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Optimal parameters for the Green-Ampt infiltration model under rainfall conditions

Li Chen[1, 2*], Long Xiang[1], Michael H. Young[3], Jun Yin[4], Zhongbo Yu[1, 5], Martinus Th. van Genuchten[6, 7]

[1] State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Hohai University, Nanjing, China.
[2] Division of Hydrologic Sciences, Desert Research Institute, 755 E. Flamingo Rd., Las Vegas, NV 89119, USA.
[3] Bureau of Economic Geology, University of Texas at Austin, Austin, TX, USA.
[4] Ministry of Forests, Land & Natural Resource Operation, Prince George, BC, Canada.
[5] Department of Geoscience, University of Nevada, Las Vegas, NV, USA.
[6] Department of Mechanical Engineering, Federal University of Rio de Janeiro, Brazil.
[7] Department of Earth Sciences, Utrecht University, Utrecht, Netherlands.
[*] Corresponding author. E-mail: li.chen@dri.edu

Abstract: The Green-Ampt (GA) model is widely used in hydrologic studies as a simple, physically-based method to estimate infiltration processes. The accuracy of the model for applications under rainfall conditions (as opposed to initially ponded situations) has not been studied extensively. We compared calculated rainfall infiltration results for various soils obtained using existing GA parameterizations with those obtained by solving the Richards equation for variably saturated flow. Results provided an overview of GA model performance evaluated by means of a root-mean-square-error-based objective function across a large region in GA parameter space as compared to the Richards equation, which showed a need for seeking optimal GA parameters. Subsequent analysis enabled the identification of optimal GA parameters that provided a close fit with the Richards equation. The optimal parameters were found to substantially outperform the standard theoretical parameters, thus improving the utility and accuracy of the GA model for infiltration simulations under rainfall conditions. A sensitivity analyses indicated that the optimal parameters may change for some rainfall scenarios, but are relatively stable for high-intensity rainfall events.

Keywords: Green-Ampt; Infiltration; Rainfall; Optimal parameters.

INTRODUCTION

The Green-Ampt (GA) infiltration model (Green and Ampt, 1911), and various modifications and extensions thereof, are used widely for analyzing relationships between rainfall, infiltration and runoff for a range of conditions and from different theoretical and experimental perspectives (Ahuja and Ross, 1973; Barry et al., 2005; Basha, 2011; Bouwer, 1978; Chen and Young, 2006; Gowdish and Muñoz-Carpena, 2009; Hilpert and Glantz, 2013; Hogarth et al., 2013; Mein and Larson, 1973; Parlange et al., 1982; Voller, 2011; among many others). Because of its simplicity and physical basis, the model has been implemented also in several codes addressing larger-scale hydrologic processes (e.g. Arnold et al., 2012; Flanagan and Nearing, 1995; O'Brien et al., 2009; Rossman, 2010).

As with most models, a critical issue in applying the GA parameterization is accurate estimation of the GA parameters. In current practice, GA model parameters are determined using both theoretical and empirical approaches (Neuman, 1976; Rawls et al., 1983; Risse et al., 1994; Saxton and Rawls, 2006). A number of studies have compared the accuracy of alternative parameterizations against numerical solutions of the Richards (1931) equation for unsaturated or variably saturated flow, mostly for ponded infiltration scenarios (e.g. Barry et al., 1993; Hsu et al., 2002; Salvucci and Entekhabi, 1994). By comparison, the accuracy of the parameterizations has not been examined extensively for infiltration under rainfall conditions. Such studies are important since GA errors in estimating infiltration can magnify considerably when runoff estimates are needed, since runoff in many situations is a small fraction of rainfall. As compared to ponded infiltration, rainfall adds additional complexity to the accuracy of parameter estimation due to the transition from flux-controlled infiltration to profile-controlled infiltration. The objective of this study was to evaluate the performance of the GA model in comparison with the Richards equation, and to determine optimal GA parameters that would improve the utility of the model for simulating infiltration under rainfall conditions. We further studied the sensitivity of the optimal parameters to various rainfall intensities and durations.

THE GA INFILTRATION MODEL AND ITS PARAMETERS

Formulation of the GA model

The GA model assumes piston-type soil water movement and a step-function for the water content in the soil profile involving two water contents, the initial water content θ_i and the saturated water content θ_s near the soil surface as shown in Fig. 1.

Using Darcy's law for these conditions, the infiltration rate i under ponded conditions is given by (Green and Ampt, 1911):

$$i = \frac{dI}{dt} = K_s\left[1 + \left(\theta_s - \theta_i\right)S/I\right] \tag{1}$$

where t is time, I is the cumulative infiltration depth, K_s is the saturated soil hydraulic conductivity, and S is the capillary pressure at the wetting front, sometimes referred to as the capillary drive (e.g. Morel-Seytoux and Khanji, 1974) or macroscopic capillary length (Philip, 1985). In some studies (Rawls et al., 1983) an effective hydraulic conductivity (K_e) is used rather than K_s, to account for hysteresis effects. This difference in interpretation will not affect the theoretical analysis in our work. Most studies directly use K_s in Eq. (6) for ponded infiltration studies (Mein and Larson, 1973; Morel-Seytoux and Khanji, 1974; Neuman, 1976). Our study also ignores the pond-

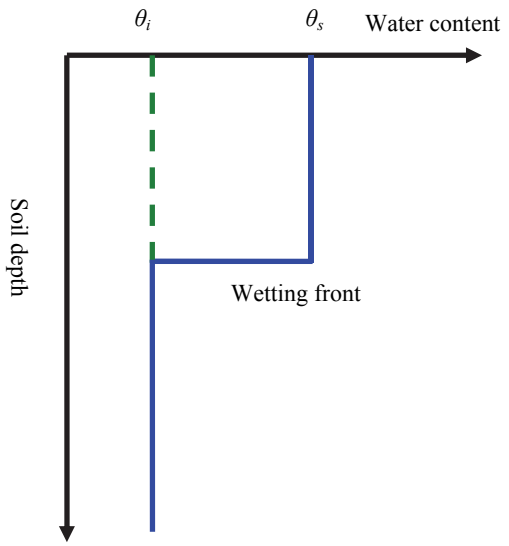


Fig. 1. Schematic soil water content profile in the GA model.

ing depth since its effect is usually very small. If large, the ponding depth can be immediately added to the right-hand side of Eq. (1).

