DE GRUYTER OPEN

GESTALT THEORY, DOI 10.1515/gth-2017-0005
 (ISSN 2519-5808); Vol. 39, No.1, 30–53

Original Contributions - Originalbeiträge

Jan Koenderink, Andrea van Doorn & Baingio Pinna

Plerosis and Atomic Gestalts

Introduction

"Plerosis" ("fullness") was introduced by Brentano (1974[1874], 1988) in his theory of *points* as parts of the intuitive continuum of visual awareness (Albertazzi, 2002, 2006, 2015). For the purpose of this paper, Albertazzi (2006) offers an especially lucid account, which we invite the reader to consult.

Mathematicians have populated the continuum with points of surprisingly diverse kinds (Bell, 2006), but it remains impossible to see the blue sky as composed of "points" of *any* kind. If there are "points" in visual awareness at all, they *have to be* "boundary objects", because uniform areas appear as undifferentiated wholes. The blue sky does not consist of "points" at all[1]. Phenomenological points are distinct from mathematical points. We consider the nature of phenomenological boundary points, which have *parts*, despite being *atomic*. But parts and atomicity make strange bedfellows. Neither mathematics, nor phenomenology, has ever arrived at a satisfactory account of points.

Brentano's concept perfectly fits the modern concept of "scale-space operators" (Koenderink, 1984). These operators have many properties in common with physiology's "receptive fields" introduced by Hartline in 1938. Because we are dealing with phenomenology here, one should speak of "perceptive fields" (Spillmann, 1971). Psychology adopts a terminology derived from some hypothetical function (such as "edge detector"; Marr & Hildreth, 1980).

The natural interpretation of Brentano's notion of "plerosis" in terms of perceptive fields renders the topic of major interest to the theory of Gestalt. *Perceptive fields are the primordial, "atomic" perceptual Gestalts*.

We first give a summary introduction to Brentano's concept of plerosis. We then explain the formal structure of scale-space operators, which is the formal account

[1] You might object that there are "locations" in the blue sky, which surely are "points"? We agree with the first, though not with the latter. A "location" is a formal reference to some region of interest by means of a reference to a frame (e.g., two approximate real numbers as Cartesian coordinates in the plane) and a scale (e.g., Asia, France, Amsterdam, your backyard, your desk and so forth). In contrast, a "point" is an individual, a thing. A single location might include infinities of "points" – if there indeed are such things as points. If "location" is used as synonymous to "point", it is hard to understand why you hold that there are locations in the blue sky. Can you point them out to us?

of perceptive field structures. Finally, we analyse a simple, paradigmatic visual Gestalt, the *square* (Pinna & Albertazzi, 2011; Pinna & Grossberg, 2005), in terms of the plerosis of its boundary points.

Brentano's Notion of "Plerosis"

Points occur both in synthetic geometry, such as Euclid's plane, and in topological accounts. Euclid (1956 [fl. 300 BCE]) faces the problem of the continuum several times in the *Elements*. He often skips problems as in Book I, Proposition 1: "To construct an equilateral triangle on a given finite straight line", where he assumes a point of intersection of two circular arcs to *exist*. Hilbert's (1980 [1899]) circle–circle intersection property corrects this "oversight".

Consider Fig. 1 (left), does point **P** exist? To the contemporary mathematician, it intuitively would be a miracle if it did. In Riemann's (1854) spaces, it usually does not. Empirically, it does not exist in human visual space awareness (Koenderink et al., 2010). The problem involves ideal planes, lines and points, whereas intuition deals with tangible (volumetric) entities. Euclid can only be "saved" by adding an axiom, as done by Pasch (1912[1882]); Fig. 1, left).

Euclid also addresses the continuum directly, as in "Magnitudes are said to have a ratio to one another which can, when multiplied, exceed one another" (Book V, Definition 4). This "Archimedean Property" (term due to Stoltz (1883), refer Ehrlich, 2006; Fig. 1, right) plays a key role in formal accounts. It implies that any point is just a number (maybe a trillion, but some number) of steps away from any other point.

The continuum has a venerable history. Paradoxical features of the continuum as compared to the discretum[2] were evident to the early Greek philosophers (Bakalis, 2005; Bell, 2006). Unlike the discretum, the continuum has no gaps. That it can be divided without limit renders it a labyrinthine jungle. Fully invariant with respect to scale, no microscope reveals a "rock bottom". Therefore, the continuum is cohesive like a viscous syrup (Bell, 2009; van Dalen, 1997).

Euclid's point is "that which has no part" (Book I, Definition 1). A point cannot divide a line, because it would have a left and a right side. Hence parts of continua cannot be points. According to Anaximander, "things in the one world-order are not separated one from the other nor cut off with an axe" (Bell, 2006). Likewise, Democrites argues that if one can break a stick, it was from the outset not whole (Weyl, 1925, p. 135). If points have no parts, "boundary points" are nonentities.

[2] A model for the discretum is a pile of coins. Here, each coin remains an individual. It has two sides and is just some number of steps away from any other coin. In contrast, points are not "individuals" in the sense of coins. Two distinct points are not some number of steps away from each other. The notion that a point might have two sides is a hard problem that Brentano was struggling with.

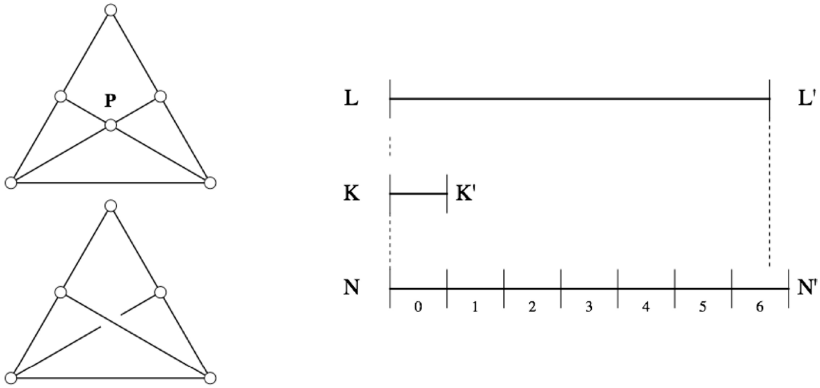

Fig. 1 *Left:* At top is the silent assumption of Euclid that the point of intersection **P** *exists*. If this appears "self-evident", look at the bottom figure. Pasch "saved Euclid" by adding an axiom. Intuitively, Pasch's doubt appears reasonable, for one readily imagines that the two lines inside the triangle will cross instead of intersect. Empirically (Koenderink et al., 2010), a point such as **P** does not exist in human visual space. *Right:* Euclid assumes that the linear segment KK' will eventually exceed the linear segment LL' (here after six additions, leading to NN'). This assumes the Archimedean Principle, essentially that KK' and LL' are *comparable*. We offer an example in footnotes 4 and 5 of a number that, arbitrarily often added to itself, will never exceed one. Intuitively, *any* two segments that are both *visible* seem *comparable* to us. However, Veronese (1894) uses arguments from visual intuition to explain the non-Archimedean continuum. Intuitions vary.

Brentano departed from the extensive account given by Aristotle (refer Roeper, 2006). Most important for this paper, he holds that the boundary between two parts of a line belongs *equally to either part*. But how can *one* be *two*?

Brentano concentrates on *coincident boundary points* as a reaction to Dedekind's (1872) interpretation of the real numbers. The Pythagoreans discovered that the length of the diagonal of the unit square (√2) cannot be expressed as a rational number. This was shocking to them, because it implies that you cannot indicate a point as being so many paces (of some given size) from a given point. Instead, not all points can be "pointed at" or constructed. It poses a threat to their very existences.[3] The rational numbers fail to "fill" the number line. Dedekind filled these "gaps", defining √2 *as* a gap, a sleight of hand that Brentano considered intuitively absurd. Why? Well, as Gödel (1960 [1947]) mentions, because of "*something like perception*".

[3] Of course, √2 can be approximated arbitrarily well by a rational number, for instance, the fraction 17/12 is less than 1% off. But that is not the issue. Why is √2 not rational anyway? Well suppose it is, then there are two natural numbers n, m (say), not sharing a common factor, such that $\sqrt{2} = n/m$. This would imply $n^2 = 2m^2$, thus n^2 is even, hence n is even, and consequently m is odd (n and m have no common factor 2 by assumption). Since n is even, there is a natural number k such that $n = 2k$, which implies $m^2 = 2k^2$, thus m^2 is even, hence m is even. But m cannot be both even and odd, so the initial assumption must be wrong, implying that √2 is not a rational number. This proof hangs on the Law of the Excluded Third. The irrationality of √2 was discovered long before the Common Era.

Consider the gap between 0.999… and 1.000…, in the decimal notation of Stevin (1585). The "…" indicates that these numbers are only *in statu nascendi*, a notion exploited in the form of "choice sentences" by another Dutchman, Brouwer (1918), who dealt a severe blow to the self-confidence of pre-twentieth century mathematicians. Brouwer fills the gaps with "mystery", since it is up to fancy to carry on the "…" *ad libitum*. Brouwer's continuum is so viscous that you may even delete all rational numbers without causing it to fall apart (van Dalen, 1997). Anaximander would nod approvingly.

