Tjalling C. Koopmans Research Institute

Discussion Paper Series nr: 07-18

R&D over the Life Cycle

Jaap Bos, Claire Economidou, Mark Sanders

Universiteit Utrecht

Utrecht School of Economics

Tjalling C. Koopmans Research Institute
Utrecht School of Economics
Utrecht University

Janskerkhof 12
3512 BL Utrecht
The Netherlands
telephone +31 30 253 9800
fax +31 30 253 7373
website www.koopmansinstitute.uu.nl

The Tjalling C. Koopmans Institute is the research institute and research school of Utrecht School of Economics. It was founded in 2003, and named after Professor Tjalling C. Koopmans, Dutch-born Nobel Prize laureate in economics of 1975.

In the discussion papers series the Koopmans Institute publishes results of ongoing research for early dissemination of research results, and to enhance discussion with colleagues.

Please send any comments and suggestions on the Koopmans institute, or this series to s.mook@econ.uu.nl

ontwerp voorblad: WRIK Utrecht

How to reach the authors

Please direct all correspondence to the first author.

Mark Sanders
Utrecht University
Utrecht School of Economics
Janskerkhof 12
3512 BL Utrecht
The Netherlands
Max Planck Institute of Economics
D-07745 Jena
Germany
E-mail: m.sanders@econ.uu.nl

Jaap Bos
Utrecht University
Utrecht School of Economics
Janskerkhof 12
3512 BL Utrecht
The Netherlands
E-mail: j.bos@econ.uu.nl

Claire Economidou
Utrecht University
Utrecht School of Economics
Janskerkhof 12
3512 BL Utrecht
The Netherlands
E-mail: c.economidou@econ.uu.nl

Utrecht School of Economics
Tjalling C. Koopmans Research Institute
Discussion Paper Series 07-18

R&D over the Life Cycle

Mark Sanders[a]
Jaap Bos[b]
Claire Economidou[c]

[a]Utrecht School of Economics
Utrecht University
and Max Planck Institute of Economics

[b]Utrecht School of Economics
Utrecht University

[c]Utrecht School of Economics
Utrecht University

June 2007

Abstract
This paper presents a model of the life cycle that drives and is driven by R&D. In the model, firms have the option to improve their quality or to invest R&D resources in efficiency gains. Faced with this tradeoff, young firms opt for quality instead of efficiency improvements, whereas more mature firms will do both. This switch is endogenous and depends on past R&D choices. We explore these two hypotheses empirically using a panel of manufacturing industries across six European countries over the period 1980-1997. Our empirical results provide support for the model's predictions.

Keywords: Growth, Life Cycle, Innovation, Stochastic Frontier Analysis, Manufacturing Industries

JEL classification: C23, L20, L60, O32

1 Introduction

The role of innovation as a key source of long-run economic growth has long been emphasized by the endogenous growth theory (Romer, 1990; Grossman and Helpman, 1991b; Barro and Sala-i Martin, 2004). A related literature stresses the role of R&D-activity in creating these innovations (Griliches, 1980; Aghion and Howitt, 1998) while a companion strand of research suggests that R&D may also contribute to productivity growth by raising the "absorptive capacity" of firms (Cohen and Levinthal, 1989). The empirical evidence in support of these theses is mounting. Typically it is shown that productivity measures are positively correlated with R&D inputs and positive spillovers are shown to exist. [1] A common assumption in both the theoretical literature on endogenous growth and the empirical studies on the impact of R&D is that R&D produces a homogenous output. That is, all innovations are assumed to be symmetric and have the same qualitative impact on productivity. There are, however, good reasons to challenge that assumption and consider the implications of having different types of innovations in one model. [2] Likewise it can be argued that different firms doing R&D, aim for different types of outcomes at different stages of their life cycle.

A broad literature on the life cycle (Vernon, 1966) has been developed in the fields of international trade theory (Krugman, 1979; Grossman and Helpman, 1991a) and industrial dynamics (Utterback and Suarez, 1993; Klepper, 1996, 1997; Audretsch, 1995; Adner and Levinthal, 2001). This literature has explored both the theoretical implications of and empirical regularities over the life cycle of products and industries. Among other things, it has shown that innovation follows a typical pattern over the life cycle. Initially, firms and industries focus on quality improvement and market development turning to cost reductions and rationalization of production only later, when the product is established. In that stage, the product is also liable for outsourcing and off-shoring, exposing the industry to foreign competition. Most of the life cycle literature, however, does not provide theoretical foundations for its observations and the hypotheses it tests.

The purpose of this paper is therefore to present and empirically test a model that explains the interaction between innovation and the industry life cycle. [3]

[1] See, for instance, Coe and Helpman (1995), Coe et al. (1997) for country-level studies and Keller (2002), Scarpetta and Tressel (2002), Griffith et al. (2004), Cameron et al. (2005) for industry-level studies.

[2] For instance, the introduction of a new general purpose technology like lasers is qualitatively different from introducing the 6 bladed safety razor.

[3] Grossman and Helpman (1991a) were among the first to build a model in which the life cycle is driven by endogenous innovation. They used the empirical evidence and theoretical insights from economic growth that suggest R&D is a key factor

Our model follows endogenous growth theory in assuming that industries invest R&D resources to create innovations. In addition, the model allows R&D to generate either quality improvements or efficiency improvements. Then it shows how that choice is affected by the life cycle stage of an industry, but also how the life cycle itself is driven by these two types of innovation and the knowledge spillovers they create. Thereby, we provide an endogenous explanation for the occurrence of industry life cycles that has implications for economic growth. We therefore make a theoretical contribution to two important strands of the literature.

More specifically, we contribute to the endogenous growth literature by accounting for different types of R&D and the endogenous switching among them. Most endogenous growth models (Romer, 1990; Grossman and Helpman, 1991b; Aghion and Howitt, 1998; Barro and Sala-i Martin, 2004; Jones, 2004) treat R&D as a process with a single output. Introducing the life cycle, allows us to link the choice for one or the other type to the life cycle stage of the industry in an intuitive way. Knowledge spillovers between the two types of innovation in turn allow us to explain the endogenous emergence of the industry life cycle. Thereby, this paper contributes to the literature on the life cycle and industrial dynamics as well.

The model provides us with two clear and testable hypotheses. First, efficiency as a result of R&D effort is expected to increase with industry maturity. Second, quality improvements resulting in technical change are expected to decrease with industry maturity. We test our hypotheses using a technique that is relatively new to the empirical literature on growth and innovation (Koop, 2001). As our model distinguishes two types of innovation, we use stochastic frontier estimation, which allows us to separate efficiency gains from technical change. The application of this method to the empirical literature on economic growth and industry productivity analysis is a further contribution we intend to make.

