A life of learning in Leiden

The mathematician Frans van Schooten (1615–1660)

Cover design by Janneke Visser

Printed by GVO drukkers & vormgevers B.V. | Ponsen & Looijen
ISBN 978-90-6464-743-7

A life of learning in Leiden

The mathematician Frans van Schooten (1615–1660)

Een geleerd leven in Leiden

De wiskundige Frans van Schooten (1615–1660)

(met een samenvatting in het Nederlands)

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Universiteit Utrecht
op gezag van de rector magnificus, prof. dr. G.J. van der Zwaan, ingevolge
het besluit van het college voor promoties in het openbaar te verdedigen op
woensdag 12 februari 2014 des middags te 4.15 uur

door

Jantien Gea Dopper

geboren op 7 maart 1981 te Veendam

promotor: Prof. dr. J.P. Hogendijk

Contents

List of figures

Figures 2.1, 2.2, 2.4, 2.5, 2.8, 3.1, 3.5, 3.6, 4.1, 5.8, 5.9, 8.6, and 8.11 are reproduced by courtesy of Leiden University Library.
Figures 2.3, 8.4 , A.1, A.2 and A.3 are reproduced by courtesy of University of Groningen Library.
Figures 2.6 and 2.7 are reproduced by courtesy of Rijksmuseum, Amsterdam.
Figures 2.10 and 2.11 are reproduced by courtesy of Museum De Lakenhal, Leiden.
Figure 2.9 is reproduced by courtesy of Musée du Louvre, Paris.
Figure 8.5 is reproduced by courtesy of Tresoar, Leeuwarden.

List of tables

CHAPTER 1

Introduction

In his work on the delights of Holland, Jean Nicolas de Parival (1605–1669) gives the following description of the teaching of mathematics in Leiden:

> And in this church (...) public lectures in the mathematical sciences are held in Dutch, daily from eleven until twelve o'clock (except Wednesdays and Saturdays), for the benefit of all illiterate people, such as masons, carpenters and the like. A good many of them attend at this time, without coats, but equipped with their sticks and leather aprons etc., which is a very funny sight. The professor, who lectures in Dutch in his ordinary professorial robe or over-garment (just like the other Latin professors), is the erudite and widely famous Mr Franciscus van Schooten.[1]

De Parival describes the teaching practice at the Duytsche Mathematicque, an institution which was associated with Leiden University and which offered mathematical lectures in the vernacular. The lectures were attended by an audience which consisted for a large part

[1] "En in die Kercke, waer de Engelsche nu predicken, in dit Bagijne-Hoff, worden alle dagen, (behalven 's Woensdaeghs, en Saterdaeghs) van elf tot twaelf uuren, openbare Lessen gedaen in de Neerlandsche Tael, in de Mathematische Konsten, tot gerief van alle ongeletterden, als Metselaers, Timmer-luyden, en diergelijcke meer; die haer dan met hoopen in die tijdt hier vinden; sonder mantels, maer met hare stocken, en schoots-vellen, &c. versien; dat dan seer kluchtigh om sien is. Den Professor, die duytsche lessen voor haer doet, evenwel in sijnen gewoonlijcken aensienlijcken Professors-Tabbaert, ofte Rock, (soo wel als alle de andere Latijnsche Professoren de hare doen,) is den Hoogh-geleerden, en Wijdt-vermaerden D. Franciscus van Schooten." [Parival, 1661, 188-189]. The description by De Parival of Leiden is studied in [Strien-Chardonneau, 2006].

of craftsmen. De Parival portrays the situation as a comic contrast between the craftsmen in their daily working clothes, who had just left their workshops in order to attend the lectures, and the neatly dressed professor in his robe, which was the common dress of professors at Leiden University. The board of the university ordered them to wear the robe during public lectures.[2]

A second contrast also emerges: the audience is described as illiterate, which should be understood as unable to read and write in Latin, whereas the professor is called "erudite" and "widely famous". The professor, Frans van Schooten jr. (1615–1660), belonged to the world of learning, and was able to read and write in Latin. He nevertheless delivered his daily lectures in the vernacular, as was necessary because of the difference in background and education between him and his audience.

In the history of mathematics, Van Schooten is best known for his activities in connection with the *Géométrie* (1637) of René Descartes (1596–1650). In this work, Descartes presented a new method for solving geometrical problems which had a profound impact on the development of mathematics. The dissemination of the new method was largely due to Van Schooten. The *Géométrie* was written in French, and in 1649 Van Schooten published a Latin translation so that the work became available in the scholarly language that was understood all over Europe. By adding his own comments he made the obscure text of Descartes comprehensible. He also propagated the new method among his students, who included young men from high-born families. The best known examples are Christiaan Huygens (1629–1695), the son of a diplomat, Johan de Witt (1625–1672), born in a patrician family in Dordt and the later grand pensionary and de facto head of state, and Johannes Hudde (1628–1704), son of a wealthy merchant and the later burgomaster of Amsterdam. Van Schooten added their contributions to the second edition of *Geometria* which was published in 1659–1661 in two volumes. This seminal work became the standard handbook of Cartesian geometry in the second half of the seventeenth century. Thus we can understand why Van Schooten was considered "erudite and widely famous".

In the quoted passage, De Parival describes another side of the mathematical activities of Van Schooten: his daily lecturing in the vernacular to craftsmen. This aspect of Van Schooten's work is little known but interesting, and shows that different types of mathematical activities took place in the sphere of Leiden University.

The aim of this thesis is to provide a new picture of the life and mathematical work of Frans van Schooten jr. He was one of the most influential Dutch mathematicians of the seventeenth century. The first chapter is a biography, and in the subsequent chapters I will study his contrasting mathematical activities which have been mentioned above. My study is based on original sources: published books, preserved correspondence, as well as some mathematical manuscripts and archival material related to Van Schooten which has rarely if ever been consulted before.[3]

In this introduction I give a number of historical and mathematical contexts and discussions which can serve as a background for interpreting Van Schooten, and to which

[2] The professors of the Duytsche Mathematicque wore these robes since 1633 during their public lectures. Leiden University Library (UBL), Archief van Curatoren, 1574–1815, inv. nr. 22, f. 100v.-101r.

[3] The manuscripts are located in University of Groningen Library (UBG). In appendix A a detailed description of these manuscripts is provided.

this study may contribute. Then I provide an overview of the existing literature on Van Schooten. Subsequently I explain my motivation and approach in this thesis and I give an overview of the structure of this thesis. Finally I list some conventions which may facilitate reading.

1.1 Contexts and themes

Early modern mathematics

In the early modern period, the word *mathematics* had a much broader scope than today. Mathematics denoted the study of number and magnitude, or of quantity in general, and within this general definition, a distinction was made between mathematics in the abstract sense and mathematics in relation to the material world. The former was called *pure mathematics*, and the latter *mixed mathematics*. Traditionally, pure mathematics consisted of arithmetic, dealing with discrete quantity, and geometry, dealing with continuous quantity. Following Aristotelian views, (classical) geometry and arithmetic were two distinct fields with their own characteristics. Mixed mathematics encompassed all disciplines in which real objects were counted and measured. Traditionally, astronomy and music were in this domain, and mixed mathematics gradually expanded and came to include chronology, geography, gnomonics, fortification, mechanics, optics, perspective, statics and surveying.[4]

The early modern concept of mixed mathematics does not coincide with the modern concept of applied mathematics. The modern term "applied mathematics" presupposes a division of the domain of mathematics into pure and applied, and the application of mathematics to domains (physics, astronomy, etc.) that are no longer considered to be mathematical themselves. In addition, applied mathematics has a connotation of usefulness and relevance for society.[5]

In the seventeenth century, both pure mathematics and mixed mathematics underwent rapid developments. By 1500 the various sub-domains of mixed mathematics were relatively few, whereas by 1700 mixed mathematics consisted of an extensive array of disciplines. In 1661, the Jesuit Gaspar Schott (1608–1666) counted almost thirty disciplines among mixed mathematics in his *Cursus mathematicus*.[6] In the domain of pure mathematics, new sub-disciplines emerged as well: the analysis of finite quantities (letter algebra and analytic geometry), and the analysis of infinite quantities (differential and integral calculus).[7]

The most significant internal development in mathematics during the life of Van Schooten was the emergence of Cartesian geometry, which was the first step towards analytic geometry. The publication of the *Géométrie* of Descartes in 1637 changed the methods and scope of mathematics in a fundamental way. The major contribution of

4 [Dear, 2011, 150], [Andersen and Bos, 2006, 969], and [Epple et al., 2013, 658].

5 On the changing perception of mathematics see [Mulder, 1990] and [Siegmund-Schultze, 2013].

6 [Remmert, 2013, 673].

7 [Andersen and Bos, 2006, 969].

the *Géométrie* is the idea that many geometrical problems can be translated into algebraic equations in two unknowns (the well-known x and y), and that the solution of such geometrical problems can be facilitated by successive algebraic manipulations of these equations. The new link between geometry on the one hand and equations in two unknowns on the other opened up new perspectives and led to the study of curves by means of their equations.

My views on early modern mathematics and Cartesian geometry in particular have been strongly influenced by the work of Henk Bos. His work is characterized by a careful examination of original sources and by an emphasis on the issues early modern mathematicians deemed important. Bos tries to understand the methodological problems which mathematicians met in their own time, and his work on Descartes's *Géométrie* in particular has been a major source of inspiration for me.[8]

Frans van Schooten jr. was a key figure in the dissemination and understanding of Cartesian geometry in the seventeenth century. In Holland, he brought people together, attracted students, and encouraged them in exploring the new possibilities which Cartesian geometry offered. In this process he made a major contribution to the development of mathematics.

The work and teaching of Frans van Schooten cover a substantial part of mathematics in the early modern sense. He was well versed in pure mathematical subjects such as classical geometry and arithmetic, and had been introduced to the major Greek mathematical works by the humanist scholar Jacob Golius (1596–1667). In the domain of mixed mathematics, he published a work on perspective, wrote a manuscript on fortification and in his role of professor at the Duytsche Mathematicque he lectured on fortification, sundials, surveying and other subjects.

The Dutch Republic

Van Schooten spent most of his life in the Dutch Republic. This Republic was the unanticipated result of the revolt of the Dutch against the absolutist ambitions of the King of Spain, who was also the ruler of the provinces of the Netherlands. The political, religious and social structures of the Republic were fundamentally different from those in the rest of Europe. A strong central authority was absent and the different provinces were in effect autonomous entities. The social culture was dominated by a burgherculture and the society was more egalitarian compared to the surrounding countries. The first half of the seventeenth century also saw a high degree of social mobility, offering possibilities for inhabitants to climb the social ladder. Frans van Schooten jr. and his father are exponents of this social mobility.[9]

The inhabitants of the Republic enjoyed a relative high degree of freedom of expression compared to other parts of Europe. This freedom was reflected in the limited extent of

[8] In his magnum opus *Redefining geometrical exactness*, Bos combines the themes which he explored in his earlier work, and provides a profound insight in the ideas on problem solving and construction in the early modern period as well as the impact of the work of Descartes, [Bos, 2001]. Other articles on Cartesian geometry include [Bos, 1981], [Bos, 1992], and [Bos, 1997a].

[9] On the general conditions in the Dutch Republic see [Israel, 1995], [Frijhoff and Spies, 2004], and [Prak, 2005].

(pre)censorhip. Publications were tolerated as long as the content was not considered to lead to a disturbance of the public and social order.[10]

These conditions offered a favourable atmosphere for the development and dissemination of ideas, knowledge and practices and offered possibilities for various activities. They also attracted foreigners. Amongst these people we find the French thinker and mathematician René Descartes who lived over twenty years in the Dutch Republic and who also published his writings in the Republic.

Klaas van Berkel has argued that the contribution of the Dutch Republic to the changing perceptions in knowledge (i.e., the scientific revolution) has been underestimated. He argues that the Dutch Republic was a valuable laboratory for the study of the practice of science and he claims that scientific developments in the Dutch Republic were stimulated by the lack of barriers to the circulation of ideas and the travel of people. He addresses two items which are of interest for the case of Van Schooten. These are the interaction between craftsmen and scholars, and the role of universities in the development of knowledge.[11]

From this perspective Van Schooten is a very interesting figure in the Dutch landscape. At the Duytsche Mathematicque, he maintained contacts with practitioners and craftsmen, but at the same time, he had close connections with the Leiden academic community and he organized private lectures for talented students. He furthermore had close relationships with leading thinkers and mathematicians outside the academic community such as René Descartes and Christiaan Huygens.

Dissemination of knowledge

In the practice of science and scholarship, the communication of ideas is essential. This is true today[12] and it was true in the seventeenth century as well. Having a brilliant idea is not enough. A new idea had to be communicated to the learned audience, by means of letters, private conversations, books, lectures or other means of communication. New ideas had to be circulated, studied, explained and discussed in order to be understood and in order to have influence and impact.[13]

In this process of circulation, explanation and discussion of new ideas, Van Schooten made a major contribution by his editions of the complete works of the mathematician François Viète (1540–1603) and by his translation of and commentary on Descartes's *Géométrie*. Van Schooten was well aware of the importance of publishing and circulating ideas and he actively encouraged talented young men in his network to publish their findings by offering them all kinds of practical and intellectual assistance.

Leiden University played an important role in the development and the dissemination of knowledge. It was a place where young, intelligent and ambitious people met and contacts were made.[14] The contact between Van Schooten and Descartes was probably

[10] [Rasterhoff, 2012, 122-125] and [Eijnatten, 2004].

[11] [Berkel, 2010].

[12] The importance of communicating scientific results is summarized by the phrase "Publish or perish".

[13] [Netten, 2012].

[14] On Leiden University in the seventeenth century see [Otterspeer, 2000] and [Hulshoff-Pol, 1975]. Zoeteman studied the student population of Leiden University and Sluijter has paid attention to the organizational structures of Leiden University, [Zoeteman, 2011] and [Sluijter, 2004].

arranged within this academic setting by Jacob Golius, professor of Eastern languages and mathematics. Golius had a good relationship with Descartes – he had drawn Descartes's attention to the Pappus problem, the problem which played a major role in Descartes's *Géométrie* as we will see in chapter 3 – and he counted Van Schooten amongst his pupils in mathematics.

In the seventeenth century, Leiden University had a good international reputation and attracted students from all over Europe, primarily from protestant regions. It was also a place where new ideas could be discussed. Although new ideas where not always part of the official lectures, the university was open to new views, and recent developments in mathematics could be shared with students during private lectures. As a young student, Van Schooten was introduced to the work of Viète in the private lectures of Golius. When Van Schooten was a professor himself, he gathered around him young men who were registered as students at Leiden University – but not necessarily as students in mathematics – and who were mostly members of relatively high-born families. They were attracted to mathematics by its intellectual challenge.

1.2 Previous work on Van Schooten

Frans van Schooten jr. is a good example of someone who stayed mostly outside the limelight of historians and mathematicians. Only a limited number of studies pay attention to his person and his work.

The work of Van Schooten, and especially his two volumes on Cartesian geometry, was widely read in the seventeenth century. The *Geometria* was reprinted twice, in 1683 and in 1695, so the work was still popular in the two final decades of the seventeenth century. Also in the seventeenth century, some letters by Van Schooten were published in publications on Descartes's correspondence.[15] In the eighteenth century, Montucla mentioned Van Schooten in his survey of the history of mathematics. Montucla considered the two editions of *Geometria* as very useful and even "containing everything needed for understanding the *Géométrie* of Descartes".[16] During the rest of the eighteenth and the first half of the nineteenth century, interest in Van Schooten was limited and remained confined to encyclopedias and biographical dictionaries.[17]

Interest in Van Schooten revived in the last quarter of the nineteenth century, when historians showed a renewed concern with original sources other than printed works. The donation of a Van Schooten manuscript to the collection of Leiden University Libary in 1875 was the occasion for the Leiden professor of mathematics Bierens de Haan, who had a strong interest in history and bibliography of mathematics, to write an article about the Van Schooten family. His article included a bibliographical overview of the printed works of Frans van Schooten sr. and Frans van Schooten jr.[18] and the first description of a collection of manuscripts associated to the Van Schooten family in the Groningen University

[15] See for instance [René Descartes, 1692].

[16] [Montucla, 1758, 123-124].

[17] See for instance [Kampen, 1826] and [Nieuwenhuis, 1840].

[18] [Haan, 1878]. The donated manuscript is nowadays known as UBL, BPL 1013. On Bierens de Haan, see

Library.[19]

In 1894 the prominent Dutch mathematician D.J. Korteweg delivered a lecture at the anniversary of the University of Amsterdam on the "Golden Age of mathematics in the Netherlands". He addressed the role of Van Schooten and his students in the dissemination of Cartesian geometry and briefly discussed the relation between Van Schooten and Descartes, a theme which I will further explore in this thesis.[20]

The decades around the turn of the century saw an increasing number of publications of original sources. The correspondence and complete works of the main scholars of the seventeenth century such as Descartes and Huygens appeared in print, and letters to and from Van Schooten were included in some of these publications. Over a hundred letters from the correspondence between Frans van Schooten jr. and Christiaan Huygens between the years 1648 and 1660 were published in the *Oeuvres complètes* of Christiaan Huygens, of which Bierens de Haan was the editor.[21] Four letters by Van Schooten addressed to Constantijn Huygens were published by Worp in 1915, and Fruin published another letter by Van Schooten in 1919.[22] Three letters from the correspondence between Van Schooten and Descartes appeared in the *Oeuvres de Descartes* in 1898–1903.[23] The *Oeuvres de Descartes* furthermore contained a discussion of a manuscript by Van Schooten.[24] Yet, the letters by Van Schooten have hardly been used as sources for studies on his life and works.

The Dutch historian of science Cornelis de Waard wrote the articles on Van Schooten sr. and jr. in the *Nieuw Nederlandsch Biografisch Woordenboek* (New Dutch Biographic Lexicon, 1927), which continues to be a valuable survey of the sources and literature.[25] De Waard furthermore published an article on hitherto unknown letters by Fermat which he had discovered in a manuscript of Van Schooten.[26]

In 1962, the German historian of mathematics J.E. Hofmann published a concise study of Van Schooten and his mathematical work: *Frans van Schooten der Jüngere*.[27] This was the first study with the aim to give a complete picture of Frans van Schooten jr. Hofmann provided a biographical sketch with emphasis on Van Schooten's printed works and then discussed several mathematical contributions from these works. Hofmann's rather technical approach gives a good impression of the various geometrical problems Van Schooten was interested in. Hofmann also wrote the article on Van Schooten in the *Dictionary of*

[Ommen and Bos, 2003].

19 The main deficiency of the article is the misidentification of some work by Frans jr. as that of his father Frans sr., with erroneous interpretations and conclusions as a result. For instance, based on a comparison of the handwriting of Frans sr. and the handwriting of UBG Hs 108, Bierens de Haan concluded that UBG Hs 108 was written by two clerks for Frans sr. We now know that the manuscript was written by Frans jr.

20 [Korteweg, 1894].

21 Huygens retrieved the letters he had sent to Van Schooten after the death of the latter, see [Huygens, 1888], [Huygens, 1889], and [Huygens, 1890].

22 [Huygens, 1915] and [Witt, 1919].

23 [Descartes, 1898, 574-582] and [Descartes, 1903, 318-322, 336-340].

24 [Descartes, 1908, 635-647]. Following Bierens de Haan, the editors erroneously attributed the manuscript to Frans van Schooten sr. instead of Frans van Schooten jr.

25 [Molhuysen et al., 1927].

26 [Waard, 1917]. The letters are in UBG, Hs 110.

27 [Hofmann, 1962].

Scientific Biography,[28] where he discussed not only Van Schooten's role in the dissemination and clarification of Cartesian geometry but also some of his original contributions from the *Exercitationum mathematicarum libri quinque* (Five books on mathematical exercises).

Publications have also appeared on other aspects of Van Schooten's life and activities, unrelated to Cartesian geometry. Knappert discussed the family life of Van Schooten on the basis of notarial acts.[29] In an article on the iconographic tradition of Descartes, Nordström studied the portrait of Descartes that was made by Van Schooten.[30] Andersen studied Van Schooten's treatise on perspective and showed that it was strongly influenced by Simon Stevin.[31] In an article on Van Schooten's interest in conic sections, Dijksterhuis stressed the dual background of Van Schooten, who was raised in two mathematical traditions: the classical geometry of Golius and the Duytsche Mathematicque of his father Frans van Schooten sr.[32] Van Schooten's professorship at the Duytsche Mathematicque has not attracted much attention among historians. In their studies on the history of surveying and fortification, Van Winter and Van den Heuvel touched on the role of Van Schooten, without paying much attention to the content and practice of his lectures at the Duytsche Mathematicque.[33]

Van Schooten's contributions to the dissemination and understanding of Cartesian mathematics were studied in Le Noir's dissertation *The social and intellectual roots of discovery in seventeenth century mathematics* (1974).[34] The main concern of this work is the theoretical concept of conceptual change in mathematics in general. Le Noir identified a conceptual change in the transition from Cartesian geometry to a full-fledged analytic geometry in the later seventeenth century. According to Le Noir, Descartes used algebra and equations merely as "a repository of geometrical information". Over time, mathematicians gradually shifted towards accepting an equation of a curve as an object of study in its own right. Le Noir argued that Van Schooten never made this conceptual switch and that his use of algebra remained closely related to geometrical constructions. According to Le Noir, Van Schooten's students De Witt and Van Heuraet were the first to make the transition to a full-fledged analytical geometry.[35]

In his dissertation *Facets of seventeenth century mathematics in the Netherlands* (1987), Van Maanen investigated the significance of Van Schooten's work for the development of Cartesian geometry. On the mathematical level, Van Maanen emphasized the added value of Van Schooten's commentary to the original text of Descartes: Van Schooten explained, simplified and corrected the original text and widened its scope. Besides this, Van Maanen also identified the important social and communicative aspects of Van Schooten's activities in teaching, publishing and initiating research with his students. Van Maanen's dissertation also included a detailed article on the mathematical career of Van Schooten's

[28] [Hofmann, 1975].
[29] [Knappert, 1938].
[30] [Nordström, 1958].
[31] [Andersen, 1990] and [Andersen, 2007].
[32] [Dijksterhuis, 2011].
[33] [Winter, 1988] and [Heuvel, 2006].
[34] [Le Noir, 1974].
[35] [Le Noir, 1974, 173-253].

student Hendrick van Heuraet.[36]

Sébastien Maronne has recently studied parts of Van Schooten's work in his dissertation (2007) on the development of the theory of curves and equations within Cartesian geometry. Maronne focussed on Descartes's solution of the problem of Pappus, which will be also be discussed in Part 2 of this thesis, and also on Descartes's theory of normals, and also on a geometrical problem which was the subject of a controversy between Descartes and the Dutch mathematician Stampioen.[37] In addition, Maronne has dated one of Descartes's letters to Van Schooten in a published article.[38]

1.3 Scope

The purpose of this thesis is to shed new light on the mathematical life and work of Frans van Schooten. Van Schooten's major achievements were the interpretation, dissemination and exploration of Cartesian geometry, but there is more to Van Schooten than Cartesian geometry alone. Below I will sketch the questions and approaches which I have used in this thesis, and then I will give an overview of the content of this thesis.

1.3.1 Approach and motivation

My main concern is to study Van Schooten in his own time and in connection with contemporary developments and practice, both mathematical and extra-mathematical. My focus is on the way in which Van Schooten shaped his mathematical life of learning in Leiden. This life comprised his scholarly activities which resulted in published books, but also his teaching activities centered around Leiden University and the Duytsche Mathematicque.

My approach to the history of mathematics is inspired by my conviction that mathematics is a human activity and that developments in mathematics are driven by both internal and external factors. I aim to study mathematics within a context of people and society, because the interaction between mathematicians and society shaped mathematics and the way it was practiced. For instance, an institution such as the Duytsche Mathematicque can not be understood without taking into account the political, military and social-economic situation in the Dutch Republic.

My approach has been inspired by the recent development of the historiography of science from a history of ideas to a history of practices, and recent attention among historians to cultural aspects of science. These recent trends have also had their impact on historians of mathematics, who have increasingly been interested in other aspects of mathematics than its internal development, and in other mathematicians than the great ones. Historians of mathematics now pay more attention to the practice of mathematics and to the social and cultural aspects and meaning of mathematics in various contexts. In her study

[36] [Maanen, 1987].
[37] [Maronne, 2007].
[38] [Maronne, 2006].

on the content and use of early modern Dutch elementary arithmetic textbooks, for example, Marjolein Kool argues that the study of the minor mathematical works is as important as the study of the major works, because the minor works contributed to the dissemination of mathematics to a wide audience.[39] In England, Stephen Johnston has analyzed the use of mathematics in society by identifying the "mathematical practitioners", that is to say, the men who engaged in the active, practical use of mathematics.[40]

Although the main focus of my study is not on the change of mathematical concepts and internal mathematical developments, this does not imply a reaction on my part against internalistic studies of the history of mathematics. However, my perspective is a distinguishing feature of this thesis compared to previous work on Van Schooten, especially the recent studies on the development of Cartesian geometry. Authors such as Maronne and Le Noir[41] were motivated by a desire to understand internal developments in mathematics, and as a result their work is of an internalistic and technical nature. Yet, my focus on Van Schooten's life of learning does not imply that this thesis will be devoid of technical mathematics. I believe that a study of mathematical content as well as context is needed in order to understand the mathematician Van Schooten.[42]

The picture of Van Schooten in this thesis is based on a wider variety of sources than earlier historians have been able to use. Thus, besides Van Schooten's printed works – which are a natural and important source for any study on Van Schooten – I have also used Van Schooten's published correspondence, as well as unpublished archival material of the Van Schooten family and Leiden University. I have also studied the hitherto unpublished manuscripts by Van Schooten, his father and his half-brother. These manuscripts have hardly ever been consulted before and are a valuable source of Van Schooten's teaching at the Duytsche Mathematicque.

1.3.2 Survey

The thesis is divided into three parts. The first part is the most extensive biography of Frans van Schooten jr. that has been written so far to my knowledge. Up till now, the most detailed biography was the concise account by Hofmann.[43] Hofmann focussed on the treatises that Van Schooten had read, and the scholarly work he wrote, but I will pay more attention to the people with whom he interacted and the activities he undertook. This means also his relations with printers, students (also others than the well-known Huygens, Hudde, Van Heuraet and De Witt), the curators of the university, and his network in general. His network extended beyond the circle of mathematicians, but we will see that he needed his extended network in order to function as a mathematician. I will also discuss his family background, and his relationship with Descartes, which was important

[39] [Kool, 1991, 13].

[40] [Johnston, 1996].

[41] In my opinion, the title of Le Noir's work (*The social and intellectual roots of discovery in seventeenth century mathematics*) does not cover its contents, because this title suggests a social context whereas the work itself is a internalistic study.

[42] De Wreede has argued similarly for the case of Willebrord Snellius [Wreede, 2007, 6].

[43] This account is contained in the first chapter (8 pages) of [Hofmann, 1962].

for his own views on mathematics and which will help us to understand his role in the dissemination of Cartesian geometry.

In the second part of this thesis (chapters 3-6) I will investigate Van Schooten's attitude towards the mathematical work of Descartes by means of a case study of a geometrical problem. The problem dates back to Greek antiquity and is nowadays called the four line Pappus problem, after Pappus of Alexandria (ca. 250 AD). In the *Géométrie*, Descartes claimed to have solved the problem by means of his new method, but his solution was criticized by contemporary mathematicians and turned out to be incomplete after some time. Descartes had described the solution as one conic section only, but the complete solution to the problem consisted of two conic sections. Descartes's solution of the four line Pappus problem will be introduced in chapter 3. In the course of my analysis, I will propose a new interpretation of one of Descartes's notations, which is consistent with the text as it was printed in the *Géométrie* and in the Latin version *Geometria* which was published by Van Schooten. Hitherto, it was believed by historians of mathematics who are trained in modern concepts and notations that there were some curious printer's errors in the *Géométrie* and the Latin version *Geometria*. My new interpretation shows that the text in the *Géométrie* and the Latin *Geometria* is correct. In the following chapter I will study the debate on the four line Pappus problem after the publication of the *Géométrie*. I will show that some of Van Schooten's comments on the Pappus problem in *Geometria (1649)* were conceived in close cooperation with Descartes himself, even though Descartes denied that he was involved in the new Latin edition.

After Descartes passed away in 1651, the incompleteness of his solution of the Pappus problem became even more obvious, and, as a result, Van Schooten found himself in an awkward position. He felt that he faced an impossible choice between mathematical correctness on one hand, and Descartes's legacy on the other hand. In the end, Van Schooten admitted to some extent that Descartes's solution seemed to be incomplete, but we will see that he continued to defend the Cartesian legacy in very curious ways.

The third part of this thesis (chapters 7-10) deals with the teaching of Van Schooten. As we have seen above, Van Schooten spent a considerable amount of time in lecturing on mathematics to a predominantly non-academic audience. This was done at the Duytsche Mathematicque, the institution associated with Leiden University in order to provide mathematical instruction in the vernacular. Van Schooten had lectured there since 1635 on an irregular basis as a replacement for his father, and after the death of his father he succeeded him as professor of the Duytsche Mathematicque in 1646. The first chapter 7 introduces the reader to the institution Duytsche Mathematicque and its relation with Leiden University, and also to the manuscripts which are the main source for the next two chapters and which have hardly been studied hitherto. A detailed description of these manuscripts is given in Appendix A.

wing chapters 8 and 9 deal with the subjects of Van Schooten's lectures. The main aim of these chapters is to identify the content and the level of the courses which he taught, and to investigate how he shaped the teaching program at the Duytsche Mathematicque. The content of the courses will be examined and assessed from various perspectives. It will be shown that Van Schooten continued to lecture on the same subjects as his predecessors, but that he changed the content of his lectures on the traditional subjects, and also introduced new subjects to the curriculum. The former chapter 8 deals with the subjects which were part of the curriculum of the Duytsche Mathematicque since its establishment in 1600. In arithmetic I compare Van Schooten's lectures with the lectures in Latin by professors at other universities, as well as the lectures in the vernacular by arithmetic teachers outside the universities. For fortification, I compare the contents of Van Schooten's lectures to the ideas of contemporaries on the subject. The latter chapter 9 is about subjects which Van Schooten jr. added to the curriculum, namely logarithms and algebra. In chapter 10, a survey of Van Schooten's teaching at the Duytsche Mathematicque will be presented on the basis of the material of the three previous chapters.

The thesis concludes with a final chapter 11, in which I have collected some reflections and insights from the previous chapters in order to give a final overview and characterization of Van Schooten and his mathematical activities.

1.4 Conventions

In this thesis the following conventions will be used.

By "Van Schooten" I denote Frans van Schooten jr. (1615–1660), the main subject of this thesis. His father was also called Frans van Schooten (ca. 1582–1645) and was a professor at the Duytsche Mathematicque as well. Because the two successive professors Frans van Schooten at the Duytsche Mathematicque in Leiden have confused various historians, I add the abbreviations sr. and jr. to the family name whenever necessary. The Huygens family can be a similar cause of ambiguity, because Van Schooten had relations with the father Constantijn Huygens (1596–1687), and his two sons Constantijn (1628–1697) and Christiaan (1629–1695). I will mention the first name in case confusion can arise. In case the first name is not mentioned, it should be clear from the context which member of the Huygens family is meant.

I have decided to indicate by Duytsche Mathematicque the insitution associated to Leiden University in which mathematics was taught in the vernacular. This is the term which one will find in the sources as well. In the recent literature, this institute has also been called *engineering school*, but I have not used this term because it is not found in the contemporary seventeenth century sources.[44] Most people attending the lectures at the Duytsche Mathematicque were not official students, but they were called *auditors*. In this thesis I have avoided the word *auditors* and instead I freely use the word student to denote attendees of the Duytsche Mathematicque.

For easy understanding, I have given an English translation of the titles of works written in languages other than English. Exceptions are the three works which all translate in English as *Geometry*, and to which I will often refer in this study. These are Descartes's French *Géométrie* of 1637, the first Latin edition *Geometria* of 1649 and the second Latin edition *Geometria* of 1659. I will refer to Descartes's work by the original French title. To

[44] The word engineer was employed in sources related to the Duytsche Mathematicque, but the term engineering school was not.

distinguish between the two Latin editions I add the year of publication: *Geometria (1649)* and *Geometria (1659)*.

In the spelling of Dutch surnames I follow the Dutch conventions for upper case and lower case of the prefixes. Thus, the prefix of the surname is written in upper case (*Van Schooten*), and in the first name together with a surname the prefix is in lower case (*Frans van Schooten*). In determining an alphabetic order, I adhere to the Dutch convention of ignoring the prefix. This means that in the bibliography *Van Schooten* is placed as *Schooten, van,* before *Stevin*.

Part I

Life, work and network

CHAPTER 2

Biography

The aim of this chapter is to give a coherent picture of the life of Frans van Schooten, taking into account his activities, his contacts in his native Leiden and elsewhere, and the places he visited. Van Schooten is mainly known in the modern literature for his contributions to the dissemination and understanding of Cartesian geometry. This chapter will pay attention his other activities as well.

The main order of this chapter is chronological. In three thematic sections I elaborate on some themes which do not fit well in a purely chronological narrative. These sections include Van Schooten's family ties, his relation with Descartes, and his work as a draughtsman.

2.1 Youth

Frans van Schooten was born on 15 June 1615 in Leiden, as the son of Frans van Schooten sr. and Jannetgen Harmensdr. van Hogervorst.[1] He grew up in the heart of Leiden's

[1] The date of birth is mentioned on the commemorative medal which was made after Frans van Schooten jr.'s death. This date of birth agrees with other relevant sources: In the archive of the Orphan Chamber, Frans is said to be around 9 years of age on 14 February 1625, and on 15 May 1631 he entered Leiden University at the age of 16. If both statements are correct, Van Schooten was born between 15 May 1615 and 13 February 1616, and the date of 15 June 1615 fits nicely in this interval. See Regionaal Archief Leiden (RAL), Weeskamer van Leiden, (1343) 1397–1860 (1866), nummer toegang 518, inv. nr. 113+6, f. 336v. and UBL, Archief van Senaat en Faculteiten, 1575–1877, inv. nr. 9, 13. The cast of the commemorative medal is part of the collection of Museum De Lakenhal in Leiden, inv. nr. 3630.1 and 3630.2.

academic quarter. His parental house was located at the Rapenburg between the Houtstraat and the Kloksteeg.[2] From the windows of this house one had a nice view of the main building of Leiden University just across the canal. His father, who was professor in the Duytsche Mathematicque, lectured in the Faliebagijnkerk which was located at the same side of the canal, in a short walk of only 150 meters to the south.

The neighbours on both sides were connected to the university as well. On the north side, Claudius de Lartillier rented rooms in his house to students. On the other side, Aelius Everhardus Vorstius (1565–1624) was a professor of medicine and physics. The members of the Elzevier family of printers lived on the same part of the Rapenburg.[3] They were the printers of the University and ran a small shop across the canal at the entrance of the university main building.[4] Thus, Van Schooten was surrounded by the atmosphere of the university already as a young boy.

The Van Schooten family was of Flemish descent, just like many other families in seventeenth-century Leiden. The upheavals in the Southern Netherlands made many of its inhabitants leave for the Northern Netherlands and a substantial number settled there. Mainly due to these migrants, many cities in the Northern Netherlands boomed during the period 1575–1625. In Leiden, the population increased almost fivefold from 10,000 to 47,000 inhabitants. In the subsequent fifty years, the population stabilized around 50,000, and Leiden was the second largest Dutch city after Amsterdam.[5]

In 1584, Frans's grandfather, also called Frans van Schooten or Verschooten (1557– ca. 1640), came to Leiden together with his wife and his son Frans, the future father of Frans jr. Originally from Nieukerke, West-Flanders, the grandfather registered as a citizen in Leiden on 19 May 1584. He made a living as a baker and therefore I will refer to this Frans van Schooten as Frans the baker.[6] In Leiden three other sons were born: Johannes (unknown – before 1651), Joris (1587–1652) and Christiaen (unknown – after 1647). By 1597, a brother of Frans the baker had also settled in Leiden.[7] During his life in Leiden, Frans the baker must have made a good living, as he owned six houses in the city by 1623.[8] His four sons

[2]The house was demolished in 1650 and the current building at Rapenburg 40 was built on the same location, [Th. H. Lunsingh Scheurleer and van Dissel, 1990, 251].

[3][Th. H. Lunsingh Scheurleer and van Dissel, 1990, 251].

[4][Otterspeer, 2000, 99].

[5][Maanen and Groenveld, 2003, 45].

[6]RAL, Archief der Secretarie van de stad Leiden (1290–1575), inv. nr. 22, f. 94r.

[7][Molhuysen et al., 1927, 1108].

[8]Frans the baker owned a house at Marendorp, which was inherited by Frans sr. and eventually came in the possession of Frans jr. who sold it in 1650; see RAL, Stadsarchief van Leiden (Stadsbestuur (SA II)), (1253) 1574–1816 (1897), toegang 501A, inv. nr. 6621, f. 547v. In 1597, Frans van Schooten the baker bought a house at the Nieuwe Oosterlingplaats. This house was inherited by his son Joris, see RAL, Stadsarchief van Leiden (Stadsbestuur (SA II)), (1253) 1574–1816 (1897), toegang 501A, inv. nr. 6617, f. 463. Furthermore, Frans van Schooten the baker owned two buildings at the Doelenachtergracht, bought in 1604 and 1611 respectively, and a house at the Oude Vest acquired in 1617. After Frans's death these three houses were inherited by his son Johannes, see RAL, Stadsarchief van Leiden (Stadsbestuur (SA II)), (1253) 1574–1816 (1897), toegang 501A, inv. nr. 6615, f. 394v. and inv. nr. 6624, f. 595. In 1623, Frans the baker bought a house at the Oude Vest, which he sold in 1637, RAL, see Stadsarchief van Leiden (Stadsbestuur (SA II)), (1253) 1574–1816 (1897), toegang 501A, inv. nr. 6627, f. 647v.

pursued different careers. Christiaen succeeded his father in the bakery,[9] Joris apprenticed to become a painter, and Johannes studied at the university in Leiden and became a Remonstrant minister. Frans sr. took training in mathematics and finally became the professor at the Duytsche Mathematicque, the institution which we will study in more detail in Chapter 7 of this thesis, and at which mathematics was taught in the vernacular.[10]

Frans van Schooten sr. belonged to the first levy of students trained at the Duytsche Mathematicque after its foundation in 1600. During the first decade of its existence, he became the personal assistant of Ludolph van Ceulen, the professor of the Duytsche Mathematicque, and he worked as a tutor in Van Ceulen's private school. After his marriage in 1609 to Jannetgen Harmensdr. van Hogervorst, he intended to start a private school for arithmetic and affiliated subjects in the town of Gouda,[11] and he requested permission to start a school from the Gouda city council. He used the arithmetic school of Van Ceulen as a model for his own school.[12] The death of Van Ceulen in December 1610 changed his prospects and van Schooten sr. stayed in Leiden to fill the vacancy at the Duytsche Mathematicque. In the year 1615, when his son Frans jr. was born, Frans sr. eventually obtained the professorship of the Duytsche Mathematicque, which provided him with a fixed income and a position.[13]

On the national level, the early years of Frans jr. were marked by the intensification of the conflict between remonstrants and contraremonstrants. What had started as a purely theological conflict on predestination between Gomarus, spokesman of the contraremonstrants, and Arminius, leader of the remonstrants, gradually developed into a political conflict, in which the remonstrant land's advocate Johan van Oldebarnevelt was opposed to prince Maurice. The political conflict was eventually settled in the years 1618–1619, and Van Oldebarnevelt was beheaded. The theological side of the conflict was resolved during the Synod of Dort, which established the victory of the contraremonstrants by condemning the remonstrant ideas and emphasizing the calvinist rules.[14]

Frans jr. was raised in a remonstrant family and his religious education took place in private classes under Jacobus Batelier (1593–1672).[15] Batelier, a pastor with Arminian sympathies, was forced to abandon his calling by the Synod of Dort and he was excluded from the religious community. He subsequenty settled in Leiden where he started a private school in 1621.[16]

---

[9] Christiaen's occupation is mentioned in the registration of his marriage with Jannetgen la Mote (RAL, Doop-, trouw- en begraafboeken Leiden, 1575–1811 (1859), toegang 1004, inv. nr. 198, f. B 264) and in the registration of his real estate (RAL, Stadsarchief van Leiden (Stadsbestuur (SA II)), (1253) 1574–1816 (1897), toegang 501A, inv. nr. 6634 f. 18). Christiaen's father Frans van Schooten the baker was a witness at the marriage.

[10] On Joris van Schooten see [Molhuysen et al., 1927, 115-116] and `http://burckhardt.ic.uva.nl/ecartico/persons/6733`. Johannes matriculated on 17 February 1599 at Leiden University, [Rieu, 1875, 55]. On Johannes's activities within the Remonstrant church see [Tideman et al., 1905, 74-80].

[11] The taking out of the marriage license on 2 January 1609 is recorded in RAL, Doop-, trouw- en begraafboeken Leiden, 1575–1811 (1859), nummer toegang 1004, inv. nr. 7, G-033. His father Frans the baker was one of the witnesses; Jannetgen's witness was her mother Marijtgen Warnaerts.

[12] Streekarchief Midden-Holland, Archief van de stad Gouda, inv. nr. 2806.

[13] Frans sr. was appointed on 8 February 1615. UBL, Archief van Curatoren, 1574–1815, inv. nr. 20, f. 354v.

[14] [Netten, 2010, 24].

[15] Frans van Schooten to Christiaan Huygens, 31 March 1651, [Huygens, 1888, 140].

[16] [Berg, 1998, 25].

The aftermath of the Synod of Dort also had its implications for the life of the family. Frans van Schooten jr.'s uncle Johannes was an openly remonstrant clergyman. Just like Batelier, Johannes had to give up his position as pastor in the calvinist church after he had refused to subscribe to the rules of the Synod of Dort. In the following years, Johannes van Schooten served as a pastor for remonstrant communities in the surroundings of Leiden.[17] In 1632, Frans sr. declared in his will that Johannes would be the guardian of any minor children that would be left behind after his death.[18] That Frans sr. sent his son to a remonstrant teacher and named his brother, a remonstrant pastor, as a guardian for any minor children, is a witness of his remonstrant sympathies.

Other members of the Van Schooten family were also listed as members of the remonstrant community of Leiden in a document of 1656: uncle Christiaen, aunt Marijtgen (widow of his uncle Joris van Schooten), and cousin Anthonij.[19] Frans jr. was not mentioned in the 1656 document, but his letters to Constantijn Huygens of 1646 reveal that he was not a calvinist and suggest that he was a remonstrant as well.[20]

The family life of Frans jr. was disrupted in October 1623 by the death of his mother Jannetgen van Hogervorst.[21] Frans jr. was subsequently registered as an orphan by the Weeskamer (Orphan Chamber), a public body to supervise the property of orphans. The announcement of a second marriage of his father was the occasion for the Orphan Chamber to settle the inheritance of the son Frans jr.[22] On February 14, 1625, Frans sr. appeared in the Orphan Chamber together with the two guardians of Frans jr., who were family members of the late Jannetgen van Hogervorst. In front of the Orphan Chamber, Frans sr. testified to provide for Frans jr. and to pay his education until Frans jr. reached the age of 25, got married or was declared mature.[23] A few weeks later Frans sr. remarried Maria Gool.[24] She was the daughter of a cobbler, and a descendant of a prominent family whose members had been engaged as magistrates in the Leiden city administration for generations. The administrative ambitions of the Gool family also extended beyond Leiden, and Maria's uncles Dirck and Egbert held positions in the administrative centre of the Dutch Republic, The Hague.[25] However, Maria's branch of the family consisted of craftsmen: her father was a cobbler, her brother Pieter was a brewer, and her brother in law Adriaen Dircx Bout was a merchant.[26] The marriage to Maria meant for Frans van Schooten sr. that he

[17] [Tideman et al., 1905, 74, 141 and 156] and [Kist, 1836, 59].

[18] RAL, Heilige Geest- of Arme Wees- en Kinderhuis (HGW), toegang 519, inv. nr. 4588, f. 12v.

[19] Anthonij was a remonstrant clergyman just like his father Johannes, [Graaff, 1966, 252 and 260] and [Tideman et al., 1905, 74-80, 230-231 and 446].

[20] Frans van Schooten to Constantijn Huygens, 4 February 1646, [Huygens, 1915, 278-279].

[21] She was buried in the Hooglandse kerk on 9 October 1623. Stadsarchief van Leiden (Stadsbestuur (SA II)), (1253) 1574–1816 (1897), toegang 501A, inv. nr. 1316, f. 218r.

[22] It sometimes happened that both the Orphan Chamber and the surviving parent were negligent in the settlement of the inheritance, [Smit, 1940, 113]. It seems that this indeed was the case after the death of Frans jr.'s mother.

[23] RAL, Weeskamer van Leiden, (1343) 1397–1860 (1866), nummer toegang 518, inv. nr. 113+6, f. 336v.-337v.

[24] RAL, Doop-, trouw- en begraafboeken Leiden, 1575–1811 (1859), nummer toegang 1004, inv. nr. 9, I - 250.

[25] Egbert was a clerk at the Council of State in The Hague, responsible for the administration of the fiefs of Holland and Dirck held a position of *kastelein* meaning he was the keeper of the princely court and as such responsible for guests at the court and hostages. See [Juynboll, 1931, 119-120].

[26] RAL, Heilige Geest- of Arme Wees- en Kinderhuis (HGW), toegang 519, inv. nr. 4588, f. 13r.

was related to a well-known family, with access to networks in The Hague. Furthermore, the marriage established a kinship relation between Frans sr. and the future professor of mathematics and Eastern languages at Leiden University, Jacob Golius, who was a son of Dirck, Maria's uncle.

2.1.1 Mathematical education

Three men played a major role in the mathematical education of Frans van Schooten jr. : his father Frans van Schooten sr., Jacob Golius, and the German mathematician Christiaan Otterus. We will now take a closer look at these three men.

Frans van Schooten sr.

Frans jr. received his early mathematical education from his father Frans sr., who was at that time professor in the Duytsche Mathematicque. This institution had been created in 1600 within Leiden University, to train men as fortification engineers for the army.[27] The teaching at the Duytsche Mathematicque was in Dutch, and the people attending the lectures were not considered to be proper students of the university, although the lecturers were extraordinary university professors. The main focus at the Duytsche Mathematicque was on surveying and fortification, two subjects in which Frans van Schooten sr. was experienced. By the time of his marriage in 1609 he described himself as a surveyor,[28] and in 1612 he made a map of the estates of the university located in the manor of Abcoude.[29] In later years, Van Schooten sr. was involved in various fortification projects. In September 1628, he drew a map of the recently established fortification works near Bergen op Zoom at the request of stadtholder Frederick Henry.[30] Furthermore, a printed map of the siege of Grol (1627) has been attributed to Van Schooten sr., perhaps incorrectly.[31]

Van Schooten sr. published two works which were clearly aimed at a Dutch audience with a practical mathematical interest. Presumably he wrote these works with his own students in mind. They were printed as booklets in the handy duodecimo format, so that surveyors could carry them in their pockets. The first is a Dutch translation of the *Elements*

[27] For a more detailed discussion of the early years of the Duytsche Mathematicque and its position within the university see chapter 7.

[28] RAL, Doop-, trouw- en begraafboeken Leiden, 1575–1811 (1859), nummer toegang 1004, inv. nr. 7, G-033.

[29] RAL, PV98010.

[30] These fortification works were the so called "West-Brabantse waterlinie". The map shows the strip of land flooded as a defence line between Steenbergen and Bergen op Zoom. Between the inundated area and Bergen op Zoom three additional fortresses were erected. The map drawn by Van Schooten shows the inspection by Frederick Henry of the fortification works, and the boats on the lake are identified as Frederick Hendrick's retinue who sailed the artificially created lake on 9 September 1628, [Sinke, 2001]. This manuscript map was the model for the maps which were printed on a smaller format by the Amsterdam printers Blaeu, Jansonius and Schenk & Valk, [Donkersloot-de Vrij, 1981, 117]. A detailed history of the West-Brabantse waterlinie can be found at `http://www.westbrabantsewaterlinie.nl`.

[31] Because there exists a similar map attributed to the engineer Theodoor Niels, it is questionable if Van Schooten was really involved in the siege of Grol. This is even more doubtful because the manuscript map (UBL, COLLBN Port 11 N 224) which seems to be the model for the printed map was definitely not made by Van Schooten. On the basis of the handwriting I conclude that Van Schooten was not the maker of this manuscript map. On the various extant maps of the siege of Grol and their publications see [Pluijm, 2006, 101-113].

of Euclid, published in 1617, and containing only the definitions and propositions without the proofs.[32] The second booklet appeared with the Amsterdam printer Willem Blaeu in 1627. It contained trigonometric tables, obviously intended for calculations in surveying and fortification.[33]

Van Schooten sr. probably introduced the young Frans jr. to the basics of mathematics as follows. First Frans jr. learned to count and to calculate, and he mastered some elementary propositions and constructions in Euclid's *Elements*. As the young Frans grew up, his father paid more attention to the subjects that were important in the Duytsche Mathematicque: arithmetic, practical geometry and surveying, and fortification. The father put the emphasis on actual geometrical calculations,[34] in which numbers were freely assigned to arbitrary line segments or areas. Frans jr. must also have learned how to use decimal fractions and trigonometric tables. In 1629, at the age of fourteen, he was so well trained in mathematics that he could assist his father and two Leiden surveyors in a fortification project in Utrecht.[35] We can assume that by this time the young Frans also attended the lectures by his father in the Duytsche Mathematicque.

Frans jr. pursued his education at Leiden University, where he enrolled on 15 May 1631 as a student of mathematics.[36] He was the second member of his family to register as a proper student, after his uncle Johannes.[37] At the university, Frans jr. entered an intellectual environment which contrasted with the practical fortification activities of 1629. He became a student of Jacob Golius, professor of mathematics and Arabic, and he made frequent visits to the German mathematician Christiaan Otterus who organized private lectures when he was in town.

Jacob Golius

In 1625, the humanist scholar Jacob Golius (1596–1667) succeeded his tutor Erpenius as professor of Arabic. In 1629, Golius also became professor of mathematics as the successor of Willebrord Snellius.[38] The combination of philology and mathematics was not unusual in that time, because scholarship was largely based on classical Greek texts. In the *Collection* of Pappus of Alexandria (ca. 250 AD), which had been published with Latin translation in 1588, several lost Greek mathematical works were mentioned, and scholars had an interest in discovering and reconstructing such classical works. Golius's teacher Erpenius had stressed the importance of Arabic because some texts that had been lost in the original Greek were known to exist in Arabic translations. These considerations may

[32] [Euclid, 1617].

[33] [Schooten sr., 1627]. Blaeu also published French and Latin versions of the work, which were reprinted numerous times throughout the seventeenth century.

[34] The manuscripts of Frans van Schooten sr. shed light on his lectures; these manuscripts are kept in UBL and UBG, for an inventory see [Maanen, 1987, 147-241] (UBL) and Appendix A of this thesis (UBG).

[35] [Bordes, 1856, 116].

[36] [Rieu, 1875, 235] and UBL, Archief van Senaat en Faculteiten, 1575–1877, inv. nr. 9, 13.

[37] Johannes van Schooten enrolled as a student at the age of 15 on 17 February 1599, [Rieu, 1875, 55].

[38] Snellius died on 30 October 1626. The chair of mathematics remained vacant for over three years until Golius was appointed on 21 November 1629. On Snellius's life and work see [Wreede, 2007]; for Golius's appointement [Molhuysen, 1918, 146].

have motivated Golius, who started as a student in mathematics, to take up Arabic studies as well.[39]

The connection between Arabic and ancient Greek learning was one of the reasons why Golius set out on two journeys to the Maghreb and the Levant in the 1620s. During the first journey (1622–1624), he was attached as an engineer to a Dutch diplomatic mission to Morocco. In the Maghreb he improved his Arabic language skills and collected several manuscripts. Golius went on his second trip after he had accepted the chair of Arabic in Leiden in May 1625. He persuaded the curators of the university to postpone his teaching duties and to allow him to leave Leiden for a journey while receiving his salary. He also arranged an additional budget for the acquisition of manuscripts on behalf of the university. He left Leiden in 1625 and first went to Aleppo, from where he traveled through Syria and Mesopotamia. In 1627 he arrived in Istanbul, where he resided with the Ambassador Haga. During his stay in the Levant he used various contacts of the ambassadors and the local elites to build a network which gave him access to scientific knowledge and manuscripts.[40] On his return to Leiden in 1629, Golius had acquired a fine collection of Arabic manuscripts obtained by purchase, gift or copy. After his return in Leiden he managed to extend his collection through agents.

