impropriety must be determined. Therefore, in the next chapter I will try to show why the impropriety of this notion of exemplarity—that the examples precede that of which they are an example—is not fortuitous, in fact precisely in the same way in which Plato's lack of proper determination of *chora* is not fortuitous but necessary. I wish to show that precisely this impropriety, by which Lautman can be said to interpret ontological difference, constitutes the moment of coincidence of Lautman's work with the philosophy of Heidegger, and the reason why Lautman's conception of mathematics in terms of examples and dissociation can serve as an alternative way of questioning in Heidegger's sense.

Conclusions

The aim of this chapter was to determine how Lautman's philosophy of mathematics should be understood so as to possibly provide a viable alternative mode of philosophical questioning; whether or not Lautman's conception of mathematics diverged from Heidegger's own, and whether Lautman's conception of mathematical existence might be compatible with Heidegger's philosophy of being, so as to be able to extend or perhaps even partially replace the notion of questioning within Heidegger's philosophy. In order to do so, a general overview of Lautman's philosophy was given, which furthermore was analyzed from the point of view of philosophy, of mathematics, and by discussing the secondary literature. Thereafter, Lautman's philosophy was compared to the results concerning Heidegger's conception of mathematics and questioning as determined in the first chapter, and the question was put forward how Lautman's philosophy is to be understood if it is to serve as an alternative mode of questioning.

Lautman, it was shown, does not believe the foundation of mathematics is to be sought in the foundation of mathematical entities—it is not a question of finding a small amount of general and rich mathematical entities from which all the others can be deduced. Rather, mathematical existence can be seen from four different points of view: that of facts, entities, theories and Ideas. This conception of mathematical reality already departs from Heidegger's conception of it, for according to Heidegger, mathematics is founded upon a few axioms which determine objects retrieved beforehand.

Lautman does not deny that mathematical entities are in the end wholly dependent, for their existence, upon the system of axioms within which they emerge. But the conception of basic objects does not precede, as in Heidegger, their determinations in axioms, rather, *systems of axioms* determine mathematical entities relationally. The whole of mathematics, furthermore, is secured in its existence through its participation within the dialectic of Ideas.

Lautman's Platonism does not consist of an 'in-itself existence' of mathematics through its participation in an immutable universe. While mathematics receives its existence through its participation in the dialectic of Ideas, the Ideas only exist insofar as they are incarnated in mathematical theory in its *living movement*, in mathematical experience or practice.

One amendment of Lautman's philosophy can be made following a criticism of Brendan Larvor; the method of division need not be limited to division in *two* categories, only to finite amounts; furthermore, Laura Fontanella's discussion of axiomatic systems as "definitions in disguise" showed that *single* notions can be seen to incarnate in multiple ways. Indeed, there are many fruitful notions of "set" which cannot simply be hierarchically ordered. Lautman's Platonism can provide an account of this situation by arguing that the dialectical Idea of a set—the *being* of the entities called "set"—cannot be exhausted by any amount of incarnations, in precisely the same way as liaisons between multiple notions. The dialectic is thus characterized by an *essential insufficiency*, which, it was shown throughout the chapter, can be thought to coincide with Heidegger's *inexhaustibility* of self-questioning. That is, the dialectic must be understood as posing only problems, while any attempt at an answer already belongs to mathematical theory. Lautman here takes over from Heidegger the thought that posing a question itself already 'delimits the field of the existent', which is equally at the heart of the constant questioning in Heidegger: knowledge needs to 'first and constantly' question its own presuppositions in order 'to preserve things in their inexhaustibility' (Heidegger 1967, 65).

This results in two questions, which were treated in §4 and §§6-8, respectively. First was the question of how mathematics develops—how we are to conceive of mathematical theory in its movement. Second was the question how we are to conceive of Lautman's philosophy as foundational for mathematics, seeing as how one might argue him to fall back on a form of circular reasoning—granting mathematical entities existence through their participation in the dialectic of Ideas, and simultaneously granting the Ideas existence insofar as they are incarnated.

In order to think through mathematical practice, Lautman turns go Georges Bouligand's notion of a causal proof and the necessary connection between requirements and results of such a proof. The notion of necessary connection, existing 'between the psychology of the mathematician and logical deduction', without being reducible to either, perfectly complements Lautman's philosophy. The measure of causality of a proof—which itself is a notion which cannot be feasibly formalized, but is instead useful in comparing different existing proofs—is the measure in which the necessity of connection between entities required by a proof and conclusions derived about those entities is expressed by the proof. Given that, for Lautman, entities are defined through the axiomatic system as a whole, the notions of causality of proofs and necessity of connection thus allow a mathematician in practice to more truly grasp what a certain mathematical being *is*.

The causality of proofs might be improved through *dissociation*. For Lautman, mathematics progresses through the emergence of new entities, which at times force a working mathematician to dissociate certain meanings which before were either the same or perhaps largely coextensive. The search for greater causality of proofs, which lies at the heart of the intimate connection between mathematical practice and the progress of generalization, is thus aided by a movement of dissociation which is as much due to the mathematician as the theory itself; while the mathematician might try to "rig" theory in order to retrieve from mathematical theory precisely what was, so to speak, "put into it", *what is there to put in* does not depend on their whim. It is by considering the difference between fruitful fruitless rigging by which, *'within* the possibilities of axiomatic definition', 'creative conceptions' can be distinguished from 'those which lead to nothing new'.

The second question—how Lautman conceives of the foundation of mathematics, if mathematical theory depends on its participation in a dialectic of Ideas but this dialectic of Ideas depends on its incarnation in mathematical theory—lead to the consideration of the question of *chora*, or the "matter" onto which Ideas are 'effectively drawn'. Lautman univocally refuses any conception of this matter as a third term. Rather, Lautman reinterprets the notion of participation in terms of *exemplarity*. Mathematical entities serve *as* necessary matter onto which Ideas are drawn, while any entity or mathematical theory will always only remain an *example* of the Idea, which does not by necessity have to be such or such.

Concerning this, Pierre Cassou-Noguès observed a problem. If incarnation does not take place chronologically, does it ever truly take place at all? How can Lautman insist on the anteriority of the dialectic over mathematical theory while simultaneously emphasizing the *regressiveness* of analysis through which one observes the Ideas, even writing that Ideas only exist insofar as they are incarnated? In other words, this notion of *exemplarity*—which replaces the 'contingent interposition' of some Matter 'heterogenous' to the Ideas—seems improper. In a sense, it must be said that the examples precede that of which they are an example, but that these examples simultaneously *are what they are* inasmuch as they are an example *of something*.

Two things can thus be concluded. First of all, that Lautman's conception of mathematics is different from Heidegger's, and does not fall pray to what Heidegger calls 'the mathematical', which is his characterization of modern thought. Lautman is much closer to the conception of *chora* which Heidegger wonders might provide an alternative to the Platonic anticipation of modern thought, that of *chora* being 'that which separates itself from every particular, that which withdraws, and in this way admits and "makes room" precisely for something else'. Indeed, the conception of the Idea as that which only ever incarnates in an *example* of it might equally be described as

Ideas *withdrawing* from every *particular* incarnation of them, making room for mathematical theory precisely by letting mathematical theory *be* this example which does not have to be such and such—which is, in fact, the condition of possibility for the progress of mathematics through the dissociation of the meaning of these examples.

This means, secondly, that it has to made clear how Lautman's notion of exemplarity is to be understood so as to not simply seem the term masking some circular reasoning. That is what I wish to do in the following chapter.

Chapter 3: Exemplarity

Dissociation as Questioning

From the first chapter, it became clear that the condition which any alternative mode of Heideggerian questioning has to satisfy can be summarized as being that it conforms to the alternative reading of *chora* put forward by Heidegger in *Introduction to Philosophy*. That is, instead of anticipating modern thought—which finds its culmination in the philosophy of Kant—by thinking *chora* as a difference in place between being and beings, with beings thus participating in a being which itself is 'elsewhere' (be it the realm of ideas, in Plato, or the a priori structures of the transcendental subject in Kant), *chora* should be thought as something other than a split or cut, as 'that which separates itself from every particular, that which withdraws, and in this way admits and "makes room" precisely for something else'.

The second chapter gave an overview of Lautman's philosophy of mathematics, and showed that Lautman's conception of mathematics does not fall within what Heidegger calls modern thought. Lautman's Platonism does not conceive of *chora* as difference in place, and the cut between dialectics and mathematics, or being and beings, can, for Lautman, 'not be envisioned'. *Chora*, the "matter" onto which the Ideas are drawn, must for Lautman too be thought otherwise.

At the end of the second chapter, it became clear that Lautman's alternative reading of participation—although he does not frame it explicitly as such—is given by his notion of *exemplarity*. Mathematical theory, mathematical entities, serve as 'examples' of the Ideas which they incarnate, and are 'perhaps foreign' to the 'very nature' of these Ideas. A seeming problem with this notion of exemplarity is that it seems to 'short-circuit', as Pierre Cassou-Noguès wrote: either the Ideas are chronologically anterior to mathematical theory, which Lautman denies, or they seem to never truly incarnate at all, since in that case the mathematical 'examples' provide the existence of that of which they are supposed to be an example, they seem to precede or at least coincide with them.

If this impropriety of the notion of example in Lautman's Platonism is not shown to be something else than a failure, Lautman cannot be said to provide a notion of mathematical existence, for, in his work, this notion of existence is provided precisely by the relation of participation and incarnation of mathematics and dialectics.

