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Harsh environments: Multi-player cooperation with excludability and congestion

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ABSTRACT

The common-enemy hypothesis of by-product mutualism proposes that organisms are more likely to cooperate when facing the common enemy of a harsher environment. Micro-foundations of this hypothesis have so far focused on the case where cooperation consists of the production of a pure public good. In this case, the effect of a harsher environment is ambiguous: not only a common-enemy effect is possible, but also an opposite, competing effect where the harsher environment reduces the probability of cooperation. This paper shows that unambiguous effects of a harsher environment are predicted when considering the realistic case where the collective good produced is excludable (in the sense that whether or not a player benefits from the collective good depends on whether or not he is contributing) and/or congestible (in the sense that the benefits the individual player obtains from the collective good are affected by the number of contributing players). In particular, the competing effect is systematically predicted for club goods, where defectors are excluded from the benefits of the collective good. A common-enemy effect is instead systematically predicted for charity goods, where cooperators are excluded from the benefits of the collective good. These effects are maintained for congestible club goods and for congestible charity goods. As the degree to which a collective good is excludable can be meaningfully compared across different instances of cooperation, these contrasting predictions for public good, charity goods and club goods yield testable hypotheses for the common-enemy hypothesis of by-product mutualism.

1. Introduction

An important theme in evolutionary biology is to explain the evolution of cooperation, as the existence of cooperation between organisms would at first sight seem at odds with individual selection (for overviews, see Dugatkin, 2002a; Sachs et al., 2004; Lehmann and Keller, 2006; Nowak, 2006). Key explanations that have been provided for the evolution of cooperation include kin selection (Hamilton, 1964) and reciprocity (Trivers, 1971). Yet, cooperation is also observed among non-related organisms (Dugatkin, 2002b), and evidence for the type of scorekeeping linked to reciprocity is scarce (Clutton-Brock, 2009). By-product mutualism provides a simple alternative explanation for the evolution of cooperation, and posits that organisms cooperate simply because this contributes to their individual fitnesses (West Eberhard, 1975; Brown, 1983). The common-enemy hypothesis of by-product mutualism argues that in particular, organisms are more likely to increase their individual fitnesses by cooperating when facing "the common enemy of a sufficiently adverse environment" (Mesterton-Gibbons and Dugatkin (1992, p.273)), where the common enemy can both be biotic (e.g. a predator, Mesterton-Gibbons and Dugatkin 1992, p.274) and abiotic (e.g. harsh weather conditions, Dugatkin, 1997a, p.84). Put otherwise, this hypothesis predicts that the higher the degree of adversity facing groups of organisms, the more likely they are to cooperate.

Such a common-enemy effect has been independently proposed across several disciplines (e.g., Simmel (1908), Coser (1956), Heider (1958), Muller and Opp (1986), Bornstein et al., 2002)), highlighting its intuitive appeal. Yet, the underlying mechanism by which a harsher environment causes a common-enemy effect is not immediately clear. It is here that evolutionary game theory is useful, which has so far modeled the common-enemy hypothesis of by-product mutualism in two main ways. In a first model, namely the private-good model (see Appendix B), by-product mutualism means that a cooperating player produces a private good to himself, which happens to create a by-product benefit to players in the same group. In a second model, namely the collective-good model, which is the focus of this paper, by-product mutualism takes the form of the production of a collective good by the players in a group. There are two main reasons for considering such a model on top of the private-good model. First, in some of the key examples suggested for the common-enemy hypoth-

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Table 1
Common-enemy effect and competing effect for players that produce a public good, as a function of the level of cooperation costs and of the degree of adversity. For more than two players, there is an additional rage of intermediate cooperation costs, where the effect of an increase in the degree of adversity is non-monotonic.

	Small degree of adversity	Large degree of adversity
Small cooperation costs	*Common-enemy effect*	*Competing effect*
Large cooperation costs	*Competing effect*	*Common-enemy effect*

esis of by-product mutualism (Mesterton-Gibbons and Dugatkin 1992; Dugatkin 1997b), players clearly produce a collective good rather than a private good, such as a prey to be consumed in the case of cooperative hunting (e.g. Scheel and Packer, 1991; Stander, 1992), or a common territory in case of collective defense (e.g. Grinnell et al., 1995). Second, while the private-good model illustrates a case where a harsher environment systematically promotes the evolution of cooperation, such an effect is unlikely to always occur given that cooperative behavior differs both across species (e.g., Packer and Ruttan, 1988), and within species across contexts (e.g., Muller and Mitani, 2005). A model is then needed that is both able to predict when a larger degree of adversity promotes the evolution of cooperation, and when it does not promote it. More so, a plausible alternative hypothesis to the common-enemy hypothesis is that a harsher environment makes it *less* likely that cooperation evolves (Krams et al., 2010, p.513), which we refer to as the competing hypothesis (in the sense of opposite hypothesis to the common-enemy hypothesis). Indeed, as pointed out by Sandoval and Wilson (2012), results on the effect of the risk of predation on the frequency of mobbing behavior are mixed. A competing hypothesis has also been proposed in other disciplines that formulate a version of the common-enemy hypothesis (Coser (1956); Muller and Opp (1986)).

De Jaegher and Hoyer (2016a) treat a public-good variant of the collective-good model. The degree of adversity here is specified as the degree of complementarity between the players' efforts in producing the public good, namely the extent to which a last cooperating player contributes to the value of the public good (referred to as the added benefit of cooperating jointly), compared to a first cooperating player (referred to as the added benefit of cooperating alone). The reasoning is that a larger degree of adversity makes each player's cooperative effort more pivotal in producing a public good. It is shown that whether the common-enemy hypothesis or the competing hypothesis applies, as represented in Table 1, depends on whether the degree of adversity is high or is low to start with, and on whether cooperation costs are small or large.[1] Additionally, when there are more than two players, the typical nonlinearities associated with multiple-player games arise (cf. Peña et al., 2014; Gokhale and Traulsen, 2014; Broom and Rychtář, 2016), and the critical cooperation costs dividing the cases of large and small cooperation costs are themselves a function of the degree of adversity. Because of this, for intermediate cooperation costs, the effect of the degree of adversity on the probability of cooperation is non-monotonic in multi-player games (De Jaegher, 2017).

Looking at the literature so far, whereas the private-good model focuses on the extreme case with production of a good that is both excludable and congestible, the public-good variant focuses on the other extreme with production of a good that is both non-excludable and non-congestible (Sandler, 1992; Dionisio and Gordo, 2006; Nunn and Lewis, 2001). A good is non-excludable if no player in the group can be excluded from benefiting from it, whether or not he cooperates. A good is non-congestible (also referred to as non-rivalrous, non-diminishable, or not subject to crowding), if the fact that more group members are enjoying the good does not diminish its value. Yet, as indicated in Table 2, one can consider less extreme collective goods that are non-congestible but still excludable, or are non-excludable but still congestible. Indeed, two key examples given for the common-enemy hypothesis of by-product mutualism may not fit the scenario of a pure public good. In cooperative hunting, it may be that only the predators in a group who participate in the hunt get benefits from the large prey caught. If cooperative hunting involves chasing prey sufficiently far away from the common territory, and if the prey is consumed on the spot, then predators within a group who do not leave the territory to join in the collective hunt, do not benefit from the prey. Thus, in a game where predators decide whether or not to participate in a collective hunt, the predators may decide whether or not to contribute to a so-called club good (Sandler and Tschirhart, 1997; only cooperators benefit from the collective good), rather than to a public good. Also, in collective defense, where defense may be produced by sentinels, it may be only the group members who do *not* act as sentinels that benefit from the collective good that sentinels produce by their behavior, so that sentinels produce a charity good (Peña et al., 2015; only defectors benefit from the collective good) rather than a public good. The reasoning is that the individual sentinel produces a private benefit to itself (Bednekoff, 2001; Clutton-Brock et al., 1999), namely being secure from predators, with a positive by-product collective good for the players who do not act as sentinels, namely collective security. The individual sentinel does not get better off if the number of sentinels increases, if its own sentinel behavior already perfectly alerts this sentinel about approaching predators. Non-sentinels from their side may be alarmed when an individual sentinel flees; non-sentinels are better off the more sentinels there are, if they do not always notice sentinels fleeing.

Additionally, collective goods may be congestible: e.g. if the prey predators catch in a collective hunt is not too large, more participating predators may mean that each predator consumes a smaller share of the prey (Packer and Ruttan, 1988), so that a congestible public good is obtained (where as we will later show a congestible public good can be reframed as a so-called commons good). Also, in the defense of a collective territory, more non-sentinels in a group who rely on sentinels may cause congestion, as a large number of non-sentinels may obstruct each other when fleeing in response to the fleeing of a sentinel. Finally, collective goods may be both congestible and excludable, and private goods are not the only goods that have both these properties.[2] Given the relevance of these alternative types of collective goods, in the context of the common-enemy hypothesis of by-product mutualism, it is important to analyze whether the common-enemy effect or competing effect are predicted for such goods. While as illustrated

[1] For a precise definition of cooperation costs, see Section 3.

[2] In standard taxonomies of collective goods, if a good is both excludable and congestible, it simply becomes a private good. Yet, as will become clear from the analysis, equivalence with a private good is only obtained for a congestible club good if the impact function relating the number of cooperators to the value of the good is linear; this shows that the combination of congestion and excludability has non-trivial aspects as long as one deviates from the simple case of a linear impact function.

Table 2
Congestion, excludability, and types of goods.

	Excludable	Non-excludable
Congestible	*Private good/Congestible club good/Congestible charity good*	*Congestible public good/Commons good*
Non-congestible	*Club good/charity good*	*Public good*

in Table 1 the effect of a harsher environment is ambiguous in the case of public good, we will show that the effect of harsher environment is straightforward for most of the alternative categories of collective goods.

The paper is structured as follows. We start with a general framework in Sections 2 and 3, and then shortly repeat the results for a public good (Section 4); as shown in Appendix C, the case of a congested public good is qualitatively identical. Section 5 treats the production of a club good, and Section 6 the production of a charity good, with the corresponding congestible goods treated in Appendices D and E. For all the goods we consider, we first look at the simple case of two players in each group, which can be read separately, and then at the more complex case of any number of players. We end with a discussion in Section 7. Appendix F contains a list of symbols, and a list of definitions of the concepts used.

2. General framework: multiplayer cooperation and degree of adversity

We start by providing a general framework for the different model variants in this paper. Each player can do A (=cooperate) or B (=defect). We consider an infinitely large, well-mixed population reproducing asexually. Repeated and random matching of the population in groups of n players takes place.[3] At each point of time, the fraction of cooperating players is denoted as x, and the fraction of defecting players as $(1-x)$. Denote by $f_A(x)$ a player's fitness of cooperating, and by $f_B(x)$ a player's fitness of defecting, each time when a fraction x of players in the population cooperates. Then under continuous replicator dynamics, the change in the fraction of cooperating players is given by

$$\dot{x} = x(1-x)g(x) \tag{1}$$

where $g(x) = [f_A(x) - f_B(x)]$ is referred to as the gain function (Bach et al., 2006).

More in detail, following the notation of Peña et al. (2014), denote by a_k a cooperator's payoff when k other players in his group cooperate, and denote by b_k a defector's payoff when k other players in his group cooperate, each time for $k = 0, 1, \ldots, n-1$. Following the approach of these authors, the gain the focal player makes by switching from defecting to cooperating when k other players cooperate (or in short: the gain from switching) then equals $d_k = a_k - b_k$ for $k = 0, 1, \ldots, n-1$. Alternatively, following the approach of e.g. Mesterton-Gibbons and Dugatkin (1992, p. 269), $\mathcal{C}_k = b_k - a_k = -d_k$ measures a cooperator's net cost of cooperating rather than defecting when k other players cooperate. These two approaches are equivalent, as the gain from switching may be negative so that its absolute value measures a positive net cost, and as the net cost may be negative so that its absolute value measures a positive gain. The advantage of considering $-d_k$ as a net cost is that this net cost of the focal player can then be compared with the by-product benefits the other players obtain from the fact that the focal player cooperates rather than defects. In particular, when k other players currently cooperate, the individual player who cooperates obtains a by-product benefit of $\mathcal{B}_{a,k} = a_{k+1} - a_k$ when a defecting player switches to cooperating; for the individual player who defects, this by-product benefit equals $\mathcal{B}_{b,k} = b_{k+1} - b_k$. It follows that, when a total of k players currently cooperate in a group and one player now switches from defecting to cooperating, the sum of the by-product benefits the other players obtain from this (referred to as the indirect gains from switching in Peña et al., p.125)) equals $k\mathcal{B}_{a,k-1} + (n-k-1)\mathcal{B}_{b,k}$ (for $k = 0, 1, \ldots, n-1$, where for $k = 0$, $\mathcal{B}_{a,k-1} = 0$). When $d_k < 0$ (positive net cost to the individual player from cooperating), but $k\mathcal{B}_{a,k-1} + (n-k-1)\mathcal{B}_{b,k} + d_k > 0$, it is not in the interest of the individual player to cooperate, even though this is in the interest of the group as a whole. More generally, $k\mathcal{B}_{a,k-1} + (n-k-1)\mathcal{B}_{b,k} + d_k$, denoted as D_k, can be seen as the sum of the gains the n players obtains when one player switches from cooperating to defecting (taking as a starting point the situation where a total of k players currently cooperate; these are referred to as the inclusive gains from switching in Peña et al., p.125)). When $D_k > d_k$, too little cooperation may take place in the evolutionary stable strategy (henceforth ESS, Maynard Smith and Price, 1973) from the group's perspective (i.e., considering the sum of the fitnesses in the group); when $D_k < d_k$, there may be too much cooperation from the group's perspective.

The gain function in (1) can be rewritten as

$$g(x) = \sum_{k=0}^{n-1} \binom{n-1}{k} x^k (1-x)^{n-1-k} d_k. \tag{2}$$

For instance, consider the well-known two-player case ($n = 2$), represented in (3), where traditionally R denotes the reward payoff when both players cooperate, T the temptation payoff of defecting when the other player cooperates, S the sucker payoff of cooperating while the other player defects, and P the punishment payoff of both defecting.

$$\begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array} \tag{3}$$

Then $a_1 = R$, $a_0 = S$, $b_1 = T$, $b_0 = P$, $d_1 = R - T$, and $d_0 = S - P$. Also, $\mathcal{C}_1 = T - R$, $\mathcal{C}_0 = P - S$, $\mathcal{B}_{a,1} = R - S$, $\mathcal{B}_{b,0} = T - P$. By (2), the gain function becomes

$$g(x) = (1-x)(S-P) + x(R-T). \tag{4}$$

Consider now the sequence $\boldsymbol{d} = (d_0, d_1, \ldots, d_{n-1})$, referred to as the gain sequence. Define $\Delta d_k = d_{k+1} - d_k$ as the first forward difference of d_k, where $\Delta\boldsymbol{d} = (\Delta d_0, \Delta d_1, \ldots, \Delta d_{n-2})$ is the sequence of first forward differences. For instance, with $n = 2$, $\boldsymbol{d} = (d_0, d_1) = (S-P, R-T)$, and $\Delta\boldsymbol{d} = (\Delta d_0) = ((R-T) - (S-P))$. In all the cooperation games we consider, either $\Delta\boldsymbol{d} > 0$ (sequence $\boldsymbol{d}$ is increasing), $\Delta\boldsymbol{d} < 0$ (sequence $\boldsymbol{d}$ is decreasing), or $\Delta\boldsymbol{d} = 0$ (sequence $\boldsymbol{d}$ is constant, in which case the so-called equal-gains-from-switching property Nowak and Sigmund, 1990) applies). It follows that $\boldsymbol{d}$ has either zero or one sign changes. Given this fact, applying Result 3 in Peña et al. (2014) (which makes use of the fact that ((2) is a polynomial in Bernstein form (Farouki, 2012)), depending on whether $\boldsymbol{d}$ is increasing or decreasing, and depending on whether $\boldsymbol{d}$ changes sign, the game takes on several forms, following taxonomies of cooperation games such as Doebeli and Hauert (2005), Archetti et al. (2011), or Archetti and Scheuring (2012).