A way to fill the gaps are the *nil-square infinitesimals e*, the non-trivial solutions of $e^2 = 0$ (Giordano, 2001).[4,5] Infinitely many of these exist, whereas their sign is *not decidable*. Thus, any number is enveloped in a cloud of numbers that are neither larger nor smaller than it!

Remarkably, Brentano considered Dedekind's number-theoretic exercises empty sophistry, because *the continuum is given directly in intuition:*

> "… I affirm that… the concept of the continuous is acquired not through combinations of marks taken from different intuitions and experiences, but through abstraction from unitary intuitions... Every single one of our intuitions—both those of outer perception as also their accompaniments in inner perception, and therefore also those of memory—bring to appearance what is continuous" (Brentano, 1988, p. 6).

Intuitively, no notion of "adjacent points" is required because it is natural enough to intuit distinct points as *coincident*. Brentano illustrates this with a quote from Galileo Galilei:

> "It is commonly believed that if four different-coloured quadrants of a circular area touch each other at its centre, the centre belongs to only one of the coloured surfaces and must be that colour only. Galileo's judgment on the matter was more correct; he expressed his interpretation by saying paradoxically that the centre of the circle has as many parts as its periphery" (Brentano, 1974[1874], p. 357).

[4] Consider the nil-square number *e*, a non-trivial ($e \neq 0$) solution of the equation $e^2 = 0$. It is clearly very small, near to zero. Yet, it is not zero by construction. Does it lie to the left or to the right of zero on the number line, or – more formally – is *e* negative or positive? Suppose that $e > 0$, then $e^2 > 0$, which is false by assumption. Now suppose $e < 0$, then $e^2 > 0$ too, which is again false by assumption. Thus, *e* lies neither to the left nor to the right of zero, yet it is different from zero. Again (note 2), the proof hangs on the Law of the Excluded Middle.

[5] The "nil-square infinitesimals" (introduced in footnote 4) offer a simple example of numbers that do not conform to the Archimedean Principle. Let *e* be defined as a non-trivial solution (meaning $e \neq 0$) of the equation $e^2 = 0$. If it were Archimedean, there should exist a natural number *n* such that $(n\ e)^2 > 1$. But $(n\ e)^2 = n^2\ e^2 = 0$ (because $e^2 = 0$ by assumption), thus contradicting the initial assumption. Thus, there is no such natural number *n*, hence *e* violates the Archimedean Principle. Notice that the proof hangs on the Law of the Excluded Middle.

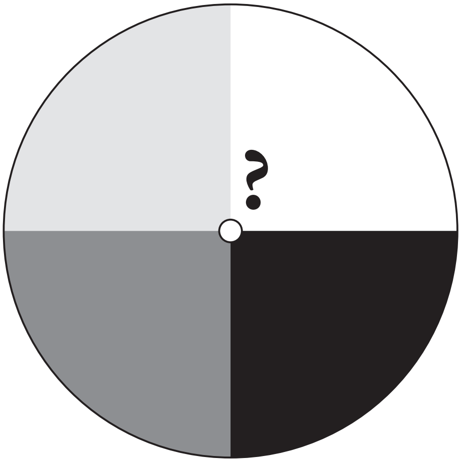

Fig. 2 Galileo's circle. The disk has been painted white, light grey, as well as grey and black. Any location receives just one colour. What is the colour of the centre? It is easy enough to think of half a dozen reasonable answers to that question. Brentano's – surprising to many – answer is that the centre has *all four colours*. It has a fourfold plerosis or can be said to consist of four coincident points.

Notice that the centre of the circle is a "point"[6] that apparently *has several colours simultaneously* (Fig. 2).

These views of Brentano closely resemble those of the Veronese (1894), whose approach was geometrical, rather than number-theoretical. In Veronese's view, it is our imagery that supplies us with basic geometric objects. Veronese's disapproval of the mainstream (forcefully led by Georg Cantor, who lived between 1845 and 1918) constructions derives from similar intuitions as Brentano's.

Thus, Brentano arrives at his notion of *plerosis*. He writes: "The spatial point cannot exist or be conceived of in isolation. It is just as necessary for it to belong to a spatial continuum as for the moment of time to belong to a temporal continuum." (Brentano, 1974[1874], p. 354).

Indeed, the now is just as much part of the past as it is of the future. Think of Husserl's (1991, sec. 40) retention and protention. Saint Augustine (Book XI; 397–400 CE, see Henry Chadwick, 1992) already had much the same intuition. The moment "now" is a Janus-faced entity that has a two-sided plerosis (Koenderink,

[6] Why the scare quotes? Well, what is meant by "point" here? What is "the centre" like? Is it a mere location or is it something? A point would be something, but a location is nothing but an indication of some whereabouts. A location is a necessarily vague region of interest as indicated by such conceptual crutches as coordinates and scale, which are references to some pre-established frame. Here, the location is "the centre of the circle". The issue of whether there is some special point of that region of interest is left open here.

2002). In that sense, Brentano's "points" have parts, but only because these are "coincident points".

The above quote by Brentano may be taken as his definition of "point" and "plerosis". It is a definition just as much as a research proposal. This makes sense, because Brentano is talking phenomenology, which is an empirical science. In formal endeavours, like logic or mathematics, one defines one's concepts before using them. In the empirical sciences, it is the other way around. Concepts are not defined but acquire an intuitive meaning through use. (A good explanation is seen in the paper by Feynman, 1966.) It is also how we approach the problem of making sense of Brentano's notion of "plerosis". After considering Brentano's uses, we attempt to extend the notion in various directions in order to see how far it takes one. Any concept of importance should have the potential for development or should be replaced. It is how Brentano approached such matters.

Scale-Space: the Proper Formal Substrate of the Visual Field

The "visual field" is the common substrate of intensive qualities, such as colour, and also apparently extensive qualities, such as direction or orientation. A direction is defined by two distinct points. Consider Euclid "The extremities of lines are points" (Book I, Definition 3) and "To draw a straight line from any point to any point" (Postulate 1). A direction is thus an *extensive* object. But when the points are distinct, *yet coincident*, the extension becomes nil, only the direction survives as a *trend*.

Brentano regards the continuum of directional trends as a secondary continuum whose manifestation is dependent on the primary continuum of places. He offers the example of a curved line, whose direction changes continuously from point to point: "In the double continuum that presents itself to us in the line, it is this continuum of directions that is to be referred to as the secondary, the manifold of differences of place as such as the primary continuum." (Brentano, 1988, p. 21).

This is essentially the modern view of differential geometry, the continua of places and directions together subtend the "tangent bundle" of the visual field. A directional trend is the "derivative of a point", essentially the difference of two points, normalised by their common separation, in the limit of coincident points. This intuition of Cartan (1923) has proven to be extraordinarily fruitful in theoretical physics (Misner, Thorne, & Wheeler, 1973).

To formalise Brentano and Veronese's "intuitions" one needs to consider the psychogenesis of visual awareness from phenomenological and biological perspectives. We follow Brown (1977), based on the phenomenology of mental disorders. Brown recognises that qualities and meanings cannot be computed from optical structure – "vision as inverse optics" (Marr, 1982) – but ultimately derive from autogenous processes of the core self. Thus, *vision is controlled hallucination*. The primary visual system of physiology (cortical area V1 in man) is interpreted as a "blackboard" on which abstracted optical structures are continually overwritten; it is in no way a "neural centre of consciousness". Psychogenesis adapts to the structure of the *Umwelt* by checking its imagery, like von Uexküll's (1909, 1920) "seek images", to what is on the blackboard. "Action" is a probing *from the inside out*.

Formally, geometrical objects such as points occur in two mutually complementary ways (Koenderink, van Doorn, & Pinna, 2015; Koenderink, van Doorn, Pinna, & Wagemans, 2016). In filling the blackboard, points enter as *operators* on optical structure. Given a retinal irradiance distribution, a point yields a sample value that represents the irradiance at the location of the point, averaged over the extent (*vide infra*) of the point. This defines a point as a *receptive field* in the sense of neurophysiology. In imagery, points enter as *brushes* in the sense of "brush" as familiar from image processing applications such as Photoshop.[7] A point contributes a "touch" – as in a painting – to the "canvas" of the awareness (Koenderink, van Doorn, & Pinna, 2015; Koenderink, van Doorn, Pinna, & Wagemans, 2016).

Biological fitness requires psychogenesis to "account for" the blackboard data: this is "controlled hallucination". Thus awareness will be adapted to the life world: the tiger and the lamb have different visual awarenesses for the same optical structure.