The possibility of interpreting Lautman's conception of mathematics as an alternative mode of Heideggerian questioning hinges on the interpretation of *chora* put forward in this conception. It is already clear that this interpretation is not incompatible with Heidegger's thought. What I wish to show, in this chapter, is that the seeming impropriety of Lautman's notion of exemplarity is in fact not a mistake, but is precisely that by which Lautman's philosophy can be seen to amend or extend Heidegger's notion of questioning.

In sum, in this chapter I wish to do two things. First, to show how Lautman's notion of exemplarity is to be understood so as to see it as something other than a failure, and so as to read Lautman as providing an alternative mode of questioning. In the process, and secondly, it will finally be cleared up what must be meant by speaking of an alternative "mode" or "notion" of questioning, for it will become clear in what measure Lautman serves as an *extension* of Heidegger's thought, and in what measure it can be seen to *amend* it.

To do so, I turn to analyses of Jacques Derrida. Derrida in fact has taken up Heidegger's comment whether not *chora* might be thought otherwise, and through Derrida's work on the notion of *metaphor* in Heidegger, a situation analogous to Lautman's use of exemplarity can be shown to exist in Heidegger's conception of the relation of language and truth. I will thus first work through Derrida's analyses of *chora* and *metaphor* in Heidegger (§1, 2), and finally of the relation of language and truth (§3), and then compare the results with Lautman's notion of exemplarity (§4), in order to argue that not only that Lautman does provide a notion of mathematical existence through it, but that the way in which he does makes his philosophy a viable alternative mode of Heideggerian questioning.

In short, the question answered in this chapter is how we are to understood Lautman's notion of exemplarity so as to see it as providing a notion of mathematical existence which is compatible with Heidegger's philosophy, while providing a different conception of mathematics which lies closer to what Heidegger envisions when he says that philosophy should consist of questioning.

§1. *Chora*

Derrida takes up the question of *chora* within the context of Heidegger's philosophy. Given that *chora* is to determine the *relation* of the duality of being and beings—as Heidegger tells us—Derrida to begin with writes that '*chora* is nothing (no being, nothing present)'.[4] It is a word for 'that which is not reappropriable' (Derrida 2002, 59).[5] This is also clear, since if *chora* is rendered as *something*—a being—it cannot be that which determines the relation of being and beings.

Given that Derrida does not supply this interpretation of *chora* as 'a Heideggerian reading', it is immediately clear that he does not follow Heidegger in his determination of *chora* in Plato as 'difference in place'. Derrida in fact emphasizes that Heidegger's determination of Plato's conception is made 'outside of all quotation and all precise reference', and Derrida thus seems to be skeptical with respect to this determination. In other words, if Derrida's work seems to follow up on Heidegger's question whether or not *chora* might not be thought differently, he also argues that this different interpretation is in fact already *present* from the start. He emphasizes that "Platonism", whatever it is, must always remain the name of an interpretation which dominates and renders invisible 'other motifs of thought which are also at work in the text' of "Plato." Which isn't to say that Platonism is illegitimately or arbitrarily called so, and that it is not for *necessary* reasons the 'dominant effect' of those texts, but that this effect is always one among many, and is thus 'always turned back against the text' in some measure (Derrida 1995, 119-120).

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[4] Some translations of Derrida render *chora* as *khora*. I have replaced all such occurrences with *chora* for reasons of consistency.

[5] To be sure, this is not the only term of which Derrida throughout the course of his philosophy argues that it is 'no being' or 'nothing present'. For present purposes, however, what is important is not whether this interpretation of *chora* is distinct from interpretations of other terms, but how we are to understand the possible alternative meaning of *chora* within Heidegger's philosophy.

In various texts, Derrida comes back to what he calls this ‘place name’, a name for ‘*place itself*, the place of absolute exteriority’ within ‘the open interior of a corpus, of a system, of a language or culture’ (Derrida 2002, 57). *Chora*, it can be understood, is a name for that which makes possible naming at all. Since *chora* determines the duality of being and beings, and we always, according to Heidegger, speak *within* this duality, it is, indeed, the ‘place of absolute exteriority’ within an ‘interior’ which thus must be ‘open’ from this inside. This also makes it clear why it is not reappropriable by ‘any theological, ontological or anthropological insistence’, that it is ‘without age, without history and more “ancient” than all opposition’, that it cannot even be said to be ‘beyond being’ (Derrida 2002, 58). That is, since it is a *name*—if that is the word—‘for *who* or *what* will have given place to all this’ as a kind of ‘receptacle’, it is always spoken of in a way that is seemingly improper, since whatever notions of concepts are used to determine it are already themselves determined *by* it. (The circularity in *chora*, then, is similar to the circularity in Kant’s proof regarding his highest principle—a circularity which Heidegger associates with the “open.”)

This, however, means that one is perhaps mistaken when one speaks of *chora* as providing a name of a *relation* between being and beings. Heidegger, it was shown, writes that Plato’s interpretation of the duality—as he saw it—is not the only possible one, that the ontological difference can be thought otherwise. But perhaps it is already wrong to speak of a *duality* of being and beings, of ontological *difference* between this and that. Indeed, since *being* is not itself a being, it is not to be, in any proper sense, *related* as something distinct to other beings.

Chora, then, is not so much a name for the relation *between* being and beings, but must itself by an attempt to name *how being provides being*. And indeed, Heidegger himself has taken up his remark whether *chora* might be thought as that which ‘withdraws from every particular’ and in that way ‘makes room’ as a description of being itself; in *Contributions to Philosophy* he writes that ‘being itself is essentially determined as [the] self-withdrawing concealment [which *conceals itself* in the manifestness of beings]’, that ‘being is withdrawing from beings and yet lets them appear as what “is”’ (Heidegger 2012, 88, 92).

It is already becoming clear, then, that this interpretation of *chora* is in fact a determination of *being* itself, or an attempt to question a particular determination of being. It almost seems like *chora* is another word for *being*. This is quite characteristic of Derrida, who often, as Geoffrey Bennington writes, takes terms from texts 'read not according to a program or a method', but by 'the chance of encounters with what is bequeathed or repressed by the tradition' (Bennington 1993, 267). These terms remain at once connected to the texts from which they were taken, and simultaneously 'cannot for all that remain enclosed in a supposed immanence of the text' (Bennington 1993, 267-268).

The discourse on *chora*, as what makes place for beings to be, in any case, by withdrawing—as that which separates itself from every particular, as Heidegger wrote—is confronted with a problem. It 'plays for philosophy a role analogous to the role which *chora* "herself" plays for that which philosophy speaks of, namely, the cosmos formed or given form according to the paradigm', and that 'it is from this cosmos that the proper—but necessarily inadequate—figures will be taken for describing *chora:* receptacle, imprint-bearer, mother, or nurse.' In other words, 'philosophy cannot speak directly … about what these figures approach' (Derrida 1995, 126).

In other words, since we are speaking of *that* which makes possible speaking of this or that to begin with—speaking of what *allows* determination of things *as* things, which itself then cannot *simply* conform to such determination—how we speak is 'necessarily inadequate'. If one says that it is anterior to everything, one must remember that is also anterior to this notion of anteriority, or similarly that it precedes precedence, that it is prior to any a priori, and so on. Speaking of *chora*, then, a *necessary* impropriety of terms emerges.

Philosophy thus, as Derrida writes, takes these figures for describing *chora* from that which is made possible *by* it. What must be done to understand the *necessary impropriety* of these terms is to clarify the sense in which these are "figures."

§2. Metaphoricity

Saying that the figures for *chora* are metaphors fails to account for the fact that the proper notion of metaphor can be said to be founded upon the distinction between sensible and intelligible which *chora* itself makes possible. That is, if we say that these figures are metaphors in this proper philosophical sense, we have already determined *chora* as itself being a being (cf. Derrida 1982), since in that case we are using *known* beings to determine an *unknown* being, as proper metaphors do. Derrida emphasizes this fact multiple times:

> Almost all the interpreters of the *Timaeus* gamble here on the resources of rhetoric without ever wondering about them. They speak tranquilly about metaphors, images, similes. They ask themselves no questions about this tradition of rhetoric which places at their disposal a reserve of concepts which are very usueful but which are all built upon this distinction between the sensible and the intelligible, which is precisely what the thought of the *chora* can no longer get along with (Derrida 1995, 92).

> It is not a question here of criticizing the use of the words *metaphor, comparison* or *image*. It is often inevitable [and it] will sometimes happen that we too will have recourse to them. But there si a point, it seems, where the relevance of this rhetorical code meets a limit and must be questioned as such, must become a theme and cease to be merely operative. It is precisely the point where the concepts of this rhetoric appear to be constructed on the basis of "Platonic" oppositions … from which *chora* precisely escapes (Derrida 1995, 147).

There are several texts in which Derrida comments upon this notion of metaphor which would not be reducible to its proper, rhetorical sense (cf. Derrida 1982; Derrida 1991; Derrida 2007). It is beyond the scope of this thesis to go through all of them. A text which we might privilege is 'The *Retrait* of Metaphor', for it links precisely the question of improper metaphoricity with the matter if *withdrawal* which Heidegger associates with the alternative possibility of *chora* and, subsequently, his own thought of being. In 'The *Retrait*', Derrida is dealing with Heidegger's 'treatment of language' (Derrida 2007, 52). Every withdrawal of being, Derrida explains, corresponds with a *determination* of being: by being determined in such or such a way, while not *being* such or such, being is simultaneously revealed and concealed (Derrida 2007, 64-65). That is, while every determination of being is necessarily *inadequate*, in some sense—since being is not a being, and determining it as this or that thus makes the mistake of treating it as a being—it is also *necessary* that it is inadequately determined.

