Guest Editorial

PERCEPTION

The • of Awareness

Perception
2016, Vol. 45(9) 969–972

DOI: 10.1177/0301006616655916
pec.sagepub.com
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A point is that which has no part. (Euclid, 1956, Book I, Def. 1)

Euclid was born ca. 300 BCE at Alexandria, Egypt, so why worry about his definition? Here's why: Euclid's point is a natural kind defined by the absence of parts. Bizarre! Just *imagine* an object lacking of natural parts! Think of a cat and try to "put a blind eye" to its head. Now go on with its tail and any other part you can imagine. What you end up with is a cat without parts. Cheshire cat (Carroll, 1865) perhaps? More likely you lost the cat in the process. Truly *atomic* objects are *nothing*. But, according to Democritus (Bakalis, 2005), *reality* itself is nothing but "atoms and void," pretty much our modern perspective. Such a reality can't account for much!

Euclid's definition led people to say that "points have no size." They intend "zero size," a notion that opens up a can of worms Bell (2005).[1] Euclid's definition only makes sense if you understand "points" as objects whose existence is "beyond being or non-being." Such objects include the round square and other goodies (Meinong, 1899). Although no one can visualize such "subsisting" objects, academics enjoy reasoning about them. I know physicists who take electrons for real. Sure, but do I see them? No!

Try to visualize a light point (Figure 1, left). I imagine a white circular disk on a dark background. It *has to be circular*, or it would have parts and it has to be of some *visible* diameter. The background has to contrast with light, for a light point on an equally light background is invisible. Surely, the background *belongs to the point*, the point cannot exist without it, but isn't the background itself "a large point"? Is a light point actually *two* points then? So what about the dark one—and so forth?

A blue sky background is not composed of points, it is an object whose interior has qualities (blueness, etheriality, luminosity, etc.), but no parts. Not in my perception. My iPhone wastes up to eight million pixels on the blue sky, which seems pointless to me. Although somewhat less than infinity, it exceeds the number of fibers in my optic nerve. But I'm not aware of even a single "blue sky point."

How large should the background be? Large enough that it acts like the blue sky that should leave me with the light point. What is still irksome is the circular edge, for an "edge" is part-like. If I increase my (mental) viewing distance, the light disc appears smaller and smaller to my eye, until I can no longer make out its outline. Yet I do not necessarily lose the light point, for if its luminous flux is fixed and the background dark, it remains perfectly visible: *it becomes like a star*. This is indeed an apt model of a point, a "Dirac (1958) δ–function," a "star point."[2] Its having "no size" has led to problems in the proto–psychophysics of visual acuity.[3]

Here is an alternative train of thought: I rephrase Euclid's definition into "A point is that of which one ignores the parts." That makes great sense! At least to me. It implies that Utrecht (I happen to live at the center of my world) is a mere *token* on the globe (Figure 1, center, central white dot). This type of "point" would reveal internal structure when you scrutinize it, or put a microscope on it. Utrecht certainly looks more than a white

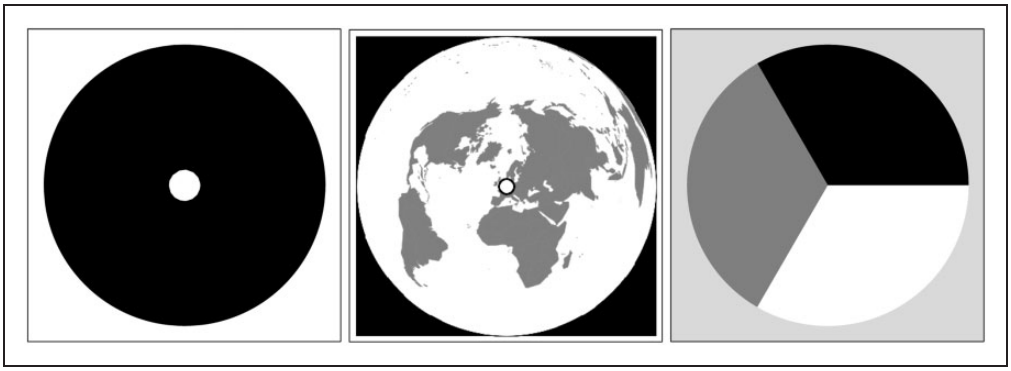

Figure 1. Left a white point on a black ground; Center the point on the globe where I live; Right is the center point white , gray, black, the average of white , gray and black, no color at all, or white, gray and black at the same time, or . . . ? Take your pick!

dot from where I'm writing now. Such points evidently come in many sizes, given that my galaxy is but a (central) point in my universe. I'll call them "token points."

Token points may well have properties, not just color, but even geometrical symmetries like a direction, or orientation. This reminds me of Euclid's second definition of points: "The ends of a line are points."[4] The latter points have the geometrical quality that they are "sided." Let me call them "structural points."

Mathematics has struggled for centuries with such sided points (Bell, 2005) and they remain a headache. Structural points were introduced in psychology by Brentano (2009) to give a phenomenological account of points. Brentano—one founding father of psychology—granted points a "plerosis" (or "fullness"), which is exactly the "sidedness" of Euclid's endpoints of lines.