The different notions of point meet at the stage where psychogenesis probes the blackboard. The *brush* is like a question, to which the result of an *operator* may yield an answer. Psychogenesis seeks for *resistance* to probing; it is "corrected by the world" where imagery fails to account for blackboard content. Thus, imagery adapts to the *Umwelt*, which subserves biological fitness. The *meaning* of answers is in the *questions*. Thus, mere *structure* in the blackboard is turned into *meaning* in visual awareness. The key insight that intentional probing creates awareness is due to Schrödinger (1944), though it is implicit in von Uexküll's (1920) notion of the "new loop".

Is it paradoxical to speak of the "extent" of a point? It seems so at first blush. But remember Euclid's "a point is that which has no part". Euclid never says a point should have a particularly small size. Consider a geographical atlas. Paris is *a point*

[7] Photoshop is a computer application used by many people to edit their photographs or images. For people not familiar with it the notion of a painter's "touch" might be more useful. Where a receptive field as considered in neurophysiology is a sampler that performs a local measurement, the "touch" does not sample or measure, but puts something were initially was nothing but blank canvas or paper. Think of the dot made by a pencil on paper, or a little stroke of paint on canvas. Here we consider something tangible (a visible dot of some sort) in visual awareness.

on maps of the Northern hemisphere, because *one chooses to ignore its parts*. The point as operator does exactly that: it yields only an intensive value. The optical structure inside the extent of the point is fully lost in the averaging over it. Operators necessarily have finite extent, because "the irradiance at a point of zero extent" is physically absurd. A point must have *some* finite size, *any* size except none (Kandinsky, 1926). A general formalism avoids any *specific* choices. All sizes are equal! This leads to the notion of "scale-space" (Florack, 1997; Koenderink, 1984), the de facto formal basis of modern image processing (ter Haar Romeny, 2003).

In this paper, we refer to the formal scale-space operators either as such, or as "receptive fields", for which they are models. We may also use the functional denotations "edge detector", "line detector" and so forth, as common in neurophysiology and artificial intelligence.

In scale-space formalism, the receptive field of a point is uniquely characterised as a Gaussian weight (Florack, 1997; Fig. 3, left). Its weight is concentrated in a finite region, but nowhere vanishes. Thus, all points overlap with each other everywhere, and one obtains a very viscous structure. Since the point operators are explicitly represented as scalar functions of location, it is immediate to implement abstract intuitions. Cartan's (1923) "directional derivative of a point" can be executed right away by differentiating the weight function, yielding the "edge detector" (Florack, 1997; ter Haar Romeny, 2003; Fig. 4, left). Differentiating once more yields Hubel and Wiesel's (1968) "line detector" (Fig. 4, right). This identifies the receptive fields of physiology as scale-space operators.

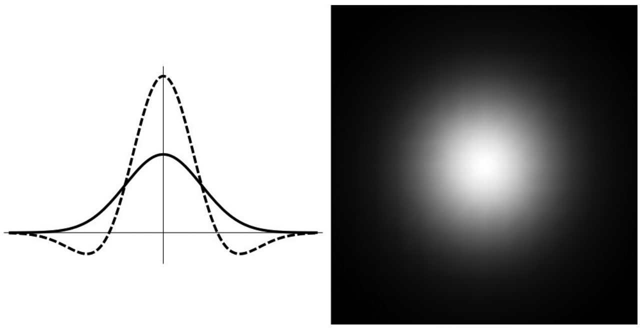

Fig. 3 At left, 1D profiles of a "point" (drawn) and a "location" (dashed) at the same scale. When considered as perceptive entities, the point has a blotch of colour (figure at right), whereas the location does not, because its total weight is zero. The point needs a background, whereas the location does not because it has the local contrast. When considered as receptive fields, or scale-space operators, the location is known as the "Laplacian". The Laplacian receptive field is the average of line detectors (Fig. 4, right) over all orientations. Like the point, the plerosis of the location is isotropic, what Brentano calls "full".

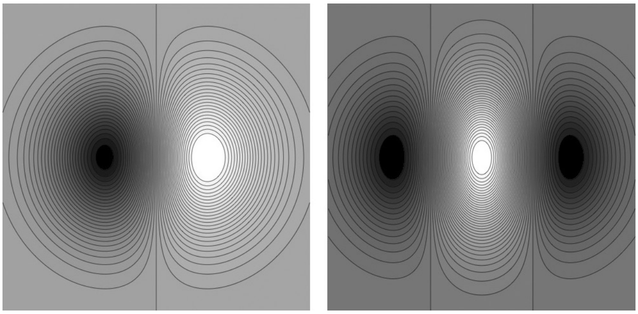

Fig. 4 At left, we show the "edge detector" and at right, the "line detector" receptive fields. Here, we use contour plots, which makes it much easier to understand the structure. The edge detector occurs for all directions and the line detector for all orientations. Another interpretation of these figures is in terms of perceptive fields. Then, the edge brush (left) is a typical boundary point with twofold plerosis in Brentano's sense. At the literal coincidence of two points, it is also *atomic*, just like the point.

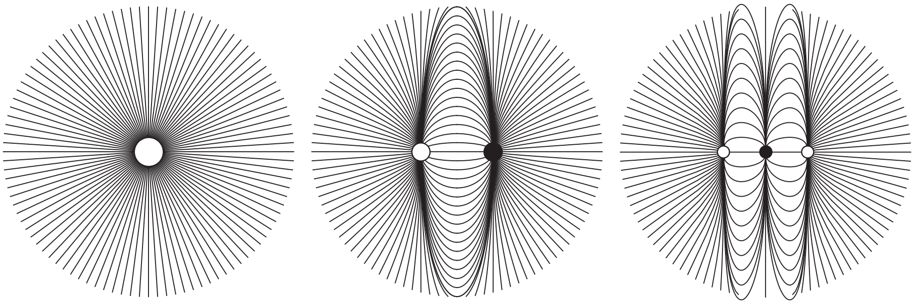

Fig. 5 At left, the "full plerosis" of a generic point, which faces all directions equally, is shown. At the centre, the plerosis of an atomic "edge point" is shown. It has twofold plerosis. The atomic "line point" at the right has twofold plerosis too, but of a different kind, because it is bilaterally symmetric, whereas the "edge point" is antisymmetric. These pictures show a "microscopic view"; at a macroscopic view, the orbits that are "internal to the atom" are not visible.

Brentano's plerosis considers the areas "faced" by the point. An edge point faces two abutting areas, half the rays departing from the point goes into one area, the other half into the other area. This is formalised in the scale-space formalism. One shoots "rays" from a small area about the point to all directions (Fig. 5, left). As "rays", it is natural to use the streamlines of the gradient of the receptive field profile (Fig. 5, centre and right). One finds the plerosis from counts of where the rays end up. This allows a measure of plerosis. Something similar applies to boundary points that are parts of larger, grouped entities (Fig. 6).

The scale-space formalism of point *operators*, or receptive fields, may double for the scale-space theory of *brushes*. But, although formally equivalent, brushes

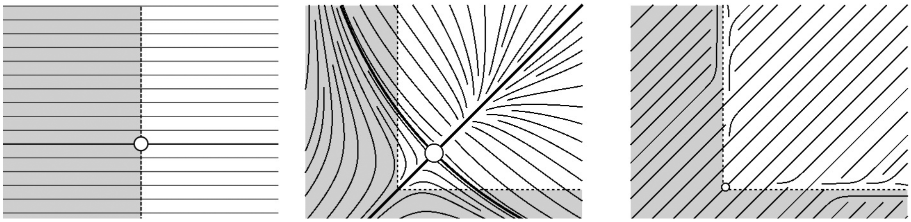

Fig. 6 At left, the plerosis of an edge point is shown, and at the centre and right, that of a corner point is shown. These are not isolated similar to the entities in Fig. 5, for instance, the edge point belongs to an extended edge. Here, the "rays" are streamlines of the gradient of the grey tone relative to the grey tone of the point. These points do not have "full plerosis"; they face only two singular directions. Notice that the "corner point" lies slightly on the inside of the "ideal" corner (centre). The notion of "plerosis" becomes intricate for cases like this. Brentano's notion – ignoring scale – seems simpler, but is actually trickier, because infinite resolution does not exist. At right, it is obvious that the corner point has a split plerosis in the ratio 1:3. (The centre and right figures show the same configuration; in order to intuit the structure, one needs to consider both. The plot at the centre is only a few "point sizes" wide, whereas the one at the right, many point sizes; so one has a micro and a macro view.)

and operators are categorically distinct (Koenderink, van Doorn, & Pinna, 2015; Koenderink, van Doorn, Pinna, & Wagemans, 2016). When we consider elements of imagery, we will use "brush" as described by a formal expression, or "perceptive field" as used in psychology. Psychologists also use functional denotations such as "edges", "lines" and so forth, which refer to specific perceptive fields.

What is interesting about Brentano's plerosis is that one has "multi-point properties" that are *atomic* (Fig. 2), due to the existence of "coincident points". This is Brentano's key insight. Atomicity *implies a Gestalt nature*. Atoms have no parts; they are necessarily wholes, thus Gestalts. *Brentano's boundary points are the primordial Gestalts of visual awareness*.

