



# Making and Implementing a Mathematics Day Challenge as a Makerspace for Teams of Students

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## Abstract

This study reports on a way to address twenty-first-century skills in mathematics education by organizing one-day mathematics challenges in the Netherlands. During such a day, students work in teams in school on an open-ended problem which aims to elicit skills like problem-solving, modeling, collaboration, and communication. The framework and the methodological approach of the maker movement are used to describe and analyze the design of these learning spaces for students and the practices they become engaged in. In this study, two design teams are interviewed and two assignments, including student work, are analyzed. The results show that the maker perspective bears similarities with the problem-solving perspective, but also enriches the problem-solving perspective by emphasizing the importance of tinkering, making something, and working as a community of practice. Emerging task characteristics that afford students' making processes are the use of a context that is meaningful for students, the low-floor-high-ceiling character of the open problem, and the request for a product. The extent to which the requested product is more context-related or more mathematical depends on the intentions of the task and the interest of the target group. Maker characteristics of the design teams elicit the importance of brainstorming with professionals, time for tinkering with the problem situation, and time for exploring possible student strategies, before the final assignment is developed.

**Keywords** Maker movement · Mathematics education · Problem-solving · Task design

## Introduction

The importance of developing twenty-first-century skills is nowadays emphasized in many policy documents around the world. However, addressing these skills in

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mathematics education is still a major challenge for our community. This is partly caused by a mismatch between what policy defines as important in education, and what is measured and valued in terms of outcomes (Wake & Burkhardt, 2013). The key obstacle for implementing twenty-first-century skills is high stakes assessment, which is generally conceived in terms of focusing on procedural competence and technical fluency (Noyes, Drake, Wake, & Murphy, 2010). Consequently, school mathematics mainly directs student focus on how to pass these procedure-oriented tests, rather than supporting students in developing critical, creative, and flexible skills in mathematics (Ofsted, 2012).

The struggle with implementing twenty-first-century skills in mathematics education is not new. Many of these skills are related to long-existing notions like problem-solving and inquiry-based learning in mathematics education (Artigue & Blomhøj, 2013). The importance of problem-solving in mathematics education is rather obvious, as it creates opportunities for students to learn to find a way out of a difficult mathematical problem, a way around an obstacle, and in the end to reach a result which was not immediately visible (Polya, 1962). Problem-solving is considered as the “art” of dealing with non-trivial problems which do not yet have a known, routine solution strategy to the student, but which provide opportunities to create and communicate new solution strategies (Schoenfeld, 2007). Recently, the maker movement added another dimension to the relevancy of giving students the experience of working on open problems, which is closely connected to product development (Halverson & Sheridan, 2014; Martinez & Stager, 2013). The maker movement has its roots in the ideas of Seymour Papert who already emphasized the relation between learning and constructing in the 1980s. Learning is most effective when students experience (part of) the learning activity as constructing a meaningful product (Papert, 1986).

In this movement, learners are turned into makers. The challenge for educational designers is to create makerspaces that fit both school practices and learning goals (Sheridan et al., 2014). With reference to the problem-solving tradition, this could be realized by connecting makerspaces, which employ the potential of making and also include less goal-directed processes like tinkering and play (Martinez & Stager, 2013), with the affordances of problem-solving in mathematics education. What the maker movement adds to the problem-solving tradition is the acknowledgment of the importance of free play and the transmission of ownership from solving problems posed by someone else to creating your own product. The problem-solving tradition and the maker movement share the focus on processes of construction and inquiry. One of the limitations in current school systems is the implementation of this notion of makerspaces for students, because making needs more time than regular lesson tables in secondary schools allow for. In addition, more expertise is needed for designing such spaces and maker challenges for students that both are motivating and contribute to learning mathematics (Sheridan et al., 2014). In this paper, we explore the notion of makerspaces for educators (teachers, educational researchers, and professional experts) who aim to design mathematical makerspaces for students in secondary schools (see Fig. 1). This exploration is expected to provide a better understanding of the process of making makerspaces for students, as well as insight into the characteristics of these makerspaces and the opportunities they provide for students’ mathematical practices.

## Theory

The potential of makerspaces and making is explored in the context of fostering problem-solving skills in mathematics. We focus on problem-solving since it provides opportunities for students to create, use, and critically reflect on representations in mathematics. In addition to the importance of these opportunities for developing twenty-first-century skills, the ability to create, use, and reflect on representations in mathematics also appears to be a significant indicator for mathematics literacy (De Lange, 2003) and contributes to higher achievement in mathematics (Cai, 2013). Routine tasks offer opportunities to practice and become fluent by deliberate effort, while problem-solving tasks require much longer reasoning lengths and more creative skills (Liljedahl, 2008). Problem-solving tasks are often less structured than textbook tasks and allow for various solution strategies and multiple solutions (Jones, Swan, & Pollitt, 2015).