[3] Our analysis in this sense relates to literature that considers multi-player evolutionary matrix games (e.g. Peña et al., 2014; Gokhale and Traulsen, 2014; Peña et al., 2015; Broom and Rychtář, 2016), rather than the stylized two-player matrix games that are often considered in evolutionary game theory.

When $\boldsymbol{d} > 0$ (meaning that the gain from switching is positive, and the net cost of cooperating negative, whatever the number of cooperating players), the game has a single ESS where all players cooperate (see Result 3.1(b) in Peña et al. (2014)), and is referred to as a Harmony Game (Martinez et al., 1999; also referred to as by-product mutualism (Hauert et al., 2006)). For $n = 2$, such a case is obtained when $(R - T) > 0$, $(S - P) > 0$. When $\boldsymbol{d} < 0$ (meaning that the gain from switching is negative, and the net cost of cooperating positive, whatever the number of cooperating players), the game has a single ESS where all players defect (see Result 3.1(a) in Peña et al. (2014)), and is referred to as a Prisoner's Dilemma (Tucker, 1950).[4] For $n = 2$, such a case is obtained when $(R - T) < 0$, $(S - P) < 0$. For $\Delta\boldsymbol{d} = 0$ (equal gains from switching), these first two cases are the only two possible cases. Yet, for $\Delta\boldsymbol{d} \neq 0$, the game can take on further forms when $\boldsymbol{d}$ changes sign.

When $\Delta\boldsymbol{d} < 0$ and $\boldsymbol{d}$ changes sign (meaning $d_0 > 0$ and $d_{n-1} < 0$), the game is a Snowdrift game Sugden, 1986; also referred to as Chicken game (Russell, 1959) or Hawk-Dove game (Maynard Smith and Price, 1973)), and as $\partial g(x)/\partial x < 0$ (Property 11 of polynomials in Bernstein form (Farouki, 2012, p.391)) has a single interior ESS for $x^* \in [0, 1]$ such that $g(x^*) = 0$. In such a case, the gain from switching is positive, and the net cost of cooperating negative, when few players cooperate; when many players cooperate the opposite is the case. x^* now measures the ESS fraction of cooperating players in the population (Result 3.2(b) in Peña et al. (2014)). For $n = 2$, this case is obtained when $(S - P) > 0 > (R - T)$, in which case by ((4) we obtain that $x^* = (S - P)/[(S - P) - (R - T)]$. Finally, when $\Delta\boldsymbol{d} > 0$ and $\boldsymbol{d}$ changes sign (meaning $d_0 < 0$ and $d_{n-1} > 0$), the game is a Stag Hunt Skyrms 2004; also referred to as bistability (Hauert et al., 2006)), and as $\partial g(x)/\partial x > 0$ (Farouki, 2012) has a single interior fixed point at $x^* \in [0, 1]$ such that $g(x^*) = 0$, where this interior point is unstable (Result 3.2(a) in Peña et al. (2014)); additionally the game has both an ESS where all players cooperate, and one where all players defect. In this case, the gain from switching is negative, and the net cost of cooperating positive, when few players cooperate; when many players cooperate the opposite is the case. Assuming then that in the initial population, each initial x is equally likely, the size of the basin of attraction of the joint cooperation equilibrium equals $(1 - x^*)$, and measures the probability that the joint cooperation ESS will evolve. For $n = 2$, this case is obtained when $(R - T) > 0 > (S - P)$, in which case by ((4) we obtain that $(1 - x^*) = (R - T)/[(R - T) - (S - P)]$.

We are interested in the effect of the degree of adversity, denoted by a parameter α, on the probability of cooperation. The degree of adversity may, first, affect the probability of cooperation, because it has a game-changing effect, in the sense that the type of the game is changed (in this sense, our paper relates to a strand of literature (e.g., Hauert et al., 2006) that does not consider stylized collective-action situations such as Prisoner's Dilemmas, Stag Hunts, and Snowdrift games in isolation, but considers them as part of one and the same situation, with the type of game being played depending on one or more parameters (in our case the degree of adversity). Second, the degree of adversity may affect the probability of cooperation within a specific type of game.

We separately look at the case with and without an interior fixed point. When there is an interior fixed point x^*, the effect of the degree of adversity on the probability of cooperation is determined by the sign of $\partial x^*/\partial\alpha$. This sign can be derived by applying implicit differentiation to the condition $g(x^*) = 0$, from which it follows that $\partial x^*/\partial\alpha = -[\partial g(x)/\partial\alpha]/[\partial g(x)/\partial x]|_{x=x^*}$. We repeat that $\partial g(x)/\partial x < 0$ when $\Delta\boldsymbol{d} < 0$, and $\partial g(x)/\partial x > 0$ when $\Delta\boldsymbol{d} > 0$. The following effects of an increase in α are now possible. When $\Delta\boldsymbol{d} < 0$ and $\boldsymbol{d}$ changes sign, the game is a Snowdrift game, and x^* measures the ESS fraction of cooperating players in the population, so that there is a common-enemy effect when $\partial x^*/\partial\alpha > 0$, and a competing effect when $\partial x^*/\partial\alpha < 0$. As $\partial g(x)/\partial x < 0$, an increase in α has a common-enemy effect iff $\partial g(x)/\partial\alpha > 0$, and a competing effect iff $\partial g(x)/\partial\alpha < 0$. When $\Delta\boldsymbol{d} > 0$ and $\boldsymbol{d}$ changes sign, the game is a Stag Hunt game, and $(1 - x^*)$ measures the size of the basin of attraction of the joint cooperation ESS, and therefore measures the probability that cooperation will evolve. It follows that there is now a common-enemy effect when $\partial x^*/\partial\alpha < 0$, and a competing effect when $\partial x^*/\partial\alpha > 0$. As $\partial g(x)/\partial x > 0$, an increase in α has a common-enemy effect iff $\partial g(x)/\partial\alpha > 0$, and a competing effect iff $\partial g(x)/\partial\alpha < 0$. We conclude that whether the game is a Snowdrift game or a Stag Hunt, a common-enemy effect is obtained when $\partial g(x)/\partial\alpha > 0$, and a competing effect when $\partial g(x)/\partial\alpha < 0$.

To determine the sign of $\partial g(x)/\partial\alpha$, consider the sequence $\partial\boldsymbol{d}/\partial\alpha = (\partial d_0/\partial\alpha, \partial d_1/\partial\alpha, \ldots, \partial d_{n-1}/\partial\alpha)$. The cases that will arise once we model the micro-foundations behind the payoffs, are the following:

- Case 1. It is the case that $\partial\boldsymbol{d}/\partial\alpha > 0$. By (2), it is clear that for this case $\partial g(x)/\partial\alpha > 0$, meaning that the common-enemy effect always applies.
- Case 2. It is the case that $\partial\boldsymbol{d}/\partial\alpha < 0$. By (2), it is clear that for this case $\partial g(x)/\partial\alpha < 0$, meaning that the competing effect always applies.
- Case 3. $\partial\boldsymbol{d}/\partial\alpha$ is an increasing sequence and changes sign. In this case, by (2), for small x^* (i.e. small probability of cooperation for a Snowdrift game and large basin of attraction of joint cooperation for a Stag Hunt), it is the case that $\partial g(x)/\partial\alpha < 0$ (competing effect), whereas for large x^* (i.e. large probability of cooperation for a Snowdrift game and small basin of attraction of joint cooperation for a Stag Hunt), it is the case that $\partial g(x)/\partial\alpha > 0$ (common-enemy effect).[5] For instance, for $n = 2$, when it is the case that $\partial(S - P)/\partial\alpha < 0$ and $\partial(R - T)/\partial\alpha > 0$, by (4) it follows that $\partial g(x)/\partial\alpha = (1 - x)[\partial(S - P)/\partial\alpha] + x[\partial(R - T)/\partial\alpha]$, so that indeed the expression is negative for small x and positive for large x.

When there is no interior fixed point for $\Delta\boldsymbol{d} < 0$ or $\Delta\boldsymbol{d} > 0$, it is clear that a change in α can still have a game-changing effect, and turn the game from a Harmony Game or a Prisoner's Dilemma into a Stag Hunt or Snowdrift game, but never directly from a Harmony Game into a Prisoner's Dilemma. Such cases may simply be treated as limit cases of the cases with an interior fixed point, with the interior solution approaching either $x^* = 0$ or $x^* = 1$. When $\Delta\boldsymbol{d} = 0$ (equal gains from switching), there is never an interior fixed point, and the game can only change from a Harmony Game into a Prisoner's Dilemma, or vice versa.

3. Collective good production (complementarity, excludability and congestion)

We now model the micro-foundations of the payoffs in the previous section. In the collective-good model, players who cooperate contribute to the production of a collective good (such as the catch of a large prey, or the defense of a common territory). Denote by c the cost to the individual player of contributing to the collective good, or in short the cooperation costs. Also, denote by I_k (referred

[4] Formally, to obtain a Harmony Game or a Prisoner's Dilemma, it should additionally be the case that $R > P$ and that $R > (S + T)$. We will also use these terms to refer to games with a dominant strategy to cooperate or defect even when these additional conditions are not valid, because we want to avoid defining even additional categories of games.

[5] The result follows from the fact that $\partial g(x)/\partial\alpha$ is also a polynomial in Bernstein form. By the variation diminishing property of these polynomials (Property 2 in Peña et al., 2014, p.26), when the sequence $\partial\boldsymbol{d}/\partial\alpha$ has zero or one sign changes, so does the polynomial.

to as the impact function) the value of the collective good produced when exactly k players within the group cooperate. In the broadest setting we consider, the only restriction imposed on the impact function is that I_k it strictly increases in k, and weakly decreases in the degree of adversity α (meaning that $\partial I_k/\partial\alpha \le 0$).

The focus in this paper is on how excludability and congestion of the collective good affect the incidence of the common-enemy and competing effects defined in Section 2. Different types of goods are obtained depending on whether or not the collective good is excludable and/or congestible. It is useful for the analysis of these collective goods to define δ_k as the added benefit a focal player obtains from cooperating rather than defecting when k other players cooperate; this is the gain from switching, excluding the cooperation costs c (i.e., $\delta_k = d_k + c$). When the collective good is a *public good*, the collective good produced within a group is neither excludable, in that players obtain the value of the collective good whether or not they cooperate, nor congestible, in the sense that if one player benefits from the collective good, this does not affect the benefits of any other player within the group. We have $a_k = I_{k+1} - c$, and $b_k = I_k$, so that the gain from switching d_k equals $I_{k+1} - I_k - c$, and that $\delta_k = I_{k+1} - I_k$. Furthermore, $\mathcal{C}_k = c - (I_{k+1} - I_k)$, $\mathcal{B}_{a,k-1} = I_{k+1} - I_k$, $\mathcal{B}_{b,k} = I_{k+1} - I_k$, and $D_k = n(I_{k+1} - I_k) - c$. As $D_k > d_k$, underprovision of the public good may take place. We note that $\delta_k = \mathcal{B}_{a,k-1} = \mathcal{B}_{b,k}$: with a public good, player i obtains the same added benefit from the fact that player j cooperates rather than defects, no matter whether or not players i and j are one and the same player, and when they are different players, no matter whether i cooperates or defects himself.

We deviate now from the public good model to consider excludability in two forms, following Peña et al. (2015). When the collective good is a *club good*, excludability takes the form that only cooperating players benefit from the collective good; in this case, $a_k = \beta_{k+1} - c$, and $b_k = 0$, so that $d_k = I_{k+1} - c$ and $\delta_k = I_{k+1}$. Also, $\mathcal{C}_k = c - I_{k+1}$, $\mathcal{B}_{a,k-1} = I_{k+1} - I_k$, and $\mathcal{B}_{b,k} = 0$. We note that $\delta_k > \mathcal{B}_{a,k-1} > \mathcal{B}_{b,k} = 0$: with a club good, defectors do not obtain by-product benefits, and the added benefit to a player from one extra player cooperating is larger when it is the player himself who switches to cooperation. As $D_k = I_{k+1} + k(I_{k+1} - I_k) - c$, which is larger than d_k, there may be underprovision of the club good.

When the collective good is a *charity good*, excludability takes the form that only defecting players benefit from the collective good (while it continues to be the case that the value of the collective good from which the defecting players benefit, depends on the number of cooperating players). Cooperating players do not benefit from the collective good, but cooperation costs c are negative, so that a cooperating player obtains a fixed benefit $-c$ from cooperating. With a charity good, $a_k = -c$ and $b_k = I_k$, so that $d_k = -I_k - c$, and $\delta_k = -I_k$. It is the case that $\mathcal{C}_k = c + I_k$, $\mathcal{B}_{a,k-1} = 0$, and $\mathcal{B}_{b,k} = I_{k+1} - I_k$. We note that $\mathcal{B}_{b,k} > \mathcal{B}_{a,k-1} = 0 > \delta_k$: with a charity good, cooperators do not obtain by-product benefits, and the added benefit to a player from one extra player cooperating is larger when it is another player who switches to cooperation. As $D_k = -I_k - c + (n-k-1)(I_{k+1} - I_k)$, which is larger than d_k, there may again be underprovision of the charity good.