The collection which Golius took to Leiden included several Arabic translations of Greek mathematical and astronomical works which had been considered lost until that time. The most important of these manuscripts was the Arabic version by Thābit ibn Qurra (826–901) of Books V-VII of the *Conics* of Apollonius (ca. 200 BCE). The *Conics* consisted of eight books, of which only the books I-IV were known in Greek. Book VIII is still lost today. In Aleppo, Golius discovered Books V-VII of the *Conics* in an Arabic manuscript which dates back to the eleventh century, and which also contains Books I-IV in Arabic translation. He had a copy made of this manuscript,[41] and in 1627, by intermediary of the Dutchman David le Leu de Wilhem (1588–1658), he was able to acquire the original eleventh-century manuscript as well.[42] Back in Leiden, Golius entrusted the copy to the university library, while keeping the original in his private collection.[43]

Golius expressed his intention to publish a Latin translation of Books V-VII of the *Conics*, but to the regret of the mathematical community over Europe he never did. The works which Golius did publish were not of a mathematical nature but primarily contributions to the teaching of Arabic. His masterpiece was his *Lexicon Arabico-Latinum*, an Arabic dictionary on which he worked for many years before finally publishing it in 1653. The work became the standard Arabic-Latin dictionary soon after its publication and remained so until the nineteenth century.[44]

[39] [Vermij, 2002, 23-24].

[40] The missions to Morocco and the Levant are discussed in more detail in [Dijksterhuis, 2011, 96-103].

[41] This copy is nowadays in UBL, Or. 14.

[42] This manuscript is currently kept in Oxford, Boldleian Library, MS. Marsh 667. After Golius's death, this manuscript was auctioned in 1596. The connection with Wilhem is established by Golius's notes at the beginning of the manuscript, see [Apollonius, 1990, lxxxvi].

[43] [Witkam, 1980, 57].

[44] [Toomer, 1996, 49-50].

Golius introduced the young Frans van Schooten to the treasures of Greek mathematics. Thus Frans jr, who was familiar with practical geometry as taught by his father, now discovered classical Greek geometry with its focus on constructions and proofs. In practical mathematics, numbers were freely assigned to any length or area, so that calculations could be made. In classical geometry, numbers were absent, and instead of calculating lengths or surfaces, one had to solve problems by means of constructions, and give proofs. Contemporaries of Van Schooten and Golius were at the time exploring the use of algebra in geometry, but they met with conceptual difficulties in representing arbitrary line segments or areas as numbers.[45]

With Golius, Van Schooten studied parts of Pappus's *Collection* and he even copied parts of Pappus's text in his private notebook.[46]. Golius also discussed classical Greek geometrical problems with Van Schooten, such as the trisection of an angle.[47] In 1661, Golius lectured on spherical triangles on the basis of an Arabic manuscript of the *Spherics* of Menelaus, which is lost in Greek.[48] It is quite probable that Golius also used Arabic manuscripts at the time when Van Schooten was his student. In any case, Golius gave him access to the three lost books V-VII of Apollonius's *Conics*, and even hired him to make the drawings of the figures for the Latin translation of these books, which never came off the press.[49] I will postpone a further discussion of Van Schooten's work as a draughtsman to section 2.10.

Golius also paid attention to more recent developments in mathematics. In his public lectures he discussed the algebra of Viète.[50] The notes by Van Schooten reveal that these lectures included an elaborate treatment of the work *De aequationum recognitione et emendatione (Concerning the recognition and emendation of equations)* (1615) with emphasis on the techniques for removing the second highest order term from a polynomial.[51] Furthermore, Golius and Van Schooten also studied *Apollonius Gallus*, a treatise in which Viète reconstructed Apollonius's lost treatise *De Tactionibus (On tangencies)*. This treatise was about the geometrical problem of constructing, by means of ruler and compass, a circle that is tangent to three given objects (lines, points or circles) in a plane.[52]

In addition to the official lectures at the universities, Van Schooten must have attended private lectures as well. These were given by university professors and also by lecturers who did not have an official position at the university but nevertheless made a living by training students. Since these lecturers were not members of an official institution, their activities are often difficult or impossible to trace for the historian.[53] One of these lecturers was Christiaan Otterus.

[45] For a full discussion of these conceptual difficulties for the geometers wanting to use algebra see [Wreede, 2007, 185-186].

[46] UBG, Hs 108, f.22v.-25r

[47] UBG, Hs 108, f. 6v.

[48] [Juynboll, 1931, 140].

[49] [Malcolm and Stedall, 2005, 415].

[50] Van Schooten refers to these lectures in his dedication letter to Golius in the collected works of Viète, [Viète, 1646, *2r.].

[51] UBG, Hs 108, f. 30r-v.

[52] UBG, Hs 108, f. 21v.-22r.

[53] [Goudeau, 2005, 44].

Christiaan Otterus

Christiaan Otterus, or Otter (1598–1660), was a private lecturer whom Van Schooten frequently visited. Originally from Ragnitz, Prussia, Otterus came to the Dutch Republic in 1619 for the first time, and spent some time in Leiden.[54] In the subsequent years, he traveled in the Republic, and studied at Franeker University as a student of Adriaan Metius (1571–1635). In the years 1627–1639 he worked at least part-time in Leiden, where he earned a living as a tutor in mathematics.[55] Otterus and Frans van Schooten sr. were good friends, with a common interest in mathematics and military architecture.[56] Like Van Schooten sr., Otterus had spent time in the army and he was present at the siege of Den Bosch of 1629.[57] In 1635, Otterus showed up in Franeker to attend Metius's funeral, and he stayed in Franeker for some time to give lectures in mathematics, including a thorough exposition of fortification. He was a candidate for the vacant chair of mathematics in Franeker, but the position was ultimately given to Bernhard Fullenius, and Otterus returned to Holland.[58] He left the Republic in 1647 to become the court mathematician of Frederick William, Elector of Brandenburg. In 1658, he returned to the Republic as the first professor of mathematics in Nijmegen.[59]

Otterus had experience with engineers in the army, and he knew what knowledge an engineer needed in his actual work. Like Van Schooten sr., Otterus lectured on the subjects known as mixed mathematics. During his lifetime, he published a short treatise *Specimen problematum* in which he presented solutions to 22 geometrical problems in a context of fortification.[60] This treatise reveals that Otterus had a theoretical interest in fortification, which went beyond the usual contents of fortification courses at the Duytsche Mathematicque. Otterus's solutions to the 22 problems are in classical Greek geometrical style, not involving any numbers or calculations. The extensive title of the work illustrates the important role of a theoretical problem, namely the rational division of straight lines.[61] Thus for Otterus fortification was not only a practical affair but also a source of inspiration for theoretical problems.[62] Otterus elaborated on these theoretical ideas in his lessons with Van Schooten.[63]

[54] Otterus's presence in Leiden is documented by the entries in his album amicorum, [Buck, 1764, 209-210]. Buck states that Otter was a student, but I have not found traces of his matriculation at Leiden University in the archives in 1619. The first official registration of Otterus in Leiden is dated 18 November 1626, [Rieu, 1875, 197].

[55] Otterus registered as a student of the university in Leiden on 18 July 1634 and 22 October 1636, [Rieu, 1875, 264 and 281]. In 1634 he lived at the house of Claudius de Lartillier. This house was located at the Rapenburg, at the spot of the present house numbered 38. The owner of the adjacent house was Frans van Schooten sr., who let this house, [Th. H. Lunsingh Scheurleer and van Dissel, 1990, 250].

[56] Van Schooten sr. signed Otterus's album amicorum on 20 July 1629, [Kowalewski, 1935, 269].

[57] [Buck, 1764, 223].

[58] [Dijkstra, 2012, 166-167].

[59] [Buck, 1764, 251].

[60] [Otterus, 1646].

[61] The complete title reads *Specimen problematum hercotectonico-geometricorum quo ut fortificationis (vulgo ita dictae) modi universalis ita sectionis rationalis linearum vestigium exhibetur* (Example of geometrical fence-building problems, in which manners of universal fortification (as it is commonly called) as well as a trace of the rational section of lines are displayed).

[62] I will elaborate further on Otterus's ideas on fortification in section 8.2 of this thesis.

[63] UBG, Hs 108, f. 12r. and 13r.

Otterus also had a lively interest in mathematical instruments. During his lifetime, he made paper models of around fifty instruments.[64] The models roughly fall in two categories: models of instruments for division of angles and models of instruments for the generation of curves. Otterus produced various instruments for the generation of conic sections, but he also designed instruments for drawing more complex curves.[65] We do not know when Otterus made those instruments. In 1646 Van Schooten stated that Otterus invented "many things" for drawing conic sections but that these inventions had not been published.[66]

Van Schooten sr., Golius and Otterus played a crucial role in the sharpening of Van Schooten jr.'s mathematical mind. The education by his father had familiarized him with "mixed" mathematics and the use of mathematics in practical affairs such as fortification. With Golius, Van Schooten entered into the humanist world of learning and learned about pure mathematics in the style of the Greeks. Otterus formed the bridge between the realm of mixed mathematics and pure mathematics, with his theoretical interest in subjects such as fortification.

By June 1635, when he was nearly 20, Van Schooten was so well versed in the curriculum of the Duytsche Mathematicque that the board of curators and burgomasters of the university allowed him to replace his father during illness.[67] He had yet to meet the man with the most influence on his career: René Descartes.

2.2 Meeting the master: Descartes

2.2.1 Printing the *Discours de la méthode* and the essays

The first contact between Frans van Schooten and René Descartes took place before or in the summer of 1636. The initiator of the encounter must have been Jacob Golius, who knew both Descartes and Van Schooten well and was aware of their mutual interest in mathematics. Golius had also introduced Descartes to other men of learning, such as the poet and musician Constantijn Huygens (1596–1687), who worked as a diplomat under Frederick Henry, and whom Descartes met in 1632.[68]

In 1636 Descartes was busy with the preparation of his essay on *Dioptrique* for the press of the Leiden printer Joannes Maire. The printing of his two other essays *Météores* and *Géométrie* and of the *Discours de la méthode* would follow in the upcoming months.

[64] Before World War II, these paper instruments were part of the collection of the Stadtgeschichtliche Museum (museum of city history) in Königsberg (now Kaliningrad, Russia). What has become of these models after the war remains unknown. The collection of instruments was described by Kurt Reidemeister and Theodor Peters in [Reidemeister and Peters, 1933].

[65] The instruments discussed by Reidemeister and Peters generate a curve of maximum degree 6, [Reidemeister and Peters, 1933].

[66] [Schooten, 1646, Preface].

[67] UBL, Archief van Curatoren, 1574–1815, inv. nr. 22, f. 187v.-188r. This replacement was suggested by Van Schooten sr. and it was a first step towards the succession of Frans sr. by Frans jr. For more details on the succession see below, page 44.

[68] [Jorink, 2008, 21-22].

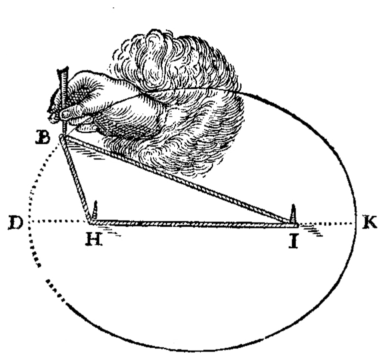


Figure 2.1 – Gardener's construction of an ellipse, [Descartes, 1637b, 90].

Descartes struggled with the illustrations and figures which he wanted to add to his essays. Constantijn Huygens supported Descartes in the practical aspects of the publication process, including the illustrations. He recommended to Descartes to use woodcuts instead of copperplates, and to insert the figures in the main text rather than in a separate section.[69] On 15 June 1636, he recommended to Descartes to look for an engraver who had affinity with the content of his essays, and who possessed the appropriate drawing skills.[70] Less than a month later, Van Schooten jr. had been enlisted for tracing the is of the *Dioptrique*.[71] The combination of his mathematical talent with his draughtsmanship made him the ideal candidate, and his work was well appreciated by Descartes. I will discuss the qualities of Van Schooten as a draughtsman in more detail below in section 2.10.

Among the illustrations of the *Dioptrique*, Van Schooten had to draw two figures showing the generation of an ellipse and a hyperbola by means of motion. These figures were printed in the *Dioptrique* as 2.1 and 2.2, and his private notebook contains an illustration very similar to the second figure, namely figure 2.3.[72] The accompanying notes by Van Schooten reveal that he was involved in the design of these constructions. On the hyperbola he noted:

> The demonstration for describing a hyperbola to the question of the distinguished Mr Decartius [sic]. FaSi.[73]

The abbreviation 'FaSi" stands for "Franciscus a Schooten invenit" (i.e., "Frans van

[69] In seventeenth century works on natural philosophy, illustrations were not common at all, and if they were used, they were put in a separate section at the end, [Jorink, 2008, 31]. However, in books on mathematics, geometrical figures were printed in the text at the spot where they were discussed, because these figures were necessary to understand the text.

[70] Constantijn Huygens to Descartes, 15 June 1636, [Descartes and Huygens, 1926, 21].

[71] Descartes to Constantijn Huygens, 13 July 1636, [Descartes and Huygens, 1926, 25].

[72] Note the similarity in the lettering in Van Schooten's figures 2.3 and 2.2. The drawing and accompanying note were made somewhere between 5 December 1632 and 1642; the exact date is not known.

[73] "Ad quaestionem illam d. ill. decartij demonstratio pro describenda linea hyperbole. FaSi." UBG, Hs 108, f. 13v.

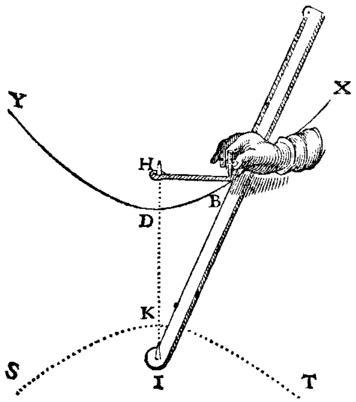


Figure 2.2 – Method using strings for generating a hyperbola, [Descartes, 1637b, 101].

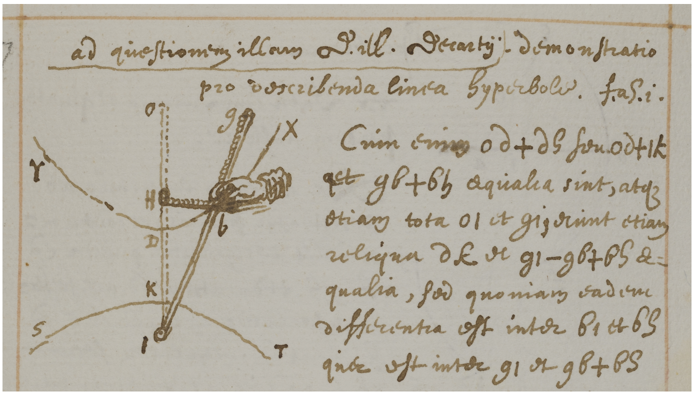


Figure 2.3 – Method using strings for generating a hyperbola drawn by Van Schooten. UBG, Hs 108, f. 13v.

Schooten has invented").[74] Here, Van Schooten claims his own contributions to the development of methods for generating conic sections. Moreover, he presents his own contribution as an answer to a question posed by Descartes. This is an important observation for it shows that at least some of the notes by Van Schooten are the result of an exchange of ideas between him and Descartes.

Descartes also put Van Schooten in contact with other scholars who would be important for his further career. On 22 March 1637, he sent Van Schooten to The Hague to deliver a letter to Constantijn Huygens. This was probably the first time when Van Schooten met Huygens, for Descartes found it appropriate to include the following note on Van Schooten in the letter:

> It is the young Schooten who will deliver this letter to you, but I beg you not to judge him on his attitude.[75]

In spite of the shy, and maybe inappropriate, attitude of Frans van Schooten, Descartes considered it worthwhile to bring him in contact with Huygens.[76] Because Descartes discussed his projects with both men, it is likely that they were aware of each other's interest in Descartes's work.

The first encounter between Huygens and Van Schooten was the beginning for a lasting contact. Huygens had a deep interest in scientific activities and highly appreciated the ideas of Descartes, but he wondered whether he possessed enough knowledge of mathematics in order to understand the *Géométrie* in depth. It was clear to Huygens that Descartes had "transmitted something to the mind of the young Van Schooten,"[77] and that Van Schooten had a profound knowledge of the mathematical ideas of Descartes.

The relation with Huygens opened new opportunities for Van Schooten. By intermediary of Huygens, Van Schooten was assigned the task of producing a map of the siege of Breda (1637) for which Van Schooten was rewarded a sum of 300 guilders by the stadholder Frederick Henry. This was a considerable amount of money, equal to 75% of Van Schooten's yearly pay in 1646.[78]

Van Schooten's work on the publication of Descartes's treatises went beyond the preparation of the illustrations and mathematical figures. Just like Huygens, Van Schooten

[74] This abbreviation has been identified by Dijksterhuis, [Dijksterhuis, 2011, 110]. In the early modern period, it was not uncommon for a painter and craftsmen to add the words "invenit et fecit" to his name on a painting, etching or instrument. See for instance [Vlieghe, 2001, 178], [Bowen and Imhof, 2008, 235] and [Baarsen and Estié, 2000, 36].

[75] "C'est le jeune Schooten qui vous presentera cete lettre, mais vous prie de ne point juger luy par sa contenance." Descartes to Constantijn Huygens, 22 March 1637, [Descartes and Huygens, 1926, 39]. On Van Schooten's attitude, see also his behaviour when meeting princess Elisabeth in 1645, see page 39.

[76] My view on the contacts between Van Schooten on one hand and Descartes and Huygens on the other hand differs from the view of Fokko Jan Dijksterhuis. Dijksterhuis claims that Van Schooten was first involved in learned circles around Constantijn Huygens, and that he was introduced to Descartes in this way, [Dijksterhuis, 2011, 109].

[77] "Transmis quelque chose dans l'esprit du jeune Schooten", Constantijn Huygens to Descartes, 24 March 1637, [Descartes and Huygens, 1926, 40].

[78] Constantijn Huygens to Frans van Schooten sr., 31 July 1640, UBL, HUG 37. I have not found any traces of this map of the siege of Breda.

proofread parts of the printer's proofs.[79] In his notebook, Van Schooten made comments on parts of the *Géométrie* based on a manuscript or printer's proofs. He later edited some of these comments and included them in the Latin *Geometria*.[80] As we have seen, he also contributed to the generation of conic sections by means of motion. From these elements taken together we obtain the picture that by 1637, Van Schooten was well versed in the mathematics of the *Géométrie* and that he was not just an illustrator, but a valuable partner for Descartes in the preparation of the *Discours de la méthode* with its essays for publication.

2.2.2 Making lenses

The combination of Van Schooten's mathematical talents and his draughtsmanship also made him an ideal participant in the lens making project of Descartes. In Paris, Descartes had been involved in making a perfect lens, not suffering from spherical aberration. An ordinary spherical lens does not focus parallel incident rays in a single point and produces a blurred image as a result. The challenge was thus to produce a lens which focuses parallel incident rays into a single point. To meet the challenge, one would need knowledge of mathematics and of the laws of refraction of light for the determination the shape of such a lens, mathematical draughtsmanship for the accurate drawing of such a shape, and finally lens grinding expertise for the production of the lens.[81]

From the theory of dioptrics, Descartes knew that the surface of such a lens had to be a rotation surface of one of the three conic sections. In Paris, he had worked on the production of elliptical and hyperbolic lenses together with the mathematician and draughtsman Claude Mydorge (1585–1647) and the lens grinder Jean Ferrier. In the Dutch Republic, Descartes tried to continue this project in close collaboration with Constantijn Huygens from 1635 onwards.[82] However, their first two attempts at lens making did not produce the desired result. In the subsequent attempt of autumn 1637, Huygens asked Van Schooten to draw the required hyperbola.[83] Thus, Van Schooten was given the task which had been given to Mydorge in Paris, namely the very precise drawing of conic sections.[84] Although Van Schooten drew the hyperbola accurately and Huygens praised his draughtsmanship,

[79] Descartes to Constantijn Huygens, 20 April 1637, [Descartes and Huygens, 1926, 44]. On Huygens's participation in proofreading and making corrections, see [Otegem, 2002, 8-9].

[80] These comments in the notebook are found in UBG, Hs 108, f. 4v., 14r., 51v., 52v., and 55v.-58v. Van Schooten must have used a manuscript or printer's proofs because the page numbers and references in the notebook differ from the page numbers in the 1637 printed edition. At some instances, the sentences differ from the final text of the 1637 edition although the line of thought is the same. For instance, the text of Van Schooten on f. 57v. reads "Soit donné $x^3 - \sqrt{3}xx + \frac{26}{27}x - \frac{8}{27}\sqrt{3} = 0$ en j'en demande un autre en sa place dont tous les termes s'expriment par des nombres rationaux" whereas the final text of the *Géométrie* reads "Comme si on a $x^3 - \sqrt{3}xx + \frac{26}{27}x - \frac{8}{27}\sqrt{3} = 0$ & qu'on veuille en avoir une autre en sa place, dont tous les termes s'expriment par des nombres rationaux", [Descartes, 1637c, 379].

[81] [Dijksterhuis, 2007, 62].

[82] On Descartes's projects of lens making in France and the Dutch Republic see [Burnett, 2005], especially pages 41-70. Dijksterhuis pays attention to the lens making project in the Dutch Republic in [Dijksterhuis, 2007, 61-67].

[83] Constantijn Huygens to Descartes, 18 September 1637, [Descartes and Huygens, 1926, 55-56].

[84] [Dijksterhuis, 2011, 109].

the attempt failed again because the lens grinder did not deliver the quality demanded by Huygens and Descartes.[85]

2.3 To France and back: 1641–1645

2.3.1 Leaving Leiden: 1641–1642

In the seventeenth century it was common practice for young men of wealthy families to make a journey at the end of their education. Such a journey, often known as grand tour, also marked the beginning of a career in society. The purpose of the journey was to be introduced to other languages, cultures and habits, and the itinerary always included France, and often Italy as well. For youngsters with a special interest in academia, a trip along various universities was appropriate. Such a *peregrinatio academica* was often completed by a doctoral degree at some university.

Van Schooten's journey to France, England and Ireland has to be seen in this context of travels of young men, although his journey was neither a real grand tour nor a proper peregrinatio academica. Van Schooten did not prepare himself for an administrative position in society. His journey had the focus on learning and scholarship in common with a peregrinatio academica, but was not confined to the boundaries of university institutions. It can be better understood as a *peregrinatio mathematica*, a travel aimed at mathematics and meeting capable and interesting mathematicians.

For a young mathematician, France was a suitable destination. Paris had an active mathematical community in which the latest achievements and results were discussed. The pivot of this community was Marin Mersenne (1588–1648), who organized regular meetings and who corresponded with a large number of mathematicians and other men of learning in France and abroad, including Pierre de Fermat (1607–1665). Descartes had connections with these Parisian circles and with Mersenne in particular. In September 1641 he recommended Van Schooten to Mersenne and thus Van Schooten was introduced to the Parisian scholars.[86]

Van Schooten arrived in France in the autumn of 1641. In Paris, he became familiar with mathematical works and results hitherto unknown to him, in particular the ideas of Pierre de Fermat. He made transcriptions of various letters in the correspondence between Fermat, Mersenne and Roberval dating from the period June 1636 until November 1642. He also copied parts of treatises by Fermat, including: *Ad locos planos et solidos isagoge*, an introduction to analytic theory written before Descartes's *Géométrie* was published; *Loci ad tres lineas demonstratio*, a demonstration of the three line locus; and two treatises dealing with the method of maxima and minima.[87]

Van Schooten's contacts with Claude Mylon (1615–1660) also date back to his journey to Paris. Originally, Mylon had been trained as a lawyer, and he had a position in the Parisian parliament during the 1640s. Nevertheless, Van Schooten found in him a young

85 [Dijksterhuis, 2007, 66].
86 Descartes to Mersenne, September 1641, [Descartes, 1899, 437].
87 These transcriptions are in UBG, Hs 110. See also [Waard, 1917].

man of his age with whom he shared an interest in mathematics. Van Schooten and Mylon spent quite some time in Paris together, and they may have attended lectures by Roberval, who is believed to have been Mylon's teacher.[88] It is certain that Van Schooten was present at some lectures of Roberval at the Collège de France; he noted that Roberval began his lectures in academic Latin, and later explained the subjects in more detail to the students in French.[89] From at least 1644 Mylon was part of the circle around Mersenne, and he used to make detailed notes of the subjects and problems that were discussed during the meetings.[90] Van Schooten maintained a friendly relationship with Mylon over the years.

One of the mathematical problems which Mylon and Van Schooten discussed was the so-called *Problema Astronomicum*. In 1638 the Dutch mathematician Jan Jansz. Stampioen had challenged his fellow mathematicians, and Descartes in particular, by proposing this problem: For three sticks perpendicular to the horizontal plane, we are given their lengths and certain pieces of information on their mutual location and the trajectories of their shades during a day. It is required to determine which day of the year it is and where on earth these sticks were located. The problem played a role in the controversy between Stampioen and Descartes in the years 1638–1640. By the end of 1639 this controversy became known in Paris, and the mathematicians were interested because of the involvement of Descartes. As the exchange of arguments in Holland was in Dutch, it was hard for the French community to fully understand what was at stake. Mylon had shown Van Schooten the solution of a related problem, which involved only one stick. In 1657, Van Schooten published Mylon's solution in the *Exercitationum mathematicarum libri quinque*.[91]

Via Mylon, Van Schooten had a nodding acquaintance with Pierre de Carcavi (ca. 1600–1684), whom he met at Mylon's home. After Van Schooten had returned to Leiden, Carcavi took the initiative to start a correspondence. For some time the two mathematicians exchanged letters, which are now lost. Van Schooten however did not pay much attention to the correspondence because he considered the issues which Carcavi brought up not very interesting.[92] One of these issues was the publication of the works of Fermat, which Carcavi tried to publish with Elzevier through the intermediary of Van Schooten.[93]

Another man whom Van Schooten probably met in Paris is the printmaker, book illustrator and author Abraham Bosse.[94] Bosse was active at the crossroads of mathematics, crafts and drawing techniques, and this combination of subjects had also caught the attention of Van Schooten. Bosse was influenced by the ideas of Desargues on perspective, and had attended his classes in Paris. During the 1640s, Bosse published several works on the use of the theory of Desargues in perspective and drawing, and on gnomonics.[95]

[88] [Mesnard, 1991, 245].

[89] Van Schooten to Constantijn Huygens, 4 juni 1646, [Huygens, 1915, 317].

[90] [Mesnard, 1991, 245, 251].

[91] The solution of Mylon is found in [Schooten, 1657, 507] and [Schooten, 1660a, 467]. For a detailed account of the historical and mathematical aspects of the *Problema Astronomicum* and the roles of Descartes, Van Schooten and Mylon, see [Maronne, 2007, 361-436].

[92] Van Schooten to Christiaan Huygens, 3 May 1656, [Huygens, 1888, 410].

[93] Carcavi to Christiaan Huygens, 22 June 1656, [Huygens, 1888, 432].

[94] In 1657, Bosse sent his regards to Van Schooten through Mylon. Claude Mylon to Frans van Schooten, 5 January 1657, [Huygens, 1889, 4].

[95] For a detailed study of Bosse see [Goldstein, 2012].

Outside Paris, Van Schooten went to Blois to see Florimond de Beaune (1601–1652).[96] De Beaune had an official position as counselor at the court of justice in Blois, but he devoted a lot of his time to astronomy and mathematics. In 1639, he had worked on the construction of a lens grinding machine for Descartes, and the two men had a good relationship. Therefore Van Schooten's visit to De Beaune may have been arranged by Descartes. At his home, De Beaune had a large library with a fine collection of astronomical and mathematical books.[97]

De Beaune was among the few who understood the contents of the *Géométrie* soon after it had been published, and this is the reason why Descartes rewarded him with a separate printed copy of the work.[98] De Beaune's notes on the *Géométrie*, entitled *Notes brèves*, were among the first explanatory texts in which difficult passages in the *Géométrie* were clarified. De Beaune must have finished his notes near the end of 1638, and Mersenne offered to send them to Descartes on 15 January 1639,[99] so it is likely that Van Schooten had seen the notes of De Beaune via Descartes before he started his journey. In any case, De Beaune and Van Schooten must have discussed the ideas of the *Géométrie* in great depth in Blois. De Beaune's deep understanding of the *Géométrie* as well as his other interests in mathematics and astronomy made him for Van Schooten an inspiring person to meet, and in 1649, Van Schooten added a Latin version of the *Notes brèves* to his Latin translation of the *Geometria*.[100]

On his way to or from France, Van Schooten paid a visit to Dublin and London. In Dublin he met the Irish mathematician William Purser, who showed him how to determine the angles of a triangle from the lengths of the sides by means of logarithms.[101]

The journey enriched Van Schooten with new contacts and new ideas on mathematics. He learned about the work of Fermat, attended lectures of Roberval, discussed mathematical problems with Mylon, and made the aquaintance of Mersenne. He had left Leiden as a student and returned as a scholar, informed of the latest developments in mathematics.

[96] [Descartes, 1913, 20].

[97] "Debeaune." Complete Dictionary of Scientific Biography. 2008. Encyclopedia.com. (retrieved September 26, 2012). `http://www.encyclopedia.com/doc/1G2-2830901102.html`.

[98] [Otegem, 2002, 102].

[99] Descartes to Mersenne, 9 February 1639, [Descartes, 1898, 498-499] and [Descartes, 1941, 176].

[100] [Beaune, 1649]. The original manuscript of De Beaune's *Notes brèves* of 1638 is lost, but two copies of the French text are extant, one in Paris (Bibliothèque nationale de France, Ms. français 9556, f. 84-104) and one in London (British Library, Harleian M.S. 6796, art 23). For a discussion of these manuscripts see [Maronne, 2006, 211]. Van Otegem claims to have discovered a French manuscript of the *Notes brèves* in the handwriting of De Beaune. His claim is debatable, because he erred in the shelf mark of the manuscript (he gave Bibliothèque nationale de France, Ms. français 3930, f. 84r.-101r., but I have not been able to find the *Notes brèves* in the manuscript, `http://gallica.bnf.fr/ark:/12148/btv1b9060535w`, retrieved 26 September 2012) and he seems to not have noticed the publication of the text of De Beaune in [Descartes, 1941, 356-401], see [Otegem, 2002, 146, footnote 30].

[101] [Schooten, 1660a, 468].

2.3.2 Return to the Republic

After his return to Leiden, Van Schooten went to see Descartes in his residence at that time, castle Endegeest.[102] Van Schooten discussed his experiences in France, especially the mathematical ideas of Fermat.[103] After moving to Egmond, Descartes asked Van Schooten to trace the figures of his treatise *Principia philosophiae*.[104]

It is not clear how Van Schooten jr. made a living in the early years after his return from France. He did not have an official position at the university. The chair of mathematics was still held by Golius, and Van Schooten sr. taught at the Duytsche Mathematicque, though Van Schooten jr. replaced his father from time to time during illness of the latter. Presumably Van Schooten jr. made some money by private teaching, as was common in Leiden.[105]

Descartes had career plans for Van Schooten and in December 1644, he invited Van Schooten to visit him in Egmond where he was living at the time. Descartes was wondering whether Van Schooten was willing to move to The Hague the following summer, in order to teach mathematics to Christiaan Huygens, Constantijn's second son, who had shown a great talent in mathematics. Since the summer of 1644, Constantijn Huygens had employed Jan Jansz. Stampioen as instructor for his children. This was a situation which Descartes must have regretted because he considered Stampioen incompetent in mathematics.[106] If Van Schooten were to move to The Hague, the children of his friend Huygens would receive proper training by a capable mathematican and teacher, and Van Schooten would have a paid position.

Van Schooten however preferred to stay in Leiden, in the academic setting which suited him best. He told Descartes that he wanted to continue his studies in mathematics, which gave him a pleasure that he would not want to give up. He feared that a move to The Hague would deprive him of this possibility, because he would have so many pupils that he could not continue any serious studies in mathematics. But he made clear to Descartes that he was willing to render his services to Christiaan once Christiaan had been enrolled as a student in Leiden.[107]

Soon afterwards, on 12 May 1645, the two brothers Constantijn and Christiaan

[102] As Descartes moved from Endegeest to Egmond aan den Hoef in early May 1643, Van Schooten must have been back in Leiden before May 1643. Descartes's move is documented in [Descartes, 2003, 61 and 65].

[103] Van Schooten to Christiaan Huygens, 19 September 1658, [Huygens, 1889, 221].

[104] Descartes to Constantijn Huygens, 20 September 1643, [Descartes and Huygens, 1926, 213]. The work appeared in 1644 [Descartes, 1644a].

[105] In Leiden other private instructors like Nicolaus Goldmann, Samuel Kechel and Jean Gillot gave private lectures to students on various mathematical subjects. On Gillot see [Witkam, 1967] and [Witkam, 1969]. The career of Goldmann is extensively studied by Goudeau in [Goudeau, 2005].

[106] In 1638, Stampioen had challenged Dutch mathematicians to solve two problems involving third-degree equations. Descartes took up the challenge through his straw man Jacob van Waessenaer. Polemic pamphlets were published from both sides, and the polemic culminated in 1640 when the challenge took the form of a wager. A jury consisting of Jacob Golius, Frans van Schooten sr., the Utrecht professor of mathematics Bernardus Schotanus (1598–1652) and the secretary of Rotterdam and amateur mathematician Andreas van Berlicom (ca.1587–1656) decided in favour of Descartes. For a more detailed account see [Descartes, 2003, 301-302].

[107] Descartes to Constantijn Huygens, 21 December 1644, [Descartes and Huygens, 1926, 234].

Huygens registered as students in law at Leiden University and Van Schooten started instructing both young men.[108] The lectures by Van Schooten to Christiaan started with elementary algebra, but soon included more intricate algebra from the work of Viète and algebra and geometry from Descartes's *Géométrie*.[109] Van Schooten also instructed Christiaan in the art of fortification.[110]

2.3.3 The new edition of Viète's work

By the second decade of the seventeenth century, Viète's works had been sold out, and because they had been printed in small numbers, only few copies circulated. The Elzevier publishing house initiated a new edition of the collected works in order to make them available to the community of mathematicians, and they estimated the public to be large enough to generate a profit. Van Schooten was eventually involved as the editor of the volume. The new edition appeared in 1646 with minor comments by Van Schooten.[111]

The first traces of this edition project date back to 1637.[112] To prepare the new edition, Elzevier had to collect the existing works by Viète, and Mersenne served as an intermediary. He had contacts with Fermat, who had access to the manuscripts of Viète which were in the possession of Jean d'Espagnet, an old friend of Viète. The project was well known in the mathematical circles in Leiden. In July 1639, the German student Woldeck Weland (1614–1640) wrote from Leiden to his mentor Jungius (1587–1657) in Hamburg that Elzevier would start printing the collected works of Viète within a few weeks.[113] The French mathematician Jean de Beaugrand (ca. 1586–1640) had been invited to make additional comments and corrections to the original texts.[114] Beaugrand had a deep understanding of Viète's scholarly work and he had published an annotated version of Viète's *In artem analyticem isagoge* in 1631.[115]

The project however took more time than foreseen. Beaugrand did not deliver his notes in time and he refused to send any hitherto unpublished work by Viète.[116] In the summer of 1640 Elzevier made arrangements for the drawing of the mathematical figures. The identity of the draughtsman is unknown, but Van Schooten is a likely candidate, as

[108] [Rieu, 1875, 358] The father Constantijn Huygens had carefully prepared the education and the daily routine of his two sons. The emphasis of their education in Leiden was on law: they had to spend four and a half hour per day on studying law or attending lectures on law by Vinnius. One hour a day, from ten to eleven in the morning, was reserved for mathematical instruction by Van Schooten, [Huygens, 1888, 4-5].

[109] UBL, Hug 12. For a description of the manuscript see [Maanen, 1987, 204-205].

[110] UBL, Hug 16. The content and structure of these lectures were similar to Van Schooten's lectures on fortification at the Duytsche Mathematicque. For details on fortification at the Duytsche Mathematicque and Van Schooten's ideas on the subject see chapter 8.

[111] [Viète, 1646].

[112] On 23 January 1638, Mersenne wrote the Leiden professor of theology André Rivet that he had sent the works of Viète to Elzevier about a year ago but did not have any information concerning the progress of the printing. Mersenne to Rivet, 23 January 1638, [Mersenne, 1972, vol. 7, 32-33].

[113] Weland to Jungius, July 1639, [Jungius and Avé-Lallemant, 1863, 285].

[114] The involvement of Beaugrand is apparent from a letter of Mersenne to Rivet, 23 January 1638, [Mersenne, 1972, vol. 7, 33].

[115] The treatise had originally published in 1591, [Viète, 1591]; the new version with Beaugrand's notes is [Viète and de Beaugrand, 1631].

[116] [Mersenne, 1972, vol. 14, 261].

he was in Leiden at the time and had experience in tracing mathematical figures. In any case the production of the figures did not lead to an immediate start of the printing, and by August 1640, Elzevier was unable to announce the date of printing. Mersenne, who was annoyed by the delay of the publication, urged Descartes to withdraw his copies of Viète's works. Elzevier refused to return the copies, as they still planned to publish the treatises.[117] The project was further delayed by the death of Beaugrand at the end of 1640, which forced Elzevier to look for another editor and commentator.

Some scholars have connected Van Schooten's journey to France in 1641–1643 to the new edition of Viète's works.[118] However, the connection between Van Schooten's journey and the editing process was not as straightforward as it might seem. Van Schooten left Leiden in 1641, and stayed in Paris with Mersenne during the autumn of 1641.[119] In September 1642, while Van Schooten was still on his journey, the Elzevier publishing house had not yet found a competent mathematician to take care of the edition of Viète's works.[120] Thus it is implausible that Van Schooten had already been approached for the editing and commenting work at that time. Van Schooten may have collected some new material on Viète during his trip in France, but I have not found evidence that he actually did so. In any case, the edition project of Viete's works was not the main motivation for Van Schooten to leave Leiden. It was common for young men of well-to-do families to make a grand tour, and Paris with the Mersenne circle was the place to be for a young and ambitious mathematician such as Van Schooten.

Van Schooten probably came into the picture as an editor and commentator only after returning from his journey. In 1642, the English mathematician Pell stressed to Elzevier the importance of having a capable mathematician on the spot to oversee the printing process of the text and to add the necessary commentaries, because otherwise the publication would lose quality. Pell suggested Golius for this task.[121] Elzevier may have accepted Pell's argument, but no such mathematician was available in Leiden at that time. Golius was the only man in town who was up to the task, but he was busy with the preparations of his Arabic–Latin dictionary.[122] After Van Schooten had returned to Leiden in 1643, he was charged with the editorial work and with the writing of the commentaries. The printing started somewhere after May 1645, and by March 1646 the whole work had almost been printed.[123]

[117] Descartes to Mersenne, 30 August 1640, [Descartes, 1899, 167-168] and [Descartes, 1947, 147-148].
[118] Dijksterhuis suggests that Elzevier may have contributed to the costs of the journey, [Dijksterhuis, 2011, 110].
[119] Descartes to Mersenne, September 1641, [Descartes, 1899, 437], [Descartes, 1951, 62] and Descartes to Mersenne, 17 November 1641, [Descartes, 1899, 450] and [Descartes, 1951, 73].
[120] [Malcolm and Stedall, 2005, 98].
[121] Pell to Hartlib, 12 October 1642 [Vaughan, 1839, 353].
[122] This dictionary eventually appeared in 1653, [Golius, 1653].
[123] Pell to Cavendish, 19 May 1645, [Malcolm and Stedall, 2005, 415] and Varenius to Jungius, 20 March 1646, [Jungius and Avé-Lallemant, 1863, 318].

2.4 Establishment as a scholar and professor

The year 1646 marked the beginning of Van Schooten's professional career as a mathematician. In that year, he succeeded his father as professor at the Duytsche Mathematicque. The same year saw the publication of the collected works of Viète with his commentary, and the publication of his first scholarly treatise: the *De organica conicarum sectionum in plano descriptione* (On the tracing of conic sections in the plane by means of instruments). With his position at the university and the publication of two books, Van Schooten was now recognized as a professional mathematician.

2.4.1 Appointment as professor

After Frans van Schooten sr. passed away on 11 December 1645, Frans jr. immediately started a lobby in order to obtain the position of professor at the Duytsche Mathematicque which his late father had held.[124] Frans jr. probably remembered that his father had to make similar efforts in 1611–1615 in order to eventually obtain the position.

At that time, the succession of the professorship of the Duytsche Mathematicque was related to the question in whose sphere of influence the institution belonged. The outcome in 1615 showed that the influence of Prince Maurice had decreased in comparison to the influence of the council of curators and burgomaster of the university. In 1600, when the Duytsche Mathematicque was founded, Prince Maurits had taken the initiative and his recommendation of Van Ceulen and Van Merwen as professors led immediately to the appointment of both men. After both professors had passed away in 1610, Maurice's candidate was Samuel Marolois, a mathematician living in The Hague.[125] The other candidate for the position was Frans van Schooten sr., who had been Van Ceulen's assistant, and who continued the lessons at the Duytsche Mathematicque after Van Ceulen's death. Frans sr. sent at least five requests asking for his appointment as professor of Duytsche Mathematicque. His candidature was supported by at least three petitions, signed by auditors of the Duytsche Mathematicque and by other men who attended the lectures.[126] Among the subscribers, one finds several surveyors and other men with important professions in the city of Leiden, such as the main carpenter, the main mason, and the main stonemason in service of the city.[127] Thus the competition was between a Leiden citizen, who was supported by a fair part of the audience of the lectures, and a court-based mathematician supported by Prince Maurice. The outcome was the appointment of Frans van Schooten sr. on 8 February 1615.[128] This appointment shows the diminished influence of

[124] Frans van Schooten sr. was buried on 14 December 1645 in the Hooglandse kerk in Leiden. RAL, Stadsarchief van Leiden (Stadsbestuur (SA II)), (1253) 1574–1816 (1897), toegang 501A, inv. nr. 1320, f. 344v.

[125] Requests of Marolois, asking support from Maurice and the Committed States, and Maurice's recommendation of Marolois are found in UBL, Archief van Curatoren, 1574–1815, inv. nr. 42/2 and 42/3.

[126] The requests by Frans sr. as well as the petitions are found in UBL, Archief van Curatoren, 1574–1815, inv. nr. 42/2 and 42/3.

[127] In the period 1611–1615, the main carpenter was Jan Ottozn. van Zeyst, the main mason Hendrik Cornelisz. van Bilderbeeck and the main stonemason Harman Claeszn, [Pelinck, 1967, 61-64]. All three of them signed petitions in favour of Frans van Schooten sr.

[128] UBL, Archief van Curatoren, 1574–1815, inv. nr. 20, f. 354v., [Molhuysen, 1916, 59].

Maurice on his own project, and the increasing significance of the Duytsche Mathematicque for craftsmen and the city of Leiden.

When Frans van Schooten jr. started his lobby for the professorship, the ordinary procedure for the appointment was that the council of curators and burgomasters first identified capable candidates and then decided during a meeting which candidate would be approached for the position.[129] Thus the decision was made by the council of curators and burgomasters, and Van Schooten jr. tried to influence the members of this council by means of his network.

On 13 December 1645, just two days after the death of his father and one day before the burial, Van Schooten sent a letter to Constantijn Huygens, indicating that he would like to succeed his father as professor of Duytsche Mathematicque. He asked Huygens to support his candidature and to use his own network for this purpose. Van Schooten suggested that his competence in the mathematical ideas of Descartes should not be mentioned, in order to make sure that Amelis van den Bouckhorst did not know about his connection with Descartes.[130] Van den Bouckhorst was one of the four curators and probably the most influential of them.[131] By 17 December 1645, Van Schooten had also asked his acquaintances to put in a good word for him at the burgomasters.

Van Schooten was already aware that he was not the only candidate for the vacant position: his rival Jan Jansz. Stampioen was in the running as well. On 17 December 1645, Van Schooten wrote to Constantijn Huygens that he was most worried by the preferences of the curators and stadholder Frederick Henry. He also mentioned that he had understood that one of the curators, Amelis van den Bouckhorst, was on the hand of Stampioen. This preference of Van den Bouckhorst must have been the reason why Van Schooten did not want his connections with Descartes to be known to Van den Bouckhorst. The controversy between Stampioen and Descartes of the years 1638–1640 was still fresh in the memory, so knowledge of his contacts with Descartes might diminish his chances on obtaining the professorship. Furthermore, Fredrick Henry would also support Stampioen, who was the tutor of the young prince William. Van Schooten probably overestimated the influence of Frederick Henry, who had never interfered in university matters.[132] Van Schooten asked Huygens to discuss the matter with Pollot, and to contact George Gleser.[133] Both Pollot and Gleser had made career in the army and were confidents of stadholder Frederick Henry.[134]

Van Schooten also tried to use Descartes's network. Provided with a letter of recommendation by Descartes,[135] Van Schooten visited princess Elisabeth of Bohemia (1618–1680) on 27 December 1645 in The Hague. Descartes and Elisabeth maintained

[129] [Sluijter, 2004, 131].

[130] This letter is lost today, but its contents are known from an auction catalogue, [Veilingcatalogus, 1882, 55]. In this auction catalogue, the man referred to as Lord of Wimmenum is erroneously identified as Nicolaes van den Bouckhorst. By 1645, Nicolaes had passed away and his son Amelis had become lord of Wimmenum.

[131] [Otterspeer, 2000, 427].

[132] [Sluijter, 2004, 53].

[133] Frans van Schooten jr. to Constantijn Huygens, 17 December 1646, [Huygens, 1915, 265].

[134] Pollot had contacts with Descartes; for more information on Pollot see [Descartes, 2003, 289-292]. On Gleser, see [Huygens, 1913, 97].

[135] This letter of Descartes has been lost.

a correspondence from the spring 1643 onwards. Their exchange initially focused on mathematics, natural philosophy and metaphysics but gradually became more personal.[136] Van Schooten requested Elisabeth to recommend him to the curators for the vacant position. Out of loyalty for Descartes, Elisabeth was willing to make an effort for Van Schooten's sake. However, Van Schooten was highly impressed by the status of Elisabeth, and he was not able to conceal his uneasiness. Immediately after having requested Elisabeth's support, he rushed out of the room, and Elisabeth had to follow him all the way to the door in order to ask him to whom she should send her recommendation.[137]

According to Elisabeth, the main danger for Van Schooten was not his competitor Stampioen, who was not even considered to be anywhere near Van Schooten in mathematical knowledge, but the closing down of the Duytsche Mathematicque. The curator Van den Bouckhorst was initially in favour of closing down, but the intervention of Elisabeth made him change his mind and support Van Schooten.[138] Van Schooten wrote to Constantijn Huygens that he was surprised by the possible closing of the Duytsche Mathematicque. Because so much had been achieved by the Duytsche Mathematicque, the whole idea of closing was in his opinion only a pretext for not having to select one of the candidates. With respect to his own candidature, Van Schooten thought that the only possible obstacle was his religion, because he was not a calvinist but a remonstrant. But since "two times three makes six" for a calvinist as well for others, the religion should not cause a problem for the professorship. The professor of the Duytsche Mathematicque was not a member of the Senate and would therefore not be involved in theological discussions within the university anyway.[139]

Van Schooten's lobby proved successful and he was appointed professor on 8 February 1646 by the board of curators and burgomasters on the yearly pay of 400 guilders.[140] From this moment onwards, the Duytsche Mathematicque occupied a permanent position in Van Schooten's life. He devoted a lot of time and energy to the Duytsche Mathematicque, developing the curriculum, preparing lectures and reading them four days a week. Thus the Duytsche Mathematicque structured this daily routine for more than fourteen years, from 1646 until his death in 1660. He revised the courses his father had taught, altered the curriculum, introduced new subjects like perspective and logarithms, and developed his own views on algebra and fortification. This work is unrelated to the contents of the *Géométrie* of Descartes, which was not part of the public courses at the Duytsche Mathematicque. Because the Duytsche Mathematicque played a prominent role in Van Schooten's life, the subject deserves more space than this biography allows, and will be further discussed in part III of this thesis.

[136]On the contacts between Elisabeth and Descartes, see [Descartes, 2003, 258-261]. Descartes dedicated his *Principia philosphiae (Principles of philosophy)* to Elisabeth, [Descartes, 1644a].

[137]Elisabeth of Bohemia to Descartes, 27 December 1645, [Descartes, 1901, 339-340] and [Descartes, 1956, 338-339]; an English translation has appeard in [Palatinate and Descartes, 2007, 128-129].

[138]Elisabeth of Bohemia to Descartes, 27 December 1645, [Descartes, 1901, 339-340] and [Descartes, 1956, 338-339].

[139]The Senate was the council of ordinary professors. As the professor of the Duytsche Mathematicque was an extraordinary professor, he was not present in the meetings of the Senate. Frans van Schooten to Constantijn Huygens, 4 February 1646, [Huygens, 1913, 278-279].

[140]UBL, Archief van Curatoren 1574–1815, inv. nr. 23, f. 302v.-303r. and [Molhuysen, 1916, 304].

2.4.2 The first published works

The collected works of Viète

The year 1646 also marked the establishment of Van Schooten as a scholar by the publication of his first two works. The collected works of Viète appeared at the end of the summer of 1646. The edition included explanatory notes by Van Schooten at the end of the volume, and a dedication at the beginning, dated August 1646 and addressed to his teacher Jacobus Golius. In this dedication letter, Van Schooten reveals that he first encoutered the mathematical ideas of Viète in the public lectures of Golius. Van Schooten states that Golius would have been the right person for editing the works of Viète, because he had explained these works in a clear way. But since Golius's duties prevented him from taking up the job, Van Schooten was eventually approached for the task, and he expresses his gratitude to Golius for his advice and assistance during the editing process.[141]

A treatise on tracing conic sections: *De organica conicarum sectionum in plano descriptione*

Later in the same year, an Schooten's first independent work *De organica conicarum sectionum in plano descriptione* appeared with Elzevier. In this work Van Schooten introduced several new instruments for tracing conic sections in the plane. He dedicated the book to the curators of the university and the burgomasters of the city of Leiden, who had appointed him as professor of the Duytsche Mathematicque earlier in 1646.

In the dedication letter, Van Schooten described the following analogy between the study of mathematics in Antiquity and at Leiden University: the Ancients studied mathematics in Greek, which was their mother tongue, and at Leiden University it was also possible to study mathematics at the Duytsche Mathematicque in the vernacular.[142] By emphazising this parallel between Antiquity and his own time, Van Schooten placed his professorhip in the tradition of the Ancients and legitimized the use of the vernacular in his teaching at the Duytsche Mathematicque. The curators were pleased by his book (and probably by the dedication letter as well) and rewarded Van Schooten with a one-time grant of 60 guilders.[143]

This was the first independent work of Van Schooten and with it, he presented himself to the mathematical community. Therefore it is appropriate to discuss the content and background of the work in some detail.

The subject of the work fitted nicely in the agenda of the mathematical community as conic sections were a popular theme of study in the seventeenth century, for the following reasons. The publication of Pappus's *Collection* in 1588 drew the interest of mathematicians to the fact that some Greek mathematical works were lost. These included Books V-VII of the *Conics* by Apollonius, a standard work on conic sections. In the seventeenth century, conic sections gradually became acceptable means of construction in solving geometrical problems, in addition to the Euclidean straight lines and circles. In the proof of

[141] [Viète, 1646, dedication letter].
[142] [Schooten, 1646, dedication letter].
[143] UBL, Archief van Curatoren, 1574–1815, inv. nr. 24, f. 1r.-v.

the correctness of such a construction, properties of conic sections were used, although little or no attention was usually given to the actual drawing of the conic sections themselves.[144] The conic sections were defined in the theoretical way of Apollonius of Perga as the intersections of an (oblique) cone and a plane. Yet, some geometers became interested in the explicit drawing of conic sections because of the use of these curves for practical purposes such as lens grinding and gnomonics. Often they used pointwise constructions of a type which had been explained by the mathematician and draughtsman Mydorge in 1631 in the second book of his *Prodromi catoptricorum et dioptricorum* (Preliminaries to catoptrics and dioptrics).[145]

Van Schooten had personally encountered this revived interest in conics sections in its different aspects. For Golius, he had drawn the geometrical figures for the projected Latin translation of the lost Books V-VII of the *Conics*.[146] It is tempting to assume that he also studied the content of these lost books with Golius. As we have seen above, van Schooten discussed with Descartes ways of generating conic sections by means of motions or strings, and he even proposed such constructions to Descartes.[147] The generation of curves by motion was a pivotal ingredient of Descartes's geometrical doctrine in the *Géométrie*. The main criterion for Descartes for accepting curves in geometry was based on the way in which the curve could be traced; he only admitted curves in geometry which could be generated by what Henk Bos has called a "coordinated and continuous motion".[148] For Descartes, curves were generated by kinematic models to be contemplated in the mind rather than by actual instruments, and for him, his geometrical doctrine was not an incentive for actually making concrete instruments for drawing curves.