Non-atomic configurations can be "Gestalts by fiat". These are "atomic" because psychogenesis ignores their structure, like that of Rome on the map of Europe. Scrutiny allows such Gestalts by fiat to be further analysed. An example is the square, a good Gestalt, which has various points of diverse plerosis: full plerosis for the body, two-sided but equal plerosis for the edges as well as a more intricate type of two-sided plerosis for the corners.

The Whole and Its Parts I: Filling the Blackboard

Consider a white square on a black background (Fig. 7). It is a Gestalt with good *Prägnanz* (Wertheimer, 1922, 1923), it certainly appears as a whole. But equally certain, scrutiny reveals "parts", such as edges and corners. The square must be

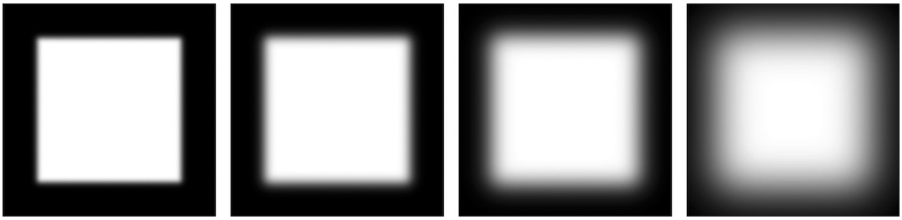

Fig. 7 A white square at various point sizes; the points grow in size by factors of two from left to right. This is only part of a series from "perfectly sharp" to "totally blurred".

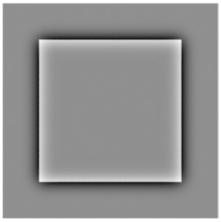

Fig. 8 This is the difference between the square and a blurred version of it, showing the articulation at a limited range of point sizes. (In the limit of an infinitesimal range, one obtains the Laplacian.) This leaves only the boundary articulation. The *macchia* is lost, because the centre and the background have equal tones. Consequently, you cannot see the colour of the square. However, you can still see the contrast with its background. It still looks like a "light square" to us, an instance of Pinna's "water colour illusion" (Pinna, 2008; Pinna, Brellstaff, & Spillmann, 2001; Pinna & Deiana, 2015; Pinna & Reeves, 2006; Pinna, Spillmann, & Werner, 2003).

considered a fiat Gestalt. We investigate the relation of the whole to its parts both from the perspective of *sensing,* i.e., signal detection, and of *imagery*. It is important to distinguish these dual perspectives on "the same" subject matter (here, "a white square") at all times.

The white square possesses at least a location, an extent and a colour. The extent and colour are left when the square is viewed with a point-size of about the square's size. This leaves the background, which is vital to the existence of the white square, for *white* backgrounds annihilate it! The background is *part of* the square. A Laplacian operator (Fig. 3, left) removes the overall background, leaving the contrast at the edge (Fig. 8). This analysis reveals the square as a *macchia* (blob of colour, *tâche*; see Boime, 1993; Broude, 1987; Imbriano, 1868; Panconi, 1999) characterised by location, extent and colour contrast. The figure (as defined by a sharp outline say) and the *macchia* have only a tenuous relation (Fig. 9), as is evident from the phenomenological fact that it is unnecessary to "paint within the lines". The charm of many works of the visual arts derives from intentional "sloppiness".

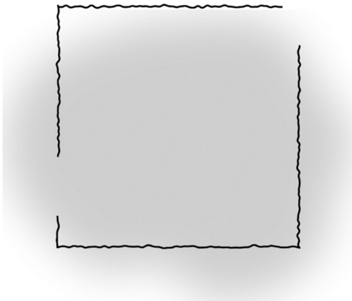

Fig. 9 A grey square. The square has been drawn in black outline, to which the grey *macchia* was added. Notice that this latter "painting" does not take any notion of the squareness, but only of location and size. Yet, this is evidently a "grey square". Instead of the outline, one might use four corners, etc. Nor need the outline be very precise, or even closed.

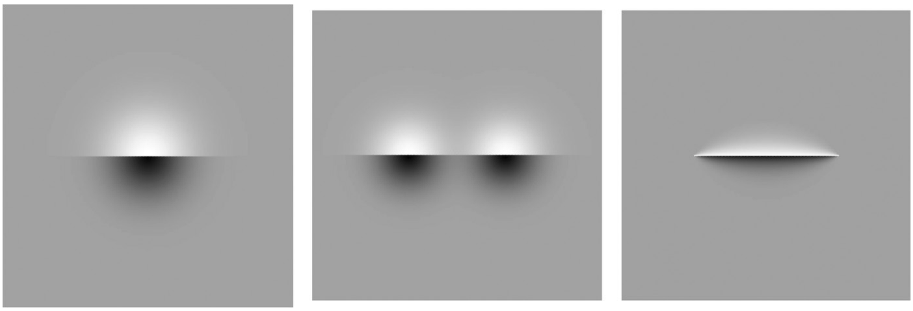

Fig. 10 From left to right: a local edge: bilateral plerosis, two collinear local edges and a continuous grouping of local edges, i.e., a global edge.

An "edge brush" is a bilocal object; it has a symmetrical two-sided plerosis (Fig. 4, left; Fig. 5, centre; Fig. 10, left). Edge brushes reveal the sides when their size is appreciably less than the size of the square. Coincident edge brushes with the same directionality but different sizes will react similarly to the sides, because the white square is "ideal" and has "true", i.e., infinitely sharp, edges. Edge detector activity does not "look like an edge" but is mere *edginess* (Fig. 11, left and centre). In order to obtain something that indeed *looks* like an edge, one has to paint with edge brushes, using the edginess to determine the weight of the touch. Combining such "edge touches" at many scales, one obtains local edge imagery (Koenderink, van Doorn, & Pinna, 2015; Koenderink, van Doorn, Pinna, & Wagemans, 2016). It is a typical Brentano-type two-sided boundary point. Multi-scale edges are the bread and butter of painting (examples and discussions can be found in Cateura, 1995; Jacobs, 1986).

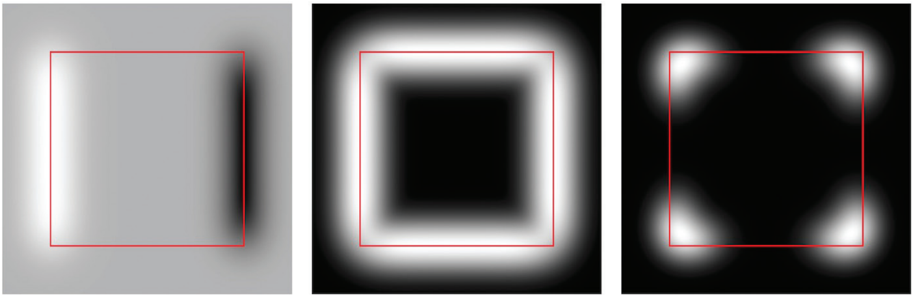

Fig. 11 From left to right: vertical edginess (note polarity), overall squared edginess [typical result of running "edge detection" in an application such as Photoshop (2014)] and cornerness (result from a "corner detector").

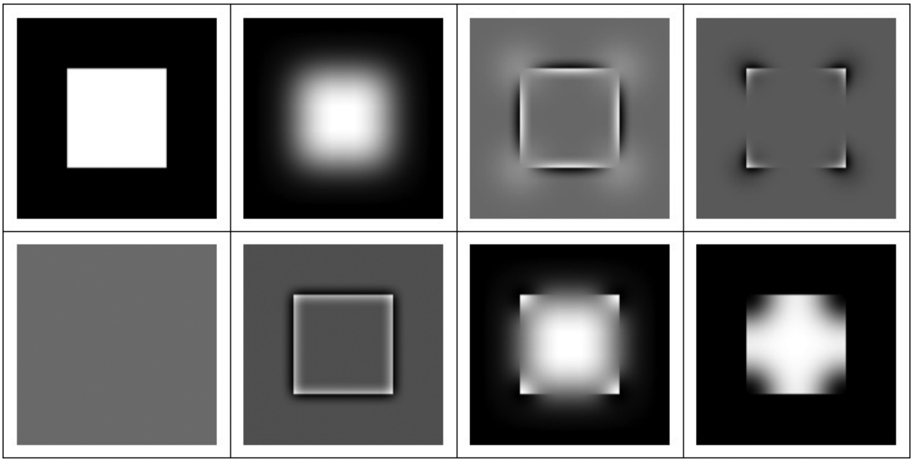

Fig. 12 At top, from left to right, white square, macchia only, edges only and corners only. At bottom, from left to right, nothing (no macchia, no edges and no corners), no macchia, no edges and no corners.

Mutually collinear edge touches naturally group into global edge imagery (Fig. 10). A continuous series of them groups (Metzger, 1936) into awareness of a side. There are four of such groups in a particular configuration. The latter group of four yields a somewhat weak impression of a white square, so does the white square with its corners removed (Fig. 12).