These determinations, then, are neither simple *metaphors* nor simply *proper* determinations; one can speak of being only '*quasi*-metaphorically, according to a metaphor of metaphor' (Derrida 2007, 66). It is from here that the *necessary* impropriety of the notion of metaphor emerges. That is, insofar as any concept of metaphor emerges from *some* determination of being, it is improper, but insofar as one can speak of being only from *within* one such determinations of being, this impropriety is not something which one could have avoided, which could be corrected by some *proper* discourse—in fact, the *properness* of discourse itself is only produced through the determination of being which simultaneously produced the notion of metaphor to which it is opposed. 'The so-called metaphysical discourse [i.e. the determination of being anticipated by the interpretation of *chora* as "cut"] can be exceeded', Derrida thus writes, '… only according to a withdrawal of metaphor *insofar as* it is a metaphysical concept, thus according to a withdrawal of metaphysics, a withdrawal of the withdrawal of being'. Only insofar as one *improperly* uses the notion of metaphor produced by metaphysics—improper *precisely from the point of view of this metaphysical determination of being,* that is—can being be spoken of. 'But because this withdrawal of the metaphoric does not free up the place of a discourse of the proper or the literal,' because the *proper* or *literal* itself is only produced as such within the metaphysical determination one tries to exceed, 'it will at the same time have the sense of a re-folding … and of a re-turn, … of yet another metaphor' (Derrida 2007, 66).

Speaking of being thus takes the form of a movement of metaphoricity, with certain metaphors replacing other metaphors. Indeed, the notion of taking being as *withdrawal*, of letting beings be by withdrawing from them, itself begins to look like a metaphor for being—for in this description, ‘withdrawal’ cannot mean withdrawal in any proper sense, it cannot be the withdrawal of *a* being from some interior populated with other beings.

It is clear, then, why being can only be spoken of ‘improperly’, or perhaps *not even improperly*, through a *movement* of metaphoricity wherein previous metaphors are shown to be improper, but only by way of ever *new* metaphors. This, in fact, is characteristic of Heidegger’s conception of the relation of language, being and truth.

§3. Language, Being and Truth

Heidegger writes in *Pathmarks* (2008) that ‘the proximity of being is produced as language’, or, translated alternatively, ‘realizes itself historically as essence’, both of which are attempts to translate the term *west* in Heidegger, a verb made out of the term *Wesen*, essence (as cited in Derrida 2016, 56). Given the importance of the term *chora* within the current context, and of *chora* as that which “makes place” by “withdrawing,” one might translate that “the nearness of being *takes place* as language.” Derrida tries to elucidate what that means, and because of its importance for the current considerations, I cite his explanation almost in full:

> Being is nothing, is not a being; it does not belong to the totality of beings. Its meaning can appear only if beings come to be declared as what they are in their being (i.e., if being is said). But this does not mean that being belongs only to language in the sense in which one might speak of a linguistic being in a pejorative sense. A being that was only in and through the world would be nothing but a verbal phenomenon, and so it would not be Being. The co-belonging of being and language … forbids us from making the being of language dependent, as a simple character or power among others, on a being that might be called man, for example (Derrida 2016, 56).

It is then, also, this conception of language by which Heidegger distances himself from Kant. The alternative understanding of *chora* or being is, as was shown in the first chapter, dependent on a thought which would not make beings dependent on another being—the transcendental subject, for example. Given that beings are insofar as they are *declared* as what they are, i.e. that being takes place as language, the being of language may thus not be made dependent upon man. Continuing the quotation:

> So the essence of language must be rethought in the light of the meaning of being. In doing so, one will go beyond that philosophy of language that makes of language the original character of man. To the contrary, one will come … to determine man on the basis of language as language of being, and this is not a simple nuance or a simple logical inversion of the procedure. This inversion is hugely important since it liberates being from the ontic determination and allows one no longer to "tell stories"—that is no longer to think the question of being as the product, or the idea, or the character of a being already known, or that one believes one already knows: namely, man. Which would rule out the very possibility of a truth in general (Derrida 2016, 56).

This inversion of the being of language, which is no *simple* inversion, is thus necessary to 'no longer "tell stories" about being, or *chora*, or—if Lautman is to provide an alternative mode to all this—*Ideas.* That is, this other conception of language is necessary to bear witness to the fact that being is not spoken of by *metaphors*, pure and simple; what must be understood is the way in which these are only 'quasi-metaphors', and to escape the recourse to a privileged being, like the subject in Kant.

This inversion of language—whereby it is not because language is a capacity of man and the meaning of being is dependent on the language by which we grasp it, but whereby *we* are only what we are through a language that is thought as the taking place of being—is in fact the only way in which we do not rule out 'the possibility of a truth in general'. For otherwise, truth would become by necessity dependent upon a being supposedly already known and itself unquestioned. Truth would become 'subjective' not in the sense of being merely an opinion or dependent on whim, but in the sense of being founded upon a subject which we would be unable to elucidate any further.

From this inversion of language follows a reversal of the notion of metaphor. That is, while 'traditionally philosophers use metaphors to illustrate an as-yet-unknown by reference to the known or the familiar', Derrida is interested, in the words of Bennington, ‘in cases where he feels able to claim that apparent metaphors are *not* in fact being used in this way at all’ (Bennington 2016, 259-260), as could be seen in the discussion of *chora*.

As an example from Heidegger, Derrida takes the phrase ‘language is the house of being’, which he does not deny one *can* read as a metaphor, but ‘the problem … is that of knowing if it is *only* a metaphor’, he now writes (Derrida 2016, 57). This is precisely the question in what sense it is what Derrida later calls a quasi-metaphor (and in fact, Derrida refers to this sentence again in ‘The *Retrait* of Metaphor’ (Derrida 2007, 69)). We can, of course, always take such a sentence as being a metaphor, but only if we forget that that which is thus described is in fact that which we must know beforehand to grasp the metaphor at all. In the case of the example, we can only understand what a house *is,* what language *is*, if being is already understood.

The problem Heidegger is dealing with then, according to Derrida, is that ‘the meaning of being is … the condition of possibility of language on which it nonetheless depends’ (Derrida 2016, 57). Only if the relation of being and language is thought through in this way can we attain something like the truth in general, not dependent upon some other being.

§4. Conclusion: Dissociation as an Alternative Mode of Questioning

The resulting impropriety of the notion of metaphor shows the coincidence of Heidegger’s thought with that of Lautman. Heidegger does not want to give assent to a thought whereby the existence of all beings is, in the end, dependent on one privileged being which itself must by necessity remain unelucidated. Taking on Lautman’s terms when it comes to taking *chora,* Heidegger distances himself from a thought wherein a privileged being is ‘contingently interposed’ and itself ‘heterogenous to being’—as according to him the subject is in Kant. From this it follows that one must conceive of ‘being as the condition of possibility of language on which it nonetheless depends’—which is what Lautman precisely does by conceiving of the dialectic of Ideas as that which *governs* mathematical theory and from which mathematical theory receives its existence *even as the dialectic is dependent on theory for its incarnation*.

From this necessary conception of being and language follow the quasi-metaphors or "reversed metaphors" whereby the unknown term is in fact the one which would be required to understand the metaphors in the first place. In Lautman, similarly, it is only by way of the Idea that one can truly understand what mathematical entities are, even if we only have the mathematical entities as *examples* from which to grasp the Idea. The impropriety inherent in Lautman's notion of exemplarity is thus not a fault but a necessary consequence of thinking the relation of being to beings in such a way so as to hold open the possibility of a truth in general. One has to resort to such an 'improper' use of example in precisely the same way as one has to resort to quasi-metaphors of "withdrawal."

It is then precisely by this notion of exemplarity, which Cassou-Noguès as something which in a sense 'short-circuits' Lautman's whole analysis, that the condition of possibility of a truth in general resides. It is only by way of such a short-circuited notion that the possibility of a truth in general becomes available to any knowledge or thought. This is why Derrida spoke of this 'place' as 'the place of absolute exteriority' *within* 'the open interiority of a system': the system has to be open *from within* to its exterior in order to allow for the fact that *we* are able to understand it while at the same time it provides us with a truth that in a radical sense *would not be dependent upon us*. This is why Lautman must emphasize that these examples only serve as matter *for thought*, that they are not *the* matter of *the* ideas, but insofar as mathematics is thought, the mathematical entities are examples of the Ideas that make them possible. Mathematics can only be experienced, while the Ideas which must be grasped through it have to be 'something more and something else' than the experience of mathematics, and it is precisely by experiencing mathematics as "merely" being an example of the Ideas incarnated in it that we remain thoughtful of the fact that the Ideas never coincide with any single mathematical incarnation of it.