An implicit function for I can be derived by integrating Eq. (1) to obtain:

$$K_s t = I - S(\theta_s - \theta_i)\ln\left(1 + \frac{I}{S(\theta_s - \theta_i)}\right) \tag{2}$$

For infiltration under constant rainfall intensity, Eq. (2) can be modified as (Chu, 1978; Mein and Larson, 1973):

$$K_s\left[t - (t_p - t_s)\right] = I - S(\theta_s - \theta_i)\ln\left(1 + \frac{I}{S(\theta_s - \theta_i)}\right) \tag{3}$$

where t_p is the time to ponding given by

$$t_p = \frac{I_p}{p} = \frac{(\theta_s - \theta_i)S}{p^2/K_s - p}t \tag{4}$$

in which p is the rainfall intensity, and I_p the infiltration depth at time t_p. The variable t_s in Eq. (3) is a virtual time defined by:

$$t_s = \frac{I_p}{K_s} - \frac{S(\theta_s - \theta_i)}{K_s}\ln\left[1 + \frac{I_p}{S(\theta_s - \theta_i)}\right] \tag{5}$$

This model was extended further by Chu (1978) to unsteady rainfall events.

Model parameters and parameterization approaches

The GA model parameters include the saturated soil hydraulic conductivity K_s, the capillary pressure at the wetting front S, as well as the initial (θ_i) and saturated (θ_s) water contents. Of these, K_s is a relatively standard hydraulic parameter that can be measured in the laboratory or field using a variety of approaches. For ponded infiltration, S is given by (Morel-Seytoux and Khanji, 1974; Neuman, 1976):

$$S = \frac{1}{K_s}\int_{-\infty}^{0} K(\psi)\,d\psi \tag{6}$$

where ψ is the capillary pressure under unsaturated conditions. Eq. (6) assumes that the effect of the hydraulic conductivity K_i at the initial capillary pressure head (ψ_i) or water content (θ_i) is very small and can be ignored. For relatively wet initial conditions, K_s in Eq. (6) formally should be replaced by ($K_s - K_i$), with the integration being carried out between ψ_i and 0 (e.g. Smith et al., 2002).

Equation (6) requires integration of the hydraulic conductivity K as a function of the capillary pressure ψ. A number of analytical expressions for $K(\psi)$ may be used for this purpose (e.g. Brooks and Corey, 1964; Kosugi, 1996; van Genuchten, 1980). Because of their complexity, most of these functions require numerical integration. An exception is the power law function of Brook and Corey (1964), which has been used in a number of GA applications, notably by Rawls and coworkers (Rawls et al., 1989, 1983). In our study we used the van Genuchten (1980) function for $K(\psi)$ and integrated Eq. (6) numerically. Numerical integration may be too cumbersome for many practical studies. A useful approximation of S based on Eq. (6) for the $K(\psi)$ function of van Genuchten (1980) is given by Morel-Seytoux et al. (1996).

$$S = \frac{0.046m + 2.07m^2 + 19.5m^3}{(1 + 4.7m + 16m^2)\alpha} \tag{7}$$

where α and $m = 1–1/n$ are parameters in the original van Genuchten-Mualem formulation (van Genuchten, 1980).

The use of analytical functions for $K(\psi)$ has the advantage that many of the parameters in these functions are often immediately available, including through the use of pedotransfer functions which estimate the parameters from soil texture and related data (Rawls et al., 1983; Schaap et al., 2001; Vereecken et al., 2010). Alternatively, GA parameters could be estimated using field-based approaches to better account for site-specific conditions. Some studies aimed at deriving GA model parameters from field experiments, mainly focusing on the hydraulic conductivity. Single- and double-ring infiltrometer experiments (Nimmo et al., 2009; e.g. Stahr et al., 2004) and rainfall simulation (e.g. Risse et al., 1994) have been used for this purpose to back-calculate K_s from continuous measurements of the infiltration depth. Unfortunately, S cannot be determined easily using these approaches.

Most GA applications in the literature consider ponded infiltration, whereas many practical applications involve conditions in which the soil surface is initially unsaturated. Mein and Larson (1973) modified the model to approximate rainfall-infiltration cases using the ponded GA model equation before the onset of ponding. The appropriateness of parameters determined for ponded infiltration conditions may be reduced further due to this approximation. In general, little attention has focused on the performance of the GA model when theoretical parameters are employed. Only Mein and Larson (1973) conducted limited tests to compare the theoretically parameterized GA model to the Richards equation.

An issue with most or all existing approaches is that model performance using estimated GA parameters is not quantified by comparisons to actual infiltration data or numerical predictions, or evaluated relative to other combinations of parameter values, especially under rainfall conditions. Therefore, infiltration simulation using the resulting parameters may not ensure optimal results. Prior work seeking optimal GA infiltration parameters is limited. To our knowledge, only Hsu et al. (2002) addressed this issue through comparisons with infiltration rates based on the Richards equation. However, as acknowledged in

their paper, the approach was "less rigorous" in that GA parameters were determined by fitting to only three points rather than the entire infiltration curve. In addition, an artificial time shift parameter was used by Hsu et al. (2002). According to Eqs (3–5), this parameter is not independent since it can be calculated from other parameters.