In this article, we explore the potential of maker concepts for investigating the collaborative and creative aspects in the *design* and *implementation* of problem-solving tasks as one-day-challenges for teams in Dutch mathematics education (Doorman et al., 2007). We build upon two dimensions of the maker movement: *Makerspaces* as communities of practice for makers, and (the process of) *making* in education according to related learning practices. Berman, Garcia, Nam, Chu, and Quek (2016) describe how a community of practice may eventually emerge through making-based activities. The theoretical framework of communities of practice is therefore often used as a lens for analyzing makerspaces (Halverson & Sheridan, 2014; Hsu, Baldwin, & Ching, 2017; Sheridan et al., 2014).

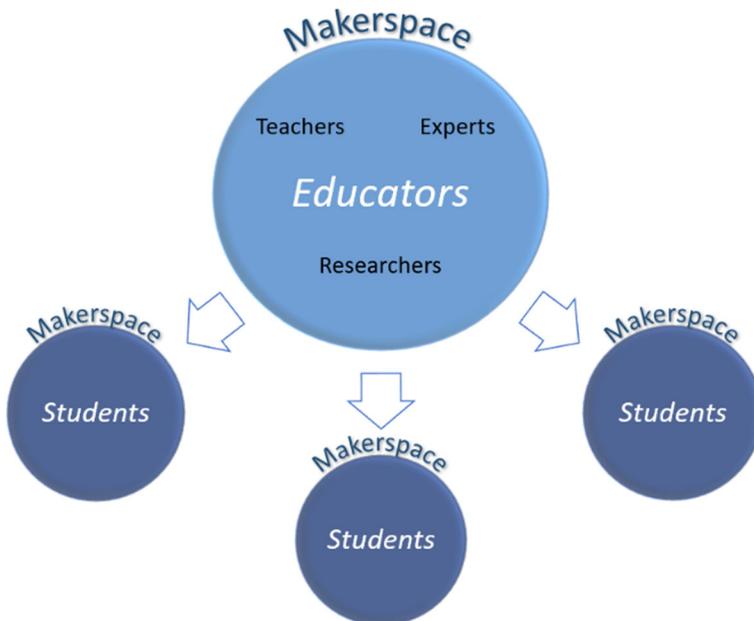


Fig. 1 Makerspaces to create makerspaces

Communities of practice can be described using three characteristics: mutual engagement, a joint enterprise, and a shared repertoire (Besamusca & Drijvers, 2013; Wenger, 2007). The first characteristic, mutual engagement, ensures that there is a sense of coherence within the community, by which participants feel included in what matters. Besides this individual need, community engagement needs to be fostered by diversity and partiality, because mutual engagement involves not only our own competence but also the competence and knowledge of others. The second characteristic, a joint enterprise, is focused on the goal of the community, which gives participants a shared purpose. During the realization of this goal, the connection between the community and the “real world” is made by the production of boundary objects (Akkerman & Bakker, 2011). These are products made by, or within, the community which can be used outside the community. The third characteristic, a shared repertoire, is generated by the use and adaptation of resources by the community when working towards their goal. This results in a shared repertoire consisting of resources, knowledge, and practices (Besamusca & Drijvers, 2013).

Communities of practice offer a lens for the collaborative and goal oriented work in makerspaces. This lens in itself is however not sufficient in describing characteristics of the—sometimes undirected—process of making. A second dimension of the maker movement is oriented on this process of making. Wardrip and Brahms (2015) deliver a framework for learning through making based on theory and practice. This framework is built on seven learning practices which together describe the process of making: inquire, tinker, seek and share resources, hack and repurpose, express intention, develop fluency, and simplify to complexify. Perhaps, the most characteristic to making are tinker, which describes the learners’ purposeful play with the resources, and hack and repurpose, which describes the harnessing and salvaging of resources by learners in enhancing or creating a product. All seven learning practices can however arise when going through the process of making (Wardrip & Brahms, 2015).

These two frameworks are used to investigate and better understand the characteristics and the potential of making and implementing a mathematics day challenge as a makerspace for teams of students. The following research questions frame the study:

1. What are successful characteristics of makerspaces and making for students working in teams during a one-day challenge in mathematics education?
2. What are successful characteristics of makerspaces and making for designers creating these makerspaces for teams of students?

## Method

### Context of the Study

Since the 1990s, mathematics days have been organized yearly for secondary schools in the Netherlands. Students work in teams of three or four on an open-ended problem that elicits problem-solving skills, modeling, creativity, reasoning, structuring, collaboration, and communication (Doorman et al., 2007). These skills are described in the Dutch curriculum and schools are responsible for assessing them in their school exams. School exams account for 50% of the students’ final grades, in combination with the

content-oriented national exams that account for the other 50%. Students in Dutch upper secondary education can choose different streams based upon their interests and abilities. These streams also impact the kind of mathematics being taught. In the social sciences stream, mathematics A is application oriented and involves a lot of statistics, while in the natural and life sciences stream, mathematics B is more formal and its core component is calculus. The yearly mathematics day assignments are organized for both streams to offer schools the opportunity to assess these so-called 21st C skills in mathematics.