What does this come down to in practice? As was already said, Derrida will often reveal or deconstruct the quasi-metaphoricity of a certain statement by way of *another* statement, which itself however could then be shown to be similarly quasi-metaphorical. (That is, if one describes the quasi-metaphoricity of *chora* in terms of "withdrawal", the notion of withdrawal will start to occupy the place of a non-being which was before occupied precisely by the term *chora*.) The paradoxical statements which follow from such a procedure are not meant to mystify, but precisely to bear witness to the fact that we will not once and for all determine the relation of the system within which we work to the truth in general. That we can only reveal the metaphoricity of something known by way of yet other metaphors, however, is not futile:

> The work of thinking is basically nothing other, in what is called science or elsewhere, than [an] operation of destruction of metaphor. … Which does not mean that one leaves the metaphorical element of language behind, but that in a new metaphor the previous metaphor appears as such. … Given that, it could happen that there is more thinking in the gesture of a scientist or a poet or a non-philosopher in general when he gives himself up to this, than there is in the philosophical-type gesture that moves around in metaphorical slumber. … So it is not a matter of substituting one metaphor for another … but of thinking this movement as such, thinking metaphor in metaphorizing it as such, thinking the essence of metaphor (this is all Heidegger wants to do) (Derrida 2016, 190).

Heidegger, it was said, eventually takes recourse to poetry and the fine arts for this task. That is, questioning must be understood as being none other than 'thinking metaphor in metaphorizing it as such', as perpetually remaining aware that one can reveal previous quasi-metaphors only by way of new quasi-metaphors, and that it is not a matter if simply substituting one metaphor for another, but of thinking this *movement* of substitution.

My argument, finally, is that this happens precisely in the *movement* of mathematics through the occurrence of *dissociation.* Lautman wrote that the fourfold reality of mathematics—facts, entities, theories, Ideas—are intimately related in that the facts 'consist in the discovery of new entities', which, through there organization within the mathematical theory within which they emerge, give rise to the dissociation of meanings of mathematical notions. This dissociation of meaning precisely causes one to *reconceive* of mathematical beings—since the mathematical beings *are* nothing other than the ways in which they are related, and dissociations affect the *necessary connection* which might be grasped by the most causal proofs available, these beings are indeed through dissociation reconceived in their very being.

Lautman thus supplies the understanding of mathematics as a system of axioms and formal deduction with the notions of *fruitfulness*, *necessary connection*—the fruitfulness of some "rigging" of axioms or proofs over and against others, which is dependent on *what is there to rig at all*, and the *necessary connection* between the axioms and requirements which a proof uses in order to arrive at some conclusion, which makes one grasp better what it is that there is to rig. These notions can hardly be properly formalized—in order to do so, mathematics would need to be complete. They are notions, rather, which bear precisely on mathematics *in its movement*. Through its movement, mathematics realizes the measure in which its own notions are merely provisional, and as such are merely *examples* of the Ideas which they incarnate. Future dissociations will affect them—to the core.

What remained there to answer is the relation between this "mode" of questioning and the questioning proposed by Heidegger himself. The one crucial difference which emerged throughout these considerations was that Lautman does not conceive of dissociation as being caused solely by the practicing mathematician. Indeed, dissociations can only occur in the *practice* of mathematics—mathematical theory will not dissociate itself. But it is the emergence of mathematical entities within the deductive structure of mathematics which, for the mathematician, *must* give rise to these dissociations of meaning. This is why Lautman writes that these entities are 'accepted by an incomprehensible necessity of fact' throughout the history of mathematics. Heidegger, on the other hand, seems to conceive of questioning as something done on the part of the philosopher: it is Heisenberg, or Heidegger, whom holds out into the questionable, and thus preserves beings in their inexhaustibility; it is an individual achievement.

The other difference, or consequent difference, is that for Lautman, notions can never simply fall away into namelessness. Dissociations will only ever *multiply* the meanings of the past. Which is not to say that *nothing* *changes*—indeed, the notions of continuity and derivability of a function can from the emergence of Weierstrass's continuous function without derivatives on never again be thought to be almost coextensive; these notions *do* change and a previous notion thus in some sense falls away. It does not fall away, however, into *namelessness*, but instead dissolves—*dissociates*—into new names. Thus while Heidegger, when pondering what would happen to our current terms, like "volume", assumed that what we understand under that name must for some time become nameless, Lautman shows the possibility of seeing the activity of questioning as something which will only ever dissociate names, and take up old notions into new notions, into which they are dissolved.

Lautman's conception of mathematics, then, as distinct from Heidegger's, might serve as an example of a mode of questioning which responds more intimately to the matter which is questioned. Instead of questioning in such a way so as to undo beings of their being, beings are conceived as *different* beings through the slow movement of mathematics as a whole, and through the *collective* effort of mathematicians rather than the individual effort of a pondering philosopher—mathematicians whom, furthermore, are at times *forced* to "question" the beings with which they deal in precisely Heidegger's sense, and whom deal with beings with they treat as always only being *provisionally* such or such, never as being the final and true incarnation of some notion.

The inexhaustibility which characterized the need to question in Heidegger is thus taken up in the work of Lautman as the essential insufficiency as dialectical Ideas. The Ideas are drawn onto examples which serve like a matter as envisioned by Plato, but the examples are ever only examples, even if the Ideas are dependent on them to exist at all. This is not simply a case of circular reasoning, but a necessary consequence of thinking through the relation of being and language. If one understands, as I argue one should, Lautman's notion of exemplarity analogously to the notion of quasi-metaphor as clarified in Derrida's readings of Heidegger, Lautman can be seen to provide a foundation of mathematics and a notion of mathematical existence, furthermore, which opens the practice of mathematics to the possibility, rather than being merely subjective or a merely human endeavor, of truth in general.

Given that Heidegger’s notion of questioning was “tested” as to its usefulness in practice, it is no more than fair that we show how Lautman notion holds up. Therefore, to end this thesis, in the final chapter we will take a look at the history of mathematics through the lens of Lautman’s philosophy thus understood.

Chapter 4: Mathematics

Dissociative Questioning in Practice

In the previous three chapters, the main research question has, in fact, already been answered. Lautman's notion of dissociative mathematics can be understood to extend and amend Heidegger's notion of questioning. If one understands his notion of exemplarity in the way I have proposed in the third chapter, it coincides with Heidegger's quasi-metaphorical use of language, and Lautman's project can thus be seen to ground mathematics in a Heideggerian manner. In this way, Lautman's work can be seen as an extension of that of Heidegger. Lautman's work can be seen to *amend* Heidegger's work inasmuch as he envisions *dissociation* to happen not merely on account of exceptional individual mathematicians, but rather as being an activity *forced* onto mathematicians by their collective development of mathematics itself.

Heidegger's considerations of being, as Derrida wrote, attempt to relieve ontology of the repeated foundations of being upon some privileged being; the subsequent conception of quasi-metaphorical language, and of the meaning of being—and with it the possibility of a truth in general—being preserved only in a metaphorical *movement* was shown to be analogous to the dissociative movement of mathematics as put forward by Lautman, wherein entities emerge which make possible the dissociation of the notions which nevertheless are necessary to make them intelligible in the first place. Mathematical examples allow dialectical notions to *be*, even if they never *are* these notions in a definite, final way. Conceived in this way, then, does the movement of mathematics remain open to a truth in general which would not depend on the mathematician, even though, *in its movement*, this truth would become accessible for mathematical practice, and even though this truth *is not* beyond this movement.

Given that Heidegger's notion of questioning was "tested" as to its usefulness or fruitfulness in practice, it seems no more than right that Lautman's conception of mathematics, too, will shortly be viewed from the perspective of its practice.

This chapter can merely offer an outline of mathematical practice as seen through the lens of Lautman's philosophy as I have argued it must be understood. It will not be comprehensive enough to deserve even the name of a partial "Lautmanian history of mathematics", nor will it be structured as to attempt to be such a thing. Rather, I wish to elucidate several aspects of this philosophy by way of various examples. These examples are not rigorously ordered, and do not exhaust the aspects of Lautman's philosophy which could be elucidated.

First, I will look at the notion of meaning with respect to mathematical existence (§1). Second, the primacy of dissociation over generalization will be considered (§2). Then, I will look at the relation of *causality* and *descriptiveness* of proofs (§3) and finally, I will look at a case of the emergence of new entities and their role beyond mathematics (§4).

§1. Meaningful Existence: Roots of Negative Numbers in Cardano

In order to clarify the notion of *meaningfulness* of mathematical procedures, one might turn to the passage in Cardano's *Ars Magna* in which square roots of negative numbers emerge. The notion of meaningfulness is important since, in Lautman's philosophy, it is not simply a matter of arguing for the existence in-itself of this or that mathematical entity, but a characterization of mathematical reality which partakes both in logical deduction and in the psychology of the mathematician without being reduced to either. Thus only in light of what mathematical entities—logically defined—*mean* to the mathematician, can they be said to incarnate certain Ideas, and thus only insofar as we grasp mathematical entities as meaningful in some way do we deal with them as they truly are.

The square root of a negative number, in Cardano's book, comes up through the example of a pair of numbers the sum of which would yield 10, and the product 40. In order to solve this question, Cardano divides 10 into two equal parts of 5, and squares the 5 to make 25. From the 25, the 40 is subtracted – and, in an operation he has shown earlier, adding and subtracting the square root of the result from the parts of 10 will yield a product of 40 (Cardano 1968, 219).

That is, the solution to the question is $5 - \sqrt{(-15)}$ and $5 + \sqrt{-15}$. The sum of these is obviously 10, and the product is $5^2 + 5\sqrt{-15} - 5\sqrt{-15} - \sqrt{-15}^2$, whereby all the remaining negative roots cancel out, and the result is $5^2 - -15$ is 40. (Cardano 1968, 219-220).