In much of our analysis, we specifically consider the impact function $I_k = \beta_k$, with β_k defined in (5) below (De Jaegher, 2017). With this impact function, the value of the collective good is 0 when all players defect ($k = 0$), and V when all players cooperate ($k = n$). For both public goods and club goods, whether congested or not, we assume for this case that $V > c$ (meaning that if cooperating leads a player to get the maximal value of the collective good rather than nothing, this player is better off cooperating). For charity goods, whether congested or not, we assume that $V > -c$, so that a player who by cooperating would forego the full value of the collective good, would prefer keeping this value rather than obtaining the benefit from cooperating. The impact function β_k is defined as follows:

$$\beta_0 = 0,\ \ \beta_k = \frac{V\left(w + w^2 + \ldots + w^k\right)}{w + w^2 + \ldots + w^n} \text{ for } 1 \le k \le n. \tag{5}$$

In (5), w is the degree of complementarity, and ranges from 0 to $+\infty$. Our interest then is in the effect of the degree of complementarity on the probability of cooperation, and in this sense our analysis relates to literature investigating the effect of the shape of the impact function of the collective good on the type of cooperation game played, such as Motro (1991), Bach et al. (2006), Hauert et al. (2006), Peña et al. (2014) and Peña et al. (2015) (see Hirshleifer (1983) for an early treatment). The impact function in (5) is a variant of the geometric impact function introduced by Hauert et al. (2006), where our impact function differs from the one of these authors because of the addition of the denominator.[6] In order to write (5) more concisely for $w \neq 1$, we note first that $\sum_{i=1}^{\infty} w^i = w[1 + \sum_{i=1}^{\infty} w^i]$, meaning that $\sum_{i=1}^{\infty} w^i = w/(1-w)$. Furthermore, $\sum_{i=1}^{z} w^i = \sum_{i=1}^{\infty} w^i - \sum_{i=z+1}^{\infty} w^i = \sum_{i=1}^{\infty} w^i - w^z \sum_{i=1}^{\infty} w^i$, meaning that $\sum_{i=1}^{z} w^i = w(1-w^z)/(1-w)$. Applying this to (5) for $z = t$ and $z = n$, the first part of (6) below follows. The specification for $w = 1$ in (6) follows simply by substituting $w = 1$ into (5).

$$\beta_k = \frac{V\left(1 - w^k\right)}{1 - w^n} \text{ for } w \neq 1,\ \ \beta_k = \frac{kV}{n} \text{ for } w = 1 \text{ (with } 0 \le k \le n). \tag{6}$$

As shown in De Jaegher (2017), the impact function in (6) is concave for $w < 1$, linear for $w = 1$, and convex for $w > 1$. Moreover, the impact function lies lower the higher w. Fig. 1 compares the effect of an increase in the degree of complementarity to an increase in the degree of synergy (Hauert et al., 2006). With a larger degree of synergy, adding one extra cooperating player increases the quantity of the collective good to a larger extent, because the efforts of the cooperating players reinforce each other to a larger extent. A larger degree of synergy therefore makes the impact function to a larger extent convex (or, at first, less concave), and at the same time shifts the impact function upwards, as illustrated in the right part of Fig. 1. An increase in the degree of complementarity also makes the impact function more convex (or less concave), but on the contrary shifts the impact function downwards, as illustrated in the left part of Fig. 1.

The reasoning for a larger degree of complementarity being in line with a harsher environment is the following. First, in the context of collective-good production, a harsher environment may make the impact function to a larger extent convex (or less concave), because a harsher environment makes each player's role in contributing to the collective good more pivotal. When all players contribute, as each player's contribution becomes more pivotal, one deviating player who defects makes the amount of the collective good produced decrease to a larger extent (boomerang effect; Mesterton-Gibbons and Dugatkin 1992). At the same time, as each player's contribution becomes more pivotal, when all players defect, the first player to contribute makes the level of the collective good increase to a lesser extent (sucker effect; De Jaegher and Hoyer 2016a); this is because, when each player's role in contribut-

[6] When using our model to look at the effect of group size on the probability of cooperation, the impact function in (6) leads to counterintuitive results, where the probability of cooperation evolving may be larger the larger the number of players in a group. This is because with (6), when the number of players is increased, the shape of the impact function is also inadvertently changed. A variant of (6) can be treated, with the only difference that the degree of complementarity is adjusted to the number of players, so as to preserve the shape of the impact function, and eliminate the counterintuitive results. As our focus is on the effect of the degree of adversity, we focus on the more straightforward model with an unadjusted degree of complementarity, as our focus is not on the effect of group size.

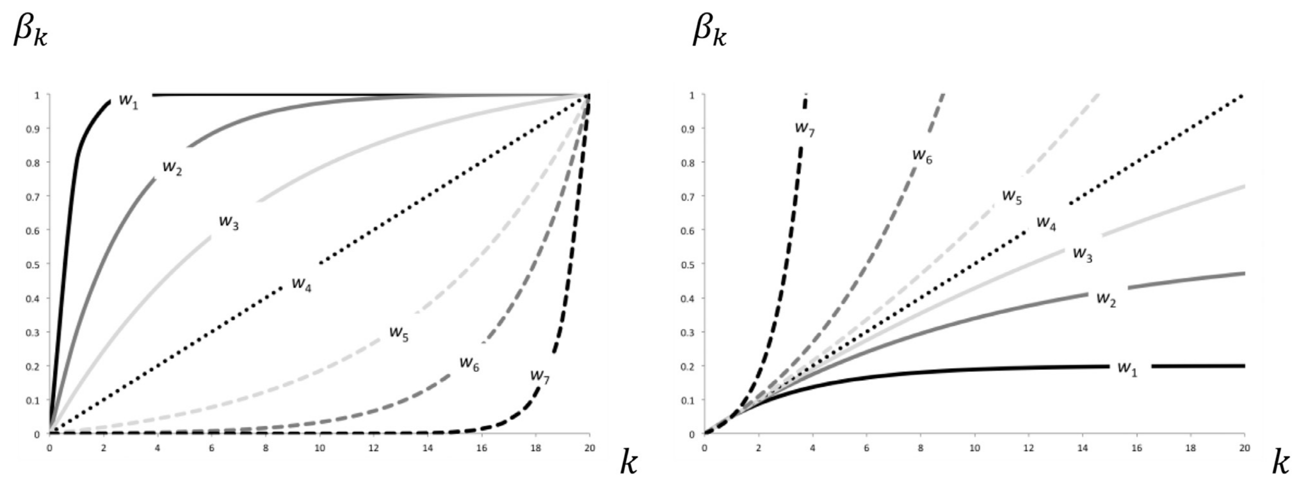

Fig. 1. Value of the public good as a function of the number of cooperating players for several degrees of complementarity (left), and for several levels of synergy/discounting (right), where $w_1 < w_2 < w_3 < w_4 < w_5 < w_6 < w_7$. The example represents the case $n = 20$.

ing to the collective good becomes more critical, a single cooperating player will be able to contribute less. Second, a necessary condition for the degree of complementarity to serve as a measure of the degree of adversity, is that it shifts the impact function downwards, meaning that for the same number of cooperating players, a lower level of the production good is typically produced. Indeed, the environment could hardly be seen as having become harsher if more is produced for an equal number of cooperating players. This necessary condition is valid for the degree of complementarity, as for any number of cooperating players from 1 to $(n-1)$, less is produced the higher w.

In this way, the least harsh environment is obtained as w approaches zero, in which case a single cooperating player in a group suffices to produce a large value of the collective good; the collective good is then produced through a best-shot impact function (Hirshleifer, 1983). The harshest environment is obtained as w approaches infinity, in which case no value of the collective good is produced unless all players in a group cooperate; this is known as a weakest-link impact function (ibid). For an intermediate level of harshness we have $w = 1$, and we obtain summation (or: linear) impact function.[7]

The impact function in Eq. (5) is normalized such that, whatever the degree of complementarity, zero value of the collective good is produced when no player cooperates, and the same value V of the collective good is produced whenever all players cooperate. The former normalization makes sense, as nothing should be produced if nobody does any effort. The reasoning behind the latter normalization is that an increase in the degree of complementarity corresponds to each of the players in a group taking on to a larger extent specialized roles, in a division of labor (e.g., in the case of lion cooperative hunting, drivers and catchers, or wings and centres (Stander, 1992)). When a larger prey makes the predators contributions more complementary, the prey may still be caught when each predator contributes, but is less likely to be caught when one predator defects, because one of the critical roles in cooperative hunting is not filled in.[8]

We now treat the different types of goods one by one, where in the body of the paper we focus on the case without congestion. Congestible goods are confined to the Appendices C, D and E. While congestible goods have some unique features, the comparative-statics results for the degree of complementarity, which are the main focus of this paper, are similar as for the corresponding non-congestible goods.

4. Public good

As a benchmark, we shortly repeat the results for the public-good case in De Jaegher (2017). Cases fitting a pure public good include cooperative hunting when the prey is sufficiently large for one predator's consumption of the prey not to affect any other predator's consumption, and when prey are caught in the proximity of the territory of the group, so that it is not possible to exclude predators not participating in the hunt from consuming prey. Another example may be collective defense, where cooperating takes the form of being vigilant to predators. The benefits players obtain from collective defense are the benefits of being able to forage. If both vigilant and non-vigilant players are predated upon with the same probability, and if vigilant players can continue to forage equally well as non-vigilant players (which is possible if being vigilant is not incompatible with foraging; Caro, 2005, p.127), the players may be seen as producing a public good when being vigilant. A final example is siderophore production by bacteria, which is costly to produce for the individual but equally benefits all individuals in the group, including oneself (Griffin et al., 2004).

For the public good, with a generic impact function I_k, the gain from switching d_k equals $I_{k+1} - I_k - c$, and thus depends on the

[7] De Jaegher (2017) also treats a model where contributing to a collective good consists of taking defensive efforts against random attacks directed at a pre-existing collective good (such as a common territory). As shown there, an increase in the number of random attacks has a similar effect as the degree of complementarity in the production model. Indeed, it can be checked that all results in the current paper, extend to the defense model when $n = 2$. While it is difficult to provide results in closed form in the defense model when $n > 2$, this suggests that the results in this paper extend to the defense model.

[8] In economics, the most commonly-used impact function that allows for varying degrees of complementarity is the so-called constant-elasticity-of-substitution (CES) impact function (Solow, 1956). Applied to the current model, this function can be written as $V\{[\frac{1}{n}\sum_{i=1}^{n}(y_i+1)^{\pi}]^{1/\pi} - 1\}$ (De Jaegher and Hoyer, 2016b), where $y_i = 1$ ($y_i = 0$) means that player i cooperates (defects). As pointed out by McAvoy and Hauert (2016), this impact function takes the form of a Hölder, or generalized average. When π approaches minus infinity, we obtain a weakest-link impact function; when π approaches one, we have a summation impact function; when π approaches plus infinity, we have a best-shot impact function, making π an inverse measure of the degree of complementarity (Ray et al., 2007). Just as is the case in our impact function, it is the case that the value of the collective good is not affected by the degree of complementarity if all players take the same action. We do not use the CES impact function, because it does not allow us to derive the interior fixed point in closed form.

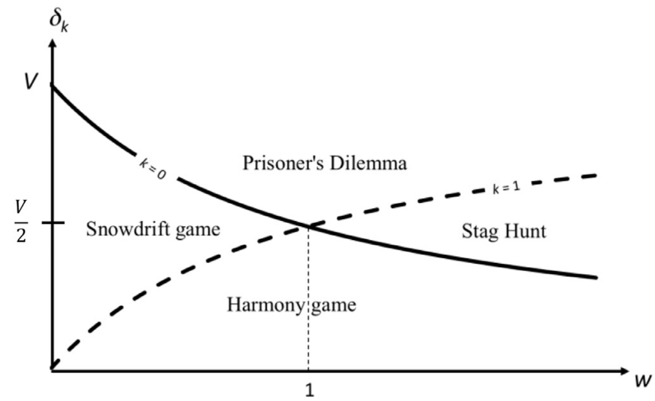

Fig. 2. Public good: for the case of two players, added benefit (δ_k) of cooperating jointly ($k = 1$) and of cooperating alone ($k = 0$) as a function of the degree of complementarity w. Areas where, as a function of cooperation costs, the game is a Prisoner's Dilemma, a Stag Hunt, a Harmony Game, or a Snowdrift game.

change in the value of the public good as one extra player cooperates. As the direction of this change is not restricted by the fact that impact function increases, the type of the public-goods game is thus also ambiguous. Moreover, the fact that increases in the degree of adversity decreases production, also does not imply any restrictions on the effect of the degree of adversity on the gains from switching, and therefore it is ambiguous whether the common-enemy effect or the competing effect applies. Focusing on the case $I_k = \beta_k$, since $d_k = \frac{Vw^{k+1}}{w+w^2+\ldots+w^n} - c$, it is clear that $\Delta \boldsymbol{d} \gtreqless 0$ for $w \gtreqless 1$, meaning that by the analysis in Section 2, if $\boldsymbol{d}$ changes sign, the game is a Snowdrift game[9] for $w < 1$, and a Stag Hunt for $w > 1$.[10] The results for the effect of the degree of complementarity on the probability of cooperation are summarized in Proposition 1 below. A useful starting point to understand these results is the case of two players in each group ($n = 2$) (De Jaegher and Hoyer, 2016a):

$$\begin{array}{ccc} & A & B \\ \begin{array}{c} A \\ B \end{array} & \multicolumn{2}{c}{\begin{pmatrix} V - c & V/(1+w) - c \\ V/(1-W) & 0 \end{pmatrix}} \end{array} \tag{7}$$

With two players, it is the case that $\delta_1 = Vw/(1+w)$, and $\delta_0 = V/(1+w)$. As represented in Fig. 2, the former increases in w, and the latter decreases in w, where $\delta_0 \gtreqless \delta_1$ iff $w \lesseqgtr 1$; this corresponds to Case 3 in Section 2. For the effect of an increase in the degree of complementarity, four quadrants can now be distinguished in Fig. 2 as a function of the relation between cooperation costs c (measured along the Y-axis), and δ_0 and δ_1. For $w > 1$, a common-enemy effect (switch from Prisoner's Dilemma to Stag Hunt) applies for $c > 1/2V$ (large cooperation costs in Proposition 1 below), and a competing effect (switch from Harmony Game to Stag Hunt) for $c < 1/2V$ (small cooperation costs in Proposition 1 below); for $w < 1$, a competing effect (switch from Snowdrift game to Prisoner's Dilemma) applies for $c > 1/2V$ (large cooperation costs), and a common-enemy effect (switch from Snowdrift game to Harmony Game) for $c < 1/2V$ (small cooperation costs). These game-changing effects of the degree of complementarity are confirmed for the probability of cooperation within the Snowdrift and Stag Hunt games, given the fact that $x^* = \frac{V-c(1+w)}{V(1-w)}$, where it is the case that $\partial x^*/w \gtreqless 0$ iff $c \lesseqgtr 1/2V$ (De Jaegher and Hoyer, 2016a).

[9] For w approaching 0, the game then approaches a Volunteer's Dilemma (Diekmann, 1985), where a single cooperating player produces the maximal value of the public good, and the dilemma is which player will cooperate.

[10] For $w = 1$, the game is equivalent to a linear public good game (e.g., Archetti et al., 2011).

Proposition 1 shows that the same results apply for $n > 2$, with the exception that for intermediate cooperation costs, the effect of the degree of complementarity is non-monotonic (for a detailed formulation of Proposition 1, and the proof of Proposition 1, see De Jaegher (2017), Results 1 and 2

Proposition 1. *Consider the production of a* ***public good*** *with the impact function β_k in (5), and with $n > 2$. As the degree of complementarity is increased:*

with **low complementarity** ($w < 1$):

- for **large cooperation costs**, the *competing* effect applies (first Snowdrift game with decreasing probability of cooperation, then Prisoner's Dilemma);
- for **intermediate cooperation costs**, first the *common-enemy effect* applies, and then the *competing effect* (first Snowdrift game with increasing, and then decreasing probability of cooperation; finally, Prisoner's Dilemma);
- for **small cooperation costs**, the *common-enemy effect* applies (first Snowdrift game with increasing probability of cooperation, then Harmony Game).

with **high complementarity** ($w > 1$):

- for **large cooperation costs**, the *common-enemy effect* applies (first Prisoner's Dilemma, then Stag Hunt with increasing probability of cooperation);
- for **intermediate cooperation costs**, first the *common-enemy effect* applies, and then the *competing effect* (first Prisoner's Dilemma, then Stag Hunt with first increasing, and then decreasing probability of cooperation);
- for **small cooperation costs**, the *competing effect* applies (first, Harmony game, then Stag Hunt with increasing probability of cooperation).