Edge curvature is obtained from a certain differential invariant involving a mixture of edge and line detectors (Koenderink, 1988, 1990, 1993; Koenderink & Richards, 1988; Koenderink & van Doorn, 1990, 1992a, 1992b, 1997). Such a compound operator makes a "corner detector" (ter Haar Romeny, 2003). Coincident corner detectors with the same directionality but different sizes will react similarly to the corners, because the white square is "ideal" and has "true", i.e., infinitely sharp, corners. The corner detector activity does not "look like a corner"; it is mere *cornerness* (Fig. 11, right). In order to obtain something that indeed *looks* like a corner, one has to paint with corner brushes, using the cornerness to determine the weight of the touch. Combining such "corner touches" at many scales, one obtains corner awareness. The corner is a Brentano-type two-sided boundary point, with split plerosis in a ratio of 3:1.

The four corners immediately group to a good Gestalt (Attneave, 1954), a "partially covered white square". Edges are "implied", perhaps even "amodally present". This Gestalt is reminiscent of the well-known Kanizsa square (and triangle; Kanizsa, 1980). The edges group to a square too, the corners seeming at least "visually implied". Although the *macchia* captures location and colour, it fails to distinguish between a circular disc and a square and "implies" neither corners nor edges.

There exist various other relations that no doubt help define "square-hood". For instance, consider the four bilateral reflection symmetries (Fig. 13).

Edge imagery typically occurs in sets of opposites. In the case of the square, these are the pairs of parallel sides and the pairs of sides that meet at a corner. In such cases, one has an implied medial axis (Blum, 1967), which is an axis of bilateral symmetry. In drawing, artists treat opposite edges as a unit, for instance, in drawing roughly cylindrical or ovoidal anatomical parts (Jacobs, 1986).

Corners also typically occur in sets of opposites. Especially, the case of corners at a diagonal evokes strong intuitions and already suggests a square of rather low *Prägnanz*. In artistic composition, it is common to let two diagonally opposite corners define a "grip" (Dunn, 1995, pp. 36–37).

The Whole and Its Parts II: Psychogenesis of Visual Awareness

Psychogenesis of visual awareness originates as imagery, perhaps based on a seek image (von Uexküll, 1920), that probes the blackboard for evidence of its concrete actuality. It is an investigation like the Game of Twenty Questions

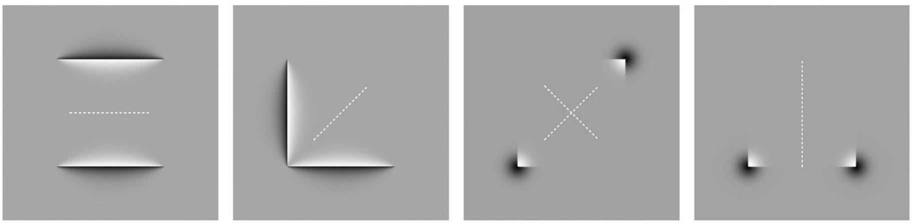

Fig. 13 Medial axes due to bilateral symmetry can be due to a pair of either edges or corners. Here are some possibilities.

(Richards, 1982; Richards & Bobick, 1988), asking questions and probing for answers in the blackboard. We investigate points from this perspective.

The set of optical structures is unbounded, only constrained by the observer's *Umwelt*, whereas the set of possible imagery is limited by the observer's "cue world" and "self world". Imagery fits the optical structure of an infinite set of possibilities quite satisfactorily, although an immensely larger set of possibilities is willy-nilly ignored. There is no fixed map that psychogenesis might possibly apply. Psychogenesis needs to cut down on alternatives very fast in order to let vision be a crucial life-saver. Ethology reveals that animals use very specific coincidences of cues, called *releasers*. A famous example of von Uexküll involves the female tick, which "becomes aware of a mammal" through the coincidence of warmth and the smell of butyric acid. *Effective questioning* aims at cutting down alternatives to a potential actuality in as fast and cheap a way as possible (Richards & Bobick, 1988; Riedl, 1975). Rare coincidences and easy targets are the key to success, as the example of the tick illustrates.

Let psychogenesis consider a white square as a potential actuality. A cheap way to cut down on numerous alternatives is to check for a white *macchia* within a rough range of sizes in the blackboard. If so, it checks whether the *macchia* is roughly isotropic, with full plerosis. This is a good start, because it is cheap. It simply terminates the process when there are no signs of a white square on the blackboard. It is cheap because it is handled in low-resolution mode, greatly decreasing the structural complexity. This may well be sufficient. For example, the selection of suitable males for mating by female stickleback fishes is limited to identifying a red *macchia* within a narrow range of sizes (Tinbergen, 1952). There is no attempt to ascertain the presence of a male more precisely.

Next step is to check for the presence of edges and/or corners. If there are none, the process is terminated: there is no white square. If there are edges, rough statistics on directions may rule out squares, for there have to be two modes of roughly equal size. Corners are also informative; if there are none, the white square seek image fails. If present, their *number* is considered. Four corners suggest squareness, while more corners suggest something else. Do three corners indicate triangularity? Not necessarily, for part of a square might be occluded. How to deal with this depends on ecology and lifestyle. If triangles rarely occur, three corners may well be taken to indicate squareness. The cost for a mistake should be taken into account. Prior notions of probabilities and costs are crucial, as they derive from the life world.

We did not mention specific geometrical properties, but once there is sufficient trust in the existence of a white square in the optical structure, more costly investigation makes sense. This requires complicated and focussed probing of a geometrical nature, mere presence or global statistics being insufficient. Examples include testing for coincidences, collinearities and symmetries. The process terminates when the imagery fits the optical structure "beyond reasonable doubt". Then, white square imagery becomes *manifest actuality*, i.e., visual awareness.

In human visual awareness, the geometrical step is limited to a focal area at fixation and requires a "good look". In eccentric vision, the geometrical analysis is skipped, leading to confusions known as "crowding" (Bouma, 1970). Crowding implies confusion due to excessive structural riches. Eccentric vision serves to select likely regions of interest for the next involuntary fixation. Launching saccades is part of psychogenesis over slightly extended periods. A "glimpse" does not allow for costly geometrical probing, even in focal vision. When the scene is relatively static, a glimpse may be extended to a "glance", or even a "good look". Psychogenesis is well suited to handle different time frames, as there is no need to wait for completion of "inverse optics computations".

There is no "top–down computation" in the blackboard and psychogenesis does not even attempt to compute the presence and geometry of objects – perhaps to be identified as "a white square" – from structure. Such is this a hopeless task, to which "controlled hallucination" offers a viable alternative. It acts similarly to the way a forensic investigator solves a case. It can be implemented in any one of the various ways of "soft computing". For instance, the generic algorithm of "harmony seeking" is perfectly suited (Geem, Kim, & Loganathan, 2001) and has the right biological flavour to it.

"Concrete actuality" is categorically different from "reality". Reality is the environment as seen by the All-Seeing Eye (Koenderink, 2014). This assumes that there is a way the world really is, a notion that seems far fetched. Concrete actuality has the advantage of being well defined. It presents the actual state of the observer's self-world. Since biological observers are by the very fact of their survival overall successful in their behaviour, one may conceive of manifest objects as elements of an "optical user interface" (Hoffman, 2009; Koenderink, 2011). They have an adaptable template character and are akin to Gestalts. Evidence for such template structures in the case of human observers is rapidly accumulating (Koenderink, 2015), whereas ethology teaches us that in the case of animals, it is a basic fact of life.

The interface depends upon the animal's *Umwelt* and lifestyle. From ecology, we learn of many spectacular mistakes involving vertebrates. A toad snaps at match sticks instead of worms, a lapwing goose takes a football for an egg and a stickleback fish female takes arbitrary red objects for males, all behaviours that cannot be explained on the basis of physiology and anatomy. Certainty cannot be had and unnecessary precision is costly. Effective interfaces hide any complexity not worth its cost. In case of innate abilities, the cost of a single individual is negligible from an evolutionary perspective.

The white square in visual awareness is an *ideal object*. It originates from drives in the core self, becomes visual imagery as affordance captured by a seek image and finally objectifies as a white square "out there" when the seek image sufficiently accounts for the current blackboard contents. This is what a "worm" is to a toad. A "worm" is properly described as an elongated tasty object and optically match sticks are in that class. Only new dimensions (e.g., swallow experience) will change the toad's concrete actuality.

As psychogenesis terminates, "white square" imagery has become *manifest* and psychogenesis starts from scratch. In humans, this is a systolic, legato-style process that repeats about a dozen times a second (Brown, 1977).

To believe that the awareness is the causal consequence of a "real white square being out there" is to fall for the All-Seeing Eye delusion. Awareness is a mental *achievement*, an imaginative construction, not something "received".

This *imaginative constructivism* differs from mainstream notions of vision as inverse optics and mind as computation, in that it relies on the notion of a *creative subject*. The connection with the intuitionistic continuum is immediate. Brouwer introduced the "creative subject" in mathematics in the early twentieth century and it is at least dormant in Stevin's (1585) number system.

The Whole and Its Parts

The white square is a prägnant Gestalt by fiat, evidently a "whole". The question arises whether *macchia*, edges, corners and so forth are "parts" of the white square at all.