Van Schooten had also encountered conic sections in the setting of dioptrics, while working with Descartes on the grinding of lenses. Here Van Schooten experienced the difficulties of tracing a conic section accurately. He had practical experience with the instruments which were used in the Duytsche Mathematicque for different purposes such as measuring angles. He was also familiar with the way in which the craftsmen who attended the lectures at the Duytsche Mathematicque used various instruments in their daily work. In addition, Van Schooten knew about the instruments which his former teacher Otterus had designed for tracing conic sections, and also with the paper models of these instruments which Otterus had made.[149]

In *De organica conicarum ... descriptione*, Van Schooten related results from the classical Greek theory on conic sections to his own insights on the generation of curves by means of motion and to the fabrication of concrete instruments. In the preface, he explained to his reader that the principal aim of his book was to provide a description of conic sections in a plane by means of motions in that plane. The motivation for this

[144] [Bos, 2001, 217].
[145] [Mydorge, 1631] and [Bos, 2001, 217].
[146] For more details see below, p. 70.
[147] See page 27 above.
[148] See [Bos, 2001, 339]. A coordinated continuous motion has four properties: (1) the moving objects are straight or curved lines; (2) the point which traces out the curve is the intersection of two of such moving lines; (3) the movements are continuous; and (4) the movements are coordinated by one initial motion. On the Cartesian criteria for accepting curves in geometry see [Bos, 2001, 335-354].
[149] See page 26 above.

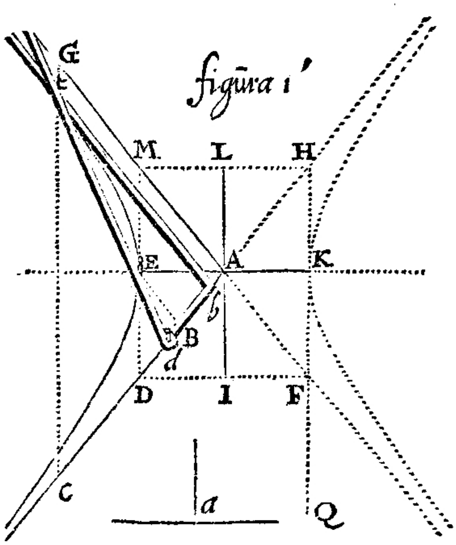


Figure 2.4 – The stylized construction of the motions generating a hyperbola, [Schooten, 1646, 38].

project was essentially twofold. First, as conic sections were employed in the construction of geometrical problems, the question arose how these conic sections themselves were obtained in practice. Van Schooten was one of the first mathematicians to systematically address this question. According to Van Schooten, one would need a proper description of the conic sections and for him such a proper description involved motion. For him it was unsatisfactory to use Apollonius's definition of the conic section as the intersection of an oblique cone with a plane. Secondly, a deeper understanding of conic sections was also useful in more practical branches of mathematics such as dioptrics, catoptrics, perspective and gnomonics. As mentioned above, a perfect lens had a parabolic, elliptical or hyperbolic rotation surface. In perspective, a circle can be transformed in an ellipse and in gnomonics, the endpoint of a shadow of the gnomon describes a conic section on the plane of the sundial. Therefore the production of lenses and sundials required the actual tracing of conic sections. For this matter, Van Schooten considered it appropriate to discuss the actual construction of instruments for tracing conic sections, an activity which he classified as part of mechanics to distinguish it from mathematics.[150]

The structure of Van Schooten's treatise reflects the dichotomy between motion as a mental activity for the proper description of conic sections and the actual design of concrete instruments for drawing the conics. The first four chapters deal with the mental contemplation of the tracing of curves. In these theoretical chapters, Van Schooten shows that the curves produced by his abstractly defined instruments satisfy properties which Apollonius proved in Book 1 of the *Conics* and therefore are conic sections in the Greek

[150] [Schooten, 1646, Preface to the reader].

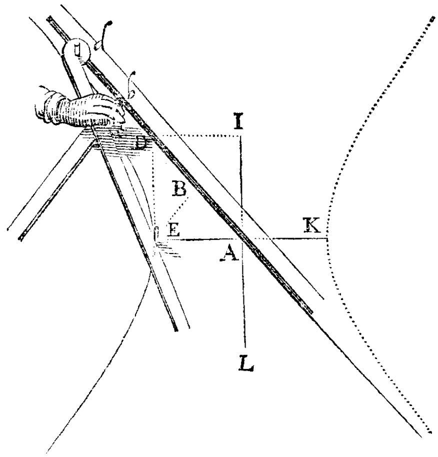


Figure 2.5 – An instrument for drawing a hyperbola, [Schooten, 1646, 45].

sense. Thus the theoretical chapters serve as the justification of the ten more practical chapters which follow. These chapters are about the practical construction of instruments and their use in the drawing of a conic section. Van Schooten gives details on how to construct the instruments, what material to use, and how to arrange the transfer of movement from one part of the instrument to another.

The same dichotomy can be seen in the titles of the chapters and in the figures accompanying the discourse. The titles of the theoretical chapters are of the following type: "On ellipses described by implicated motions," whereas the practical chapters bear titles such as "On the mode of describing ellipses". The figures show the difference between mental and material process as well. The figures on motions in the theoretical chapters are depicted in a stylized way (see figure 2.4.) whereas in the chapters on drawing, actual instruments are depicted with little hands showing the working of these instruments (see figure 2.5).[151]

2.5 Mathematics: A family affair

The Van Schootens managed to keep the chair of the Duytsche Mathematicque in the family for over sixty years. In the period between 1615 and 1679, the professors were Frans sr., Frans jr. and Petrus van Schooten respectively. In Leiden, this practice was not at all uncommon. Between 1575 and 1672, twenty percent of the sons of Leiden professors became professors themselves, in most cases in Leiden as well. Three-quarters of these "son-professors" occupied the same chair as their fathers.[152] A similar thing happened to

[151] [Dijksterhuis, 2011, 116-117]
[152] [Otterspeer, 2000, 305].

the chair of mathematics at the University of Leiden, where Willebrord Snellius succeeded his father in 1613.[153] In the University of Franeker, Bernard Fullenius sr. held the chair of mathematics in the period 1636–1657 and his son Bernard jr. was professor of mathematics in the years 1684–1707.[154]

Such a continued occupation was also common among surveyors. An investigation of admissions of surveyors reveals typical families of surveyors, practicing the same profession over generations.[155] Leiden had the prominent surveyor Jan Pietersz Dou, who also held a position as public notary. His sons and grandson followed his footsteps.[156] The rival of the Van Schooten family, Jan Jansz. Stampioen, was also a member of a family with a mathematical and surveying background. His father Jan Jansz. sr. was also a surveyor, and his son Nicolaes must have possessed mathematical knowledge as well, because he was consulted concerning a method for determining longitude on sea.[157]

In early modern society, the extended family played an important role in social relations. The blood relatives and the in-laws formed the most natural network that could be used to support the family. Such support could involve major decisions in life, for instance help in finding a suitable marriage partner or assistance in finding a suitable job. The family also played an important role after the death of a family member, especially when minor children became orphans; then the children were initially supported by the family.[158] In such cases, the continuation of a chair at the university was in the interest of the family, although the successful implementation also depended on the abilities of the sons. Frans jr. had already shown his interest and talent in mathematics at an early age.

We have reason to believe that Van Schooten sr. carefully planned the succession of his chair during his life, with his son as his intended successor. Early in 1635, Van Schooten sr. addressed a petition to the board of curators and burgomasters, requesting permission for his son Frans jr. to take care of his public lectures during his illness. The main reason for the request were the heavy headaches, the attacks of dizziness and the severe pains caused by kidney stones which tormented Van Schooten sr. to such an extent that he had been temporarily unable to fulfill his obligations as a professor. For the benefit of the students of the Duytsche Mathematicque, Van Schooten proposed his son as a temporary substitute. In the petition he recommended the qualities of his son and stated that he had trained his son in the mathematical sciences from childhood.[159]

The board of curators and burgomasters did not approve the petition immediately, but decided to undertake a further investigation into the qualities of Frans jr., who was only 19 years old at the time. Although the appointment of a new professor was the joint competence of the curators and burgomasters together, it was decided that investigation should be carried out by the burgomasters. This decision may be an indication of the significance of the Duytsche Mathematicque for the city. No details of the investigation are

[153] [Wreede, 2007, 74].
[154] On the careers of the two Fullenii see [Dijkstra, 2012, 162-170 and 285-310].
[155] [Muller and Zandvliet, 1987, 149].
[156] [Pouls, 2004, 26].
[157] [Descartes, 2003, 299-300].
[158] [Frijhoff and Spies, 2004, 217].
[159] UBL, Archief van Curatoren, 1574–1815, inv. nr. 22, f. 187v.-188r.

known, but the outcome must have been positive, because the board approved the substitution by Frans jr. on 8 June 1635. They attached only a single condition to their approval of the petition of Van Schooten sr.: permission was only granted orally and no confirmation in writing would be given to Van Schooten sr. [160] Perhaps the board wanted to avoid any formal documentation which Frans jr. could use later on for claiming the chair of the Duytsche Mathematicque. Nevertheless, Van Schooten jr. succeeded his father in 1646.

Yet, family ties could be defaced. The settling of the inheritance of Van Schooten's sr. was a problematic affair, straining the family ties. Frans jr. was the son of Van Schooten sr.'s first marriage. After the death of his wife, Van Schooten sr. contracted a second marriage with Maria Gool. This marriage produced three children reaching maturity: Petrus, Cornelia and Johanna.[161] When Frans sr. passed away in 1645, he left his wife and four children, of whom Frans jr. was from his first marriage. Van Schooten sr. and his wife had drawn up a will on 1 November 1632, with detailed arrangements of their inheritance.[162] In this will, Frans sr. specified that in case he was the first one to pass away, his inheritance would have to be divided in the following way: Frans jr. would inherit a house on the Oude Rijn in Leiden and all the books, manuscripts, maps and two instruments, or the sum of one hundred guilders instead of the two instruments.[163] The rest of Van Schooten sr.'s possessions, both movable and immovable, would devolve upon Maria Gool.

Settling the inheritance took several years and was the main issue in a conflict between Frans jr. and his stepmother Maria Gool. After the death of his father, Frans jr. was not allowed to see the complete will, and he was not granted access to the inventory which must have been drawn up shortly after Frans sr. had died in 1645. The drawing up of such an inventory was common practice after a death. A copy of the inventory was only given to Frans jr. on 27 November 1650, five years after the death of his father. Another copy of the inventory, made in 1651, has been preserved. This copy contains critical remarks in the hand of Frans jr. concerning the inheritance.[164]

Frans jr. criticized the behaviour of Maria Gool concerning the inheritance. Whereas Maria claimed that Frans jr. had obtained all books and instruments of his father, in agreement with the will, Frans jr. stated that this was not the case: he had only obtained 35 or 36 books, whereas the total collection of his father must have consisted of 300 to 400 books. He also noted that Maria did not grant him access to his own possessions in the parental home, and that he was not even allowed to take his best coat with him. Moreover, he accused Maria of making a profit from the inheritance, by liquidating non-existing debts with money of his late father, by withholding goods from the inventory, and by putting erroneous total sums in the inventory. We do not know the details of the final settlement

[160] UBL, Archief van Curatoren, 1574–1815, inv. nr. 22, f. 188r.

[161] RAL, Heilige Geest- of Arme Wees- en Kinderhuis (HGW), toegang 519, inv. nr. 4591. Petrus was born in 1634, [Molhuysen et al., 1927, 1116], but the dates of birth of Cornelia and Johanna are unknown.

[162] RAL, Heilige Geest- of Arme Wees- en Kinderhuis (HGW), toegang 519, inv. nr. 4588.

[163] No further details on the instruments are found in the will. Probably they were surveying instruments.

[164] The notes by Frans van Schooten jr. are in pencil in the margin of the copy. I identified Frans jr. by his handwriting and by the phrase "Eeniger dagen naer mijn vader zal overlijden, heeft mij myn stiefmoeder (...) het testament getoont" ("A few days after the death of my late father, my stepmother showed me the will"). RAL, Heilige Geest- of Arme Wees- en Kinderhuis (HGW), toegang 519, inv. nr. 4589.

of the inheritance, but the accusations which Frans jr. made against his stepmother must have troubled the relationship with the Gool family for quite some time.

This history of the inheritance of Van Schooten sr. shows the limitations which a historian encounters when relying on inventories: parts of the property may not have been registered, as was the case with the books and the instruments which Van Schooten sr. owned.[165] Nevertheless, the inventory contains interesting information on the family and on the property they owned. Van Schooten sr. possessed four houses, estimated at a total value of 9,350 guilders. The most valuable house was the dwelling house on the Steenschuur which was estimated at 6,000 guilders in 1645.[166] Moreover, Van Schooten sr. owned bonds (*obligaties)* of a total value of 19,142 guilders. With this property, the Van Schootens belonged to the class of prosperous citizens.[167]

What can be said about the books possessed by the Van Schootens? Frans sr. seems to have owned 300 to 400 titles, of which at least 35 were inherited by his son Frans jr. A collection of 300 to 400 titles is considerable, but minor if compared to the collections of other professors. University professors in Leiden often owned collections containing over a thousand titles, and the same was the case in Groningen.[168] Compared to the book collections of citizens, the collection of Van Schooten sr. was large. Keblusek analysed one hundred inventories of citizens in The Hague, and of these only one contained a book collection of more than 400 titles.[169] A comparison can also be made with the 1644 inventory of Jan Jansz. Stampioen, which was made after the death of his wife Anna Pieters by order of the orphan's chamber. Stampioen owned 95 bound books, and a considerable amount of unbound books, which included unsold copies of his own books. His collection consisted to a large extent of mathematical titles, including works by Ramus, Van Ceulen, De Decker, Vlacq and Stevin.[170] Thus the collection of Stampioen was much smaller than that of Van Schooten sr.

Eventually, the books, maps, manuscripts and instruments of Frans sr. as well as the collection of Frans jr. were inherited by Petrus van Schooten.[171] After the death of Petrus the books and instruments were auctioned on 4 March 1680 by the bookseller Felix Lopez de Haro at the Rapenburg in Leiden. A catalogue of the collection has been preserved.[172] The collection consisted of 861 titles: 185 in folio, 44 in quarto, 156 in octavo, and 76 in

[165] For a thourough discussion on the use of inventories in historical research see [Wijsenbeek-Olthuis, 1995].

[166] RAL, Heilige Geest- of Arme Wees- en Kinderhuis (HGW), toegang 519, inv. 4589. The purchases of the houses on the Steenschuur in 1623 and 1629 are documented in RAL, Stadsarchief van Leiden (Stadsbestuur (SA II)), (1253) 1574–1816 (1897), toegang 501A, inv. nr. 6614, f. 133v. and 134v. These houses were located on what is nowadays the Rapenburg, somewhere between Rapenburg 80 and Rapenburg 96.

[167] I follow here the distinction made by Marika Keblusek. She distinguishes five categories of citizens in The Hague in the seventeenth century: (1) indigents with property under 1,000 guilders; (2) small owners with property between 1,000 and 10,000 guilders; (3) prosperous citizens with property between 10,000 and 50,000 guilders; (4) wealthy citizens with property between 50,000 and 100,000 guilders and (5) very wealthy citizens with a property over 100,000 guilders, [Keblusek, 1997, 46]. Compared to The Hague, Leiden citizens were somewhat less wealthy, [Noordam, 1994, 87].

[168] [Hoftijzer, 2007, 138] and [Huisman, 2003, 320].

[169] [Keblusek, 1997, 146].

[170] [Keblusek, 1997, 150].

[171] RAL, Heilige Geest- of Arme Wees- en Kinderhuis (HGW), toegang 519, inv. nr. 4591.

[172] [Haro, 1680].

duodecimo. The majority of the books dealt with mathematics in its seventeenth-century sense: geometry, arithmetic, astronomy, astrology, fortification, optics etc. Of the 185 folio titles, 85 % were mathematical books, and approximately the same percentage of the other formats was on mathematics as well. The remaining fifteen percent were mainly books on alchemy, medicine and history. It is noteworthy that the collection hardly included religious literature, and that the religious books were two treatises by the remonstrant minister and protestant leader Wtenbogaert (1557–1644), works by the remonstrant theologicians Simon Episcopius (1583–1643) and Conradus Vorstius (1569–1622), and two treatises by Hugo Grotius (1583–1645).[173]

Van Schooten jr.'s own family life is poorly documented. In 1652 he married at the age of 37 Margaritgen Wijnants, his former handmaiden.[174] Nothing is known about the background of Margaritgen. At the time of their marriage, the couple must have been together for some years, because they had made a will on 22 July 1649 instituting each other as heirs. This will was revised shortly before their marriage in 1652.[175] In May 1651 Frans van Schooten jr. bought a house in the Herensteeg and he moved in shortly afterwards.[176] We do not known where he lived before he moved to the Herensteeg. No children are known from his marriage.

2.6 Towards a Latin version of the *Geometria*

The history of Van Schooten's translation and publication of the Latin version of the *Geometria* is closely linked to the history of Descartes's own work. As Descartes and Van Schooten were in regular contact, the story of the *Geometria* concerns their relationship as well, and the extent of Descartes's involvement in the project. The question also arises who initiated the translation project: Van Schooten, Descartes himself, the printer or even someone else? Another question concerns the chronology: when did Van Schooten start his work on translating the *Géométrie*?

At its publication in 1637, the *Géométrie* was one of the four parts of a work which consisted of an introduction and three accompanying essays. The *Discours de la méthode*

[173] The works with a remonstrant nature were Wtenbogaert's *Twaelf Predicatien, over verscheyden aenmerckelicke Texten ende Materien: dienende tot ware Godtsalicheydt ende Christelicke Vrede der Kercken* and *Responsio necessaria ad contra-remonstrantium contrariam declarationem: continens perspicuam & solidam eiusdem refutationem; in qua etiam demonstratur doctrinam remonstrantium, si nascentis Christianismi secula, & subsecuta postmodum reformationis tempora respicias, non novam sed antiquam esse*; two works by Episcopius: *Apologia pro confessione sive declaratione sententiae eorum, qui in Foederato Belgio vocantur Remonstrantes, super praecipuis articulis religionis christianae. Contra Censuram quatuor professorum Leidensium: inscripta nobilibus, prudentibus ac potentibus d.d. deputatis et consiliariis Ordinum Hollandiae et West-Frisiae* and *Responsio ad duas Petr. Wadingi, Jesuitae Antverpiensis, epistolas*; and one work by Vorstius: *De immutabilitate & simplicitate Dei.*

[174] Leiden Archief, Doop-, trouw- en begraafboeken Leiden, inv. nr. 14, f. 294r. Her name was also spelled "Magrijchen" or "Greychen", [Knappert, 1938].

[175] For a discussion of these notarial documents see [Knappert, 1938].

[176] RAL, Stadsarchief van Leiden (Stadsbestuur (SA II)), (1253) 1574–1816 (1897), toegang 501A, inv. nr. 6614, f. 111r. Van Schooten refers to his recent move in the letter dated 30 June 1651 to Christitaan Huygens, [Huygens, 1888, 144]. This house was located at the modern address Herensteeg 21. RAL, PV1142.7.

served as the introduction in which Descartes discussed his philosophical method on using the mind and discovering truths. The three essays illustrated the use of this method in particular domains: dioptrics, meteorology (i.e., atmospheric phenomena) and geometry. The strong link of the *Géométrie* with the philosophic method appears in the following quotation from Descartes in a letter to Mersenne:

> I have only tried to persuade that my method is better than the ordinary by the *Dioptrique* and the *Metéores*, but I claim to have shown it by my *Géométrie*.[177]

Thus, Descartes emphasizes the prominent role of the *Géométrie* in his project, and he even claims that it was more significant than the other two essays.

The unity of the four texts was broken by the Latin translation of the *Discours de la méthode* in 1644, which included only the essays on dioptrics and meteorology. The *Géométrie* was omitted. Van Schooten also contributed to the detachment of the *Géométrie* from the *Discours de la méthode* and the two other essays by his 1649 publication of the translation as a geometrical text on its own. He continued the separation process by the addition of work of his students to the second edition of 1659. Thus the *Géométrie* was no longer an essay illustrating Descartes's philosophical method, but an independent mathematical treatise with its own merits, to be read separately from the *Discours de la méthode*.

This separation from the original four-part publication is related to the nature of the *Géométrie*, which was (and still is) hard to understand due to the difficult mathematical content, the notation which was new for the time, and the novel mathematical ideas and techniques. At the time of the publication of the *Discours de la méthode* in 1637, Descartes was well aware of the difficulties which the *Géométrie* might pose to his audience, and he inserted an explicit warning to the reader indicating that this last essay presupposed mathematical knowledge.[178] He had six separate copies printed on large paper, to be distributed to the first six people who showed a good understanding of the treatise.[179] The *Géométrie* was only accessible to mathematicians, and even they had severe difficulties in understanding it in depth.

In the summer of 1638, Descartes had the idea to publish a Latin edition of the four treatises together with the objections he had received to his work and his replies. In the summer of 1639 it was known in Leiden that Maire was going to print the Latin edition, but this project never materialized. Eventually the Latin translation was printed as *Specimina philosophiae* (Specimina of philosophy) in 1644 with Elzevier. This translation did not include a Latin version of the *Géométrie*.[180] The manuscript on which the *Specimina* was based must have been finished in the summer of 1638 at the latest. The translator has not been identified with certainty although the name of Etienne de Courcelles is mentioned in this connection.[181]

[177] Descartes to Mersenne, end of December 1637, [Descartes, 1897, 478] and [Descartes, 1939, 65].

[178] [Descartes, 1637c, 296].

[179] Among the recipients one finds Florimond de Beaune and Godefroy van Haestrecht. For the other recipients of these six copies see [Otegem, 2002, 100-102].

[180] For a detailed account of the course of events with respect to the printing of objections, the second edition of the French *Géométrie* and the ideas on a Latin edition, see [Descartes, 2007, 1-18] and [Otegem, 2002, 34-37].

[181] In her study on the *Specimina*, Vermeulen does not dare to identify Courcelles as the translator; she

The manuscript did not include a translation of the *Géométrie* either. May the absence of the *Géométrie* then indicate that this task had already been claimed by Van Schooten by 1638? Vermeulen has convincingly argued that this was not the case, as a letter of Descartes to Mersenne on the possible printing of De Beaune's *Notes brèves* (Short notes) reveals that at that time no Latin translation existed and that there were no concrete plans for making one.[182] If Van Schooten had planned to translate the *Géométrie*, Descartes would have been aware of such plans in view of the close relationship between the two. Thus it seems unlikely that Van Schooten had any plans to translate the *Géométrie* before his journey to France.

Once he had returned to Leiden in early 1643, Van Schooten was busy with several projects. In the spring of 1643, he was hired by Descartes for the drawing of the illustrations for the *Principia Philosophiae*, and in the years until 1646 he prepared his edition of Viete's work for the press and finished his own *De organica conicarum ... descriptione*. In the meantime, he started to write a treatise on fortification, and also started working on a restitution of Apollonius's lost work *Plane loci*.[183] Thus Van Schooten was a busy man and he had ambitious plans for the near future. Yet the two treatises he intended to publish were delayed for a long time. The restoration of Apollonius saw the light only in 1657 and the treatise on fortification was never printed at all.[184] His work on a translation and commentary on the *Géométrie* may well have been the main reason for this long delay. If this is true, Van Schooten probably started serious work on his translation by the end of 1646 at the earliest.[185]

In the preface of the 1649 edition of the *Geometria*, Van Schooten claims that the initiator of this Latin translation was the publisher Jean Maire, who had also published Descartes's *Géométrie* in 1637.[186] This may well be a faithful account of the events. After the publication of Descartes's *Specimina Philosophiae*, the *Géométrie* was the only one of the three essays that had not been translated into Latin yet. Maire possessed the woodcuts which Van Schooten had made for the original French edition, and thus the publication of a Latin version would be relatively easy once a translation had been made.[187] Furthermore, it seems that Maire considered a Latin edition of the *Géométrie* to be profitable, despite his relatively bad experience with the French edition. In any case the Latin edition might have deteriorated the sales of the original French edition.[188]

---

states that there is a possibility that the translator was someone else from among Descartes's acquaintances, [Descartes, 2007, 8-14].

[182] [Descartes, 2007, 16-17]. The letter under consideration is Descartes to Mersenne, 25 December 1639, [Descartes, 1898, 638-639] and [Descartes, 1941, 307-308].

[183] Van Schooten mentioned these two treatises in the preface of the *De organica conicarum ... descriptione*, [Schooten, 1646, Preface].

[184] Van Schooten's restitution of *Plane loci* was printed as Book III of his *Exercitationum mathematicarum libri quinque* in 1657, [Schooten, 1657, 191-292]. A Dutch version was published in 1660, [Schooten, 1660c, 184-272]. Van Schooten's ideas on fortification will be discussed in chapter 8.

[185] Van Schooten's manuscript Hs 108, dated by me in the period 1632–1641, contains some notes in Latin on the *Géométrie*, but the manuscript does not bear traces of a systematic translation project of the *Géométrie*. UBG, Hs 108.

[186] [Schooten, 1649b, Preface to the reader].

[187] [Otegem, 2002, 37]

[188] [Netten, 2012, 184].

If Maire was indeed the initiator, he must have approached Van Schooten at the latest in 1647. It seems that Van Schooten's translation was finished before April 1648, because Descartes wrote to Mersenne on 4 April 1648 that

> I have by no means wanted to see the version of Schooten, even though he had desired it.[189]

It remains questionable if Descartes gave a truthful account by stating that he had not seen Van Schooten's *Geometria*, because Descartes participated in the preparation of a explanatory note in Van Schooten's comments.[190]

The news of the upcoming Latin edition of the *Géométrie* soon made its way to the French mathematical community, resulting in several reactions. One such reaction concerned the mathematical details of the so-called Stampioen affair of the years 1638–1640, in which Jan Jansz. Stampioen and Descartes (through a spokesman called Waessenaer) were involved.[191] As the publications related this affair were in Dutch, their contents remained inaccessible for the French community. Some curious French mathematicians, probably including Mylon, therefore urged Van Schooten to publish a translation of the *Problema Astronomicum*, which Stampioen had announced, and a solution in the *Geometria*. Van Schooten accepted this request.[192] Secondly, Mersenne informed Van Schooten in the spring of 1648 on the latest developments in France concerning the so-called problem of Pappus, which Descartes claimed to have solved in the *Géométrie*.[193]

By June 1648, Van Schooten had finished his Latin translation of De Beaune's *Notes brèves*. He had not informed De Beaune of his translation nor of his project of publishing the notes. When De Beaune heard of these plans, he cooperated and even provided Van Schooten with some extra observations to be added to the notes.[194]

The printing of the *Geometria* was finished before 17 August 1649.[195] The dedication letter is dated 12 July 1649 and the dedicatee was princess Elisabeth of Bohemia. The reason for dedicating the work to her must have been her efforts in 1646 for the appointment of Frans van Schooten as professor. Elisabeth had shown a great interest in mathematics, and she had quite some mathematical talent, as her corrrespondence with Descartes reveals.[196]

The edition of 1649 contained several treatises. First, there was the Latin translation of Descartes's *Géométrie*. This was followed by the Latin version of the *Notes brèves* of De Beaune and by Van Schooten's own comments on the *Géométrie*. The final part was

[189] Descartes to Van Schooten, 4 April 1648, [Descartes, 1903, 143] and [Descartes, 1963, 27].

[190] See section 4.4 below and [Maronne, 2007, 115-120].

[191] On the Stampioen affair see [Descartes, 2003, 301-302].

[192] On the interest of the French mathematicians in the Problema Astronomicum see [Maronne, 2007, 383-387]. Rasmus Bartholin described in a letter to his former teacher Ole Worm (1588–1655) how Van Schooten was approached by French mathematicians [Worm, 1751, 981]. On Van Schooten's solution of the *Problema Astronomicum* see [Maronne, 2007, 361-436].

[193] I discuss the role of Mersenne in more detail in section 4.4.2.

[194] [Maronne, 2006, 216-218] and see the letter of De Beaune to Van Schooten, dated June 1648 by Maronne, published in [Descartes, 1941, 321-322]. Adam and Milhaud date the letter in 1648–1649.

[195] Descartes to Carcavi, 17 August 1649, [Descartes, 1903, 392] and [Descartes, 1963, 253].

[196] On Elisabeth's mathematical interest see [Bos, 2003].

an *Additamentum* in Latin, containg the solution on the *Problema Astronomicum* and a discussion on the method of extracting the roots of binomial expressions.

2.7 Van Schooten and Descartes

The encounter with Descartes around 1636 had a profound impact on the young Van Schooten. Van Schooten was receptive to the mathematical ideas of Descartes and he was one of the few mathematicians who could understand the content of the *Géométrie* and judge the potential of the work for the development of mathematics.[197] Van Schooten's scholarly work reflects his commitment to Cartesian geometry. He contributed to its dissemination by the Latin translations of the *Géométrie* in 1649 and 1659–1661, and by the publication of material from his own lectures in the *Principia matheseos universalis* (Principles of universal mathematics) in 1651.

Van Schooten's cooperation with Descartes in the second half of the 1630s was the basis of a relationship which would last until the death of Descartes in 1650. Throughout the 1640s, Van Schooten paid regular visits to Descartes. Just before and after Van Schooten's travel to France, it was easy for him to meet Descartes, who lived at the castle Endegeest just outside Leiden.[198] In 1643 Descartes moved to Egmond; he first lived in Egmond aan den Hoef and later in Egmond Binnen.[199] Van Schooten travelled to Egmond several times to visit Descartes.[200] These personal contacts with Descartes enriched Van Schooten, and contributed to his developments as a scholar. Through Descartes's network, Van Schooten was introduced to mathematicians in France in the early 1640s, and in 1646 Van Schooten used Descartes's network in order to organize support for his candidature as professor of the Duytsche Mathematicque.

The surviving correspondence between Descartes with Van Schooten together with Van Schooten's manuscripts and published works show that Van Schooten's interest in the ideas of Descartes was limited to mathematics. Van Schooten did not exhibit any deep insights in natural philosophy and in the other fields Descartes had worked on.[201] After Van Schooten's return from his stay in Paris he discussed the mathematical work of Fermat with Descartes.[202] In 1646 Descartes informed Mersenne that Van Schooten

[197] Descartes wrote Mersenne in March 1638 that among the few people understanding his mathematical ideas of the *Géométrie* were two men with a background in teaching military mathematics. It is likely that Descartes had Frans van Schooten jr. and Jean Gilot in mind. Descartes to Mersenne, 1 March 1638, [Mersenne, 1972, VII, 82-83].

[198] In a letter to Christiaan Huygens, Van Schooten remembers a meeting with Descartes at Endegeest shortly after his return from France in spring 1643. Van Schooten to Christiaan Huygens, 19 September 1658, [Huygens, 1889, 221-222].

[199] [Bos, 2002, liii-liv].

[200] In September 1643 Descartes invited Van Schooten to come to Egmond to work on the illustrations of the *Principia philosophiae*.Van Schooten and Descartes also met in December 1644, and shortly before 10 March 1649 Van Schooten paid a visit to Descartes in Egmond Binnen. Descartes to Constantijn Huygens, 21 December 1644, [Descartes and Huygens, 1926, 234], and Van Schooten to Descartes, 10 March 1649, [Descartes, 1903, 320] and [Descartes, 1963, 189].

[201] A similar picture emerges from the correspondence between Van Schooten and Christiaan Huygens.

[202] Van Schooten to Christiaan Huygens, 19 September 1658, [Huygens, 1889, 221-222].

had given a short overview of the content of the mathematical work of Cavalieri's *Geometria indivisibilibus continuorum nova quadam ratione promota* (Geometry developed by a certain new consideration using the indivisibles of the continuous magnitudes) to him. Van Schooten thought that the method of Cavalieri only produced results that had been proved before, and his judgement convinced Descartes that he should not spend time on Cavalieri's work.[203] Van Schooten also consulted Descartes on recent mathematical developments. In March 1649, Van Schooten sent the work of Gregorius Saint-Vincent on the quadrature of the circle to Descartes, with the request to look at it and give his opinion.[204]

Descartes shared parts of his correspondence and his private papers with Van Schooten. Most of these letters and papers dealt with mathematical subjects, though occasionally Descartes showed Van Schooten letters on other subjects as well.[205] In 1649 Van Schooten published extracts of Descartes's letters in the *Geometria* and it is likely that Descartes had given these letters to him directly.[206] Van Schooten also owned a manuscript copy of Descartes's *Compendium musicae (Compendium of music)*, which he probably copied from Descartes's original.[207]

Van Schooten also lent a helping hand to Descartes from time to time. According to the philosopher and physician Henricus Regius (1598–1679), Van Schooten was put in charge of the distribution of the complimentary copies of the *Principia philosophiae* to the friends of Descartes in 1644. Descartes left a list with the names of these friends with Van Schooten.[208] The report of Regius is plausible because Descartes left the Republic by June 1644 for France, while the printing of his treatise had not been finished yet by Louis Elzevier, and Van Schooten had been involved in the preparation of the *Principia philosophiae* for the press. However, other letters seem to indicate that Anthony Studler van Zurck (ca. 1608–1666), lord of Sweyburg and Bergen, actually distributed the copies.[209] However this may be, the fact that Regius mentioned Van Schooten in this connection indicates that Van Schooten was considered an intimate friend of Descartes. In April 1649 Van Schooten provided Descartes with a large number of quills, according to Descartes enough for a hundred years of writing.[210]

After the death of Descartes, Van Schooten was involved in the liquidation of Descartes's affairs in Leiden. Before his departure to Sweden in autumn 1649, Descartes had

[203] Descartes to Mersenne, 20 April 1646, [Descartes, 1901, 394-195] and [Descartes, 1960, 43-44].

[204] Van Schooten to Descartes, 10 March 1649, [Descartes, 1903, 318-319]. The work Van Schooten refers to is [Saint-Vincent, 1647].

[205] Van Schooten saw the letter sent to Descartes by the Queen Christina of Sweden and the French ambassador Chanut. Frans van Schooten to Dirck Rembrantsz van Nierop, 27 November 1653, [Nierop, 2012, 83].

[206] These include an extract from a letter by Descartes to Mersenne on the tangent to the trochoid, compare Descartes to Mersenne, 23 August 1638, [Descartes, 1898, 309-313] and [Descartes, 1941, 28-32] with [Schooten, 1649c, 227-229]. Van Schooten indicated explicitly that this construction is in "the words of the author" (i.e., the words of Descartes). It is difficult to imagine that Van Schooten published the extracts of this letter without the consent of Descartes. See also the quadrature of the cycloid: Descartes to Mersenne 7 July 1638 and [Schooten, 1649c, 226].

[207] Van Schooten's copy is found in UBG, Hs 108, f. 60r.-83v. On Descartes's treatise see [Wardhaugh, 2008].

[208] Henricus Regius to René Descartes, late July/August 1644, [Bos, 2002, 176].

[209] This information is based on the following letters: Elizabeth to Descartes, 1 August 1644, [Descartes, 1901, 131-133] and [Descartes, 1956, 159-160]; Constantijn Huygens to Mersenne, 16 August 1644, [Descartes, 1901, 133]; Van Zurck to Constantijn Huygens, 30 August 1644, [Descartes, 1901, 134] and [Huygens, 1915, 54].

[210] Descartes to Van Schooten, 9 April 1649, [Descartes, 1903, 336] and [Descartes, 1963, 201].

entrusted a suitcase containing letters and papers to his friend Cornelis van Hogelande (1590–1662). The suitcase was opened on 4 March 1650 at the request of Anthony Studler van Zurck. It can be assumed that Van Zurck made a claim on Descartes's possessions as Descartes owed him a considerable sum of money. The opening of the suitcase took place at the house of Van Hogelande, in the presence of the public notary Frans Doude and three witnesses. Two of these witnesses were Cartesians: Frans van Schooten and the young doctor Johannes de Raey (1622–1702), and the third witness was the Frenchman Louis de la Voyette who was a friend of Constantijn Huygens.[211]

Van Schooten held Descartes and his work in great admiration and has to be regarded as a zealous Cartesian with respect to Descartes's mathematical heritage. This attitude towards Descartes became prominent in the 1650s in several contexts, and it stood in the way of a critical reflection on the work of Descartes. The clearest example is the way in which Van Schooten dealt with criticisms by other mathematicians on Descartes's solution of the so-called Pappus problem. I will analyse these criticisms and Van Schooten's reactions in detail in chapter 5 of this thesis.

Van Schooten's uncritical admiration of Descartes's work also shines through in the correspondence between him and his talented student Christiaan Huygens. In a letter of 29 October 1652 Huygens informed Van Schooten that he had discovered a new method for producing a lens without spherical aberration. Descartes had indicated in the *Dioptrique* that a lens in the shape of a rotation solid of an ellipse or hyperbola could produce a sharp image without spherical aberration, whereas it was known that spherical lenses always produced blurred images. Huygens argued that under certain conditions a circular lens could produce a perfect image without spherical aberration. The main obstacle in the production of a perfect lens by then was the construction of the surface of the lens, as a rotation surface of a conic section. The construction of a spherical lens is much simpler and therefore Huygens's finding opened up new prospects for the fabrication of perfect lenses.[212]

Van Schooten realized of course the possible implications of Huygens's finding, but he doubted Huygens's discovery, because he attached more importance to the reasoning of Descartes than to the abilities of his student. The sceptical reaction of Van Schooten speaks volumes:

> ...of which Descartes shows that this can be achieved most easily by using plane and hyperbolic surfaces, or by spherical and elliptical ones, whereas you wish to achieve this by using spherical surfaces only. I do not know whether you have examined with sufficient thoroughness what he [i.e., Descartes] has communicated on the laws of refraction. I have come to know his mental power so profoundly, and I have always found it to be so clever, that I am quite confident that it is not easy (to find) anything by him which is erroneous or insufficiently understood; on the contrary, that same thing must be certain in

211 For a discussion of the possible content of the Leiden suitcase see [Descartes, 2003, xi-xv].

212 Christiaan Huygens to Van Schooten, 29 October 1652, [Huygens, 1888, 186]. On Huygens's work on dioptrics in the early 1650s see [Dijksterhuis, 2004, 11-52].

every respect, and seems to be equivalent to the truth itself.[213]

Huygens trusted his own discovery and welcomed this answer by Van Schooten as an opportunity to tackle this uncritical praise of Descartes. He replied to Van Schooten:

> I was much delighted [by your response], not only because this discovery seems to me even more important, but also because I have obtained at the same time an opportunity to drive out this pernicious prejudice of yours, that you do not hesitate to swear on the words of Descartes. Whose divine talent I always respect, but I do not esteem it so highly that I believe it to be useless to ascertain, with the standard of truth, the things which he often asserts without proof.[214]

Huygens also doubted the collision theory in Descartes's *Principa philosophiae* of 1644, and over the years, he became more and more convinced that Descartes's collision laws were incorrect.[215] In autumn 1654 Huygens mentioned to Van Schooten that he believed the laws of collision of Descartes to be incorrect.[216] Van Schooten's response followed a similar pattern as two years before: he could hardly believe that such an outstanding mind as Descartes could have published incorrect statements and he advised Huygens to drop the subject and focus his mind on other matters. Continuing the subject would be a waste of time and talent.[217]

These exchanges with Huygens show that Van Schooten's admiration of Descartes had assumed vast, almost divine proportions. Full of wonder for the outstanding intellect of the late Descartes, Van Schooten incorrectly believed that the man who had considered everything so fully and thoroughly could not err in his thinking. On the other hand, his zealous belief in the Cartesian mathematical ideas must have made him an inspiring teacher and communicator of these ideas.

[213] "Quae Cartesius simplicissime omnium per superficies planas et hyperbolicas, aut per sphaericas et Ellipticas fieri posse ostendit, tu autem id per solas superficies sphaericas factum vis, nescio an satis accuratè, quae de Refractionum legibus tradidit, examinaveris. Quippe tam planè ingenium ejus perspectum habeo, idque semper tam perspicacissimum deprehendi, ut planè confidam non facilè quisquam ab ipso commissum aut non satis perspectum, sed illud ipsum ex omni parte constare debere, et vel cum ipsa veritate videri certare." Van Schooten to Christiaan Huygens, 4 November 1652, [Huygens, 1888, 188].

[214] "Valde gavisus sum tam quod eo majoris momenti inventio visa est quam quod unam saltem aliquam occasionem nactus sum quâ possim damnosum istud praejudicium tibi exigere, quo ductus in Cartesij verba jurare non dubitas. Cujus quanquam ingenium divinum semper suspiciam, non tamen tantum tribuo ut non ea sine demonstratione saepe adfirmare est solitus, ad veritatis normam exigere utile credam." Christiaan Huygens to Frans van Schooten, 7 November 1652, [Huygens, 1890, 457].

[215] For the development of the ideas of Huygens on collisions see [Andriesse, 2005, 104-114].

[216] The original letter in which Huygens makes this statement is lost.

[217] Here is the Latin text: "Quod caeterarum est rerum, quaeso ut unam aut alteram demonstrationem communicare mihi digneris, quae regulas motus Cartesij, quas falsas autumas, omnino refutent. Vix enim fieri puto quicquam a sublimi ac perspicacissimo isto ingenio in lucem proditum esse, quod veritati non sit consentaneum. Praesertim cum dictae regulae ei tam perspectae fuerint, ut sibi mirum videri haud semel asseruerit, quo pacto aliquis de illarum veritate ambigere possit. Quapropter Tibi autor esse minime nolim, ne quid existimationi tuae detrimenti moliaris, ut demonstrationibus illis concinnandis supersedeas potius, quam tempus atque industriam tuam inutiliter impendas. Novi enim in abstracto tria haec distincte et a se invicem plane diversa quoad motum ab eo considerari: nimirum, pondus, spatium, et celeritas; quae duo posteriora (ut plurimum distinctu difficilia) autores dum vulgo confundunt, quid mirum si inter se dissentiant." Van Schooten to Christiaan Huygens, 25 October 1654, [Huygens, 1888, 301].

2.8 Exploring Cartesian mathematics with students

As a professor of the Duytsche Mathematicque, Van Schooten was supposed to lecture on topics in mixed mathematics in the vernacular.[218] This meant that Van Schooten's official lectures were not about the ideas of Descartes in the *Géométrie*, but about subjects such as commercial arithmetic and fortification. Discussions of the new mathematical ideas and techniques of Descartes were limited to the private lectures which Van Schooten held outside the official structure of the university. At Leiden university, such private lectures were common and held by many professors. Students gradually considered the private lectures even more important than the public lectures, with conflicts with the board of the university as a result.[219]

Over time, Van Schooten gathered a number of students around him who had a lively interest in Descartes's new mathematics. Van Schooten taught them the necessary knowledge of algebraic manipulations and geometry that was necessary to fully understand the ideas of Descartes. With his most outstanding students he widened the scope of Cartesian mathematics and explored the new possibilities which it offered.

2.8.1 Early years

Christiaan Huygens was by far the most talented student of Van Schooten. Above, we have seen that Van Schooten instructed the two sons of Constantijn Huygens from 1645 onwards. Christiaan stayed in Leiden until March 1647 and in this period a close and warm relationship developed, which continued in the years afterwards. When Christiaan moved to Breda in 1647, he started a correspondence with Van Schooten. In the following years, Van Schooten used this correspondence as a means to discuss the mathematical problems which he received from other mathematicians in the course of time. The correspondence continued until the death of Van Schooten, and although he and Huygens had different opinions from time to time, especially concerning the ideas of Descartes, they maintained an intense exchange of ideas. Van Schooten often encouraged Huygens to publish his work, and in some instances advised him not to continue with certain subjects.

Marcus Meibom (1621–1710) was another student in the circle around Van Schooten. Meibom, a self-willed and obstinate young man from the city of Tönning, Germany, matriculated in Leiden on 29 September 1645 at the age of 24 as a student of medicine.[220] His interest was not only in medicine but also in mathematics and the theory of music. He owned a copy of the introductory text *Recueil du calcul, qui sert à la geometrie du Sieur Des-Cartes*.[221] With Van Schooten he intensively studied Descartes's *Géométrie*. In the 1649 Latin edition, Van Schooten included a geometrical example proposed by Meibom on the reduction of equations.[222] Meibom stayed in the Dutch Republic until 1652, when he moved to the court of Queen Christina in Sweden.

[218] The lectures at the Duytsche Mathematicque will be analyzed in part III of this thesis.
[219] [Otterspeer, 2000, 231-233].
[220] [Rieu, 1875, 362]
[221] His copy is preserved in the Koninklijke Bibliotheek in The Hague with signature 73 J 17. It is not known how he obtained this copy.
[222] [Schooten, 1649c, 273]. The example was a geometrical problem in which it is required to construct in line

By 1651, Van Schooten considered Christiaan Huygens and Marcus Meibom as the two men most dedicated to the development of the geometrical methods of Descartes. In the beginning of that year, Van Schooten had received three complimentary copies of his introductory lectures on the Cartesian method *Principia matheseos universalis* from the printer Elzevier. He decided to keep one copy and present the two others to Huygens and Meibom.[223]

This introduction to Cartesian geometry *Principia matheseos universalis* was not written by Van Schooten himself, but by his Danish student Rasmus Bartholin (1625–1698). Bartholin had arrived in Leiden in May 1646 where he matriculated as a student of mathematics.[224] He stayed more than four years in Leiden and during this period he made an intensive study of the techniques employed by Descartes in the *Géométrie*, under Van Schooten's supervision. As part of his studies, Bartholin undertook a detailed investigation of the *Problema Astronomicum* and the solution that had been published by Descartes's straw man Waessenaer.[225] Van Schooten added some notes by Bartholin to the translation of Waessenaer's solution in the 1649 *Geometria* edition, in order to make Waessenaer's solution more accessible to the reader.[226]

A profound understanding of the *Géométrie* required a sound knowledge of symbolic algebra, which was a subject which mathematicians still had to learn in the first decades after the publication of the *Géométrie*. This is clear from the existence of introductory texts on symbolic algebra such as the *Recueil du calcul, qui sert à la geometrie du Sieur Des-Cartes*. Thus, Bartholin considered it useful to make the introductory lectures of Van Schooten available to a larger public in the form of the *Principia matheseos universalis*.[227] The work has to be considered as a joint venture of Bartholin and Van Schooten.

The *Principia matheseos universalis* is a well-structured text explaining the basics of symbolic algebra and the manipulation of elementary equations. The structure of the text resembles arithmetical texts, in the sense that the basic operations are discussed for various symbolic expressions (monomials, polynomials, fractions and roots of algebraic expressions). Following Descartes, letters represent straight line segments, because they are the most "simple and distinct" quantities. The same words simple and distinct were used by Descartes for explaining straight line segments as the most suitable mathematical objects for study.[228] Some of the examples in the *Principia matheseos universalis* were taken from algebraic manipulations in the *Géométrie*.[229]

segment. A first analysis of the problem leads to an equation of the fourth degree, suggesting that the problem cannot be solved by ruler and compass. The reduction techniques of Descartes however show that such a construction of the problem is possible. On the reducibilty of equations see [Bos, 2001, 383-397].

[223] Frans van Schooten to Christiaan Huygens, 26 February 1651, [Huygens, 1888, 138-139]. The Leiden Elzeviers were known to be parsimonious and they distributed only a minimal number of complimentary copies to their authors, [Netten, 2012, 183].

[224] UBL, Archief van Senaat en Faculteiten, 1575–1877, inv. nr. 10, 66.

[225] Waessenaer had published this solution in the pamphlet *Den on-wissen Wis-konstenaer I.I. Stampioenius ontdeckt* (The unreliable mathematician I.I. Stampioenius uncovered), [Waessenaer, 1640, 63-77].

[226] [Schooten, 1649a, 318-323]. The notes were included in unchanged form in the 1659 edition of *Geometria* [Schooten, 1659a, 385-389].

[227] [Bartholin, 1651, Preface to the reader].

[228] Compare [Descartes, 1637a, 20] and [Bartholin, 1651, 2].

[229] See for instance the division of polynomials, [Bartholin, 1651, 16], which is explained by means of examples

Another student of the early years of Van Schooten's professorship is Johan de Witt (1625–1672), the later grand pensionary of Holland, who was the most important official of the Republic from 1653 onward.[230] De Witt was born in Dordt in a wealthy family whose members had held several positions in the city government. He arrived in Leiden on 24 October 1641, when Van Schooten had just left the city for France.[231] De Witt stayed in Leiden until October 1645, when he left on a journey through France and England together with his brother.[232] After his return he took the oath as a lawyer before the Court of Holland on 11 November 1647, and settled in The Hague.

Although De Witt was a student in law, he shared an interest in mathematics with Van Schooten, and in a small mathematical community such as Leiden it is hardly conceivable that two men with such a common academic interest did not meet. His first meeting with Van Schooten probably took place in Leiden between 1643 and 1645, when De Witt was still a student and Van Schooten was not yet a professor. In any case, they must have met well before the summer of 1649, because Van Schooten included a geometrical problem proposed by De Witt in the Latin *Geometria* of 1649.[233] Around the same year, De Witt wrote a draft version of his treatise on conic sections which was eventually published in 1661 as *Elementa curvarum linearum* (The elements on curved lines) in the second edition of the *Geometria*.[234]

The contact seems to have diminished after De Witt took up his political positions. In a letter of 20 April 1654, Van Schooten congratulated De Witt with his appointment of grand pensionary, a position which De Witt had held by then for four months already. Van Schooten furthermore pointed out to De Witt that he had included his geometrical problem in the 1649 *Geometria* edition.[235] The contact was restored by the beginning of 1658, when Van Schooten received the manuscript of De Witt's treatise on conic sections. Van Schooten immediately got enthousiastic about the text and he pushed De Witt to publish the treatise as part of the second edition of the *Geometria*. Between February 1658 and 4 March 1660, Van Schooten and De Witt exchanged over twenty letters regarding the preparation of the text for the press. Van Schooten offered to De Witt to review the text and to take care of the illustrations. The result of the cooperation between Van Schooten and De Witt is the treatise *Elementa curvarum linearum* in the final form which appeared in 1661.[236]

---

referring to [Descartes, 1637c, 87-88].

[230] His political career started with his appointment as pensionary of the city of Dordt in December 1650, and in 1653 he became the grand-pensionary of Holland. He held this position until he was murdered in 1672. On the political career of De Witt see [Rowen, 1986] and [Panhuysen, 2005].

[231] Johan de Witt matriculated on the same day as his older brother Cornelis, [Rieu, 1875, 327].

[232] During this travel he obtained his doctorate in law at the university of Angers on 22 December 1645, [Molhuysen and Blok, 1914, 1460].

[233] The problem leads to an irreducible equation of the fourth degree, hence it is not constructible using straight lines and circles only, but requires the use of conic sections as well. The problem is a variant on the problem which Meibom proposed. Thus Meibom and De Witt studied similar problems and they may have attended private lectures of Van Schooten together. [Schooten, 1649c, 276-278].

[234] [Schooten, 1649b, Preface to the reader **3].

[235] Frans van Schooten to Johan de Witt, 20 April 1654, [Witt, 1919, 144-145].