Each assignment consists, in general, of three parts, starting with an introduction of the problem situation and the provided resources to foster a shared goal and repertoire within the teams. The introduction is followed by more explorative part to experience the context and the scope of the problem with its links to the world of mathematics, and finally by the main part of the assignment that describes a requested product.

Examples of products are design for security cameras in a museum or a plan for container logistics in a harbor, each including a mathematical underpinning. Many participating schools have a whole grade joining this challenge. For one full day, the students are free for other subjects and work in teams on the assignment in a separate location of the school. The main responsibility for the teachers is to keep the students on task and to support the teamwork during the whole period. Furthermore, they are responsible for grading the products of their students.

The mastery within the design teams increased while they developed assignments for these one-day challenges in collaboration with the teaching community (in the Netherlands, but also with participating teachers from countries like Denmark, Germany, Iran, and Japan). However, the articulation, iteration, and refinement of design principles and ways of working have hardly been documented by the designers.

### **Participants, Data, and Analysis**

In this study, we focus on two different design teams. One design team constructs the one-day challenge for the social sciences stream, the so-called Alympiad,<sup>1</sup> and the other design team constructs the challenge for the natural and life sciences stream, the Mathematics B-day.<sup>2</sup> The Alympiad design team has eight members varying from teachers, teacher educators, didactical designers, a mathematician, and a representative from the world of work. The B-day design team has sixteen members and also consists of teachers, mathematicians, and didactical and professional experts.

We used the frameworks from learning practices and communities of practice to analyze the design of these one-day challenges and the work of students on two example assignments. The student work was available, because schools joining the challenge send in example work. The frameworks guided the construction of schedules for the design teams. We first introduced the characteristics of maker-related learning practices and of communities of practice. Next, we asked each team to jointly answer to what extent they recognize the learning practices in their design processes, and to what extent the assignments and the student work on these assignments support the students to engage in these learning practices (see Fig. 2).

<sup>1</sup> <https://www.uu.nl/en/education/mathematics-a-lympiad>

<sup>2</sup> <https://www.uu.nl/en/education/mathematics-b-day>

<i>Learning Practices</i>	<i>Members of the design team</i>	<i>Students supported by the design</i>
Inquire		
Tinker		
Seek & Share Resources		
Hack & Repurpose		
Express Intention		
Develop Fluency		
Simplify to Complexify		

**Fig. 2** Schedule for maker-related learning practices

In addition, we asked the design teams to reflect on their own work as a community of practice and to what extent their assignments support the teams of students to work on the assignments as a community of practice during this one-day challenge (see Fig. 3).

After the design teams submitted the two schedules, we interviewed a representative of each team with their schedules and an example assignment at hand. The interviews were structured by their answers and intended to overcome misunderstandings and to enrich the provided texts with illustrative references to the design of the example assignment.

We analyzed and compared the answers in the cells of the schedules (the units of analyses) of these two design teams on the structured interviews. The analysis is oriented on identifying two-step characteristics of (i) the assignments and the ways of working in the design teams as communities of practice creating tasks as makerspaces that (ii) let students collaborate as a community of practice and solve a problem or explore a situation mathematically and report their results. For identifying these characteristics, we used a grounded theory approach by highlighting maker-related keywords in the answers and looking for similarities and differences in the answers of both design teams. We triangulated the answers by providing actual results of student work, i.e. snapshots of their products and quotes from their reports with reflections on the one-day challenge.

<i>Community of practice</i>	<i>In the design team</i>	<i>In the teams of students</i>
Mutual engagement		
Joint enterprise		
Shared repertoire		

**Fig. 3** Schedule for identifying a community of practice characteristics

to from	rest area	forest	playground	festival area	Pond
rest area	-	150	40	50	100
Forest	100	-	120	140	140
Playground	30	160	-	60	110
festival area	40	150	30	-	120
Pond	-	-	-	-	-

Fig. 4 Costs for redeveloping an area in the park

## Two Example Assignments

### Example from Alympiad

One of the Alympiad assignments concerns the reparceling of a park including a festival area, forest, playground, rest area, and a pond.<sup>3</sup> Costs are introduced related to the redevelopment of parts of the park (in euros per square meter) in a table (Fig. 4).

The main task for the students is to create more space for a festival. They are confronted with a rather realistic situation and a complete cost-redevelopment table. Figure 5 shows the map of a large municipal park, which is bounded on one side by a river. It is crossed by two paved trails. Organizers of an annual festival have asked the local authority to expand the area for their festivals. They would prefer at least twice as large an area.

The local authority is willing to agree to an extension of the festival area, but the festival organization will have to come up with a good plan and take into account some constraints like the following: the interests of other visitors to the park, the pond cannot be replaced, and the rest area must be (at least partly) adjacent to the pond. In their plan, students should be clear how the various uses are spread around the park and how much it costs in total.

The illustrative mathematics B-day assignment<sup>4</sup> was based on a famous open problem in the mathematics community: the moving sofa problem. Even though in real-life sofas are three-dimensional, the problem for these students is mathematized and reduced into a two-dimensional problem situation. What is the area of the largest sofa that can move through a hall with width one and one straight angle?