Intuitively, when we make each player's cooperative effort more pivotal (i.e., when we increase w), this makes it less attractive to deviate from joint cooperation, as the defector then becomes to a larger extent the victim of his own defection (boomerang effect; Mesterton-Gibbons and Dugatkin 1992). Applied e.g. to the context of cooperative hunting, where the public good is the catch of a prey, when all predators initially cooperate and one predator in the group defects, this causes a larger reduction in the probability of catching the prey (=added benefit of cooperating jointly)) the larger the prey, where the size of the prey reflects the degree of adversity. In particular, the common-enemy effect occurs when cooperation costs c are close to the added benefit of cooperation jointly (δ_{n-1}). The fact that the added benefit of cooperating jointly increases in the degree of complementarity explains why the common-enemy effect occurs for low cooperation costs when the degree of complementarity is low, and for high cooperation costs when the degree of complementarity is high.

At the same time, when we make each player's cooperative effort more pivotal, it becomes less attractive to deviate from joint defection, and unilaterally cooperating will have less impact (sucker effect; De Jaegher and Hoyer 2016a). Again applied to cooperative hunting, when all predators initially defect and one predator in the group deviates by cooperating, this causes a smaller increase in the probability of catching the prey (=added benefit of cooperating alone) the larger the prey. The competing effect occurs when cooperation costs c are close to the added benefit of cooperating alone (δ_0). The fact that the added benefit of cooperating alone decreases in the degree of complementarity explains why the competing effect occurs for high cooperation costs when the degree of complementarity is low, and for low cooperation costs when the degree of complementarity is high.

For $n = 2$, these are the only effects of the degree of complementarity, as these are the only added benefits (given the fact

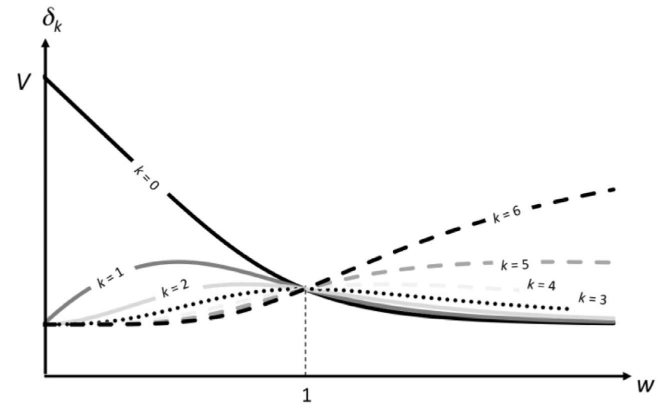

Fig. 3. Public good: added benefits of cooperating (δ_k) when k players in a group of 7 players cooperate, as a function of the degree of complementarity w.

that $\delta_{n-1} = \delta_1$). Yet, for $n > 2$, there are additional intermediate added benefits δ_k for k between 1 and $(n-1)$, and these determine the non-monotonic effect of the degree of complementarity for intermediate cooperation costs. The added benefits are represented as a function of w in Fig. 3 for the case $n = 7$. Define $\boldsymbol{\delta} = (\delta_0, \delta_1, \ldots, \delta_{n-1})$ as the added benefit sequence. For $w = 1$, $\boldsymbol{\delta}$ is constant, with each δ_k equal to V/n. For $w < 1$, $\boldsymbol{\delta}$ is decreasing, and for $w > 1$, $\boldsymbol{\delta}$ is increasing. δ_{n-1} is increasing in w, and δ_0 decreasing in w. Moreover, for intermediate k, as a function of w, δ_k reaches a maximum for intermediate w. The effect of the degree of complementarity on the probability of cooperation can now be inferred by fixing a c along the Y-axis, and increasing w. When an added benefit turns from being smaller than c into being larger than c, this suggests an increase in the probability of cooperation; for a change from being larger than c to being smaller than c, the opposite is obtained. For a fixed intermediate c, it can be seen now that as we increase w, for $w < 1$, first δ_1 and δ_2 become larger than c, suggesting that more players will act in the ESS, but then again become smaller than c, suggesting that fewer players will act in the ESS; for $w > 1$, as we increase w, first δ_4 and δ_5 may become larger than c, suggesting a larger basin of attraction of the joint cooperation ESS, and then again become smaller than c, suggesting a smaller basin of attraction of the joint cooperation ESS. Thus, because the intermediate added benefits are non-linear, for intermediate cooperation costs a non-monotonic effect of the degree of complementarity is possible (cf. Case 3 in Section 2).[11] In order to study the effect of excludability and congestion, we now consecutively investigate how the results for the public good are modified for club goods and for charity goods, where the treatment of congestible public goods, congestible club goods and congestible charity goods is relegated to Appendices C to E, as the results for these goods are similar.

[11] Our model is binary, in the sense that players can only either take a fixed cooperative effort, or take no effort. The probability that cooperation evolves is measured by the size of the basin of attraction of the equilibrium where all players take the fixed cooperative effort, or by the equilibrium fraction of players that take the fixed cooperative effort. Yet, one may expect that in a model with continuous instead of discrete efforts, the degree of complementarity also affects the level of the equilibrium cooperative effort itself. The CES impact function (see Footnote 8) with continuous efforts, which shares characteristics with the impact function in (5), suggests that this expectation is invalid; as shown by Cornes (1993), the level of the mutual best-response nonzero efforts with a CES impact function are not affected by the degree of complementarity. From this perspective, it makes sense to focus on a binary model.

5. Club good

When the collective good is a club good, it is the case that cooperators obtain the value of the collective good produced, whereas defectors do not obtain anything. Examples of club goods may include the cooperative hunting of a prey, if consuming the prey is only possible when participating in the hunt. This may occur if the prey is caught at a sufficiently large distance from the common territory of the group, so that the prey is consumed on the spot, meaning that defectors who stay behind do not benefit from the prey. In the context of collective defense, an example may be found in circular defense (Jolivet et al., 1990), where individuals in the group can be seen as forming a circle to cover each other's backs. When the size of the group is large, it may be possible to take up position on the inside of the circle, and benefit from the efforts of those positioned on the circumference of the circle; as defectors benefit from the efforts of cooperators, circular defense has public-good features in this case. When the group is instead small, there may be insufficient space on the inside of the circle, and it may only be possible to benefit from collective defense by taking up a position on the circumference of the circle; in this case, circular defense is a club good.

With a club good, for generic impact function I_k, as the gains from switching consist of the full value of the club good, it is clear that levels for the cooperation costs exist such that the game is a Stag Hunt; in this case, as increases in the degree of adversity mean that weakly less is produced, if such increases have any effect, this can only take the form of a competing effect.[12] Specifically with impact function β_k, as $d_k = \frac{V(1-w^k)}{1-w^n} - c$, it is the case that $\Delta \boldsymbol{d} > 0$, and by the analysis in Section 2, when $\Delta \boldsymbol{d}$ changes sign, the game is a Stag Hunt. When all other players cooperate, a focal player who switches from defecting to cooperating obtains a benefit V at a cost c, and is therefore better off cooperating; it follows that when $\Delta \boldsymbol{d}$ does not change sign, the game cannot be a Prisoner's Dilemma, but can only be a Harmony Game. Intuitively, as a player who does not cooperate does not obtain any benefit from the collective good, the added benefit of cooperating increases (or: the net cost of cooperating decreases) the larger the number of cooperating players in the population, and this independently of whether the impact function is convex, linear, or concave. To look at the effect of the degree of complementarity on the probability of cooperation, we again look first at the simple case $n = 2$, which is represented in Eq. (8):

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} V - c & V/(1+w) - c \\ 0 & 0 \end{pmatrix} \end{array} \tag{8}$$

We now have $\delta_1 = V$, and $\delta_0 = V/(1+w)$. As represented in Fig. 4, δ_1 is fixed at V, whereas δ_0 decreases in w, with $\delta_0 = \delta_1$ when $w = 0$. Considering now the relation between δ_0 and δ_1 and cooperation costs c (as measured on the Y-axis), it follows that, as the degree of complementarity is increased, the game switches from a Harmony Game to a Stag Hunt, meaning that there is therefore a competing effect.

This game-changing competing effect of the degree of complementarity is general for any n, and is confirmed by the marginal effect of the degree of complementarity within the Stag Hunt. To see why this is so, note that the benefit part of the gain from switching is the value of the collective good itself. As this value decreases in the degree of complementarity, it follows that $\partial \boldsymbol{d}/\partial w < 0$, so that Case 2 in Section 2 applies. Graphically, as represented in Fig. 5 for the case $n = 7$, for any given w, each δ_k decreases in w (with

[12] I am grateful to one of the anonymous referees for encouraging me to express the results for club goods and charity goods for generic impact functions.

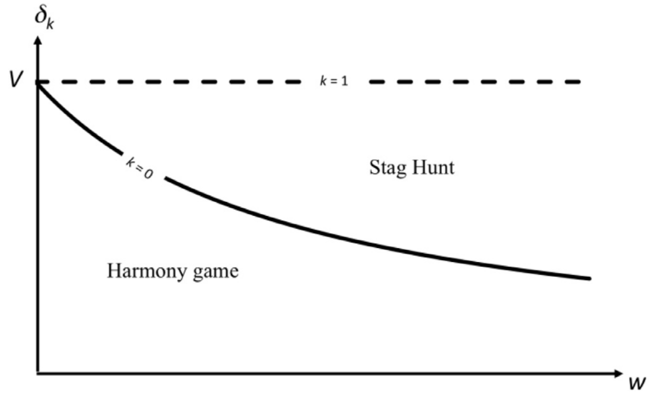

Fig. 4. Club good: for the case of two players, added benefit (δ_k) of cooperating jointly ($k = 1$) and of cooperating alone ($k = 0$) as a function of the degree of complementarity w. Areas where, as a function of cooperation costs, the game is a Harmony Game, or a Stag Hunt.

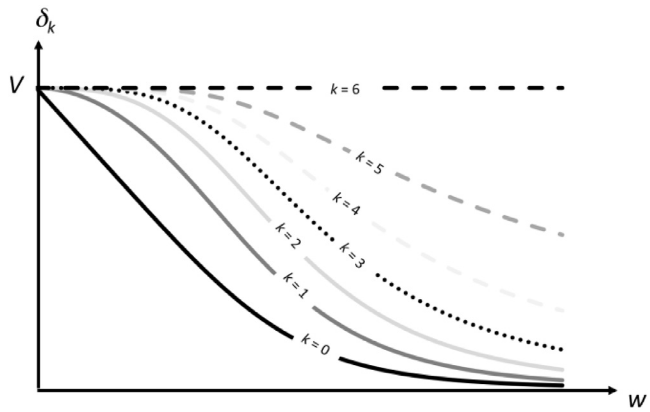

Fig. 5. Club good: added benefits of cooperating (δ_k) when k players in a group of 7 players cooperate, as a function of the degree of complementarity w.

the exception of δ_{n-1}, which is flat at V). Fixing a c on the Y-axis in Fig. 5, for small w, it is the case that $c < \delta_0$, and the game is a Harmony Game.[13] As w is increased such that $c \geq \delta_0$, the game becomes a Snowdrift game; as w is further increased, consecutively $\delta_0, \delta_1, \ldots$ become smaller than c, in line with the competing effect. Intuitively, given that each added benefit decreases in the degree of complementarity, a sucker effect is the only possible mechanism: when each player's cooperative effort becomes more critical, there is always less reason to cooperate. Moreover, as the club-good feature means that a player who switches from defecting to cooperating can gain anything from a minimal added benefit to the maximal value of the collective good, the sucker effect applies over the entire range of cooperation costs. Our results for the club good are summarized in Proposition 2 (for a precise statement of Proposition 2 and for the proof, see Appendix A).

Proposition 2. *Consider the production of a* ***club good*** *with* $n \geq 2$*. Then, with any impact function* I_k *that strictly increases in the number of cooperating players k, if an increase in the degree of adversity has any effect on the probability of cooperation, it is a competing effect. Specifically with impact function* β_k *in (5), as the degree of complementarity is increased, the* competing *effect applies where at first we have a Harmony Game and then a Stag Hunt with decreasing probability of cooperation.*

6. Charity good

When the collective good is a charity good, cooperators (which can be seen as contributors) obtain an invariable benefit, whereas defectors (which can be seen as recipients) obtain a value of the collective good that depends positively on the number of contributors. An example in the context of collective defense may be found in sentinel behavior (Bednekoff et al., 2001; Clutton-Brock et al., 1999). If a sentinel needs to take up a high position to be vigilant, then acting as a sentinel means foregoing the benefits of other activities such as foraging; at the same time, a sentinel may obtain the fixed benefit of always being safe from predation. Non-sentinels can continue foraging, and are more likely to notice fleeing sentinels, the more individuals in their group act as sentinels. At the same time, the individual sentinel does not benefit from the presence of other sentinels in the group, if he is completely safe from predation by acting as a sentinel. Sentinels then produce a charity good for non-sentinels, as each non-sentinel is attacked with the same probability, and therefore obtains the same benefits. Peña et al. (2015) offer as examples of charity goods eusociality in Hymenoptera, with sterile workers providing a benefit to queens; furthermore, several instances of microbial cooperation involving sacrifice and self-destruction. Yet, relatedness may be the driving factor behind these examples, where the payoff obtained from cooperating consists of inclusive fitness, and depends on how many individuals are benefiting within the same group. For this reason, we consider sentinel behavior as the motivating example for this section.

With a charity good, for generic impact function I_k, as switching from defecting to cooperating causes the loss of the full value of the collective good, and as more is lost the more players currently cooperate, it is clear that levels for the cooperation costs exist such that the game is a Snowdrift game. If the game indeed takes this form, as an increase in the degree of adversity mean that weakly less is produced, if such an increase has any effect, it can only take the form of a common-enemy effect, because weakly less is lost when switching from defecting to cooperating.