Brentano cannot conceive of a point as isolated from the space around it, nor of the continuum as constituted of points. He feels that continuity is concrete actuality given in perception, not a mathematical construct. He may have thought of my blue sky. Thus, he focuses on the notion of *boundary*. For simplicity, consider the one-dimensional time axis, with *now* as the boundary between *past* and *future*. The *now* is both the end of the past as well as the beginning of the future, it is not that the *now* is isolated *between* past and future, because that would imply the times axis to be composed of points. *Now* is half plerosis in past and half plerosis in future. It is akin to the retention and protention of the now of Husserl (1991). Thus, boundary points have parts in respect to their plerosis! Either Euclid's two definitions are at odds, or Euclid is talking of different objects.

Brentano would hold that (Figure 1, right)

> if a circular area is made up of three sectors, a red, a blue, and a yellow, then the mid-point is a whole which consists to an equal extent of a red, a blue, and a yellow part.

Thus, the center point has *parts* despite its lack of dimension! He quotes Galileo as "saying paradoxically that the centre of the circle has as many parts as its periphery." Don't we all agree? Well, we *should*, I feel, but mathematicians are not of one mind.

Brentano feels the notion of mathematicians that the center point would be red, blue, or yellow to be an abomination. In his refutation of "Dedekind's cut,"[5] he says

> the whole circular surface would then be conceivable as having been deprived of its mid-point, like Dedekind's number–series from which only the number 1/2 has fallen away. One sees

> immediately that this is absurd if one keeps in mind that the true concept of the continuum is obtained through abstraction from an intuition, and thus also that the entire conception has missed its target.

From the perspective of the phenomenology of vision, I can only agree. A mathematician might say that Brentano denied the *Law of the Excluded Middle*[6] and was an early intuitionist (Bell, 2005). The intuitionistic continuum is very "syrupy" (van Dalen, 1997), so deleting a point fails to cut a line in two.

I might add a few more points to the point, but I feel three will do. *Star points* certainly occur in concrete actuality, but they seem to be somewhat singular occurrences. *Token points* are all over the place in my vision, but they seem to exist in a shady region between awareness proper and—perhaps still visual, certainly nonverbal—reflective thought. They are the kind of points artists like Kandinsky (1926) seem to talk about. Then there are the *structural points*.

In my view, the *structural points* are the most interesting of all. They are evidently a concrete actuality in my vision and imagery. They seem like a kind of "atomic Gestalts" to me. There also exists an obvious link to mind theory, in that structural points might be implemented as *perceptive fields*—or, if you prefer *brain* theory, substitute *receptive fields*. Think of an "edge detector," which evidently implements a "point" (zero degrees of freedom because a single axon) that has two-way plerosis. It has a foot in either side, so to speak, much like the now is part of both past and future, an edge is part of its left and right uniform areas.

So Brentano is really the inventor of the "points" of visual awareness. His notion of "plerosis" exactly captures the concept of "perceptive field." This was the early twentieth century, long before the modern concept of the "receptive field" was proposed by Hartline (1938) in neurophysiology.

Theoreticians often hold that physics will ultimately reduce to geometry (Lisi & Weatherill, 2010). I have speculated in the past (Koenderink, 1990) that this is the natural way for mind and brain science to go. So the academic question "What is a •?" is quite to the point in studies of *visual awareness*: A few minutes of thought lead to a dozen unanswered questions. There is work to do!

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Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

Notes

1. Just consider: How many points on a line? Can they touch? What is between the points? Does a line "split" when I remove a point? And so forth! Such points were discussed since the Pre-Socratics and they're still being discussed in contemporary mathematics.

2. The Dirac *b*–function is everywhere zero, except at some location. It has unit total weight, despite the fact that it vanishes virtually everywhere. It is used as a sieve to obtain the value of smooth functions at that location.
3. Helmholtz (1867) discusses historical methods to measure visual acuity and notices that the size of stars is not acceptable. Indeed, the angular diameter of a bright star like Sirius (0.00593600) severely overestimates acuity.
4. Perhaps remarkably, neither definition is ever used in the Elements! One speculates that the definitions are not by Euclid, but were added by later authors.
5. Dedekind (1872) (my translation):

   > I find the nature of smoothness in this: When all points of a line are divided such that each point of one part lies to the left of any point of the other part, then there exists a uniquely determined point that causes the cut.

   This is not necessarily intuitive, for example, Anaxagoras (ca. 510–428 BCE) famously held that a continuum defies the chopping off of its parts with a hatchet.
6. The Law of the Excluded Middle would say that a non-zero (real) number is either positive or negative: no "third" possibility. But the number ε defined as $\varepsilon \neq 0$ and $\varepsilon^2 = 0$ is neither positive nor negative, although non-zero by definition. There are infinitely many of such "nil square infinitesimals" at zero distance from 0. They yield a "third" possibility. Such a number is approximately $0.000\ldots0001\ldots$, where the number of zeroes remains undecided, an example of Brouwer's "choice sequences" (van Dalen, 1997).

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