[236] [Witt, 1661]. A modern translation of the treatise together with an extensive commentary and a discussion

2.8.2 A Hamburg connection

Daniel Lipstorp (1631–1684) was another student of Van Schooten. He arrived in Leiden in summer 1652 and matriculated at the university on 4 July 1652 as a student in philosophy.[237] Before coming to Leiden, he had spent some time in Hamburg studying with the mathematician Tassius who was professor of the Hamburg Gymnasium. Lipstorp came to Leiden to continue his studies, and Van Schooten taught him Cartesian geometry with special attention to symbolic algebra.[238] Lipstorp's interest in Cartesian philosophy and mathematics resulted in the publication of *Specimina philosophiae Cartesianae* (Specimina of Cartesian philospphy) in 1653 which mainly consists of a discussion of air. The book includes an introductory chapter in which Lipstorp argues that mathematical proofs are required in physics as well as in mathematics.[239] In 1653 Lipstorp became the court mathematician in Weimar, from where he maintained a correspondence with Van Schooten.[240]

Lipstorp was not the only student of the Hamburg Gymnasium who studied mathematics in Leiden. Meibom had also been a student in Hamburg,[241] and another student of the Hamburg Gymnasium who came to Leiden was Woldeck Weland (1614–1640). He stayed in Leiden in the second half of the 1630s, when Van Schooten was not a professor yet. Previously he had studied mathematics and philosophy with Joachim Jungius in Hamburg. In May 1635 Weland matriculated in Leiden, and after some time he continued his education by travelling to Oxford, Paris and Orleans. In the spring of 1639 he returned to Leiden to study medicine. During his stay in Leiden, he published *Strena mathematica sive elegantiorum problematum triga* (Mathematical new year's gift, or a triad of elegant problems), a mathematical treatise which appeared in 1640 in Leiden, and with which Van Schooten was familiar.[242] It is hardly conceivable that Van Schooten and Weland did not maintain any contacts, because the Leiden mathematical community was small. Golius may have made the connection but Descartes may also have fulfilled this role as both men were in contact with Descartes.[243] However, I have not found sources with explicit evidence of contacts between Weland and Van Schooten.

Van Schooten and Weland shared an interest in the restoration of lost works of Apollonius of Perga. A number of lost Greek mathematical works were mentioned in the *Collection* of Pappus of Alexandria, and it had become a popular activity for early modern mathematicians to reconstruct such lost works.[244] Both Weland and Van Schooten worked

of the cooperation between Van Schooten and De Witt was published by Grootendorst, see [Witt, 2000] and [Witt, 2010].

[237] [Rieu, 1875, 422].

[238] [Lipstorp, 1653, Preface to reader].

[239] For a discussion of Lipstorp's views see [Vermij, 2002, 142-146].

[240] Frans van Schooten to Christiaan Huygens, 3 March 1654, [Huygens, 1888, 271-272]. Van Schooten had introduced Lipstorp to Huygens by sending a copy of *Specimina philosophiae Cartesianae* to Huygens in spring 1653, [Huygens, 1888, 228].

[241] Dansk Biografisk Leksikon, `www.denstoredanske.dk/Dansk_Biografisk_Leksikon/Medier/Bibliotekar/Marcus_Meibom`, retrieved September 2013.

[242] UBG, Hs 108, f. 98r.-v.

[243] Weland corresponded with Descartes by the end of 1638, [Elsner, 1988, 34-35].

[244] See for an overview of the mentioned works in Pappus's *Collection* and the reconstructions [Elsner, 1988,

on a restoration of Apollonius's treatise on plane loci, a work which had also gained the attention of Fermat. Fermat had started working on his reconstruction in 1628, and the work was completed in spring 1636 and circulated in Parisian circles from 1637 onwards.[245] Weland began his work on the reconstruction by 1638, whereas the earliest evidence of Van Schooten's interest dates back to 1646.[246] It is likely that Weland was unaware of Fermat's reconstruction but that he was inspired by his former teacher of the Hamburg Gymnasium, Joachim Jungius, who had also started a reconstruction of *De Locis Planis* but had never finished the work.[247]

Weland died in 1640 and his reconstruction was finished by Johannes Müller (ca. 1634 – 1671),[248] another student of Van Schooten. Müller finished the treatise and prepared it for publication. When he died in 1671, the work had not yet appeared and it remained unpublished for centuries, until the publication by Elsner in 1988.[249]

Müller's interest in mathematics led him from his native Hamburg, where he studied with Joachim Jungius and Adolph Tassius, to Leiden, where he worked with Van Schooten and Golius. In Leiden, Müller prepared a disputation *Disputatio mathematica de luna* (Mathematical disputation of the moon) which he defended on 28 June 1655.[250] Müller dedicated his disputation to his teachers Jungius, Golius and Van Schooten. From Leiden, Müller went to Heidelberg to continue his studies, and he sent the fruits of his studies to Van Schooten.[251] In 1660, Müller was offered the chair of mathematics at the Gymnasium in Hamburg, a position he held until his death in 1671.

2.8.3 The students Hudde, Van Heuraet, and others

Throughout the 1650s, Van Schooten continued to gather students in his private circle, introducing them to Cartesian mathematics and encouraging them to investigate the new possibilities of the Cartesian methods. Van Schooten found a receptive ground in two extraordinarily gifted students: Johannes Hudde (1628–1704) and Hendrick van Heuraet

9-10].

245 [Mahoney, 1994, 94-95, 416-147].

246 [Elsner, 1988, 23-24] and [Schooten, 1646, Preface to the reader].

247 On the relation between Jungius and Weland see [Elsner, 1988, 24-31].

248 According to Elsner, Johann Müller was born in 1611, first made a career as a lawyer and pursued studies in mathematics from 1650 onwards, [Elsner, 1988, 36]. I believe that Elsner mistakenly identifies two men called Johann Müller who were not the same person. My argument is as follows. In 1658, Van Schooten describes Müller as a "juvenis" at the time he defended his thesis in Leiden in 1655, see Frans van Schooten to Christiaan Huygens, 15 June 1658, [Huygens, 1889, 184-185]. It would be very unlikely that Van Schooten would call a man, who is older than himself, and who already had a doctorate in law, a "young man". I suggest that the Johannes Müller who originated from Hamburg, studied mathematics in Leiden, and later became professor of mathematics in Hamburg, is the same as the Johann Müller who matriculated in Leiden on 7 July 1654 at the age of 20, [Rieu, 1875, 437]. This leads to a year of birth of ca. 1634.

249 Elsner published the Latin treatise *Apollonius Saxonicus* (Saxon Apollonius) together with a German translation and extensive commentaries, [Elsner, 1988].

250 A copy of his thesis is found in UBL, Archief van Senaat en Faculteiten, 1575–1877, inv. nr. 290, f. 38, [Müller, 1655]. Müller's request for his disputation was discussed in the Senate on 29 April 1655, [Molhuysen, 1918, 96]. On the practice of disputations see [Miert, 2005, 118-127] and [Otterspeer, 2000, 236-242].

251 Frans van Schooten to Christiaan Huygens, 15 June 1658, [Huygens, 1889, 184-185]. The treatise he sent to Van Schooten is [Müller, 1656].

(1634 – [1660]).[252] Both men were born in well-to-do merchant and patrician families in Holland and they had the financial means allowing them to spend their time on study and travel. It is likely that Hudde initially studied law in Leiden but soon turned his attention to other subjects, in particular mathematics. In 1656, when he was a student in Leiden, he published an anonymous treatise on dioptrics and two anonymous pamphlets. In these pamphlets he refuted the argumentation which the Leiden reformed minister Du Bois had given against Copernicanism and Cartesianism.[253]

Hudde and Van Heuraet both had an interest in studying curves by means of equations. Whereas Descartes focussed on the use of "geometrical" curves for solving geometrical problems, the attention of mathematicians gradually shifted towards the properties of these curves themselves. In 1657 Van Schooten presented to van Heuraet and Hudde several problems which were the subject of discussion in his own correspondence with Sluse (1622–1685) and Huygens. These problems concerned the quadrature of curves, tangents to a curve through a given point, and centers of gravity.[254]

Hudde and Van Heuraet left Leiden together by May or June 1658 for a grand tour through France and Switzerland, and they both returned to the Republic in the second half of 1659.[255] Hudde subsequently settled in Amsterdam were he became more and more involved in the city government. Because no traces of Van Heuraet have been found after the beginning of 1660, it is assumed that he died early that year.[256]

The best known contributions by Hudde and Van Heuraet are the treatises which Van Schooten published in the second edition of the *Geometria* in 1659. Hudde's contributions deal with the theory of equations and the determination of maxima and minima of polynomials. Van Heuraet showed that the rectification of a curve is equivalent to the quadrature of an associated curve. Earlier contributions by Hudde were published by Van Schooten in 1657 in the *Exercitationum mathematicarum libri quinque*. These include the determination of the maximal width of a curve nowadays known as the folium of Descartes,[257] and the generation of higher order curves by the intersection of a solid and a plane.[258] Van Schooten sent Hudde a complimentary copy of the *Exercitationum mathematicarum libri quinque* with a personal note, which has been preserved.[259]

To finish this section I will briefly mention two other students of Van Schooten from the same period. Theodoor Craanen (ca. 1633 – ca. 1689)[260] studied in Leiden in 1655 and 1656 where he attended Van Schooten's private lectures and lent a helping hand to Van

[252] On the mathematical work of Hudde see [Haas, 1956], [Atzema and Vermij, 1995], and [Vermij, 1995]. The contributions of Van Heuraet are discussed by Van Maanen in [Maanen, 1987, 43-106].

[253] The treatise on dioptrics is discussed in [Atzema and Vermij, 1995]. The polemic with Du Bois is discussed in detail by Vermij in [Vermij, 2002, 288-294].

[254] For a detailed discussion of the problems see [Maanen, 1987, 48-62] and [Yoder, 2004, 119-129].

[255] [Maanen, 1987, 69] and [Molhuysen and Blok, 1911, 1173].

[256] The last reference to Van Heuraet is found in a letter from Van Schooten to Christiaan Huygens of 17 January 1660, [Huygens, 1890, 10-11]. Besides an early death, Van Maanen also keeps the possibility open that Van Heuraet left Leiden and died elsewhere, [Maanen, 1987, 70].

[257] This curve has the equation $x^3 + y^3 = axy$ for given a, [Schooten, 1657, 493].

[258] See [Schooten, 1657, 475-480 and 497-499].

[259] New York, Columbia University Libraries, shelf number PLIMPTON 510 1657 Sch6, see `http://mathdl.maa.org/mathDL/46/?pa=content&sa=viewDocument&nodeId=2591&bodyId=3105`.

[260] [Schooten, 1657, 412].

Schooten.[261] He was receptive to Cartesian ideas, and later became professor of medicine and mathematics in Duisburg, Germany. Alexander Soete de Villers was born in a wealthy family.[262] He had a particular talent in mathematics and registered as a student in mathematics in Leiden at the age of 15 on 28 March 1654. He lived in the house of Pieter Smits, a mathematics instructor who rented rooms to students, mainly in mathematics.[263] Villers attended the public lectures of Golius and the private lectures of Van Schooten on Cartesian geometry. Just like Craanen, he served as a carrier between Van Schooten and Huygens for mail.[264] After his study in Leiden, Alexander joined the army. He fell at the battle of Seneffe on 11 August 1674.[265]

2.9 The publications of the 1650s

2.9.1 *Specimina philosophiae* (1656)

In 1653, Van Schooten initiated the project to publish a collective work on dioptrics. This work was to include a Latin translation of the *Dioptrique* of Descartes together with commentaries by the Louvain professor of mathematics Gerard van Gutshoven and a treatise on dioptrics by Christiaan Huygens.[266] Apparently Van Schooten was not completely satisfied with the earlier Latin translation of the *Dioptrique*, which had appeared in the *Specimina Philosophiae* in 1644, together with Latin versions of the *Discours de la méthode* and the *Météores*. He thought it useful to publish a new Latin edition of the *Dioptrique* structured along the same lines as his own Latin *Geometria* of 1649.

Van Schooten approached the Leiden printer and bookseller Johannes Maire, who owned the woodcuts of the original publication of Descartes's *Dioptrique* of 1637. These woodcuts had been made by Van Schooten himself.[267] Maire was sympathetic to the project and he asked Van Schooten to put him in contact with Van Gutshoven and Huygens in order to arrange an agreement for publication.[268] Huygens had played with the idea of publishing the work elsewhere, and his friend Lipstorp informed him that Elzevier was also interested. However, he preferred to publish his work on dioptrics together with the treatises of Descartes and Van Gutshoven, and he promised Van Schooten to finish the treatise as soon as possible.[269] Despite this positive reaction by Huygens, the collective work on dioptrics did not materialize, for reasons which remain unknown. It seems

[261] Craanen served as a mailman, bringing a parcel with books Van Schooten had acquired for Huygens to Huygens in The Hague in 1656. Frans van Schooten to Christiaan Huygens, 19 October 1656, [Huygens, 1888, 509].

[262] His father was Alexander de Soete de Laecke, heer van Villers, captain in the army. His two brothers made a journey in France accompanied by their tutor [Scherf and Dekker, 1994, 35].

[263] Pieter Smits lived in the Breestraat. UBL, Archief van Senaat en Faculteiten, 1575–1877, inv. nr 10, f. 371.

[264] Frans van Schooten to Christiaan Huygens, 4 February 1658, [Huygens, 1889, 130].

[265] [Bos, 1675, 433].

[266] The original version of this treatise by Huygens is lost today. The version in the *Oeuvres Complètes* of Huygens is based on a copy made in Paris by Niquet, probably in 1666 or 1667, [Huygens, 1916, 1-271]. For an extensive study of Huygens's treatise see [Dijksterhuis, 2004, 11-52].

[267] Van Schooten's 1649 *Geometria* appeared with Maire as well.

[268] Frans van Schooten to Christiaan Huygens, 5 June 1653, [Huygens, 1888, 233-234].

[269] Christiaan Huygens to Frans van Schooten, 8 June 1653, [Huygens, 1888, 234].

that the project had been abandoned by April 1654, when Huygens considered offering his manuscript on dioptrics to Elzevier for publication.[270]

Later in 1654, Van Schooten saw a second chance for the collective work on dioptrics, this time in connection with the publication of revised versions of Descartes's philosophical works. The Amsterdam printer Louis Elzevier took the initiative for the project and asked the Leiden professor of philosophy Johannes de Raey, a convinced Cartesian, to oversee the new publication of the *Discours de la méthode* and of the *Principia philosophiae*.[271] Elzevier approached Frans van Schooten for the field of dioptrics.

Van Schooten immediately turned to Huygens with the request to send his comments and corrections on the works of Descartes. He also asked Huygens to consider a publication of his own work on dioptrics in the projected new edition of the Cartesian works with Elzevier. To persuade Huygens that Elzevier was a good choice by now, Van Schooten mentioned the fact that Elzevier recently bought the woodcuts of the original 1637 editions of the *Discours de la méthode* and the accompanying essays. Thus the quality of the illustrations in the new edition would be better than in Elzevier's previous editions of the works of Descartes.[272] In a letter of 29 October 1654, Huygens provided Van Schooten with the requested comments on Descartes's work, but he declined the invitation to include his own treatise on dioptrics in the new edition of Descartes's works. He believed that there would be a limited audience for the new edition, because the readers interested in the works of Descartes already owned a copy.[273]

Whether Van Schooten also approached Van Gutshoven for the Elzevier project remains unknown. In any case, the new Latin edition of Descartes's *Dioptrique* appeared without the treatises by Huygens and Van Gutshoven. Van Schooten limited himself to correcting the 1644 Latin translation, and he added a short appendix entitled *Animadversiones in Dioptricam* (Observations in dioptrics), in which he included one demonstration by Van Gutshoven and one by Huygens.[274] Thus in the end, Van Schooten was unable to realize the comprehensive work on dioptrics, and had to satisfy himself with an appendix of just five pages.[275]

2.9.2 A second edition of the *Geometria*

The willingness of Elzevier to publish new editions of Descartes's work together with Van Schooten's enthusiasm in the dissemination and propagation of the mathematical ideas of Descartes came together in the plan for a second edition of the *Geometria*.[276] It is not

[270] Christiaan Huygens to Frans van Schooten, 1 April 1654, [Huygens, 1888, 280]. The translation and commentaries by Van Gutshoven are considered lost today, [Descartes, 2007, 72].

[271] [Descartes, 2007, 72]. During Descartes's life, Louis Elzevier had published most of the philosophical treatises, see [Kingma, 2000] and [Otegem, 2002].

[272] Frans van Schooten to Christiaan Huygens, 25 October 1654, [Huygens, 1888, 301].

[273] Christiaan Huygens to Frans van Schooten, 29 October 1654, [Huygens, 1888, 303].

[274] According to Vermeulen, Van Schooten overlooked most of the errors that had been made in the 1644 Latin translation, and his corrections to the translation only concern typographical errors and some minor mistakes in grammar, syntaxis and style [Descartes, 2007, 72].

[275] [Descartes, 1656, 244-248].

[276] Already during his lifetime, Descartes published a considerable number of his works with Louis Elzevier, see [Otegem, 2002].

known whether Van Schooten or Elzevier took the initiative for the second edition, but perhaps the new project was a natural consequence of the work on Descartes's dioptrics in which Van Schooten had been engaged for Elzevier. In any case, by June 1656 it was widely known that Van Schooten was working on a second edition of the *Geometria*, enriched with additional treatises. The news had also spread to Paris because Van Schooten had written his friend Mylon about his project.[277]

The second edition consisted of two volumes: the first appeared in the summer of 1659 and the second in 1661. The publication of the second volume was overseen by Petrus van Schooten after the death of Frans van Schooten in May 1660. Before his death, Frans had almost completely prepared the treatises for printing.

The first volume resembled the first edition of the *Geometria*. It included the Latin translation of the *Géométrie*, followed by the *Notae breves* (Short notes) of De Beaune and Van Schooten's comments, together with the two additional treatises by Van Schooten on cubic equations and root extraction of binomials. Compared to the first edition, Van Schooten revised his translation in some instances and he enlarged and modified his comments. It seems that he already revised some of his comments to the first edition by 1655, or even earlier.[278] The new additions to the first volume included some new discoveries by Van Schooten and Huygens, two treatises by Hudde, and one treatise by Van Heuraet.

Hudde communicated his findings to Van Schooten in two letters, dated July 1657 and February 1658. The first letter is a contribution to the theory of equations and the second is on the determination of maxima and minima of polynomials. In April 1658, Van Schooten asked Hudde for permission to include the two letters in his second edition of the *Geometria*, and Hudde provided Van Schooten with a dedicatory letter, just before he left the Republic in May/June 1658 to travel through France together with his fellow student Van Heuraet.[279]

The final contribution to the first volume is a letter sent by Van Heuraet to Van Schooten on the rectification of curves.[280] Van Heuraet sent the letter on 13 January 1659, and it reached Van Schooten just in time to be included in the first volume.[281] Both Hudde and Van Heuraet were unable to oversee the printing of their contributions, as they were travelling in France when the work was printed.[282]

The second volume of the *Geometria* included a reprint of the 1651 *Principia matheseos universalis* by Van Schooten en Bartholin, two treatises by Florimond De Beaune, prepared for the press by Rasmus Bartholin, the treatise by Jan de Witt on conic sections,

[277] Claude Mylon to Christiaan Huygens, 23 June 1656, [Huygens, 1888, 439].

[278] This can be deduced from the references to a geometrical problem which Van Schooten discussed with John Wallis. Van Schooten refers to this problem and his correspondence with Wallis about it in [Schooten, 1659c, 234-236] and indicates that this correspondance had taken place three years before. Wallis refers to the same correpondence in his *Treatise on algebra*, explicitly dating this correspondence to the first months of 1652, [Wallis, 1685, 260].

[279] The contributions of Hudde are analyzed by Haas, [Haas, 1956].

[280] A detailed account of Van Heuraet's method is found in [Maanen, 1987, 88-95].

[281] Van Schooten immediately passed the letter on to Elzevier, Frans van Schooten to Christiaan Huygens, 13 February 1659, [Huygens, 1889, 353].

[282] Frans van Schooten to Christiaan Huygens, 13 February 1659, [Huygens, 1889, 353].

and a treatise by Van Schooten on the use of algebra in geometrical problems. According to the title pages, the treatises by De Beaune and De Witt were printed in 1659 and the other treatises in 1661. It is likely that the sudden death of Van Schooten delayed the publication of the second volume for quite some time.

De Beaune had died in 1652, and just before his death he had consigned part of his manuscripts to Rasmus Bartholin, Van Schooten's former student. On his way to Copenhagen in 1656, Bartholin paid a visit to Van Schooten in Leiden and showed him the manuscripts of De Beaune: two treatises on equations, a treatise on mechanics and a treatise on solid angles. These treatises had all been written in French by De Beaune and Bartholin had committed himself to translating them into Latin for publication.[283] The two treatises on equations were published in the second edition of the *Geometria*: *De natura et constitutione aequationum* (On the nature and constitution of equations) and *De limitibus aequationum* (On the boundaries of equations).[284] Bartholin must have prepared the treatises for publication soon after he had seen Van Schooten, as his dedication letter is dated 1657. The two other treatises by De Beaune were published neither by Van Schooten nor by Bartholin.[285]

2.9.3 Publishing in Latin and Dutch: the case of the *Exercitationum mathematicarum libri quinque*

In 1657 Van Schooten published with Johannes Elzevier the work *Exercitationum mathematicarum libri quinque*, consisting of five Books on various mathematical subjects and an appendix by Huygens entitled *De ratiocinis in ludo aleae* (On reasoning in games of chance), on probability theory. A Dutch version of the same work appeared in 1659–1660 as *Mathematische oeffeningen, begrepen in vijf boecken* with the Amsterdam printer Gerrit van Goedesbergh. The Dutch version contained two appendices: Huygens's treatise on probablity theory in Dutch and a treatise by Van Schooten on perspective.

The *Exercitationum mathematicarum libri quinque* contains two works which Van Schooten had announced earlier. In 1646 he announced that he planned a reconstruction of Apollonius's *De locis planis*. The reconstruction was included as Book III of the *Exercitationum mathematicarum libri quinque*. It involves problems that can be solved by means of ruler and compass; in his solution of some of these, Van Schooten also uses algebraic equations in the way of Descartes. Book II contains a treatise on geometry in which Van Schooten's experiences in pure geometry in Euclidean style and practical geometry of surveyors are combined in a theoretical framework for (practical) geometry. Earlier, Van Schooten had communicated his views on geometry in this book to his friends, and he used elements of the treatise in his public lectures at the Duytsche Mathematicque.[286] The existence of the treatise was announced by Lipstorp in 1653, but without technical

[283] Frans van Schooten to Christiaan Huygens, 20 November 1656, [Huygens, 1888, 512].

[284] See [Beaune, 1661b] and [Beaune, 1661a]. These treatises have been translated into English by Robert Schmidt, [Viète et al., 1986].

[285] The treatise on solid angles was lost until it was rediscovered and published by Pierre Costabel in [Beaune, 1975]. The treatise on mechanics is still considered lost today.

[286] [Schooten, 1660a, 155].

mathematical details.[287]

The three other books in *Exercitationum mathematicarum libri quinque* were different in character. Book I dealt with 50 arithmetical and 50 geometrical problems, of the same kind as the problems that were used in the courses at the Duytsche Mathematicque.[288] Book IV was a reprint of the 1646 treatise *De organica conicarum ... descriptione* on the tracing of conic sections by means of instruments. The final Book V was about miscellaneous mathematical subjects, including problems on combinatorics and on the application of algebra to geometry.

The publication process of the *Exercitationum mathematicarum libri quinque* was a peculiar one, because the works were originally intended to be published in Dutch, but appeared in Latin first. At least some of the treatises which Van Schooten intended to publish in the work were already finished by 1652. By that time, Van Schooten sent his reconstruction of Apollonius's treatise *De locis planis* to Christiaan Huygens for review.[289] Huygens replied to Van Schooten that he enjoyed reading the text and the subtle and elegant proofs. He encouraged Van Schooten to have the work printed, and to publish it in the vernacular. Huygens argued that students who only mastered the vernacular did not have any other good literature on ancient Greek geometry at their disposal beyond the *Elements* of Euclid.[290] The response by Huygens suggests that the treatise which Van Schooten had sent to him was written in the vernacular. Further evidence is in a letter of October 1653 to the astronomer and mathematician Dirck Rembrantsz van Nierop (1601–1682), in which Van Schooten states that he had written three treatises in the vernacular in which he used the mathematical methods of Descartes in some instances.[291] The reconstruction of Apollonius's *De locis planis* must have been one of these treatises.

Although the reconstruction was finished in 1652, the publication took a long time. In April 1653, Huygens reminded Van Schooten that he should not deprive readers with an interest in mathematics of his finished mathematical treatises.[292] Apparently Van Schooten had not yet found a printer at that time. However, he now took up the work of finding a printer, and he prepared the treatise for printing. He finished the dedication letter and sent it to Huygens for revision. This letter was dedicated to Pierre Chanut, the French ambassador in the Dutch Republic during the years 1653–1655. Before his years in the Republic, Chanut had resided at the Swedish court and he had organized the transfer of Descartes to Sweden.

Van Schooten had difficulties in arranging the publication of his treatises in the vernacular. Johannes Elzevier was only willing to publish the treatises in Latin, and refused to print an edition in Dutch, because he did not see any possible commercial benefits in such a project.[293] As a result, Van Schooten had to translate the works into Latin and he

[287] [Lipstorp, 1653, 10].
[288] For a detailed overview of the arithmetic courses of the Duytsche Mathematicque I refer to chapter 8.
[289] Frans van Schooten to Christiaan Huygens, 28 July 1652, [Huygens, 1888, 183-184].
[290] Christiaan Huygens to Frans van Schooten, 13 August 1652, [Huygens, 1888, 184-185].
[291] Frans van Schooten to Dirck Rembrandtsz van Nierop, 9 October 1653, [Nierop, 2012, 81-82].
[292] Christiaan Huygens to Frans van Schooten, 7 April 1653, [Huygens, 1888, 228].
[293] Frans van Schooten aan Christiaan Huygens, 1 October 1657, [Huygens, 1889, 62].

published the five treatises in 1657 in Latin with Johannes Elzevier under the title *Exercitationum mathematicarum libri quinque*.[294] It took Van Schooten another couple of years to find a printer who was willing to print the Dutch version. Eventually, the Amsterdam printer Gerrit van Goedesbergh published the work in 1659–1660.[295] Van Goedesbergh had some experience in the publication of mathematical treatises in the vernacular, as he had printed several works of the Dutch astronomer and mathematician Dirck Rembrantsz van Nierop (1610–1682) in the years before.[296] Van Schooten and Van Nierop had exchanged letters in 1653, and it may well be that the experiences of Van Nierop suggested to Van Schooten to turn to Goedesbergh.

2.9.4 Contribution to *Commercium epistolicum* in 1658

In 1658, the English mathematician John Wallis published a volume entitled *Commercium epistolicum de quaestionibus quibusdam mathematicis nuper habitum* (Correspondence by letters on certain recent mathematical problems), and containing a correspondence between French, Dutch and English mathematicians.[297] The volume consists of a collection of letters exchanged by mathematicians after Fermat had challenged them with two problems, which will be discussed below. The volume contains one letter by Frans van Schooten.

In 1658, Wallis and Van Schooten had already been in contact for years. We know that Wallis wrote a letter to Van Schooten in 1649 on the subject of extracting roots of cubic binomials, a topic that had been discussed in the first edition of the *Geometria*.[298] They maintained a regular correspondance from at least the year 1652 onwards and and sent complimentary copies of their publications to one another.[299] In the second edition of the *Geometria*, Van Schooten referred to his previous correspondence on a geometrical problem with Wallis.[300]

The *Commercium epistolicum* is about the following two challenge problems on number theory, which were published by Fermat on 3 January 1657:

1. To find a cube number which, added to all its aliquot parts, makes a square.
2. To find a square number which, added to all its aliquot parts, makes a cube.

[294] [Schooten, 1657].
[295] [Schooten, 1660a].
[296] On the publication of Dirck Rembrantsz van Nierop see [Nierop, 2012, 343-386].
[297] [Wallis, 1658]. A French translation of the *Commercium epistolicum* is found in [Fermat, 1896, 399-602].
[298] [Wallis et al., 2003, 12-13]. A cubic binomial is an algebraic expression of the form $\sqrt[3]{a} \pm \sqrt[3]{b}$ or a variant of this form.
[299] An exchange of letters of the year 1652 is mentioned by Wallis in [Wallis, 1685, 260]. Most of the letters which Van Schooten and Wallis exchanged are lost today, but the existence of the correspondence can be inferred from references in printed works or other letters; see for a good overview of the exchanges [Wallis et al., 2003]. Wallis sent a copy of his *Arithmetica infinitorum (The arithmetic of infinitesimals)* to Van Schooten in 1656, and he sent him the first volume of his *Operum mathematicorum* in 1657. In return, Van Schooten wanted to provide Wallis with a copy of his *Exercitationum mathematicarum libri quinque*, but the book did not reach Wallis. In 1659 Van Schooten sent a copy of the second edition of *Geometria* to Wallis. [Wallis et al., 2003, 176, 590] and [Fermat, 1896, 554].
[300] [Schooten, 1659c, 234-236]. Wallis refers to the same problem in a different wording in his *Treatise on algebra*, [Wallis, 1685, 260-262].

Fermat addressed these problems to "mathematicians from Gallia, England, Netherlands and the rest of Europe".[301] In all likelyhood, Fermat had the Dutch mathematicians Jacob Golius and Frans van Schooten in mind, and also the French mathematician Bernard Frénicle de Bessy (ca. 1605–1675), the only Frenchman (except Fermat himself) who had shown an interest in problems on number theory. The English mathematicians whom Fermat addressed must have included John Wallis, whose work had been the occasion for Fermat to put such number theoretical problems on the agenda again.

I now focus on Van Schooten's contributions to the problem, which were part of a larger discussion between mathematicians which does not concern us here.[302]

In a letter of 18 March 1658, Van Schooten wrote to Wallis about his work on the two problems which Fermat had proposed. The letter containing the problems, which had been sent by the Dutch envoy in Paris, Willem Boreel, arrived on 7 February 1657 at the address of Golius, but because Golius was busy those days the letter was not opened until 11 February 1657 in the presence of Van Schooten.[303]

On 17 February 1657, Van Schooten replied by sending a letter on the problems to Boreel in Paris. Van Schooten did not give a solution, but only indicated that there were no prime numbers under 98 other than 7 satisfying the requirements of the first problem. In the rest of his letter, he sketched in broad outlines how one would have to proceed to find a solution.[304] Because he found the actual compuations according to his method too laborious, he did not investigate the matter any further. Van Schooten then proposed two problems to Fermat of the same kind (i.e., on number theory). The first problem was: to solve the equation $x^3 + y^3 = z^3$ in integers, or to show that are no solutions. The second problem was: to determine whether or not it is possible to find other perfect numbers than those resulting from Euclid's method in Book IX of the *Elements*. The problems are interesting because Van Schooten also demanded a proof for non-existence, if no solutions exist. It remains unknown whether Van Schooten's response ever reached Fermat.[305]

In the meantime, Mylon had informed Van Schooten in a letter of 9 March 1657 that Frénicle had solved the proposed problems of Fermat, and that plans were made in Paris for the publication of this solution. By May 1657, these plans had been further developed and Frénicle wanted to include Van Schooten's method as well. Van Schooten supported the publication, including the simplifications in his own method which Frénicle had found. Van Schooten furthermore urged Frénicle to add what the main motivation of his own work had been: to promote the use of algebraic methods for solving problems.[306]

Frénicle published his solution as *Solutio duorum problematum* (The solution of two

[301] Problemata duo mathematica, tamquam indissolubilia Gallis, Anglis, Hollandis, nec non ceteris Europae Mathematicis proposita . . . , see [Fermat, 1894, 333].

[302] Fermat's side of the story, with major emphasis on his motivation and the underlying mathematical agenda, is extensively discussed by Mahoney, [Mahoney, 1994, 332-347]. The English side of the story, with major actors Wallis and Brouncker, is studied by Stedall in [Stedall, 2000]. For a detailed view on the mathematical aspects of the problem see also [Hofmann, 1944].

[303] Golius was about to resign as the rector magnificus of the university during the yearly meeting of professors to be held on 8 February 1657.

[304] He elaborated on the number of aliquot parts of non-prime integers by means of a combinatorial reasoning he had published before in his *Exercitationum mathematicarum libri quinque*, [Schooten, 1657, 383-387].

[305] [Fermat, 1896, 556-559].

[306] [Fermat, 1896, 563-569].

problems on cubic and quadratic numbers)[307], and the Dutch envoy in Paris sent a copy to Van Schooten on 26 October 1657. Van Schooten was not at all content with this publication as Frénicle had undertaken what Van Schooten calls an "inquisition" of his method. He ironically mentioned to Wallis that he never expected such a thing from France.[308]

2.10 Draughtsmanship

As has been noticed above, Van Schooten had a particular talent for drawing. He used the combination of his skills as a mathematician and a draughtsman in the preparation of Descartes's work for the press, and also in the lens making project of Constantijn Huygens and Descartes in the years 1636–1637. In the later years, Van Schooten continued to use his talents as a draughtsman in geometrical figures, woodcuts, engravings and etchings.

We do not know to what extent Van Schooten had any formal training or education in drawing or painting. It is natural to assume that he was introduced to this area by his uncle Joris van Schooten (1587–1651/2), a well-known Leiden painter and one of the founders of the Leiden Saint Luke guild of painters in 1642.[309] This assumption is plausible because there exists a portrait, which was made by Frans van Schooten of his uncle Joris van Schooten (figure 2.6). This portrait has hardly been noticed in the literature. The author of the portrait can be identified by the engraving at the left bottom side: "F. a. Schooten J. fecit". Furthermore, the name is written in a style which resembles the handwriting of Frans jr.[310]

By the intermediary of his uncle, Van Schooten had access to the community of draughtsmen and painters in Leiden. Some members of this community also showed an interest in mathematics. The Leiden painter Abraham van den Tempel, who was one of the founders of the Guild, matriculated on 7 February 1652 as a student in mathematics at Leiden university.[311] It is worth noticing that perspective was considered a part of mixed mathematics, and was therefore taught by Frans van Schooten at the Duytsche Mathematicque.

Descartes seems to have been content with the illustrations which Van Schooten produced for his essays in 1637. In September 1643, he invited Van Schooten to prepare the illustrations of his treatise *Principia philosophiae*.[312]

For the treatise of Johan de Witt on conic sections, included in the second edition of the *Geometria*, Van Schooten not only edited the manuscript, but also prepared the figures. In

[307] [Frénicle de Bessy, 1657].
[308] [Fermat, 1896, 569].
[309] `http://collectie.lakenhal.nl/kunstenaars?maker[naam]=Schooten,+Joris+van`. Retrieved 24 December 2014
[310] The portrait was described by Christiaan Kramm, who wrongly identified the author as Frans van Schooten sr. Kramm identified the portrayed man as Joris van Schooten on the basis of his name which is written at the bottom of the engraving in a seveneenth century hand, [Kramm, 1861, 1490]. The collection of the Rijksmuseum in Amsterdam contains two prints of the engraving, object number RP-P-OB-59.061, and RP-P-OB-59.062. The latter contains the identification of the man as Joris van Schooten. A high resolution digital reproduction of the engravings is on `https://www.rijksmuseum.nl/en/collection/RP-P-OB-59.061` and `https://www.rijksmuseum.nl/en/collection/RP-P-OB-59.062`. It has to be noted that the face of the portrayed man resembles the features of Frans van Schooten sr. as they are known by other portraits of him.
[311] [Rieu, 1875, 425].
[312] Descartes to Constantijn Huygens, 20 September 1643, [Descartes and Huygens, 1926, 213].

Figure 2.6 – Portrait of Joris van Schooten by Frans van Schooten jr. Rijksmuseum, object number RP-P-OB-59-062.

a letter to De Witt of February 1658 Van Schooten offered his help, and the figures were produced in the course of 1658.[313] It is unclear whether Van Schooten only designed the drawings, or whether he also made the woodcuts.

A reference to the draughtsmanship of Van Schooten was furthermore made during the controversy between Jacob Golius and John Pell on the publication of Books V-VII of Apollonius's *Conics*. As mentioned above, Golius possessed an eleventh-century Arabic manuscript containing Books V-VII of Apollonius's *Conics*, and Golius intended to publish these books in a Latin translation.[314] The publication however was delayed, and by 1642 Golius did not have the monopoly on the Arabic text of the 'lost' books V-VII anymore. The German scholar Christian Ravius (1613–1677) had acquired a manuscript copy of an Arabic epitome by 'Abd al-Malik al-Shirazi (ca. 1150) of Books V-VII of the *Conics* and he had taken this manuscript to Utrecht where he lectured as professor of oriental languages. In 1644, the mathematician John Pell, who resided in Amsterdam at the time, borrowed this manuscript from Ravius and began to translate the treatise into Latin.[315] Pell had already contacted the Amsterdam publisher Joan Blaeu, who was willing to publish Pell's translation of Books V-VII together with a reprint of Commandino's Latin translation of Books I-IV.

When Golius got wind of these plans in the spring of 1645, he went to Amsterdam to see Blaeu and Pell. Pell described his meeting with Golius in a letter to Charles Cavendish. According to Pell, Golius brought with him the Arabic manuscript with the seven books of Apollonius. Golius told Pell that:

> He [i.e., Golius, JGD] hath beene at some cost to pay for the delineating of all the figures and indeed they are exceeding neately drawen in paper by his Collegues sonne & some of them are also graven.[316]

Thus Golius showed not only the manuscript but also the drawings of the figures to Pell, and told that these had been made by the son of his colleague (Frans van Schooten sr.), that is, by Frans van Schooten jr. The whole collection of figures was engraved on 93 plates, and consisted of a little over 200 figures.[317] Although Golius printed a specimen page containing his translation of the beginning of Book V, he never published his translation.[318]

The plates of the engravings were kept by Golius during his life. A year after Golius's death in September 1667, these plates were bought for the university library of Oxford by the English philologist Thomas Marshall (1621–1685). The plates were transported to England on a ship which unfortunately sank in sight of the harbour, so they were lost. Yet, Marshall knew that some prints made of these plates were still in Golius's notes. In 1669 these notes, including the prints, were obtained by the English astronomer and scholar

313 [Witt, 2000, 5-7].
314 See page 23.
315 [Malcolm and Stedall, 2005, 292-293]. Ravius's own competence in Arabic was contested by his contemporaries. Golius also had a copy of this manuscript of Ravius made by intermediary of his copist Nicolaus Petri who had previously worked for Ravius, [Schmidt, 2005, 34-35].
316 Pell to Cavendish, 19 May 1645, [Malcolm and Stedall, 2005, 415].
317 [Toomer, 1996, 239].
318 [Apollonius, 1990, lxxxvi-lxxxvii].

Edward Bernard (1638–1697), who had come to Leiden to copy the manuscript of the lost books of Apollonius's *Conics* in the university library.[319]

Unfortunately I have not been able to find any evidence on the origin of the figures in Van Schooten's own published work. Because Van Schooten made the drawings for two books by Descartes, the planned translation of the *Conics* by Golius as well as the treatise on conics by De Witt, it is a natural assumption that he also produced the drawings for the printed versions of his own treatises.

Van Schooten made a major contribution to the iconographic tradition of Descartes by a portrait which he made in 1644. Today, scholars identify two main iconographic traditions of Descartes: one descending from the engraving by Van Schooten (figure 2.7) and the other originating from the portrait made by Frans Hals, copies of which are in the Ny Carlsdad collection in Copenhagen and the Louvre museum in Paris (figure 2.9).[320]

The making and distribution of the portrait made by Van Schooten has been investigated by Nordstöm, who believes that the Hals portrait was based on the Van Schooten engraving combined with the artist's own imagination. Nordstöm thinks that Descartes never sat for Hals, and that the portrait was made on the instigation of Hals's friend Suyderhoef, who wanted to make a profit from the death of Descartes. The following account is to a large extent based on Nordström's work.[321]

Van Schooten's portrait of Descartes was printed as the frontispiece in the 1659 *Geometria*. The engraving carries the inscription "Franciscus à Schooten PrMat ad vivum delineavit et fecit Anno 1644" (Franciscus à Schooten Pr[ofessor of] Mat[hematics] drew lifelike and made [this] in the year 1644.) The reference to the professorship in combination with the year 1644 is remarkable, as Van Schooten was appointed professor in 1646. The engraving which was printed in 1650 only bears the letters "P.M.", which must have been changed to"PrMat" after 1650.[322]

The engraving bears the year 1644 as the year of production. The Latin verse by Constantijn Huygens jr. was added later, in 1649 at the earliest. Van Schooten was invited by Descartes in the autumn of 1643 to draw the figures for the *Principia philosophiae*, which was initially intended to be published before Easter 1644. The printing was delayed and not even finished when Descartes left the Republic for a visit to France later in 1644. It is likely that Van Schooten and Descartes had additional meetings in 1644 and that van Schooten made the portrait during one of these sessions. We do not know why the portrait was made. By 1648, however, Van Schooten planned to include the portrait in

---

319 [Toomer, 1996, 239, 242]. These prints are today kept in Oxford, Bodleian library, MS Lat. Class. e. 2. Dijksterhuis has recently investigated the claim that Van Schooten made figures of Books V-VII of the *Conics* for Golius. Apparently unaware of the history of the plates, he was tempted to identify the figures in Golius's seventeenth-century copy of the Arabic manuscript (the copy is nowadays UBL, Or. 14.) as the figures drawn by Frans van Schooten. Because the figures in this seventeenth-century Arabic manuscript are rather sloppy, Dijksterhuis doubts whether Van Schooten was involved at all [Dijksterhuis, 2011, 105-106]. The seventeenth-century Arabic manuscript copy had been made for Golius by a scribe, and as pointed out by Witkam, the figures were drawn by Golius himself. [Witkam, 1980, 53] and [Witkam, 2007, 19-20].

320 [Hynes, 2010, 575]. The description of the Hals portrait in the Louvre reads: Portrait du philosophe René Descartes (1596–1650), copie ancienne d'après un original perdu peint en 1649 et gravé en 1650, inv. 1317.

321 [Nordström, 1958].

322 Compare figures 2.7 and 2.8.

Figure 2.7 – Portrait of Descartes by Frans van Schooten, distributed by Van Schooten in 1650. Rijksmuseum object number RP-P-1910-4460.

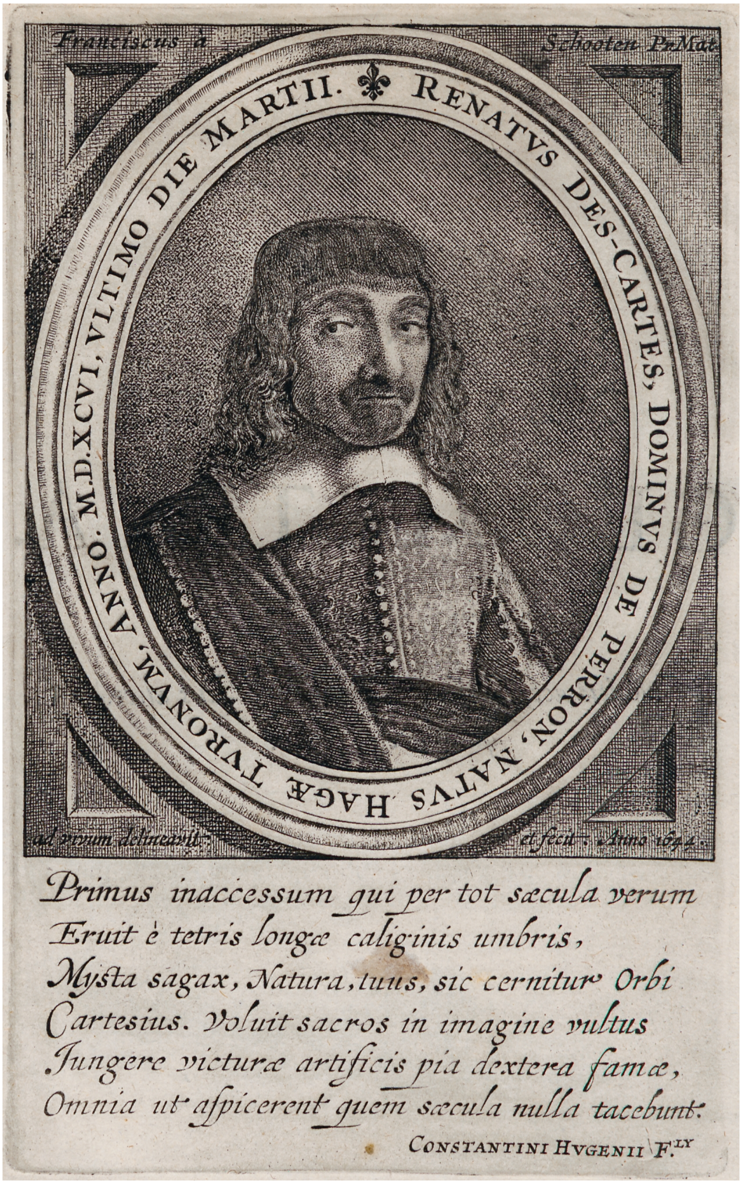


Figure 2.8 – Portrait of Descartes by Frans van Schooten. Frontispiece of *Geometria* 1659.

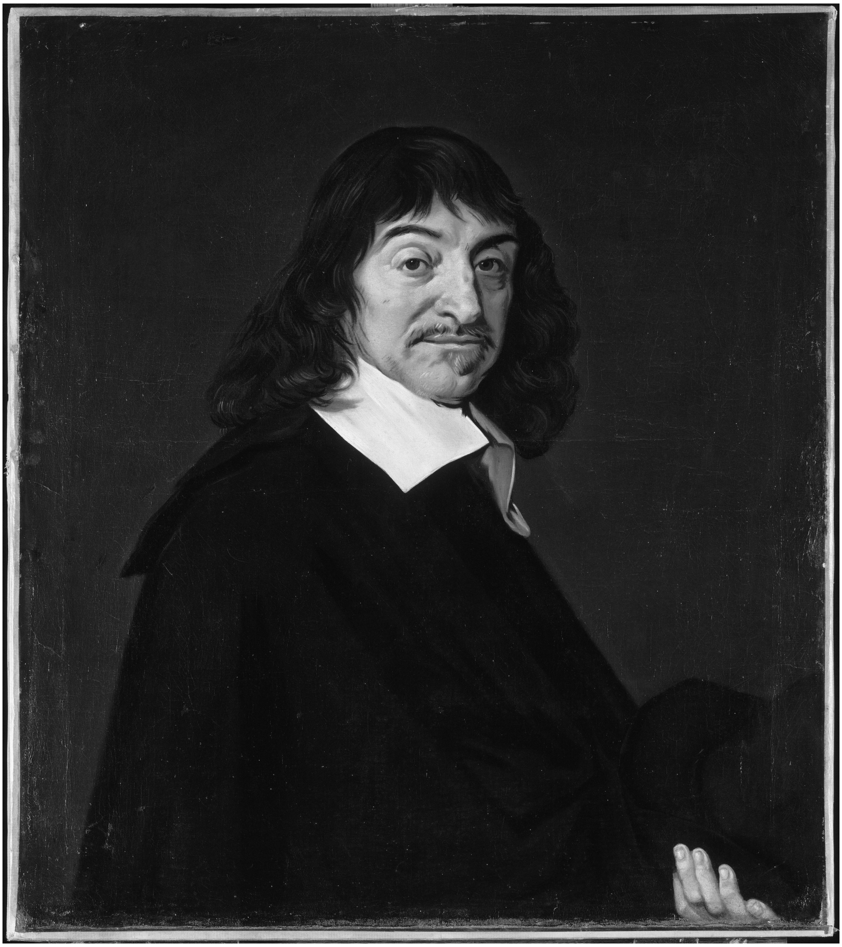


Figure 2.9 – Portrait of Descartes, after a work by Frans Hals. Louvre, inv. 1317.

the first edition of the Latin *Geometria*, and he requested his former student Constantijn Huygens jr. to write a verse for the portrait.[323] In the mean time, Van Schooten had also received a verse on Descartes by his student Rasmus Bartholin, who was also a Cartesian. On 10 March 1649, Van Schooten sent the verses of Huygens jr. and Bartholin to Descartes, together with a copy of the engraving. Descartes replied by 9 April 1649, writing that the engraved portrait is "very well done", though he had some reservations regarding the beard and the clothing. He also expressed his enthousiasm regarding the verses which Van Schooten had sent him.[324] It seems that Van Schooten had changed his mind by then,

[323] The verse was composed by May/June 1648, according to a letter of Frans van Schooten to Constantijn Huygens jr. of 3 November 1648. The letter is published in [Descartes, 1908, 628-629] and in [Huygens, 1915, 504]. In the latter publication, the addressee is errounously identified as Constantijn Huygens sr.

[324] [Nordström, 1958, 197] and Descartes to Frans van Schooten, 9 April 1649, [Descartes, 1903, 338] and

because Descartes adds that he agrees with Van Schooten's decision to omit the portrait on the grounds that Huygens jr. was not competely satisfied with the verses. In case Van Schooten might change his mind again and include the portrait after all, Descartes urged him to remove the reference to him as Lord of Perron and his birthday, because he disliked titles and horoscope makers.[325] The first edition of the *Geometria* indeed appeared without the portrait of Descartes.

The death of Descartes on 11 February 1650 was the occasion for Van Schooten to print the portrait and distribute it amongst his contacts. The plate however had not been finished, as the project of including the portrait in the *Geometria* in 1649 had come to a standstill, and the place for the verses was still empty. Van Schooten decided to include the verse by Constantijn Huygens, although he was unsure about some Latin wording. By the spring of 1650 Constantijn jr. was unavailable as he had left the country in May 1649 for a lengthy journey through France and Italy. Therefore Van Schooten consulted some Latinists among his acquaintances, and he discussed their suggestions with Constantijn's father. The result was only one correction in the verse.[326]

The lay-out of the text around the portrait made it impossible to add the date of death of Descartes in a decent way, so only Descartes's date of birth is shown.

Van Schooten printed a few hundred copies of the engraving, and he distributed these copies among his acquintainces. Rasmus Bartholin, Van Schooten's student who lived in Leiden in 1650, received two copies, of which he sent one to his former teacher Ole Worm in Copenhagen.[327]

Out of this first print run of Descartes's portrait of 1650, only two copies seem to have survived. One copy is currently kept in the Royal Library in Copenhagen and the other copy is in the Rijksmuseum in Amsterdam.[328] In 1656, the portrait was printed again for an edition of the collected works *Opera philosophica* (Philosophical works).[329] In the following years, the portrait was used in various editions of Descartes's work by the Amsterdam printing house Elzevier. Van Schooten included the portrait in the second edition of *Geometria* in 1659, and in 1664 the portrait also appeared in the fourth edition of Descartes's *Opera philosophica*. After the death of Daniel Elzevier the plate eventually came into the possession of the Blaeu printing house in Amstedam, where it was used again in the 1683 edition of *Geometria* and the 1692 edition of the *Opera philosophica*. The quality of the prints deteriorated over time, as the same plate was used again and again. The first prints of 1650 were of the best quality.

[Descartes, 1963, 202-203].

325 [Nordström, 1958, 197-198].

326 [Nordström, 1958, 198-199] and Christiaan Huygens to Constantijn Huygens jr., 29 March 1650, [Huygens, 1888, 124-125].

327 [Nordström, 1958, 202].

328 Both copies have been indentified by Nordström. The copy in the Rijksmuseum has object number RP-P-1910-4460.

329 From 1656 onwards the portrait bears 'Prmat' in the upper-rightcorner of the engraving. On the printing history of the *Opera philosophica* see [Otegem, 2002, vol 2, 679-710].