The brief summary above of the GA model its parameters shows a need to systematically investigate performance of the model using existing parameterizations, as well a need for practical guidance in seeking optimal GA model parameters, especially for infiltration under rainfall conditions.

PERFORMANCE OF THE GA INFILTRATION MODEL

In this study we combined the variables S and $(\theta_s–\theta_i)$ in Eqs (1–5) into a single parameter, denoted as MS (i.e., $MS = S(\theta_s–\theta_i)$). This combination is preferred since the effect of the initial soil moisture condition is then automatically taken into account for any θ_i value. Hereafter, we report on the two parameters K_s and MS for the GA model.

We used the established approach, first suggested by Mein and Larson (1973), for comparing GA simulation results with numerical solutions of the Richards equation to evaluate the use and accuracy of the GA parameterization. The Richards equation was solved using the van Genuchten (1980) soil hydraulic property relationships for $\theta(\psi)$ and $K(\psi)$. For the evaluation we numerically integrated Eq. (6) very carefully using Romberg integration, and multiplied the resulting S values with $(\theta_s–\theta_i)$ to obtain MS.

Table 1 provides van Genuchten soil hydraulic parameters for the sand, sandy clay loam, and clay loam soils (Carsel and Parrish, 1988) used in the simulations. These soils were chosen to generate simulated infiltration data across a two-order-of-magnitude range in K_s. Fig. 2 shows the van Genuchten (1980) $\theta(\psi)$ and $K(\psi)$ relationships for these soils. Table 1 also shows the rainfall rates used in the calculations. We assumed constant rainfall intensities p, each lasting 10 hrs. To ensure that both infiltration and runoff covered a substantial part of total rainfall, rainfall intensities were set at approximately twice K_s for all soils. The soil and rainfall data were used to compare GA infiltration results obtained by numerically solving Eqs (3–5) with Hydrus-1D (Šimůnek et al., 2008) results based on the Richards equation. Only relatively simple (one-dimensional, uniform) soil profiles were considered, while ignoring such complexities as soil layering, hysteresis, hydrophobicity, preferential flow paths, and sloping soil surfaces. Hydrus-1D simulations were carried out using a relatively deep soil profile with a free drainage lower boundary condition to avoid the impact of the lower boundary, and a very fine grid size (1 mm) with small time steps (adaptive with the smallest value being 10^{-5} hr) to ensure high numerical accuracy. Soil profile thickness and a lower boundary do not need to be specified for the GA model.

Cumulative infiltration curves (normalized by total rainfall) obtained with both the GA model and the Richards equation for the three soils are presented in Fig. 3. Before ponding commenced, the curves overlapped along a straight line representing the cumulative rainfall rate. Ponding times differed for each soil depending upon soil texture and rainfall intensity. After ponding commenced, as a result of the reduced infiltration rate, the cumulative infiltration curves were situated below the straight line representing cumulative rainfall. Although cumulative infiltration continued to increase over time, the rate of increase (i.e., the infiltration rate) gradually declined and ultimately approached a constant value equal to K_s in each case. While the Richards equation and GA model showed the same general patterns, the infiltration curves obtained with the two parameterizations differed depending upon soil type.

For the sand soil, the GA model agreed closely with the Richards equation throughout the period of investigation. For the sandy clay loam soil, the GA model over-predicted the infiltration depth by up to 11% relative to the Richard equation. For the clay loam soil, the theoretical GA model parameters over-predicted the infiltration depth even more, this time by up to 25%. These results suggest that a better GA parameterization

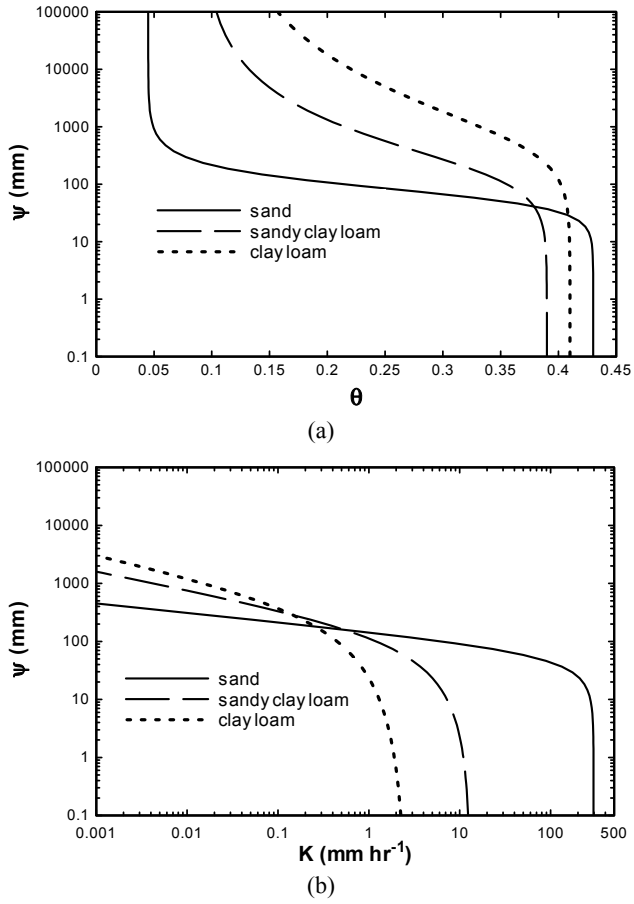


Fig. 2. van Genuchten (1980) soil hydraulic functions for the three test soils: (a) Soil water retention curves; b) Unsaturated soil hydraulic conductivity curves.