Cardano illustrates this with a geometrical example, wherein the 10 is represented by a line AB of that length, and the 25 by a square on a line one-half the length of that line, AC. From the square, written AD, is subtracted an area of 40, given by $4AB$, a rectangle on the side of AB. 'The square root of the remainder, then—if anything remains—added to or subtracted from AC shows the parts (Cardano 1968, 219). This is procedure is taken over from an earlier one, in which no negative roots emerged.

In modern notation, this general solution might be written thus: given two numbers A and B, produce numbers p and q the sum of which is A, and the product B. In order to solve this question, construct the number $C = \sqrt{1/4\,A^2 - B}$, where $1/4\,A^2$ is the square of the half of A, and B is the product sought, as in the example. Now $1/2\,A - C$ and $1/2\,A + C$ give the numbers p and q sought, since in sum they yield A, and the product is $1/4\,A^2 + 1/2\,AC - 1/2\,AC - C^2$, which is $1/4\,A^2 - (1/4\,A^2 - B)$, or B. This procedure thus—to our modern understanding—in general works for all numbers A and B.

Cardano, however, remarks that in the case where $A = 10$ and $B = 40$, the remainder produced in the construction of C 'is negative', and therefore 'you will have to imagine $\sqrt{-15}$—that is, the difference between AD [25] and $4AB$ [40]' (Cardano 1968, 219). After producing the result, he writes that 'the nature of AD is not the same as that of 40 or of AB, since a surface is far from the nature of a number and from that of a line, though somewhat closer to the latter.' The number thus used 'truly is sophisticated, since with it one cannot carry out the operations one can in the case of a pure negative and other numbers' (Cardano 1968, 220). Concerning the 'nature' of the negative root, he elsewhere writes that, since both -3 and $+3$ produce 9 when squared, '$\sqrt{-9}$ is neither $+3$ or -3 but is some recondite third sort of thing' (Cardano 1968, n220).

All of these considerations give some insight into the question of the meaning of mathematical entities. When the problem is represented geometrically, there is a difference between a pair of numbers A and B whose remainder C is positive, and in the case where it is negative. As Cardano writes, in the latter case one must ‘imagine’ it, or perhaps it would be right to say that one precisely *cannot* imagine it; the entity is *geometrically meaningless,* ‘some recondite third sort of thing’. To modern understanding, the geometrical representation is not of a primary nature: one can still understand the relation between geometrical operations and algebraic operations, but it is also possible to consider $\sqrt{-5}$ a number in much the same way as 10. One of the major differences is that Cardano puts forward is that with a number like the former, ‘one cannot carry out the operations one can in the case of a pure negative [like -5] and other numbers’—and here one is reminded of what Laura Fontanella wrote, as quoted in the second chapter: the system of axioms which can be said to define the “set” do not define it in the sense of ‘a collection of objects as a totality in its own right’, what a “set” is must rather be thought as ‘the possibility of performing specific operations on such collections’ (Fontanella 2019, 169). The *meaning* of an entity must thus be sought in *that which can be done with it*, the role the entity can have in various mathematical procedures and proofs. Thus here again, one of Lautman’s insistencies—that the mathematical real must be characterized *between* logical deduction and the psychology of the mathematician, without being mistaken for either—emerges. The meaning of the same thing—the square root of a negative number—has changed at the moment when one feels one *can* carry out the operations which one can perform on “regular” numbers.

This is also quite intuitive. The negative number could be said to be defined already when Diophantus regards an equation of which it would be the result as ‘absurd’. That is, given an equation like $2x - 10 = 2$, the solution of which would be -4, the number -4 can be said to exist insofar as it is characterized by precisely this equation, even if the equation is regarded as absurd and if no solution like -4 is given. However, this seems contrived. One would want to argue something like negative numbers exists only from the moment the way we should treat them are defined on, that is, only when we are given rules by which to operate them are there truly things like negative numbers.

What is more, these rules *have to be able to be given*. As can be seen from the example of Cardano, it is not obvious how one should calculate with these emerging mathematical entities, this "recondite third sort of thing" which is a negative root. What I am trying to get at, is that it might not at all be obvious that rules by which to deal with such "new" numbers can be set up. This is why Lautman distinguishes between fruitful and fruitless rigging of axiomatic systems: of course one can propose any sort of rule to deal with such an entity, but not every rule will yield results as fascinating and rich as others.

For Lautman, then, mathematical existence is the *meaningful* existence of mathematics. Such characterization does not signify that this existence is *subjective*, that mathematics exists insofar as one *gives it meaning*. One cannot predict which "rigging", which invention of rules or procedures, will yield rich results, results which endow the entities thus constructed to be meaningful in ways unforeseen.

How can this be understood, however, as a mode of *questioning* in Heidegger's sense? Questioning, in Heidegger, takes aim at the "duality" within which we always already speak, and whose characterization as *duality* is in fact already somewhat inadequate, even if such inadequacies are unavoidable. Questioning is a means to preserve things in their inexhaustibility, to not let any certain determination of thinghood become obscure and "natural." In Heidegger, this resulted in the possible namelessness of concepts. In Heidegger's later works, in order to move past the 'necessary impasse' of the characterization of being and beings as in terms of a duality or ontological difference, he starts emphasizing the mode of *letting* beings be, where this letting is to be thought as more originary than being (cf. Heidegger 2003, 58-61; Raffoul and Nelson 2013, 4). For Heidegger, such letting already must not be confused with *passivity*; passivity in the sense of non-action is only possible because "there is being" [*Es gibt Sein*], which is to be understood as "it *lets* being" [*Es* läßt *Sein*]. It can thus not simply be a matter of a subject "waiting and seeing", being engaged in this letting-be is something other. Letting be is the way in which *chora* allows beings to be by withdrawing.

Might we not grasp mathematics—as seen from the perspective of the philosophy of Lautman and understood as put forward in the previous two chapters—as being engaged in such letting-be? In a fundamental sense, mathematical entities are not characterized as being such or such. Even if one invents rules for operating negative numbers, this does not remove or cover over the possibility of inventing *different* rules, of trying to think a certain entity otherwise (as can be seen in the case of sets and their various axiomatizations). Is not the inexhaustibility of the being of beings, to which Heidegger attempts to be attentive, possibly reflected in this possible *richness* of the mathematical edifice? Lautman's notion of *exemplarity*, by which we understand that every determination of a mathematical entity *simultaneously* determines an Ideal which differs from this entity while it can only be grasped through such an example and does not even in any chronological or psychological sense predate it. That is, the determination through some formalization of negative numbers allows one to conceive of negative numbers as being some meaningful mathematical entity, but also already entails the possibility of thinking this *same* entity differently; the "recondite third sort of thing" which is rendered intelligible by some formalization might be rendered intelligible otherwise, and still it *is* nothing other than this intelligible thing.

§2. Dissociation and Generalization: Equality and Equivalence, Conics

Lautman, to emphasize the difference between the dissociative effort which he deems primary, and the generalizations which result from it, points toward the example of equality and equivalence. That is, in the work of Hilbert and Bernays, equality is defined by the axioms which state that an element is equal to itself, and that if two elements are equal, all arithmetic properties that apply to one apply to the other. Thus the elements must be, with respect to these properties, indiscernible. From these two axioms—the first of which already defines the reflexivity of equivalence—symmetry and transitivity too, follow (if one element is indiscernible from another, then the other is also indiscernible from the first; if one element is indiscernible from a second, and the second from a third, then the first is also indiscernible from the third in this sense) (Lautman 2011, 34).

Arithmetic equality is thus an equivalence relation, and an equivalence relation may be said to be a generalization of this notion of equality. What Lautman wishes to emphasize, is that these two—equality and equivalence—also differ in the measure of *precision* with which they can view mathematical structures. What is dissociated within equality by the notion of equivalence relations is 'the point of view of countability of classes from that of the countability of individuals', which in arithmetic equality 'are conflated' (Lautman 2011, 35).

That is, the one does not simply generalize equality, in the sense that all information which the relation of equality contains is still contained by equivalence relations: an equivalence relation no longer counts individuals. This can be made clear by thinking, for example, of the orbits of a group, which are related by the equivalence relation of conjugation classes. This equivalence relation does not in any way measure the amount of individuals of the group, it measures orbits and the amount of individuals within the subsets of the group, within which the orbits divide the group, are indiscernible through the equivalence relation (Lautman 2011, 35).

Another example of this can be found in the relation between ellipses, parabolas and hyperbolas in Euclidean geometry on the one hand, and conic sections in projective geometry on the other. Since the various conic sections become indiscernible in projective geometry, proofs in projective geometry can elucidate whether certain characteristics of ellipses, parabolas and hyperbolas in Euclidean geometry were due to the fact that they are conic sections, or are due to qualities which are specific to parabolas, for example. As the dissociation between classes and individuals becomes graspable through the distinction between equivalence and equality, and the dissociation of equivalence *within* the notion of arithmetic equality, so the dissociation between conic sections in general and specific conic sections becomes graspable through the generalization made possible by extending the Euclidean plane with the points at infinity necessary to construct the projective plane (cf. Coxeter 1974, 71-90). In other words, projective geometry makes it possible to distinguish, *within* specific conic sections, their being-conic more generally.

Take, for example, Pascals theorem which states that, given six points on a conic, three pairs of opposite sides meet in three points which are collinear. Because of the points at infinity within projective geometry, which make sure *all* pairs of lines meet in at least one point, no exceptions have to be made for cases wherein opposite sides are parallel, while, if one produces the proof in the Euclidean plane without reference to such points at infinity, one has to treat several different cases depending on possible parallel opposite sides.