We explore our two hypotheses empirically using a panel of twenty-one manufacturing industries across six European countries over the period 1980-97. In doing so, industries are classified by life cycle stage following related approaches in the literature (Audretsch, 1987). We separate industry efficiency

in generating innovation and is an economic and therefore endogenous decision on behalf of profit motivated agents.[4] Their model bridged the gap between endogenous growth theory and life cycle theories (Krugman, 1979) of international trade, in particular. In Sanders (2005), similar modeling techniques were used to explain (skilled) labor demand dynamics in a closed economy context, while Audretsch and Sanders (2007) present a model in which endogenous R&D-driven life cycle dynamics explain the international division of labor. In all these models, the life cycle is both driving and driven by R&D generated innovations. To our knowledge, decision-based theoretical models of the life cycle have so far not been tested empirically.

gains from quality improvements by estimating a stochastic production frontier that allows us to distinguish between efficiency gains (moving towards the frontier) and technical change (shifts of the frontier). In estimating the effect of maturity on efficiency and technical change, we control for country- and industry-specific fixed effects as well as industries' R&D effort.

Our results provide strong support for both theoretical predictions of the model. Specifically, we find that efficiency is positively and significantly affected by an increase in maturity. Furthermore, the marginal effect of an increase in maturity on efficiency increases with R&D effort. Technical change, however, decreases with maturity. This effect, too, becomes stronger with higher levels of R&D effort, perhaps reflecting the substitution from quality improvement to efficiency improvement that our model predicts.

The rest of the paper is organized as follows. Section 2 presents the theoretical model and the hypotheses under investigation. Section 3 is devoted to a discussion of the data, the measurement of industry maturity and the estimation of a production frontier. Empirical results confirming our hypotheses are then presented in Section 4. Section 5 summarizes the findings and concludes.

2 Theoretical Model

In this section, we introduce a model in which producers have an incentive to do both efficiency increasing and quality increasing R&D, and are allowed to endogenously switch between these two types. The incentive to do R&D in an endogenous growth model always follows from consumers' willingness to pay for quality and quantity. Moreover, to effectuate this willingness to pay, producers need to be able to appropriate the value of innovations in the form of profits.

In our model we therefore assume that consumers consume a range of goods and value both their quality and quantity. These goods are produced by monopolists who make profits and thereby can finance the investment of resources in increasing resource efficiency (generating the same output with less inputs) and increasing product quality (generating higher value-added for the same inputs). Firms then equate the marginal expected discounted value product of R&D resources in both activities by deciding on the direction of their R&D effort.

2.1 Consumers

Let us first consider the consumers. Consumers are generally assumed to maximize utility from consumption over time. Using a separable utility function and standard intertemporal budget constraint, we first solve:

$$\begin{aligned} &\max_{E(t)} \int_0^\infty e^{-\rho(\tau-t)} \log\left(\frac{E(t)}{P(t)}\right) dt \\ &s.t. \\ &\dot{A}(t) = Y(t) + rA(t) - E(t) \end{aligned} \tag{1}$$

where $E(t)$ is the minimum expenditure required at time t to purchase one unit of the direct utility index U defined below, at a price $P(t)$. Consumers discount at a subjective rate ρ, receive an interest rate r on assets A and collect Y labor income, which is taken as given. It is a standard result (see Barro and Sala-i Martin, 2004, Chapter 2) that this yields the Ramsey-rule for optimal consumption:

$$\frac{\dot{E}(t)}{E(t)} = (r - \rho) \tag{2}$$

Together with the intertemporal budget constraint, this also implies an optimal savings path. These savings are required in the model to finance R&D investments later on. Consumers now need to decide on how to spend their consumption expenditure on the n products available. The problem for the consumer at each point in time is given by: [5]

$$\begin{aligned} &\max_{c(i)} U = \left(\int_0^n c(i)^\alpha q(i)^{1-\alpha} di\right)^{1/\alpha} \\ &s.t. \\ &E \leq \int_0^n p(i)c(i)di \end{aligned} \tag{3}$$

where the direct utility is derived from consuming a range of Cobb-Douglas aggregates in quantity $c(i)$ and quality $q(i)$. The solution to this problem yields the quantity, $c(i)$, of each product variety demanded in function of its price, $p(i)$, and quality, $q(i)$, and the minimum price of one unit of U, P.

$$c^D(i) = q(i)\left(\frac{p(i)}{P}\right)^{\frac{-1}{1-\alpha}} \frac{E}{P} \tag{4}$$

where in equilibrium it can be shown that:

[5] To save on notation we drop time arguments t until they become relevant again.

$$P \equiv \left(\int_0^n p(i)^{\frac{-\alpha}{1-\alpha}} q(i) di \right)^{\frac{1-\alpha}{-\alpha}} \tag{5}$$

Thus, we have derived the demand functions for every variety i, where demand for an individual variety is positive in its quality and total expenditure on consumption, and negative in the price p. The demand is also positive in the quality-price index P, reflecting the fact that varieties are substitutes.

2.2 Producers

Turning to production, we then assume that every variety is produced by one firm that sets a price in monopolistic competition with the other firms. The problem of the producers can then be reduced to choosing the price of variety i, $p(i)$, that maximizes profits, π, and is represented by:

$$\begin{aligned} &\max_{p(i)} \pi(i) = x(i)p(i) - wl(i) \\ &s.t. \\ &x(i) = c^D(i) \\ &x(i) = b(i)l(i) \end{aligned} \tag{6}$$

where $\pi(i)$ is profits and $x(i)$ is the volume of variety i produced at quality $q(i)$ using production factor $l(i)$ bought in a competitive market at price w. This implies that $b(i)$ is the firm specific efficiency parameter. The resulting profit maximizing price is the familiar Amoroso-Robinson mark-up over marginal costs:

$$p(i) = \frac{b(i)^{\frac{-(1-\alpha)}{\alpha}} w}{\alpha} \tag{7}$$

Note that higher efficiency $b(i)$ allows firm i to set lower prices, but obviously implies higher profits. To show this, first we define the quality/efficiency index BQ:

$$BQ \equiv \left(\int_0^n b(i)q(i)di \right)^{\frac{-(1-\alpha)}{\alpha}} \tag{8}$$

Factor demand for firm i is then given by:

$$l^D(i) = \frac{b(i)q(i)}{BQ} \frac{\alpha E}{w} \tag{9}$$

Substituting demand (4), price (7) and factor demand (9) into (6) yields the equilibrium profits for firm i:

$$\pi^* = E(1-\alpha)\frac{q(i)b(i)}{BQ} \tag{10}$$

These profits are positive in both quality and efficiency. By increasing quality and/or efficiency relative to the other firms, a firm can increase its flow of rents. It is these additional rents that provide an incentive to invest R&D resources in improving quality or efficiency. To get to the value of a marginal improvement in either quality or efficiency, we first need to consider the value of the flow of profits that one obtains without increasing quality or efficiency. This is the discounted value of an infinite flow of rents at constant efficiency and quality levels, b_0 and q_0, respectively:

$$V(t) = \int_t^\infty e^{-r(\tau-t)}\pi(q_0, b_0, \tau)d\tau \tag{11}$$

In Appendix A, it is shown that (11) can be written as:

$$V(t) = \frac{(1-\alpha)E}{r - \dot{E}/E + B\dot{Q}/BQ}\frac{b_0 q_0}{BQ} \tag{12}$$