A visually evident cue to square-hood is a *part* by the very fact that it is such a cue without being the square as such. One might also inquire whether the something is a Gestalt by itself and whether the original object might survive its deletion. This yields diverse perspectives on part-hood.

A *corner* immediately hits the eye and is evidently an important cue to squarehood. An isolated corner appears as a weak Gestalt by itself, and it may certainly be used as an element in painting. One may delete one or even two corners and still be visually aware of a square. In that sense, corners are true parts of the square. They are evidently *atomic* parts.

Something similar holds for *edges*. Isolated edges yield a fairly *prägnant* Gestalt, but deleting all edges, or even a single edge, really hurts a square. Edges may indeed be considered atomic parts.

Isolated edge *points* are good Gestalts, but such a point cannot be seen in an edge as it is lost in the grouping. Moreover, you cannot delete an edge point from an edge (remember Anaximander). One should not count isolated edge points as proper parts of the square – although they may be considered parts of an edge.

The *macchia* is an obvious part (Bijl, Koenderink, & Toet, 1989). It is a good Gestalt all by itself, and the square loses its colour when it is deleted. The square survives as a geometrical entity (a flimsy skin, or a skeleton) and retains its boundary contrast.

Constructing the square from *macchia*, edges and corners (Pinna, 2010, 2012) results in a *whole*. Does it "have" parts? The act of construction suggests *yes*, but phenomenologically, it is like a gin-and-tonic. You mix it, but it is neither gin, nor tonic, yet each contributes to the gin-and-tonic. It eludes mereological account. A white square is a fiat Gestalt and – as long as psychogenesis treats it like that – it is atomic. Lifting the fiat reveals "parts" but destroys the "square".

Experiment

In an experiment (see Appendix), a large group of naive observers selected the best "white square" from each of all 28 pairs of images taken from those of Fig. 12. The group consensus is the order shown in Fig. 14.

This result suggests an order of importance for the various (would-be) parts. However, this should be considered *cum grano salis*, for – although there proved to be satisfactory inter-observer consistency in this experiment – there doubtless exist differences between observers. It seems likely that age (toddler, teenager or adolescent; see Elkind, 1964) and experience (e.g., as artist, designer, carpenter, cartographer and so on) make a difference.

We notice that the observers make a great deal out of the task "*what is most like a WHITE SQUARE?*". There is little doubt that a task like "*what is most like a SQUARE?*" might well permute the middle part of the scale. A *WHITE thing* and a *coloured SQUARE* are very different objects! There is no end to such issues. Investigating square-hood requires more than a lifetime in experimental phenomenology. What is completely ignored here is that imagery is meaning and value through and through, so "abstract squares" are nonentities.

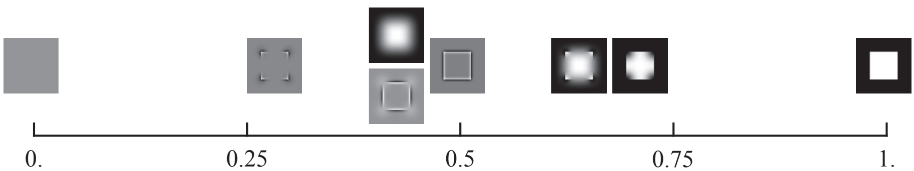

Fig. 14 The empirical voting order of "square-hood", as ranked by a hundred naive observers. It ranges from *zero* (a featureless grey field, not containing any part of the white square) to *one* (the white square itself). Notice that the square without its macchia lies halfway; the no-edges and no-corners are better; and the macchia-only, corners-only and edges-only are the worst.

Conclusions

Brentano's intuition as it has come to us in his concept of "plerosis" fits in well with a number of diverse modern notions. His conception of the continuum was similar to Veronese's; however, both were considered misguided by the mainstream as represented by Cantor (see Bell, 2006). Their intuitions were close to Brouwer's intuitionistic continuum and at least implicitly deny the Law of the Excluded Middle. Brentano's concept of primary and secondary continua is an early example of what is presently known as the "tangent bundle", a central concept of differential geometry.

Brentano's concept of plerosis had not been used in mathematics till the development of the theory of scale-space operators in the late twentieth century. The notion of a boundary point on a line as belonging to *both* parts, effectively being a coincident pair of distinct points, is somewhat reminiscent of Cartan's (1923) "derivative of a point". However, only in the theory of scale -space operators does the plerosis – although the term has not been used in that field – explicitly appear.

This implies that Brentano taught the essence of a theory of "perceptive fields", or – blurring ontological levels as in modern neurophysiology – "receptive fields". This is remarkable, since the very term "receptive field" was only introduced by Hartline in 1938. Perhaps Weber's (1846) work on the two-point threshold in touch might be suggested as a forerunner. Indeed, this defines sensory "points" on the skin, but points with other than full plerosis do not occur in the work, whereas Brentano envisaged perceptive fields of considerable complexity.

The concept of plerosis had no impact on the psychology of vision, although it is the proper counterpart to the receptive fields of neurophysiology. The latter are records of diverse neuro-anatomical observations, whereas Brentano's plerosis neatly fits a formal setting. Brain theory has much to gain from insights deriving from phenomenology. That Brentano's insight was forgotten is certainly to the detriment of psychology.

In the view of psychogenesis based on the fundamental insights of especially von Uexküll in biology and Brown in psychiatry, the notions of receptive and perceptive fields naturally come together. The receptive fields of neuroscience are *points* in the aspect of samplers or operators, whereas the perceptive fields are *points* in the aspect of brushes or touches in visual awareness. Both aspects are necessary if visual perception is to be grounded in the structure of the environment as well as in the creative, productive imagination. In many respects, this is closely analogous to the way sciences such as physics proceed. The "understanding of the physical world" is just as much due to experimentation and observation as it is due to the creative imagination. Feynman's book "*The Character of Physical Law*" is – from a slightly abstract perspective – not at all different from the structure of psychogenesis as sketched here. It seems the only way how mind might build meaning on mere structure. It puts mind in the world and world in the mind.

What about the "points" in the blue sky, do they exist? Evidently not as boundary points. They cannot be detected by "receptive fields" or "feature detectors" and thus never make a mark on the blackboard. The blue sky is a manifest psychic object. Indeed, "boundary points" only exist because of the continua they bound.

Appendix: The Experiment

In an experiment, a group of 122 observers selected the best "white square" from pairs of images taken from those in Fig. 12. The set contained all pairs of the eight images, thus $8(8 - 1)/2 = 28 = N$ in total. These were presented in fully randomised order. Participants were fully naive and new to the task (students from the University of Sassari). Participants needed only a few minutes to complete the task after reading a short instruction sheet.

Given the 28 preference orders of an observer, we constructed the best overall order. This order is easily found through voting, the number of votes for an item being the number of times it was preferred to any other item. We have shown (Koenderink, van Doorn, Albertazzi, & Wagemans, 2015) this to be equivalent to a least-squares estimate of order. We define the consistency in terms of the number of responses *according with* the voting order (n, say) and *going against* the voting order (m, say), setting $C = (n - m)/N$.

Participants responded in a self-consistent order; the median concordance was 1.0, and the interquartile range was 0.79–1.0. The inter-observer rank correlation was also high (median Kendall tau: 0.69, interquartile range: 0.52–0.82). We determined the overall order for the group. The self-consistency of the group order was 0.73. Then, we calculated the Kendall rank correlations of all individuals with the group order. These final rank correlations were all high (median Kendall rank: 0.79, interquartile range: 0.67–0.86). Apparently, the group was quite consistent and homogeneous.

In order to investigate the homogeneity further, we did a cluster analysis on the individual orders. We agglomerated using Ward linkage and squared Euclidian distance. This yielded two major clusters. The clusters differed mainly in the rank of the *macchia*.

Summary

Franz Brentano, 1838–1917, introduced the intriguing concept of "plerosis" in order to account for aspects of the continuum that were "explained" by formal mathematics in ways that he considered absurd from the perspective of intuition, especially visual

awareness and imagery. In doing this, he pointed in directions later developed by the Dutch mathematician Luitzen Brouwer. Brentano's notion of plerosis involves distinct though coincident points, which one might call "atomic entities with parts". This notion fits the modern concepts of "receptive field" in neurophysiology, "perceptive field" in psychology and "differential operator" in the formal theory of scale space. We identify Brentano's boundary points as the primordial atomic Gestalts of visual imagery. The concept deserves to play a key role in Gestalt theory.
Keywords: Plerosis, continua, receptive fields, perceptive fields, atomic Gestalt, squares

Zusammenfassung

Franz Brentano (1838-1917) führte das faszinierende Konzept der "Plerosis" ein, um Aspekte eines Kontinuums zu erklären, die von der formalen Mathematik in einer Art und Weise erklärt wurden, die er aus der Perspektive der Anschauung, insbesondere des visuellen Bewusstseins und der Bildsprache, für absurd befand. Dabei deutete er in Richtungen, die später vom niederländischen Mathematiker Luitzen Brouwer entwickelt wurden. Brentanos Begriff der Plerosis beinhaltet abgesetzte, jedoch übereinstimmende Punkte, man könnte auch sagen, "atomare Einheiten mit Teilen". Dieser Begriff passt zu den modernen Konzepten der "Empfangsfelder" in der Neurophysiologie, dem "Wahrnehmungsfeld" in der Psychologie und dem "Differentialoperator" in der formalen Theorie des Skalenraums. Wir verstehen Brentanos Grenzpunkte als die ursprünglichen, winzig kleinen Gestalten der visuellen Bildsprache. Das Konzept verdient es, eine Schlüsselrolle in der Gestalttheorie zu spielen.
Schlüsselwörter: Plerosis, Kontinuum, Empfangsfelder, Wahrnehmungsfelder, Atomgestalt, Quadrate.