In the example in Fig. 6 (on the left) the sofa is rectangular shaped, but this is known not to be optimal; any shape is allowed: round, non-smooth, non-convex. The optimal area is known to be between 2.22 and 2.37. The lower bound is based on a design by J. Gerver (Fig. 6 right side) where the edge consists of 18 different curves.<sup>5</sup>

<sup>3</sup> Full task see: <http://www.fisme.science.uu.nl/toepassingen/28634/>

<sup>4</sup> <https://www.uu.nl/en/education/mathematics-b-day/archive-of-assignments>

<sup>5</sup> [https://en.wikipedia.org/wiki/Moving\\_sofa\\_problem](https://en.wikipedia.org/wiki/Moving_sofa_problem)

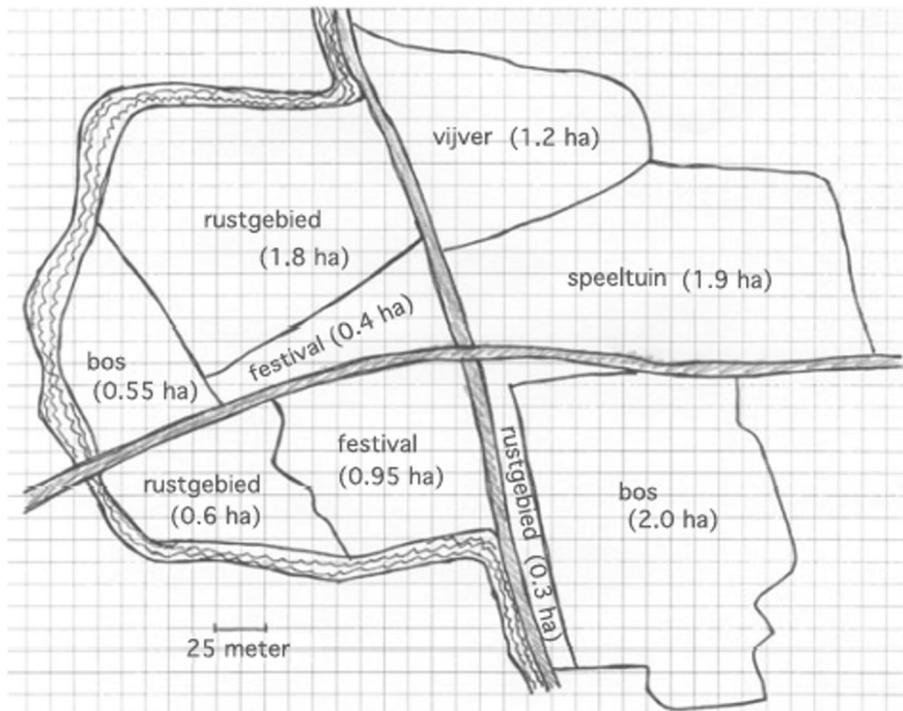


Fig. 5 A map of the municipal park

Obviously, students are not expected to solve the problem. The literature about the problem is very likely to be inaccessible to students both practically and in the sense of mathematical maturity (the papers are too hard to read for them). So students can choose their approach to the design according to their taste and abilities.

### Results

We first present the maker-related learning practices as reported by the design teams. We describe the practices within the teams, the design team as a makerspace, and the expected practices by the students when they work in teams on the assignments. Second, we present characteristics of the communities of practice as reported by the

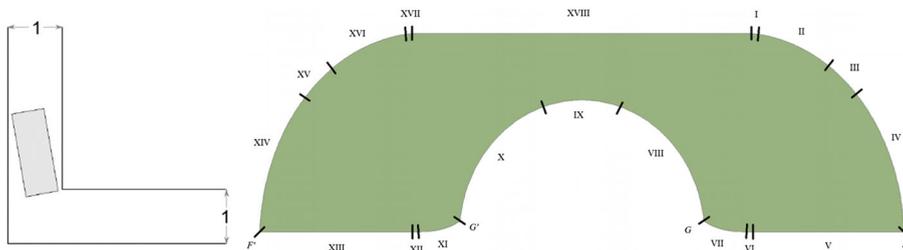


Fig. 6 The sofa problem and one of the solutions

design teams. Also, here we both focus on the design team as a community of practice and on their intentions for the teams of students when working on an assignment. Finally, we present two examples to enrich the design teams' statements and to illustrate what teams of students do and achieve when working on a one-day challenge.

### Maker-Related Learning Practices

The results of the interviews of the Alympiad and B-day design team members, with respect to maker-related learning practices as experienced by them and as created for the students, are discussed along the seven practices.

The first learning practice, *inquire*, is an obvious phase in both design teams when inquiring into potential problem situations during an intense 24-h brainstorm. In the Alympiad design team, this inquiry starts with a collection of resources (e.g. professional reports and newspaper articles) for the assignment, while the B-day team often starts with several mathematical ideas. From this follows a process of inquiry into the possibilities and affordances of the context or idea in smaller groups. This inquiry-process of circa two hours ends in a selection of two or three potential ideas. Their written answers in the schedule:

Alympiad: We first investigate for some contexts in groups of two or three whether the idea or resource can be made into an assignment.