Decision makers are assumed to expect that profits will grow at some constant rate due to increases in consumption expenditure and the quality/efficiency index BQ. Taking the derivative of this expression with respect to b_0 and q_0 respectively, yields the value of marginal increases in efficiency, V_b, and quality, V_q:[6]

$$V_b(t) = \frac{(1-\alpha)E}{r - \dot{E}/E + B\dot{Q}/BQ}\frac{q(i)}{BQ} \tag{13a}$$

$$V_q(t) = \frac{(1-\alpha)E}{r - \dot{E}/E + B\dot{Q}/BQ}\frac{b(i)}{BQ} \tag{13b}$$

2.3 R&D

Now, consider the innovation functions that specify the process of efficiency and quality improvement:

[6] It is the undiscounted value, to be precise. Solving a Hamiltonian for the firms to optimally invest in R&D over their lifetime would give these values times e^{-rt} as the shadow price of marginal innovations over time, but as the two types of R&D compete for the same resource, the discounting factor will appear on both sides of the arbitrage equation and, therefore, can be ignored.

$$\dot{b}(i) = \phi R_b(i) q(i)^{\beta} b(i)^{1-\beta} \tag{14a}$$
$$\dot{q}(i) = \psi R_q(i) q(i)^{1-\gamma} b(i)^{\gamma} \tag{14b}$$

where R_b and R_q are the quantities of R&D effort devoted to improvements in efficiency and quality, respectively, ψ and ϕ are R&D productivity parameters, and β and γ are parameters between 0 and 1 that capture the knowledge spillovers of R&D. These functions have the following symmetric properties: they are linear in R&D effort at the firm level and exhibit diminishing returns to firm specific knowledge spillover from both quality and efficiency improvements in the past, but constant returns to both. The marginal value product of R&D resources in firm i can now be obtained by taking the derivative of (14a) times (13a) and (14b) times (13b) with respect to R&D effort. We obtain:

$$MVP_b(t) = \frac{(1-\alpha)E}{r - \dot{E}/E + B\dot{Q}/BQ} \frac{\phi q(i)^{1+\beta} b(i)^{1-\beta}}{BQ} \tag{15a}$$
$$MVP_q(t) = \frac{(1-\alpha)E}{r - \dot{E}/E + B\dot{Q}/BQ} \frac{\psi q(i)^{1-\gamma} b(i)^{1+\gamma}}{BQ} \tag{15b}$$

Arbitrage will imply that all R&D effort in a firm is aimed at the activity that has the highest marginal value product. This implies that in (steady state) equilibrium, the marginal value products must equalize. To ensure that new firms start by improving quality only, we must impose the parameter restriction:

$$\psi > \phi \tag{16}$$

where we have normalized the initial quality and efficiency parameters, b_0 and q_0 to one. To avoid all R&D effort being concentrated in one firm (the least mature), we also need to assume that R&D resources are less than perfectly mobile across firms. [7] A convenient short-cut is then to simply assume R&D resources are firm specific and their supply is given. At this point, we might also have assumed that quality improvement and efficiency enhancement require very different types of R&D workers or that R&D competes for productive resource $l(i)$ at the firm level, but as our focus is on the life cycle, the above

[7] Alternatively, we may assume diminishing returns to R&D effort in the innovation production functions, such that the marginal productivity of R&D workers goes to infinity when R&D employment falls. In that case, the allocation of R&D over firms would be endogenous, and although this is a possible interesting extension to the model, it is beyond the scope of this paper.

assumption is more convenient. It leads us to the following proposition: [8]

Proposition 1 *For a given number of varieties, n, there exists a steady state equilibrium in the model in which utility grows at a constant rate. That is the case when all products are mature and, in their mature stage, all firms develop quality and efficiency at the same constant growth rate.*

In this paper, however, we are less interested in the steady state. Moreover, as we aim to test our hypotheses using industry-level data, we need to develop industry-level hypotheses. We propose that due to the constant introduction of new varieties and the demise of old ones, a steady state will never be reached in any industry. If new varieties are introduced regularly, then, at any point in time, an industry consists of new and mature firms. In the mature firms, R&D is aimed at developing both hard-to-measure quality improvements and easy-to-measure efficiency improvements through cost reductions. In fact, our model predicts an exact 50-50 split, where the knowledge spillovers will ultimately ensure that both the quality and efficiency parameter grow at the same rate. In contrast, the new firms predominantly focus their R&D effort on hard-to-measure quality improvements. In the absence of diminishing returns to R&D, they even devote all R&D effort to that purpose. [9] At the industry level, this will imply that mature industries are dominated by mature firms and R&D generates relatively more efficiency gains. In young industries, on the other hand, young firms dominate and R&D is aimed at quality improvements, causing lower efficiency levels and gains.

As a result, we can infer two testable hypotheses from our model. First, the model predicts that efficiency increases with the maturity of an industry, as a larger part of R&D effort is aimed at generating the same output with less inputs. Second, the model predicts that quality improvements decrease with an industry's maturity, as industries devote a smaller share of their R&D effort to generating a higher value-added for the same inputs. The first hypothesis is considerably stronger than the second one: not only does it follow from the model that even mature industries still do some quality improvements, but in addition, quality improvements are significantly more difficult to measure than efficiency changes.

To empirically test our hypotheses, we need to operationalize the concepts of maturity, efficiency and quality improvements, and then test whether the relationship between R&D intensity and efficiency gains and/or quality improvements depends on the life cycle stage of the industry under consideration. We will do the former in the next section before we turn to our empirical results in Section 4.

[8] See Appendix A for the proof.

[9] Note that our parameter restriction on ϕ and ψ ensures this in the model.

3 Data and Methodology

3.1 *Data*

We use data from 21 two-, three- and four-digit manufacturing industries in six European countries (Finland, France, Germany, Italy, Netherlands and Spain) over the period 1980-1997. Annual raw data are retrieved from various sources. Data on industry output (value-added) and investment (for constructing capital stock) are retrieved from the OECD (2002) *Structural Analysis Database* (STAN). Data on labor are extracted from the Groningen Growth and Development Centre (www.ggdc.net) *60-Industry Database*. Finally, data on R&D are obtained from the OECD (2002) *Business Enterprise Expenditure on Research and Development* (BERD). The same International System of Industries Classification code (ISIC, rev. 3) was used in all data sources. Definitions of the variables and data sources as well as the manufacturing industries considered in our analysis and their ISIC codes are presented in Appendix B.