Specifically with impact function β_k, as $d_k = -\frac{V(1-w^k)}{1-w^n} - c$, it is the case that $\Delta \boldsymbol{d} < 0$, and by the analysis in Section 2, when $\Delta \boldsymbol{d}$ changes sign, the game is a Snowdrift game. When all players defect, a single player who switches to cooperating does not forego any benefits as no value of the collective good is produced, but obtains a benefit $-c$. it follows that the game cannot be a Prisoner's Dilemma; therefore, if $\Delta \boldsymbol{d}$ does not change sign, the game is a Harmony Game. Intuitively, as only a player who defects benefits from the collective good produced by any cooperators, the added benefit of cooperating decreases (or: the net cost of cooperating increases) the larger the number of cooperating players in the population, because cooperating means foregoing a larger and larger value of the collective good. This is true independently of whether the impact function is convex, linear, or concave. To look at the effect of the degree of complementarity, we again take the case $n = 2$ as a starting point:

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} -c & -c \\ V/(1+w) & 0 \end{pmatrix} \end{array} \tag{9}$$

Fig. 6 represents the benefits foregone when contributing rather than receiving for the case $n = 2$. When the other player contributes, this is $-\delta_1 = V/(1+w)$; when the other player receives, this equals $-\delta_0 = 0$. The type of the game played is now determined by the relation between the fixed benefit of cooperating $-c$

[13] This differs from the standard definition of a Harmony Game. Consider the two-player case. Then in the standard definition of a Harmony Game (Martinez et al. 1999), we have $R > T > S > P$, so that for any mixed group, defectors are better off than cooperators. In general, this means that for any intermediate number k of cooperating players in a group, defectors are better off than cooperators (as is the case with a public good). With a club good, however, $R > S > P = T$, and in mixed groups cooperators are better off than defectors.

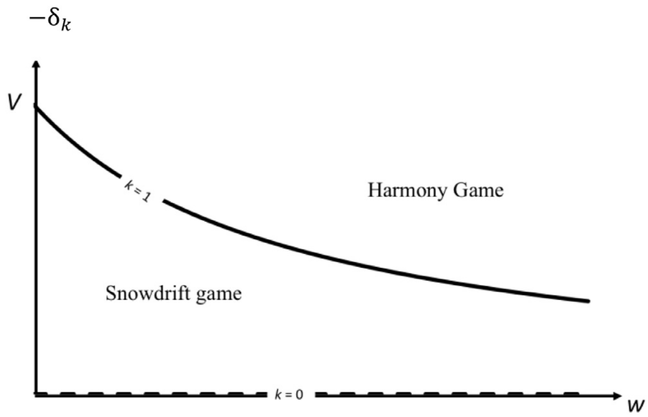

Fig. 6. Charity good: for the case of two players, benefit foregone when cooperating instead of defecting ($-\delta_k$) when one player currently cooperates ($k = 1$) and when all players currently defect ($k = 0$), as a function of the degree of complementarity w. As a function of the benefit of cooperating ($-c$), areas where the game is a Harmony Game, or a Snowdrift game.

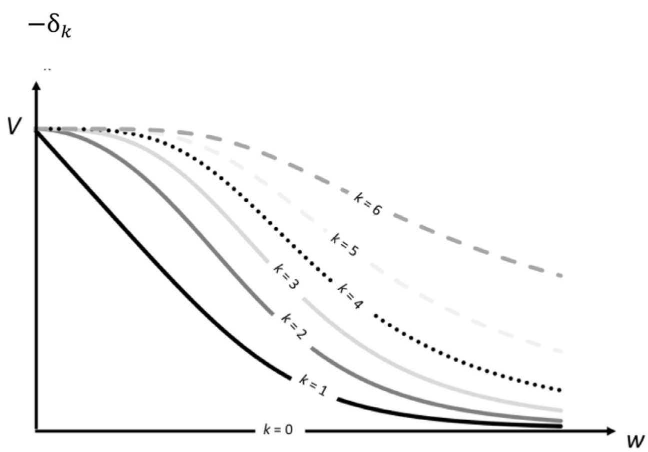

Fig. 7. Charity good: benefit foregone when contributing rather than receiving ($-\delta_k$) when k players in a group of 7 players contribute, as a function of the degree of complementarity w.

(measured on the Y-axis), and $-\delta_0$ and $-\delta_1$. It follows that as the degree of complementarity is increased, the game switches from a Snowdrift game to a Harmony Game, leading to a common-enemy effect.

This game-changing common-enemy effect of the degree of complementarity is general for any n, and is confirmed by the marginal effect of the degree of complementarity within the Snowdrift game. This is because the benefit foregone when switching from defecting to cooperating is the value of the collective good itself. As this value decreases in the degree of complementarity, it follows that $\partial \boldsymbol{d}/\partial w > 0$, so that Case 1 in Section 2 applies. Graphically, as represented in Fig. 7 for the case $n = 7$, for any given w, each $-\delta_k$ decreases in w (with the exception of $-\delta_0$, which is flat at 0). Fixing a benefit of cooperating $-c$ along the Y-axis in Fig. 7, for small w it is the case that $-c < \delta_1$, and the game is a Snowdrift game. As w is further increased, consecutively $-\delta_1, -\delta_2, \delta_3, \ldots$ become smaller than $-c$, in line with the common-enemy effect (where the game becomes a Harmony Game for $-c > \delta_6$). Intuitively, given that each benefit foregone decreases in the degree of complementarity, a common-enemy effect is the only possible effect: when each player's cooperative effort becomes more critical, less value of the collective good is foregone when cooperating. Moreover, by the charity-good feature of the collective good, a player who switches from defecting to cooperating can lose anything from a minimal to a maximal value of the collective good; the common-enemy effect therefore applies over the entire range of cooperation costs. Our results for the charity good are summarized in Proposition 3 (for a precise statement of Proposition 3 and for the proof, see Appendix A).

Proposition 3. *Consider the production of a* ***charity good****, with $n \geq 2$. Then, with any impact function I_k that strictly increases in the number of cooperating players k, if an increase in the degree of adversity has any effect on the probability of cooperation, it is a common-enemy effect. Specifically with impact function β_k in (5), as the degree of complementarity is increased, the common-enemy effect applies, where at first we have a Snowdrift game with increasing probability of cooperation, and then a Harmony Game.*

7. Discussion

In a model where organisms produce a public good, a harsher environment can both have a common-enemy effect or a competing effect, in that depending on the level of the cooperation costs and on whether the initial degree of complementarity is high or low, the probability that cooperation evolves may either increase or decrease when the environment becomes harsher (De Jaegher and Hoyer, 2016a; for a summary, see Table 1 above). Moreover, when cooperating groups contain more than two players, for intermediate cooperation costs the effect of a harsher environment on the probability of cooperation is non-monotonic (De Jaegher, 2017). As shown in the current paper, in a model where organisms instead produce a club good (only cooperators benefit from the collective good), or produce a charity good (only defectors benefit from the collective good), the effect of a harsher environment on the probability of cooperation is straightforward and unambiguous. These results are not changed when the club good or the charity good is additionally congestible.

A first manner in which the analysis in the current paper is useful, is that it helps to a better understanding of the ambiguous effects of harsher environments that are obtained for public goods. With a public good, the added benefit of cooperating alone operates in the same manner as with a club good, as no previously obtained benefit is foregone when switching from zero cooperating players to one cooperating player. This is why for a public good, a competing effect similar to the one for a club good is obtained when the costs of cooperation are close to the added benefit of cooperating alone. With a concave impact function for the production of the public good, as the added benefit of cooperating alone is larger than the other added benefits, a competing effect is therefore obtained for large costs of cooperation, whereas with a convex impact function, as the added benefit of cooperating alone is smaller than the other added benefits, a competing effect is obtained for small costs of cooperation. Also with a public good, the added benefit of cooperating jointly (i.e., with all players) operates in the same manner as with a charity good, as the benefit obtained when all players cooperate is fixed (namely the maximal value of the public good), but the benefit that would have been obtained when one player defects is variable. This is why for a public good, a common-enemy effect similar to the one for a charity good is obtained when the costs of cooperation are close to the added benefit of cooperating with all players. With a concave impact function for the production of a public good, as the added benefit of cooperating jointly is smaller than the other added benefits, a common-enemy effect is therefore obtained for small costs of cooperation, whereas with a convex impact function, as the added benefit of cooperating jointly is larger than the other added benefits, a common-enemy effect is obtained for large costs of cooperation.

A second way in which the analysis is useful, is that it may lead to predictions that are more straightforward to test. As illustrated in Table 1, for public goods the predictions on the incidence of the common-enemy and competing effects depends both on the level of the cooperation costs, and on the initial harshness of the environment. In principle, one could try to test these predictions across species, or within species across several contexts (say, cooperative hunting, and collective defense by the same species such as lions (*Panthera leo*; Grinnell et al., 1995, Scheel and Packer, 1991)), by looking for instances fitting the four scenarios in Table 1. But here, one hits upon the problem that one cannot with a species, say, compare the height of cooperation costs in cooperative hunting, to this same height in collective defense, and across species cannot compare cooperation costs in cooperative hunting. One therefore needs to focus on a single cooperative instance within a single species in order to test the public-good model. Yet, as the public-good model allows for several directions of the effect of increases in the degree of adversity on the probability of cooperation (increasing, decreasing, inverse U-shaped), finding one such effect does not confirm the theory, and one needs enough variance in the degrees of adversity and in the cooperation costs to cover all the cases in Table 1. This may be difficult to achieve: for instance, in a specific cooperative context such as the cooperative hunting of a particular large prey, it may be difficult to find enough variance in prey characteristics for it both to be possible that a single hunting predator suffices to catch the prey (low adversity), or that all predators need to hunt (high adversity); also, it may be difficult to find sufficient variance in predator contextual variables such that for low degrees of adversity, either cooperative hunting is evolutionarily stable (small cooperation costs), or it is not (large cooperation costs).

As our analysis shows, relaxing the assumptions that collective goods are non-excludable and non-congestible leads to unambiguous predictions. While some authors recognize that in reality collective goods produced by organisms may not be pure public goods, they consider only pure public goods for simplicity (Archetti et al., 2011, p.1305; Nunn and Lewis, 2001), arguing that this simplification does not lead to different results. Our analysis, however, shows that in the context of the common-enemy hypothesis of by-product mutualism, it can be essential to deviate from the model of a pure public good. In particular, while adding congestion does not make any difference in that the results for the public good and the congestible public good are analogous, adding excludability means that the competing effect is always predicted for club goods (e.g., situations of cooperative hunting where one can only benefit from the prey when participating in the hunt), and the common-enemy effect is always predicted for charity goods (e.g., sentinel behavior when the individual sentinel does not benefit from other sentinels). The importance of these predictions is that, while it may be difficult to compare cooperation costs or degrees of adversity across different instances of cooperative behavior, there are qualitative differences between cases with and without excludability, and with different types of excludability (club goods, charity goods). These qualitative differences make it possible to test our predictions by comparing the effect of the degree of adversity across several instances of cooperative behavior.

For instance, in the case of cooperative hunting, one should be able to identify the extent to which defectors do or do not benefit from a successful hunt: if the prey is caught in the proximity of the territory of the group, it will not be possible to exclude defectors (public good); if cooperating instead means participating in a hunt that takes place at a distance from the territory, where the prey is locally consumed, defectors are automatically excluded (club good). Also, in the case of collective defense, where cooperating means being vigilant to intruders or predators, one should be able to identify the extent to which vigilant group members benefit from any other group members' vigilance: if being vigilant is incompatible with other activities such as foraging, as is the case for sentinel behavior, the individual sentinel does not benefit from any other sentinels, and only non-sentinels obtain by-product benefits (charity good); if being vigilant does not exclude other activities such as foraging, individual vigilance is more likely to be imperfect, and vigilant individuals benefit from other group members' vigilance (public good). In this manner, one should be able to identify instances where club goods, charity goods or public goods are produced, and one can then test whether indeed a harsher environment has an opposite effect for club goods and for charity goods, and has an opposite effect when comparing club goods to public goods, and charity goods to public goods.

A third way in which our results in this paper are useful, is that they show that our predictions for (congested) club goods and (congested) charity are less restrictive, in that they apply for a wider range of harsher environments than is the case for public goods. Indeed, in the broadest sense, in the context of collective-good production, a harsher environment (or: higher degree of adversity) may be defined as any circumstance that decreases the production of the collective good, all else equal (in particular, holding fixed the number of cooperating players in a group). For public goods, this concept of harsher environment is too broad to come to unambiguous predictions about the incidence of the common-enemy effect and the competing effect. This is because for public goods, it is important to know how the shape of the impact function changes, as a harsher environment shifts it downwards. It is for this reason that our analysis of public goods is focused on specific instances where the harsher environment makes each player's contribution to the public good more pivotal, so that the impact function becomes more convex when it shifts downwards. Yet, with (congested) club goods and (congested) charity goods, the manner in which the shape of the impact function changes as a harsher environment shifts it down, does not matter for the results. This suggests that the common-enemy effect and the competing effect are more robust in the context of such goods.

We end by noting that we have deliberately focused on harsh environments as a rationale for the evolution of cooperation, and have excluded either relatedness or repeated interaction as rationales, in order to identify the effect of a harsher environment in isolation. In reality, multiple of these rationales may be at work at the same time. For instance, providers of charity goods may benefit from providing such a good because they are related to other group members. Integrating the effect of harsh environments into models of kin selection and reciprocity, may be the subject of future research.[14]

Acknowledgements

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Appendix A: Exact formulation of Propositions in the body the paper, and proofs

This Appendix formulates more detailed versions of Propositions 2 and 3 in the body of the paper, and proves these propositions. To read the more detailed version of the propositions, define w_0 as degree of complementarity such that $c = \delta_0$, and w_{n-1} as the degree of complementarity such that $c = \delta_{n-1}$

[14] For a recent model that studies the interplay between relatedness, responsiveness as a measure of reciprocity, and the degree of synergy, see Van Cleve (2017).

Proposition 2*. *Consider the production of a* ***club good*** *with* $n \geq 2$. *Then for any impact function* I_k *that strictly increases in k and for which* $\partial I_k/\partial\alpha \leq 0$, *if an interior fixed point* x^* *exists, we have a Stag Hunt (the fixed point is unstable), and it is the case that* $\partial x^*/\partial\alpha \geq 0$. *Specifically with impact function* β_k *in (5), when* $w \leq w_0$, *the game is a Harmony Game; when* $w > w_0$, *the game is a Stag Hunt and* $\partial x^*/\partial w > 0$ *(where* x^* *approaches 1 as* $w \to +\infty$*). It is the case that* $w_0 \lesseqgtr 1$ *for* $c \gtreqless V/n$.

Proof. For generic I_k, given that $d_k = I_{k+1} - c$, and given that I_k strictly increases in k, it follows that $\Delta \boldsymbol{d} > 0$ and (by the shape-preserving properties of polynomials in Bernstein form) that $g(x)$ increases in x, meaning that the game is a Stag Hunt if $\boldsymbol{d}$ changes sign (Result 3.2(a) in Peña et al. (2014)). Moreover, as $\partial I_k/\partial\alpha \leq 0$, meaning that $\partial d_k/\partial\alpha \leq 0$, it follows from (2) that $\partial g(x)/\partial\alpha \leq 0$. As $g(x)$ decreases in x, and may shift down as α is increased, if there is any effect of the degree of adversity, it is a competing effect in the form of a decrease in the size of the basin of attraction of the joint cooperation ESS.