2.11 Death and beyond

The last sign of life of Van Schooten is a letter of 14 March 1660 to Christaan Huygens. Van Schooten sent a copy of his treatise *Tractatus de concinnandis demonstrationibus geometricis ex calculo algebraïco* (Treatise on putting together geometrical demonstrations from algebraic calculation) to Huygens and asked his opinion of it. Van Schooten stressed that the printer wanted to finish the printing of the second volume of the *Geometria*.[330] He was not able to witness the printing of the treatise as he died on 30 May 1660.[331] The cause of his death remains unknown, but it seems that he passed away quite unexpectedly, as he was in the middle of a lecture series on irrational quantities at the Duytsche Mathematicque and busy with the preparations of the printing of the treatise when death overtook him.[332] He was buried on 5 June 1660 in the Hooglandse kerk in Leiden.[333]

Shortly after his death, the Leiden student Samuel Tennulius wrote an eulogy on Van Schooten. This eulogy circulated in Paris by the end of July 1660 in circles of the French astronomer Ismael Boulliau and was printed eight years later in Tennulius's Latin translation of the commentary by Iamblichus on the *Introduction to Arithmetic* of Nicomachus of Gerasa.[334]

The death of Van Schooten was communicated to the mathematical community in Europe by Christiaan Huygens. On 15 July 1660 Huygens sent letters to John Wallis and Pierre de Carcavi with the sad news. It is not known exactly in what wording Huygens described the death of Van Schooten to these men, because only the drafts of the letters have survived.[335]

Van Schooten had made his will in 1649 when he appointed his handmaiden Margaritgen Wijnants as his heir, but he kept his printed books manuscripts, maps and instruments out of the provisions of the will. In 1652, when he married, he had his will redrafted and made his new wife his sole heir. Shortly after Frans van Schooten's death, Margaritgen made a new will, instituting her niece and her late husband's brother Petrus as her heirs, both for one half of her possessions.[336] Eventually, the books, manuscripts, maps and instruments of Frans jr. came into the possession of Petrus van Schooten, though it is not clear whether this happened after the death of Frans jr. or only after his wife Margaritgen had died.[337]

Shortly after the death of Van Schooten a commemorative medal was made (figures

[330] Van Schooten to Christaan Huygens, 14 March 1660 [Huygens, 1890, 41]. Huygens replied on 19 March 1660, [Huygens, 1890, 43-33].
[331] His brother Petrus gives 30 May 1660 as the date of his death, UBG, Hs 437, f. 218v. This date is also mentioned on Van Schooten's commemorative medal.
[332] UBG, Hs 437, f. 218v.
[333] RAL, Stadsarchief van Leiden (Stadsbestuur (SA II)), (1253) 1574-1816 (1897), toegang 501A, inv. nr. 1323, f. 97r. Samuel Tennelius gives 4 June 1660 as date of the burial in his eulogy, [Tennulius, 1668, 209].
[334] Boulliau to Heinsius, 30 July 1660, [Huygens, 1890, 508] and [Tennulius, 1668, 209-211].
[335] Christiaan Huygens to Wallis, 15 July 1660, [Huygens, 1890, 96], and Christiaan Huygens to Carcavi, 15 July 1660, [Huygens, 1890, 96]. In the letter which he sent to Carcavi, Huygens described the circumstances of Van Schooten's death.
[336] [Knappert, 1938, 165-168].
[337] RAL, Heilige Geest- of Arme Wees- en Kinderhuis (HGW), toegang 519, inv. nr. 4591.

Figure 2.10 – Cast of the front of the commemorative medal of Frans van Schooten. Leiden, Museum De Lakenhal, inv. nr. 3630.1. The circumscription reads: "Franciscus a Schooten in Academia Leydensi Patria Matheseos Professor Natus 15 Junij 1615 Denatus 30 Maij 1660".

2.10 and 2.11).[338] Such medals were common during the seventeenth century in the Dutch Republic, to commemorate family affairs like birth, marriage and death. In case of death, these medals were use to preserve, disseminate and cherish the memories of the deceased among relatives and friends, and sometimes to a broader public. Thus, these medals were a medium of communication equivalent to printed pictures and pamhlets. Medals were an accessible medium as they were easy to transport, and relatively easy to multiply. The two sides of the medal were usually designed along established lines, with one side having the portrait of the deceased along with further information on his identity, titels and family connection. The reverse side contained specific information on the person.[339]

The medal of Van Schooten roughly follows this pattern. The front side bears the weapon of the Van Schooten family, instead of a picture of Van Schooten. The circumscription gives details on Van Schooten's life and his position at Leiden university. The reverse side refers to his mathematical career. The figure displays a cone and a cylinder which circumscribe a sphere. The circumscription compares Van Schooten's mathematical achievements to those of Archimedes and Descartes.

[338] A cast of the front and back of this medal is in the collection of Museum De Lakenhal in Leiden, inv. nr. 3630.1 and 3630.2. A medal is part of the collection of the Geldmuseum, Utrecht, inv. nr. PE-01655.
[339] [Scher, 1997, 9-11].

Figure 2.11 – Cast of the back of the commemorative medal of Frans van Schooten. Leiden, Museum De Lakenhal, inv. nr. 3630.2. The circumscription reads "Hic Archimedes imo erat Cartesius" (This Archimedes was even more Descartes).

2.11.1 Legacy: teaching

The teaching of practical mathematics in the vernacular continued in Leiden in the first decades after Van Schooten's death. The way in which he arranged his lectures influenced his former students in the design of their own lectures.

Petrus van Schooten succeeded his half-brother Frans van Schooten jr. as professor of the Duytsche Mathematicque. In the summer of 1660, Petrus made a petition to the board of curators and burgomasters to obtain the chair. As other mathematicians, whose names remain unknown, had shown their interest in the position as well, the curators and burgomasters decided to ask the opinion of Jacob Golius.[340] Apparently Golius was in favour of Petrus, who was provisionally appointed on 8 February 1661; a permanent appointment followed on 7 November 1661.[341]

Petrus simply continued the lessons in the way of his half-brother. He lectured on the same subjects as Frans jr.: fortification, practical geometry, algebra, sundials and perspective.[342] The course by Petrus on fortification had a similar structure as that of Frans jr., and

[340] The curators and burgomasters discussed the succession of the Duytsche Mathematicque in their meeting of 7 August 1660, [Molhuysen, 1918, 165].
[341] [Molhuysen, 1918, 171, 173].
[342] [Molhuysen, 1918, 167*, 174*-177*, 197*-202*, 208*, 211*, 224*-226*, 232*-234*, 236*-237*].

Petrus used the same principles for the design of the fortification works.[343]

Petrus van Schooten died in 1679, and no successor was appointed after his death. Two years later, on 8 May 1681 the Duytsche Mathematicque was officially closed by the board of curators and burgomasters.[344]

The influence of Van Schooten's teaching was not restricted to Leiden or the Dutch Republic. The most prominent example outside Leiden was in Sweden, in the teaching of the polymath and scholar Olaf Rudbeck (1630–1702) at Uppsala University. As one of the pioneers in the study of lymphatic vessels he received a stipend of Queen Christina in 1652 to study at Leiden University. He subsequently traveled to Leiden where he stayed from autumn 1653 untill early summer 1654. Here he attended Van Schooten's lectures at the Duytsche Mathematicque. These lectures inspired him and served as a model for his own teaching of practical mathematics at Uppsala University in the period 1660–1702. In particular, Rudbeck valued the teaching of mathematics in the vernacular to craftsmen, because he believed that mathematical knowledge would improve their understanding and ability in their own profession. Rudbeck explicitly mentioned the influence of Van Schooten on his ideas on what an academic education was supposed to be. His courses covered similar subjects as the courses by Van Schooten such as surveying, fortification and gnomonics.[345]

2.11.2 Legacy: printed works

Frans van Schooten passed away while he was preparing the second volume of the second edition of *Geometria* for the press. Thanks to his half-brother Petrus, the second volume finally came out in 1661. Petrus considered it his duty to finish the work of his late half-brother and to see the second volume through the press. He composed the final list of errata to the treatise of Johan de Witt on conic sections and sent the list to De Witt on 17 December 1660.[346] Furthermore, Petrus prepared the treatise *Tractatus de concinnandis demonstrationibus geometricis ex calculo algebraïco* of his late half-brother for print. As Frans had sent it to Huygens just before his death, the content of the treatise was finished, but the front matter still had to be done. Petrus composed the dedication letter of the treatise, and addressed it to the board of the curators and burgomasters of the university. The printing of the second volume was completed by the end of January 1661.[347]

The two volumes of the *Geometria* of 1659–1661 were the standard reference works for analytic geometry in the second half the seventeenth century. The Latin translation

[343] This is clear form a comparison of a manuscript UBL, BPL 1993, which is assumed to contain notes of lectures by Petrus, with UBG, Hs 442, containing notes, made by Petrus, of lectures by Frans jr.
[344] [Molhuysen, 1918, 364-366].
[345] On Rudbeck's teaching of practical oriented mathematics see [Kallinen, 2006, 118-122] and [Dahl, 1995]. The stay in Leiden inspired Rudbeck to undertake other activities in Uppsala as well, including the building of a an anatomical theatre and the creation of a botanical garden, [Eriksson, 2004].
[346] The treatise itself had been printed in 1659 according to the titlepage [Witt, 1661, titlepage] and [Grootendorst and de Witt, 1997, 6].
[347] Elzevier informed Johan de Witt on 2 February 1661 that the printing of the second volume had just been finished and that they would sent a copy to De Witt, [Witt, 1922, 91].

of Descartes's *Géométrie* made the work accessible to scholars not mastering French, and Van Schooten's additional comments and explanations facilitated the understanding of Descartes's intricate text. Moreover, the additional treatises by Van Schooten's students in the *Geometria* illustrated the further possible uses of algebra in geometry. The work saw two reprints in the course of the seventeenth century. After the death of Daniel Elzevier in 1680, the Blaeu printing house obtained the rights to print the works of Descartes,[348] and they published a third edition of the *Geometria* in 1683. This edition was simply a reprint of the second edition of 1659–1661. A fourth edition appeared in 1695 in Frankfurt, and was the only edition which appeared outside the Dutch Republic. The second volume of the fourth edition contained a new commentary *Notae et animadversiones tumultuariae in universum opus* (Collected notes and observations regarding Cartesian geometry) by Jacob Bernoulli (1654–1705).[349]

The works of Van Schooten found their way to libraries of scholars and universities all over Europe. In England, the young Isaac Newton (1642–1727) was introduced to the *Géométrie* of Descartes through the translation and commentaries by Van Schooten. He carefully read the second edition of the *Geometria* during the summer and autumn of 1664 and he made detailed notes on the work. In addition, he studied Van Schooten's edition of Viète's collected works, and he wrote extensive notes on Book V of Van Schooten's *Exercitationum mathematicarum libri quinque*.[350]

[348] [Otegem, 2002, 69].

[349] For a detailed discussion of these two editions see [Otegem, 2002, 130-145].

[350] Newton's notes on the works of Van Schooten are published in [Newton and Whiteside, 2008, 19-22 and 29-87].

Part II

Scholarship: Descartes's geometry

Introduction

Frans van Schooten played an important role in the dissemination of the new mathematical methods of the *Géométrie* (1637) of Descartes. Van Schooten did this by publishing a Latin translation of the *Géométrie* with his own commentaries and explanatory notes as well as relevant texts by other mathematicians in 1649, and by putting out a second edition with additional commentaries and texts in 1659. In this part of the thesis I will investigate for a specific case how Van Schooten commented on the new mathematical method of Descartes, and the way in which he dealt with criticisms of Descartes which were made by other mathematicians. The specific case is a geometrical problem which had already been studied in Greek antiquity and which was solved by Descartes in the *Géométrie* by means of his own methods: the so-called Pappus problem in four lines.

Descartes devoted a fair part of his *Géométrie* to the solution of the Pappus problem. The problem had been suggested to him in 1631 by Jacob Golius, professor of mathematics and Eastern languages at Leiden University, and also Van Schooten's mathematics teacher. In the *Géométrie*, Descartes used the Pappus problem as a vehicle to demonstrate how his new techniques could be used, and also to show that his method was able to surpass the achievements of the ancients Greeks.

I have selected a special case of this problem, the so-called Pappus problem in four lines, for several reasons. First, the problem has a prominent role in the *Géométrie*. Secondly, Descartes's solution was incomplete, and it was criticized by other mathematicians between 1637 and 1659. This is precisely the period when Van Schooten worked on his two editions of *Geometria*. The criticisms became increasingly strong in the course of time and they illuminate some of the difficulties mathematicians met in understanding Descartes's new methods and techniques. I will investigate the way in which Van Schooten dealt with these critical assesments of Descartes's work and in what way he consulted other mathematicians in this process, as part of his preparation of the two editions of the *Geometria*. I will show that Van Schooten tried to ignore the criticisms in *Geometria (1649)* and reluctantly took some account of them in *Geometria (1659)*.

Descartes's solution of the Pappus problem has been studied by modern scholars. In his book *Redefining geometrical exactness*, Henk Bos analyzed the development of Descartes's solution of the Pappus problem in relation with his ideas on geometrical construction and exactness.[351] The work of Bos is characterized by a strong interest in the roles of algebra and geometry in the mathematics of Descartes, and by a profound analysis of extramathematical aspects of early modern mathematics. Emily Grosholz studied the Pappus problem in relation to the Cartesian way of reasoning, and she critically examined the role of reductionism in the solution.[352] In Part I of his dissertation,[353] which has much facilitated the research in this chapter, Sébastien Maronne discussed Descartes's solution of the Pappus problem in the *Géométrie* and in his correspondence from the year 1631 until 1649, in the framework of the general development of a Cartesian theory of algebraic curves. The focus of my study of the Pappus problem differs from that of Maronne, because I present an analysis of all aspects connected to Van Schooten, who naturally plays a minor role in Maronne's account. In some cases my analysis will lead to interpretations different from Maronne, as will be discussed below.

In order to understand the choices which Van Schooten made in writing his comments on the Pappus problem, and the influence of other mathematicians on these choices, it is necessary to dive into the mathematical content of the Pappus problem and the solution by Descartes. This will be the subject of chapter 3. In the course of this chapter, I will present a new interpretation of Descartes's (and Van Schooten's) use of the plus and minus sign in algebraic substitutions. This new interpretation seems to have escaped historians of mathematics until now but will facilitate the understanding of the texts by Descartes and Van Schooten. The rest of this part of the thesis covers in chronological order the period from the publication of the *Géométrie* in 1637 until the year 1660 in which Van Schooten passed away.[354] I divide the period 1637–1660 into two shorter periods: 1637–1649 and 1649–1660, separated by the publication of the *Geometria (1649)*. This event was soon followed by the death of Descartes on 11 February 1650, and thus the period 1649–1659 belongs for a greater part to the post-Cartesian period.

The organization of the chronological sections is as follows: In chapter 4 I discuss the relevant criticisms by other mathematicians on Descartes's solution of the Pappus problem from 1637 until the publication of the *Geometria (1649)* and Van Schooten's comments in the *Geometria (1649)*. I discuss how these comments are related to the debate in the previous years. Chapter 5 deals with the period after 1649. In the first two sections 5.1 and 5.2, attention is paid to the criticisms that were made in the years 1649–1660. As we will see, the relation between Van Schooten and Christiaan Huygens was important in this connection. In section 5.3 I will discuss how the criticisms influenced the enlarged 1659 edition of the *Geometria*. In the final chapter 6 I draw conclusions on the basis of the three previous chapters.

[351] See especially chapters 15, 19 and 23 of [Bos, 2001]. The Pappus problem also plays a major role in [Bos, 1992].

[352] [Grosholz, 1991b, 15-37].

[353] [Maronne, 2007, 31-139].

[354] After Van Schooten's death, two more editions of the *Geometria* appeared in 1683 and 1695, but these contained the same text as the 1659 edition, and do not shed any new light on the views and comments of Van Schooten.

CHAPTER 3

The four line Pappus problem and Descartes's solution

The subject of this chapter is the Pappus problem, which Descartes used in the *Géométrie* in a prominent way to illustrate the power of his new mathematical methods and techniques. First I will introduce the general Pappus problem and the Pappus problem in four lines, and then I will discuss in detail how Descartes solved the Pappus problem in four lines. I will show that Descartes modelled his discussion of the four line Pappus problem after a specific numerical example. A separate section at the end of the chapter will be devoted to a new interpretation of the way in which Descartes and Van Schooten used plus and minus signs in algebraic substitutions.

3.1 Definition of the Pappus problem

The Pappus problem in its general form is as follows in modern notation:

Problem 1.
Let n be a natural number ≥ 4, and suppose that the following objects are given in the plane: n straight lines ℓ_i, n angles $\theta_i \neq 0$, a ratio τ, and if n is odd, a line segment c. For any point C in the plane, define the oblique distance d_i as the length of the line segment joining C and ℓ_i which meets line ℓ_i at an angle equal to θ_i.

It is required to determine the collection of points C such that:

$$\textit{for n even, } n = 2m: \qquad d_1 d_2 \dots d_m : d_{m+1} \dots d_n \qquad = \tau \tag{3.1}$$

$$\textit{for n odd, } n = 2m+1: \qquad d_1 d_2 \dots d_{m+1} : c\, d_{m+2} \dots d_n \qquad = \tau \tag{3.2}$$

The Pappus problem for $n = 3$ differs from the general form, since for $n = 3$ it is required to determine the collection of points C such that $d_1 d_2 : d_3^2 = \tau$. This is a variant of the problem for $n = 4$, with $\ell_3 = \ell_4$ and $\theta_3 = \theta_4$.

The general problem is named after the Greek mathematician Pappus of Alexandria (fl. ca. 300-350), who mentioned it in Book VII of his *Collection*. This book was translated into Latin by Commandino and became available in print in 1588.[1] The work was widely known among seventeenth century mathematicians and had a profound influence on the subfield of geometry which was dedicated to geometrical problem solving. Pappus provided statements on the aims and rules of geometrical problems solving together with many examples and problems, including the problem that is now named after him.[2]

Pappus did not provide a solution of his general problem, but he stated that two cases ($n = 3$ and $n = 4$) had been solved centuries before him by Euclid (ca. 300 BC) and Apollonius (ca. 200 BC), in works that have not come down to us. Pappus did not use the modern notation and he called the cases $n = 3$ and $n = 4$ "the locus of three lines" and "the locus of four lines".[3] The word *locus* (plural: *loci*), meaning "place", is a technical term of ancient Greek and seventeenth-century mathematics, referring to what is in modern terms any collection of points in the plane satisfying a certain property. *Locus* is the Latin translation of the ancient Greek *topos* with the same meaning. In the terminology of the Latin translation of the *Collection*, Pappus distinguished between *plane loci*, meaning straight lines and circles, *solid loci*, meaning conic sections, and *linear loci*, meaning curves that are not straight lines, circles, or conic sections. The term *solid loci* also appears in the *Géométrie* of Descartes and in the criticisms by other mathematicians that we will study below. According to Pappus, the ancient mathematicians Euclid and Apollonius showed that the *locus of three lines* and the *locus of four lines* are *solid loci*. This means in modern terms that for $n = 3$ and $n = 4$, the collection of points C satisfying $d_1 d_2 : d_3^2 = \tau$ or $d_1 d_2 : d_3 d_4 = \tau$ are conic sections.

3.2 Descartes's solution

Descartes treated the general Pappus problem in Book I and Book II of the *Géométrie*. In Book I, he argued that the solutions of the general problem are curves and he discussed the way in which the nature of these curves depends on the number n of given straight lines ℓ_i and on the different configurations of these lines. He furthermore started a detailed investigation of the case $n = 4$, the locus of four lines. This problem is nowadays called the Pappus problem in (or: for) four lines, or simply the four line Pappus problem. In Book II he continued his investigation and provided a construction of the solution curve,

[1] [Pappus, 1588]. The work was reprinted in [Pappus, 1589] and [Pappus, 1602].

[2] [Bos, 2001, 37].

[3] For the text of Pappus on what is nowadays called the Pappus problem see [Pappus, 1986, 118-123].

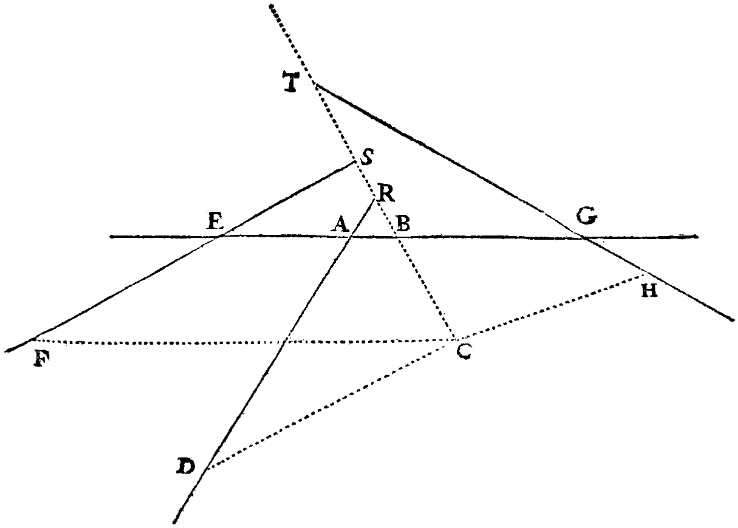


Figure 3.1 – The Cartesian Pappus problem. [Descartes, 1637c, 311].

to be presented below. He then solved a special case of the Pappus problem for $n = 5$, in which four given lines $\ell_1, \ldots, \ell_4$ are parallel and equidistant and the fifth given line ℓ_5 is perpendicular to the other four.

I will only discuss Descartes's solution of the four line locus because this is the only case which Van Schooten considered in his commentary. Throughout the entire exposition of this problem, Descartes used the same configuration of the given lines and angles. This configuration is displayed in figure 3.1, which is taken from the *Géométrie*. In this figure, the four given lines are drawn as solid lines and the dotted line segments are the oblique distances d_i. In our modern notation we have $\ell_1 = AB$, $\ell_2 = EF$, $\ell_3 = AD$, and $\ell_4 = GH$. The points E, A and G are the points of intersection of ℓ_1 with ℓ_2, ℓ_3 and ℓ_4 respectively. Note that the line segments $\ell_1, \ldots, \ell_4$ and their points of intersection are given, but not the positions of points B, F, D and H on the given lines, because these positions depend on the point C which has to be determined. Descartes was looking for the points C that satisfy the property $CB \cdot CF = CD \cdot CH$, so he assumed in modern notation $\tau = 1$. From now on I will refer to this particular case of the four line locus, including the configuration of lines of figure 3.1, as the *Cartesian Pappus problem.* This problem plays a prominent role in the way Descartes structured his exposition of problem solving in general.

Descartes's solution consists of two parts. In the first part he introduced algebraic notation and he investigated the distances CB, CF, CD and CH in order to obtain an equation in two unknowns which all points of the solution satisfy. The second part consists of the actual construction of the solution curve based on the equation which he had found.

3.2.1 Finding the equation

Descartes supposed C to be a point satisfying $CB \cdot CF = CD \cdot CH$. He then introduced two unknown line segments $x = AB$ and $y = BC$ in order to relate the point C to the line AB. A modern mathematician is reminded of an oblique coordinate system with origin in A, one axis along AG and the second axis the line through A parallel to BC. This coordinate system depends only on given objects, namely the line ℓ_1, its point of intersection A with ℓ_3, and the given angle $\angle ABC = \theta_1$. The use of the letters x and y by Descartes may remind a modern mathematician of real numbers, but as we will see below, x and y denote (lenghts of) line segments and were therefore positive quantities for Descartes.

Now the line segments AE and AG are known because they are determined completely by the given lines ℓ_i. Descartes put $AE = k$ and $AG = l$. The points of intersection of the extended line segment BC with ℓ_2, ℓ_3 and ℓ_4 are called S, R and T respectively. Then the shapes of triangles ABR, CRD, BES, CSF, BGT, and TCH in the figure are determined by the ℓ_i and the θ_i.[4] Descartes expressed them in the following way. He chose an arbitrary known line segment z and put

$$z : b = AB : BR \tag{3.3}$$

$$z : c = CR : CD \tag{3.4}$$

$$z : d = BE : BS \tag{3.5}$$

$$z : e = CS : CF \tag{3.6}$$

$$z : f = BG : BT \tag{3.7}$$

$$z : g = TC : CH \tag{3.8}$$

Now the quantities k, l, b, c, d, e, f, and g are also known, that is to say, of known positive magnitude. Descartes expressed all oblique distances (d_i in our terminology) in terms of these known quantities, the known quantity z, and the unknowns x and y. By the choice of the coordinate system,

$$d_1 = BC = y. \tag{3.9}$$

For $d_2 = CF$, Descartes proceeded in the following way. From the figure, Descartes concluded $EB = AB + AE = x + k$. Combining this with (3.5) Descartes got

$$BS = \frac{dx + dk}{z}.$$

The figure also shows that

$$CS = BS + BC = \frac{dx + dk + yz}{z},$$

[4] If $\theta_1 \neq 90^o$, it is possible to find for any point C in the plane two points B_1 and B_2 on ℓ_1 such that CB makes an angle θ_1 with ℓ_1. Then triangle CB_1B_2 is isosceles and $CB_1 = CB_2$. Pappus (and Descartes) considered only one of these points in such a way that for different points C the corresponding line segment CB is always in the same direction.

and by combining this with (3.6) Descartes obtained the expression for CF:

$$d_2 = CF = \frac{ezy + dek + dex}{z^2}. \tag{3.10}$$

The distances d_3 and d_4 were obtained in a similar way:

$$d_3 = CD = \frac{cyz + bcx}{z^2}, \tag{3.11}$$

and

$$d_4 = CH = \frac{gzy + fgl - fgx}{z^2}. \tag{3.12}$$

Substituting these expressions into $CB \cdot CF = CD \cdot CH$ and isolating the y^2 term Descartes obtained:[5]

$$y^2 = \frac{(cfglz - dekz^2)y - (dez^2 + cfgz - bcgz)xy + bcfglx - bcfgx^2}{(ez^3 - cgz^2)}. \tag{3.13}$$

Descartes was aware of the fact that equation (3.13) depends on the position of point C with respect to the four given lines ℓ_i, as a consequence of the fact that he was working with positive quantities only. In the determination of EB, he wrote that in the case of figure 3.1, $EB = k + x$, but if B is between A and E, then $EB = k - x$ and when E is between A and B, then $EB = x - k$. This may occur when point C is supposed to be at an other position in the plane with respect to the four given lines. Figure 3.2 displays a point C' in such a way that E is located between A and the corresponding point B'. Descartes did not provide a similar figure in his *Géométrie*. In that case $EB' = AB' - AE = x - k$, so

$$B'S' = \frac{dx - dk}{z}.$$

Since in this case $C'S' = B'S' - B'C' = \frac{dx - dk - yz}{z}$, we obtain

$$C'F' = \frac{dex - ezy - dek}{z^2},$$

which is different from (3.10).

Maronne has shown that if one computes only with positive line segments, as Descartes did, one has to distinguish 23 different regions of the plane to take account of all possible positions of C.[6] The argument of Descartes shows the validity of equation (3.13) only for C in one of these regions, namely the shaded region in figure 3.3. This region is bounded at the top by the line segment AG, on the left by the line through A parallel to BC, and on the right by the line through G parallel to BC.

[5] Here Descartes made the tacit assumption $ez^3 - cgz^2 \neq 0$. In a letter to De Beaune of 20 February 1639, he admitted that he had overlooked this case, [Bos, 2001, 320 footnote 18].

[6] [Maronne, 2007, 46-64].

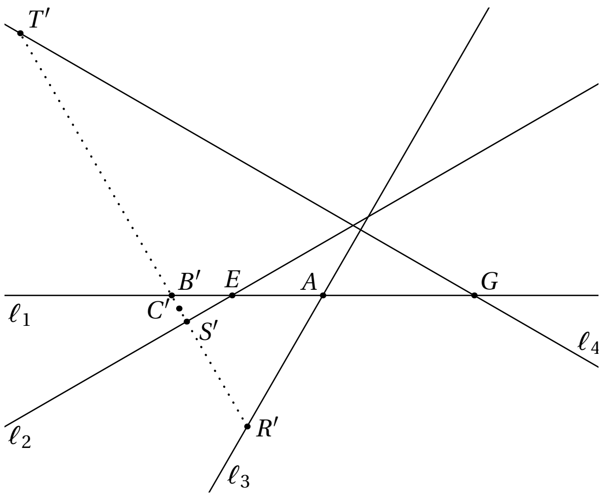


Figure 3.2 – Altered position of C.

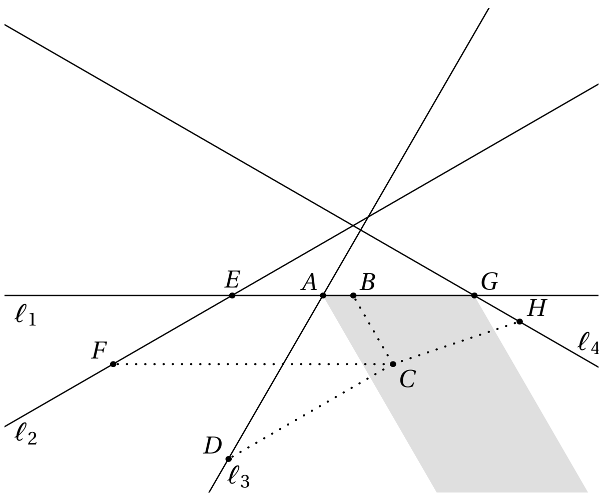


Figure 3.3 – The region to which the equation (3.13) belongs.

In modern terms, the analysis of Descartes consists of expressing the distances d_i as

$$d_i = \alpha_i x + \beta_i y + \gamma_i,$$

where x and y are positive unknown quantities, and the quantities α_i, β_i, and γ_i are determined by ℓ_i, θ_i, and τ.[7] This led Descartes to a final equation of the form

$$y(\alpha_2 x + \beta_2 y + \gamma_2) = (\alpha_3 x + \beta_3 y + \gamma_3)(\alpha_4 x + \beta_4 y + \gamma_4),$$

and to one conic section as the solution curve. However, the solution of a Pappus problem in four lines consists of two conic sections. This can be seen by considering the problem in modern terms in an oblique coordinate system in which x and y are real numbers, which may be positive, zero or negative. Then the oblique distance can be expressed as $d_i = \left|\alpha_i x + \beta_i y + \gamma_i\right|$, and hence the complete solution is

$$\left|y(\alpha_2 x + \beta_2 y + \gamma_2)\right| = \left|(\alpha_3 x + \beta_3 y + \gamma_3)(\alpha_4 x + \beta_4 y + \gamma_4)\right|.$$

This last equation represents two conic sections, namely

$$\pm y(\alpha_2 x + \beta_2 y + \gamma_2) = (\alpha_3 x + \beta_3 y + \gamma_3)(\alpha_4 x + \beta_4 y + \gamma_4).$$

Since Descartes in his calculation implicitly assumed the point C to be in the shaded region in figure 3.3, he obtained only one of the two conic sections. The error is due to his restriction to positive quantities, so his equations are not sufficiently general to include the full solution.

I now return to Descartes's determination of the equation. After having found equation (3.13), Descartes used substitution in order to simplify the equation. He introduced the quantities m and n by putting[8]

$$2m = \frac{cfglz - dekz^2}{ez^3 - cgz^2}, \tag{3.14}$$

and

$$\frac{2n}{z} = \frac{dez^2 + cfgz - bcgz}{ez^3 - cgz^2}. \tag{3.15}$$

Inserting them in equation (3.13) he obtained

$$y^2 = 2my - \frac{2n}{z}xy + \frac{bcfglx - bcfgx^2}{(ez^3 - cgz^2)}. \tag{3.16}$$

Descartes solved this equation for y and took the positive root

$$y = m - \frac{n}{z}x + \sqrt{m^2 - \frac{2mn}{z}x + \frac{n^2}{z^2}x^2 + \frac{bcfglx - bcfgx^2}{(ez^3 - cgz^2)}}. \tag{3.17}$$

[7] In the Cartesian Pappus problem $\gamma_3 = 0$; compare with equation (3.11).

[8] Again, Descartes tacitly assumed $ez^3 - cgz^2 \neq 0$.

Descartes further abbreviated by putting

$$o = \frac{-2mn}{z} + \frac{bcfgl}{ez^3 - cgz^2}, \tag{3.18}$$

$$\frac{p}{m} = \frac{n^2}{z^2} - \frac{bcfg}{ez^3 - cgz^2} \tag{3.19}$$

and he concluded

$$y = m - \frac{n}{z}x + \sqrt{m^2 + ox - \frac{p}{m}x^2}. \tag{3.20}$$

By taking only the positive root in (3.20), Descartes only obtained part of the conic section. The other part of the conic section is found by taking the negative root. In the *Géométrie*, Descartes did not discuss the question how one can be sure that the whole conic section serves as a solution, instead of only a part.

Comparison of (3.17) with (3.19) reveals a difficulty with respect to the sign in front of the term $\frac{p}{m}x^2$ in equation (3.20). Using the substitution (3.19), one would expect a plus sign instead of a minus sign in front of the term $\frac{p}{m}x^2$. I will explain this peculiar minus sign in section 3.3.

3.2.2 Construction of the solution

The next step in the solution was its explicit construction. From ancient Greek geometry Descartes took the notion that any geometrical object has to be constructed from objects that are known at the beginning. A geometrical problem is solved once the solution has been actually constructed. Therefore, Descartes had to construct the conic section which he had found as a solution of the Pappus problem. For this construction, Descartes referred to the end of Book I of the *Conics* of Apollonius. There, Apollonius showed how to construct a conic section from the following data: the type of conic section (parabola, hyperbola, ellipse), a point on the conic section and the diameter of the conic section through that point, the corresponding angle of ordinates, the *latus rectum*, and, in the case of the hyperbola and ellipse, also the *latus transversum*. Using these data, Apollonius constructed a three-dimensional cone which intersects the given plane in the required conic section. The reader who is not familiar with the Apollonian terminology of conics can find the necessary explanations in Appendix B of this thesis. In *Geometria (1659)*, comment CCC, Frans van Schooten gave a three-dimensional Apollonian construction so that the reader of the *Geometria* could dispense with the *Conics*.[9]

In order to use the Apollonian construction, Descartes had to indicate how to obtain from the equation (3.20) the data which are required for that construction. In the following, I use the notation of figures 3.1, 3.2 and 3.3 above.

Descartes first (see figure 3.4) constructed K on BC such that $BK = m$ and then he drew a line through K parallel to AB and constructed I on this line such that $IK = x$. This means that the point I is determined by the given line segments and given angles. In modern

[9] [Schooten, 1659c, 206-220].

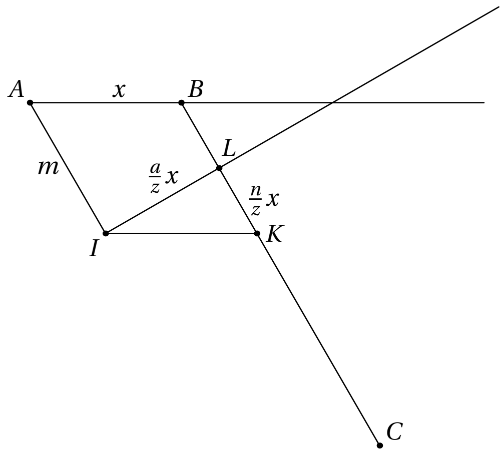


Figure 3.4 – The construction of the line IL.

terms, the oblique coordinates of I are $x = 0$ and $y = m$. Then he drew a straight line through I which intersects BK in L such that $IK : KL = z : n$. I call this line ℓ. Since $IK = x$, we have $KL = \frac{n}{z}x$ and $BL = m - \frac{n}{z}x$. Since $\angle LKI = \angle BAI = \theta_1$, and $IK : KL = z : n$, the shape of triangle KLI is known, so the ratio $KL : IL$ is known. Descartes put $KL : IL = n : a$. Now a is a known quantity, and $IL = \frac{a}{z}x$.

We have $BL = m - \frac{n}{z}x$, and we see from (3.20) that we now need to consider

$$LC = \sqrt{m^2 + ox - \frac{p}{m}x^2} \tag{3.21}$$

in order to obtain the solution.

Equation (3.21) has been derived for the Cartesian Pappus problem. In a more general four line Pappus problem, LC will be of the form

$$LC = \sqrt{m^2 \pm ox \pm \frac{p}{m}x^2}, \tag{3.22}$$

and some of the terms might be missing (i.e., equal to zero); otherwise the quantities m, o and p in (3.22) are always positive or possibly zero, as we will explain in more detail in the next section 3.3. The construction of the conic section then depends on the signs in front of the terms m^2, ox, and $\frac{p}{m}x^2$ in (3.22). I note that Descartes also considered some cases where m^2 has a negative sign, although he was aware of the fact that such cases cannot occur in the four line Pappus problem.[10] Apparently he included cases with $-m^2$ because he wanted to treat as many types of quadratic equations as possible.

Descartes distinguished three cases, which I summarize as follows:

1. The first case where $m = o = p = 0$. Then $LC = 0$ and point C will be on the line IL, therefore the line ℓ is the desired locus.

[10] [Descartes, 1637c, 329].

2. The second case where $\pm m^2 \pm ox \pm \frac{p}{m}x^2$ is a perfect square $(\alpha + \beta x)^2$. Descartes listed the conditions under which this happens. In this case he stated that point C will be on a straight line other than IL. This line is the desired locus.

3. In the third case, point C will be on a conic section or a circle. The type of the conic section can be inferred from the expression for LC. Descartes said:[11]

   (a) If $\frac{p}{m}x^2 = 0$, the conic section is a parabola with diameter along IL, and LC is an ordinate to this diameter.

   (b) If, in the words of Descartes, $\frac{p}{m}x^2$ has the minus sign, the conic section is an ellipse or a circle with diameter along IL, and LC is an ordinate to this diameter.

   (c) If $\frac{p}{m}x^2$ has the plus sign, the conic section is a hyperbola. The position of the diameter is dependent on the expression for LC. If $LC = \sqrt{m^2 \pm ox + \frac{p}{m}x^2}$ and $o^2 < 4mp$, or $LC = \sqrt{m^2 + \frac{p}{m}x^2}$, the diameter is parallel to LC and the corresponding ordinate is a straight line parallel to IL, as in the "variant construction" below. If $LC = \sqrt{m^2 \pm ox + \frac{p}{m}x^2}$ and $o^2 > 4mp$, the diameter is along IL, and LC is the corresponding ordinate, just as in the case of the ellipse and the parabola.

Then Descartes gave the necessary data in order to construct the conic section of the third case in the Apollonian way.[12] He did not give the analysis, that is, the way he found these data.[13] In case the conic section is a parabola, he first stated that one has to know the position of the vertex (in the Apollonian sense) N and the latus rectum in order to perform the Apollonian construction. Descartes defined the position of N by specifying the distance IN and the relative positions of I, L and N. For clarity, I have listed Descartes's different possibilities for the parabola in table 3.1. The table gives for each possible expression of LC^2 the corresponding relative position of the points I, L and N, the magnitude of the line segment IN, and the magnitude of the latus rectum.

If the conic section is a hyperbola, an ellipse or a circle, Descartes assumed the position of the diameter to be known, and he stated that one needs to know the position of the centre M of the conic section as well as the latus rectum and latus transversum. I list his different cases of the ellipse in table 3.2 and his cases of the hyperbola in table 3.3.[14] Note

[11] In modern terms, this can be explained as follows. Let a curve $\mathscr{C}$ correspond to the equation $y_1^2 = a + bx_1 + cx_1^2$ in coordinates x_1, y_1, with constants a, b and $c \in \mathbb{R}$ and $a \neq 0$, $b^2 - 4ac \neq 0$. If $c = 0$, the curve is a parabola; if $c > 0$ the curve is a hyperbola; and if $c < 0$ the curve is an ellipse. This characterization is independent of the angle between the axes of the underlying coordinate system x_1, y_1; the angle may be right or oblique. Descartes tacitly assumed $m \neq 0$.

[12] [Descartes, 1637c, 330-332].

[13] Descartes may have found the data by a procedure using indeterminate coefficients, as explained in [Bos, 2001, 323].

[14] In case the conic sections is an ellipse or a hyperbola, Descartes also mentioned the case "$-m^2$". I have decided to include these cases $-m^2$ also in tables 3.2 and 3.3, except for the cases where all terms have a minus sign and no solution is possible. This is in agreement with the fact that Descartes excluded such a case for the parabola.

Expression for LC^2	Relative position of I, L and N	IN	Latus rectum
$+m^2+ox$	I between L and N	$\frac{am^2}{oz}$	$\frac{oz}{a}$
$+m^2-ox$	L between I and N	$\frac{am^2}{oz}$	$\frac{oz}{a}$
$-m^2+ox$	N between I and L	$\frac{am^2}{oz}$	$\frac{oz}{a}$
$+ox^\dagger$	$I=N$	0	$\frac{oz}{a}$

† Descartes did not indicate the sign of o, he only indicated that in this case $m=0$. Because Descartes explicitly excluded the case $LC^2=-m^2$, I have also excluded the case $LC^2=-ox$.

Table 3.1 – The cases of a parabola.

Expression for LC^2	Position of L and M with respect to I	IM	Latus rectum r	Latus transversum d
$+m^2+ox-\frac{p}{m}x^2$	same side	$\frac{aom}{2pz}$	$\frac{z}{a}\sqrt{o^2+4mp}$	$\frac{ma}{pz}\sqrt{o^2+4mp}$
$+m^2-ox-\frac{p}{m}x^2$	different sides	$\frac{aom}{2pz}$	$\frac{z}{a}\sqrt{o^2+4mp}$	$\frac{ma}{pz}\sqrt{o^2+4mp}$
$-m^2+ox-\frac{p}{m}x^2$	same side	$\frac{aom}{2pz}$	$\frac{z}{a}\sqrt{o^2-4mp}$	$\frac{ma}{pz}\sqrt{o^2-4mp}$
$+ox-\frac{p}{m}x^2$	same side	$\frac{aom}{2pz}$	$\frac{oz}{a}$	$\frac{oam}{pz}$
$+m^2-\frac{p}{m}x^2$	$I=M$	0	$\frac{z}{a}\sqrt{4mp}$	$\frac{ma}{pz}\sqrt{4mp}$

Table 3.2 – The cases of an ellipse. The locus is a circle if $a^2m=pz^2$ and the angle ILC is right.

that for the circle, the same cases are obtained as for the ellipse, with the only difference that in the case of a circle the angle ILC is a right angle and $a^2m=pz^2$.

A variant construction for some hyperbolas

For the construction of most of the conic sections occurring as solutions of the four line Pappus problem, the diameter is the straight line through I and L and LC is a corresponding ordinate. This construction applies to all possible parabolas, all possible ellipses and circles, and to seven of the ten possible hyperbolas in table 3.3. Descartes illustrated the construction of these cases by figure 3.5 in the *Géométrie*.

For the three remaining cases of the hyperbola, the construction is different in the sense that diameter is a line parallel to LC, and the corresponding ordinates are parallel to IL. Descartes illustrated this construction with figure 3.6. I call the construction of a conic section with the diameter along IL and ordinates along LC as in figure 3.5 the *standard* construction, and the construction of a hyperbola with diameter parallel to LC and ordinates parallel to IL as in 3.6 the *variant* construction.[15] Note that in the two figures 3.5

[15] In modern terms, the variant construction is used if the expression for LC^2 (as function of real x) is non-zero

Expression for LC^2	Position of L and M with respect to I	IM	Latus rectum r	Latus transversum d
$+m^2+ox+\frac{p}{m}x^2$ and $o^2>4mp$	different side	$\frac{aom}{2pz}$	$\frac{z}{a}\sqrt{o^2-4mp}$	$\frac{ma}{pz}\sqrt{o^2-4mp}$
$-m^2+ox+\frac{p}{m}x^2$	different side	$\frac{aom}{2pz}$	$\frac{z}{a}\sqrt{o^2+4mp}$	$\frac{ma}{pz}\sqrt{o^2+4mp}$
$+m^2-ox+\frac{p}{m}x^2$ and $o^2>4mp$	same side	$\frac{aom}{2pz}$	$\frac{z}{a}\sqrt{o^2-4mp}$	$\frac{ma}{pz}\sqrt{o^2-4mp}$
$-m^2-ox+\frac{p}{m}x^2$	same side	$\frac{aom}{2pz}$	$\frac{z}{a}\sqrt{o^2+4mp}$	$\frac{ma}{pz}\sqrt{o^2+4mp}$
$+ox+\frac{p}{m}x^2$	different side	$\frac{aom}{2pz}$	$\frac{oz}{a}$	$\frac{oam}{pz}$
$-ox+\frac{p}{m}x^2$	same side	$\frac{aom}{2pz}$	$\frac{oz}{a}$	$\frac{oam}{pz}$
$-m^2+\frac{p}{m}x^2$	$I=M$	0	$\frac{z}{a}\sqrt{4mp}$	$\frac{ma}{pz}\sqrt{4mp}$
$+m^2+\frac{p}{m}x^2$	$I=M$	0	$\frac{2a^2m^2}{pz^2}$	$2m$
$+m^2+ox+\frac{p}{m}x^2$ and $o^2<4mp$	different side	$\frac{aom}{2pz}$	$\frac{a^2m}{z^2p}\sqrt{4m^2-o^2\frac{m}{p}}$	$\sqrt{4m^2-o^2\frac{m}{p}}$
$+m^2-ox+\frac{p}{m}x^2$ and $o^2<4mp$	same side	$\frac{aom}{2pz}$	$\frac{a^2m}{z^2p}\sqrt{4m^2-o^2\frac{m}{p}}$	$\sqrt{4m^2-o^2\frac{m}{p}}$

Table 3.3 – The cases of a hyperbola.

and 3.6, which are reproduced from the *Géométrie*, point C is located at exactly the same position.

The details of the variant construction are as follows. First, one has to construct the line IL in the way described above on page 93. The centre M of the hyperbola is defined as the point on IL with $IM = \frac{aom}{2pz}$. The position of M and L with respect to I can be read off from table 3.3. Through M, draw a line parallel to LC. The vertex (in the Apollonian sense) of the hyperbola is point O on this parallel line defined by $OM = \sqrt{m^2 - \frac{o^2m}{4p}}$. Descartes stated that the solution curve is a hyperbola with vertex O, diameter OM, and latus rectum and transversum as given in table 3.3. For any point C on the hyperbola, the line CP parallel to IL and with P on the diameter OM is the corresponding ordinate.

Proof of the construction

Descartes then sketched a proof that the Apollonian construction for the specified data produces a conic section whose points satisfy the required equation.[16] This proof is based on the fundamental properties of the conic sections which Apollonius proved in *Conics* I:11-13.[17] Descartes discussed only the varaint con where $LC = \sqrt{m^2+ox-\frac{p}{m}x^2}$. The

for all x, so no point of the conic section is located on line IL, which can therefore not be a diameter of the conic section in the Apollonian sense.

[16] [Descartes, 1637c, 332-333].

[17] These fundamental properties are discussed in Appendix B.

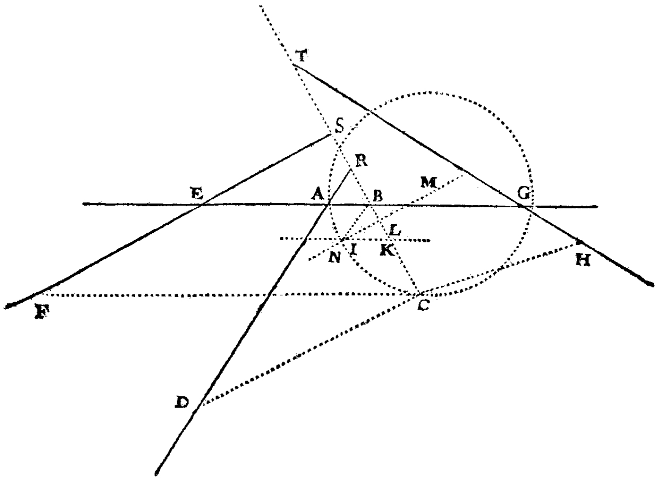


Figure 3.5 – The standard construction of a conic section, in this case a circle, with diameter IL and ordinate LC. [Descartes, 1637c, 327].

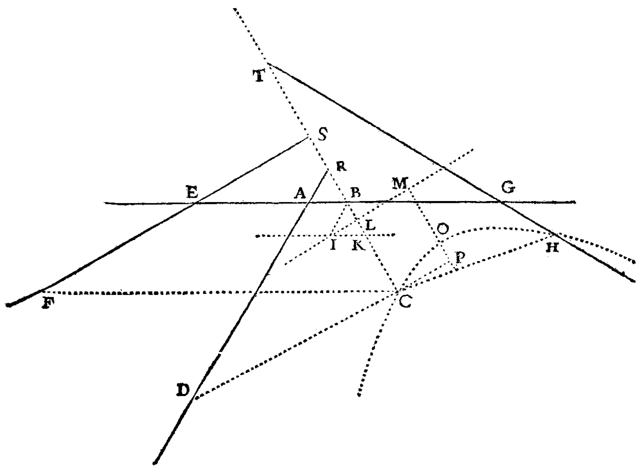


Figure 3.6 – The variant construction of the hyperbola. [Descartes, 1637c, 331].

conic section in this case is an ellipse, and from Apollonius we know that the points of the ellipse satisfy the fundamental property which I indicate in my own notation as

$$v^2 = ru - \frac{r}{d}u^2, \tag{3.23}$$

with u the abscissa measured from the vertex N along the diameter IL, v the corresponding ordinate, r the latus rectum and d the latus transversum. Descartes explicitly calculated the right hand side of (3.23) for his specified data as follows. We have

$$u = NL = NM - MI + IL = d - IM + \frac{a}{z}x,$$

and for IM, r and d we take the values from table 3.2, thus

$$IM = \frac{aom}{2pz}, \tag{3.24}$$

$$\text{the latus rectum } r = \frac{z}{a}\sqrt{o^2 + 4mp}, \tag{3.25}$$

$$\text{and the latus transversum } d = \frac{ma}{pz}\sqrt{o^2 + 4mp}. \tag{3.26}$$

Substitution of these values for u, r and d in (3.23) yields after some calculation

$$v^2 = m^2 + ox - \frac{p}{m}x^2.$$

Thus

$$v = \sqrt{m^2 + ox - \frac{p}{m}x^2}. \tag{3.27}$$

Combining this with (3.21) we obtain $v = LC$. Descartes concluded that LC is an ordinate of the ellipse which he had constructed, with corresponding abscissa NL. So the constructed ellipse possesses the required property in the sense that all its points are solutions of the Cartesian Pappus problem.

3.2.3 The numerical example

We now turn to the numerical example with which Descartes finished his discussion of the four-line Pappus problem.[18] For this particular case I will also compute the second conic section which belongs to the solution but which Descartes did not mention. This numerical example is important because we will see that it determined the structure of Descartes's general exposition of the four line Pappus problem to some extent.

Descartes chose as his numerical example $EA = 3$, $AG = 5$, $AB = BR$, $BS = \frac{1}{2}BE$, $BG = BT$, $CD = \frac{3}{2}CR$, $CF = 2CS$, $CH = \frac{2}{3}CT$ and $\angle ABR = 60°$. From these data we can deduce the given angles θ_i in the Pappus problem as:

$$\theta_1 = \angle CBG = 60°, \qquad \theta_3 = \angle CDA = \arcsin\left(\frac{1}{\sqrt{3}}\right), \tag{3.28}$$

$$\theta_2 = \angle CFE = 30°, \qquad \theta_4 = \angle CHG = \arcsin\left(\frac{3}{4}\right). \tag{3.29}$$

[18] [Descartes, 1637c, 333-334].

For the angles between the ℓ_i we find

$$\angle BES = 30^\circ, \qquad \angle EAR = 120^\circ, \qquad \angle EGT = 30^\circ. \tag{3.30}$$

Descartes supposed the point C at the position shown in figure 3.5. For the four distances, one finds:

$$\begin{aligned} CB &= y, \\ CF &= x + 3 + 2y, \\ CD &= \frac{3}{2}x + \frac{3}{2}y, \\ CH &= \frac{10}{3} - \frac{2}{3}x + \frac{2}{3}y. \end{aligned}$$

By substitution of these expression in $CB \cdot CF = CD \cdot CH$, Descartes produced the equation

$$y^2 = 2y + 5x - xy - x^2, \tag{3.31}$$

and by solving this equation for y and taking the positive root, he found

$$y = 1 - \frac{x}{2} + \sqrt{1 + 4x - \frac{3}{4}x^2}. \tag{3.32}$$

If we put with Descartes $z = 1$ we can compute his algebraic quantities defined above (see page 88) as $b = 1, c = \frac{3}{2}, d = \frac{1}{2}, h = 3, e = 2, f = 1, l = 5, g = \frac{2}{3}$ and hence, if we substitute these quantities in the equations (3.14), (3.15), (3.18), (3.19), we find $m = 1, n = \frac{1}{2}, o = 4, p = -\frac{3}{4}$. Descartes stated $m = 1$, $o = 4$ but also $p = \frac{3}{4}$, which differs from the computed value of p by a minus sign.[19]

According to the construction, $BK = m = 1$ and $KL = \frac{n}{z}x = \frac{1}{2}AB$. Since IK is parallel to AB it follows that $\angle IKL = 60^\circ$. Because L bisects BK, it follows that $\angle IKL = 30^\circ$. Hence, $\angle ILK$ is a right angle and thus the ordinates are perpendicular to the diameter. From the Pythagorean theorem, it follows that $IL = \frac{\sqrt{3}}{2}x$, and thus $a = \frac{\sqrt{3}}{2}$. We observe that $a^2m = pz^2 = \frac{3}{4}$ and thus the solution curve is a circle. The circle has centre M at distance $IM = \frac{aom}{2pz} = \frac{4}{3}\sqrt{3}$ from M, and its radius is half of the latus transversum, hence the radius is equal to $\sqrt{\frac{19}{3}}$.