3.2 *Maturity*

For testing our hypotheses, we require a measure of maturity that is industry-specific, monotonic in maturity and continuous.[10] Audretsch (1987) classified industries as "mature" or "young" by looking at their sales dynamics. We follow his approach and first estimate the following equation:

$$ln(S_{ijt}) = \beta_i + \beta_1 t + \beta_2 t^2 + \varepsilon_{ijt} \tag{17}$$

where $ln(S_{ijt})$ is the log of real sales in country i, industry j at time t, and t and t^2 is time (1 in 1980) and time squared, respectively.[11] We include a country-specific fixed effect and estimate the above equation for each industry.

Audretsch then suggests to consider the sign and significance of β_1 and β_2 in order to classify industries. For example, he writes that an industry is "classified as growing when either β_2 was positive and statistically significant at the 90% level, or β_2 was statistically insignificant, but β_1 was positive and statistically significant" (Audretsch, 1987, p. 301).

[10] In the data, mature industries can become young again. Think, for example, of the phone industry that was rejuvenated with the introduction of mobile phones.
[11] Note that Audretsch (1987) used real sales. We use the log of real sales here, so we can calculate semi-elasticities that are comparable across industries.

For our purpose, rather than having a binary measure, we construct a measure that is continuously increasing in maturity. We therefore consider the effect of an increase of t on the log of real sales: $\partial \ln(S_{ijt})/\partial t = \beta_i + \beta_2 * t$. By evaluating this semi-elasticity at the mean of t for all industries, we now have a measure for industry growth in real sales over time. Maturity, M_{jt}, is then defined as: $[(\partial \ln(S_{ijt})/\partial t] * (-1)$.

Industry-specific maturity estimates are shown in Table 3 in Appendix B. [12] Previous studies have ranked industries according to high/low-tech classification (Scarpetta and Tressel, 2002; Silverberg and Verspagen, 1994). Our maturity classification is in line with their results, if one assumes that low-tech industries are more mature than high-tech industries. Among the most mature industries are the oil industry, the textile industry and the ship-building industry. The least mature industries include the office machinery, the communication and the precision instruments industry.

3.3 Efficiency

The efficiency parameter, $b(i)$, of our model, intends to capture the efficiency at which firms transform inputs into output. Empirically, we can only proxy this efficiency at the industry level by assuming that all firms in an industry, across countries, in principle have access to the same production technology. However, we also assume, as the empirical literature Koop (2001)has shown, that many firms can operate below their industry's best practice production frontier. We measure their inefficiency relative to that benchmark over time to construct a proxy for $b(i)$. [13] Such inefficiencies can be measured by means of a stochastic frontier production model. [14] A frontier production function defines the maximum output achievable, given the current production technology and available inputs. If all industries produce on the boundary of a common production set that consists of an input vector with two arguments, physical capital (K) and labor (L), then output can be described as:

$$Y^*_{ijt} = f(K_{ijt}, L_{ijt}, t; \beta) \exp\{\nu_{ijt}\} \tag{18}$$

[12] Note that we assume that an industry, in different countries, is at the same stage of the life cycle.
[13] Inefficiency, thus measured, is by definition bounded from above as no firm can be more efficient than the benchmark. Our theoretical model does not impose that restriction on efficiency for computational convenience. The discontinuity that would otherwise be introduced seriously complicates the mathematical exposition of our model and adds little in terms of intuition.
[14] Stochastic frontier analysis (SFA) was introduced by Aigner et al. (1977), Battese and Corra (1977), and Meeusen and Broeck (1977).

where Y_{ijt}^* is the frontier (maximum) level of output in country i, in industry j at time t, production technology is characterized by function f and parameter vector β, t is a time trend variable that captures Hicks-neutral technological change (Barro and Sala-i Martin, 2004), and ν_{ijt} is and i.i.d. error term distributed as $N(0, \sigma_\nu^2)$, which reflects the stochastic character of the frontier.

Two points are noteworthy regarding equation (18). First, the frontier, as it is defined, represents a set of maximum outputs for a range of input vectors. Therefore, at any moment in time, it is estimated from observations on a number of industries. Conventional growth empirics (Scarpetta and Tressel, 2002; Griffith et al., 2004; Cameron et al., 2005) that study inefficiency usually just benchmark all industries to one - the leader industry, i.e., the industry with the highest level of productivity in the sample. An implicit, however non-trivial, assumption in this literature is that the leading industry itself constitutes the frontier and the single benchmark for all other industries. Second, our modeling approach treats the frontier as stochastic through inclusion of the error term ν_{ijt}, which accommodates noise in the data, and therefore allows for statistical inference. In this respect, our modeling approach fundamentally differs from other (non-parametric) frontier analyses (Färe et al., 1994) that do not allow for random shocks in the frontier. [15]

However, some industries may lack the ability to employ existing technologies efficiently (e.g. due to mismanagement) and therefore produce less than the frontier output. If the difference between maximum and actual (observable) output is represented by an exponential factor, $\exp\{-\upsilon_{ijt}\}$, then the actual output, Y_{ijt}, produced in each country i in industry j at time t can be expressed as a function of the stochastic frontier output, $Y_{ijt} = Y_{ijt}^* \exp\{-\upsilon_{ijt}\}$, or equivalently:

$$Y_{ijt} = f(K_{ijt}, L_{ijt}, t; \beta) \exp\{\nu_{ijt}\} \exp\{-\upsilon_{ijt}\} \tag{19}$$

where $\upsilon_{ijt} \geq 0$ is assumed to be i.i.d., with a normal distribution truncated at zero $N(0, \sigma_\upsilon^2)$, and independent from the noise term, ν_{ijt}. [16]

An industry is inefficient if it fails to absorb the best-practice technology. The advantage of our framework is that it enables us to distinguish between efficiency changes and technical change. This contrasts with conventional (non-frontier) studies where a single productivity measure captures both efficiency change and technical change.

[15] Comprehensive reviews of frontier approaches can be found in Kumbhakar and Lovell (2000) and Coelli et al. (1998).

[16] When estimating equation (19), we obtain the composite residual $\exp\{\varepsilon_{ijt}\} = \exp\{\nu_{ijt}\} \exp\{-\upsilon_{ijt}\}$. Its components, $\exp\{\nu_{ijt}\}$ and $\exp\{-\upsilon_{ijt}\}$, are identified by the λ $(= \sigma_\upsilon / \sigma_\nu)$ for which the likelihood is maximized (for an overview, see Coelli et al., 1998).