One could take many more analogous examples; one could view the extension of the natural numbers by negative numbers, or of whole numbers by all rational numbers, as extensions which make it possible to grasp consequences which in fact hold for equations on whole numbers generally, while, when working within the natural numbers only, several cases might emerge dependent on 'how many' solutions a given equation has. For example, given the equation $(x + a)^2 = 9$, if one works only with natural numbers, the treatment differs depending on the choice of a. For $a = 4$, the equation has no solutions. For $a = 2$, the only solution is $x = 1$. If one extends the natural numbers by the negative numbers, however, there are always two solutions which can be given in general, namely $x = 3 - a$ and $x = -3 - a$, since $x = \sqrt{9} - a$. Such a generalization is always also related to a certain dissociation of meanings of a notion, since the original distinction between several cases might be meaningful in a different situation, and from the point of view of the extended, generalized case, this distinction disappears (similar to how various differences between circles and parabolas do in fact have meaningful consequences which disappear in the case of conics in general).

From the point of view of the meaning of mathematical entities, then, dissociation is prior to generalization: it is the dissociation of notions which is meaningful within mathematical practice, and generalization is only a consequence—even if it is closely related. As Lautman writes,

> the passage from notions said to be 'elementary' to abstract notions doesn't present itself as a subsumption of the particular under the general but as the division or analysis of a 'mix' which tends to release the simple notions with which this mix participates. It is therefore not Aristotelian

> logic, that of genera and species, that plays a part here, but the Platonic method of division (Lautman 2011, 41)

That is, 'the fact that the relation of equality, like any relation of equivalence, determines within the set in question a division into classes … relates to the structure that the relation of equality imposes on the set', while the relation of equality, too, has bearing on the cardinality of a set. Equality, then, is a 'mix' of the notions of classes and individuals, or, stated more mathematically, equality is the equivalence relation in which every class contains only one individual.

Lautman's insistence on the priority of the Platonic method of division over the Aristotelian logic of genera and species plays a somewhat hidden role in his relation to Heidegger, for it is because of this insistence that Lautman's understanding of the *existence* of mathematics is not founded upon its axiomatic structure as in Heidegger's conception of mathematics. For Heidegger, mathematics receives its ontological foundations from the basic concepts or objects put down into axioms, and from there on, mathematics only produces *results* which have no real ontological consequence. In Lautman's conception, the ontological richness is not reduced to the tautological or deductive character of the mathematical edifice, but rather penetrates the whole of mathematical theory equally. Rather than being 'a language that is indifferent to the content that it expresses', the nature of the mathematical real 'is different from the too simplistic schema that is used to try to describe it' when mathematics is conceived of as in the work of Heidegger (Lautman 2011, 87). The primacy of dissociation of mathematical notions by way of specific examples over the generalization of mathematical entities is thus closely connected to Lautman's Platonism generally, wherein exemplarity interprets the participatory relation that is the ontological ground for the whole of mathematics at once, rather than for a few basic concepts which serve as a foundation for the rest.

§3. Causality and Descriptiveness: The Pythagorean Theorem

Georges Bouligand, at one point, describes an example of thinking in terms of causality within a proof. He refers to the Pythagorean theorem: given a right-angled triangle ABC, the area of the square built on the hypothenuse BC is equal to the sum built on the sides of the right angle. He remarks that the theorem does not merely hold for the area of squares, but for the area of all similar polygons comparably built on the sides of a right-angled triangle (Bouligand 1934, 25).

He then constructs an intuitive example. Given a right-angled triangle ABC, consider the triangle itself the polygon built on the hypothenuse BC. Draw the altitude through A, and let D be the point where the altitude meets the hypothenuse. Since ABC, ABD and ACD share a side, a corner, and all have a right angle, the polygons are similar. Furthermore, the sum of the areas of ABD and ACD obviously amounts to the area of ABC, since these areas literally coincide. (Bouligand 1934, 25). Given 'the well-known property of the ratio of areas of two similar polygons'—that is, the relation of the ratio of the area of two similar polygons to the ratio of the sides is independent from the specific polygon—this result can immediately be extended to other cases (Bouligand 1934, 25).

What is interesting about this example, wherein the similar polygons chosen are the triangle ABC itself, and ABD and ACD, the triangles built on the sides of the right angle by way of the altitude, is that the conclusion—that the area of the two smaller polygons together yields the area of the larger polygon—hardly seems like a *conclusion*: since the two smaller similar triangles are actually *inscribed* in the larger triangle, their area's together literally amount to the larger area. The conclusion or proof thus almost sounds more like a *description*: even someone unversed in mathematics, without any knowledge of the Pythagorean theorem, would probably give assent, precisely because the proof, and especially its conclusion, more or less coincides with a description of the figure.

This fact, that in a causal proof, the *deductions* in a sense start to coincide with *descriptions* of what is simply there before us, makes intuitive the way in which the Lautmanian conception of mathematics lies closer to Heidegger's descriptions of pre-modern mathematics than it does to modern mathematics, i.e., the particular kind of mathematics whose conception, according to Heidegger, became both possible and necessary with the constitution of the transcendental subject. Before that point, Heidegger wrote, propositions were the simple saying which contained what was simply present. Within modernity, for Heidegger, the proposition instead retrieves all of its truth only from being "correctly deduced" from earlier established proofs, starting with the posited "I" at the core of thought. By turning to meaningful existence, and by showing the role of causal proofs and dissociation which lie at the heart of mathematical generalizations and deductions, Lautman escapes this understanding of the proposition and instead turns back, somewhat, to the simple saying of the simple present. That is, the measure of *causality* of a proof is in fact, in a sense, the measure of *descriptiveness* of its deductions; and, according to Lautman, only insofar as we turn to mathematical entities with the question of what they *are*—their meaningful existence within the mathematical edifice—do we truly grasp them. In this way too, then, does mathematics in Lautman's conception depart from the modern mathematical projection as envisioned by Heidegger.

§4. Beyond Mathematics: Cantor and the Actual Infinite

One of the kinds of entities which, according to Lautman, were like a 'fact to be explained', a paradox which was 'accepted by an incomprehensible necessity of fact' before there was a proper deductive theory of it—that is, one of the entities which gave rise to necessary dissociations of mathematical notions, is 'the transfinite' (Lautman 2011, 88). To end this chapter, I will look at the transfinite as it emerges in the work of Georg Cantor.

In the 'Contributions to the Foundations of Transfinite Set Theory', a late article in which Cantor presents his thoughts once more as a whole, the transfinite is, in first instance, defined negatively: 'The sets with a finite cardinal number', he writes, 'are called "finite sets", we want to call all others "transfinite sets" and their corresponding cardinal numbers "transfinite cardinal numbers"' (Cantor 1932, 292-293) As an example, he gives the set of all finite cardinal numbers v, the cardinal number of which he calls aleph zero, or $\aleph_0$. The prove that this is a transfinite number, 'that is, it is unequal to any finite number m', follows from the fact that the set of all finite cardinal numbers does not "grow" when a single element is added, i.e., $\aleph_0 + 1 = \aleph_0$, while earlier, Cantor had already proven for finite cardinal numbers m that $m + 1 \neq m$. Therefore, $\aleph_0$ cannot be a finite number. After this, Cantor defines a multitude of transfinite numbers on the basis of $\aleph_0$ (Cantor 1932, 293). This proof makes it clear, furthermore, that the study of transfinite sets allows the dissociation of ordinality and cardinality (for in the case of an ordinal number x, $x + 1$ is the next ordinal number, even if they have the same cardinality)—a distinction which, as Cantor writes elsewhere, 'in finite sets is hardly noticable' (Cantor 1994, 102).

What is interesting is how Cantor takes up this notion of the transfinite within discussions which are not confined to mathematics. Cantor defends this notion of infinite cardinalities within discussions with philosophers and especially theologians. In a brief essay to Cardinal Franzelin, he defends 'actual infinite numbers' and argues that all earlier objections against it suffer from the fact that 'they from the outset demand or rather impose upon the numbers in question all properties of the finite numbers', while they should in fact 'constitute an entirely new species of number' (Cantor 1994, 99). In other words, precisely the dissociation of cardinality and ordinality will make it possible, according to Cantor, to defend the actual infinite numbers against earlier objections.

The transfinite cannot be considered, in the Aristotelian distinction, *potential* infinity, for potential infinity 'signifies a *changeable* finite magnitude', while the transfinite is 'fixed in itself' and 'constant', 'situated however beyond all finite magnitudes'. Furthermore, the transfinite is often mixed with the Absolute—the actual infinite as it is 'in God'—while they should be distinguished since 'the former is to be conceived as an *indeed Infinite*, but nevertheless a *yet increasable*, the latter however essentially as *unincreasable* and therefore mathematically *indeterminable*' (Cantor 1994, 100-101). Since Kant, Cantor argues, 'the false notion has come into vogue … as if the *Absolute* were the ideal boundary of the *Finite*, while in truth this boundary can only be thought of as a *Transfinitum* and indeed as the *minimum of all Transfinites*' (Cantor 1994, 101). There thus starts occurring another dissociation on the basis of the study of the transfinite between what is increasable and what is finite: Cantor can describe something infinite in itself which, however, shares with potential infinity that it can always be increased. The transfinite is infinite however not in the sense that it can grow beyond bounds, as with the potential infinite; it really is infinite as is. Cardinal Franzelin first believes Cantor's ideas to 'contain the error of pantheism', but after a clarification by Cantor, Franzelin is satisfied precisely by the fact that the transfinite is 'yet increasable' (Cantor 1994, 103).

