

A first mathematics curriculum: Stevin's *Instruction* for engineers (1600)

Jenneke Krüger

Freudenthal Institute, University of Utrecht, Netherlands

Introduction

Teaching of mathematics has a history which spans thousands of years, see e.g. Robson, (2009); Siu (2009); Keller (2014). However, texts which contain a plan for the teaching of mathematics, a formal curriculum, are of far more recent date. Volkov (2014) describes prescriptions for the School of Computations in China, which started in 656. The matters mentioned are, among others, the textbooks to be used, the number and grouping of students and the duration of the study. In Europe, medieval universities might have lists of books to be read by the lecturers to the students. The *Ratio studiorum* of 1599, which provided a curriculum for the system of education by the Jesuits, contained some remarks on mathematics teaching, but only a few and not very detailed (Paradinas Fuentes, 2012).

In the Netherlands the oldest known plan for a mathematics course is the *Instruction* by Simon Stevin (1549-1620), dating from 9 January 1600. It is remarkable because of the detailed specification of aims, content, teaching methods, learning activities, materials and teaching language. The *Instruction* was written, in the Dutch language, for the Engineering Course, which was attached to the University of Leidenⁱ in 1600, on request of Prince Maurice (1567-1625), commander of the Dutch army. The course received its name, *Duytsche Mathematique* (Dutch Mathematics), to indicate that the lectures were in Dutch, instead of in Latin (Van Maanen, 1987). For more information on the *Duytsche Mathematique* see e.g. Krüger (2010; 2014).

Simon Stevin was born in Bruges, in the Southern Netherlands. After 1577 he migrated to the Northern Netherlands; in 1581 he was inscribed in the municipal register of Leiden. In 1583 he enrolled at the university in Leiden, where Maurice of Orange was a student as well. Stevin was a prolific author, who published on a range of mathematical, science and engineering subjects. He also was an inventor and an engineer; he worked as a military engineer for Prince Maurice in the army. During a number of years he was mathematics tutor of Prince Maurice (Devreese & VandenBerghe, 2008; Dijksterhuis, 1943). For a better understanding of the *Instruction* it is important to realize that Stevin was a proponent of the teaching of theory preceding practical work, of the use of Dutch language instead

of Latin for teaching and for mathematics and of the use of decimal notation by mathematical practitioners.

In this paper I present a translation of the *Instruction* into English. Some of the words Stevin used have become obsolete and other expressions now have a different meaning, e.g. instead of 'rondt' we use 'cirkel' (circle); 'dadelyck' at present means 'at once', but in 1600 meant something like 'practising'. So before translating I transcribed a few expressions into modern Dutch. When translating into English I maintained the original syntax as much as possible and also some specific expressions, such as 'counting' which Stevin used as a synonym for arithmetic. 'Thyende (ge)tal' which we know as decimal fractions, I translate with 'decimal numbers'. The reason to omit the word 'fractions' in this case is that Stevin himself stressed in *De Tiende* that these numbers were not fractions, as they could be used in the same way as integers. The word fractions ('gebroken getallen') was reserved for the notation with a slash (Sems & Dou, 1600; Stevin, 1585).

The translation is based on two texts. Molhuysen (1913) presents the text of the addendum, which was on 10 January 1600 added to the Resolutions of the curators of the university; the second text is a manuscript in the Regional Archives in Leiden (Fig. 1), which is a copy provided by the curators for the printer Jan Paedts Jacobsz. (RAL LB 36949).

There are two small but significant differences between the text in Molhuysen (1913) and the manuscript in the Regional Archives; the latter is a copy from slightly later in 1600.

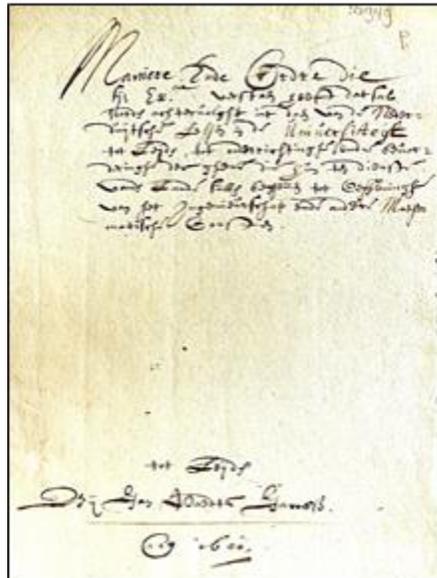


Fig. 1. Front page of the copy of the *Instruction*, 1600 (RAL LB 36949)

The last paragraph (It is also understood...) is present in Molhuysen (1913), but not in the manuscript. On the other hand, on the front page of the manuscript are

'engineer and other mathematical Arts' mentioned as aims of the course; in Molhuysen only 'engineer' is mentioned. These small differences suggest that the curators looked further than a purely military course. There also was a demand for mathematically trained civilians, such as surveyors, wine gaugers, building masters and mathematics teachers.