Table 1
Frontier estimation results

	Coeff.	Std.Err.	
k	-1.181	0.038	***
l	1.938	0.060	***
$\frac{1}{2}k^2$	0.206	0.011	***
$\frac{1}{2}l^2$	-0.310	0.016	***
kl	0.011	0.012	
t	0.260	0.011	***
$\frac{1}{2}t^2$	0.000	0.001	
kt	-0.024	0.002	***
lt	-0.011	0.002	***
$\sigma = (\sigma_\upsilon^2 + \sigma_\nu^2)^{1/2})$	1.247	0.023	***
$\lambda = \sigma_\upsilon / \sigma_\nu$	3.135	0.091	***

Notes: Lower case letters y, k and l denote logs. Log likelihood value = -1904.333, fixed effect estimations, coefficients significant at the 1/5/10% level (*/**/***). Obs.=2,268.

We estimate equation (19) with a translog specification and true fixed effects (Greene, 2005). Table 1 contains our results. A value of 3.135 of parameter λ indicates that a significant part of our estimation residual consists of inefficiency.[17] In the left panel of Figure 1 below, we plot the efficiency distribution.[18] The most efficient industry is 84% efficient, whereas the least efficient industry is 6.8% efficient.[19] On average, industries are 50% efficient and efficiency grows with 0.8% per year.

3.4 Technical Change

Quality improvement as represented by increases in $q(i)$ in our model, can best be proxied by computing the outward shifts of the production frontier we previously estimated. Quality improvements as conceptualized in the model, increase the value-added for all possible input-vectors and therefore shift the production frontier out. In the empirical literature, however, such shifts are referred to as technical change, a slightly broader concept than quality improvements. This is why, in our operationalization, we prefer to refer to technical

[17] To be precise, it tells us that slightly more than 75% of our total standard error (σ) consists of inefficiency (σ_υ), whereas 25% is noise (σ_ν) - hence the ratio of 3.135/1.
[18] Figure 1 contains kernel density estimates.
[19] Most efficient was the machinery industry in Germany in 1997, least efficient was the Finnish office machinery industry in 1983. Industry efficiency scores, defined as $\mathit{eff}_{ijt} = \exp\{-\upsilon_{ijt}\}$ ($0 \leq \mathit{eff}_{ijt} \leq 1$ where $\mathit{eff}_{ijt} = 1$ implies full efficiency), are available from the authors.

change, tc. We follow Altunbas et al. (1999) and calculate technical change by taking the derivative of equation (18), $\partial \ln Y^{*}_{ijt}/\partial t = \beta_t + \beta_{\frac{1}{2}t^2} * t + \beta_{kt} * k + \beta_{lt} * l$. As it is clear from the right panel of figure 1, we observe both technical progress ($tc > 0$) and technical regress ($tc < 0$).[20]

Figure 1. Distributions of efficiency and technical change

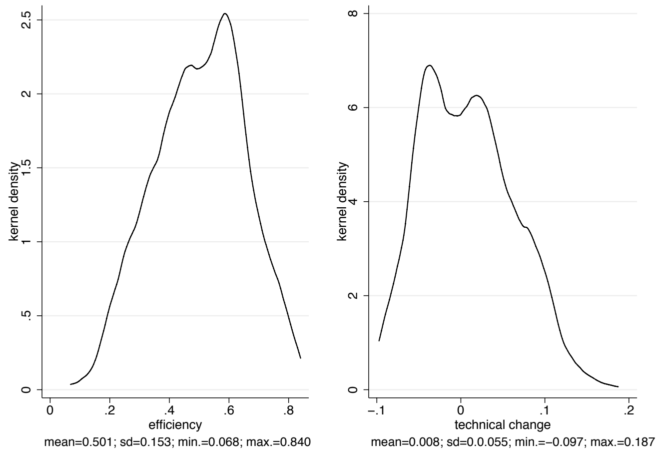


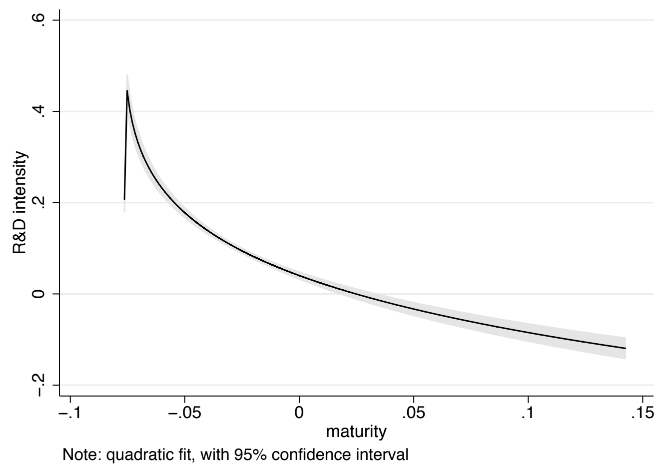
Figure 1 shows the distributions of our efficiency and technical change measure. Both measures approximate the normal distribution very well, which facilitates our empirical testing in the next section.[21] In addition, we check how our measures relate to the business cycle, since the latter may also affect our measures and we may have a spurious relationship. To do so, we compare the development in efficiency, technical change, maturity and R&D intensity with growth in real GDP.[22] As it turns out, none of our measures has a correlation with GDP growth that is significantly different from zero.[23] Consequently, we feel confident about our hypotheses tests in the next section.

[20] Correlation between efficiency and technical change is negative (-0.447), but insignificant.
[21] Since efficiency scores are often truncated, some studies have opted for truncated regressions when explaining efficiency (Bos and Kool, 2006). In our study, this is not necessary.
[22] Real GDP data derived from the OECD (2006) *Main Economic Indicators*.
[23] We calculate correlations both for our whole data set, and on a country-by-country basis. For the whole data set, correlations of the real GDP growth with eff_{ijt} (- i.e. the transformed υ), tc_{ijt}, and M are -0.0470, 0.1019, and 0.0153, respectively.

4 Empirical Results

In this section, we test our two hypotheses. Our first hypothesis predicts that efficiency change decreases with maturity. Our second hypothesis predicts that technical change increases with maturity.

Figure 2. R&D effort over the life cycle



In order to test these hypotheses, we need to control for the actual R&D effort of an industry. We do so by controlling for R&D intensity, defined as the ratio of R&D and value-added. [24]

Figure 2 shows that R&D effort decreases with maturity, although some industries increase their R&D effort as they become highly mature. [25]

In order to test our hypotheses, we estimate the following equations [26] :

$$\upsilon_{ijt} = \beta_i + \beta_1 M_{jt} + \beta_2 R\&D_{ijt} + \beta_3 (M_{jt} * R\&D_{ijt}) + \varepsilon_{ijt} \tag{20a}$$

$$tc_{ijt} = \beta_i + \beta_1 M_{jt} + \beta_2 R\&D_{ijt} + \beta_3 (M_{jt} * R\&D_{ijt}) + \varepsilon_{ijt} \tag{20b}$$

[24] In using R&D intensity we aim to control for size differences, which may be strongly correlated with maturity. We also estimated with R&D flows, and results are qualitatively similar.