Acknowledgement

This work was supported by a visiting scientist grant from the University of Sassari and the Methusalem program by the Flemish Government (METH/14/02), awarded to Johan Wagemans.

References

Albertazzi, L. (2002). Towards a neo-Aristotelian theory of continua: Elements of an empirical geometry. In L. Albertazzi (Ed.), *Unfolding perceptual continua* (pp. 29–80; esp. pp. 44–52). Amsterdam, Netherlands: Benjamins.

Albertazzi, L. (2006). *Immanent realism: An introduction to Brentano*. (pp. 233–268; esp. pp. 248–267). Dordrecht, Netherlands: Springer.

Albertazzi, L. (2015). Spatial elements in visual awareness. Challenges for an intrinsic "Geometry" of the visible. *Philosophia Scientiæ, 19*(3), 95–125.

Attneave, F. (1954). Some informational aspects of visual perception. *Psychological Review, 61*, 184–193.

Bakalis, N. (2005). *Handbook of Greek philosophy: From Thales to the Stoics analysis and fragments*. Victoria, BC, Canada: Trafford Publishing.

Bell, J. (2006). *The continuous and the infinitesimal in mathematics and philosophy*. Monza, Italy: Polimetrica.

Bell, J. (2009). Cohesiveness. *Intellectica, 9*(1), 51.

Bijl, P., Koenderink, J. J., & Toet, A. (1989). Visibility of blobs with a Gaussian luminance profile. *Vision Research, 29*(4), 447–456.

Blum, M. (1967). A transformation for extracting new descriptors of shape. In W. Wathen-Dunn (Ed.), *Models for the perception of speech and visual form* (pp. 362–380). Cambridge, MA: MIT Press.

Boime, A. (1993). *The art of the Macchia and the Risorgimento*. Chicago and London: The University of Chicago Press.

Bouma, H. (1970). Interaction effects in parafoveal letter recognition. *Nature, 226*(241), 177–178.
Brentano, F. (1974, [1874]). *Psychologie vom empirischen Standpunkt*. Leipzig, Germany: Duncker und Humblot. (*English quotations from: "Psychology from an Empirical Standpoint"*. New York: Humanities Press).
Brentano, F. (1988). *Philosophical investigations on space, time and the continuum* (Smith, Trans.). London, England: Croom Helm.
Broude, N. (1987). *The Macchiaioli: Italian painters of the nineteenth century*. New Haven and London: Yale University Press.
Brouwer, L. E. J. (1918). Begründung der Mengenlehre unabhängig vom logischen Satz vom ausgeschlossenen Dritten. Erster Teil, Allgemeine Mengenlehre. *KNAW Verhandlungen, 5*, 1–43.
Brown, J. W. (1977). *Mind, brain and consciousness*. New York, NY: Academic Press.
Cartan, É (1923). Sur les variétés à connexion affine et la théorie de la relativité généralisée (première partie). *Annales Scientifiques de l'École Normale Supérieure, 40*, 325–412.
Cateura, L. (1995). *Oil painting secrets from a master*. New York, NY: Watson & Guptill.
Chadwick, H. (1992). *St. Augustine, Confessions. [orig. 397-400CE]*. Oxford, England: Oxford University Press.
Dedekind, R. (1872). *Stetigkeit und irrationale Zahlen*. Braunschweig, Germany: Vieweg, p. 18.
Dunn, C. (1995). *Conversations in paint, a notebook of fundamentals*. New York, NY: Workman Publishing.
Ehrlich, P. (2006). The rise of non-Archimedean mathematics and the roots of a misconception. I. The emergence of non-Archimedean systems of magnitudes. *Archive for History of Exact Sciences, 60*(1), 1–121.
Elkind, D. (1964). Studies in perceptual development II, part-whole. *Child Development, 35*(1), 81–90.
Euclid. (1956, [fl. 300BCE]). The thirteen books of Euclid's elements, (Translation and Commentaries by Heath, T. L.), Three volumes. New York, NY: Dover Publications.
Feynman, R. (1966). *The character of physical law*. Cambridge, MA: MIT Press.
Florack, L. (1997). *Image structure*. Dordrecht, Netherlands: Kluwer Academic.
Geem, Z. W., Kim, J. H., & Loganathan, G. V. (2001). A new heuristic optimization algorithm: Harmony search. *Simulation, 76*(2), 60–68.
Giordano, P. (2001). Nilpotent infinitesimals and synthetic differential geometry in classical logic. In P. Schuster, U. Berger, & H. Osswald (Eds.), *Reuniting the antipodes–constructive and nonstandard views of the continuum* (pp. 75–92). Dordrecht, Netherlands: Kluwer.
Gödel, K. (1960, [1947]). What is Cantor's continuum problem? *The American Mathematical Monthly, 54*, 515–525. (originally published in 1947, Page references are to the reprinting of the expanded 1960 version in Benacerraf and Putnam 1983, pp. 420–485).
Hartline, H. K. (1938). The response of single optic nerve fibers of the vertebrate eye to illumination of the retina. *The American Journal of Physiology, 121*, 400–415.
Hilbert, D. (1980, [1899]). *The foundations of geometry* (2nd ed.). Chicago, IL: Open Court.
Hoffman, D. D. (2009). The interface theory of perception. In S. Dickinson, M. Tarr, A. Leonardis, & B. Schiele (Eds.), *Object categorization: Computer and human vision perspectives* (pp. 148–165). New York, NY: Cambridge University Press.
Hubel, D. H., & Wiesel, T. N. (1968). Receptive fields and functional architecture of monkey striate cortex. *Journal of Neuroscience, 195*, 215–243.
Husserl, E. (1991, [1893-1917]). *On the phenomenology of the consciousness of internal time (1893-1917)* (J. B. Brough, Trans.). Dordrecht, Netherlands: Kluwer.
Imbriano, V. (1868). *La quinta promotrice 1878-1868: Appendici di Vittorio Imbriani*. Napoli, Italy: Tipografia Napolitana.
Jacobs, T. S. (1986). *Light for the Artist*. NewYork, NY: Watson-Guptill.
Kandinsky, W. (1926). In W. Gropius & L. Moholy-Nagy (Eds.), *Punkt und Linie zu Fläche. Bauhaus Bücher, Schriftleitung* München, Germany: Verlag Albert Langen.
Kanizsa, G. (1980). *Grammatica del vedere. Saggi su percezione e Gestalt*. Bologna, Italy: IL Mulino.
Koenderink, J. J. (1984). The structure of images. *Biological Cybernetics, 50*, 363–370.
Koenderink, J. J. (1988). Operational significance of receptive field assemblies. *Biological Cybernetics, 58*(3), 163–171.
Koenderink, J. J. (1990). The brain a geometry engine. *Psychological Research, 52*(2–3), 122–127.
Koenderink, J. J. (1993). What is a "feature"? *Journal of Intelligent Systems, 3*(1), 49–82.
Koenderink, J. J. (2002). Continua in vision. In L. Albertazzi (Ed.), *Unfolding perceptual continua, advances in consciousness research 41* (pp. 101–118). Amsterdam and Philadelphia: John Benjamins.
Koenderink, J. J. (2011). Vision as a user interface. In B. E. Rogowitz & T. N. Pappas (Eds.), *Proceedings of SPIE-IS&T Electronic Imaging, SPIE, Vol. 7865: Human Vision and Electronic Imaging XVI* (pp. 1–13). San Francisco: SPIE-IS&T.