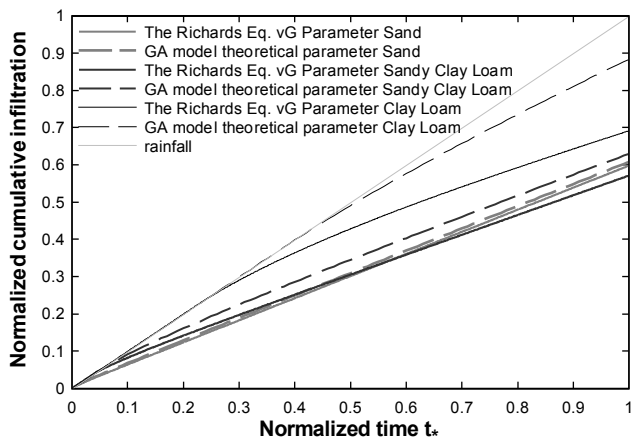


Fig. 3. Comparison of infiltration curves (normalized by total rainfall) obtained with the Richards equation and the GA model.

Table 1. Soil and rainfall parameters used for the numerical comparison.

	vG parameters for the Richards equation			Theoretical GA parameters		Rainfall intensity
	K_s (mm hr^{-1})	α (mm^{-1})	n	K_o (mm hr^{-1})	MS (mm)	p (mm hr^{-1})
Sand	297	1.45×10^{-2}	2.68	297	14.8	500
Sandy clay loam	13.1	5.90×10^{-3}	1.48	13.1	10.4	25
Clay loam	2.6	1.90×10^{-3}	1.31	2.6	17.3	5

is needed, or that parameters in the GA model should be adjusted (optimized), to provide improved infiltration predictions. The latter option is addressed next.

OPTIMAL GA PARAMETERS

We next seek to determine optimal GA parameters through a least square analysis. A straightforward approach to optimize the GA parameters is to calculate the overall error between the GA infiltration curve and the Richards equation for all possible K_s and MS parameter combinations, and then to identify those combinations corresponding to a minimum root mean squared error (RMSE). A similar approach was followed by Yin (2008). The RMSE was used as the objective function (F) to be minimized:

$$F=\sqrt{\frac{1}{k}\sum_{i=1}^{k}\left[y_i-f\left(t_i,K_s,MS\right)\right]^2} \tag{8}$$

where y_i represents cumulative infiltration depths corresponding to times t_i obtained with the Richards equation, $f(t_i, K_s, MS)$ are GA cumulative infiltration depths (i.e., I in Eqs (1) through (3)) for parameters K_s and MS, and k is the number of samples for each infiltration curve. The objective function thus serves as an indicator of the overall accuracy of the GA model compared to infiltration behavior identified using Hydrus-1D. The minimum RMSE of F corresponds to optimal K_s and MS values. Ten-hour rainfall durations were used in all simulations, considering that rainfall duration is independent of soil type. This approach provided performance of the infiltration parameters across a wide range of GA parameter values.

To show results of different conditions on the same scale, we used normalized dimensionless variables. As dimensionless objective function we used $F_* = F/I_F$, where I_F is the final cumulative infiltration rate. We further used dimensionless (scaled) values $K_* = K_o/K_s$ and $MS_*=MS_o/MS$ ($=S_o/S$), where K_o and MS_o are the adjusted (or scaled) values of the original (theoretical) GA parameters K_s and MS.

Plots of the dimensionless objective function F_* for the sand, sandy clay loam, and clay loam soils are shown in Figs 4–6, respectively. F^* represents the average relative error between the GA model and the actual infiltration curve obtained with solutions of the Richards equation. Note that errors in the final infiltration amount may be significantly higher than the average error given by F^*. Substantial computational effort was required to obtain each of these maps since hundreds of thousands of GA model solutions were generated for each case. To show a wide range of F_* values on the plots in Figs 4–6, the upper bounds on F_* were set to 0.1 (10%), which is higher than acceptable errors in most applications. Also shown in the figures are comparisons of infiltration curves obtained with solutions of the Richards equation and the GA model using the optimal parameters.

The results in Figs 4–6 reveal several interesting features of the objective function. Most importantly, F_* for all soils converged to single minima that can be located on the plots. This indicates that optimal GA parameters exist and can be obtained using the optimization analysis. The GA simulation results with optimal parameters were notably better than those generated using the theoretical parameters for especially the medium-and fine-textured soils (Fig. 3). Therefore, for practical applications, the standard (theoretical) parameterization can be replaced with an optimization approach. In addition, the results in Figs 4–6 show that F_* is more sensitive to K_* than to MS_*, thus indicating that K_s is a more important parameter than S in GA model applications (especially for coarser soils).

Searching for optimal parameters may be straightforward for coarse-textured soils but difficult for fine-textured soils. For the sandy soil, the elliptical shape of F_* contoured a large area of K_*-MS_* space, thus ensuring that the optimization algorithm could reach the minimum. As soil texture becomes finer, however, the difficulty of locating optimal parameters increased since the shape of the F_* contours tended to become more stretched so that the direction of local gradients started to deviate from the minimum. For such cases one may obtain multiple acceptable solutions with the optimization although the objective function still has only one global minimum.