[25] Note that, if we assume that competition is most fierce for very young and very mature industries, the relationship in Figure 2 is in line with recent work by Aghion et al. (2005) who find an inverse U-shaped relationship between innovation and competition.

[26] We have also estimated equation (20b) using efficiency scores, eff_{ijt} - i.e. the transformed υ that is bounded between zero and one. Results are qualitatively similar, but somewhat less significant, as the transformed measures are truncated.

where tc_{ijt} is technical change, υ_{ijt} is the efficiency term, M_{jt} is our maturity measure and $R\&D_{ijt}$ is R&D intensity. Our hypotheses are confirmed if $\partial \upsilon_{ijt}/\partial M_{jt} > 0$ and $\partial tc_{ijt}/\partial M_{jt} < 0$ for every level of R&D intensity. We estimate equations (20a) and (20b) with country-specific fixed effects, and calculate conditional marginal effects following Brambor et al. (2006). [27] Table 2 contains our estimation results. [28]

Table 2
Explaining technical change and efficiency change

Hypothesis 1				Hypothesis 2			
efficiency	Coef.	Std. Err.		technical change	Coef.	Std. Err.	
M	-0.617	0.343	*	M	-0.114	0.042	***
$R\&D$	-0.141	0.095		$R\&D$	-0.041	0.012	***
$M \cdot R\&D$	6.210	2.481	***	$M \cdot R\&D$	-2.139	0.307	***
intercept	-0.726	0.009	***	intercept	0.003	0.001	***
σ_u	0.221			σ_u	0.045		
σ_e	0.291			σ_e	0.036		
ρ	0.367			ρ	0.605		
F-test coef.	26.57	(3,2259)	***	F-test coef.	73.90	(3,2259)	***
F-test fe	218.38	(5,2259)	***	F-test fe	575.52	(5,2259)	***

M is maturity, defined as (-1) times the conditional marginal effect calculated in section 3.2; $R\&D$ is R&D intensity, defined as the ratio of R&D expenses to value-added; fixed effect estimations, with country-industry specific fixed effects u_i; σ_u and σ_e represent the variance of the fixed effect and error term, respectively; ρ is fraction of variance due to u_i; F-tests for joint significance of coefficients and fixed effects; degrees of freedom between brackets; significance at the 1/5/10% level (*/**/***)

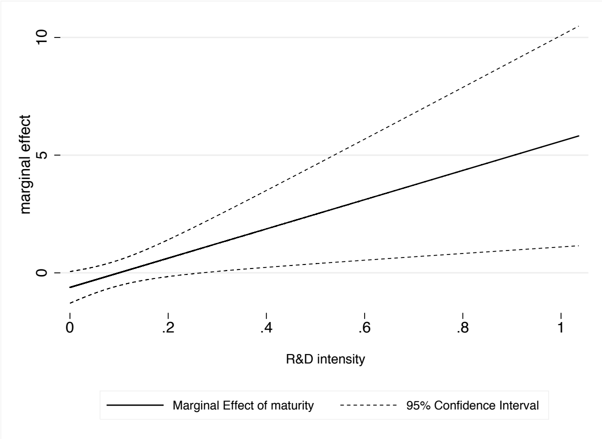
Figure 3 shows the (conditional) marginal effect of an increase in maturity on efficiency, for every level of R&D intensity. On average, an increase in maturity has a positive effect on efficiency for all levels of R&D intensity. Hence, we find that our first hypothesis is confirmed.

In fact, mature industries with a high R&D intensity, benefit from an even larger increase in efficiency. However, the effect of an increase in maturity is most significant at low levels of R&D intensity.

Likewise, Figure 4 shows the (conditional) marginal effect of an increase in maturity on technical change, for every level of R&D intensity. Clearly, maturity positively and significantly affects technical change for all levels of R&D

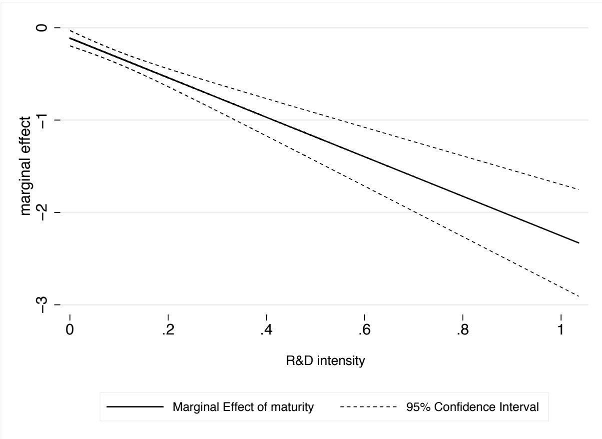
[27] Since our maturity measures are industry-specific, we cannot include industry-specific fixed effects.
[28] Our results are qualitatively similar when we use R&D flows instead of R&D intensities. However, in the former case, since mature industries tend to be larger as well, we would not be sure whether we were measuring a size effect or the impact of R&D.

Figure 3. Efficiency and maturity



intensity. The more mature industries become, the lower their technical change is. This confirms our second hypothesis.

Figure 4. Technical change and maturity



Interestingly, this negative effect becomes even more negative with increasing R&D intensity. In line with our model, this may reflect the fact that mature industries substitute a significant part of their R&D effort, previously aimed at technical change, to increase their efficiency. Summing up, we have found significant evidence in support of both of our hypotheses.

5 Conclusion

In this paper, we have developed a model of innovation over the industry life cycle. We show that industries engage in R&D to generate quality and efficiency improvements. The aim of R&D, either to improve quality or efficiency, depends on the life cycle stage of an industry. However, the life cycle itself is endogenous, and driven by these two types of innovation and the knowledge spillovers they create.

Our model leads to two testable hypotheses. First, efficiency is expected to increase with maturity. Second, technical change is expected to decrease with maturity. Both hypotheses are tested empirically for a sample of twenty-one manufacturing industries across six European countries over the period 1980-97. In order to measure efficiency and technical change, we estimate a stochastic production frontier.

Our empirical results support both of our hypotheses. The marginal effect of an increase in maturity on efficiency is positive and increases with R&D effort. The marginal effect of an increase in maturity on technical change is negative and becomes stronger with R&D effort, as industries substitute efficiency improvement for quality improvement.

Our findings have important policy implications. Policy makers generally agree that higher R&D spending is desirable and are willing to subsidize and/or give tax credits to firms who do R&D. Both our model and our empirical results show that the impact of an increase in R&D effort, however, depends on *where* innovation takes place. Innovation policies aimed at mature industries will increase efficiency, but cannot be expected to result in large technological advancements. The latter, originate primarily in new industries. However, the pace at which they can be realized decreases sharply with R&D effort, suggesting that the returns to investments in innovation in new industries decrease rather rapidly with the level of investment. Designing an effective and efficient innovation policy requires careful consideration of these results.