I want to draw attention to this discussion, for here one might see that the progress of mathematics is not incompatible from theological or philosophical discussions which one would usually consider to be beyond it. One of the distinctions between philosophy and science for Heidegger was that philosophy is always *one*, while science is always *specialized*—precisely because science, for Heidegger, must progress from given basic objects which themselves are beyond question, and these basic objects form the unquestionable essence of science. The theological discussions of Cantor, however, show that the dissociated meanings emerging in the progress of mathematics can also be taken up in other fields; it is very much possible to think of the *transfinite* within theology or philosophy. Furthermore, there are, in this scheme, no unshakable basic objects of mathematics; mathematics does not found itself, it is founded by the dialectics, which is a "pure problematics", as Lautman wrote, over and against which all mathematical attempts of 'solving' its problems remains provisional. The progress of mathematics, then, and the meaningful existence of mathematical entities, is not necessarily some specialized and closed-off field of thought. Of course, the arguments and definitions Cantor uses in his theological correspondence differ from his mathematical definitions and proofs, but nevertheless he refers to those proofs precisely in order to argue for the proven *existence* of this actual infinite, and proceeds to show which philosophical notions should be dissociated in order to "make room" for the transfinite within other discussions. In other words, for Cantor himself clearly the *Idea* of the transfinite is not confined to axioms and deductions: it lies beyond mathematics. In this way, Lautman's dialectic of Ideas also provides the ground for the possible *unity* of sciences. That is, while Heidegger distinguishes between the unity of philosophy over and against the dispersed specialized sciences, Lautman's philosophy—whereby the axiomatic structure of mathematics does not coincide with its ontological foundation—

allows for these two to once again be brought together, as they were for Heidegger in *Being and Time*: the unity of philosophy, in Lautman's thought, *is* the unity of science: even if sciences are specialized, the dialectics, as "pure problematics" beyond mathematical theory, can thus equally give rise to answers in theology (and, as Lautman himself has shown, physics) (cf. Lautman 2011, 229-262).

Conclusions

Of course, these examples cannot exhaust all the ways in which conceiving of mathematics along the lines of Lautman's philosophy, as understood in this thesis, shows its effects in mathematical practice. I do hope to have shown, however, how Lautman's conception of mathematics distinguishes itself from the conception of mathematics which, for Heidegger, became both possible and necessary with the emergence of modernity.

Four aspects of this were shown. First of all, the fact that within Lautman's conception of mathematics, mathematical existence amounts to *meaningful* existence allows for the inexhaustibility of the being of mathematical beings. Given the *exemplarity* of mathematical entities—that they only ever incarnate *one* way of conceiving of an Idea, even if the Idea is only accessible to thought through its examples, and even if there might, for now, be only *one* example of it—they are not confined to a single determination of their being. Mathematical entities are to be thought as being not simply some in-itself defined existing entity, but are only grasped as they truly are when they are thought as examples of some meaningful notion, and understood as being not simply some static thing, but as being all its meaningful relations to the rest of the mathematical edifice—the various operations which can be performed on them. Exemplarity, in other words, splits an entity within itself: it is simultaneously a logically and rigorously defined, theoretical entity, and it is an *example* of a meaningful being which the practicing mathematician understands as being able to be used.

Given, furthermore, that Lautman envisions *dissociation* as being the activity more at the heart of mathematical practice than generalization, the meaning of entities is not given to them purely by the definitions and deductions which give rise to them. That is, contrary to Heidegger's conception of sciences as being founded upon a few basic objects, from which the rest of the science springs forth purely as produced results, for Lautman existence permeates the whole of the mathematical edifice equally. Dissociation allows the distinction of several aspects of mathematical entities, and allows one to achieve greater causality in proofs, showing with more clarity the necessary connection of the requirements of a proof with the conclusion. This necessary connection, given that an entity is only truly grasped when grasped as it exists within mathematical *practice*, thus including all its meaningful relations to other entities, allows one to understand better what an entity actually *is*. Dissociation, then, allows one to grasp *what aspect* of an entity establishes these meaningful relations. Through the establishment of the notion of an equivalence relation, the aspects of equality of counting classes and counting individuals can be distinguished, which without the notion of an equivalence class are conflated; through the notion of transfinite sets can one dissociate cardinality and ordinality, which in finite sets are all but indistinguishable; through projective geometry and the extension of Euclidean geometry with points at infinity, one can distinguish within specific conic sections the quality of being a conic section generally. The generalization thus provides not merely a more general entity, which can simply be specified, and grounds other entities; rather, generalizations provide *within* more specific entities dissociations of various aspects of their being, if the being of such entities is understood, as in Lautman's thought, as their *meaningful* existence.

Proofs with greater causality, moreover, present themselves as being *more (merely) descriptive of the entities they speak of*. That is, a proof of great causality starts looking a lot as a mere description of a certain situation. Lautman's emphasis, then, on dissociation, causality of proofs and the necessary connection which they clarify, allows his conception of mathematics to escape the conception of mathematics as positing only what is in the subject, as Heidegger thought.

Lastly, the conception of mathematical entities as incarnating Ideas, understood by way of the notion of *exemplarity* put forward in Lautman's work, allows for the *unity* not only of various mathematical fields, but also of mathematical fields with *other* fields, such as theology. The meaningful existence of sets, as established by this or that formalization, incarnates an idea of *set* which might serve a meaningful role beyond the mathematical edifice; the construction of the transfinite can play a role within theological or philosophical discussions on infinity. Thus, by conceiving of the whole of mathematics as merely an *example*, while simultaneously conceiving of the realm of ideas as being dependent on mathematics for its own existence—the notion of exemplarity as explained in the third chapter—the unity of mathematics is regained, which, within Heidegger's conception, science lost. Where Heidegger conceived of a unified philosophy and necessarily specialized sciences, Lautman shows the unity of philosophy to be the very unity of science; philosophy, as characterized by philosophical *questioning*, must preserve the "between," which, in Lautman's conception, precisely happens by conceiving of the practice of mathematics as performing various dissociations upon meaningful entities. The provisionality of all these entities, thought as "examples" of Ideas which are only ever incarnated Ideas, allows one to think of *different incarnations of the same Idea*, even if one can only conceive of the Idea on the basis of different incarnations which only *become* different incarnations of the same in their comparison. The circularity inherent in such reasoning is a circularity not resulting from some fault, but from the attempt to allow for the possibility of a truth in general, rather than a *subjective* truth—understood as a truth dependent on ascription of mathematic entities to the forms of intuition of the subject. In other words, the relation between being, truth, and man is "reversed."

All of this shows, then, that the practice of mathematics, through the lens of Lautman's philosophy and especially his notion of exemplarity, shows itself as being much richer than Heidegger's characterization of as (like all sciences) being merely result-producing, and not actually thinking. In fact, given the results from the previous chapters, it can even be put forward as a mode of philosophical questioning in Heidegger's sense. The ascription of one determination of the being of beings, caused by the privileging of one being among them, can be sidestepped by the seemingly circular notion of exemplarity put forward by Lautman, which in fact, through its circle—that the examples are only ever examples of what comes in existence *because of them*—allows one to conceive of mathematics in such a way so as to understand the truth of mathematics to be *independent* of man, of for example the practicing mathematician, even if this truth is only graspable—thinkable at all—precisely through this mathematical practice.

Conclusion

In this thesis, Lautman's conception of dissociative mathematics was put forward as a mode of questioning which can extend and partially amend that of Martin Heidegger. That is, first Heidegger's own conception of mathematics, and its distinction from philosophical questioning, was shown to hinge on the ascription of the *being* of mathematics (and indeed of all things) to a heterogenous other being—the transcendental subject in Kant, *chora* in Heidegger's conception of Plato. Questioning, for Heidegger, has to remain attentive to the fact that the interpretation of the relation between being and beings is not at all self-evident. Experience, Heidegger wrote, 'is in itself a circular happening through which what lies within the circle'—the circular reasoning whereby that which is said to make possible experience must itself be proven *in* experience, and thus in a way be presupposed—'become exposed (*eröffnet*). This open (*Offene*), however, is nothing other than the between (*Zwischen*)—between us and the thing' (Heidegger 1967, 242). Ontological difference is not to be covered over, even if even the determination "ontological difference" it itself such a covering. Questioning must be a radical self-questioning because we can deal with being only by determining it in some way while all determinations are by necessity inadequate.

The shape this circularity took on in Heidegger's philosophy was the fact that, as Derrida characterized it, 'the meaning of being is … the condition of possibility of language on which it nonetheless depends' (Derrida 2016, 57). This resulted in a conception of language whereby statements which try to remain attentive to this exposed "open" or "between" cannot be said to be either proper or metaphorical. Rather, they are quasi-metaphors in that they presuppose the unknown which they might seem to characterize, and their (quasi-)metaphoricity was shown to be able to be elucidated only by way of always yet other metaphors. The meaning of being—and with it the possibility of a truth in general—is only preserved in such a metaphorical *movement* of which every moment can be said to be merely provisional.