Another significant aspect of our results relates to the difference between the theoretical and optimal values of K_s and MS. The dimensionless optimal parameters for the three test cases are included in Table 2. The data indicate that for the coarse- and medium-textured soils (the sand and sandy clay loam) in our study, the optimal hydraulic conductivity (K_o) was relatively close to K_s, such that K_* approached unity, while the optimal MS_o was substantially smaller than the theoretical MS value (i.e., $MS_* \ll 1$). For the fine-textured (clay loam) soil, the optimal K_o was about 40% of K_s, while MS_o was larger than the theoretical value of MS.

From the F_* plots in Figs 4–6 one can deduce that the overall infiltration errors (i.e., F_* values) using theoretical GA parameters were several orders of magnitude higher than those for the optimal parameters. These results can be explained by considering the different infiltration mechanisms for the three soils. The infiltration rate for the coarse- and medium-textured soils approximated K_s most of the time since gravitational forces for these soils should dominate the capillary pressure gradients. The optimal K_o for those soils is hence expected converge quickly to K_s. By comparison, the effects of capillarity on infiltration remain important for a much longer period of time. As such, the oversimplified moisture profile (Fig. 1) in the GA model inaccurately reflects the effects of capillary forces on infiltration of especially fine-textured soils. Still, the use of optimal parameters (K_o and MS_o)greatly improved the GA predictions, with especially K_o becoming differently from K_s when fine-textured soils are considered, as shown by the entries of K_* in Table 2.

Table 2. Sensitivity of optimal parameters K_o and MS_o to rainfall intensity p and duration t_r.

Case	Soil texture	Rainfall intensity p (mm hr^{-1})	Duration t_r (hr)	p_*‡	t_{r*}	Optimal parameters		Theoretical parameters	
						K_*	MS_*	K_*	MS_*
1	Sand	300	3	1.010	60.203	0.997	0.136	1	1
2			5		100.338	0.998	0.094		
3			10		200.676	0.999	0.066		
4		500	3	1.684	60.203	0.985	0.536		
5			5		100.338	0.991	0.473		
6			10†		200.676	0.995	0.399		
7	Sandy clay loam	15	3	1.145	3.779	0.938	0.286	1	1
8			5		6.298	0.992	0.159		
9			10		12.596	1.008	0.117		
10		25	3	1.908	3.779	0.855	0.881		
11			5		6.298	0.908	0.714		
12			10†		12.596	0.954	0.556		
13		50	3	3.817	3.779	0.741	1.522		
14			5		6.298	0.840	1.122		
15			10		12.596	0.924	0.798		
16	Clay loam	5	3	1.923	0.451	0.296	3.524	1	1
17			5		0.751	0.385	2.546		
18			10†		1.503	0.419	2.254		
19		10	3	3.846	0.451	0.481	1.876		
20			5		0.751	0.439	2.112		
21			10		1.503	0.439	2.105		

†Cases corresponding to Figs 4, 5, and 6.
‡Subscript * denotes non-dimensional parameters, $p_* = p/K_s$, $t_{r*} = t_r/t_c$, $K_* = K_o/K_s$ and $MS_* = MS_o/MS$.

In view of the above results, we also studied the time scale that characterizes the driving forces in the system to better explain their impact on the objective function and the optimized values of K_s and MS. Following Smith (2002), a characteristic time scale can be defined as:

$$t_c = \frac{(\theta_s - \theta_r)S}{K_s} \tag{9}$$

where S is the capillary pressure of the wetting front as used previously. The time scale characterizes the dominant force driving infiltration. Smith (2002) showed that for ponded cases, gravitational forces dominate infiltration and that the infiltration rate approaches K_s for t values larger than about $10t_c$, while gravitational forces are negligible when t is much smaller than t_c. In the current study (Table 1), values of t_c can be obtained easily for the three soils: t_c = 0.05, 0.79, and 6.6 hr for the sand, sandy clay loam, and clay loam soils, respectively. The two orders of magnitude difference in t_c reflects the wide range in soil hydraulic conditions for the three soils.

If we use t_r for rainfall duration, then the ratios of t_r/t_c (i.e., the dimensionless rainfall duration, denoted further as t^*) for a ten-hour event are 200, 12.7, and 1.5 for the three soils, respectively. This explains why the optimal value of K_s is close to the theoretical K_s value for the two coarser-textured soils when the rainfall event extends to 10 hrs. These results also indicate that for short-duration events (i.e., events with a $t_* << 10$), the GA parameters that best describe the infiltration depth are considerably different from the theoretical values.

Figure 6a also suggests that small t_* values can result in a narrow valley in parameter space where the GA parameters produce very similar values of the objective function, with all parameters being far superior than the theoretical values. This result has practical meaning, especially for arid environments where runoff generating rainfall events are often very intense and of short duration, thus yielding small t_* values. For such short-duration cases, optimal parameter sets should perform far better than the GA theoretical parameters, thus providing more accurate predictions of runoff.

Sensitivity of optimal parameters to rainfall intensity and duration

We next conducted a sensitivity test to evaluate changes in the optimal parameters as a function of rainfall intensity and duration. Two or three rainfall intensities and three durations were tested for each soil type (Table 2). Optimal parameters for each case were identified directly from plots of the objective function (F_*). Our objective of these tests was to found out if optimal parameters applicable for the various scenarios exist and could be identified.