References

Adner, R. and D. Levinthal (2001). Demand heterogeneity and technology evolution: Implications for product and process innovation. *Management Science 47*, 611–628.

Aghion, P., N. Bloom, R. Blundell, R. Griffith, and P. Howitt (2005). Competition and innovation: An inverted-u relationship. *Quarterly Journal of Economics 120(2)*, 701Ð728.

Aghion, P. and P. Howitt (1998). *Endogenous Growth Theory*. Cambridge, MA: MIT Press.

Aigner, D., C. Lovell, and P. Schmidt (1977). Formulation and estimation of stochastic frontier production function models. *Journal of Econometrics 6(1)*, 21–37.

Altunbas, Y., J. Goddard, and P. Molyneux (1999). Technical change in european banking. *Economics Letters 64(2)*, 215–221.

Audretsch, D. (1987). An empirical test of the industry life cycle. *Weltwirtschaftliches Archiv 123(2)*, 297–308.

Audretsch, D. (1995). Innovation, growth and survival. *International Journal of Industrial Organization 13(4)*, 441–457.

Audretsch, D. and M. Sanders (2007). Globalization and the rise of the entrepreneurial economy. *Jena Economic Research Papers 2007-003*.

Barro, R. and X. Sala-i Martin (2004). *Economic Growth*. Cambridge, MA: MIT Press.

Battese, G. and G. Corra (1977). Estimation of a production frontier model, with application to the pastoral zone of eastern australia. *Australian Journal of Agricultural Economics 21*, 169–179.

Bos, J. and C. Kool (2006). Bank efficiency: the role of bank strategy and local market conditions. *Journal of Banking and Finance 30(7)*, 1953–1974.

Brambor, T., W. Clark, and M. Golder (2006). Understanding interaction models: Improving empirical analyses. *Political Analysis 14(1)*, 63–82.

Cameron, G., J. Proudman, and S. Redding (2005). Technological convergence, r&d, trade and productivity growth. *European Economic Review 49(3)*, 775–807.

Coe, D. and E. Helpman (1995). International r&d spillovers. *European Economic Review 39(5)*, 859–887.

Coe, D., E. Helpman, and A. Hoffmaister (1997). North-south r&d spillovers. *Economic Journal 107*, 134–149.

Coelli, T., D. Prasada Rao, and G. Battese (1998). *An Introduction to Efficiency and Productivity Analysis*. Boston/Dordrecht/London: Kluwer Academic Publishers.

Cohen, W. and D. Levinthal (1989). Innovation and learning: Two faces of r&d. *Economic Journal 99*, 569–596.

Färe, R., S. Grosskopf, M. Norris, and Z. Zhang (1994). Productivity growth, technical progress, and efficiency change in industrialized countries. *American Economic Review 84(1)*, 66–84.

Greene, W. (2005). Reconsidering heterogeneity in panel data estimators of the stochastic frontier model. *Journal of Econometrics 126*, 269–303.

Griffith, R., S. Redding, and J. v. Reenen (2004). Mapping the two faces of r&d: Productivity growth in a panel of oecd industries. *Review of Economics and Statistics 86(4)*, 883–895.

Griliches, Z. (1980). *New Developments in Productivity Measurement*, Chapter Returns to R&D Expenditures in the Private Sector. Chicago, IL: Chicago University Press.

Grossman, G. and E. Helpman (1991a). Endogenous product cycles. *The Economic Journal 101*, 1214–1229.

Grossman, G. and E. Helpman (1991b). *Innovation and Growth in the Global Economy.* Cambridge, MA and London: MIT Press.

Jones, C. (2004). Growth and ideas. *NBER Working Paper No. 10767.*

Keller, W. (2002). Trade and the transmission of technology. *Journal of Economic Growth 7(1)*, 5–24.

Klepper, S. (1996). Entry, exit, growth, and innovation over the product lifecycle. *American Economic Review 86*, 562–583.

Klepper, S. (1997). Industry life cycles. *Industrial and Corporate Change 6*, 145–182.

Koop, G. (2001). Cross-sectoral patterns of efficiency and technical change in manufacturing. *International Economic Review 42*, 73–103.

Krugman, P. (1979). A model of innovation technology transfer and the world distribution of income. *Journal of Political Economy 87*, 253–266.

Kumbhakar, S. and C. Lovell (2000). *Stochastic Frontier Analysis.* Cambridge, UK: Cambridge University Press.

Meeusen, W. and J. v. d. Broeck (1977). Efficiency estimation from cobb-douglas production functions with composed error. *International Economic Review 18*, 435–444.

Romer, P. (1990). Endogenous technical change. *Journal of Political Economy 98(5)*, 71–102.

Sanders, M. (2005). *Technology and the Decline of Unskilled Labour Demand: A Theoretical Analysis of the US and European Labour Markets.* Cheltenham, UK, Northampton, MA (USA): Edward Elgar Publishers.

Scarpetta, S. and T. Tressel (2002). Productivity and convergence in a panel of oecd industries: Do regulations and institutions matter? *OECD Working Paper No. 342.*

Silverberg, G. and B. Verspagen (1994). Learning, innovation and economic growth: A long-run model of industrial dynamics. *Industrial and Corporate Change 3*, 199–223.

Utterback, J. and F. Suarez (1993). Innovation, competition, and industry structure. *Research Policy 22*, 1–21.

Vernon, R. (1966). International investment and international trade in the product cycle. *Quarterly Journal of Economics 80*, 190–2007.

Appendix A: Proofs

Derivation of equation (12)

First, we show that equation (11) can be written as (12) when agents expect rates of aggregate spending and quality and efficiency improvement to be constant. Substituting equation (10) into (11) yields:

$$V(t) = \int_t^\infty e^{-r(\tau-t)} E(\tau)(1-\alpha)\tfrac{q(i)b(i)}{BQ(\tau)} d\tau \tag{21}$$

For a constant (expected) growth rate of the quality and efficiency index, BQ, and expenditures, E, we can use:

$$BQ(T) = \int_t^T BQ(t) e^{B\dot{Q}(\tau)/BQ(\tau)\tau} d\tau = BQ(t)e^{B\dot{Q}/BQ(T-t)} \tag{22}$$

and

$$E(T) = E(t)e^{\dot{E}/E(T-t)} \tag{23}$$

to write:

$$V(t) = \int_t^\infty e^{(-r+\dot{E}/E-B\dot{Q}/BQ)(\tau-t)} E(t)(1-\alpha)\tfrac{q(i,\tau)b(i,\tau)}{BQ(t)} d\tau \tag{24}$$

As every new firm starts with exogenously given starting values for $b(i)$=b_0 and $q(i)$=q_0, and R&D wage costs exactly exhaust the discounted gains from future innovations, the value of a firm at time t can be written as in equation (12):

$$V(t) = \frac{(1-\alpha)E}{r - \dot{E}/E + B\dot{Q}/BQ} \frac{b_0 q_0}{BQ} \tag{25}$$

Proof of proposition 1

Proposition 1 reads: For a given number of varieties, n, there exists a steady state equilibrium in the model in which utility grows at a constant rate. That is the case when all products are mature and, in their mature stage, all firms develop quality and efficiency at the same constant growth rate.