Specifically for $I_k = \beta_k$, using (2), and substituting for $d_k = \beta_{k+1} - c$, for $w \neq 1$, it follows that

$$g(x) = \sum_{k=0}^{n-1} \binom{n-1}{k} x^k (1-x)^{n-1-k} \left[\frac{V\left(1 - w^{k+1}\right)}{1 - w^n} - c \right]$$

Given that by the binomial theorem, $\sum_{k=0}^{n-1} \binom{n-1}{k} x^k (1-x)^{n-1-k} = 1$, this is re-expressed as:

$$g(x) = \frac{V}{1-w^n} - \frac{wV}{1-w^n} \sum_{k=0}^{n-1} \binom{n-1}{k} (wx)^k (1-x)^{n-1-k} - c$$

Again applying the binomial theorem, note that $\sum_{k=0}^{n-1} \binom{n-1}{k} (wx)^k (1-x)^{n-1-k} = (wx + 1 - x)^{n-1}$. We conclude that:

$$g(x) = \frac{V\left[1 - w(wx + 1 - x)^{n-1}\right]}{1 - w^n} - c \text{ for } w \neq 1 \tag{A.1}$$

Using (2), and plugging in the value for d_k calculated in Section 3 for the club good, for $w = 1$, it follows that

$$g(x) = \sum_{k=0}^{n-1} \binom{n-1}{k} x^k (1-x)^{n-1-k} \left[\frac{(k+1)V}{n} - c \right]$$

Applying again the binomial theorem, this is re-expressed as:

$$g(x) = \frac{V}{n} - c + \frac{V}{n} \sum_{k=1}^{n-1} \binom{n-1}{k} k x^k (1-x)^{n-1-k}$$

Using the fact that $\binom{n-1}{k} k = (n-1)\binom{n-2}{k-1}$, it follows that:

$$g(x) = \frac{V}{n} - c + \frac{V}{n} x(n-1) \sum_{(k-1)=0}^{n-2} \binom{n-2}{k-1} x^{k-1} (1-x)^{n-2-(k-1)}$$

which applying once again the binomial theorem, we finally write as

$$g(x) = \frac{V[1 + (n-1)x]}{n} - c \text{ for } w = 1 \tag{A.2}$$

Using the fact that for any interior fixed point x^*, it must be the case that $g(x^*) = 0$, solving for x^* in (A.1) and (A.2), we obtain that:

$$x^* = \frac{1}{1-w} - \frac{1}{1-w} \left[\frac{V - c(1-w^n)}{Vw} \right]^{1/(n-1)} \text{ for } w \neq 1 \tag{A.3}$$

and

$$x^* = \frac{cn - V}{V(n-1)} \text{ for } w = 1 \tag{A.4}$$

In both (A.3) and (A.4), it is the case that $x^* < 1$ given our assumption that $V > c$. Also, $x^* > 0$ as long as $c > \frac{V(1-w)}{1-w^n} = \delta_0$. It follows that for $0 < w \leq w_0$, the game is a Harmony Game, and for any $w > w_0$, the game is a Stag Hunt. As $\delta_0 = \frac{V(1-w)}{1-w^n} = \frac{V}{(1+w+\ldots+w^{n-1})}$, which equals V/n for $w = 1$, it follows that $w_0 \lesseqgtr 1$ for $c \gtreqless V/n$. Once w is large enough for the game to be a Stag Hunt, given the fact that $\partial \boldsymbol{d}/\partial w < 0$, it follows that $\partial g(x)/\partial w < 0$, which along with the fact that $\partial g(x)/\partial x > 0$ means that $\partial x^*/\partial w > 0$ (see Section 2); this implies a competing effect where an increase in w decreases the size of the basin of attraction of the joint cooperation ESS. The game only approaches a Prisoner's Dilemma in the limit, as w approaches infinity. □

Proposition 3*. *Consider the production of a* ***charity good*** *with* $n \geq 2$. *Then for any impact function* I_k *that strictly increases in k and for which* $\partial I_k/\partial\alpha \leq 0$, *if an interior fixed point* x^* *exists, we have a Snowdrift game (the fixed point is stable), and it is the case that* $\partial x^*/\partial\alpha \geq 0$. *Specifically with impact function* β_k *in (5), when* $w < w_{n-1}$, *the game is a Snowdrift game and* $\partial x^*/\partial w > 0$ *(common-enemy effect); when* $w \geq w_{n-1}$, *the game is a Harmony Game. It is the case that* $w_{n-1} \lesseqgtr 1$ *for* $-c \gtreqless [(n-1)/n]V$.

Proof. For generic I_k, given that $d_k = -I_k - c$, and given that I_k strictly increases in k, it follows that $\Delta \boldsymbol{d} < 0$ and (by the shape-preserving properties of polynomials in Bernstein form) that $g(x)$ decreases in x, meaning that the game is a Snowdrift game if $\boldsymbol{d}$ changes sign (Result 3.2(b) in Peña et al. (2014)). Moreover, as $\partial I_k/\partial\alpha \leq 0$, meaning that $\partial d_k/\partial\alpha \geq 0$, it follows from (2) that $\partial g(x)/\partial\alpha \geq 0$. As $g(x)$ increases in x, and may shift upwards as α is increased, if there is any effect of the degree of adversity, it is a common-enemy effect in the form of an increase the fraction of cooperating players in the unique interior ESS.

Specifically for $I_k = \beta_k$, using (2), and plugging in the value for d_k calculated in Section 3 for the charity good, for $w \neq 1$, it follows that

$$g(x) = \sum_{k=0}^{n-1} \binom{n-1}{k} x^k (1-x)^{n-1-k} \left[-\frac{V\left(1 - w^k\right)}{1 - w^n} - c \right]$$

Using the binomial theorem, this is re-expressed as:

$$g(x) = -\frac{V}{1-w^n} - c + \sum_{k=0}^{n-1} \binom{n-1}{k} (wx)^k (1-x)^{n-1-k}$$

Applying once more the binomial theorem, we finally obtain that

$$g(x) = -\frac{V\left[1 - (wx + 1 - x)^{n-1}\right]}{1 - w^n} - c \text{ for } w \neq 1 \tag{A.5}$$

Using (2), and plugging in the value for d_k calculated in Section 3 for the charity good, for $w = 1$, it follows that

$$g(x) = \sum_{k=0}^{n-1} \binom{n-1}{k} x^k (1-x)^{n-1-k} \left[-\frac{kV}{n} - c \right]$$

Applying again the binomial theorem, this is re-expressed as:

$$g(x) = -c - \frac{V}{n} \sum_{k=0}^{n-1} \binom{n-1}{k} k x^k (1-x)^{n-1-k}$$

From the proof of Proposition 2*, we know that $\sum_{k=0}^{n-1} \binom{n-1}{k} k x^k (1-x)^{n-1-k} = x(n-1)$, so that we finally obtain

$$g(x) = -\frac{V(n-1)x}{n} - c \text{ for } w = 1 \tag{A.6}$$

Using the fact that for any interior fixed point x^*, it must be the case that $g(x^*) = 0$, solving for x^* in (A.5) and (A.6), we obtain that:

$$x^* = \frac{1}{1-w} - \frac{1}{1-w}\left[\frac{V + c(1-w^n)}{V}\right]^{\frac{1}{n-1}} \text{ for } w \neq 1 \tag{A.7}$$

and

$$x^* = \frac{-cn}{V(n-1)} \text{ for } w = 1 \tag{A.8}$$

It follows from (A.7) and (A.8) that $x^* > 0$ as soon as $c < 0$; clearly, taking joint defection as a starting point, a first player always has an incentive to contribute, as this means obtaining positive benefit $-c$ without foregoing any benefits from the collective good. At the same time, as follows from (A.7), $x^* < 1$ as soon as $-c < \frac{V(1-w^{n-1})}{(1-w^n)} = -\delta_{n-1}$ (the benefit foregone by being the n-th player who cooperates exceeds the benefit obtained from contributing to the collective good). As δ_{n-1} equals V for $w = 0$ and decreases in w, and given our assumption that $0 < -c < V$, it follows that for any c we consider, there is always a range of smaller w for which there is an interior fixed point. As $\delta_{n-1} = \frac{V(1-w^{n-1})}{(1-w^n)} = \frac{V(1+w+\ldots+w^{n-2})}{(1+w+\ldots+w^{n-1})}$, which equals $[(n-1)/n]V$ for $w = 1$, it follows that $w_{n-1} \lesseqgtr 1$ for $-c \gtreqless [(n-1)/n]V$. For $w < w_{n-1}$, given the fact that $\partial \boldsymbol{d}/\partial w > 0$, it follows that $\partial g(x)/\partial w > 0$, which along with the fact that $\partial g(x)/\partial x < 0$ means that $\partial x^*/\partial w > 0$ (see Section 2); this implies a common-enemy effect where an increase in w increases the ESS fraction of cooperating players. □

Appendix B: Private good

Mesterton-Gibbons and Dugatkin (1992) provide the example of a sparrow (*Passer domesticus*) that finds a food source, and makes a chirrup call to attract other sparrows (Elgar, 1986; Newman and Caraco, 1989). The private good that the sparrow produces is a reduction in its vulnerability to predation (caused by having a larger number of sparrows at the food source); the by-product benefit produced is letting the other sparrows share in the food source. The degree of adversity is specified as the risk of predation, where a higher risk of predation means that a sparrow that does not make a chirrup call is more likely to face predation. With a higher risk of predation, an individual sparrow that fails to cooperate becomes the victim of its own defection, a mechanism that Mesterton-Gibbons and Dugatkin (1992) refer to as the boomerang effect. It is this mechanism that leads to the common-enemy effect. As the benefit from producing the private good does not depend on the extent to which other players cooperate (Newman and Caraco, 1989), the boomerang effect applies no matter how many players currently cooperate. A further example proposed by Mesterton-Gibbons and Dugatkin (1992) refers to communally breeding birds, which feed non-related fledglings to ensure that their begging does not attract predators, as these could otherwise attack the birds' own fledglings. Finally, Detto et al. (2010) propose as another example fiddler crabs (*Uca mjoebergi*), who assist neighboring crabs in their defense against intruders in order to avoid the risk that the current neighbor is replaced by a strong crab.

The private-good model may be seen as a multi-player version of the donor-recipient game (also referred to as mutual aid game, or donation game). In this game, the net cost of cooperating when k other players cooperate is independent of k, and is denoted as $\mathcal{C}$ (where the subscript is dropped). The by-product benefit the focal player obtains from the fact that another player cooperates rather than defects, does not depend on whether the focal player cooperates or defects himself, and does not depend on the number of other cooperating players; this benefit is denoted as $\mathcal{B}$ (where the subscript is again dropped). A focal player who defects does not incur any cost, and does not produce any benefit for the other players in her group. It follows that $a_k = k\mathcal{B} - \mathcal{C}$, $b_k = k\mathcal{B}$, $d_k = -\mathcal{C}$, and $D_k = (n-1)\mathcal{B} - \mathcal{C}$, where we assume that $(n-1)\mathcal{B} > \mathcal{C}$. The net cost $\mathcal{C}$ can be further decomposed into the cost c of producing the private good, minus the benefit u the focal player obtains from the private good, so that $\mathcal{C} = c - u$. This illustrates the fact that the net cost $\mathcal{C}$ can be negative (McAvoy and Hauert, 2016). The private-good model has the equal-gains-from switching property, and following the general framework in Section 2 is a Prisoner's Dilemma for positive net cost ($\mathcal{C} > 0$) and a Harmony Game for negative net cost ($\mathcal{C} < 0$). An increase in the degree of adversity increases the benefit u the focal player obtains from the private good,[15] meaning that the gain from switching increases (boomerang effect), and can thus turn the game from a Prisoner's Dilemma into a Harmony Game, resulting in a common-enemy effect.

Appendix C: Congestible public good

With a *congestible public good*, the value of the collective good is divided equally over the n players in the group, so that $a_k = I_{k+1}/n - c$, and $b_k = I_k/n$, meaning that the gain from switching equals $d_k = (I_{k+1} - I_k)/n - c$, so that the added benefit of cooperating equals $\delta_k = (I_{k+1} - I_k)/n$. Furthermore, $\mathcal{C}_k = c - (I_{k+1} - I_k)/n$, $\mathcal{B}_{a,k-1} = (I_{k+1} - I_k)/n$, and $\mathcal{B}_{b,k} = (I_{k+1} - I_k)/n$. It is the case that $\delta_k = \mathcal{B}_{a,k-1} = \mathcal{B}_{b,k}$, and that $D_k = I_{k+1} - I_k - c > d_k$. Thus, this case is comparable to the public good, except that benefits are divided by n. For $I_k = \beta_k$, all results are therefore analogous to those of the public good; however, because the value of the congested public good is divided by n, the case of large cooperation costs quickly becomes the dominant one, with all other cases vanishing.

We further point out that in the standard taxonomy of collective goods, a good which is congestible but which is nonexcludable is usually referred to as a commons good (e.g., Dionisio and Gordo, 2006; Rankin et al., 2007). The congestible public good can easily be reframed as a commons good in the following way. Let defecting correspond to an individually beneficial action such as polluting, and let this yield an individual benefit θ. Let cooperating correspond to refraining from pollution. Let the value of the commons good be equal to I_k/n if k players refrain from polluting. Taking now $\theta = c$ (where c can be seen as the opportunity cost of not polluting), the analysis is identical as for a congestible public good.

Appendix D: Congestible club good

With a *congestible club good*, only cooperating players benefit from the value produced of the collective good, and addition-

[15] Analytically, it does not make a difference whether u is considered as the benefit of producing the private good, or is considered as a cost that is incurred when not producing the private good (e.g., cost of facing possible predation). Considering u as the cost of not producing the private good, makes it clear why an increase in u can be considered as an increase in the degree of adversity.

ally, the value of the collective good is divided over them. In the case of cooperative hunting, such a case is obtained if the prey is not only caught at a distance from the common territory of the group, but is also sufficiently small such that one cooperator's consumption of the prey diminishes the consumption possibilities of the other cooperators. It is the case that $a_k = I_{k+1}/(k+1) - c$, $b_k = 0$, meaning that $d_k = I_{k+1}/(k+1) - c$ and $\delta_k = I_{k+1}/(k+1)$. Also, $\mathcal{C}_k = c - I_{k+1}/(k+1)$, $\mathcal{B}_{a,k-1} = I_{k+1}/(k+1) - I_k/k$, and $\mathcal{B}_{b,k} = 0$. Just as with the standard club good, it is the case that $\delta_k > \mathcal{B}_{a,k-1} > \mathcal{B}_{b,k} = 0$. Moreover, it is the case that $D_k = I_{k+1}/(k+1) + k[I_{k+1}/(k+1) - I_k/k] - c$.

For the congestible club good, with a generic impact function I_k, because of congestion, the effect of a larger number of cooperators on the value of the club good is ambiguous, and we may have a Stag Hunt, a Snowdrift game, or a more complex game with multiple interior fixed points and both Stag Hunt and Snowdrift-game features. Yet, as shown in the first part of Proposition D.1 below, because it continues to be the case that only cooperators obtain value from the congested club good, and because this value decreases in the degree of adversity, a higher degree of adversity decreases the probability that cooperation evolves.