Koenderink, J. J. (2014). The all seeing eye. *Perception, 43*, 1–6.
Koenderink, J. J. (2015). Ontology of the mirror world. *Gestalt Theory, 37*(2), 119–140.
Koenderink, J. J., Albertazzi, L., van Doorn, A. J., van Ee, R., van de Grind, W. A., Kappers, A. M. L., … de Vries, S. (2010). Does monocular visual space contain planes? *Acta Psychologica (Amst), 134*(1), 40–47.
Koenderink, J. J., & Richards, W. A. (1988). Two-dimensional curvature operators. *Journal of the Optical Society of America A, 5*(7), 1136–1141.
Koenderink, J. J., & Van Doorn, A. J. (1990). Receptive field families. *Biological Cybernetics, 63*(4), 291–297.
Koenderink, J. J., & Van Doorn, A. J. (1992a). Generic neighbourhood operators. *IEEE Transactions on Pattern Analysis and Machine Intelligence, 14*, 597–605.
Koenderink, J. J., & Van Doorn, A. J. (1992b). Receptive field assembly pattern specificity. *Journal of Visual Communication and Image Representation, 3*(1), 1–12.
Koenderink, J. J., & van Doorn, A. J. (1997). Local image operators and iconic structure. In G. Sommer & J. J. Koenderink (Eds.), *Proceedings of the AFPAC'97. Algebraic Frames for the Perception-Action Cycle* (pp. 66–93). Berlin, Heidelberg: Springer.
Koenderink, J. J., van Doorn, A. J., & Pinna, B. (2015). Psychogenesis of Gestalt. *Gestalt Theory, 37*(3), 287–304.
Koenderink, J. J., van Doorn, A. J., Albertazzi, L., & Wagemans, J. (2015). Relief articulation techniques. *Art & Perception, 3*(2), 151–171.
Koenderink, J. J., van Doorn, A. J., Pinna, B., & Wagemans, J. (2016). Boundaries, transitions and passages. *Art & Perception, 4*(3), 185–204.
Marr, D. (1982). *Vision: A computational investigation into the human representation and processing of visual information*. New York, NY: Freeman.
Marr, D., & Hildreth, E. (1980). Theory of edge detection. *Proceedings of Royal Society of London B, 207*, 187–217.
Metzger, W. (1936). *Gesetze des Sehens*. Frankfurt am Main, Germany: W. Kramer & Co.
Misner, C. W., Thorne, K. S., & Wheeler, J. A. (1973). *Gravitation*. San Francisco, CA: W. H. Freeman.
Panconi, T. (1999). *Antologia dei Macchiaioli, La trasformazione sociale e artica nella Toscana di metà 800*. Pisa, Italy: Pacini Editore.
Pasch, M. (1912). *[first edition 1882], Vorlesungen uber neuere Geometrie* (2nd ed.). Leipzig, Germany: B.G. Teubner, p. 21.
Photoshop. (2014). Adobe, Retrieved from http://www.adobe.com/mena_en/products/photoshop.html
Pinna, B. (2008). Watercolor illusion. *Scholarpedia, 3*(1), 5352.
Pinna, B. (2010). New Gestalt principles of perceptual organization: An extension from grouping to shape and meaning. *Gestalt Theory, 32*(1), 11–78.
Pinna, B. (2012). Perceptual organization of shape, color, shade and lighting in visual and pictorial objects. *i-Perception, 3*, 257–281.
Pinna, B., & Albertazzi, L. (2011). From grouping to visual meanings: A new theory of perceptual organization. In L. Albertazzi, G. van Tonder, & D. Vishwanath (Eds.), *Information in perception* (pp. 288–344). Cambridge, MA: MIT Press.
Pinna, B., Brelstaff, G., & Spillmann, L. (2001). Surface color from boundaries: A new 'watercolor' illusion. *Vision Research, 41*, 2669–2676.
Pinna, B., & Deiana, K. (2015). The syntax organization of shape and color and the laws of coloration in vision, art and biology. *Art & Perception, 3*(3), 319–345.
Pinna, B., & Grossberg, S. (2005). The watercolor illusion and neon color spreading: A unified analysis of new cases and neural mechanisms. *Journal of the Optical Society of America A, 22*, 2207–2221.
Pinna, B., & Reeves, A. (2006). Lighting, backlighting and watercolor illusions and the laws of figurality. *Spatial Vision, 19*, 341–373.
Pinna, B., Spillmann, L., & Werner, J. (2003). The watercolor effect: A new principle of grouping and figure-ground organization. *Vision Research, 43*, 43–52.
Richards, W. R., & Bobick, A. (1988). Playing twenty questions with nature. In Z. W. Pylyshyn (Ed.), *Computational processes in human vision: An interdisciplinary perspective* (pp. 3–26). The Canadian Institute for Advanced Research Series in Artificial Intelligence. Norwood, NJ: Ablex Publishing.
Richards, W.R. (1982, December): *How to play twenty questions with nature and win* (M.I.T. A.I. Memo 660).
Riedl, R. (1975). *Die Ordnung des Lebendigen. Systembedingungen der Evolution*. Hamburg, Germany: Paul Parey.
Riemann, B. (2013). *[Habilitationsvortrag 10. Juni 1854]. Über die Hypothesen, Welche der Geometrie zu Grunde Liegen*. Berlin, Germany: Springer.

Roeper, P. (2006). The Aristotelian continuum. A formal characterization. *Notre Dame Journal of Formal Logic, 47*(2), 211–232.
Schrödinger, E. (1944). *What is life?*. Cambridge, UK: Cambridge University Press.
Spillmann, L. (1971). Foveal perceptive fields in the human visual system measured with simultaneous contrast in grids and bars. *Pflügers Archiv, 326*(4), 281–299.
Stevin, S. (1585). In A. J. E. M. Smeur (Ed.), *De Thiende* (p. 1965). Nieuwkoop, Netherlands: De Graaf.
Stoltz, O. (1883). *Zur Geometrie der Alten, insbesondere über ein Axiom des Archimedes*. Mathematische Annalen, *22,* 504–519.
ter Haar Romeny, B. M. (2003). *Front-end vision and multi-scale image analysis*. Dordrecht, Netherlands: Kluwer Academic.
Tinbergen, N. (1952). The curious behavior of the stickleback. *Scientific American, 187*(6), 22–26.
van Dalen, D. (1997). How connected is the intuitionistic continuum? *The Journal of Symbolic Logic, 62*(4), 1147–1150.
Veronese, G. (1894). *Grundzüge der Geometrie von mehreren Dimensionen und mehreren Arten gradliniger Einheiten in elementarer Form entwickelt* (Uebersetzt von A. Schepp). Leipzig, Germany: B.G. Teubner.
von Uexküll, J. (1909). *Umwelt und Innenwelt der Tiere*. Berlin, Germany: Springer.
von Uexküll, J. (1920). *Theoretische Biologie*. Berlin, Germany: Springer.
Weber, E. H. (1846). Tastsinn und Gemeingefühl. In R. Wagner (Ed.), *Handwörterbuch der Physiologie mit Rücksicht auf physiologische Pathologie* (Band 3 ed., pp. 481–588). Braunschweig, Germany: Vieweg.
Wertheimer, M. (1922): Untersuchungen zur Lehre von der Gestalt, I: Prinzipielle Bemerkungen. Psychologische Forschung, 1, pp. 47–58. [Translated extract reprinted as "The general theoretical situation." In W. D. Ellis (Ed.), (1938). *A source book of Gestalt psychology* (pp. 12–16). London, UK: Routledge & Kegan Paul Ltd.].
Wertheimer, M. (1923): Untersuchungen zur Lehre von der Gestalt, II. Psychologische Forschung, 4, pp. 301–350. [Translated as "Investigations on Gestalt principles, II. In L. Spillmann (Ed.), (2012). *On motion and figure-ground organization* (pp. 127–182). Cambridge, MA: M.I.T. Press].
Weyl, H. (1925). On the current epistemological situation in mathematics [English translation of *Die Heutige Erkenntnislage in der Mathematik, Symposion*, 1, 1925–1927, 1–32.] In P. Mancosu (Ed.) (1998), pp. 123–142.

Jan Koenderink (b. 1943) is Professor Emeritus (in physics) from the University of Utrecht. He has worked in physics, mathematics, psychology, biology, philosophy and computer science. His main interests are centred on the nature of awareness, especially for the case of vision. Much of his work is related to his interests in artistic expression.
Address: Laboratory of Experimental Psychology, University of Leuven (K.U. Leuven), Tiensestraat 102 - Box 3711, Leuven B-3000, Belgium
Email: KoenderinkJan@gmail.com

Andrea van Doorn (b. 1948) is an Emeritus Associate Professor at the Technische Universiteit Delft in Industrial Design. Her research interests are cognitive science, ecological physics, human interfaces, non- verbal communication and visual phenomenology. She has a keen interest in the visual arts.
Address: Faculteit Sociale Wetenschappen, Psychologische Functieleer, Universiteit Utrecht, Heidelberglaan 2, 3584 CS Utrecht, The Netherlands
Email: andrea.vandoorn@gmail.com

Baingio Pinna (b. 1962) has been Professor of Experimental Psychology and Visual Perception at the University of Sassari since 2002, was Research Fellow at the Alexander Humboldt Foundation, Freiburg, Germany, in 2001/02, was winner of a scientific productivity prize at the University of Sassari in 2007 and was winner of the International "Wolfgang Metzger Award to eminent people in Gestalt science and research for outstanding achievements" in 2009. His main research interests concern Gestalt psychology; visual illusions; psychophysics of perception of shape, motion, colour and light; and vision science of art.
Address: Department of Humanities and Social Sciences, University of Sassari, Via Roma 151, 07100, Sassari, Italy
Email: baingio@uniss.it