The proof of this proposition follows from the fact that for any given n, firms will allocate all R&D labor to the R&D activity with the highest marginal value product (equations (17) and (18) respectively). However, as these marginal value products are negatively related to the level of quality and efficiency achieved in that industry, all industries eventually end up in a situation where the marginal value products are equalized and henceforth remain equal. As n

is constant, this situation will arise for all industries in the end and at that point a steady state is reached in which quality and efficiency expand at the same rate. To see this one can equalize equations (17) and (18) and solve for the steady state ratio of quality to efficiency:

$$\frac{b(i)}{q(i)} = \left(\frac{\varphi}{\psi}\right)^{\frac{1}{\beta+\gamma}} \tag{26}$$

In any industry where this ratio is lower (higher) all R&D effort goes into increasing efficiency (quality) until the ratio is (re)established. For given n all industries must converge to this ratio and equations (15) and (16) then tell us that quality and efficiency expand at:

$$\frac{\dot{b}(i)}{b(i)} = \frac{\dot{q}(i)}{q(i)} = \varphi^{\frac{\gamma}{\beta+\gamma}} \psi^{\frac{\beta}{\beta+\gamma}} \frac{R^*(i)}{2} \tag{27}$$

where $R^*(i)$ is the level of total R&D in industry i. Assuming, for computational convenience and without loss of generality that these levels are equal across industries at R^*/n, BQ expands at that rate by the definition in equation (8).

From equation (2) we know that expenditure on consumption will grow at a constant rate for any constant interest rate. Given the budget constraint and a fixed labor supply that also implies wages will grow at that rate. Normalizing nominal expenditure to one implies that interest rate equals ρ in equilibrium and together with the constant growth rate of BQ this implies that prices will fall at rate $-\frac{1-\alpha}{\alpha}G_{BQ}$, where G_{BQ} represents the growth rate of index BQ. By equation (5) this implies that the growth rate of P is $-G_{BQ}$ and, consequently, utility grows at a constant rate of $2\frac{1-\alpha}{\alpha}G_{BQ} = \frac{1-\alpha}{\alpha}\varphi^{\frac{\gamma}{\beta+\gamma}}\psi^{\frac{\beta}{\beta+\gamma}}\frac{R^*}{n}$ in the steady state.

Appendix B: Data and Sources

Value-Added (*Y*): gross value-added expressed in 1995 constant prices (euros). Gross value-added was deflated by implicit value-added deflators to yield deflated gross value-added expressed in 1995 constant prices (euros). We follow the OECD (2002) practice for the construction of the implicit value-added deflators. Data on gross value-added are retrieved from the OECD (2002) *Structural Analysis Database*(STAN).

Physical capital (*K*): gross capital stock expressed in 1995 constant prices (euros). Following common practice in the literature (e.g. Hall and Jones, 1999), we employ the perpetual inventory method to construct a proxy for capital stock, using data on gross fixed capital formation (GFCF). The initial value for the 1980 capital stock is specified as $\mathrm{K}_{1980} = \mathrm{GFCF}_{1980}/(\mathrm{g} + \delta)$, where g is the average geometric growth rate of the gross fixed capital formation (constant prices) series from 1970 to 1980 and δ is the depreciation rate. Instead of assuming a constant depreciation rate, we use the average service life (ASL) of capital per industry (ISDB98-*methods used by OECD countries to measure stocks of fixed capital*, OECD, 1993). Each industry's capital stock is constructed as capital stock minus depreciated capital stock plus gross fixed capital formation ($\mathrm{K}_\mathrm{t} = (1-\delta) * \mathrm{K}_{\mathrm{t}-1} + \mathrm{GFCF}_\mathrm{t}$). Data on gross fixed capital formation are retrieved from the OECD (2002) *Structural Analysis Database*(STAN).

Labor (*L*): annual total hours worked in an industry (in thousands). Data are retrieved from the Groningen Growth and Development Centre (GGDC, 2006) *60-Industry Database.*

Research and Development Intensity (*R&D*): R&D expenditures to value-added ratio. Data on R&D expenditure are retrieved from the OECD (2002) *Business Enterprise Research and Development* (BERD).

Table 3
Manufacturing industries ranked by maturity

Industry	Abbr.	ISIC code	b0		b1		b2		R^2	M_{jt}
Petroleum products	COK	23	10.004	***	-0.149	***	0.006	***	0.178	0.045
Textile products	TEX	17-19	9.754	***	-0.013		0.000		0.014	0.009
Ships and boats	SHI	351	7.903	***	-0.035	***	0.002	***	0.040	-0.002
Food products	FOD	15-16	10.781	***	-0.007		0.001	***	0.170	-0.004
Non-ferrous metals	NFM	272+2732	8.373	***	-0.021		0.002	*	0.039	-0.008
Mineral products	ONM	26	9.188	***	0.010		0.000		0.302	-0.012
Wood products	WOD	20	8.633	***	-0.001		0.001	***	0.418	-0.014
Iron and steel	IAS	27+2731	9.159	***	-0.015		0.002	***	0.407	-0.015
Manufacturing n.e.c.	MA	36+37	8.957	***	0.003		0.001	***	0.629	-0.018
Chemicals	CHE	24 less 2423	9.822	***	0.008		0.001	*	0.576	-0.018
Paper products	PAP	21-22	9.952	***	0.018	***	0.000		0.657	-0.019
Machinery	MAC	29	9.766	***	0.007		0.001	***	0.679	-0.025
Motor vehicles	MOT	34	9.347	***	0.055	***	-0.002	***	0.299	-0.025
Fabricated metal	FAB	28	9.511	***	0.019	***	0.000		0.660	-0.026
Aircraft + spacecraft	AIR	353	7.108	***	0.021		0.000		0.307	-0.029
Rubber/plastics	RUB	25	8.710	***	0.041	***	0.000	*	0.871	-0.034
Pharmaceuticals	PHA	2423	8.041	***	0.050	***	0.000		0.813	-0.042
Electrical machinery	ELE	31	8.579	***	0.040	***	0.000		0.779	-0.042
Precision instruments	MED	33	7.818	***	0.052	***	-0.001		0.584	-0.042
Communication	RAD	32	8.220	***	0.017		0.003	*	0.499	-0.059
Office machinery	OFF	30	6.969	***	0.055		0.001		0.418	-0.075

Notes: fixed effect estimations, coefficients significant at the 1/5/10% level (*/**/***); Abbr. = Abbreviation; ISIC code (rev. 3); M(aturity) = $[\partial \ln(S_{ijt})/\partial t] * (-1)$ evaluated at mean value of t.