The *Instruction*

His Excellency has deemed it useful for the sake of the country and for those who will enroll to become an engineer, that there will be a plan for the teaching thereof which will take place at the Academy in Leiden, as follows:

1

The general opinion is that the auditors should be trained as quickly as possible to serve the country as engineers. To this goal they shall learn arithmetic or the counting and surveying, but of each of these only so much as is necessary for a practising ordinary engineer. Those who have come thus far are allowed to study more in depth if they wish to do so.

This is the general idea, the specifications are as follows:

2

In the counting will be taught the four operations in whole numbers, in broken numbers and in decimal numbers, also the rule of three in each type of number.

3

When they have become skilled in this way of counting, they know sufficient arithmetic for ordinary engineering. They will then start with surveying on paper in the way of surveyors, that is not to construct any lines through known lines, but only to find the area of plane figures through measured lines, drawn to scale, using arithmetic in decimals.

4

Measurements of circles, sectors and segments, also the area of a globe; the curves which are named ellipse, parabola, hyperbola etc. are not necessary, as engineers rarely have to do such measurements; but they shall only learn with rectilinear figures, thereafter with curved figures in the way of surveyors, how to measure a plane by means of various subdivisions, such as triangles or other figures to check the different calculations.

5

After this follows the measuring on paper of dykes, ramparts and earthworks, in order to learn how many 'schachten'ⁱⁱ or feet the proposed work contains.

6

Once they have sufficient skills in measuring on paper and understand on a small scale what has to be done in reality, they shall start with the actual surveying in the field, they will be shown that instead of using ruler, compass and square on paper, one uses other tools in the field, with the same purpose. They shall practise measuring the height and width of earthworks and their volume, and how many 'schachten' or feet of soil are needed.

7

After that they will learn to draw on paper the perimeter of the land they have measured, and again the reverse, from a drawing on paper, in the open field lay out the plan using pickets.

8

Once they have mastered these matters they will be ready to learn fortification, to which goal wooden or clay models of entrenchments and bastions will be made and with the help of these having learnt the proper names, the drawing of ground plans or maps of towns will be easy for them, so they can practise that working in towns.

9

Also they will draw on paper the perimeter of entrenchments or towns with four, five and more bastions, they will then be given the measurements and in the open field lay out such fortifications with pickets.

10

Once they have come thus far, they are allowed in summer to go to the army or to building sites of fortifications, which will be most advantageous for those who as soldiers have to return to the army and so can observe these matters for themselves and even help with the actual work.

11

As they have learnt so much, that they can thus usefully serve the country, those who wish to do so are allowed, as has been mentioned before, to continue to study at Leiden in winter, in order to become more accomplished engineers.

12

The lessons in counting and geometry will in general take half an hour; the other half hour will be filled with answering questions from individual students and explanations about what they have not understood from the lectures.

13

As those who work in engineering, rarely talk in Latin with each other, but instead in each country the vernacular is used, so the lessons will not be given in Latin, French or similar languages, but only in Dutch.

It is also understood that all those who are admitted to this course of the art of engineering, first have to promise and to swear that they will not use what they learn in the service of the enemy of this country.

Actum the 9th January 1600.
MAURICE DE NASSAU.

Some comments

In the following account (N) indicates the paragraph in the *Instruction* with the same number N.

Prince Maurice and Simon Stevin intended a course, which provided a modern and efficient training for surveyors and military engineers (1). In the independence war against the Spanish army (re)capturing and defending fortified towns were of the utmost importance. Modern military techniques made use of mathematics: geometry and also trigonometry. There was a lack of well-trained surveyors and engineers in the Netherlands. As there were no examples of such a course, Stevin specified what content and teaching methods should be used.

The content was to consist of arithmetic (2, 3), the geometry as used by surveyors (3,4), calculation of volumes, dykes, entrenchments, etc. (5), practice of surveying (6) and fortification (8, 9). Arithmetic was limited to the four main operations and the rule of three (proportionality) but included using the very new notation of decimal fractions (1, 2, 3). In *De Thiende* (1585) Stevin had stressed the advantages for mathematical practitioners of using a decimal notation for fractions.

In teaching surveying (3, 5, 6) as well as in teaching fortification (9, 10), theory should precede practice. Stevin specified that conic sections, which interested many mathematicians at the time, were not part of the *Duytsche Mathematique* (4).

Geometry should be mainly practical; of Euclid only that which was necessary for practitioners and also learning to take measurements and to check calculations. We may learn something about Stevin's ideas in this respect from his *Mathematical Memories*, published between 1605 and 1608. In the first book he discusses trigonometry, in his opinion the basis for (practical) geometry; in the second book he treats geometry and some surveying techniques.