B-day: Many assignment begin with a single mathematical idea (like a simple game). From this follows a process of inquiry into the possibilities and affordances of the idea. The team explores it from both a mathematical and a didactical perspective: Can this become interesting and feasible for students? Does it have a low floor and a high ceiling?

One of the resources during the Alympiad brainstorm was a picture of a plan for reparceling land in Exloo (Fig. 7). This context was selected for further exploration in the tinkering phase due to its visual characteristics and the option to add costs for various steps in the reparceling process.

Both design teams mention that they also try to support this inquiry for the students by providing them opportunities to explore the context and the mathematics in it. The Alympiad uses simplifications of the context to support students in focusing on ways to mathematize the problem by providing a schematized version of the problem in one of the introductory tasks with rest, playground and festival areas, and a reparceling cost table (Fig. 8).

The B-day-assignment includes explicit attention for initial observations, possible hypotheses and useful proving techniques. In preparation for the actual design work, during the first few hours of the day, students do some "exercises." Some are of an explorative character, like in Fig. 9.

The purpose of this type of exercise is to invite students to tinker, to purposely play with shapes, and to realize the importance of a process of designing, testing, and evaluating. Additional tasks provide students with mathematics tools (e.g. Fig. 10). In this exercise, students are provided with a mathematical result that should be very useful later on.

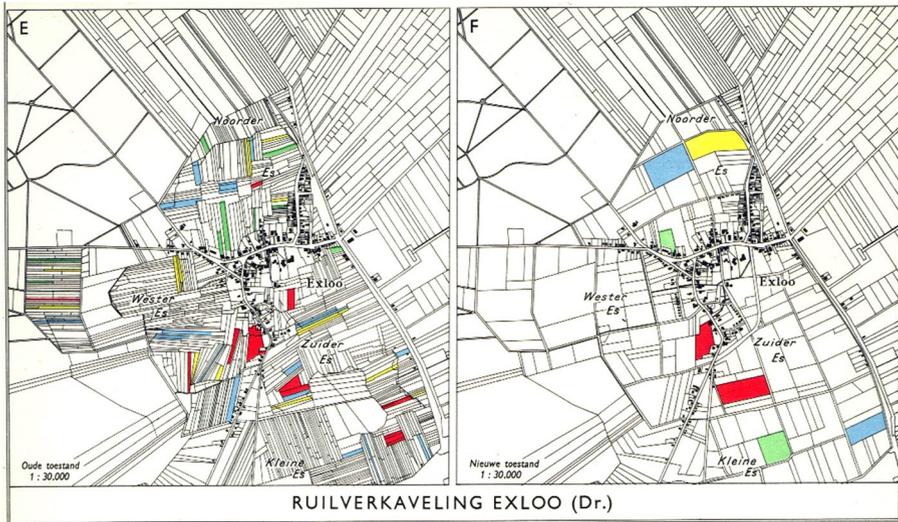


Fig. 7 Reparceling land in Exloo

*Tinkering* by the Alympiad design team can be recognized when they tinker with available data (adapt authentic data or even create datasets). Furthermore, the design of the final assignment is a tinkering process of constructing the assignment in several rounds by changing pairs of designers. In the B-day team, the writing of the actual assignment is coordinated by one person delivering several versions of the assignment and discussions over e-mail or in one-to-one settings with other team members. As an example, in the sofa problem, many options for posing the final problem were explored: give the right-angled hallway and ask for the largest shape, allow variations in the hallway (e.g. U- or Z-shaped), or ask for a supermarket plan including the optimal shape of the shopping cart.

The *seek and share resources* phase is especially important for the Alympiad, since they value the realistic character of the assignment and often consult professional experts (e.g. container logistics specialist, statisticians). In addition, for that team, hack and repurpose is connected to the adaptation of authentic practices and resources for the assignments. As an example, in the playground assignment, the initial context concerned a reparceling plan for a municipality (including areas for agriculture and industry). This was adapted to a self-designed situation in which the location for festivals became the central issue as the design team expected that to be of interest for these students (Fig. 5). In general, this is reported as an important but sometimes unpredictable process of balancing resources, the boundary objects, still being authentic enough and yet accessible to students in the context of this one-day challenge.



Fig. 8 Schematized land reparceling plan with costs

**Exploration 3 (Half disk)**

- a) What is the maximum radius for a semi-circular disk that you can move through the corridor?
- b) Describe, as precise as you can, how to get the largest possible semi-circular disk through the corridor. For example: first put the semi-circular disk into position ..., then move it over distance ..., then turn ... degrees around rotation point..., etc.