We further focus on the case $I_k = \beta_k$. We first show that $\beta_{k+1}/(k+1) - \beta_k/k \gtreqless 0$ for $w \gtreqless 1$. For $w < 1$, $\frac{\beta_{k+1}}{k+1} - \frac{\beta_k}{k} < 0 \Leftrightarrow \frac{1-w^{k+1}}{k+1} < \frac{1-w^k}{k} \Leftrightarrow kw^k(1-w) < 1 - w^k$. Given that the right-hand side of the latter inequality can be seen as a difference of k-th powers, it can be rewritten as $kw^k(1-w) < (1-w)(1+w+w^2+\ldots+w^{k-1}) \Leftrightarrow kw^k < (1+w+w^2+\ldots+w^{k-1})$. Note now that in this inequality we have k terms on the left-hand side (k times w^k), and we have k terms on the right-hand side. Given that $w < 1$, each of the terms on the left-hand side is smaller than each of the terms on the right-hand side. For $w > 1$, $\frac{\beta_{k+1}}{k+1} - \frac{\beta_k}{k} > 0 \Leftrightarrow \frac{w^{k+1}-1}{k+1} > \frac{w^k-1}{k} \Leftrightarrow kw^k(w-1) > w^k - 1$. Given that the right-hand side of the latter inequality can be seen as a difference of k-th powers, it can be rewritten as $kw^k(w-1) > (w-1)(1+w+w^2+\ldots+w^{k-1}) \Leftrightarrow kw^k > (1+w+w^2+\ldots+w^{k-1})$. Given that $w > 1$, each of the terms on the left-hand side is larger than each of the terms on the right-hand side.

A first implication of the fact that $\beta_{k+1}/(k+1) - \beta_k/k \gtreqless 0$ for $w \gtreqless 1$, is that $D_k \gtreqless d_k$ iff $w \gtreqless 1$. With a congestible club good, underprovision is therefore possible when the impact function is convex, but overprovision is possible when the impact function is concave; in the latter case, the fact that an additional cooperator increases the value of the good is more than compensated by the fact that the cooperator creates extra congestion. A second implication is that, as $d_k = \frac{V(1-w^{k+1})}{(k+1)(1-w^n)} - c$, for $w > 1$ it is the case that $\Delta\boldsymbol{d} > 0$, and by the analysis in Section 2 when $\Delta\boldsymbol{d}$ changes sign, the game is a Stag Hunt. For $w > 1$, when $\Delta\boldsymbol{d}$ does not change sign, the game cannot only be a Harmony Game, but can also be a Prisoner's Dilemma, because when all other players cooperate, cooperating rather than defecting only yields a benefit of V/n. Intuitively, for $w > 1$, the fact that additional cooperating players add increasingly to the collective good, more than compensates for the fact that there is increased congestion, and the added benefit of cooperating increases (i.e., the net cost of cooperating decreases) in the number of players. For $w < 1$ it is the case that $\Delta\boldsymbol{d} < 0$, and by the analysis in Section 2 when $\Delta\boldsymbol{d}$ changes sign, the game is a Snowdrift game. In this case, the fact that additional cooperating players add increasingly to the collective good, is more than compensated by the fact that there is increased congestion, and the added benefit of cooperating decreases (i.e., the net cost of cooperating increases) in the number of players. For $w < 1$, when $\Delta\boldsymbol{d}$ does not change sign, the game can again both be a Harmony Game and a Prisoner's Dilemma.

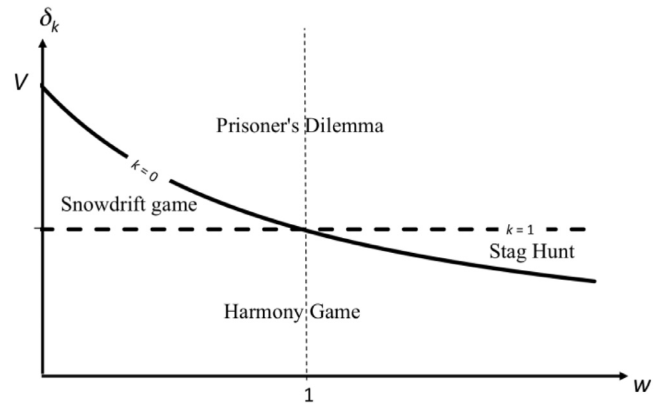

Fig. D.1. Congestible club good: for the case of two players, added benefit (δ_k) of cooperating jointly ($k = 1$) and of cooperating alone ($k = 0$) as a function of the degree of complementarity w. Areas where, as a fucntino of cooperation costs, the game is a Prisoner's Dilemma, a Stag Hunt, a Harmony Game, or a Snowdrift game.

To look at the effect of the degree of complementarity, we again take the case $n = 2$ as a starting point, represented in (D.1):

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} V/2 - c & V/(1+w) - c \\ 0 & 0 \end{pmatrix} \end{array} \tag{D.1}$$

It is now the case that $\delta_1 = V/2$, and $\delta_0 = V/(1+w)$. As represented in Fig. D.1, δ_1 is now fixed at $V/2$, whereas δ_0 decreases in w, with $\delta_0 = \delta_1$ when $w = 1$. Again, the type of the game played is determined by the relation between cooperation costs c, and δ_0 and δ_1. It follows that, as the degree of complementarity is increased, with low complementarity, for sufficiently large cooperation costs, the game switches from a Snowdrift game to a Prisoner's Dilemma, leading to a competing effect; for sufficiently small cooperation costs, the game is a Harmony Game, whatever the degree of complementarity. With high complementarity, for sufficiently small cooperation costs, the game switches from a Harmony Game to a Stag Hunt, leading again to a competing effect; for sufficiently large cooperation costs, the game remains a Prisoner's Dilemma.[16]

These game-changing competing effects of the degree of complementarity are general for any n, and are confirmed by the marginal effect of the degree of complementarity within the Stag Hunt and the Snowdrift game. To see why this is so, note that with a congestible club good, the benefit part of the gain from switching is the value of the collective good divided by the number of cooperators. As this benefit part decreases in the degree of complementarity, it follows that $\partial\boldsymbol{d}/\partial w < 0$, so that Case 2 in Section 2 applies. Graphically, as represented in Figure B.2 for the case $n = 7$, for any given w, each δ_k decreases in w (with the exception of δ_{n-1}, which is flat at V/n). For $w < 1$, fixing a c(with $c > V/n$) on the Y-axis in Fig. D.2, as w is increased, gradually added benefits for a smaller initial number of cooperating players become smaller than c, in line with a competing effect within the Snowdrift game; when finally $\delta_0 < c$, the game becomes a Prisoner's Dilemma. For $w > 1$, fixing a c(with $c < V/n$) on the Y-axis in Fig. D.2, for small w, initially all added benefits exceed c (Harmony Game). As w is increased, consecutively $\delta_0, \delta_1, \ldots$ become larger than c, in line with

[16] Formally, in a Harmony Game, total fitness is maximized when all players cooperate. This is not the case when $w < 1$, as congestion is the dominating factor in this case. Also, formally, in a Prisoner's Dilemma, total fitness is again maximized when all players cooperate. This is not the case with a congestible club good as long as $c > V/2$. As noted in Footnote 3, to avoid having to formulate further categories of games, we talk of a Harmony Game (Prisoner's Dilemma) as soon as cooperating (defecting) is the dominant strategy.

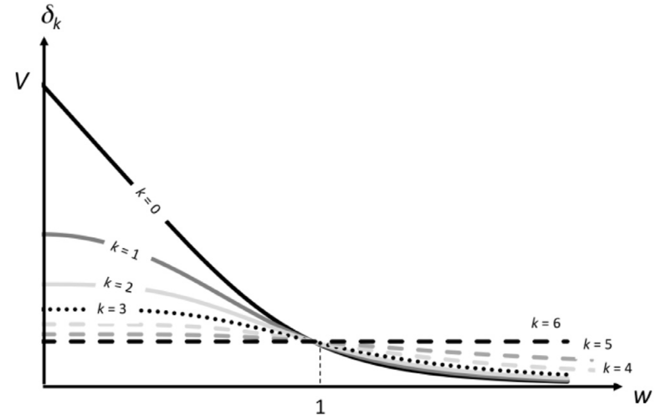

Fig. D.2. Congestible club good: added benefits of cooperating (δ_k) when k players in a group of 7 players cooperate, as a function of the degree of complementarity w.

the competing effect within the Stag Hunt Game. Our results for the congestible club good with $I_k = \beta_k$ are summarized in the second part Proposition D.1. We note that, given that as shown in Section 3 for $w < 1$, because of congestion, the sum of the gains the n players obtain when a focal player switches from defecting to cooperating is lower than the gain the focal player makes by switching from defecting to cooperating. As the game is a Snowdrift game, this means that from a group perspective, the fraction of cooperating players is inefficiently large. For this reason, the fact that the competing effect reduces the fraction of cooperating players, may be beneficial from a group perspective.

Proposition D.1. *Consider the production of a* ***congestible club good*** *with $n \geq 2$. Then for any impact function I_k that strictly increases in k and for which $\partial I_k/\partial\alpha \leq 0$, for any unstable interior fixed point x^*, it is the case that $\partial x^*/\partial\alpha \leq 0$, and for any stable interior fixed point x^*, it is the case that $\partial x^*/\partial\alpha \geq 0$. Specifically with impact function β_k in (5)*:

For $w < 1$ (low complementarity):

- for $\frac{V}{n} < c < V$ (large cooperation costs), when $0 < w < w_0$, the game is a Snowdrift game and $\frac{\partial x^*}{\partial w} < 0$ (competing effect); when $w_0 \leq w < 1$, the game is a Prisoner's Dilemma;
- for $0 < c < \frac{V}{n}$ (small cooperation costs), the game is a Harmony Game (no effect).

For $w > 1$:

- for $\frac{V}{n} < c < V$ (large cooperation costs), the game is a Prisoner's Dilemma (no effect);
- for $0 < c < \frac{V}{n}$ (small cooperation costs), when $1 < w \leq w_0$, the game is a Harmony Game; when $w > w_0$, the game is a Stag Hunt and $\frac{\partial x^*}{\partial w} > 0$ (competing effect).

Proof. With a generic impact function I_k, as the effect of k on $d_k = I_{k+1}/(k+1) - c$ is ambiguous, d_k can have one or more sign changes as k is increased, and because of the variation diminishing property of polynomials in Bernstein form (Property 2 in Peña et al. (2014)), this means that $g(x)$ can also have one or more sign changes, and therefore can have any combination of stable fixed points ($g(x)$ decreases around x^*) and unstable fixed points ($g(x)$ increases around x^*). Because $\partial I_k/\partial\alpha \leq 0$, it is the case that $\partial d_k/\partial\alpha \leq 0$, and therefore by (2), it is the case that $\partial g(x)/\partial\alpha \leq 0$. As around a stable fixed point x^* it is the case that $g(x)$ locally decreases in x, as $\partial g(x)/\partial\alpha \leq 0$, and as $g(x)$ is continuous for any given α, it follows that $g(x)$ must continue to locally decrease in x, meaning that $\partial x^*/\partial\alpha \leq 0$. Similarly, as around an unstable fixed point x^* it is the case that $g(x)$ locally increases in x, as $\partial g(x)/\partial\alpha \leq 0$, and as $g(x)$ is continuous for any given α, it follows that $g(x)$ must continue to locally increase in x, meaning that $\partial x^*/\partial\alpha \geq 0$.

For $I_k = \beta_k$, using (2), and plugging in the value for d_k for the congested club good, for $w \neq 1$, it follows that

$$g(x) = \sum_{k=0}^{n-1}\binom{n-1}{k}x^k(1-x)^{n-1-k}\left[\frac{V\left(1-w^{k+1}\right)}{(k+1)(1-w^n)} - c\right]$$

Given that by the binomial theorem, $\sum_{k=0}^{n-1}\binom{n-1}{k}x^k(1-x)^{n-1-k} = 1$, this is re-expressed as:

$$g(x) = \frac{V}{1-w^n}\sum_{k=0}^{n-1}\frac{1}{k+1}\binom{n-1}{k}x^k(1-x)^{n-1-k}\frac{\left(1-w^{k+1}\right)}{k+1} - c \tag{D.1}$$

where we note that $\frac{1}{k+1}\binom{n-1}{k} = \frac{1}{n}\binom{n}{k+1}$. Given that $\sum_{k=-1}^{n-1}\binom{n}{k+1}x^{k+1}(1-x)^{n-k-1} = (1-x)^n + x\sum_{k=0}^{n-1}\binom{n}{k+1}x^k(1-x)^{n-k-1} = 1$, it follows that $\sum_{k=0}^{n-1}\binom{n}{k+1}x^k(1-x)^{n-k-1} = \frac{1-(1-x)^n}{x}$. We therefore obtain that:

$$\sum_{k=0}^{n-1}\frac{1}{k+1}\binom{n-1}{k}x^k(1-x)^{n-1-k} = \frac{1-(1-x)^n}{nx} \tag{D.2}$$

Similarly, note that $\sum_{k=-1}^{n-1}\binom{n}{k+1}(wx)^{k+1}(1-x)^{n-k-1} = (1-x)^n + wx\sum_{k=0}^{n-1}\binom{n}{k+1}(wx)^k(1-x)^{n-k-1} = (wx+1-x)^n$, meaning that $\sum_{k=0}^{n-1}\binom{n}{k+1}(wx)^k(1-x)^{n-k-1} = \frac{(wx+1-x)^n-(1-x)^n}{wx}$. It follows that $\sum_{k=0}^{n-1}\binom{n-1}{k}x^k(1-x)^{n-1-k}\frac{w^{k+1}}{k+1} = w\sum_{k=0}^{n-1}\frac{1}{k+1}\binom{n-1}{k}(wx)^k(1-x)^{n-1-k} = \frac{w}{n}\sum_{k=0}^{n-1}\binom{n}{k+1}(wx)^k(1-x)^{n-1-k} = \frac{(wx+1-x)^n-(1-x)^n}{nx}$. Using this fact and using (D.2) to substitute into (D.1), we conclude that

$$g(x) = \frac{V\left[1-(wx+(1-x))^n\right]}{nx(1-w^n)} - c \text{ for } w \neq 1 \tag{D.3}$$

Using (2), and plugging in the value for d_k calculated in Section 3 for the club good, for $w = 1$, it follows that

$$g(x) = \sum_{k=0}^{n-1}\binom{n-1}{k}x^k(1-x)^{n-1-k}\left[\frac{V}{n} - c\right]$$

Applying the binomial theorem, it directly follows that

$$g(x) = \frac{V}{n} - c \text{ for } w = 1 \tag{D.4}$$

While we cannot solve (D.3) for x^* explicitly, and x^* is only implicitly given by the value of x such that the right-hand side in (D.3) equals zero, we note that $g(x)$ is continuous function that either monotonically increases or decreases in x. As $g(0) = \delta_0 - c$ and as $g(1) = \delta_{n-1} - c = \frac{V}{n} - c$, and given the fact that $\delta_{n-1} < \delta_0$ for $w < 1$ and $\delta_{n-1} > \delta_0$ for $w > 1$, from the intermediate value theorem, it follows that for $w < 1$ an interior fixed point exists when $\frac{V}{n} < c < \delta_0$, and that for $w > 1$ an interior fixed point exists when $\delta_0 < c < \frac{V}{n}$. Denoting by w_0 the degree of complementarity such that $c = \delta_0$, given the fact that δ_0 is decreasing in w, it follows that for $w < 1$ an interior fixed point exists when $0 < w < w_0$, and for $w > 1$ an interior fixed point exists when $w > w_0$.