Results of the sensitivity analysis are summarized in Table 2 for a total of 21 cases we examined. The results and input conditions are provided in dimensionless form. They show that

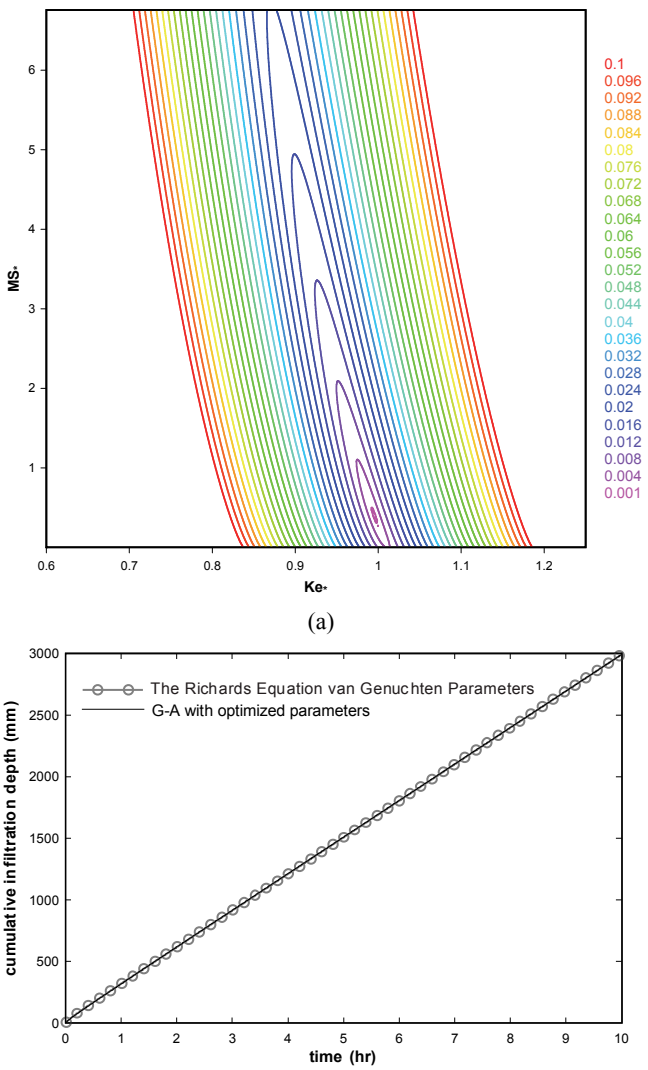


Fig. 4. (a) Distribution of the objective function F_* for the sandy soil; (b) Comparison of the optimal GA infiltration curve and solution of the Richards equation for the sandy soil.

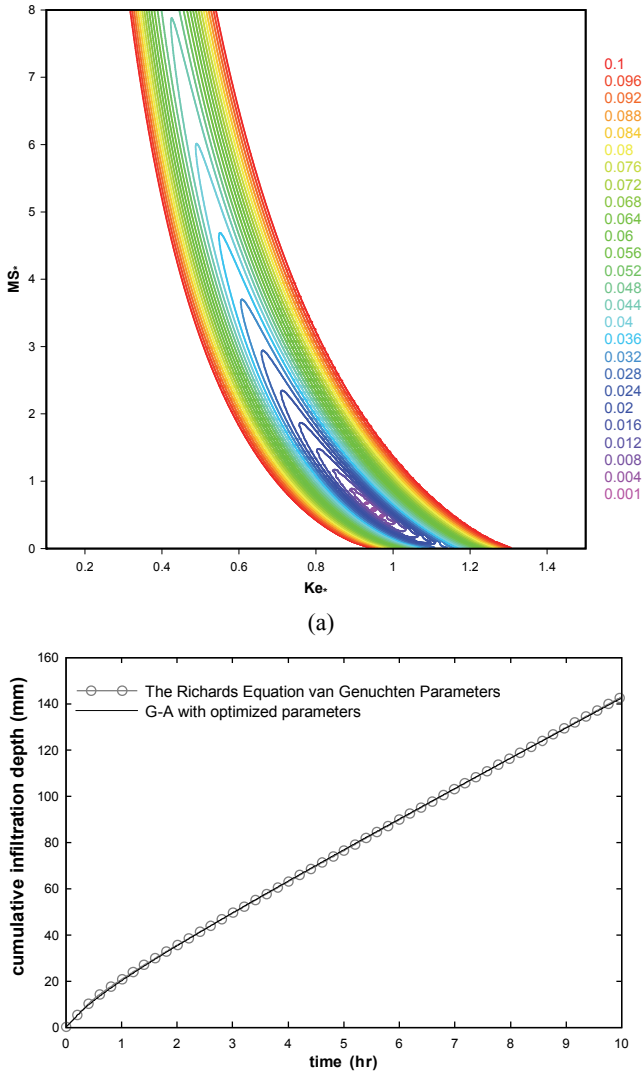
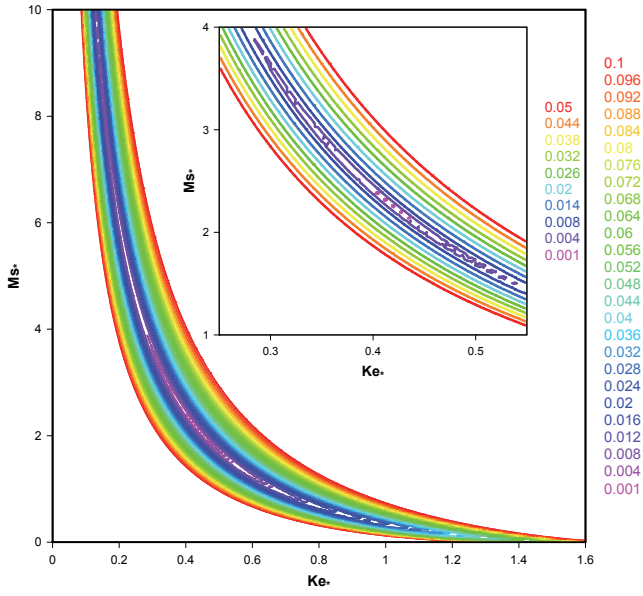


Fig. 5. (a) Distribution of the objective function F_* for the sandy clay loam soil; (b) Comparison of the optimal GA infiltration curve and solution of the Richards equation for the sandy clay loam soil.