It turns out that the configuration of this numerical example agrees exactly with figure 3.1 of the *Géométrie* and also that the solution circle is exactly the circle depicted in figure 3.5. Also, the position where the point C was supposed at the outset in figure 3.1 turns out to be exactly on the circle. Furthermore, we note that the angles between the lines ℓ_i, the angles θ_i, the circle through three given points A, G and the point of intersection of ℓ_2 and ℓ_3 can all be constructed with ruler and compass. Descartes thus illustrated his discussion with an instance of the four line Pappus problem which was relatively easy to draw for the illustrator (i.e., Van Schooten).[20] Thus the figures employed in the *general*

[19] In the next section 3.3 I will discuss Descartes's use of plus and minus signs in more depth.
[20] [Warusfel, 2010, 240-241].

exposition of the Pappus problem in four lines were the ones which fit the *particular* case of the numerical example.

After having found the circle, Descartes declared that it is easy to examine all the other cases in the same way.[21] To me, it is not clear what he means by this statement. It might refer to other instances of the four line Pappus problem, that is to say, with other configurations of lines. But it might also refer to other positions of the point C in the plane, with the same configuration of lines and angles as in figure 3.1. Above, we have seen that Descartes was well aware that the position of the point C in the plane with respect to the four given lines affected the expression of d_i. Maronne carried out the same analysis as above, but assumed that the point C is in the triangle with vertices A, G and the point of intersection of AD and GH, and thus he obtained the equation of the second conic section in the solution.[22] We note that in this case the orientation of y is reversed with respect to the case above, since CB is above the line AG. We will repeat his construction for future reference. Although Maronne believed that Descartes knew about the second conic section when he wrote the *Géométrie*,[23] there is no unambiguous evidence that this was the case; on the contrary, there are good grounds for believing that Descartes did not know about it, as I will show in the next chapter.

The details are as follows. For the four distances d_i we find:

$$d_1 = CB = y, \tag{3.33}$$

$$d_2 = CF = x + 3 - 2y, \tag{3.34}$$

$$d_3 = CD = \frac{3}{2}x - \frac{3}{2}y, \tag{3.35}$$

$$d_4 = CH = \frac{10}{3} - \frac{2}{3}x - \frac{2}{3}y. \tag{3.36}$$

Inserting these expression in $CB \cdot CF = CD \cdot CH$ and solving this equation for y by taking the least positive root we find

$$y = \frac{4}{3} + \frac{1}{6}x - \sqrt{\frac{16}{9} - \frac{11}{9}x + \frac{13}{36}x^2}. \tag{3.37}$$

By comparing the coefficients of this equation with those of (3.20) we observe that $m = \frac{4}{3}$, $o = \frac{11}{9}$ and $p = \frac{13}{27}$. Hence, $o^2 < 4mp$ and the conic section is a hyperbola with diameter parallel to LC, and corresponding ordinates parallel to IL. I note that this hyperbola has to be constructed using the variant construction.

The two solutions, the circle and the hyperbola, are shown in figure 3.7. It is clear that the hyperbola in Descartes's figure 3.6 is unrelated to the hyperbola of figure 3.7 which belongs to the solution of the four line Pappus problem, in agreement with my conjecture that Descartes was unaware of the second conic section.

[21] "Et on peut facilement examiner tous les autres cas en mesme sorte", [Descartes, 1637c, 334].

[22] [Maronne, 2007, 72-73].

[23] [Maronne, 2007, 138,139].

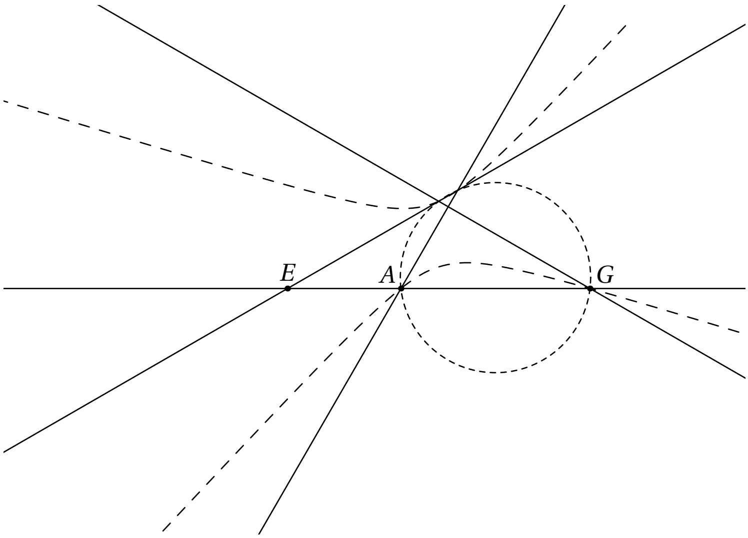


Figure 3.7 – The two conic section solutions (dashed) of the Cartesian Pappus problem.

3.3 Descartes's use of substitutions and of plus and minus signs

As we have seen above, Descartes introduced letters to denote positive quantities.[24] The letters actually denote line segments, for in the beginning of the *Géométrie*, he wrote that in solving a geometrical problem,

> one should name all the lines, those that are unknown as well as the others [that are known][25]

This use of only positive quantities makes Descartes's exposition complicated because he had to make many case distinctions, as we have seen above in his solution of the four line Pappus problem.[26] There is also a problem with substitutions, because a newly introduced quantity could become negative. For instance, if c and d are positive quantities, and we define $q = c - d$, then q can be negative, depending on the values of c and d. This problem has not yet been addressed by historians of mathematics but the following investigation will help to explain some passages in Descartes's reasonings which would otherwise cause difficulties.

[24] That Descartes used letters for positive quantities is the standard view among historians of mathematics, see for example [Bos, 2001, 304, 319] and [Maronne, 2007, 46]. Some mathematicans disagree, for instance [Warusfel, 2010, 244].

[25] [Descartes, 1637c, 300].

[26] Early modern mathematicians were familiar to such case distinctions. See for instance the discussion of the solution of the third degree equation by Cardano.

Recall the following difficulty in Descartes's solution of the four line Pappus problem in the *Géométrie*. From the equation

$$y = m - \frac{n}{z}x + \sqrt{m^2 - \frac{2mn}{z}x + \frac{n^2}{z^2}x^2 + \frac{bcfglx - bcfgx^2}{(ez^3 - cgz^2)}}, \tag{3.17}$$

and the two substitutions

$$o = \frac{-2mn}{z} + \frac{bcfgl}{ez^3 - cgz^2} \tag{3.18}$$

and

$$\frac{p}{m} = \frac{n^2}{z^2} - \frac{bcfg}{ez^3 - cgz^2}, \tag{3.19}$$

Descartes concluded

$$y = m - \frac{n}{z}x + \sqrt{m^2 + ox - \frac{p}{m}x^2}, \tag{3.20}$$

whereas the modern reader would expect

$$y = m - \frac{n}{z}x + \sqrt{m^2 + ox + \frac{p}{m}x^2}. \tag{3.38}$$

This curious minus sign has attracted the attention of historians of mathematics, and they have given explanations which involve a change of sign in Descartes's printed text, either in the definition of $\frac{p}{m}$ (thus in formula (3.19)) or in the final equation (thus in formula (3.20)).

Tannery added a minus sign to the left hand side of the formula (3.19), since he was convinced that Descartes had accidentally omitted the minus sign, and calculated further as if the minus sign had been there.[27] His interpretation has been followed by Bos and Maronne.[28] Bos stated that Descartes assumed all letters to be positive quantities but that he was aware of the fact that substitutions like (3.19) could produce negative quantities. Bos argued that Descartes's terminology was not sufficiently developed to distinguish between the positivity (or negativity) of a quantity and the sign that precedes the quantity.

Smith and Latham, on the other hand, decided to change the minus sign of $\frac{p}{m}$ in (3.20) to a plus sign, but they did not give any explanation for the change of sign.[29] Their choice is inconvenient because many formulas in the following pages in their translation do not match the originals in the *Géométrie*. Warusfel also decided to change the sign in equation (3.20) rather than in the formula (3.19), and he even concluded that the quantities used by Descartes could have negative values.[30]

I find these interpretations unsatisfactory because they all require a change of sign in the original printed text, so we would have to assume that Descartes erred or that the minus sign in equation (3.20) was a misprint. The possibility of a misprint seems unlikely, because the minus sign is repeated throughout the rest of the exposition. The assumption

[27] [Descartes, 1902, 399].

[28] [Bos, 2001, 319-320] and [Maronne, 2007, 65].

[29] [Descartes, 1954, 63].

[30] [Warusfel, 2010, 244].

that Descartes erred seems unlikely if we take into account how Van Schooten dealt with this particular minus sign. In his Latin editions of 1649 and 1659, he followed the original text of the *Géométrie* and did not change any of the plus and minus signs in (3.19) and (3.20).[31] Elsewhere in the Pappus problem, Van Schooten did correct an error in signs made by Descartes in his *Geometria (1659)*.[32] This is a first indication that the minus sign was placed deliberately by Descartes.

Another indication, which has not hitherto been noticed, is in Descartes's numerical example of the four line Pappus problem which I have discussed above. From the numerical data $b = 1, c = \frac{3}{2}, d = \frac{1}{2}, h = 3, e = 2, f = 1, l = 5, g = \frac{2}{3}, z = 1$ and the formulas $2m = \frac{cfglz - dekz^2}{ez^3 - cgz^2}$, and (3.19) above, the modern reader would conclude $m = 1, p = -\frac{3}{4}$, but Descartes stated $m = 1,\ p = \frac{3}{4}$. Again there is a difference in the minus sign at the same place in the argument. I will now propose a new explanation of the substitutions which does not involve changes in the original text of Descartes and the translations of Van Schooten and which explains their use of the plus and minus signs in these substitutions. I begin with an easy example.

Suppose that Descartes or Van Schooten had already introduced quantities c and d, en then introduced a substitution as $q = c - d$. Then they meant, in modern terms, that $q = |c - d|$, so they took the difference in absolute value and q is therefore positive or possibly zero, but never negative. The following substitution rules had to be used:

- For $c - d$ in an algebraic expression, substitute $+q$ if $c \geq d$ and $-q$ if $c < d$.
- For $-(c - d)$ in an algebraic expression, substitute $-q$ if $c \geq d$ and $+q$ if $c < d$.

Van Schooten introduced the rule in his own words in the course of comment N on Book I of the *Géométrie* (he used a instead of our q). The context of the passage does not concern us here. Van Schooten said:

> $z^2 = (+c - d)z + cd$; if we assume in this equation c to be greater than d, so that the excess is on the part of c with the sign +, and we interpret $+c - d$ by $+a$, and $+cd$ by $+b^2$, we have the same equation as before: $z^2 = az + b^2$. ...But if we put d greater than c, such that the excess is on the part of d with sign −, and we interpret $+c - d$ by $-a$, and $+cd$ by b^2; an equation of the second form will appear: $z^2 = -az + b^2$.[33]

[31] [Descartes, 1649, 31] and [Descartes, 1659, 27].

[32] Van Schooten changed the text of the translation compared to the original text of Descartes and the 1649 translation, and he also changed the text of his comment. I discuss this error in signs in more detail on page 124 below.

[33] "$z^2 = (+c - d)z + cd$. In qua aequatione si statuamus c majorem esse quam d, ita ut excessus sit penes c cum signo +, atque $+c - d$ interpretemur per $+a$, et $+cd$ per $+b^2$, habemimus eandem aequationem, quam prius, nimirum $z^2 = az + b^2$. ...At vero si ponamus d majorem quam c, ita ut excessus sit penes d cum signo −, atque $+c - d$ interpretemur per $-a$, et $+cd$ per b^2; prodibit aequatio secundae formae: nimirum $z^2 = -az + b^2$." [Schooten, 1649c, 180] and [Schooten, 1659c, 166].

In general, suppose that Descartes and Van Schooten wrote a substitution as $q = f(c, d, \ldots)$ where $f(c, d, \ldots)$ is any rational function of quantities $c, d, \ldots$, which function can take positive and negative values in modern terms.[34] Then they meant in modern terms $q = |f(c, d, \ldots)|$, so their new quantity q is positive, or possibly zero, but never negative. The substitution rules now became as follows:

In any algebraic expression one can put instead of $f(c, d, \ldots)$ the symbol $+q$ if in modern terms $f(c, d, \ldots) \geq 0$ and $-q$ if in modern terms $f(c, d, \ldots) < 0$. Instead of $-f(c, d, \ldots)$, one can put $-q$ if in modern terms $f(c, d, \ldots) \geq 0$ and $+q$ if in modern terms $f(c, d, \ldots) < 0$.

The $c, d, \ldots$ can have given numerical values or they can be undetermined variables; in the latter case it has to be decided whether $f(c, d, \ldots) > 0$ or $f(c, d, \ldots) < 0$ in order to determine the sign of the substituted quantity. Descartes made this decision by means of the concrete figure and the numerical example which he used in his general exposition.

I will now apply this new interpretation to the substitutions which define the positive quantities o and $\frac{p}{m}$ in the solution of the four line Pappus problem in the *Géométrie*. Descartes's definitions of these quantities can now be rendered in modern notation as follows:

$$2m = \left| \frac{cfglz - dekz^2}{ez^3 - cgz^2} \right|, \tag{3.39}$$

$$\frac{2n}{z} = \left| \frac{dez^2 + cfgz - bcgz}{ez^3 - cgz^2} \right|, \tag{3.40}$$

$$o = \left| \frac{-2mn}{z} + \frac{bcfgl}{ez^3 - cgz^2} \right|, \tag{3.41}$$

and

$$\frac{p}{m} = \left| \frac{n^2}{z^2} - \frac{bcfg}{ez^3 - cgz^2} \right|. \tag{3.42}$$

The quantities o and p have to be substituted into the equation

$$y = m - \frac{n}{z}x + \sqrt{m^2 - \frac{2mn}{z}x + \frac{n^2}{z^2}x^2 + \frac{bcfglx - bcfgx^2}{(ez^3 - cgz^2)}}. \tag{3.43}$$

In the numerical example of the Cartesian Pappus problem, we have $b = 1, c = \frac{3}{2}, d = \frac{1}{2}, h = 3, e = 2, f = 1, l = 5, g = \frac{2}{3}$, and $z = 1$, and we find from these formulas $m = |1| = 1, n = |\frac{1}{2}| = \frac{1}{2}, o = |4| = 4$, and $p = |-\frac{3}{4}| = \frac{3}{4}$ as in the text of the *Géométrie*. According to the substitution rules, the signs of o en p are $+$ and $-$, as in the text, and Descartes found

[34] In Books II and III, Descartes employs "less than nothing" to describe a negative value, [Descartes, 1637c, 326, 372], and he calls negative roots of an equation "false roots". On [Descartes, 1637c, 372] he discusses an equation with a "false root" 5, meaning -5 in modern terms. Other examples are: three "false" roots $2, 3, 4$ (meaning $-2, -3, -4$) in [Descartes, 1637c, 374]; three "false" roots $5, 6, 7$ in [Descartes, 1637c, 376]. In the *Géométrie*, algebraic quantities are always positive with the sole exception of [Descartes, 1637c, 384] where Descartes put a quantity p in one case equal to -4; in all other cases, the quantities p, q etc. on this and the following pages are positive, and the discussion of signs by Descartes is also in agreement with my interpretation.

$$y = 1 - \frac{x}{2} + \sqrt{1 + 4x - \frac{3}{4}x^2}. \tag{3.44}$$

I now turn to the substitution in Descartes's more general presentation. We have already seen before that the general presentation is structured according to figure 3.1 and the numerical example. In the figure of the Cartesian Pappus Problem 3.1, we have $EB = k + x$, and not for example $EB = x - k$.[35] Thus, Descartes uses in his general solution the distance $CF = \frac{ezy+dek+dex}{z^2}$, and not, for example, $CF = \frac{dex-ezy-dek}{z^2}$ which he would have obtained from $EB = x - k$.

We can assume that the same principle was used for the substitutions. Because for the numerical example of the figure, $\frac{-2mn}{z} + \frac{bcfgl}{ez^3-cgz^2} > 0$ and $\frac{n^2}{z^2} - \frac{bcfg}{ez^3-cgz^2} < 0$, Descartes must have assumed this to be true for his general presentation, and therefore the sign of o is + and the sign of $\frac{p}{m}$ is −. Therefore Descartes wrote his general formula as $y = m - \frac{n}{z}x + \sqrt{m^2 + ox - \frac{p}{m}x^2}$. In other words, it is no accident that the signs in his general formula are the same as in his numerical example $y = 1 - \frac{x}{2} + \sqrt{1 + 4x - \frac{3}{4}x^2}$.

Descartes realized that the sign of $\frac{p}{m}$ was not necessarily a minus sign in other cases than what I have called the Cartesian Pappus problem. He wrote that[36]

if $\frac{p}{m}$ is equal to 0	then the curve is a parabola,
if $\frac{p}{m}$ has the sign +	then the curve is a hyperbola,
if $\frac{p}{m}$ has the sign −	then the curve is an ellipse or circle.

The concern with positive quantities can also be seen in Van Schooten's Comment B on Book II of the *Géométrie*.[37] In his solution of the four line Pappus problem, Descartes stated that in the equation

$$y^2 = \frac{(cfglz - dekz^2)y - (dez^2 + cfgz - bcgz)xy + bcfglx - bcfgx^2}{ez^3 - cgz^2},$$

the quantity ez has to be supposed greater than cg so the denominator is positive. If ez is less than cg, he prescribed that all the signs + and − have to be changed.[38] Thus in such a case, the equation has to be

$$y^2 = \frac{(-cfglz + dekz^2)y - (-dez^2 - cfgz + bcgz)xy - bcfglx + bcfgx^2}{-ez^3 + cgz^2}.$$

Apparently, Descartes did not like a negative denominator. In his comment, Van Schooten set

$$y = \frac{fe - dk}{d - e}$$

[35] Above we have seen that the expression for EB depends on the position of C in the plane and is not always equal to $k + x$. Figure 3.2 on page 90 displays a point C' such that $C'B = x - k$.

[36] Descartes made this distinction in [Descartes, 1637c, 328].

[37] [Schooten, 1649c, 193-194] and [Schooten, 1659c, 178].

[38] [Descartes, 1637c, 325-326].

with $d < e$, so that the denominator is negative. He multiplied by $d - e$ and obtained $y(d - e) = fe - dk$, and rearranged the terms as

$$-fe + dk = -dy + ey,$$

from which he concluded

$$y = \frac{-fe + dk}{-d + e}.$$

Thus Van Schooten showed by successive algebraic manipulations that $y = \frac{-fe+dk}{-d+e}$ is equivalent to $y = \frac{fe-dk}{d-e}$, which has a positive denominator. For a modern reader who is used to calculate with negative quantities, the comment may seem superfluous. The modern reader should realize that Descartes and Van Schooten used plus and minus signs in a way which is confusing for modern readers who are trained in modern algebra. Their procedures belong to an early stage in algebraic computation, when more effective ways of dealing with negative quantities still had to be developed.

CHAPTER 4

Discussions of the four line Pappus problem up to the *Geometria* of 1649

Van Schooten's Latin *Geometria (1649)* and his commentaries came from the press in the summer of 1649[1], twelve years after the publication of the original French edition of Descartes's *Géométrie*. In these twelve years, the ideas of Descartes were criticised by other scholars. Initially, in 1637, Descartes intended to collect the remarks and criticisms in order to publish them together with his own responses, as his words in the *Discours de la méthode* show.[2] However, this project was abandoned and thus the objections and responses were not published.

I will now investigate some relevant criticisms that were made by one or more mathematicians in the years 1637–1648, in connection with Descartes's solution of the four line Pappus problem.[3] Since Descartes and Van Schooten stayed in contact during these years, it is likely that they discussed these criticisms, and such discussions may have influenced Van Schooten when working on his commentary on the *Géométrie*. The relation between Descartes and Van Schooten will be studied in subsection 4.4.

[1] Descartes stated that Van Schooten's *Geometria (1649)* had been printed in a letter to Carcavi of 17 August 1649, [Descartes, 1903, 392] and [Descartes, 1963, 252].

[2] "Mais ie seray bien ayse qu'on les [i.e., the *Discours de la méthode* and the essays] examine, & affin qu'on en ait d'autant plus d'occasion, ie supplie tous ceux qui auront quelques obiections a y faire, de prendre la peine de les enuoyer a mon libraire, par lequel en estant auerti, ie tascheray d'y ioindre ma response en mesme tems", [Descartes, 1637a, 75].

[3] These criticisms have also been studied in some detail by [Maronne, 2007].

Throughout the period 1637–1648, the main criticisms of Descartes's solution of the Pappus problem were made by Gilles Personne de Roberval (1602–1675). Roberval was a French mathematician who came to Paris in 1628 and immediately came in contact with Marin Mersenne and his circle. From 1634 until his death in 1675, Roberval occupied the chair of mathematics at the Collège de France in Paris.

4.1 Criticisms in 1638

The first criticisms were made less than half a year after the publication of the *Géométrie*. According to a letter of Descartes to Mersenne dated 31 March 1638, Roberval had told Mersenne that he wished that Descartes had explained in the *Géométrie*

> To which locus the obtained equation belongs.[4]

In his response to Mersenne,[5] Descartes referred Roberval to his discussion of the relationship between equation (3.21) and the three conic sections in pages 326-332 of the *Géométrie*, which we have summarized above. Descartes declared that the whole matter which Roberval would like to know had already been explained in his book. Since we do not know the actual wording of Roberval's criticism, we cannot check whether Descartes's response was adequate.[6]

In the same letter, Descartes also discussed the way in which he presented his solution to the four line Pappus problem. He asserted that he had only given the construction of the problem, and the demonstration of its correctness, but that he had omitted part of the analysis. By analysis he must have meant the way to determine the necessary data for the Apollonian construction from the algebraic equation (3.21). According to Descartes, he was misunderstood by other mathematicians who believed that he had only provided the analysis, without the construction and the demonstration. He compared his own role in the construction with that of an architect, who only prescribes what should be done, without performing the manual work which could be left to the bricklayers. He claimed that the mathematicians did not recognize his demonstration as such because he used the language of algebra, which allowed him to write down his reasoning in a far more succinct way than the mathematicians who did not use algebra were able to do.

Descartes went on to say that he had intentionally omitted part of the analysis. The reason was his suspicion that if the whole analysis were included, other mathematicians would claim that they already knew this matter for a long time and that Descartes's ideas were not original after all. Since part of the analysis was missing, the mathematicians would have to figure out the missing part by themselves, and in trying to do so, they would be confronted with their own ignorance.

[4]"Pour ce qui est de connoistre a quel lieu l'equation faite appartient, que vous dites que Mr de Roberval eust desire que j'eusse mis en ma *Géométrie*", Descartes to Mersenne, 31 March 1638, [Descartes, 1898, 83-84], [Descartes, 1939, 214].

[5]Descartes to Mersenne, 31 March 1638, [Descartes, 1898, 83] and [Descartes, 1939, 213-214].

[6]If Roberval somehow hinted at the second conic section in the solution, Descartes may have misunderstood his question.

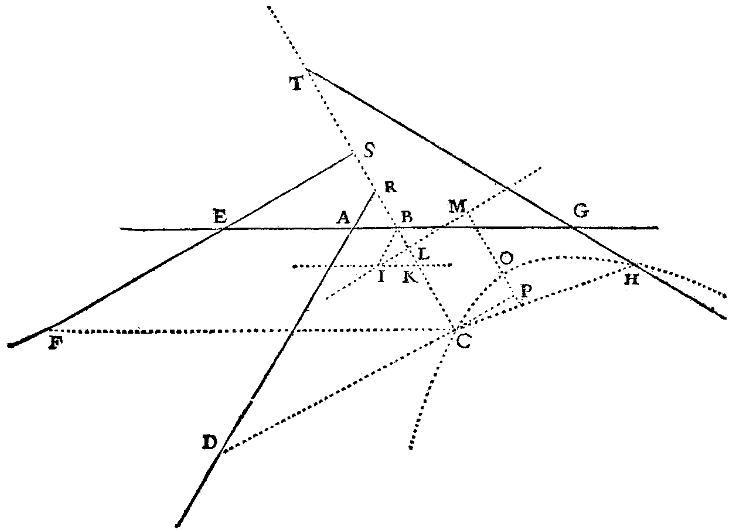


Figure 4.1 – The variant construction of the hyperbola. [Descartes, 1637c, 331].

4.2 Criticisms in 1642

The Pappus problem turned up for a second time in a letter of Descartes to Mersenne of 13 October 1642.[7] In this letter, Descartes reacted to criticisms by an anonymous mathematician on the illustrations of both the *Dioptrique* and the *Géométrie*. Because Descartes sent his response to Mersenne, it is very likely that these remarks had been made by a Frenchman. Two of the remarks refer to figures in the *Géométrie*, but only the first remark is relevant to us because it concerns the Pappus problem. The figure under consideration is the one of page 331 of the 1637 edition, which has been reproduced in the previous chapter as figure 3.6, and is shown here as figure 4.1. It displays a hyperbola as the solution of the four line Pappus problem, using the variant construction.[8]

Before turning to the criticism by the anonymous mathematician and the response to it by Descartes, I mention two possible objections that could be made to figure 4.1. In the first place, this figure displays only one branch of a hyperbola. In the classical Apollonian theory of conics, a hyperbola consists of one branch only, but the two branches of a hyperbola in the modern sense (that is, the two "opposite hyperbolas" of Apollonius) both belong to the solution of the four line Pappus problem.

The second possible objection concerns the position of the hyperbola relative to the four given lines of the problem. It can be seen without any calculation that four of the points of intersection of the given lines ℓ_i belong to the solution. Thus, point A is a part

[7] [Descartes, 1899, 583] and [Descartes, 1951, 224-225].

[8] For more information on the variant construction I refer the reader to page 95.

of the solution of the problem, since in A the distance to both the line AD and the line AB is equal to zero, and hence the condition $d_1 d_2 = d_3 d_4$ is fulfilled. The same holds for the point G, and for the points of intersection of GH with EF and of EF with AD. But in figure 4.1, the hyperbola does not pass through any of these four points, but instead, it intersects GH in point H, which is clearly not a point of the solution.

Now I turn to the anonymous critic of figure 4.1, who must have made his criticisms in a letter to Mersenne that is now lost. Descartes did not cite the criticism but we do get some idea of its nature by reading his reaction:

> And wanting, on page 331, that one indicates all the points where the straight line intersects the hyperbola, is wanting something impertinent, since these intersections do not serve any purpose in this matter; and one can never trace a hyperbola completely, for it is a figure without end.[9]

From this response the content of the criticism is not entirely clear, but one may guess that Descartes responded to one or both of the objections to figure 4.1 which have been mentioned above. The "straight line" is probably the line $\ell_4 = GH$, which intersects the hyperbola in the figure only in H. If this hyperbola is supposed to be a solution for the Pappus problem, it should intersect ℓ_4 not in H, but in two other points, namely in G and in the intersection point with EF. Descartes waved the remark aside by stating that "these intersections do not serve any purpose in this matter," but this response may well betray his own lack of understanding. It is possible that the anonymous mathematician asked for all points of intersection because he realized that the second branch of the hyperbola was missing but did not wish to reveal this fact to Descartes. Descartes's response that a hyperbola is a "figure without end" which can never be completely traced, is inadequate and reveals a condescending attitude on his part.

The identity of the critic is not mentioned in the letter, but following Maronne he can be identified as Roberval because the criticisms are in line with the objections which Roberval made in 1648, and which will be discussed below.[10]

4.3 Criticisms in 1646

Early in 1646, Roberval criticised Descartes's treatment of the Pappus problem again. In a letter to Mersenne, dated 2 March 1646, Descartes wrote:

> M. de Roberval says that I have not solved the locus of Pappus, and that there is another sense than the one I gave. About this I implore you very politely to

[9] "Et de vouloir, page 331, qu'on marquast tous les poins ou la ligne droite coupe l'hyperbole, c'est vouloir une chose impertinante, a cause que ces intersections ne servent de rien au sujet; & l'hyperbole estant une figure sans fin on ne la peut jamais tracer toute entière." Descartes to Mersenne, 13 October 1642, [Descartes, 1899, 583] and [Descartes, 1951, 224-225].

[10] [Maronne, 2007, 111-112].

> ask him, on my behalf, what this other sense is, and that he takes the trouble to put it in writing, so that I can better understand it.[11]

The main problem here is to determine what Roberval meant by "another sense". I will first discuss the interpretation by Tannery and Maronne, and then present my own alternative interpretation.

Tannery related the quoted passage to a later letter by Descartes to Mersenne of 12 October 1646.[12] In this letter Descartes stated that Roberval's remark about "another sense" concerns grammar, not geometry. Tannery therefore suggested that the remark refers to the Pappus problem in more than four lines, and that Roberval had the interpretation of an obscure passage in Book 7 of the *Collection* of Pappus in mind. On page 305 of the *Géométrie*, Descartes cited Pappus,[13] and on page 307 he gave a rather free translation of this passage. Descartes stated that the ancient Greek geometers who studied the Pappus problem in five or more lines, had imagined a curved line

> which they had shown to be useful here, but which seemed the most manifest one, and which was nevertheless not the first one.[14]

In the *Géométrie*, Descartes also solved the Pappus problem for five given lines, of which four are equidistant parallel lines and the fifth line is perpendicular to these four. From his own solution, he identified a third degree curve (later called a Cartesian parabola), as the "most manifest" curve that was mentioned in Commandino's translation of Pappus. Descartes then used this Cartesian parabola in his construction of the roots of fifth and sixth-degree equations in Book III.[15]

Tannery claimed that a more common seventeenth century interpretation of the Latin passage in Commandinus's translation is that the ancients "had invented a curved line of which they showed the utility, but which seemed to be neither the first nor the simplest."

[11] "M. de Roberval dit que ie n'ay pas resolu le lieu de Pappus, & qu'il a un autre sens que celuy que ie luy ay donné. Sur quoy ie vous supplie tres-humblement de luy vouloir demander, de ma part, quel est cet autre sens, & qu'il prenne la peine de le mettre par écrit, afin que ie le puisse mieux entendre." Descartes to Mersenne, 2 March 1646, [Descartes, 1899, 363] and [Descartes, 1960, 17].

[12] [Descartes, 1899, 365-366].

[13] The passage is part of the following longer quotation: "Quod si ad plures quam quatuor, punctum continget locos non adhuc cognitos, sed lineas tantum dictas; quales autem sint, vel quam habeant proprietatem, non constat: earum unam, neque primam, et quae manifestissima videtur, composuerunt ostendentes utilem esse."[Descartes, 1637c, 305]. Descartes's citation is taken from Commandino's Latin translation of the work of Pappus, see [Pappus, 1588, 165].

[14] "Les anciens en avaient imaginé une qu'ils montraient y estre utile, mais qui sembloit la plus manifeste, & qui n'étoit pas toutefois la première", [Descartes, 1637c, 307]. The same passage was translated by Alexander Jones in his study on Book 7 of the *Collection*. It is part of the following longer passage: "If they (are drawn) (...) onto more than four (lines), the point will touch loci that are as yet unknown, but just called 'curves', and whose origins and properties are not yet (known). They have given a synthesis of no one, not even the first and seemingly the most obvious of them, or shown it to be useful." [Pappus, 1986, 120]. The two translations are different: in Commandino's translation, the ancient Greeks had actually found a solution curve, whereas the translation of Jones states that they had not found a solution at all. We may conclude that the erroneous translation of Pappus by Commandino was the basis of Descartes's idea that the ancients had found a solution curve, and also of Descartes's idea that there was one solution curve which was somehow more prominent than the others.

[15] See [Descartes, 1637c, 408-411]. The prominent role which Descartes attributes to this curve is based on the erroneous translation by Commandino, see the previous footnote.

According to Tannery, the existence of these two different interpretations of the Latin passage is the reason why Roberval states that the Pappus problem had "another sense".[16]

I find it difficult to see how these two different textual interpretations can lead to "another sense" of the problem, which in my opinion is most naturally interpreted as a difference in mathematical meaning. Thus I disagree with Tannery, and also with Maronne who follows Tannery in this matter.[17] A more plausible interpretation is suggested by a letter of Roberval which he probably wrote in September 1646, and in which he expresses some criticism of Descartes.[18] The letter was addressed to Cavendish, but Roberval let the letter circulate in Paris so Mersenne was well aware of Roberval's opinion. In autumn 1646 Mersenne had offered Descartes to send him Roberval's letter, but Descartes told Mersenne not to send him any of Roberval's writings because they were not even worth the postage.[19] It is not clear whether Mersenne did in fact send Roberval's letter to Descartes.[20]

Roberval's letter suggests a more plausible interpretation of the "other sense". Roberval noted that one can

> reproach him [Descartes] for the same with respect to the locus of three and four lines, as Apollonius reproached Euclid on the same subject.[21]

Roberval moreover stated his intention to write a letter to Mersenne with a more explicit explanation of his reproach. Such a letter has not been preserved; it may never have been written at all.

How did Apollonius reproach Euclid? In the preface of Book I of the *Conics*, Apollonius blamed Euclid who

> did not work out the syntheses of the locus with respect to three and four lines, but only a chance portion of it, and that not successfully.[22]

In the *Collection* of Pappus, words to the same effect were included,[23] and these were cited by Descartes in the first book of the *Géométrie*.[24] So, by referring to the ancient Greeks, Roberval was blaming Descartes that his solution of the locus of three and four lines (i.e., the Pappus problem in three and four lines) was not complete. This is a poignant remark, because Descartes used his solution of the four line Pappus problem as a demonstration of the power of his own method. Now Roberval accused Descartes of making the same errors as some ancient Greeks, and thus he implicitly suggested that Descartes was perhaps not able to surpass the Greeks after all, and certainly not in the Pappus problem in three and four lines.

[16] [Descartes, 1899, 364-366].

[17] [Maronne, 2007, 113-115].

[18] The letter lacks a date; the date of September 1646 is proposed by Adam and Tannery, [Descartes, 1899, 502] and [Descartes, 1960, 174].

[19] Descartes to Mersenne, 12 October 1646, [Descartes, 1899, 525-526] and [Descartes, 1960, 192].

[20] Adam and Tannery assert that the letter was sent to Descartes by Mersenne in October 1648, [Descartes, 1899, 502], but in my opinion their evidence is not convincing.

[21] "On luy peut faire le mesme reproche touchant le lieu ad très & quatuor lineas qu'a faict, sur le mesme suiect Apollonius à Euclide." [Descartes, 1899, 507-508] and [Descartes, 1960, 178].

[22] [Heath, 2006, 129].

[23] [Pappus, 1986, 118-120].

[24] [Descartes, 1637c, 304].

In my opinion, Roberval's reproach is most naturally explained by assuming that he knew about the second conic section in the four line Pappus problem already in 1646, if not earlier. If Roberval considered Descartes's solution as incomplete due to the absence of the second conic solution, we can interpret the "other sense" of the Pappus problem as a reference to the missing conic section in Descartes's solution in the Pappus problem in four lines. For different positions of point C in the figure can lead to equations of two different conic sections. Therefore I disagree with Maronne's opinion that by 1646, Roberval was not able to understand the Pappus problem in depth. Maronne claims that in 1646, Roberval was not convinced of Descartes's solution but at the same time unable to put him in serious trouble mathematically, and that Roberval only learned about the second conic section in 1648 through a new geometric solution of the four lines Pappus problem by Blaise Pascal (1613–1662).[25] If my interpretation is correct, then Roberval must have used the second conic section as a source of criticisms on Descartes, without giving him access to the full details of the solution. Perhaps Roberval hoped to continue for some time in this way, with nasty criticisms which Descartes could not easily refute. We will see below that Roberval still used the same policy in 1649.

4.4 The year 1648: Van Schooten enters the scene

In the spring of 1648, when Van Schooten was preparing his Latin translation of the *Géométrie*, he received a letter by Descartes and one by Mersenne dealing with the four line Pappus problem. In this section I will discuss the content of both letters, and by a reconstruction of the events of the spring of 1648, I will show that the letter of Mersenne to Van Schooten was the reason why Descartes intervened.

4.4.1 Descartes's involvement in the *Geometria (1649)*

In March/April 1648, Descartes sent Van Schooten a letter which is to a large extent devoted to the Pappus problem.[26] The letter concerns a remark on figure 4.1 made by an anonymous mathematician whom Descartes called "N", and who will be identfied as Roberval below. The criticisms by N have not come down to us otherwise, so they have to be reconstructed from Descartes's response. Descartes defended himself against N's criticisms, although he had to admit that his own presentation in the *Géométrie* was obscure. At the end he proposed a note in Latin to Van Schooten which could be included in the Latin *Geometria* in order to avoid an erroneous interpretation of the figure.

The letter is important because it shows that Descartes was involved in the production

[25] [Maronne, 2007, 115].

[26] This letter is dated by Adam-Tannery in September 1639, see [Descartes, 1898, 574-582]. This dating was altered by Adam-Milhaud, who assigned the interval end of 1647 – end of 1648 as date to this letter, [Descartes, 1941, 315 and 353-356]. In a biographic notice on Jan Jansz. Stampioen de Jonghe, Jeroen van de Ven suggested that the letter should be dated 1648 [Descartes, 2003, 302]. This suggestion has been worked out by Sébastien Maronne. He claims that this letter should be dated March/April 1648. For a complete survey of the different datings and the arguments for setting the date on March/April 1648, see [Maronne, 2006].

of Van Schooten's Latin edition and therefore I will discuss it in some detail. First, I will describe Descartes's reaction on the remarks of N, and after that I will present the note which Descartes included for Van Schooten. Then I will discuss the way in which Descartes was involved with the Latin *Geometria* in general.

Descartes began by saying the following on N's remark:

> It is impertinent but not entirely wrong. For it is well known that, the same lines being given and the question not being changed, the locus cannot be together a circle and a hyperbola. Nor does one need great learning to know that the curved line needs to pass in this example through the four points of intersection as he [i.e., N] remarks.[27]

The passage shows that N's remarks concerned figure 4.1 of the *Géométrie*, in which Descartes explained what I have called the variant construction[28] of a hyperbola. From Descartes's reaction we infer that N thought the figure to be erroneous, on the grounds that the hyperbola should pass through four points of intersection of the given lines. Since this letter is our only source, it is not known whether N provided other arguments to support his claim. Because N's criticism resembles that by Roberval of 1642, Adam and Tannery identified N as Roberval and this identification has been followed by Maronne.[29]

In the reaction which we have quoted, Descartes mentioned two arguments why N's criticism was not completely unfounded. In the first place he stated that the solution of the four line Pappus problem cannot consist of both a circle and a hyperbola at the same time. Consequently, Descartes thought that the hyperbola in figure 4.1 is incorrect in the sense that the solution of the Pappus problem is a circle for the configuration of lines and angles for which the figure was drawn. I conclude that he was unaware of the hyperbola in the solution of the Cartesian Pappus problem which we have discussed in Section 3.2.3 above. Concerning the four line Pappus problem, Pappus himself wrote that the required point "will touch a conic section given in position".[30] Perhaps Descartes had inferred from this passage that the solution consisted of only one conic section or circle. In his second argument, Descartes admitted that the curve should pass through the four intersection points, and in the subsequent passage of his letter (not quoted here), he explained in detail to Van Schooten that these are the points A and G and the points of intersection of AD with EF and of GH with EF, since in these four points $CB \cdot CF = CD \cdot CH = 0$.

Descartes then went on to tell Van Schooten why he had included this figure in the *Géométrie*. His main reason was the pursuit of succinctness and therefore he had chosen

[27] "[E]lle est impertinente, encore qu'elle ne soit pas tout à fait fausse. Car on sçait bien que, les mesmes lignes droites estant posées & la question n'estant point changée, le lieu ne peut pas estre tout ensemble au cercle & à l'hyperbole. Et il ne faut pas aussi avoir grande science pour connoistre que la ligne courbe doit passer en cet exemple par les quatre intersections qu'il remarque." Descartes to Van Schooten, March/April 1648, [Descartes, 1898, 576] and [Descartes, 1941, 316].

[28] In the variant construction, the diameter of the hyperbola is parallel to LC and the corresponding ordinates are parallel to IL, in contrast to the standard construction where the diameter is along IL and ordinates along LC, see page 95.

[29] Recall that the anonymous author of 1642 had also been identified as Roberval by Tannery and Maronne, see page 110. [Descartes, 1898, 580] and [Maronne, 2007, 118].

[30] "Similiter punctum datam coni sectionem positione continget." [Pappus, 1588, 165].

to explain the four line Pappus problem on the basis of only one example, which I have called the Cartesian Pappus problem above. Descartes stressed that since the configuration of the four given lines stays the same, it is easier to understand the text. Furthermore, he stated that he had introduced the hyperbola only "after having given the true locus of this example, which is the circle".[31] This again shows that Descartes was not aware of the existence of the second conic section in the solution.

In order to avoid problems with the interpretation of figure 4.1, Descartes then proposed to Van Schooten the following solution for the Latin edition:

> Thus it seems to me that you should not at all put another figure there; since [in that case] one should also change the discussion, and the solution would be much more complicated. But you could place this note on page 331, or something similar [followed by the note in Latin which we will translate below].[32]

Because Descartes explicitly told Van Schooten to keep the same figure, it seems to me that his letter was an answer to a request that Van Schooten had made, asking how to deal with this figure. I will provide additional evidence for this conjecture below. Descartes gave two reasons for keeping the same figure: First, a change of figure would also lead to a change in the text, which Descartes did not want to change, and secondly, a change of figure would make the solution more complicated.

I conclude that Descartes intended figure 4.1 to illustratie the construction of a solution *in the hypothetical case* of a hyperbola with the variant construction. Descartes did not consider the displayed hyperbola as an actual solution of the Cartesian Pappus problem. In order to obtain the hyperbola with the variant construction as the actual solution, Descartes would have had to prepare a new figure for a configuration of given lines and angles different from that of figure 4.1. The mathematician N, however, wanted to interpret the hyperbola in figure 4.1 as an actual solution of the Cartesian Pappus problem.

To me, Descartes's argumentation is not very convincing, because a second figure would not require a major change in the discourse. On the contrary, one could keep the original text and use almost the same figure by removing in figure 4.1 three of the given straight lines, namely EF, AD, and GH. Such a new figure would make the construction of the diameter and ordinates clear, but without giving the impression that the new hyperbola is a solution of the four line Pappus problem for the same configuration of given lines and angles.

Now it is time to investigate the draft note which Descartes presented in his letter to Van Schooten, in order to avoid any misinterpretations of figure 4.1. Because the draft was written in Latin, while the rest of the letter is in French, it is obvious that Descartes wanted

[31] "Apres avoir donné le vray lieu de cet exemple, qui est un cercle." [Descartes, 1898, 576] and [Descartes, 1941, 317].

[32] "Il me semble donc que vous ne devez point y mettre d'autre figure; car il faudroit aussi changer le discours, & la solution en seroit plus embroüllée. Mais vous pourrez mettre cet advertissement dans la page 331, ou quelque'autre semblable." Descartes to Van Schooten, March/April 1648, [Descartes, 1898, 576-577] and [Descartes, 1941, 317].

the note to be included in the Latin edition which Van Schooten was preparing at the time. The draft reads as follows:

> It has to be noted that the hyperbola has been applied here to this position of lines, which only a circle fits, as will be shown hereafter; this [is] done for the sake of intelligibility and brevity at the same time, since it is easier to understand what is written down here, when the letters $ABCD$ etcetera are found at the same places in all figures, than if they had to be found sometimes at one and sometimes at another place. At this point there does not follow any error, for the whole question has not yet been determined, but it is only determined on page 333. And by changing a little in this [problem], it can be arranged that a hyperbola fits the same position of lines, which a circle suits, at least a hyperbola which does not pass through any points of intersection of the given lines, as they are represented here. Such as, for instance, if the rectangle of FC by CD has to be larger than the rectangle of CB by CH by a certain given quantity, or something like it. For the sake of the same brevity, he does not mention the opposite hyperbolas, not because they are not known by the author; after all, he has exposed four mutually opposite lines which are related to hyperbolas below on page 336. But it has to be noted that he has often neglected the simpler things in this Geometry, but that he has not omitted anything of the more intricate things, out of what he has undertaken to treat. And therefore he rather shows the position of the lines which a circle fits, than others, which ellipses and hyperbolas fit, since the invention of this comprises a particular difficulty.[33]

This draft note once again shows that Descartes was not aware of the second conic section solution. He asserted that in figure 4.1 the “hyperbola has been applied to the position of lines, which *only* a circle fits” (emphasis mine), so he recognized the circle as the sole solution of the Cartesian Pappus problem. Descartes justified the choice of his figure 4.1 by arguments of clarity and conciseness. For a reader, he argues, it is easier to comprehend the argument by means of one and the same figure in which the given lines are always in the same position and only the putative solution is altered. Moreover,

[33] “Notandum hic applicatam esse hyperbolam ei positioni linearum, cui solum circulum quadrare paulo post ostendetur; quod perspicuitatis & simul breuitatis studio factum; faciliùs enim est quae hic scripta sunt intelligere, cum notae ABCD &c. in ijsdem omnium figurarum locis reperiuntur, quam si nunc in vno, nunc in alio essent quaerendae. Nec etiam hinc sequitur vllus error; tota enim quaestio nondum est determinata, sed in pagina 333 demum determinatur ; potestque fieri, paucis ex ea mutatis, vt eidem positioni linearum, cui competit circulus, | quadret hyperbola , & quidem hyperbola quae non transeat per vllas intersectiones datarum linearum, quemadmodum hic representatur : vt, exempli causa, si rectangulum ex FC in CD debeat esse majus quam rectangulum ex CB in CH quadam data quantitate, vel quid simile. Eiusdem breuitatis studio, nulla etiam hic mentio fit oppositarum hyperbolarum, non quod ab authore ignorentur, vtpote qui paulo post, in pagina 336, quatuor lineas hyperbolae affines inter se oppositas exposuit. Sed notandum est illum faciliora ferè semper in hac Geometria neglexisse, nihil autem ex difficilioribus, inter ea quae tractanda suscepit, omisisse ; atque idcirco ipsum maluisse hic exhibere positionem linearum, cui quadrat circulus, quam alias, quibus quadrent ellypses aut hyperbolae, quia eius inuentio peculiarem habet difficultatem.” Descartes to Van Schooten, March/April 1648, [Descartes, 1898, 577] and [Descartes, 1941, 317].

Descartes stated that figure 4.1 is not causing any misunderstanding since the question "is only on page 333 determined", that is, when the numerical example is introduced.

In my opinion, this reasoning of Descartes is rather weak. As we have seen in section 3.1, he deliberately structured his explanation around the Cartesian Pappus problem. I note that in figure 4.1 the point C is at the same spot as in all other figures illustrating the four line Pappus problem in the *Géométrie*.[34] This shows that all figures were modelled after the numerical example in which the circle is a solution and point C is always the same point of this circle. For the woodcutter, it must have been easy to base figure 4.1 on the figures he had already made. We have to bear in mind that when it comes to illustrations, the publisher had a fair share in how the book eventually looked. Descartes's correspondence shows that he was aware of the role of figures in the publication process of a book.[35]

Descartes denied that he was unaware of the fact that two opposite branches of a hyperbola could be part of the solution. His reference to "four mutually opposite lines which are related to hyperbolas" is not to hyperbolas but to four curves which arise in his solution of the five line Pappus problem.[36] He argued that he had not explicitly discussed easy things such as two opposite branches, because he wanted to limit himself to more complicated matters.[37]

Descartes tried to rescue figure 4.1 by asserting that the hyperbola in the figure can be the solution of a variation of the standard Pappus problem. In the standard four line Pappus problem one looks for points C such that $CB \cdot CF = CD \cdot CH$, but Descartes claimed that the hyperbola can be the locus of points C such that $CF \cdot CD$ is larger than $CB \cdot CH$ "by a certain given quantity, or something else similar," so one could have, for example, $CF \cdot CD = CB \cdot CH + q$ for a given rectangle q. As has been remarked by Maronne, this suggestion of Descartes is noteworthy for two reasons. First, Descartes interchanged the role of the line segments CD and CB. Second, he added a constant.[38]

The interchange of CD and CB can be a writing error by Descartes, but it is more likely that he interchanged the two line segments on purpose. To see this, let us first consider the original problem with the added quantity q, Thus we are required to find points C such that $CB \cdot CF = CD \cdot CH + q$, that is in our modern notation $d_1 d_2 = d_3 d_4 + q$. Supposing C in the same spot as in figure 3.1, we can derive the equation according to the method

[34] Compare for instance figure 4.1 with figures 3.1 and 3.5. In the original 1637 edition of the *Géométrie*, these figures have the same size.

[35] See for instance the correspondence between Descartes and Constantijn Huygens of the years 1635–1636 on the preparations of the publication of the *Discours de la méthode* and the essays, [Descartes and Huygens, 1926, 3, 5, 21, 25, 28].

[36] Figure 4.1 contrasts with Descartes's solution of a five line Pappus problem, which solution consists of a third degree curve with two branches. Descartes added that the solution also included an "adjoined" curve which is obtained by reflecting the third degree curve in one of the coordinate axes [Descartes, 1637c, 338].

[37] At the end of the quoted passage Descartes also mentioned, instead of the Cartesian Pappus problem, another instance of the Pappus problem involving an ellipse or hyperbola as a solution. Although he refers to a "particular difficulty" he probably considered this as something essentially easy, to be omitted in favour of more intricate things such as the Pappus problem for more than four lines.

[38] [Maronne, 2007, 123-124].

Descartes used previously:

$$y = 1 - \frac{x}{2}x \pm \sqrt{1 + q + 4x - \frac{3}{4}x^2}. \tag{4.1}$$

The coefficient of x^2 is negative, and by the results of section 3.2.2 we know that this equation represents an ellipse; the curve is in fact a circle in this case. This circle has the same centre as the circle in figure 3.5, but its radius depends on the value of q. Thus, the change of the problem from $CB \cdot CF = CD \cdot CH$ to $CB \cdot CF = CD \cdot CH + q$, or in our modern notation $d_1 d_2 = d_3 d_4 + q$, does not provide Descartes with an argument for justifying the presence of the hyperbola in figure 4.1.

Now consider the altered Pappus problem with interchanged line segments, that is, $CF \cdot CD = CB \cdot CH + q$ or in our modern notation $d_2 d_3 = d_1 d_4 + q$. If we suppose the point C at the same spot as in figure 3.1, we can produce the equation

$$y = -\frac{1}{4} - \frac{31}{8}x \pm \sqrt{\frac{1}{16} + \frac{3}{7}q - \frac{11}{8}x + \frac{457}{784}x^2}. \tag{4.2}$$

Because the coefficient of x^2 is positive, this equation represents a hyperbola. The constant $q > 0$ assures that the hyperbola does not pass through the points of intersection of the given lines ℓ_i. Perhaps Descartes thought that by choosing a suitable q, the hyperbola of the figure 4.1 could be obtained. Probably he did not perform the actual calculations, since if he had done so, he would have seen that no suitable q can be found.

The reformulation of the problem is a very weak defence of Descartes towards the critic. By adding the constant q, Descartes was not considering the Pappus problem anymore, but rather a different problem. Moreover, this new problem has its own difficulties. One can show that the complete solution of each of the problems $d_1 d_2 = d_3 d_4 + q$ and $d_3 d_2 = d_1 d_4 + q$ does not consist of two entire conic sections, but of parts of four different conic sections.[39]

Descartes's interventions in the Latin edition of the *Géométrie* were not limited to the figure of the four line Pappus problem. In the same letter to Van Schooten, he also suggested to add a note on the transformation of an equation on page 378 of the *Géométrie*.[40]

[39] Descartes's general method consisted of expressing the distances d_i as $d_i = \alpha_i x + \beta_i y + \gamma_i$. By doing so, he would obtain for the altered problem an equation of the type

$$y(\alpha_2 x + \beta_2 y + \gamma_2) = (\alpha_3 x + \beta_3 y + \gamma_3)(\alpha_4 x + \beta_4 y + \gamma_4) + q. \tag{4.3}$$

However, in order to obtain the complete solution, we note that $d_i = |\alpha_i x + \beta_i y + \gamma_i|$ and so we obtain (for $d_1 d_2 = d_3 d_4 + q$) the equation

$$|y(\alpha_2 x + \beta_2 y + \gamma_2)| = |(\alpha_3 x + \beta_3 y + \gamma_3)(\alpha_4 x + \beta_4 y + \gamma_4)| + q.$$

This last equation represents four conic sections. If one only considers the equations of the type (4.3) for the different parts of the plane where they are valid, one will find parts of those four conic sections, but not four complete conic sections.