Given the fact that $\partial \boldsymbol{d}/\partial w < 0$, it follows that $\partial g(x)/\partial w < 0$. For $w > 1$, along with the fact that $\partial g(x)/\partial x > 0$ means that $\partial x^*/\partial w > 0$

(see Section 2), this implies a competing effect where an increase in w decreases the basin of attraction of the joint cooperation ESS. For $w < 1$, along with the fact that $\partial g(x)/\partial x < 0$ means that $\partial x^*/\partial w < 0$ (see Section 2), this implies a competing effect where an increase in w decreases the ESS fraction of cooperating players. □

Appendix E Congestible charity good

With a *congestible charity good*, it is not only the case that only defecting players benefit from the value produced of the collective good, but this value is divided equally over the defecting players. Applied to sentinel behavior, it is conceivable that the more non-sentinels there are in a group, the more they obstruct each other when attempting to flee after seeing a sentinel fleeing, so that one obtains a congestible charity good instead of a charity good. Formally, it is the case that, $a_k = -c$, $b_k = I_k/(n-k)$, meaning that the gain from switching equals $d_k = -c - I_k/(n-k)$, and that $\delta_k = -I_k/(n-k)$. Moreover, $\mathcal{C}_k = I_k/(n-k) + c$, $\mathcal{B}_{a,k} = 0$, and $\mathcal{B}_{b,k} = I_{k+1}/(n-k-1) - I_k/(n-k)$. Just as is the case with a standard charity good, $\mathcal{B}_{b,k} > \mathcal{B}_{a,k-1} = 0 > \delta_k$. Finally, $D_k = -c - \beta_k/(n-k) + (n-k-1)[I_{k+1}/(n-k-1) - I_k/(n-k)]$, which is larger than d_k. In spite of the presence of congestion, overprovision is not possible; this is because additional cooperators reduce congestion, as the congestible charity good is only enjoyed by the defectors.

With a generic impact function I_k, the gains from switching decrease in the number of cooperating players because more value of the collective good is foregone by cooperating the more players cooperate, which is further reinforced by the fact that defectors suffer less from congestion the larger the number of cooperating players. For this reason, the results for a generic impact function summarized in the first part of Proposition E.1 below, are similar as for charity goods.

For $I_k = \beta_k$, we again additionally assume for this case that $V > -c$, so that a player who by cooperating would forego the full value of the collective good (and who is presently alone to defect, so that there is no congestion), would prefer keeping this value rather than obtaining the benefit from cooperating. As $d_k = -\frac{V(1-w^k)}{(n-k)(1-w^n)} - c$, it follows that $\Delta\boldsymbol{d} < 0$, and by the analysis in Section 2, when $\Delta\boldsymbol{d}$ changes sign, the game is a Snowdrift game. Just as with standard charity good, when all players defect, a single player who switches to cooperating does not forego any benefits as no value of the collective good is produced, but obtains a benefit $-c$. It follows that the game cannot be a Prisoner's Dilemma; therefore, if $\Delta\boldsymbol{d}$ does not change sign, the game is a Harmony Game.

Again taking the case of two players as a starting point to look at the effect of the degree of complementarity, we immediately note that the case of two players is indistinguishable for the charity good and the congestible charity good. This is because a fixed benefit is obtained when contributing. Moreover, when one player contributes and the other receives, with a congestible charity good, the value of the charity good is divided over one player, so that we get the same value as with a non-congestible charity good. Finally, it continues to be the case that when no player contributes, both players obtain zero. Therefore, we continue to obtain that as the degree of complementarity is increased, the game switches from a Snowdrift game to a Harmony Game, leading to a common-enemy effect.

When looking at the effect of increases in the degree of complementarity for generic n, it is clear that the direction of the effect of w on d_k is the same, whether or not the charity good is congestible. The only difference with Fig. 6 (charity good), is that because of congestion the benefits foregone when a low number of players cooperates are smaller because of congestion. We therefore immediately move to Proposition C.1, which shows similar results as for standard charity goods.

Proposition E.1. *Consider the production of a* ***congestible charity good*** *with $n \geq 2$. Then for any impact function I_k that strictly increases in k and for which $\partial I_k/\partial\alpha \leq 0$, if an interior fixed point x^* exists, we have a Snowdrift game (the fixed point is stable), and it is the case that $\partial x^*/\partial\alpha \geq 0$. Specifically with impact function β_k in (5), when $w < w_{n-1}$, the game is a Snowdrift game and $\frac{\partial x^*}{\partial w} > 0$ (common-enemy effect); when $w \geq w_{n-1}$, the game is a Harmony Game. It is the case that $w_{n-1} \lesseqgtr 1$ for $\gamma \gtreqless (n-1)V/n$.*

Proof. For generic I_k, given that $d_k = -I_k/(n-k) - c$, and given that I_k strictly increases in k, and given that furthermore more cooperating players means less congestion among recipients, it follows that $\Delta\boldsymbol{d} < 0$, with the rest of the analysis qualitatively identical to the one for standard charity goods (cf. Result 3.2(b) in Peña et al. (2014)).

For $I_k = \beta_k$, using (2), and plugging in the value for d_k calculated in Section 3 for the congestible charity good, for $w \neq 1$, it follows that

$$g(x) = \sum_{k=0}^{n-1} \binom{n-1}{k} x^k (1-x)^{n-1-k} \left[-\frac{V\left(1-w^k\right)}{(n-k)(1-w^n)} - c \right]$$

Using the binomial theorem, this is re-expressed as:

$$g(x) = -c - \frac{V}{1-w^n} \sum_{k=0}^{n-1} \binom{n-1}{k} x^k (1-x)^{n-1-k} \frac{\left(1-w^k\right)}{(n-k)} \tag{E.1}$$

Note now that $\binom{n-1}{k}\frac{1}{(n-k)} = \frac{1}{n}\binom{n}{k}$. Using the fact that $\sum_{k=0}^{n}\binom{n}{k}x^k(1-x)^{n-k} = 1$, so that $\sum_{k=0}^{n-1}\binom{n}{k}x^k(1-x)^{n-k} = 1-x^n$, we can write

$$\sum_{k=0}^{n-1} \binom{n}{k} x^k (1-x)^{n-1-k} = \frac{1}{1-x} \sum_{k=0}^{n-1} \binom{n}{k} x^k (1-x)^{n-k} = \frac{1-x^n}{1-x}$$

We conclude that

$$\sum_{k=0}^{n-1} \binom{n-1}{k} x^k (1-x)^{n-1-k} \frac{1}{(n-k)} = \frac{1-x^n}{n(1-x)} \tag{E.2}$$

In the same way, note that $\sum_{k=0}^{n}\binom{n}{k}(wx)^k(1-x)^{n-k} = (wx+1-x)^n$, so that $\sum_{k=0}^{n-1}\binom{n}{k}(wx)^k(1-x)^{n-k} = (wx+1-x)^n - (wx)^n$. It follows that

$$\sum_{k=0}^{n-1} \binom{n}{k} (wx)^k (1-x)^{n-1-k} = \frac{1}{1-x} \sum_{k=0}^{n-1} \binom{n}{k} (wx)^k (1-x)^{n-k} = \frac{(wx+1-x)^n - (wx)^n}{1-x}$$

We conclude that

$$\sum_{k=0}^{n-1} \binom{n-1}{k} (wx)^k (1-x)^{n-1-k} \frac{1}{(n-k)} = \frac{(wx+1-x)^n - (wx)^n}{n(1-x)} \tag{E.3}$$

Using (E.2) and (E.3) and substituting into (E.1), it follows that:

$$g(x) = -\frac{V}{1-w^n}\frac{(wx)^n + (1-x^n) - (wx+(1-x))^n}{n(1-x)} - c \text{ for } w \neq 1 \tag{E.4}$$

Using (2), and plugging in the value for d_k calculated in Section 3 for the congestible charity good for $w = 1$, it follows that

$$g(x) = \sum_{k=0}^{n-1}\binom{n-1}{k}x^k(1-x)^{n-1-k}\left[-\frac{kV}{n(n-k)} - c\right]$$

which using the binomial theorem can be written as

$$g(x) = -\frac{V}{n}\sum_{k=0}^{n-1}\binom{n-1}{k}x^k(1-x)^{n-1-k}\frac{k}{n-k} - c$$

Note now that $\binom{n-1}{k}\frac{k}{n-k} = \frac{(n-1)!}{k!(n-1-k)!}\frac{k}{n-k} = \frac{(n-1)!}{(k-1)!(n-k)!} = \binom{n-1}{k-1}$, so that g(x) can be rewritten as

$$g(x) = -\frac{V}{n}\sum_{k=1}^{n-1}\binom{n-1}{k-1}x^k(1-x)^{n-1-k} - c \tag{E.5}$$

Also, $\sum_{k=1}^{n}\binom{n-1}{k-1}x^{k-1}(1-x)^{n-1-(k-1)} = \frac{1-x}{x}\sum_{k=1}^{n}\binom{n-1}{k-1}x^k(1-x)^{n-1-k} = \frac{1-x}{x}\sum_{k=1}^{n-1}\binom{n-1}{k-1}x^k(1-x)^{n-1-k} + x^{n-1} = 1$.

It follows that

$$\sum_{k=1}^{n-1}\binom{n-1}{k-1}x^k(1-x)^{n-1-k} = \frac{x\left(1-x^{n-1}\right)}{1-x} \tag{E.6}$$

Plugging (E.6) into (E.5), we obtain that

$$g(x) = -\frac{Vx\left(1-x^{n-1}\right)}{n(1-x)} - c \text{ for } w = 1. \tag{E.7}$$

(E.4) and (E.7) cannot be solved for x^* explicitly, and therefore x^* is only implicitly given by the value of x such that the right-hand side in (E.4) and (E.7) equals zero. Still, we know that $g(0) = -c + \delta_0 = -c$, and that $g(1) = -c - \frac{V(1-w^{n-1})}{1-w^n} = -c + \delta_{n-1}$. Given that δ_{n-1} equals $-V$ for $w = 0$, given our assumption that $V > -c > 0$, and given that δ_{n-1} increases in w, we conclude that for any considered $-c$, we can always find a sufficiently small w such that $g(x)$ changes sign from g(0) to g(1). It follows that, for $w < w_{n-1}$(where w_{n-1} denotes the level of w such that $-\delta_{n-1} = -c$), an interior point exists. For such an interior point, just as is the case with a charity good, given the fact that $\partial \boldsymbol{d}/\partial w > 0$, it follows that $\partial g(x)/\partial w > 0$, which along with the fact that $\partial g(x)/\partial x < 0$ means that $\partial x^*/\partial w > 0$ (see Section 2); this again implies a common-enemy effect where an increase in w increases the ESS fraction of cooperating players. □

Appendix F: List of symbols used, and list of terminology used

Table F.1–F.2.

Table F.1
List of Symbols Used.

Symbol	Definition
A	strategy of cooperating
a_k	cooperator's fitness when k other group players cooperate
B	strategy of defecting
b_k	defector's fitness when k other group players cooperate
$\mathcal{B}_{a,k}$	by-product benefit the focal player obtains from the fact that another player cooperates rather than defects when k other players than the focal player currently cooperate, and when the focal player cooperates himself (equal to $a_{k+1} - a_k$)
$\mathcal{B}_{b,k}$	by-product benefit the focal player obtains from the fact that another player cooperates rather than defects when k players other than the focal player currently cooperate, and when the focal player defects himself (equal to $b_{k+1} - b_k$)
c	cooperation costs (assumed negative for charity goods)
$\mathcal{C}_k$	net cost of cooperating rather than defecting when k other players cooperate (equal to $-d_k$)
$c^*(w, n)$	level of cooperation costs separating large from small cooperation costs
d_k	gain the focal player makes by switching from defecting to cooperating when k other players cooperate
D_k	taking as a starting point the case where a total of k players in a group cooperate, the sum of the gains the n players obtain when one player switches from defecting to cooperating (equal to $d_k + k\mathcal{B}_{a,k-1} + (n-k-1)\mathcal{B}_{b,k}$)
$\boldsymbol{d}$	gain sequence, or $(d_0, d_1, \ldots, d_{n-1})$
Δd_k	first forward difference of d_k, or $d_{k+1} - d_k$
$\Delta\boldsymbol{d}$	sequence of first forward differences, or $(\Delta d_0, \Delta d_1, \ldots, \Delta d_{n-2})$
$f_A(x)$	fitness of cooperating
$f_B(x)$	fitness of defecting
$g(x)$	gain function, equals $f_A(x) - f_B(x)$
I_k	impact function (=value of collective good when k group players cooperate)
n	number of players in group
P	punishment payoff when $n = 2$, payoff when both players defect
R	reward payoff when $n = 2$, payoff when both players cooperate
S	sucker payoff when $n = 2$, payoff cooperating player when other player defects
T	temptation payoff when $n = 2$, payoff defecting player when other player cooperates
V	value of the collective good when all players cooperate
w	degree of complementarity
w_0	degree of complementarity such that $c = \delta_0$
w_{n-1}	degree of complementarity such that $c = \delta_{n-1}$
x	fraction of cooperating players
x^*	fixed point fraction of cooperating players, such that $g(x^*) = 0$
α	degree of adversity
β_k	impact function with specific functional form
δ_k	added benefit (=fitness net of cooperation costs) to individual player of cooperating rather than defecting when k other group players cooperate
θ	fixed benefit from defecting (relevant for commons good)

Table F.2
List of Definitions.

Term	Definition
boomerang effect	effect whereby defecting player becomes victim of own defection, in obtained reduced benefits
charity good	good that is non-congestible, but excludable (only defectors benefit)
club good	good that is non-congestible, but excludable (only cooperators benefit)
common-enemy effect/ hypothesis	effect/hypothesis saying that a harsher environment (higher degree of adversity) makes cooperation more likely
commons good	good that is not excludable, but congestible (referred to as congestible public good in most of the paper)
competing effect/ hypothesis	effect/hypothesis saying that a harsher environment (higher degree of adversity) makes cooperation less likely
congestible	property of a good that the individual players' benefits are affected by number of benefits enjoying the good
degree of adversity	any measure that, when increased,decreases the fitness of an individual, holding the behavior of this individual fixed and of any individuals with whom the individual interacts
degree of complementarity	extent to which the last players to cooperate in a group increase the value of the collective good compared to the first cooperating players, with the value of the collective good when *all* players cooperate fixed
degree of synergy	extent to which the last players to cooperate in a group increase the value of the collective good compared to the first cooperating players, with the value of the collective good when *one* player cooperates fixed
excludable	property of a good that the individual can be excluded from benefiting from the good depending on his strategy
Harmony Game	game with a dominant strategy to cooperate
impact function	function relating number of cooperating players to value of the collective good produced
Prisoner's Dilemma	game with a dominant strategy to defect
private good	good that is both excludable and congestible (equivalent to a congestible club good with a linear impact function)
public good	good that is both non-excludable and non-congestible
Snowdrift game	game where individual player is better off defecting when all players cooperate, and better off cooperating when all players defect
Stag Hunt	game where individual player is better off defecting when all players defect, and better off cooperating when all players cooperate
sucker effect	effect whereby cooperating player benefits less from cooperating

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