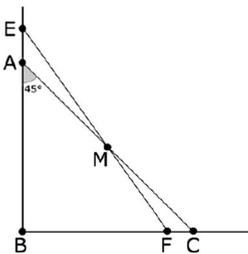
**Fig. 9** An introductory exercise for exploring the context

The Alympiad and the B-day design teams valued the *hack and repurpose* phase for the students. Students kind of redefine what is meant by mathematics. They find out it is not only about procedures and textbook problems but also about mathematical thinking, modeling, creativity, communicating, and proving.

Both design teams answered that the *express intentions* practice is implicit in the collaborative work of the design teams. The assignments of the Alympiad and the B-day always include an explicit introductory text (also for the teacher) expressing the intentions of this specific one-day challenge as being something different than working from the textbook for one day. This text includes information on the importance of teamwork and provides guidelines for the final product or report of the assignment. The information differs in the attention for the importance of context-related guidelines (in the Alympiad the reality of the advice to a municipality is important) and the underpinning of mathematical results (in the B-day, providing arguments for choices and proofs for results is important). The following text is presented to students as introduction to the Alympiad assignment:

During the day you will be working on a major open problem with a group of three to four students. The intention is that by the end of the day you will have written a paper as a result of your work. This assignment is not about the one correct answer; there isn't merely one. In the assessment the following aspects need to be taken into account: whether your strategy has been described clearly, whether the choices and results are substantiated, whether you have worked systematically, whether the use of mathematics and calculations is correct, useful and clear, whether the report/paper is coherent and stand alone, whether you used

**Assignment 1 (Triangular geometry)**



Given is an isosceles triangle  $ABC$ , where  $M$  is the midpoint of  $AC$ . Also,  $F$  is a point between  $B$  and  $C$  on the line  $BC$  and the line  $FM$  intersects the line  $AB$  in point  $E$ . Show that line segment  $EF$  is longer than line segment  $AC$ .

*Please note: even if you cannot prove it, you can still use the information that line segment  $EF$  is longer than line segment  $AC$  in later assignments.*

**Fig. 10** Introductory exercise for guiding towards mathematical tools

your creativity, and how the final assignment is executed, the FINAL ASSIGNMENT outweighs the introductory questions!

Both design teams find it important for their efficient design collaboration to have *develop fluency* in this design process. New committee members have developed their competence to design “appropriate” assignments along the way by collaborating with experienced designers. For the students, the one-day experience hardly provides enough time to have them become fluent in some mathematical skill. Although, within the task situation, this might happen, since they work for quite some time and share knowledge. The mathematical tools that students use in the assignment are usually the ones they feel most comfortable and competent with.

Finally, the *simplify to complexify* phase was only recognized by the B-day design team: Simplifying plays a role in the process of modeling a situation. Complexifying plays a role in expanding successful mathematical techniques to cover a wider range. Nevertheless, in the adaptations of the reparceling situation from the Alympiad, simplifications can also be recognized that make the assignment accessible without losing the mathematical complexity.

These results show that not every learning practice is reported by each of the teams. This might be caused by the unfamiliarity of the team members with the vocabulary. However, similarities in how both teams work can be recognized. For instance, both teams emphasize the importance of brainstorms for inquiry and tinkering the problem and the potential of a makerspace for students. Both teams also provide task characteristics like low-floor-high-ceiling, and the presence of explorative introductory tasks for supporting students in starting an inquiry and enter a tinkering phase for exploring the problem situation. The B-day focuses more on exploring mathematical tools, clarifying (mathematical) intentions and attention for supporting students in possible simplifications. In the Alympiad team, the “intention” practice was not answered, probably because intentions follow rather naturally from the (meaningful) context that dominates the making process of the students.

### **Creating a Community of Practice**

The Alympiad and B-day team members also reported on the community of practice characteristics as experienced and created by them. Within their teams, *mutual engagement* is facilitated by organizing a joint full day brainstorm meeting. Team members feel responsible for designing an assignment within the given time frame. This brainstorm is followed by rounds of pairs elaborating the task (in the Alympiad team) or by e-mail discussions (B-day). Both teams emphasize the importance of diversity within the design team for the design process. The B-day team wrote:

Diversity is essential for a good assignment for several reasons. Input for the brainstorm session will often come from diverse professional backgrounds. For example, the member working as a statistician at a hospital could inspire the assignment with ideas from his work. The team needs teachers to ensure the level is right for the participating students. The team needs highly trained mathematicians as the assignments not only need a low floor, but high ceiling and it needs to

be mathematically correct and sound. Having mathematicians on board also ensures input from the latest developments in mathematics. The didactical expertise is needed for finalizing the design and helps to involve the latest ideas on, for example, inquiry based learning.

During this *joint design enterprise*, team members need to have easy access and support to identify/develop the assignment. All feel the need to “fully” understand the mathematics underlying the problem, asking each other for proof and arguments, and critically reflect on team results. In the Alympiad team, the explicit *shared repertoire* includes the task characteristics providing a role and purpose for students, asking for a product/report and a general structure of the task (easy access in the beginning, open final task asking for creativity using mathematics and context). In the B-day team, the shared repertoire acknowledges the importance of an open final problem for the students.