[40] In the letter, Descartes wrote that his note is a reaction to an annotation made by the Dutch mathematician Godefroy van Haestrecht (ca. 1593–1659), [Descartes, 1898, 577-578] and [Descartes, 1941, 317-318]. This note by Descartes was also, with some changes in wording, included by Van Schooten in his comments. Van Schooten presents the note as the work of Van Haestrecht, and does not mention Descartes's involvement, see [Schooten, 1649c, 251-252].

Van Schooten's notebook in a manuscript in Groningen contains a second instance of his collaboration with Descartes in the preparation of the Latin version of the *Géométrie*. This instance consists of a comment written by Descartes on a paragraph of Book III of the *Géométrie*, and the correction of a misprint by Descartes.[41] In this case, Van Schooten corrected the misprint in his editions of *Geometria*, but he did not include Descartes's comment.[42]

To my knowledge, the instances that have been discussed in this section are the only written evidence of the collaboration between Descartes and Van Schooten on the Latin *Geometria*. But since Van Schooten and Descartes kept in touch during the 1640's, it is not unlikely that part of the collaboration took place orally and was not documented by letters.

4.4.2 Mersenne's involvement in the *Geometria (1649)*

Descartes did not make his work on the *Geometria* known to his correspondents in France; quite the contrary. In a letter to Mersenne dated 4 April 1648 he stated

> I have not wanted to see the version of Schooten, even if he had desired it.[43]

Descartes gave two arguments to Mersenne why he did not want to be involved in the Latin edition. First, he did not want to change the text, and if he had been involved, he could not have restrained himself from making corrections to make the text clearer. Descartes's second argument is what he calls Van Schooten's poor mastery of Latin, which would lead to an obscure translation. Descartes argued that if his own name were connected to a poorly written and obscure Latin text, he would provide his opponents (i.e., Roberval and his adherents) with additional means of attacking him.

The same letter also shows that Mersenne had written to Van Schooten in the spring of 1648 about Descartes's *Géométrie*. Descartes wrote:

> I [i.e., Descartes] was not able to read without some indignation about that what you convey me to have written to Mr Schooten concerning my *Géométrie* (...). You have seen very clearly several times by experience, that what Roberval said against my writings was false and impertinent. Nonetheless you suppose that I have to change something in them, because Roberval says that something is missing in my solution of the locus of three and four lines [i.e, the solution of the three and four line Pappus problem], as if the views of such a man should be considered.[44]

[41] UBG, Hs 108, f. 59r.-v. The passage is entitled "La facon d'exprimer la valeur de toutes les racines des Equations cubiques: & en suite de toutes celles qui ne montent jusqu'au quarré de quarré", [Descartes, 1637c, 400].

[42] The misprint was the omission of the factor z in the equation $z^3 = * - qz + p$ on line 25 of page 400 of the *Géométrie*.

[43] "Ie n'ay point voulu voir la version de Schooten, encore qu'il l'ait desiré". Descartes to Mersenne, 4 April 1648, [Descartes, 1903, 143] and [Descartes, 1963, 27].

[44] "Ie n'ai pu lire sans quelque indignation ce que vous me mandez auoir escrit au Sr. Schooten, touchant ma Géométrie (...), vous auez vu plusieurs fois trés clairement, par expérience, que ce que le Roberval disoit

The quoted passage shows that in the spring of 1648, probably in March, Mersenne wrote a letter to Van Schooten about the *Géométrie*, and that he also wrote a letter to Descartes in which he mentioned that he had written to Van Schooten. Both letters are lost today. Nevertheless, it seems probable that in the letter to Van Schooten, Mersenne discussed the criticisms by Roberval on Descartes's solution of the four line Pappus problem. Thus, in the spring of 1648, Van Schooten was aware of the debate on the four line Pappus problem that was going on in Paris.

Van Schooten could also have been informed by Constantijn Huygens of the fact that there was something missing in Descartes's solution. On 17 March 1648, Mersenne wrote to Constantijn Huygens that Blaise Pascal had given a new geometric solution of the Pappus problem in three and four lines, and that in Paris it was no longer believed that the Pappus problem had been completely solved by Descartes.[45] We know that Constantijn Huygens was in touch with Frans van Schooten in the spring of 1648,[46] and that they discussed the work of Descartes and Van Schooten's project of the Latin version.[47]

This information on the lively exchange of letters between Huygens, Mersenne, Van Schooten and Descartes in the spring of 1648 suggests the following reconstruction of what happened in that year. Early in 1648, it was well known that Van Schooten prepared a Latin edition of the *Géométrie*. Mersenne, who was familiar with Roberval's criticisms and the new solution by Blaise Pascal, decided that he had to inform Van Schooten about the problematic features of figure 4.1 in relation to Descartes's solution of the four line Pappus problem. Therefore he wrote a letter to Van Schooten in March or April 1648. In this letter, Mersenne mentioned Roberval's criticisms, and maybe also the new geometric solution of the problem by Pascal . Mersenne also informed Constantijn Huygens about this achievement of Pascal.[48] After having received this information, Van Schooten wondered how to deal with figure 4.1. He considered replacing this figure with another figure, but before

contre mes escrits estoit faux & impertinent, & toutefois vous supposez que i'y doy changer quelque chose, a cause que Roberval dit qu'il manque quelque chose en ma solution du lieu ad 3 & 4 lineas, comme si les visions d'vn tel homme deuoient estre considerables." Descartes to Mersenne, 4 April 1648, [Descartes, 1903, 142] and [Descartes, 1963, 26].

[45] Mersenne to Constantijn Huygens, 17 March 1648, [Huygens, 1888, 84]. The original solution of Pascal is lost. Almost all information on the text of Pascal is based on notes of Leibniz who had seen the solution by Pascal, see [Taton, 1962, 225-233]. In [Maronne, 2007, 125], Maronne states that Pascal's solution contained the second conic section, but this is not explicitly mentioned in the sources.

[46] Already since 1637 Constantijn Huygens and Van Schooten were in contact and discussed the work and ideas of Descartes. See the letter of Constantijn Huygens to Descartes, 18 September 1637, [Descartes and Huygens, 1926, 55].

[47] In the conversation, Huygens mentioned to Van Schooten the existence of an introductory text on the *Géométrie* entitled *Le Receuil du Calcul pour l'intelligence de la Geometrie de Monsr. des Cartes* (Collection on the calculation, serving for the geometry of Mr Des Cartes). Via Huygens, Van Schooten then received a letter by Mersenne written shortly after the conversation. Mersenne wrote to Van Schooten that he possessed a copy of *Le Receuil du Calcul*. Van Schooten immediately asked Mersenne for the text of this treatise, and for additional documents which might help in explaining and clarifying the *Géométrie*. It is very likely that Mersenne had been informed by Huygens about the fact that Van Schooten did not know about the text *Le Recueil du Calcul*, because Mersenne wrote to Van Schooten shortly after his conversation with Huygens. Most of the information in this footnote can be deduced from a letter by Van Schooten to Christiaan Huygens, 3 June 1648, [Huygens, 1888, 98].

[48] We will see below that Van Schooten in 1648 did not have an in-depth understanding of the second conic section in the solution, because in 1656 he still thought that the complete solution could consist of four conjugate hyperbolas.

taking a decision he decided to consult Descartes on the matter.[49] Descartes responded to Van Schooten in the letter which we have discussed above in detail, and which is dated in March/April 1648 by Maronne.

My conjectural reconstruction agrees well with two particular features of Descartes's letter. Descartes explicitly mentioned that Van Schooten should keep the figure; therefore Van Schooten must have wondered whether he should keep the figure or not, and so he must have known about the criticisms on the figure. That Van Schooten knew about this criticism is confirmed by Mersenne's letter to Van Schooten. Secondly, Descartes's letter to Mersenne confirms that mathematician "N" was indeed Roberval. Van Schooten had been informed by Mersenne about Roberval's criticisms, and he asks Descartes how deal with them. Descartes answered Van Schooten in March/April by the letter in which he referred to Roberval as N. After having received the response of Descartes, Van Schooten had to decide what to do with figure 4.1. The answer is found in *Geometria (1649)*.

4.5 Van Schooten's comments in the *Geometria (1649)*

In the *Geometria (1649)*, Van Schooten inserted six comments related to the four line Pappus problem, covering in total 17 pages of the total commentaries of 200 pages. All six comments refer to Book II; Van Schooten did not comment on the discussion of the Pappus problem in Book I, nor on the five line Pappus problem in Book II. In the Latin translation of the *Géométrie*, capital letters in the margin indicate sentences to which Van Schooten made his various comments. In my discussion I use the same letters to indicate these comments. The comments are printed together after the end of Descartes's text. In each comment, Van Schooten begins by citing the sentence to which the comment refers. An overview of the six comments, the original French sentences of the *Géométrie* they refer to, and the pages where the comments are found in *Geometria (1649)*, is presented in table 4.1. I now discuss the six comments, starting with comment E, followed by the comments B, C and D and finishing with F and G.

Comment E is related to figure 3.6 and also, at least implicitly, to the absence of the second conic section in the solution and to Roberval's criticisms. Above we have seen that Van Schooten turned to Descartes, who provided him with an explanation and a draft note to be inserted in the Latin version of the *Géométrie*.[50] Van Schooten chose to follow the instructions of Descartes and to disregard the views of Roberval. He kept figure 3.6 in his translation[51] and added comment E, in which he inserted almost literally the draft note which Descartes had provided. Comment E reads as follows.

[49]Van Schooten may have written him a letter, which is now lost, but he may also have posed his questions during a visit. In any case, Descartes replied in March/April, and from his the answer it is clear that Van Schooten also asked for advice on some other matters related to the *Géométrie* [Descartes, 1898, 574-582] and [Descartes, 1941, 315-320].

[50]See above, section 4.4.

[51]See [Schooten, 1649c, 36].

Comment	**Sentences of *Géométrie* the comment refers to**	*Géométrie* pp:	*Geometria* pp:
B	au moins en supposant ez plus grand que EG (...) changer tous les signes + & −.	326	193–194
C	Or cela fait, (...) pas plus malaysée a trouver qu'IL.	328	194–195
D	à sçavoir si ce costé droit est $\sqrt{\frac{o^2z^2}{a^2}+\frac{4mpz^2}{a^2}}$ le traversant est $\sqrt{\frac{a^2o^2m^2}{p^2z^2}+\frac{4a^2m}{pz^2}}$.	330	195–196
E	mais quand cete section estant une Hyperbole etc.	331	196–197
F	Car ces lieus ne sont autre chose (...) pour estre entierement determiné.	334-335	197–201
G	Et s'il manque deux conditions a la determination de ce point, le lieu ou il se trouve est une superficie, laquelle peut estre tout de mesme ou plate ou spherique ou plus composée.	335	201–209

Table 4.1 – Comments on the Pappus problem in *Geometria (1649)*.

It has to be noted that the hyperbola has been applied to this position of lines to which only a circle fits, as will be shown a little further on by the Author [i.e., Descartes]. This has been done for the sake of intelligibility and brevity; for the letters A, B, C, D etcetera can be more easily understood if they can be found at the same places in all figures written by him, than if they would have to be sought sometimes at one place and sometimes at another place.

For if it is required, that the product, which arises from the multiplication of CB with CF, is equal to that, which is the outcome of the multiplication of CD by CH, then the curved line should pass through four intersections of the given lines: namely through the intersection A of the lines DA and AB (for in this case the lines BC and CD are nothing, and then each single line on its own multiplied by the remaining lines, produces nothing); and through the intersection G of the lines AB, GH (in which case CH and CB are also nothing): and also through each of both remaining [intersections], because the [intersection] of the same lines FE and GH (in which case CF and CH are nothing) and of the same lines DA and EF (in which case CD and CF are nothing), which are not described in this figure, but are clearly discernible in the circle.

> For Mr. des Cartes wanted to reduce all instances to one example for brevity's sake; it should therefore not be surprising that, after he showed that the true locus of this example is the circle, the hyperbola does not correspond to the same position of lines as the circle does. At this point there does not follow any error, for the whole problem is not yet determined, but is only on page 38 for the first time determined. For it is after all possible that, by changing a little in this problem, a hyperbola agrees with the position of lines which corresponds to a circle; at least a hyperbola which does not pass through any intersections of the given lines. For instance, if the rectangle of FC and CD has to be larger than the rectangle of CB and CH by a certain given quantity, or something else like it: it follows that this [change] can be applied in such a way that, with the letters I, K, L, B, C, D etc. in their places, the few things which he wanted to contribute by means of the hyperbola, can be more easily understood than if the figure had been changed.
>
> For the sake of the same brevity, he does not mention the opposite hyperbolas; not because they are not known by the author – after all he showed hereafter on page 41 four mutually opposite lines which are related to hyperbolas – but because he has omitted almost all of the simpler matters in this Geometry. In the more difficult matters, which he undertook to treat, he has left out nothing. And therefore he preferred to show this position of lines which matches the circle, rather than a [position of lines] which corresponds to an ellipse or a hyperbola, since the invention of this has a particular difficulty.[52]

In Comment E, Van Schooten made only minor changes in the draft note by Descartes.

[52]"Notandum hic, applicatam esse hyperbolam ei linearum positioni, cui solum Circulum quadrare paulo post ab Authore ostenditur. Quod tam perspicuitatis quam brevitatis studio factum; quandoquidem, postquam literae A, B, C, D, &c. in iisdem omnium figurarum locis reperiuntur, quae ibidem ab eo scripta sunt, sic facilius intellegi possunt, quam si iam in vno, iam in alio essent quaerendae.

Etenim cum requiritur, ut productum, quod oritur ex multiplicatione CB per CF, aequale sit ei, quod fit ex ductu CD in CH, oportet, ut linea illa curva transeat per quatuor intersectionum puncta datarum linearum: nimirum, per inter sectionem A, linearum DA, AB (quoniam eo casu lineae BC & CD nullae sunt, ac proinde singulae, in singulas ex reliquis ductae, nihil producunt), & per intersectionum G linearum AB, GH (quo casu lineae CH & CB nullae sunt): nec non per utramq; reliquam, utpote ipsarum FE, GH (quo casu CF & CH nullae sunt), et ipsarum DA, EF (quo casu CD & CF nullae sunt), quae in hâc figurâ non sunt expressae, sed in Circulo observatae apparent. Unde, cum Domnus des Cartes, brevitati studens, referre voluerit casus omnes ad unum exemplum, figurae nempe pag. 14. mirum videri non debet, quod, postquam verum huius exempli locum Circulum esse ostendit, nec in quaestione quicquam mutavit, eidem linearum positioni non Hyperbola sicut Circulus responderit. Nec etiam hinc ullus sequitur error, quandoquidem tota quaestio nondum determinata existit, sed pag. 38. primo determinatur. Quippe fieri potest, ut paucis in eâ mutatis, eidem linearum positioni, cui Circulus competit, quadret Hyperbola; & quidem Hyperbola, quae non transeat per ullas datarum linearum intersectiones. Ut, exempli causâ, si rectangulum ex FC in CD debeat esse majus, quam rectangulum ex CB in CH, datâ quâdam quantitate, vel aliud quid simile: sequitur eam sic applicari posse, ut, manentibus literis I, K, L, B, C, D& suis locis, ea pauca, quae de Hyperbolâ afferre voluit, facilius intelligantur, quam si figura mutata fuisset.

Ejusdem brevitatis studio nulla etiam hic mentio fit oppositarum Hyperbolarum, non quod ab Authore ignorentur, utpote qui paulo post, in pag. 41. quatuor lineas Hyperbolae affines, inter se oppositas, exhibuit: Sed quod faciliora fere semper in hâc Geometriâ neglexerit. In difficilioribus certe, quae tractanda suscepit, nihil omisit. Atque idcirco hic maluit eam linearum positionem exhibere, cui conveniret Circulus, quam cui competeret Ellipsis, aut Hyperbola, quia eius inventio peculiarem habet difficultatem." [Schooten, 1649c, 196-197].

The changes concern the spelling and choice of words, but do not affect the meaning of the text. Just like Descartes, Van Schooten claimed that figure 3.6 had been inserted for reasons of clarity and conciseness, and that the hyperbola of figure 3.6 is a solution of a different problem, namely to find all points C in figure in 4.1 such that $FC{\cdot}CD = CB{\cdot}CH+q$ instead of $CB \cdot CF = CD \cdot CH$.

Comment E is considerably longer than the Latin draft note by Descartes, because Van Schooten also included his own Latin translations of Descartes's French remarks in the rest of Descartes's letter. These remarks concerned the points of intersection of the four given lines in the solution of the four line Pappus problem, and the statement that the solution in the case of figure 3.6 is only a circle and not a circle together with a hyperbola.

The three comments B, C, and D deal with the manipulation of algebraic symbols. In the first years after the publication of the *Géométrie*, it turned out that mathematicians had difficulties with these new algebraic manipulations.

Comment B refers to the use of plus and minus signs by Descartes in his manipulations of fractions. I have discussed this comment in the previous chapter in section 3.3.

Comment C deals with the cases where the locus of a Pappus problem is a straight line. Recall that in the Pappus problem, Descartes derived the equation $y = m - \frac{n}{z}x + \sqrt{m^2 + ox - \frac{p}{m}x^2}$. He stated that in case $m^2 + ox - \frac{p}{m}x^2$ is a perfect square, the square root can be extracted and the locus is a straight line, see section 3.2.2 above. Descartes distinguished three cases in which the square root of what he called $m^2 + ox - \frac{p}{m}x^2$ can be extracted. In modern terms he intended to list the cases where the square root of $\pm m^2 \pm ox \pm \frac{p}{m}x^2$ can be extracted for m, o, p positive or zero. His three cases are as follows:

1. Both m^2 and $\frac{p}{m}x^2$ have the same sign (plus or minus) and $o^2 = 4pm$.

2. Both m^2 and ox are equal to zero.

3. Both ox and $\frac{p}{m}x^2$ are equal to zero.

In the first case, Descartes erroneously stated that if $o^2 = 4pm$, the square root of $-m^2 + ox - \frac{p}{m}x^2$ can be extracted. Perhaps the minus sign was added by accident.[53] In his comment, Van Schooten repeated the statement of Descartes in his own words, including the possibility of a minus sign, and he then explained that in case (1), the expression $\sqrt{m^2 + ox + \frac{p}{m}x^2}$ can be written as $m + x\sqrt{\frac{p}{m}}$, without reference to a minus sign.[54] Van Schooten repeated cases (2) and (3), but just like Descartes, he failed to mention that the remaining term $\frac{p}{m}x^2$ or m^2 must have the sign +, for the square root to be extracted. In the 1659 edition, Van Schooten tacitly omitted the minus sign from the Latin translation of Descartes's description of case (1), and also from his own comment. For cases (2) and (3) he did not change Descartes's text but in his comment he added that the (non-zero) term m^2 or $\frac{p}{m}x^2$ should have a plus sign.[55]

[53] Descartes wrote "c'est a dire que mm & $\frac{p}{m}xx$ estant marqués d'vn mesme signe + ou −" [Descartes, 1637c, 328]. Perhaps the "ou −" was added by accident.

[54] [Schooten, 1649c, 195].

[55] [Descartes, 1659, 28-29] and [Schooten, 1659c, 182].

Comment D explains how to simplify expressions involving roots, in order to show that $\sqrt{\frac{oozz}{aa}+\frac{4mpzz}{aa}}$ can be written as $\frac{z}{a}\sqrt{oo+4mp}$. Van Schooten employed numerical examples to show the analogy between manipulation of numbers and symbols.

The two last comments F and G are related to the paragraph entitled "Which are the plane and solid loci, and the method of finding them" in the *Géométrie*.[56] In this paragraph, Descartes stated that in his previous discussion of the four line Pappus problem, he considered all second degree equations in two unknowns. This implies, he argued, that he had completely solved two problems, namely the general four line Pappus problem, and all other problems in the "composition of the solid loci" (i.e., the construction of conic sections). Descartes also stated that in the Pappus problem, one condition is missing for a complete determination of the point C, and therefore the solution consists of all the points of a line. In case two conditions are missing in a problem, the solution will be a surface. Van Schooten's comments F and G deal with these remarks on the number of missing conditions and the nature of the solution. Van Schooten provided additional examples of problems with one missing condition (F) and two missing conditions (G).

In comment F Van Schooten first discussed the general procedure for the solution of the problem. The procedure consists of first deducing an equation and then performing a pointwise construction of the curve which the equation represents. Note that in case the solution is a conic section, Descartes did not use a pointwise construction, but the Apollonian construction instead. Van Schooten then claimed that every geometrical curve is a solution of a Pappus problem. The same statement was made by Descartes in the *Géométrie*.[57] I do not understand why Van Schooten found it necessary to repeat this claim, without referring to Descartes. Finally, Van Schooten gave an example of the Pappus problem for two lines. Van Schooten found only one straight line as the solution, whereas the complete solution consists in general of two straight lines.[58]

In Comment G, Van Schooten discussed two examples of problems in which the solution is not a curve but (part of) a plane or surface. In the first example, he considered a given equilateral triangle ABC and he asked for all points E such that the sum of the distances from E to the given sides of the triangle is equal to the altitude AD of the triangle. It turns out that all points E in the interior of the triangle have the property, so now the problem turns into a theorem,[59] namely that for all points E in the interior of the triangle, the sum of the distances to the three sides are equal to AD. Van Schooten then provided a second example involving a circle.

In none of the comments did Frans van Schooten discuss the fact that in his solution of the four line Pappus problem, Descartes tacitly assumed that the resulting equation is a quadratic equation in y, that is to say that the coefficient of y^2 is non-zero. This omission

[56] [Descartes, 1637c, 334-335].

[57] See Book II at the beginning of his discussion of the Pappus problem [Descartes, 1637c, 324]. The statement is wrong; for a proof that algebraic curves exist which are not a solution of a Pappus problem see [Bos, 1981, 332-338].

[58] Van Schooten also treated this problem in his reconstruction of Apollonius's *On plane loci*, see [Schooten, 1657, 231-232] and [Schooten, 1660a, 218-219].

[59] In the modern literature, this theorem is sometimes named after Van Schooten.

had already been admitted by Descartes in a letter to De Beaune in 1639.[60] In the *Notae Breves*, De Beaune studied these cases in his fifth observation. He first explored the case where there is no y^2 term, but there are both an xy and a x^2 term with non-zero coefficients. In this case one should interchange the role of x and y. He also studied the case in which there are no y^2 term and no x^2 but there is an xy-term with a non-zero coefficient. De Beaune shows that in such a case the equation represents a hyperbola.[61] Van Schooten included De Beaune's treatise in *Geometria (1649)*, and this is probably the reason why he did not pay attention to this matter in his own comments.

[60] Descartes to De Beaune, 20 February 1639. [Descartes, 1898, 510] and [Descartes, 1941, 184-185].
[61] [Beaune, 1649, 141-146].

CHAPTER 5

Pappus in the centre again: 1649–1659

Before the publication of the second edition of the *Geometria* in 1659, Descartes's solution of the four line Pappus problem became the object of debate two more times. In 1649, when the first edition was being printed, the first debate took place between Descartes and Roberval, the latter using the French mathematician Carcavi as a mouth-piece. Shortly after this exchange, Descartes passed away in Stockholm on 11 February 1650. The second debate took place in 1656 and was related to the project to produce a second edition of the complete works of Descartes by the Elzevier printing house. The participants were Van Schooten, Roberval, and Christiaan Huygens, who initiated the debate. As we will see, Van Schooten firmly defended the intellectual legacy of Descartes, and accepted criticisms only reluctantly.

In this chapter I discuss these two debates of 1649 and 1656 in chronological order, and then the comments which Van Schooten made in his *Geometria (1659)*. Although there is no proof that Van Schooten was involved in the 1649 debate, I include it here because Roberval was involved and because in 1656 Roberval's arguments were to a large extent similar to the ones he used in 1649. The 1656 debate is of major importance for understanding a considerable number of significant changes and additions Van Schooten made in his comments in the *Geometria (1659)*.

The two debates are documented by Descartes's correspondence for the first debate and by Huygens's correspondence for the second. Since Van Schooten's correspondence is almost completely lost (apart from the Huygens–Van Schooten correspondence), no other debates between him and other mathematicians are known.

5.1 The debate of 1649

5.1.1 Roberval challenged Descartes again

During the summer of 1649, when the *Geometria (1649)* was being printed in Leiden, the Pappus problem emerged again in an indirect correspondence between Roberval and Descartes. Pierre de Carcavi (ca. 1600–1684), a French counsellor and mathematician who was in Paris at the time, served as an intermediary to express Roberval's opinions to Descartes. In a letter of 9 July 1649, Carcavi communicated to Descartes several objections that had been made by Roberval to the *Géométrie*. Three of these objections concern the four line Pappus problem. The letter by Carcavi reads as follows:

> Page 326. That the point C is through all the angles you [i.e., Descartes] have mentioned, and that you do not mention the angle where it cannot be; and that the problem can never be impossible.[1]

The first two objections concern the position of the point C. Roberval noted that points C which are solutions to the Pappus problem can be found in "all the angles you mentioned", that is, in the angles DAG, DAE, EAR and RAG in figure 3.5 of page 97. But the circle found by Descartes only passes through the angles DAG, EAR and RAG (see figure 3.5). The hyperbola also passes through angle DAE (see figure 3.7), so the circle and the hyperbola together pass through all four angles. Maronne has pointed out that Roberval is implicitly referring to the second conic section in this passage.[2]

Secondly, Roberval blamed Descartes for not noticing that there is an angle in which solutions C to the problem cannot be found. This cannot be one of the four angles around the point A, so it is most likely that Roberval meant an angle between the given lines made at one of the five other points of intersection. Figure 3.7 shows that indeed there is one such angle through which the circle and hyperbola do not pass, namely one of the angles at E between $\ell_1 = EA$ and $\ell_2 = ES$. This interpretation agrees with the observations on the Pappus problem which Roberval made in 1656 and which we will discuss below.[3] Note that Roberval's remarks now became more explicitly related to the second conic solution, as he now sketches in which regions of the plane the solution is to be found. Apparently Roberval still did not want to give the details of the second conic solution away to Descartes.

In the third place, Roberval rejected a statement by Descartes to the effect that the four line Pappus problem can be "impossible," if in all four angles (DAG, DAE, EAR and RAG) the only possible value for y is zero.[4] Roberval must have realized that this case cannot occur, and even for cases where $y = 0$ we find four of the six points of intersection of the given straight lines as part of the solution.

[1] "Pag. 326. Que le point C est par tous les angles que vous auez nommez, & que vous ne nommez point celuy où il ne peut estre; & que iamais la question n'est impossible." Carcavi to Descartes, 9 July 1649, [Descartes, 1903, 373] and [Descartes, 1963, 238].

[2] [Maronne, 2007, 127].

[3] See page 132.

[4] [Descartes, 1637c, 326].

5.1.2 Descartes's reply to Roberval

Descartes responded to Roberval's criticisms in a letter of 17 August 1649 to Carcavi.[5] In his reply, Descartes angrily remarked that the criticism is "unreasonable," that Roberval had not carefully read the text of the *Géométrie*, and that all three objections are false. Descartes tried to refute the first objection by an unconvincing linguistic argument. He emphasised that the use of the singular in the expression "*through* an angle" — whereas "*in* an angle" would have fitted better — showed the incompetence of Roberval.[6] Descartes added that in his numerical example of the four line Pappus problem, the solution circle does not pass through the angle DAE, and he concluded that Roberval's objection was false. Again we see that Descartes did not take account of the second conic section in the solution of the four line Pappus problem.

Roberval's second objection was brushed aside by Descartes in the following way:

> It is also evident that he [i.e., Roberval] errs, insofar he says that I have not mentioned the angle in which the point C can not be [found]. Since I mentioned all four angles which are made by the intersection of the two lines DR and EG, I mentioned the entire plane stretched out infinitely to all sides, and as a consequence [I mentioned] all the places, both those where the point C can be as those where it cannot be. Thus it would have been superfluous for me to consider other angles.[7]

Descartes did not realize (or did not want to realize) that Roberval's statement could also refer to angles at other points than A.

Finally, Descartes waved away Roberval's objection that the four lines Pappus problem is never impossible, by stating that "we can propose it [i.e., the problem] in several other [ways], of which some are impossible."[8] However, Descartes did not give an explicit example of an "impossible" Pappus problem, nor did he provide other arguments for his claim. Altogether, his response is unsatisfactory.

Carcavi responded to Descartes in September 1649.[9] In this letter, Carcavi provided more details on the remarks made by Roberval in the previous letter. Carcavi first apologized for a misunderstanding about the interpretation of the angle (in Roberval's first objection). He said that this was caused by himself because he used an inappropriate

[5] Descartes to Carcavi, 17 August 1649, [Descartes, 1903, 391-401] and [Descartes, 1963, 252-260].

[6] In the original French, this is about the difference between the prepositions "par" and "dans". Carcavi used "par" in his letter to Descartes in his discussion of the ideas of Roberval, whereas Descartes had used the word "dans" in the *Géométrie* and prefers this preposition also in 1649. See [Descartes, 1903, 373 and 395], [Descartes, 1637c, 326], and [Descartes, 1963, 238 and 256-257].

[7] "Il est evident aussi qu'il se trompe, en ce qu'il dit que ie n'ay pas nommé l'angle où le point C ne peut estre; car, ayant nommé tous les quatre angles qui se sont par l'intersection des deux lignes DR & EG, i'ay nommé toute la superficie indéfiniment estenduë de tous costez, & par consequent tous les lieux, tant ceux où le point C peut estre, que ceux où il ne peut pas estre; en sorte qu'il auroit eté superflu que i'eusse consideré d'autres angles." Descartes to Carcavi, 17 August 1649, [Descartes, 1903, 396] and [Descartes, 1963, 257].

[8] "On la peut proposer en plusieurs autres, dont quelques-vnes sont impossibles." Descartes to Carcavi, 17 August 1649, [Descartes, 1903, 397] and [Descartes, 1963, 257].

[9] Carcavi to Descartes, 24 September 1649, [Descartes, 1903, 412-422] and [Descartes, 1963, 273-284].

preposition in his last letter. In reality there was no difference of opinion between Roberval and Descartes about the interpretation of the angle. Then Carcavi turned to Roberval's criticisms of the *Géométrie*. Again, Roberval mentioned the errors in the figure with the hyperbola (figure 3.6), which he wanted to interpret as an actual solution of the Cartesian Pappus problem. His remarks are essentially the same as those by the anonymous critic in 1642.[10]

Subsequently, Carcavi presented Roberval's criticism on the statement by Descartes that all the required points in the four line Pappus problem are on an ellipse, parabola, hyperbola (including the opposite sections), a circle or a straight line.[11] According to Carcavi, Roberval believed this statement to be false, for the following reason:

> For it may happen that all the points are not on the same line, that is to say, when some of them will be in one of the spaces which are to be distinguished [i.e., bounded] by the four given lines, and others will be in another space.[12]

In the Cartesian Pappus problem, the four given straight lines divide the plane into eleven regions (see figure 5.1). Roberval must have observed that the required points in the interior of the same region belong to the same conic section, but that this is not necessarily true for points in the interior of different regions. We see in figure 3.7, that in four of the eleven regions part of the circle is located, and in six regions part of the hyperbola, and that there is one region where no part of a solution curve is to be found, namely region (8) of figure 5.1. Altogether, Carcavi's passage suggests that Roberval was well aware of the existence of a second conic solution.

Furthermore, Roberval repeated that there do not exist instances of a Pappus problem which are impossible, that is, which do not have solutions, but without providing further details to Descartes.

In the letter, Carcavi furthermore mentioned that Roberval had shown him a demonstration of the Pappus problem a long time ago, and that already in 1637 Roberval had presented a demonstration of the Pappus problem in which he used the four angles to mathematicians in Paris, probably the circle of mathematicians around Mersenne.[13] It is not clear exactly when Carcavi had seen Roberval's demonstration, and the story of its presentation in 1637 might be doubted. Above I have argued that Roberval knew the second conic solution in 1646 if not earlier.

The correspondence between Carcavi and Descartes ends with this letter, because Descartes never replied to it. It is not known whether Descartes shared Carcavi's letters with Van Schooten. In any case, it was too late to change anything in the *Geometria (1649)*, which had already been printed when Carcavi wrote his letter.

---

[10] See page 110.

[11] [Descartes, 1637c, 308].

[12] "Car il se pourra faire que tous ces points ne seront pas dans vne mesme ligne, sçauoir, lors que quelques-vns d'iceux seront dans l'vn des espaces qui sont distinguez par les quatre lignes données, & d'autres en vn autre espace." Carcavi to Descartes, 24 September 1649, [Descartes, 1903, 416] and [Descartes, 1963, 277].

[13] [Descartes, 1903, 415] and [Descartes, 1963, 274].

5.2 The debate in 1656: Roberval, Huygens and Van Schooten

The discussion about the Pappus problem flared up again in 1656. Van Schooten's former student Christiaan Huygens was the initiator of the debate in which Van Schooten and Roberval were the other participants. The cause for the debate was the project of producing a second edition of the *Geometria*, to which Van Schooten devoted a fair amount of his time in 1656, together with Huygens's acquaintance with Roberval.[14]

5.2.1 Roberval explained the shortcomings of Descartes's solution in detail

During the autumn of 1655, Huygens made a tour in France. He visited Angers, where he obtained his doctorate, and then Paris, where he was introduced to Roberval. After Huygens's return to the Dutch Republic in November 1655, he and Roberval maintained a correspondence. Shortly after the beginning of this correspondence, Huygens asked Roberval in a letter (dated March 1656 by Adam-Tannery), to write in detail about the "locus problem" in the *Géométrie* in which

> you have found fault, because there are people here who hold that everything can be reconciled.[15]

By then, Huygens must have been aware that Roberval had serious trouble with Descartes's solution in the *Géométrie* to the four line Pappus problem. Huygens seems to say that there were people in the Republic who tried to reconcile Roberval's criticisms with the Cartesian explanations in the *Géométrie*. Huygens must have meant Van Schooten, who was preparing his additional comments for the second Latin edition of *Geometria* at that time.

Roberval answered Huygens on the 6th of July, 1656.[16] In this letter, Roberval began by criticizing Descartes's own solution. He first commented on the following passage in the *Géométrie*:

> And if the quantity y turns out to be equal to nil, or less than nothing in this equation,[17] if the point C is supposed to be in the angle DAG, one should also suppose it to be in the angle DAE, or EAR or RAG, by changing the signs[18] + and − according to what is required for that. And if in all these four positions the value y is equal to zero, the question is impossible for the proposed case.[19]

[14] See section 2.9 above for details on the projects undertaken by Elzevier regarding the publication of revised versions of Descartes's works and the involvement of Van Schooten.

[15] "Vous avez trouve de l'abus, car il y a icy des personnes qui soustienent que tout se peut concilier." Christiaan Huygens to Gilles Personne de Roberval, [March 1656], [Huygens, 1888, 396].

[16] G.P. de Roberval to Christiaan Huygens, 6 July 1656, [Huygens, 1888, 449-452].

[17] Descartes referred to equation (3.13) of page 89.

[18] The *Géométrie* reads *lignes*, but this should be *signes*.

[19] [Descartes, 1637c, 326].

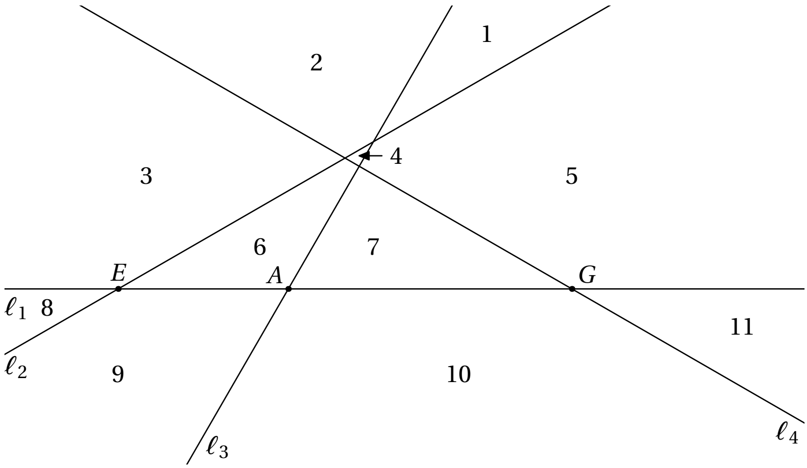


Figure 5.1 – The eleven regions in the plane.

Roberval pointed out two difficulties with this passage. In the first place, Descartes seems to say that the point C can be located in all four angles made by the lines ℓ_1 and ℓ_2, but this is in contradiction with Descartes's own solution in figure 3.5, in which the circle does not pass through angle DAE. Secondly, Descartes incorrectly states that the problem is "impossible" in case one finds $y = 0$ for all four angles.

Then Roberval turned to the relation between the position of the point C and the eleven regions determined by the four given straight lines ℓ_i in more detail. The four given lines ℓ_i divide the plane into bounded and unbounded regions. The bounded regions are triangles or quadrilaterals, such as the regions (4), (6) and (7) of figure 5.1. The unbounded regions stretch out to infinity at one side and are bounded at other sides by either two, three or four straight lines. Roberval then stated without proof that in all four line Pappus problems, solutions C can be found in:

- the bounded regions;
- the unbounded regions which are determined by three straight lines;[20]
- four of the six points of intersection between the four given straight lines.

The solution curves pass through these regions and through four points of intersection. Of the remaining regions, which are the unbounded regions determined by two lines, solutions C occur in all of them, or in all but one of them. For the Cartesian Pappus problem this means that the solutions C are in all regions, or in all regions except for one of the regions (1), (8) or (11). As we have seen, this is indeed the case, since the circle and hyperbola pass through all regions except for region (8). The observations of Roberval make

[20] Roberval considered the unbounded region (5) to be determined by three lines, whereas it is in fact determined by four straight lines. Solutions C can always be found in the unbounded regions determined by three or four straight lines.

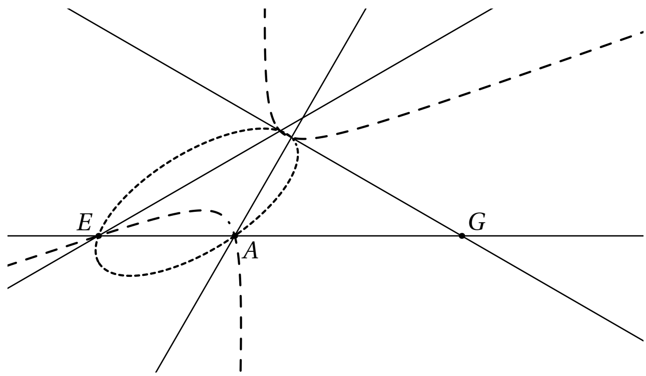


Figure 5.2 – The two conic sections, ellipse and hyperbola, which are solutions of $d_1 d_3 = d_2 d_4$.

it quite easy to sketch the second conic solution once the first conic is known, because one now knows the regions and four points through which the second conic section has to pass. Nevertheless, Roberval did not draw such a sketch in his letter.

However, Roberval also made a false statement to the following effect: if there are n unbounded regions determined by only two of the given lines, solutions C can be found in n or $n-1$ of these regions. A counterexample is the following variant of the Cartesian Pappus problem. Given are the same four straight lines ℓ_i and the same angles θ_i as in the *Géométrie*, but we now ask for all points C which satisfy $d_1 d_3 = d_2 d_4$ (i.e., $CB \cdot CD = CF \cdot CH$). The configuration of lines is the same as before, and thus we have three unbounded regions determined by two lines, namely the regions (1), (8) and (11) of figure 5.1. The solution of this case of the Pappus problem is shown in figure 5.2. We see that the two conic sections do not pass through regions (1) and (11), so Roberval's statement is false in general.

Roberval also stated that Descartes had not mastered the Pappus problem in full depth, because figure 3.6 is erroneous. He continued with the following explanation of the errors in the figure:

> The error of the good man [i.e. Descartes], in my opinion, results from [the fact] that he did not know that such a locus, to be perfect, requires two conic sections at the same time, and each [section] completely. By an entire conic section, I understand either one circumference of an entire circle, or an entire ellipse, or an entire parabola, or two entire [branches of] opposite hyperbolas which together make only one section, or two infinite straight lines which intersect each other; and in general, that [figure] which a plane can produce [by] cutting an entire conical surface, which is composed by two opposed cones with the apex of the one placed on the other, according to the definition of Apollonius. I say that two complete sections are needed, as two planes can make: in such a way that one circumference of a circle, for example, is not sufficient, but one usually needs another two opposite [branches of] hyperbolas

> for the locus to be perfect; and often four pairwise opposite hyperbolas are needed.[21]

Roberval's remark that the solution often consists of "four hyperbolas", that is in modern terminology, two hyperbolas each consisting of two branches, shows that he had studied the four line Pappus problem in depth, and for other configurations of lines than Descartes had done.

Roberval then turned towards Van Schooten's comments on the four line Pappus problem in *Geometria (1649)*. He refered in particular to comment E[22], that is, the comment on figure 3.6 for which Descartes had provided a draft in 1648. Of course, Roberval did not know that Van Schooten had modelled the comment on this draft by Descartes.

As we have seen, Van Schooten stated in comment E that figure 3.6 does not serve as an actual solution of the Pappus problem, but as an illustration of the variant construction of a hyperbola. He also stated that the hyperbola in figure 3.6 could be interpreted as a solution of an altered version of the Pappus problem with the condition $FC \cdot CD = CB \cdot CH + q$ for given $q > 0$, instead of $CB \cdot CF = CD \cdot CH$. Roberval especially attacked this second part of this defence:

> I know that Mister Schoten [on] page 197 of his commentaries on this geometry, tries to excuse the fault of his author, saying that it should sometimes be understood that rectangles are such that one is to the other (by a given greater than in ratio). But I would like for the honour of this learned man, whom I esteem highly, that he would have had less compliance with Descartes: For even if it were not true (which is nevertheless true) that the Ancients never understood it [i.e., the Pappus problem] that way, Descartes certainly did not understand it [i.e., the Pappus problem] that way; Descartes who expresses himself everywhere as the Ancients did, both in his proposition [i.e., definition] of the Problem and in his analysis and his conclusion.[23]

Thus, Roberval blamed Van Schooten for his slavish defence of Descartes, which made Van Schooten connect the figure 3.6 to a completely different problem, about rectangles

[21] "La faute du bon-homme vient, à mon auiz de ce qu'il n'a pas connu qu'vn tel lieu, pour estre parfait, demande deux sections à la fois, et chacune toute entiere. Par vne section entiere, J'entens ou vne circonference de cercle entiere, ou vne Ellipse entiere, ou vne Parabole entiere, ou deux hyperboles opposees entieres quj ne font ensemble qu'vne section, ou deux lignes droites infinies quj s'entrecoupent; et en general, ce que peut faire vn plan coupant vne superficie conique entiere, et composee des deux cornets opposez au sommet l'vn de l'autre, suiuant la definition d'Apollonius: Il faut, dis-je deux de ces sections entieres, autant qu'en peuuent faire deux plans: tellement qu'vne circonference de cercle, pour exemple, n'est pas suffisante, mais il luy faut encore pour l'ordinaire, deux hyperboles opposees, affin que le lieu soit tout parfait; et souuent jl faut quatre hyperboles opposees deux à deux." Roberval to Huygens, 6 July 1656 [Huygens, 1888, 450-451].

[22] [Schooten, 1649c, 197]. Comment E is discussed above on page 121.

[23] "Je scay que Monsieur Schoten page 197. de ses commentaires sur cette geometrie, tache d'excuser la faute de son auteur, voulant qu'il se doiue entendre quelquefois quand les rectangles sont tels que l'vn soit à l'autre (majus dato quàm in ratione). Mais Je voudrois pour l'honneur de ce scauant homme, que J'estime jnfiniment, qu'il eust eu moins de complaisance pour Descartes: car quand memes il ne seroit pas vray, ce quj est vray pourtant, que les anciens ne l'ont jamais ainsj entendu, nj Descartes aussj quj s'explique par tout comme les anciens, tant en sa proposition, qu'en son Analyse et en sa conclusion." Roberval to Huygens, 6 July 1656 [Huygens, 1888, 451].

which are “such that one is to the other (by a given greater than in ratio)”. This puzzling sentence needs some clarification. Roberval stated the part between brackets in Latin as “majus dato quàm in ratione.” The same term is found in Proposition 86 of Euclid’s *Data*,[24] as an abbreviation of definition 11 of that work:

> A magnitude is by a given greater than in ratio to a magnitude if, when the given magnitude be subtracted, the remainder has a given ratio to the same.[25]

If a magnitude M is by a given G greater than the magnitude L, which has to N a given ratio, we can say in modern terms that $M = L + G$ and $L : N$ is a given ratio, so $M - G : N$ is a given ratio.[26] Thus Roberval meant that in the context of the Pappus problem, Van Schooten suggested to replace the condition for C by $FC \cdot CD - q : CB \cdot CH = \tau$ for a given ratio τ. If we assume $\tau = 1 : 1$ we obtain the problem which Van Schooten had actually suggested, following Descartes, namely $FC \cdot CD = CB \cdot CH + q$.

Roberval continued:

> This is how he [i.e., Van Schooten] blindly defends a man [i.e Descartes] who, having the ambition of passing for impeccable, would rather have committed a thousand absurdities than willingly withdraw from an error about which I had warned him as a friend, before talking about it to anybody else.[27]

So Roberval told Huygens that he had pointed out an error to Descartes even before he shared it with other mathematicians. As we have seen above, Carcavi wrote in 1649 that Roberval had presented a demonstration about the four line Pappus problem to an assembly of mathematicians in Paris in 1637. If both statements are true, Roberval may have detected at least some errors in Descartes’s solution of the four-line Pappus problem soon after the publication of the *Géométrie*. On the other hand, Roberval may also have promoted his own case to Huygens, by presenting himself as a gentleman who first approached the author before making his criticisms publicly known.

In a letter dated July 25, 1656, Huygens informed Van Schooten about the main conclusions of Roberval’s letter, namely that the solution consists of two conic sections, and that the problem is never impossible. The letter contains a figure similar to Descartes’s figure 3.5, to which the second conic section had been added (somewhat as in figure 3.7). Huygens added that Roberval’s conclusions could be verified without “the trouble of algebraic computations,” and he insisted that Van Schooten adds Roberval’s conclusions to the comments in the second edition of *Geometria*. He even enclosed Roberval’s letter of July 6, 1656 itself, and he asked Van Schooten for his opinion on this matter.[28]

---

[24] In the 1625 edition, it reads “Si duae rectae datum spatium comprehendant in angulo dato, quadratum autem unius, quadrato alterio *maius dato, quam in ratione*, & utraque ipsarum data erit” [emphasis mine], [Euclid, 1625, 163].

[25] [Taisbak, 2003, 35]. In the 1625 Euclid edition this is formulated as “magnitudo, magnitude maior est, datâ. quàm in ratione, quando ablata datá, reliqua ad eandem habet rationem datam”, [Euclid, 1625, 18].

[26] For a more detailed discussion of the phrase I refer to [Taisbak, 2003, 57-83 and 207-213].

[27] “Voilà comme il en prent, de defendre aueuglement vn homme quj ayant l’ambition de paroitre impeccable, auroit plustost commises mille absurditez, que de se retracter de bonne grace d’vne faute dont Je l’auois aduerty en amy, auparauant que d’en parler à aucun autre.” Roberval to Huygens, 6 July 1656, [Huygens, 1888, 451].

[28] Christiaan Huygens to Frans van Schooten, 25 July 1656. [Huygens, 1888, 460-461].

5.2.2 Van Schooten tried to refute Roberval's views

On 28 July 1656, Van Schooten answered Huygens by a lengthy letter in which he tried to refute Roberval's conclusions.[29] Van Schooten did not enter into the mathematical details of the Pappus problem, rather he hid himself behind phrases of Descartes.

Van Schooten began by saying that he was unable to see what is wrong with Descartes's passage in page 326 of the *Géométrie*, concerning the impossibility of the four line Pappus problem, which Roberval had criticised so strongly. Van Schooten referred to the generality of the method of Descartes: every geometrical problem can be reduced to an equation, so every problem can be solved. If the equation does not have (positive) roots, it does not have a solution and the problem is impossible. Van Schooten concentrated on the role of the equation in the solution, and completely relied on the power of the algebraic approach, without taking into account the particular geometrical features of the problem. The point of Roberval was, however, that the four line Pappus problem can never be impossible, as can be seen by means of the geometric definition of the problem. Huygens had also stressed that Roberval's statements could be verified without the use of algebra, because the four points of intersection of the given lines are necessarily points of the solution.

Nevertheless, Van Schooten then went on to discuss the relationship between algebraic equations and the four line Pappus problem as well as the "composition of plane and solid loci", that is, the construction of circles and conic sections in general. Note that Van Schooten devoted comment F to this matter in *Geometria (1649).* In the *Géométrie*, Descartes performed for each second degree equation in two unknowns the Apollonian construction of the corresponding conic sections. Van Schooten praised Descartes for this accomplishment and added that Descartes had included the Pappus problem because this gave the "composition of solid loci". Thus he maintained that the solid loci, and not the Pappus problem, were after all the main subject matter for Descartes.[30]

Van Schooten continued that Descartes had on purpose not explained the simple matter of the Pappus problem in great detail. Here he repeated the argument which Descartes had provided in his draft note to figure 4.1 in spring 1648. With respect to figure 3.6, Van Schooten showed his irritation because Roberval raked up this topic again. In this connection, Van Schooten mentioned Descartes's aim for conciseness, and he was annoyed by Roberval's remark that "Descartes has betrayed that he has not understood this matter [i.e., the Pappus problem] in depth".[31] Van Schooten considered this remark impolite and undeserved. He responded to Roberval by a quotation which he attributes to Cato:

> While they want to reprimand someone, fools make opposite mistakes.[32]

[29] Frans van Schooten to Christiaan Huygens, 28 July 1656, [Huygens, 1888, 466-470].

[30] "E quibus manifestè perspicitur, quaestionem hanc ab ipso non aliter pertractatam esse, quàm, postquam ipsam ad aequationem perduxit, ut in eâ explicandâ Planorum simul et Solidorum Locorum compositionem exponeret." Frans van Schooten to Huygens, 28 July 1656, [Huygens, 1888, 467].

[31] Gilles Personne de Roberval to Christiaan Huygens, 6 July 1656, [Huygens, 1888, 450].

[32] "Dum culpare volunt, stulti in contraria currunt." Frans van Schooten to Huygens, 28 July 1656, [Huygens, 1888, 468]. The origin of this quotation is somewhat obscure, since the only instance where I found exactly the same passage is in the appendix by Alexander Anderson to François Viètes *De aequationum recognitione tractatus duo* of 1615, [Viète, 1615, 132]. This treatise, including Anderson's appendix, was reprinted in the

Thus Van Schooten believed that Roberval denounced Descartes but committed errors himself. Van Schooten believed that he had identified one such error in Roberval's criticism on comment E of *Geometria (1649)*. In his letter written in French, Roberval used the Latin phrase "majus dato quam in ratione", and van Schooten complained that the word *ratio* was not found in his comment E. Apparently he did not realize that Roberval's problem was even more general than the one that he had suggested in comment E.

There was more in Roberval's letter that Van Schooten did not understand correctly. In the same letter to Huygens of 28 July 1656, Van Schooten asked Roberval, by intermediary of Huygens, to explain in more detail what he meant when he wrote that

> Often 4 pairwise opposite hyperbolas are needed. By means of two such complete loci, the point C will be found in all regions which I [i.e., Roberval] have specified, and the problem can never be impossible.[33]

Though Roberval had been clear in his statement that the solution of a Pappus problem in four lines consists of two entire conic sections, Van Schooten asked for further explanation; he even thought that Roberval "doubtlessly" meant four conjugate hyperbolas. Four conjugate hyperbolas were defined in the *Conics* of Apollonius as four branches of two hyperbolas with a special relationship, which entails (among other things) that they have the same asymptotes. Because the four conjugate hyperbolas do not intersect, they cannot be the solutions of a four line Pappus problem. Van Schooten's reference to the four conjugate hyperbolas shows that he did not have a deep understanding of the second conic solution of the four line Pappus problem in 1656. Furthermore, since Van Schooten's letter is pervaded with reprimands, he may have believed that Roberval would realize by a more serious investigation that Descartes was right after all.