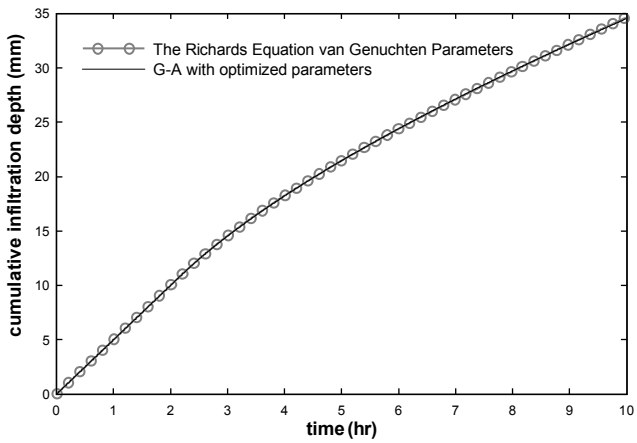
rainfall intensity and duration both affected the values of the optimal parameters. Optimal K_* values were generally close to 1 (i.e., $K_o \approx K_s$) for long-duration events (large t^*), while optimal K_o values were less than K_s, sometimes by up to 70%, for short-duration events (small t_*). Optimal MS values were less than 1.0 for low rainfall intensities or longer durations, but larger than 1.0 for short duration events ($t_* < \sim 10$). This again indicates that theoretical GA parameters are generally not the best choice for infiltration simulations, and hence that optimal parameters should be used. When K_o was set equal to K_s as a constraint in the optimization, acceptable MS_* values were generally less than 1.

Optimal parameters as a function of rainfall duration and intensity showed some general trends. For example, K_* generally increased with rainfall duration (t_{r*}) and decreased with rainfall intensity (p_*), except for small t_{r*} values when K_* increased with p_*. Conversely, MS_* generally decreased with t_{r*} and increased with p_*, except for small t_{r*} values when MS_* decreased with p_*. In addition, for long-duration rainfall events ($t_{r*} > \sim 10$), K_* approached 1 and $MS_* < 1$, while for short-duration events ($t_{r*} < \sim 1.5$), $K^* < 0.5$ and $MS >> 1$. These results show that the GA model using theoretical parameters overestimated gravitational effects for small t_{r*}, but underestimated these effects as t_{r*} increased. The optimal GA parameters compensated for such effects and could improve the simulations. In general, relatively large t_{r*} values affected the capillary component most, in which case that the optimal MS_* changed more significantly as a function of rainfall intensity.

Figure 7 shows examples of F_* distributions for different rainfall intensities and durations (Table 2). Cases for large t_{r*} values are not included since the optimal parameters did not change significantly for different rainfall conditions. Maps for medium and small t_{r*} values show F_* values below 0.025 (i.e., overall errors were less than 2.5% of cumulative infiltration), which we consider acceptable for these type of simulations. Results indicate that overall performance of the GA model was affected substantially by rainfall conditions at medium t_{r*} values (Fig. 7a) where the F_* distribution and optimal parameters changed significantly with rainfall conditions. For practical applications, optimal parameters for high rainfall intensities resulted in generally smaller errors for other rainfall conditions. This implies that optimal parameters are still applicable for the various conditions. For small t_{r*}, the performance of the GA model was not significantly affected by rainfall conditions (Fig. 7b),

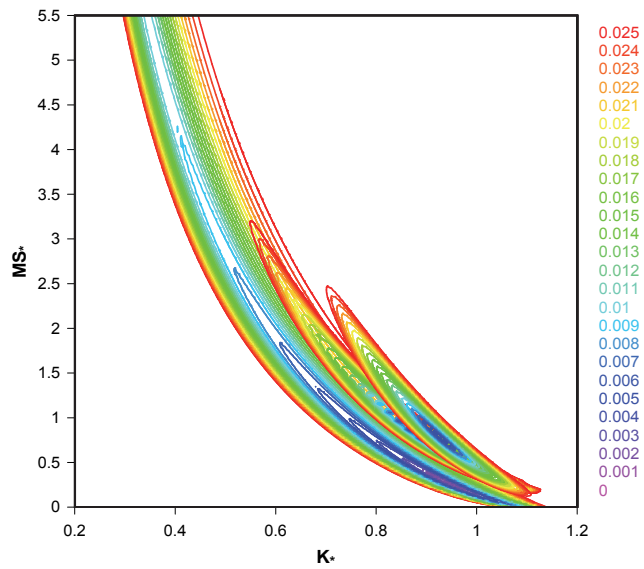


(a)

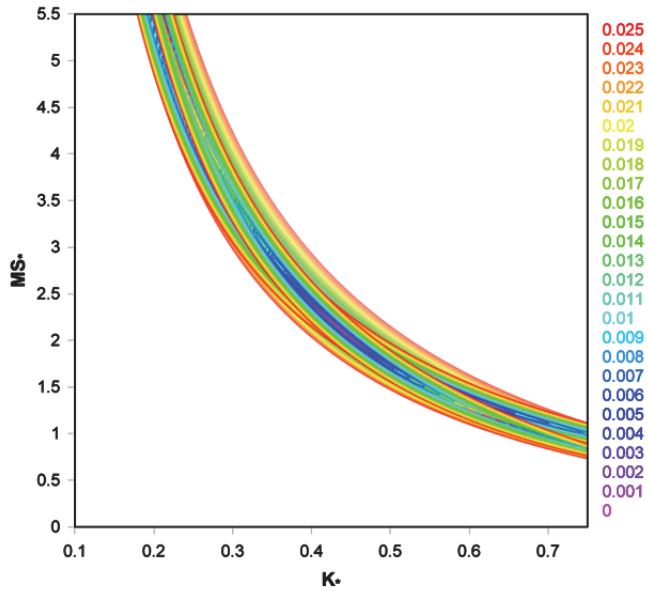


(b)

Fig. 6. (a) Distribution of the objective function F_* for the clay loam soil (including a close-up around the minimum; (b) Comparison of the optimal GA infiltration curve and solution of the Richards equation for the clay loam soil.