The community of practice for the student teams is created by the assignment. This is a challenge since, although they can work for one full day on the assignment, it is only one full day. The team members have to divide different roles: chairing the process, writing the report, modeling and performing calculations. It is both necessary to divide the different tasks and to share the results, because the final report should be a consistent whole, and it should be finished at the end of the day. The sharing encourages the mutual engagement: students are positively dependent and individually responsible for the group work. Teams that produce high-quality reports mostly consist of students who have different skills distributed amongst the team, as reported by the Alympiad design team:

Teams that produce high quality reports mostly consist of students who have different skills distributed amongst the team: overview (the "chair"), social skills to see to it that everybody's contribution to the process is recognized, mathematical thinking and modeling, creativity, communicating in writing. Since only 3 or 4 members are allowed in a team, some of these skills need to be clustered in one person.

The mathematical knowledge of the students is mostly very uniform. The diversity is found in their different ways to approach the problems in the assignment. Some students prefer to try many examples, while others prefer to reason (from an example or without one). Students are encouraged to divide the tasks of the assignment, for example, based on level, where the best students do the most demanding tasks. Some students may be made responsible for the final report because they are good at writing reports. The assignments hardly direct students on how to organize ways of engagement and participation. What helps is the shared goal provided by the assignment. In the case of the Alympiad, the context of the assignment puts the students in an authentic role, and as a team in this role, their goal is to deliver a report by the end of the day, for instance, advice for a museum (on security using cameras), including a map. Usually, the introductory part to the assignment tries to refresh and expand the shared knowledge needed for the assignment. It will offer (or let the student develop) knowledge, sometimes in the form of a tool that is useful later.

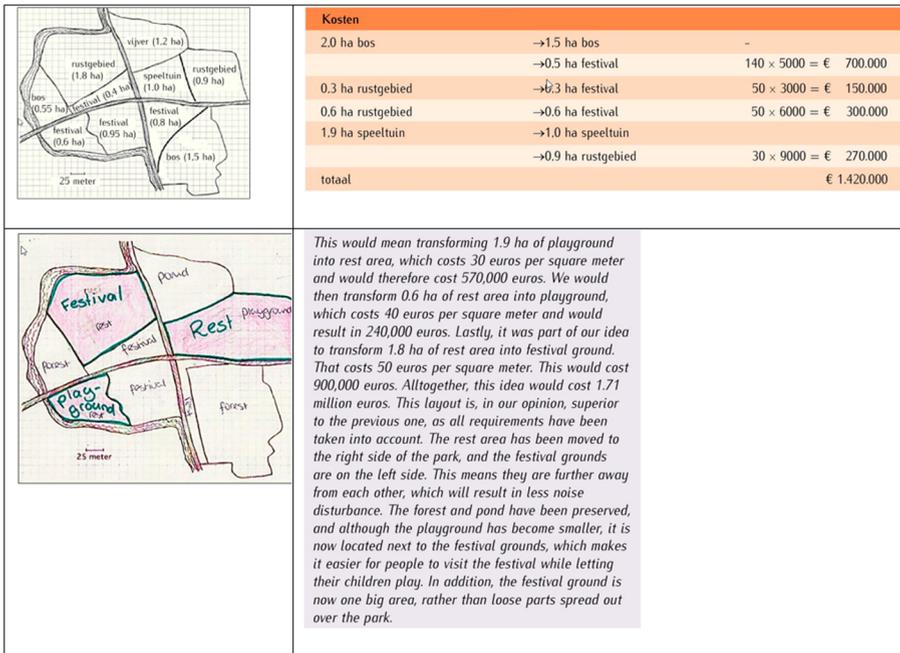


Fig. 11 Results of two teams with alternative park maps and costs underpinning

**Alympiad Student Work**

In their plan, students should be clear how the various uses are spread around the park and how much it costs in total. Excerpts of students’ team reports show the variety and context-related products they made (Fig. 11).

One team used tables to organize their results and to calculate redevelopment costs (two pictures in the top row of Fig. 11), while another team described their results in the text of their advice for the local authority (bottom row in Fig. 11). In both cases, the resulting maps are rather different, showing the freedom students experienced for designing their own advice. Furthermore, these snapshots illustrate how serious these students took the professional role and the characteristics of the contextual problem situation. In one of their reports, the students were rather explicit about the division of work within the team. In another report, the students reflected on the task and highlighted the realistic character of the problem and their teamwork:

As the Netherlands are becoming more and more of a festival country, a lot of festivals are held in summer. Due to this, existing sites are often temporarily transformed into festival grounds (...) we tried to find the best possible solution for the revitalization of the festival ground, while keeping in mind multiple requirements. Not only has electricity to be available at this location, there are podiums to be built, the soil should be reinforced, there should be toilets, catering, chill spots, etcetera. (...) we, as a team, have tried to figure out how to do it cheaply, while taking into account both the wishes of the owner of the grounds (the local authority) and the wishes of the festival organizers (...) we wrote a

letter to the municipality, in which we presented the solutions we have come up with, including the advantages and disadvantages of each and every solution.