Lautman's characterized the reality of mathematics in terms of *facts*, which consist of the discovery of mathematical *entities*, which are organized in *theories*, which participate in a dialectic of *Ideas*. The being of mathematics was then not axiomatically founded in Heidegger's sense; Lautman, rather, proposed a *participatory* interpretation of the relation between mathematical theory and the dialectic of Ideas. This interpretation was shown to be no common Platonism; Lautman, like Heidegger, does not want to resort to yet *another* being to ground the mutually dependent existence mathematics and dialectics (or being and beings). Similar to Heidegger, then, the meaning of the Ideas are the condition of possibility of mathematics on which they nonetheless depend.

This resulted in a notion of exemplarity which Pierre Cassou-Noguès showed to be 'short-ciruiting', as he said. That is, either the Ideas predate their examples, or they seemingly cannot be said to incarnate at all: the examples would have to precede and ground that of which they are an example. I proposed that this notion of exemplarity—which is a crucial aspect of Lautman's philosophy, since it replaces the heterogenous third term of the duality of being and beings—could be understood analogously to the role of language in the philosophy of Heidegger. The short-circuiting notion of exemplarity in Lautman, I showed, was analogous to the quasi-metaphors which Derrida analyzed both in Heidegger and in Plato, precisely concerning the question of *chora* and how to speak of it. It is, then, not *wrong* to say that Lautman's notion of exemplarity short-circuits, but what must be understood that it is only by way of such a deconstructed concept that one can adequately deal with being in a Heideggerian manner. This manner of "dealing with being" is one which attempts to allow for a concept of 'a truth in general', as Derrida characterized it. Only if the meaning of being is not grounded in one privileged being can the notion of truth concerning beings be thought independently of this privileged being; only when *all* beings are thought out of being, instead of vice-versa, does a truth in general remain possible.

The circularity inherent in both Heidegger's treatment of language and Lautman's treatment of the exemplarity mathematics is thus not fortuitous, but a necessary consequence given their ontological and epistemological commitments. It is a circularity, furthermore, which is reminiscent of the circularity inherent in Kant's proofs regarding his "highest principle" (that 'the conditions of possibility of experience in general are at the same time the conditions of possibility of the objects of experience') which can only proven from within the experience concerning which it is the highest principle.

The difference between Lautman's notion of *dissociation*—which was his characterization of the way mathematics progresses—and Heidegger's notion of *questioning* was twofold. Questioning, for Heidegger, happens on the side of the human being; it is through questioning that human beings preserve things in their inexhaustibility, and Heidegger constantly points out *individual* thinkers who 'hold out into the questionable.' Furthermore, the primary result of such questioning seemed to be the disappearance of current concepts into namelessness: in trying to allow for the current determinations of thinghood to be undone, they were, 'for now', replaced with nothing else.

For Lautman, the activity of *dissociation* is sometimes forced upon the mathematician. That is, entities emerge which have to be 'accepted by incomprehensible necessity of fact', like square roots of negative numbers, which already emerge within existing mathematical procedures within which they were not anticipated. The collective effort of mathematicians has to overcome the problems such entities pose by dissociative activity, pluralizing previous meanings in order to account for the entities that are not yet intelligible. This dissociative activity, furthermore, thus never leaves beings nameless; even if *older* senses dissolve, there are new senses which can be said to retake them.

The movement of metaphoricity inherent in the philosophy of Heidegger, then, by which every single statement is merely a provisional manner of moving forward, is replicated in the dissociative movement of mathematics as put forward by Lautman; all our mathematical characterizations are provisional constructions in order to advance further, and future mathematics will perhaps dissolve all meaning now given to mathematical entities. Still, this future is only made possible *by* these meanings which, in time, will be dissolved. Lautman provides a notion of mathematical existence, and thus a foundation of mathematics, which is compatible with Heidegger's philosophy and distinct from existing foundational philosophies;

furthermore, by doing so, he allows Heidegger's thought to bear on mathematics in a way which Heidegger himself did not envision: mathematics, understood as Lautman does it, in fact is a mode of philosophical questioning in Heidegger's sense, instead of a thoughtless result-producing machine. Lautman's understanding of the dissociative movement of mathematics replaces Heideggerian questioning insofar as Heidegger envisions questioning to reduce things to namelessness, and extends Heidegger's philosophy insofar as this dissociative movement coincides with the quasi-metaphorical movement of language which results from Heidegger's conception of the proximity of language, being and truth. In this way, then, does Lautman's notion of dissociation extend and amend Heidegger's notion of questioning, and, conversely, does Heidegger's philosophy provide the ground to understand the foundational, ontological relevance of Lautman's work.

This answers the question put forward at the beginning of this thesis. In order to answer it, several choices had to be made. Since Lautman provides a *mathematical* alternative, first Heidegger's own conception of mathematics was determined. Since Heidegger, however, hardly puts forward a view of mathematics, his conception was inferred from both his view of science in general and his relationship to Kant. It was made credible that Heidegger's conception of mathematics would not differ from that of Kant, and that the only thing one would have to do was show how Heidegger's relationship to Kant as a whole affected the status of Kant's conception of mathematics. This is a good strategy especially if one wants to be able to embed the resulting determined conception of mathematics within Heidegger's philosophy, and especially in his later philosophy; one could try, however, to look at Heidegger's work *before* his first major work, *Being and Time*. The advantage is that Heidegger has written more specifically on mathematical issues before turning to his existential analytic, the downside would be that these writings on mathematics are not easily relatable to his later philosophy.

Another result of the focus on Heidegger's relationship to Kant is that his later thought on technology cannot extensively be looked at. For the purposes of this thesis, the short look at the relationship between philosophical questioning and technology sufficed to argue that this relationship is still one of opposition, same as the earlier conception of the relation of questioning and scientific inquiry. It could be interesting, however, to treat Heidegger's thought on technology more extensively; indeed, Lautman's extension and amendment of Heidegger's thought might have some significant bearing on the question concerning technology.

The focus in the treatment of Lautman's work on his Platonism and his relationship to Heidegger was fruitful to arrive at an answer to the research question posed. Given more space, one could embed Lautman's work more extensively in the context of his time; his relationship to Hilbert and Brunschvicg, Bachelard and Cavaillès could be treated, and perhaps the specific aspects of his work which make it a preferable alternative could then be pointed out. Now, these relations could only be sketched: Lautman's Brunschvicgianism allowed him to conceive of a eccentric kind of Platonism, his Platonism precisely allowed him to take in a distinguished position within the heritage of Brunschvicg, and the analogy between his conception of the Platonic dialectic and Hilbert's metamathematics made it possible to envision a notion of mathematical existence which equally participates in logical rigor and actual mathematical practice. All of this could be dealt with more extensively—in order, also, to bring the conclusions more closely into contact with the discussions on the foundations of mathematics at the beginning of the twentieth century.

In other words, it was only within the scope of this thesis to show that Lautman's philosophy of mathematics indeed *can* be understood as providing an alternative mode of philosophical questioning in the sense shown.

Given that this was shown with the help of Derrida's analyses of quasi-metaphoricity, and the role these analyses have within his own philosophy, I believe it could also be interesting to turn the argument around. If one interprets the Lautmanian dissociative movement of mathematics as a mode of Heideggerian questioning, one could try and make sense of Derrida's own deconstructive reworking of such questioning in terms of mathematical dissociation. One of the most prominent presuppositions or characteristics of Derrida's thought is his use of binary oppositions; he often emphasizes that the work of deconstruction consists in *multiplying differences*: one usually takes aim at a supposedly uniform or homogenous opposition in order to show that its meaning, if rigorously analyzed, is more diffuse. This is not a matter of *blurring* the opposition but of *multiplying* differences until the simple opposition is shown to be inadequate or naive.

Such multiplication, however, quite often ends up producing aporias. If one uses the apparatus now supplied by Lautman's philosophy of mathematics, the movement of deconstruction could be shown to be similar to the development of mathematics. Derrida's attempts at showing the dissemination of notions is not unlike the inexhasutible possible incarnations of mathematical entities, and showing the aporias resulting from the rigorous analysis of notions might be similar to approaching the paradoxes which result from the consistent deduction of mathematics. The quasi-metaphorical movement of language can be interpreted in terms of the dissociative movement of mathematics instead of the other way around. This might help to make Derrida's philosophy less obscure, so to speak; its fidelity to logical reasoning and rigorous deduction could be made more explicit by treating Derrida's conception of the role of originary difference in establishing meaning analogously to the relative establishment of several mathematical entities through systems of axioms. All of these intuitions can only be sketched here—but even the idea of treating Derrida's philosophy in terms of axioms, deductions and dissociation is, I believe, a provocative.

Given Heidegger's huge influence not only on Derrida, but on the likes of Deleuze, Badiou, Foucault and so on, such mathematical reinterpretations or exegeses of continental thinkers could be, I believe, quite fruitful. Almost all continental authors at some point are accused of writing obscurely, and even if such accusations are quite often poorly researched, it is clear that there is a problem in the *reception* of these thinkers. Perhaps Lautman's conception of mathematics can help put their writing in perspective.

My own thought, in any case, has greatly profited from the connection established between mathematics and the philosophy of Heidegger though the work of Lautman. Through the conceptual apparatus provided by Lautman, I can relate my professional training as a mathematician to my philosophical endeavors. I have thus gladly combined many of my interests in working on this thesis, and it was gratifying to see it all come together. As I said: even the idea of treating the philosophers of the likes of Heidegger and Derrida in terms of axiomatics and dissociation is provocative and stimulating.

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