(a)



(b)

Fig. 7. Comparison of F_* distributions for different rainfall intensities and durations: (a) from bottom to top: cases 7, 11, and 15 in Table 2; (b) from bottom to top, cases 17, 19, and 21 in Table 2.

with optimal parameters for one case being immediately applicable to other cases. Table 3 shows the performance of the optimal parameters for clay loam soil cases included in Table 2. Results indicate that, in general, optimal parameters for high intensity rainfall events are more applicable to various conditions. This finding provides useful guidance for practical applications in terms of determining the most suitable GA parameters.

In real applications with complex rainfall conditions, the optimal parameters may be determined using as the objective function the average F_* for all significant rainfall intensities, weighted by rainfall depth. In our study, however, such an approach was not followed since the weighing factors for various rainfall conditions cannot not be determined without knowing specific rainfall characteristics. For this reason we provided only general recommendations.

CONCLUDING REMARKS

This study examined the general performance of the GA model for simulating infiltration under rainfall conditions by comparing GA calculations with numerical solutions of the Richards equation. Our results demonstrate that standard (theoretical) values of the two GA parameters (i.e., K_s and MS) will introduce significant errors in rainfall-infiltration modeling for fine- to medium-textured soils assuming rainfall events with realistic durations. We showed that optimal parameters for the GA model do exist and can be used to greatly improve the accuracy of GA simulations.

Our results further show that optimal parameters differ from theoretical values and may be affected by the rainfall characteristics. In general, both rainfall intensity and duration changed the values of the optimal parameters, with duration having a more significant impact. For long-duration (large t_{r*}) rainfall events, the optimal saturated conductivity (K_o) was close to K_s (the theoretical input value), while the optimal MS_o value was much less than the theoretical MS. For short-duration (small t_{r*}) rainfall events, optimal K_o values may become significantly smaller than K_s, whereas optimal MS_o will be much greater than the theoretical MS. Whereas optimal parameters for one rainfall

Table 3. Normalized objective function F_* for the clay loam soil assuming different parameters.

	K_*	0.2962	0.3846	0.4192	0.4808	0.4385	0.4385
	MS_*	3.5237	2.5457	2.2538	1.8757	2.1116	2.1052
Rainfall intensity p_*	Rainfall duration t_{r*}	F_* (%)					
1.9232	0.451	0.04†	0.04	0.07	0.07	0.1	0.12
1.9232	0.751	0.2	0.02†	0.07	0.13	0.11	0.15
1.9232	1.503	0.98	0.18	0.07†	0.62	0.11	0.09
3.8462	0.451	0.94	0.6	0.24	0.04†	0.1	0.13
3.8462	0.751	0.56	0.44	0.16	0.25	0.07†	0.1
3.8462	1.503	1.04	0.29	0.14	0.67	0.12	0.09†

†F_* of the optimal parameter set for the corresponding rainfall condition.

condition may not perform satisfactorily for all other conditions, especially for medium t_{r*} values, parameters feasible for various rainfall conditions may still be identified by comparing errors for the different cases. A weighted average of errors can be used for complex rainfall conditions if rainfall characteristics and their frequencies are known. Optimal parameters for high intensity rainfalls were found to be generally more applicable to different rainfall conditions.

In practical applications, parameter values other than the optimal values still may be acceptable. For short-duration rainfall events (with likely more effects when soils are fine-textured), acceptable parameters were located in a narrow valley in K_s – MS parameter space, thus implying that these two parameters should be determined concurrently to ensure satisfactory predictions of infiltration. Applying the GA model actually implies using a GA soil to approximate the real soil. A common goal of infiltration modeling is to accurately predict cumulative infiltration. If this is the sole criterion for modeling performance, a GA soil may not necessarily bear the same physical properties as the real soil (e.g., $K_o \neq K_s$). For a specific application, different GA parameters may be acceptable to approximate the behavior of the real soil, although optimal parameters would still perform slightly better than other sets. This equifinality of model parameters has been studied by Beven and Binley (1992) and Beven and Freer (2001), among others.

Practitioners may need to confront two situations when parameterizing the GA model. When a set of soil hydraulic parameters (such as the van Genuchten parameters or of other representations) are known, optimal parameters can be obtained simultaneously by fitting the GA model to the Richards equation (using the known soil hydraulic parameters). As a first approximation, pedotransfer functions could be used for this. However, if the soil hydraulic parameters are not known, and pedotransfer functions are judged to be too imprecise, infiltration experiments (preferably rainfall experiments) could be conducted to produce infiltration curves which then would replace the Richards equation in the optimization approach.

Finally, we note that the present analysis was carried out for ideal conditions involving a homogeneous soil profile. It may be interesting to conduct similar analyses for more realistic field conditions such as for layered soils, soils with sloping surfaces, or for macroporous (structured) soils exhibiting preferential flow paths. The approach followed in this paper is very much applicable also to those conditions. Our ultimate objective in all this was to obtain better estimates of infiltration for larger-scale hydrologic applications, including runoff estimation, for which numerical solutions of the Richards equation are still largely impractical.

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