### Mathematics B-day Student Work

Students came up with a range of designs for the largest sofa that can move through a hall with one straight angle (Fig. 12).

In the teamwork of the students, the variety of strategies to tackle the problem and to reach results can be recognized. This variety is an indication of space for students to explore and tinker mathematically in the problem situation. Some mathematization choices that were performed in advance by the design team, like to limit yourself to halls of width 1, did not seem to restrict the creativity of the students. Although physicists would require dimensions, the variety and authenticity of the students' strategies illustrate that such constraints did not reduce the ownership of their task for the teams. A quote from one of the team reports illustrating their way of working while exploring the problem situation:

For our own investigations we decided to use the L-shape (...) and create as much area around the L-shape. We did this by drawing a hallway of width 1. The next step was drawing on a piece of paper an L-shape with the maximal length we computed before. We cut it out and moved it through the hallway and if it crossed the boundary we cut the crossing bit of.

This quote supports the design team's intention that the one-day challenge supports students in experiencing the assignment as a maker space in mathematics.

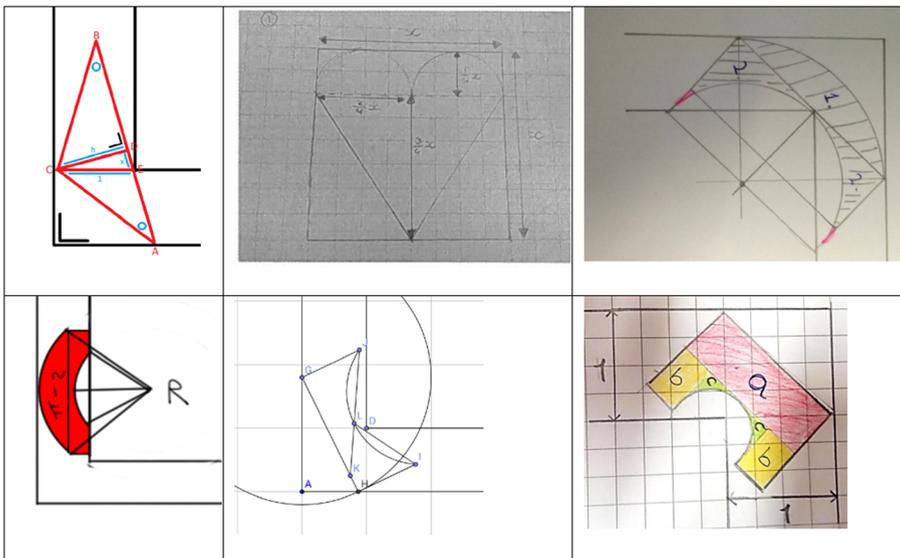


Fig. 12 Students' teamwork of the sofa problem

## Discussion

The examples of the two assignments and the student work show that students can be creative in making mathematical products. The (context of the) task is authentic and new for the students in the case of Alympiad, with not too complicated mathematics to apply, and showing how you can use and communicate your mathematical results in a report to non-mathematicians. For the mathematics B-day, the context can be more artificial, and students quickly enter the world of mathematics to invent new mathematics or new mathematical approaches. In both examples, the teams can work “on their own level,” and this gives opportunities for differentiation; the task is structured from “easy first explorative, tinkering steps” to a more complex-end task asking for a product. Students have the opportunity to do mathematics, make real “inventions,” organize, and show teamwork. For the Alympiad, the final product often is a report, advice, or design for non-mathematicians, while for the B-day, the final product is a mathematical result for mathematicians dominated by technical drawings, calculations, and proofs. These aspects are implemented by well-communicated intentions and structures of the assignments.

The learning practices framework highlights various phases in the making process like tinkering, hacking, and repurposing that enrich the traditional heuristics of problem-solving in mathematics education. These phases were rather implicit and not documented. The community of practice framework helps to make explicit what affords the creation of a team and fostering teamwork while students have to commit themselves in teams to a mathematical task in the school context. These results provide clues on how to educate and create design teams for such making-oriented problem-solving challenges in mathematics (Cohen, Monty Jones, & Smith, 2017). Furthermore, the results show how the design teams are aware of many aspects of creating a community and fostering creativity, while leaving a lot of responsibility to the students. Future designs could benefit from this perspective to support students in their teamwork and to offer them opportunities for addressing twenty-first-century skills in mathematics (Hsu et al., 2017; Sheridan et al., 2014).

This study shows the need for a further elaboration of creating makerspace for designers. Such makerspaces are quite different from developing textbook tasks in mathematics education (e.g. Kieran, Doorman, & Ohtani, 2015). Furthermore, the need has to be created for implementing such activities in the curriculum as opportunities to learn approaching and solving complex problems. The maker movement might facilitate this change from traditional educational design, mainly oriented on procedural fluency, towards a design that takes the opportunity of making and develops maker skills for students. This might contribute to bridging the often experienced gap between education and the world of work.

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