In the teaching of fortification, models were to be used (8) to learn the names of the many different parts. Stevin emphasized the importance of learning to draw ground plans and to work from those drawings (5, 7, 9).

The practice of surveying was part of the course in Leiden (6, 7); the practice of fortification had to take place while with the army or wherever fortification building was taking place (10).

For those who wanted to study further after this course for ordinary engineers, there would be opportunity to continue the study at Leiden University during the winter, when the army was not active (2, 11).

Stevin also specified that the teaching language should be Dutch (13) and that the lessons in arithmetic and geometry should consist of half an hour of theory and half an hour of opportunity for students to ask questions (12).

The *Duytsche Mathematique* was innovative in several ways.

- The formulation of a curriculum, by Simon Stevin, and preserving the text as an addendum to the Resolutions of the curators, was in itself an innovation. It provided guidance for the professors of the *Duytsche Mathematique* in the years to come.
- Dutch as a teaching language meant that capable men and boys who did not know Latin nevertheless could receive a thorough theoretical and practical training in modern methods of surveying and engineering. The university took responsibility for the implementation of a course, which could not be considered part of the regular university lectures because of the teaching language.
- The emphasis on the use of decimal notation was very unusual, but very practical for surveyors and engineers.
- The use of wooden or clay models as teaching aids was new in the Netherlands, as far as is known.ⁱⁱⁱ
- Working from plans was not customary for surveyors.
- Teaching modern fortification methods, based on geometry, was new.

Also in January 1600, two professors for this new course were appointed by the university. They were the respected mathematics and fencing teacher Ludolf van Ceulen (1540-1610) and a well-known surveyor from Leiden, Simon Fransz van Merwen (1548-1610). Stevin had not mentioned exams; but from the beginning students asked to be able to take a final exam. So within a year it became possible to take an exam on surveying. That was of value for the students, as to become a registered surveyor one had to give proof of proficiency in this field, through an exam or otherwise.

Both professors passed away in 1610; not much is known about their teaching during those first ten years. It is likely that Van Merwen undertook the practical lessons in surveying, the fieldwork, as Van Ceulen already was 60 years old when he was appointed and he was not a surveyor himself. Van Ceulen died in December 1610; in 1611 his assistant, former student and admitted surveyor Frans van Schooten Sr. (1581/82-1645), continued the lessons. Van Schooten taught both theory and practice, the first years without an official appointment from the university. During those years he received payment at the end of each year and his students lobbied to get him appointed. At the start of 1615 Van Schooten indeed became professor of *Duytsche Mathematique*; this function he exerted until his death, in December 1645.

His lecture notes, *Mathematische Wercken* (BPL 1013), are kept in the university library of Leiden. They represent a carefully planned curriculum, based on the *Instruction* by Stevin. The manuscript starts with the calculation of square and cubic roots (not mentioned by Stevin) and immediately provides an explanation of decimal fractions, with a simple and easy to use notation. Throughout the manuscript this decimal notation is consistently used. After a few calculations with decimal fractions follows some geometry, an introduction on surveying, the use of trigonometric tables (not mentioned by Stevin) and the practice of surveying. Van Schooten also treated more complicated geometrical problems, which went beyond the practice of a surveyor (Van Maanen, 1997). The calculation of volumes and introduction to wine gauging precedes the treatment of fortification.

These lecture notes were also used by the successors of Frans van Schooten Sr., at first by his eldest son Frans Jr. (1615-1660), who was succeeded by the youngest son, Petrus (1634-1679). There were some changes in the course during the 17th century, such as the public lectures on cosmic algebra and lectures on logarithms by Frans Jr.

During more than sixty years, the *Duytsche Mathematique* attracted a good many students, also from other countries. The course earned praise from important people, such as Prince Maurice and Christiaan Huygens (1609-1687). The latter was taught by Frans van Schooten Jr. during his stay in Leiden. *Duytsche Mathematique* provided inspiration for some other institutions to teach practical mathematics in the vernacular. Examples are the "Illustre School" in Amsterdam (1653) and the University of Utrecht (1659).

The *Instruction* is an early example of a formal curriculum document, in which attention is paid to many aspects of teaching. In this way Simon Stevin provided guidance to the professors who were to teach this curriculum. There was no similar course in the Netherlands or in surrounding countries, which could serve as an example for future teachers. Stevin had very clear ideas about the content

of such a course and about teaching methods; these ideas provided a solid framework for the implementation.

ⁱ Leiden is situated in the Western part of the Netherlands, her university was established in 1575 by Prince William of Orange.

ⁱⁱ A 'schacht' was 144 cubic feet.

ⁱⁱⁱ Italian engineers recommended the use of models of wood or clay.

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