Huygens communicated Van Schooten's opinions to Roberval, and observed in this connection that "he [i.e., Van Schooten] always tries to excuse his master"[34]. Huygens told Roberval that he himself would be able to explain to Van Schooten that the solution of the Pappus problem consists of two entire conic sections, so Roberval did not have to engage in this task. Huygens must have waited for some months, for at the end of November 1656 Van Schooten asked him for his findings on the Pappus problem.[35]

1646 re-edition of Vietes collected work, edited by Van Schooten, [Viète, 1646]. Yet, Anderson does not mention Cato and to my knowledge, the passage is not found in the works of Cato (the Elder). A similar passage "dum vitant stulti vitia, in contraria currunt" (afraid of mistakes they might take, fools often make the opposite mistakes, [Horace, 2008, 14]) is found in the *Satires* of Horace in Book I, Satire ii line 24, see [Flaccus, 1608, 422], but without a reference to Cato. Later in the same Satire ii on line 33-35, a passage by Cato is cited by Horace. It seems to me that Van Schooten copied his passage from Anderson's appendix, and because he knew that it was inspired by the classics, he mixed up Horace and Cato at some point.

[33] "Et souvent il faut 4 Hyperboles opposées deux à deux. Par le moyen de deux tels lieux entiers, le point C se trouvera dans tous les espaces que j'ay specifiez, sans que le Probleme puisse jamais estre impossible." Frans van Schooten to Huygens, 28 July 1656, [Huygens, 1888, 469].

[34] "Il tasche tousjours d'excuser son maistre." Huygens to Roberval, August 1656, [Huygens, 1888, 485].

[35] Van Schooten to Huygens, 29 November 1656, [Huygens, 1888, 516].

5.2.3 Huygens's investigation of the four line Pappus problem

Huygens quickly responded to Van Schooten's request. In a letter dated 6 December 1656, he presented three different configurations of the four line Pappus problem, and showed that in each instance the solution consists of two conic sections.[36] He illustrated the three different cases with three different figures, which unfortunately have not been preserved with the text of the letter. The editors of the *Oeuvres Complètes* have been able to reconstruct the first two figures. They did not give a reason why the third figure had not been reconstructed. I will now show that their reconstruction of the first figure is probably incorrect, and that it is possible to reconstruct the third figure as well.

In the first case, Huygens discussed a configuration of the four line Pappus problem in which the required points C are on the circumference of a circle and on two opposite branches of a hyperbola. The reconstruction in the *Oeuvres Complètes* is a four line Pappus problem with a circle and a hyperbola as solution, but the configuration of the given four lines differs from that of the Cartesian Pappus problem. Since the solution of the Cartesian Pappus problem also consists of a circle and a hyperbola, I believe that it is more likely that Huygens discussed the Cartesian Pappus problem, that is, the situation of figure 3.1. Huygens stated that altogether four points of the solution will be found on the straight line BC, namely points C and O on the circumference of the circle $AKLG$, and points N and P on the two branches of the hyperbola ANG and $PKLM$. Note that the labels of the points of intersection A, G, E in Huygens's text match the labels of the Cartesian Pappus problem, as is also the case for the labels F, H, D (not drawn in the reconstructed figure for sake of clarity). If we suppose the line segment BC to be in the same position as in the *Géométrie*, we obtain the reconstructed figure 5.3. Huygens mentioned that he had "done the test" in point M.[37] By doing the test he must have meant: performing the calculations by supposing the point C at position M; the resulting equation must have shown to him that the curve is a hyperbola. He did not give these computations in his letter.

In the second case (see figure 5.4) the solution is a parabola and a hyperbola, and in this case Huygens gives the computations. The four given lines ℓ_i are given as follows: two lines ℓ_1 and ℓ_3 are parallels at a given distance a. The lines ℓ_2 and ℓ_4 intersect one another at point A on ℓ_1, and ℓ_2 and ℓ_4 intersect ℓ_3 at points G and E respectively, such that $AE = AG$. From any point C, the four distances $d_1 = CB, d_2 = CD, d_3 = CF, d_4 = CH$ are drawn at right angles to $\ell_1, \ell_2, \ell_3, \ell_4$ respectively. It is required to find all points C such that $d_1 d_2 = d_3 d_4$, that is, $BC \cdot CF = CD \cdot CH$.

Supposing C in angle DGQ, Huygens showed that the resulting equation is the equation of a parabola in essentially the same way as Descartes did in the Cartesian Pappus problem. Huygens put $AB = x$ and $d_1 = BC = y$. Then $d_3 = CD = y - a$. Since $CF : CQ$ and $AB : BQ$ are given ratios, he put $CF : CQ = a : p$ and $AB : BQ = a : q$ for given p and q. He then found the distance CF in the following way: Because $AB : BQ = a : q$ it follows that $BQ = \frac{q}{a}x$. From the figure we see that $CQ = BQ - BC$ and thus $CQ = \frac{q}{a}x - y$. This yields

$$d_2 = CF = \frac{a}{p}CQ = \frac{qx - ay}{p}.$$

[36] Huygens to Van Schooten, 6 December 1656, [Huygens, 1888, 519-524].

[37] "Ego in puncto M periculum feci." Huygens to Van Schooten, 6 December 1656, [Huygens, 1888, 519].

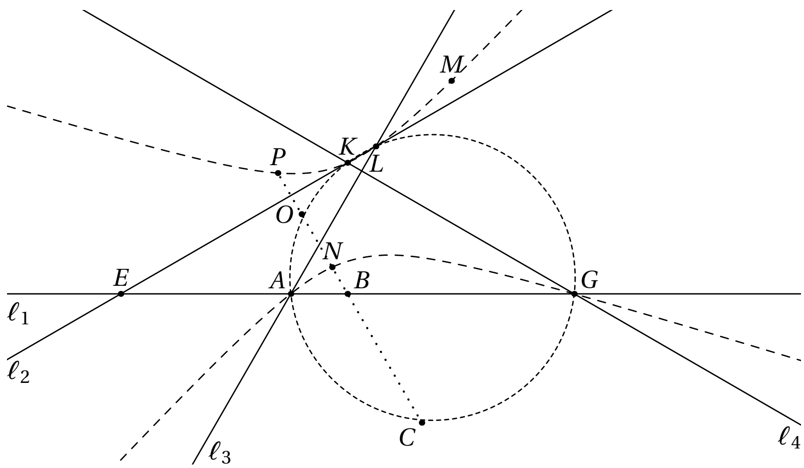


Figure 5.3 – The two conic section solutions (dashed) of the Cartesian Pappus problem, together with the extended line BC (dotted). My reconstruction of Huygens's illustration of his letter of 6 December 1656.

By using the similarity of the triangles FCQ and HCR, and the fact that $BR = BQ$ because $AG = AE$ by the definition of the problem, Huygens showed

$$d_4 = CH = \frac{qx + ay}{p}.$$

By multiplication of the four d_i's, Huygens showed that the points C satisfy the equation

$$y^2 = \frac{a}{2}y + \frac{q}{2}x. \tag{5.1}$$

Solving the equation for y yields

$$y = \frac{a}{4} \pm \sqrt{\frac{a^2}{16} + \frac{q}{2}x}. \tag{5.2}$$

Referring to Descartes, Huygens concluded that this is the equation of a parabola. He also specified the data for the (Apollonian) construction of the parabola.[38]

To obtain the second conic solution, Huygens considered point C_1 in the region between ℓ_1 and ℓ_3 with the required property. By the same reasoning as in the previous case, he deduced the expressions for the four distances d_i and he obtained the following equation of the curve:

$$xy = \frac{a}{2}x + \frac{a^2}{2q}y. \tag{5.3}$$

[38] In modern terms, the equation can be rewritten as $(y - \frac{1}{4}a)^2 = \frac{1}{2}q(x + \frac{a^2}{8q})$. Therefore, in modern coordinates the axis is $y = \frac{a}{4}$, the corresponding latus rectum is $\frac{1}{2}q$ and the vertex has coordinates $(-\frac{a^2}{8q}, \frac{a}{4})$.

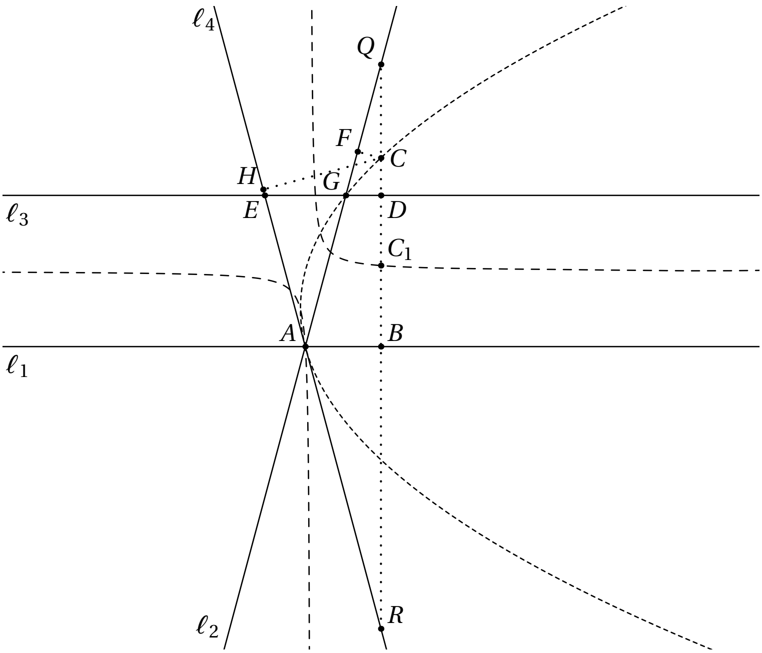


Figure 5.4 – A parabola and a hyperbola are solutions of Huygens second example. Based on the figure of [Huygens, 1888, 520].

On the basis of De Beaune's *Notae Breves*, Huygens concluded that this curve is a hyperbola with asymptotes $x = \frac{a^2}{2q}$ and $y = \frac{a}{2}$.[39] Once the asymptotes and one point on the hyperbola are known, the hyperbola can be constructed by the method of Apollonius in *Conics* II.4.

These two examples show that Huygens had carefully studied different configurations of the four line Pappus problem, in which different types of conic sections occur. Huygens's main conclusion was that

> the locus of points will always be two entire conic sections: that is, [figures, each of which is] produced by the intersection or tangency of one plane in [two] opposite cones.[40]

This conclusion of Huygens agreed with that of Roberval. Huygens mentioned that also a single line may occur as one of the conic sections, for instance if the four given lines

[39] In the fifth observation of the *Notae Breves*, De Beaune showed that quadratic equations in two unknowns x and y of the form $axy + bx + cy + d = 0$ (thus without a term in x^2 or y^2) are hyperbolas. He achieved this result by considering seventeen different cases. In each case he specified a hyperbola with given asymptotes passing through a given point P which corresponds to an equation of this form. The equation (5.3) belongs to the fourth case, see [Beaune, 1649, 141-146].

[40] "Locus puncti semper erunt duae coni sectiones integrae: hoc est, quas in oppositis conis unius plani intersectio vel contactus efficit." Huygens to Frans van Schooten, 6 December 1656, [Huygens, 1888, 522].

are "two pairs of parallels intersecting each other".[41] In that case, Huygens explained, the locus will consist of two opposite branches of a hyperbola and a single straight line. In his letter, Huygens referred to a third figure, which is now lost and which has not been reconstructed by the editors of the *Oeuvres Complètes*. In the following reconstruction I will use modern coordinates, so all quantities x, y etc. may assume all real values.

Let $\ell_1 : y = 0$ and $\ell_2 : y = a$ be two parallel lines at distance a, and let $\ell_3 : y = bx$ and $\ell_4 : y = bx + c$ also be two parallel lines, not necessarily at the same distance a, with a, b, c three constants. We assume that the distances d_i are perpendicular to ℓ_i. Then we have for a point with coordinates (x, y)

$$d_1 = |y| \tag{5.4}$$

$$d_2 = |y - a| \tag{5.5}$$

$$d_3 = \left| \frac{y - bx}{\sqrt{1 + b^2}} \right| \tag{5.6}$$

and

$$d_4 = \left| \frac{y - bx - c}{\sqrt{1 + b^2}} \right|. \tag{5.7}$$

From these expressions for the distances it is clear that the locus of the points satisfying $d_1 d_3 = d_2 d_4$ consists of one straight line together with a hyperbola, and that the same is true for the points satisfying $d_1 d_4 = d_2 d_3$, see figure 5.5.[42] Therefore I believe that the figure which Huygens included resembled 5.5b or 5.5c. The other possible Pappus problem for the same two pairs of parallel lines, with the points satisfying $d_1 d_2 = d_3 d_4$, produces a hyperbola and an ellipse as its solution.

In the rest of the letter, Huygens investigated the following difficulty in the solution of the Pappus problem:

> We do not immediately understand what makes three or even four lines useful for the proposed [question] while the equation only has two dimensions.[43]

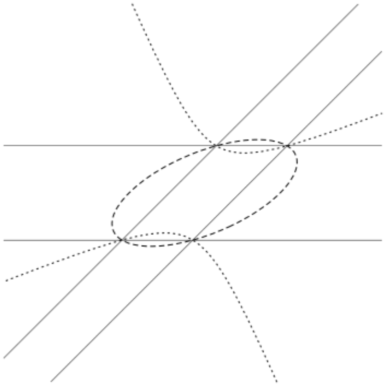
The difficulty is as follows. The algebraic equation in two unknowns is of at most second degree, so if we take one of the unknowns, say x, as a constant, the equation can be considered as a quadratic equation in y, having at most two line segments as roots. However, figure 5.3 shows that for $x = AB$, there are exactly four roots, namely the line segments BC, BO with endpoints C, O on the circle, and the line segments BN and BP with endpoints N and P on the hyperbola.

Huygens wondered what is going on here, and in order to shed some light on the matter, he first examined a problem which can be considered a Pappus problem in one dimension:

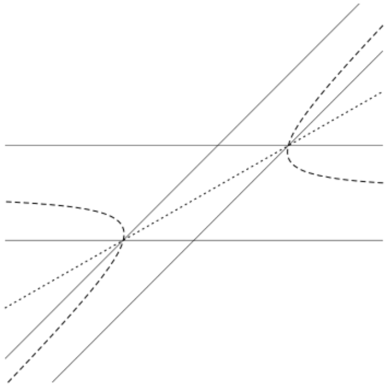
[41] "Bina sunt parallelarum paria sese intersecantia." Christiaan Huygens to Frans van Schooten, 6 December 1656, [Huygens, 1888, 522].

[42] The equations of the straight lines are $(a+c)y = ac + abx$ for $d_1 d_3 = d_2 d_4$ and $(a-c)y = abx$ for $d_1 d_4 = d_2 d_3$.

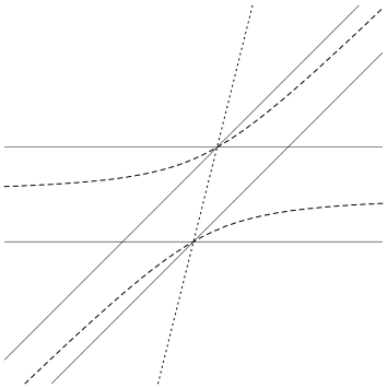
[43] "Non statim intelligamus qui fiat ut cum aequatio duas tantum dimensiones preferat, tres tamen quatuorve lineae sint proposito utiles." Christiaan Huygens to Frans van Schooten, 6 December 1656, [Huygens, 1888, 522].



(a) $d_1 d_2 = d_3 d_4$



(b) $d_1 d_3 = d_2 d_4$



(c) $d_1 d_4 = d_2 d_3$

Figure 5.5 – The three possible Pappus problems for two pairwise parallel lines (drawn) and the solution (dashed and dotted).

Problem 2 (**Division of a line (1)**[44]).
Given: four points A, B, C and D on a straight line such that $AC = 5$, $CB = 4$ and $BD = 3$, see figure 5.6. It is required to find all points E on the line such that $AE \cdot EB = EC \cdot ED$.

Huygens first supposed E between A and C, and he put $AE = x$. Then $EB = 9 - x$, $EC = 5 - x$ and $ED = 12 - x$. An easy calculation led him to the quadratic equation $13x - 30 = x^2$ with roots $x = 10$ and $x = 3$. These two roots correspond to two points E, located between

[44]This problem is a numerical example of the general problem of Apollonius's *On Determinate Section*. This work was annotated by Pappus in Book VII of his *Collection*, but the text by Apollonius is lost. In the seventeenth century, several mathematicians tried to "restore" the works of Apollonius from the annotations by Pappus, and in 1608 Willebrord Snellius published his reconstruction of *On Determinate Section* in [Snellius, 1608]. In the letter, Huygens explicitly refers to Snellius. For a recent study on Snellius see [Wreede, 2007]. The original problem solved by Apollonius was as follows: Given are four points A, B, C and D on a straight line, and a ratio τ. It is required to find the points E on the straight line satisfying $AE \cdot EB : EC \cdot ED = \tau$. In the numerical example, Huygens took $\tau = 1:1$.

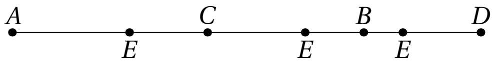


Figure 5.6 – Problem 2 (Division of a line (1)). Based on the figure of [Huygens, 1888, 520].

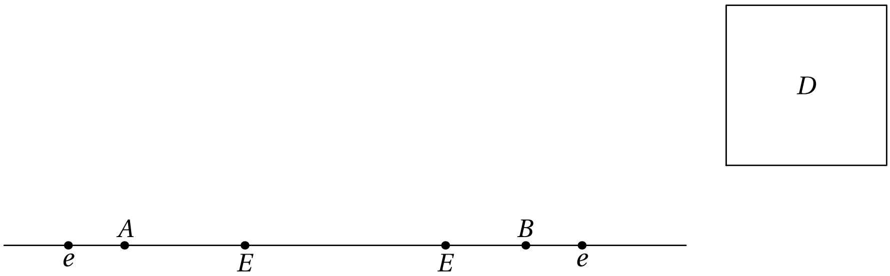


Figure 5.7 – Problem 3 (Division of a line (2)).

A and C and between B and D. However, Huygens noted that there is a third solution E between C and B such that $AE = 7\frac{1}{2}$, which also satisfies $AE \cdot EB = EC \cdot ED$.

Huygens then turned to the following problem, where the same situation is even clearer:

Problem 3 (Division of a line (2)).
Given two points A and B on a straight line, and an area D with $0 < D < \frac{AB^2}{4}$, see figure 5.7. Required: all points E on the straight line such that $AE \cdot BE = D$.

The restriction $0 < D < \frac{AB^2}{4}$ assures that the problem has two different solutions E between A and B. There is always one solution E on AB extended on the side of B and another solution on AB extended on the side of A, so the problem has four solutions. The corresponding algebraic equations will be of at most degree two, as AE and BE are linear expressions in x and D is a constant.

Huygens argued that if the point E is supposed between A and B, the resulting equation has two "true" roots, that is, two positive roots. However, if the point E is supposed either on the left side of A or on the right side of B, the equation (obtained by putting $AE = x$) will produce a "false" and a "true" root, that is, a negative and a positive root. Huygens did not give more details, but putting $AE = x$, $AB = a$ and $D = d^2$ we obtain the equation $x(a - x) = d^2$ for E between A and B, and the two equations $x(x \pm a) = d^2$ for E not between A and B. The sign depends on whether E is taken left of A (plus sign) or right of B (minus sign), but these two equations yield the same two points on the straight line, namely the two points E which are not between A and B. Huygens must have been aware of this fact and therefore he argued that two equations of degree two are needed for a complete solution of the problem. This means that one should suppose E at two different positions (between A and B, and not between A and B), and determine the corresponding equation for each position. According to Huygens, it was due to the "nature of the matter [i.e., this problem]" that two different equations are needed and that it is not possible to contain all solutions in a single equation.[45]

Huygens then stated that a similar issue is at stake in the Pappus problem if the point C is supposed in different angles, but he indicated to Van Schooten that it is hard to investigate exactly these different positions.[46] The beginning of his letter suggests that the Pappus problem kept him busy for a longer time than he had foreseen at the beginning.[47]

At the end of the letter, Huygens stated that his explanations were clear enough to take away any astonishment about the second conic solution. Furthermore, he appealed to Van Schooten to include a note on the second conic solution in the second Latin edition of the *Geometria*.

5.2.4 Van Schooten's response to Huygens

Van Schooten answered Huygens's letter on 12 December 1656.[48] He thanked Huygens for his explanations, and did not reject the examples and the results, but tried to reconcile them with the Cartesian views. He suggested that Huygens's explanations about the second conic solution were in line with the text of the *Géométrie* and also agreed with the intellectual legacy of Descartes. To strengthen his argument, Van Schooten cited the following passage from the *Géométrie*:

> And if the quantity y turns out to be equal to nil, or less than nothing in this equation[49], if the point C is supposed to be in the angle DAG, one should also suppose it to be in the angle DAE, or EAR or RAG, by changing the signs[50] + and − according to what is required for that.[51]

Van Schooten concluded that in Descartes's opinion, the problem was completely solved only if the point C was supposed to be in all four angles around A, and he considered this to be equivalent to the findings of Huygens. He also adduced Descartes's statement to the effect that when y is less than zero, one had to suppose C in another (opposite) angle. Van Schooten added that Descartes had not yet explained the concept of a "false root" (in modern terms a negative root of an equation), which belonged to the subject matter of Book III. Therefore Descartes did not mention "false roots" in Book II. Here, Van Schooten took the structure of the *Géométrie* as an explanatory factor.

[45] Christiaan Huygens to Frans van Schooten, 6 December 1656, [Huygens, 1888, 524].

[46] See Chapter 3 above, especially Section 3.3.

[47] "Praeter opinionem diutius huic rei immoratus sum." Christiaan Huygens to Frans van Schooten, 6 December 1656, [Huygens, 1888, 519].

[48] Van Schooten to Huygens, 12 December 1656, [Huygens, 1888, 526-527]. In the *Oeuvres Complètes* there is some uncertainty about the date of the letter, which is given in the text as 21 December and in the heading as 12 December. The recent inventory of the letter by Joella Yoder solves the matter by setting the date on 12 December 1656, [Yoder, 2013, 248].

[49] The equation $y^2 = \frac{(cfglz-dekz^2)y-(dez^2+cfgz-bcgz)xy+bcfglx-bcfgx^2}{(ez^3-cgz^2)}$ which is equation (3.13) of page 89.

[50] The *Géométrie* reads "lignes", but this should be "signes".

[51] The quoted passage is found in [Descartes, 1637c, 326].

The same passage from Descartes had also been quoted by Roberval in his letter of 6 July 1656.[52] Whereas Roberval used the quotation in order to draw attention to an error in Descartes's treatment of the Pappus problem, Van Schooten cited the same passage to show that the Descartes had already dealt with the issues which Huygens had investigated in detail. Thus Van Schooten was once again defending Descartes.

At the end of the letter, Van Schooten promised Huygens that his suggestions would be mentioned in the second edition of the *Geometria*. Indeed, Van Schooten added a comment BB on the above-mentioned passage of the *Géométrie*. I will discuss his comment in the next section.

The two letters by Van Schooten on the second conic solution (28 July 1656 and 12 December 1656) have in common his consistent defense of Descartes. However, there is a striking difference in the tone of both letters. In the summer of 1656. Van Schooten reacted furiously to the criticism by Roberval, but later in 1656 the tone of his reaction towards Huygens was much milder. What were the causes for the change of tone of Van Schooten?

First of all, there is the factor of the personal relationships. In the letter to Huygens, which started the debate, Roberval criticized not only the mathematics of Descartes, but also the person of Descartes,[53] and the attitude of Van Schooten. These personal attacks may have hurt Van Schooten even more than the mathematical criticisms. By calling Descartes a "bonhomme"[54] Roberval touched the person of Descartes, whom Van Schooten held in great admiration. In the same letter, Roberval also accused Van Schooten of blindly following Descartes. Of course, the letter was sent by Roberval to Huygens, who forwarded it to Van Schooten, so Van Schooten was perhaps not supposed to read the letter.

The relationship between Huygens and Van Schooten was completely different. Van Schooten had been Huygens's mathematics teacher since Huygens entered Leiden University in the spring of 1645 at the age of sixteen. Though Huygens only stayed for two years in Leiden — he was sent to the Collegium Aurasiacum in Breda in 1647 by his father — he kept in contact with Van Schooten until the death of the latter in 1660.

Secondly, there is the following difference in the mathematical explanations by Huygens and Roberval. Roberval pointed out the errors in Descartes's treatment of the Pappus problem, and he mentioned some of his own discoveries, but he did not provide proofs or explanations how these results had been found. For instance, it remains unclear on the basis of which arguments Roberval arrived at his insights on the solution curves in different regions of the plane (see page 132). It is possible that he had not found the second conic section in the solution by means of the Cartesian algebraic methods but by other (geometrical) arguments. Whereas Roberval gave the correct results, it was not clear to Van Schooten how these results had been obtained.

Huygens, on the contrary, used algebraic equations in his letter, and thus he showed that the second conic solution can be found by means of the Cartesian method. After the explicit calculations by Huygens, Van Schooten could not dismiss the second conic solution anymore. He was defeated by his own, or rather Descartes's, arms. As we will

[52] See page 131. Van Schooten omitted the last sentence that had been quoted by Roberval.

[53] It is well known that Roberval and Descartes were not on good terms; on their relationship see [Jullien, 1998].

[54] The French "bonhomme" has a rather negative connotation, [Huygens, 1888, 451].

see in the next section, Van Schooten used Huygens's explanations as an opportunity to find new interpretations of certain passages of the *Géométrie*, in agreement with the new insights.

5.3 Comments in the 1659 edition

The new insights in the four line Pappus problem challenged Van Schooten to change his commentaries. In the second edition of *Geometria* of 1659 Van Schooten added three new comments (BB, CC, CCC) of considerable length, and he also made some changes in earlier comments. I have listed all comments on the Pappus problem in table 5.1. In this section I will discuss the new comments and the changes, and I will show that the correspondence with Huygens had a major influence, in particular on comment BB in which Van Schooten acknowledged the existence of the second conic solution.

5.3.1 Van Schooten's comments related to the second conic solution

In *Geometria (1659)*, two comments deal with the second conic solution, namely BB and E. Comment BB was to a large extent inspired by the letter of Huygens of 6 December 1656, and comment E was slightly altered as compared to the text in the 1649 edition. I will first discuss comment BB and then turn to the modifications in comment E.

Comment BB relates to the passage on page 326 of the *Géométrie* (see page 131) which Roberval had criticised, and which Van Schooten had used to defend Descartes.[55] In the first part of comment BB, Van Schooten discussed the second conic solution of the four line Pappus problem, and in the second part he presented one of the problems which Huygens had introduced in his letter to Van Schooten of 6 December 1656.

Van Schooten's reasoning is essentially the same as in his letter to Huygens of 21 December 1656. Again, Van Schooten claimed that his passage should be understood in the following way: in order to find the complete solution of the problem, one has to suppose the point C in the four angles around A (that is, in angles DAG, DAE, EAR and RAG), and thus four different equations can be obtained. Once one of these equations has been found, the other three can be produced from the first one by changing plus and minus signs. Van Schooten did not explicitly indicate how to proceed in order to obtain these three equations, nor did he give an explicit calculation for one of the three other angles DAE, EAR and RAG. Moreover, he did not discuss the relation between the equations and the curve or curves.

Van Schooten then went on to defend Descartes's claim that if y is found equal to zero for all four angles, the problem cannot have a solution. He argued that Descartes had treated the four line Pappus problem only in order to present a complete description of

[55] [Schooten, 1659c, 179-181].

Letter	Sentences of *Géométrie* the comment refers to	*Géométrie* pp:	*Geometria* pp:	*Number of pages of comments*
B	au moins en supposant ez plus grand que EG (...) changer tous les signes + & −.	326	178	1
BB	Et si la quantité y (...) la question seroit impossible au cas proposé.	326	179-181	3
C	Or cela fait, (...) pas plus malaysée a trouver qu'IL.	328	181-182	1
CC	Mais lorsque cela n'est pas, ce point C est toujours en l'une des trois sections coniques, etc.	328	182-206	24
CCC	Au moyen dequoy il est (...) 1er Problesme du 1er livre d'Apollonius.	329	206-223	17
D	A sçavoir si ce costé droit est $\sqrt{\frac{o^2z^2}{a^2}+\frac{4mpz^2}{a^2}}$ le traversant est $\sqrt{\frac{a^2o^2m^2}{p^2z^2}+\frac{4a^2m}{pz^2}}$.	330	223-224	1
E	Mais quand cete section estant une Hyperbole etc.	331	224-225	2
F	Car ces lieus ne sont autre chose (...) pour estre entierement determiné.	334-335	225-227	3
G	Et s'il manque deux conditions a la determination de ce point, le lieu ou il se trouve est une superficie, laquelle peut estre tout de mesme ou plate ou spherique ou plus composée.	335	228-236	9

Table 5.1 – Comments on the Pappus problem in *Geometria (1659)*.

plane and solid loci (i.e., of all conic sections). Van Schooten stated that $y = 0$ could happen in cases where the required point C was supposed to satisfy other conditions than those in the Pappus problem. This defense is not very strong.

Furthermore, Van Schooten stated that in the configuration of the Pappus problem that had been studied by Descartes, the solution not only consists of a circle, but also of two opposite branches of a hyperbola. He indicated that one branch of this hyperbola passes through the points A and G, and that the opposite branch passes through the points of intersection of ℓ_1 with ℓ_3 and with ℓ_4. These are exactly the observations of Roberval, which Huygens urged Van Schooten to include in the second edition. Van Schooten provided neither an updated figure with the hyperbola nor an explicit equation of the hyperbola.

Then Van Schooten briefly discussed the configurations of the Pappus problem that had been worked out by Huygens in his letter of 6 December 1656, but without the explicit calculations. Without reference to Huygens, Van Schooten claimed that in case two of the given lines are parallel, the solution will consist of either a parabola and a hyperbola, or of two different hyperbolas, or of two opposite branches of a hyperbola and a straight line. He presented these claims without arguments.

In the second part of his comment BB, Van Schooten discussed other problems, which in his opinion were related to the same issues as Descartes's solution of Pappus problem. He first referred the reader to his own reconstruction of the *Plane Loci* of Apollonius,[56] and noted that this also contained a problem where the points are shown to be on some locus (straight line or circle), but can also be located on another locus if they are supposed in another part of the plane. Van Schooten did not refer to a particular problem or reconstruction, and thus the reference is rather vague.

Then Van Schooten discussed one of the problems which had been proposed to him by Huygens,[57] and which I have called Problem 3. Recall that in this problem two points A and B are given, and an area D with $0 < D < \frac{AB^2}{4}$. It is required to find the points C on the straight line through A and B such $AC \cdot BC = D$. Van Schooten put $AB = a$, $D = d$, and $AC = x$ and he discussed the corresponding equations in more detail than Huygens in his letter. Van Schooten explicitly assumed the point C in the three possible positions with respect to A and B. Each of these cases will lead to a different equation, as can be seen in table 5.2. Recall that x and a are positive quantities. In the rest of the example, Van Schooten continued with the numerical example $a = 20$ and $d = 96$, and he showed that the three equations yields six roots, which correspond to only four different points on the straight line. He noted that this is related to the fact that in the first case x is directed to the left, while in the other two cases x is directed towards the right. In modern terms, the four roots of the first and third equation differ only in sign, and correspond to only two points on the straight line.

For sake of clarity I will explain Van Schooten's reasoning for the general case. In modern terms, we can apply the transformation $x \mapsto -x$ to the roots $x = -\frac{a}{2} \pm \frac{1}{2}\sqrt{a^2 + 4d}$ of

[56] This reconstruction appeared under the title *Apollonii Pergaei Loca Plana Restituta* (Apollonius's of Perga Plane loci restored) as the third book of the *Exercitationum mathematicarum libri quinque*, [Schooten, 1657, 203-292].

[57] Huygens mentioned the problem in his letter of 6 December 1656, see above, page 143.

Case	Position of C	AC	BC	Equation	Roots
1	Left of A	x	$a+x$	$x(x+a)=d$	$x=-\frac{a}{2}\pm\frac{1}{2}\sqrt{a^2+4d}$
2	Between A and B	x	$a-x$	$x(a-x)=d$	$x=\frac{a}{2}\pm\frac{1}{2}\sqrt{a^2-4d}$
3	Right of B	x	$x-a$	$x(x-a)=d$	$x=\frac{a}{2}\pm\frac{1}{2}\sqrt{a^2+4d}$

Table 5.2 – The equations related to the position of C.

the equation of case 1, and thus obtain the roots $\frac{a}{2}\pm\frac{1}{2}\sqrt{a^2+4d}$ of the equation in case 3. Thus Van Schooten's algebraic expressions had to be interpreted within the framework of the geometrical problem. Without knowledge of the geometrical context it was not clear that the roots of the form $\alpha\pm\beta$ and $-\alpha\pm\beta$ indeed refer to the same points on the line.

Van Schooten then wanted to show that Huygens's problem could be expressed by a single equation. He knew that for two equations $p(x)=0$ and $q(x)=0$, the product $p(x)q(x)=0$ has the same roots as the two individual equations together. Van Schooten argued that for Huygens's problem, one equation with six roots can be obtained. In modern terms, Van Schooten's idea implies bringing the three equations of table 5.2 in the form $f_i(x)=0, i=1,2,3$ and then multiplying the three $f_i(x)$:

$$\begin{aligned}
0 &= \prod_{i=1}^{3} f_i(x) \\
&= (x(x+a)-d)\cdot(x(a-x)-d)\cdot(x(x-a)-d) \\
&= -x^6+ax^5+a^2x^4-(a^3+2ad)x^3+(a^2d+d^2)x^2+ad^2x-d^3
\end{aligned} \tag{5.8}$$

In order to compute this equation (5.8), one should first find the three equations $f_i(x)=0$. Thus Van Schooten's multiplication is an artificial way of arriving at a single equation, which is not at all useful in the geometric solution of the problem.

Van Schooten then introduced his new concept of *universal equation* for the same problem 3 (see above on page 143) to determine all points C on a given straight line such that $AC\cdot BC=d$. Putting $AC=x$ and $CB=\pm x\pm a$,[58] we find the equation

$$\pm x^2\pm ax=d. \tag{5.9}$$

Van Schooten manipulated this equation in order to get rid of the $\pm$-signs, in order to bring the equation in what he calls the normal mode. He isolated the terms with $\pm$-sign at one side, and then squared both sides. If there are any terms left, he repeated this procedure, until all $\pm$-signs disappeared. He illustrated this manipulation with the help of the above-mentioned equation (5.9), which I explain in general form whereas he used the numerical example $a=20, d=96$. Squaring both sides produces

$$x^4\pm 2ax^3+a^2x^2=d^2,$$

[58] Van Schooten used a special notation ♌ which is the equivalent of the modern $\pm$, [Schooten, 1659c, 181].

then isolating the term $\pm 2ax^3$ yields

$$\pm 2ax^3 = d^2 - a^2x^2 - x^4,$$

and squaring both sides again and bringing all terms to one side yields finally

$$d^4 - 2a^2d^2x^2 - (2d^2 - a^4)x^4 - 2a^2x^6 + x^8 = 0. \tag{5.10}$$

Van Schooten claimed that this equation (5.10), which he calls the universal equation, is important because it has the same roots as equation (5.9). We note that equation (5.10) can also be obtained by multiplication of all permutations of $x(\pm a \pm x) - d$, that is, as $(x^2 + ax - d)(x^2 - ax - d)(-x^2 + ax - d)(-x^2 - ax - d) = 0$.

The introduction of the universal equation is problematic for several reasons. The first problem concerns the geometrical interpretation of the algebraic expression. In the definition of equation (5.9), Van Schooten took all possible permutations $\pm x \pm a$ into account, including $-x - a$, in the expession $(x(-x - a) - d)$. In the geometrical context, this expression should be equivalent to $AC \cdot CB - d = 0$ with $AB = x$, but above in table 5.2 we have seen that we never find $CB = -x - a$. Thus, $-x - a$ does not have a geometrical interpretation in the context of this problem. Van Schooten mentioned that the two roots in his numerical example of $-x^2 - ax - d = 0$ are negative, but without giving these roots a geometrical interpretation.

A second, more serious, problem with the universal equation is the way in which Van Schooten proceeded from the equation (5.9) with $\pm$-signs to the equation (5.10) without such signs. As we have seen, his algorithm consists of isolating the terms with $\pm$- signs in equation (5.9), squaring both sides of the equation, isolating the terms with the $\pm$-sign and continuing this procedure until the $\pm$-signs have disappeared. However, this method only works when the number of $\pm$-signs is less than three, as I will now explain. Suppose we have n $\pm$-signs in an equation:

$$\pm a_n x_n \pm a_{n-1}x_{n-1} \pm \cdots \pm a_1x_1 = d. \tag{5.11}$$

Squaring the left side of the equation produces a new expression with $n + \frac{n(n-1)}{2}$ terms. Of these $n + \frac{n(n-1)}{2}$ terms, n terms are of the forms $(\pm a_i x_i)^2 = a_i^2 x_i^2$ with a plus sign. The other $\frac{n(n-1)}{2}$ terms are of the form $\pm 2a_i a_j x_i x_j$ with a $\pm$ sign. Isolating these terms, we obtain a new equation of the form

$$\sum_{\substack{i<j \\ i \neq j}} \pm 2a_i a_j x_i x_j = d'. \tag{5.12}$$

The algorithm works only when the number of $\pm$ signs decreases, that is to say, if $\frac{n(n-1)}{2} < n$. Unfortunately this is only the case when $n = 1$ or $n = 2$, and the number of $\pm$ signs will even increase if $n > 3$.

Van Schooten used Huygens's problem about points C on line AB to address the fact that two equations (for two conic sections) are necessary to obtain the complete solution of the four line Pappus problem. He suggested that the two conic sections can be obtained by a universal equation and he finished comment BB by asking why such an universal equation is not mentioned in the text of the *Géométrie*.[59] Van Schooten's explanation is as follows. Descartes had intended the equation in x and y for the four line Pappus problem to be a universal equation. While working on the equation, however, Descartes had found out that the Pappus problem was quite hard, and moreover not easy to explain. Thus he had only performed the full investigation for a point C inside the angle DAG. Descartes had omitted the rest of the explicit investigation, but he had indicated how one should proceed, by stating that one should also suppose the point C in the three other angles DAE, EAR and RAG. Van Schooten claimed that after this investigation, it would be possible to deduce the universal equation which holds for all points C which belong to the solution, and that this was what Descartes had intended.

Van Schooten did not provide further information on this universal equation, such as the degree for instance, and one may well ask what sort of universal equation he could have had obtained for the four line Pappus problem.

Van Schooten found his universal equation for Problem 3 by taking the unknown quantity x positive and by considering all possible permutations of the plus and minus signs for the other (known and unknown) quantities. In the four line Pappus problem, one would have to proceed somehow as follows. We have to take all possible permutations of the plus and minus signs in the expression for the d_i which Descartes derived in his solution on page 88 above. We will assume that y is positive, because we have

$$d_1 = BC = y. \tag{5.13}$$

For EB, Descartes found one of the expressions $x - k$, $x + k$, or $-x + k$, depending on the position of B with respect to E and A. For the universal equation, we will take

$$BE = \pm x \pm k, \tag{5.14}$$

including the impossible case $BE = -x - k$ Thus we find

$$BS = \frac{d}{z}EB = \frac{d}{z}(\pm x \pm k).$$

For CS, Descartes found one of the expressions $BS + BC$, $BS - BC$ or $-BS + BC$. Therefore

$$CS = \pm BS \pm BC = \frac{\pm dx \pm dk \pm yz}{z},$$

so

$$d_2 = CF = \frac{e}{z}CS = \frac{\pm edx \pm edk \pm ezy}{z^2}. \tag{5.15}$$

Analogously we find

$$d_3 = CD = \frac{\pm czy \pm bcx}{z^2}, \tag{5.16}$$

[59] Note that the term "universal equation" is nowhere mentioned in the *Géométrie* nor in the correspondence of Descartes.

and

$$d_4 = CH = \frac{\pm fgl \pm fgx \pm gyz}{z^2}. \tag{5.17}$$

Then the condition $BC \cdot CF = CD \cdot CH$ can be rendered in the form of the equation:

$$\frac{\pm edxy \pm edky \pm ezy^2}{z^2} = \frac{cg}{z^4}\left(\pm flzy \pm fzxy \pm z^2y^2 \pm bflx \pm bfx^2 \pm bzxy\right). \tag{5.18}$$

There are 9 $\pm$-signs, but we can eliminate one sign by division, so $2^8 = 256$ different second-degree equations are contained in this general form. We now write all of them in the form $f_i(x, y) = 0$, and then multiply the $f_i(x, y)$ to obtain the universal equation:

$$\prod_{i=1}^{2^8} f_i(x, y) = 0. \tag{5.19}$$

The degree of this universal equation in y is in general $2^9 = 512$ and its explicit calculation would be unfeasible for Van Schooten. His algorithm for the explicit calculation fails in this case, because the number of $\pm$-signs is greater than two. The universal equation offers no insight in the problem nor does it facilitate the finding of the solution curves. Therefore we can safely exclude the possibility that Van Schooten ever computed a universal equation for the four line Pappus problem.

The second conic solution also forced Van Schooten to reconsider comment E of the 1649 edition of 4.4. Recall that this comment was inspired by the letter of Descartes to Van Schooten in which Descartes defended the way in which the hyperbola in figure 3.6 was drawn. In the 1649 edition, Van Schooten had written that the solution to the Pappus problem consisted of a circle only, and that the hyperbola in Descartes's figure was the solution of an alternative problem to find points C such that $CF \cdot CD = CB \cdot CH + q$ for a given quantity q, instead of $CB \cdot CF = CD \cdot CH$. We have already identified the purpose of figure 4.1 as an illustration of the variant construction of a hyperbola.

By 1659, Van Schooten knew that the second conic solution existed and that it was a hyperbola in the configuration of the Pappus problem which Descartes had studied. Van Schooten could have determined its equation by the same method which Descartes had used for finding the equation of the circle. Recall that this hyperbola has to be constructed by the variant construction. Thus Van Schooten could have decided to adapt his comment E in this way, and to use this second conic solution as an illustration of the variant construction of a hyperbola in general. He could have added a new figure showing this hyperbola, and the reference to the alternative problem $CF \cdot CD = CB \cdot CH + q$ could have been omitted.

But this is not what Van Schooten did. In his comment E he made only two minor changes which concern the existence of the second conic solution. In *Geometria (1649)*, two sentences indicate that a circle is the sole solution of the four line Pappus problem in the configuration studied by Descartes. In the 1659 edition, these two sentences are changed. First, Van Schooten omitted the word "only" in the first sentence. In the 1649 edition we read that *only* a circle fits the straight lines, whereas the 1659 edition the text reads that *a* circle fits the given straight lines.[60] Secondly, he changed "the true locus is a circle" into "the locus is a circle". The rest of the comment is unchanged.

What may have caused Van Schooten to stick to the text of 1649? Of course, there is no evidence that he computed the equation for the hyperbola which occurs in the solution of the Cartesian Pappus problem, so there is no reason to think that he thought about changing comment E in the way we have suggested above. We should bear in mind that it was precisely this comment E for which Descartes provided the draft in 1648. It was Descartes himself who had thought about how to reply to the criticisms on the figure. Descartes had expressed these thoughts in his letter to Van Schooten, in which he explicitly rejected the insertion of a new figure.[61] Changing the words or the figure, and choosing a different explanation, would mean for Van Schooten that he had to go against the intellectual legacy of Descartes. This was something Van Schooten could not do.

5.3.2 Comments related to the construction of the problem

The two comments CC and CCC in Van Schooten's second edition concern to a large extent the construction of the conic sections. Recall that in the *Géométrie* Descartes derived the equation

$$y = m - \frac{n}{z}x + \sqrt{m^2 + ox - \frac{p}{m}x^2}$$

for the points C satisfying the property of the Pappus problem, and that $LC = \sqrt{m^2 + ox - \frac{p}{m}x^2}$. Descartes stated that for a general configuration of given lines and angles in the Pappus problem, LC can be of the form $\sqrt{\pm m^2 \pm ox \pm \frac{p}{m}x^2}$ for known quantities o and p, which can also be equal to zero. For all these cases, Descartes provided the necessary data (position of vertex, position of the diameter, and latus rectum and latus transversum) for the Apollonian construction of the conic section which belongs to the equation,[62] but he did not explain how he had found these data from the equation. Moreover, Descartes's version is very concise and therefore hard to understand.[63] The comments CC and CCC deal in detail with the construction of the conic sections which correspond to the equations.

In the lengthy comment CC[64], Van Schooten started with a solution curve with diameter along IL, vertex N on the diameter, abscissa $IL = \frac{a}{z}x$ and ordinate $LC = \sqrt{\pm m^2 \pm ox \pm \frac{p}{m}x^2}$. Van Schooten assumed that the curve is a conic section and he then set out to find the latus rectum (for a parabola) or the latus rectum, latus transversum and centre (for ellipse and hyperbola), in order to arrive at the formulas which Descartes had

[60] The first line of the comment reads: "Notandum hìc, applicatam esse Hyperbolam ei linearum positioni, cui *solum* Circuli quadrare paulo post ab Authore ostenditur", whereas the first line of the 1659 edition reads "Notandum hìc, applicatam esse Hyperbolam ei linearum positioni, cui postea Circuli quadrare ab Authore ostenditur." [Schooten, 1649c, 196] and [Schooten, 1659c, 224].

[61] See page 115.

[62] Recall that Descartes also included the case $-m^2$, which cannot occur in the Pappus problem.

[63] See [Descartes, 1637c, 328-333].

[64] [Schooten, 1659c, 182-206].

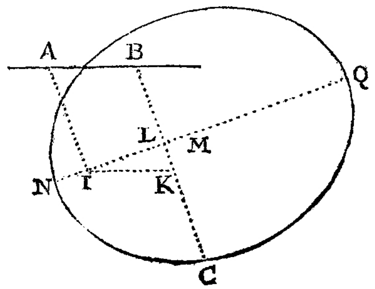


Figure 5.8 – Figure illustrating the ellipse $y = m - \frac{n}{z}x + \sqrt{m^2 + ox - \frac{p}{m}x^2}$. [Schooten, 1659c, 190].

indicated. Van Schooten gave the explicit calculations for the 24 different cases which can be derived from Descartes's discussion in the *Géométrie*. I have presented an overview of 19 possible cases in tables 3.1, 3.2 and 3.3 above. Because Van Schooten gives separate discussions for circles and ellipses, he considers 24 cases instead of 19.

I will now explain Van Schooten's methods in the example of the ellipse.[65]

Problem 4.
Given is an ellipse with diameter along IL, abscissa $IL = \frac{a}{z}x$, and ordinate $LC = \sqrt{m^2 + ox - \frac{p}{m}x^2}$ (see figure 5.8). We note that x is arbitrary. It is required to find the positions of the center M of the ellipse and of the vertex N (in the sense of Apollonius) on the diameter IL, and to determine the latus rectum and latus transversum.

We recall that in modern terms, AB can be considered as the x−axis and AI as the y−axis, and point I is given. Van Schooten first labeled the following line segments in the figure: $NM = MQ = c$ and $IM = d$. He furthermore called the latus transversum $t = 2c$, and the latus rectum r. Then

$$NL = c - d + \frac{a}{z}x \tag{5.20}$$

and

$$LQ = c + d - \frac{a}{z}x. \tag{5.21}$$

Then Van Schooten used *Conics* I:21, to the effect that all points C on the ellipse satisfy

$$LC^2 : NL \cdot LQ = r : t. \tag{5.22}$$

Substitution of (5.20), and (5.21) and $t = 2c$ in (5.22) yields after some calculation

$$\frac{rc^2 - rd^2 + \frac{2adr}{z}x - \frac{a^2r}{z^2}x^2}{2c} = m^2 + ox - \frac{p}{m}x^2,$$

[65] Van Schooten calls this the first case of an ellipse, see [Schooten, 1659c, 190-191].

which equation must be valid for all x.

Van Schooten remarked that c must be larger than d. By equating the terms in the different powers of x he obtained three equations:

$$m^2 = \frac{rc^2 - rd^2}{2c}, \tag{5.23}$$

$$o = \frac{adr}{cz}, \tag{5.24}$$

$$-\frac{p}{m} = -\frac{ra^2}{2cz^2}. \tag{5.25}$$

Van Schooten solved these three equations for c, d and r and he found

$$d = IM = \frac{aom}{2pz},$$

$$r = \frac{z}{a}\sqrt{o^2 + 4mp},$$

$$t = 2c = \frac{ma}{pz}\sqrt{o^2 + 4mp},$$

in agreement with the *Géométrie* of Descartes, see the tables above.

Van Schooten proceeded in this way for all 24 cases. Three of these 24 cases require a variant construction in which the diameter is parallel to LC (instead of IL) and the corresponding ordinates are parallel to IL instead of LC. Van Schooten's analysis makes clear why a variant construction is needed. For the hyperbola such that $IL = \frac{a}{z}x$ and $LC = \sqrt{m^2 - ox + \frac{p}{m}x^2}$, Van Schooten finds $r = \frac{z}{a}\sqrt{o^2 - 4mp}$. Thus the construction is only possible if $o^2 > 4mp$. The case $o^2 = 4mp$ is not interesting because the solution is a straight line.

For the case $o^2 < 4mp$, Van Schooten used the following construction to find Descartes's specifications of the latus rectum and transversum and the center:

Problem 5.
We consider a hyperbola with diameter parallel to AI, ordinates parallel to IL, such that, if $IL = \frac{a}{z}x$, then $LC = \sqrt{m^2 - ox + \frac{p}{m}x^2}$, where segment LC is parallel to the diameter (and to AI) and $o^2 < 4mp$ (see figure 5.9). Required to find the position of the vertex O (in the Apollonian sense) and to determine the latus rectum and latus transversum.

Van Schooten first set $IM = d$, and $OM = MQ = e$, so that $t = 2e$. Then he noted that $LM = IM - IL = d - \frac{a}{z}x$. Van Schooten then used *Conics* I:21, to the effect that for any point C on the hyperbola,

$$r : t = CP^2 : QP \cdot PO. \tag{5.26}$$

hence

$$QP \cdot PO = \frac{t}{r}CP^2 = \frac{2de^2}{r} - \frac{4ade}{rz}x + \frac{2ea^2}{rz^2}x^2. \tag{5.27}$$

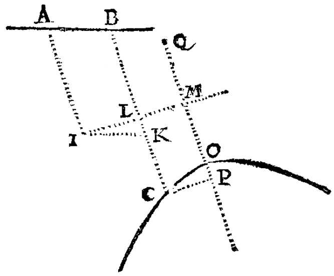


Figure 5.9 – Figure illustrating the hyperbola $y = m - \frac{n}{z}x + \sqrt{m^2 - ox + \frac{p}{m}x^2}$ with $o^2 < 4mp$. [Schooten, 1659c, 202].

Van Schooten then argued that $QP \cdot PO + MO^2 = (MP + MO)(MP - MO) + MO^2 = MP^2 = LC^2$. Hence, adding $MO^2 = e^2$ to both sides of (5.27),

$$LC^2 = \frac{2de^2}{r} - \frac{4ade}{rz}x + \frac{2ea^2}{rz^2}x^2 + e^2. \tag{5.28}$$

The remaining steps in Van Schooten's argument are similar to the case of the ellipse. Van Schooten compared this expression LC^2 with the earlier expression $LC^2 = m^2 - ox + \frac{p}{m}x^2$ and equated the terms of the different powers of x. Thus he found the latus rectum, the latus transversum and the centre as specified by Descartes.

I now turn to Van Schooten's source of inspiration for these 24 cases. These cases had not yet been mentioned in *Geometria (1649)*, and it was Huygens who inspired Van Schooten to insert comment CC in the 1659 edition. Huygens had noticed that Descartes stated the lengths of the latus rectum and the position of the vertex of the parabola without further explanation. He wrote to Van Schooten in 1654 that it would be worthwhile to explain to the reader how this ratio could be obtained from the equation.[66]

In Huygens's notes there is a more detailed explanation, which is dated 1654 by the editors of the *Oeuvres Complètes*,[67] and which is part of a general investigation of the conic sections that may result from for the equations in the Pappus problem. In these handwritten notes, Huygens treated, for one equation of a parabola and one equation of an ellipse, the way in which the data for the Apollonian construction can be found from the equation. It turns out that his method is exactly the same as the one published by Van Schooten in the *Geometria (1659)*. Huygens indicated that his notes belong to pages 33-35 of the Latin edition of the *Géométrie*. Comparison of these pages with both the 1649 and 1659 edition show that the reference is to pages of the 1649 edition, in agreement with the dating of the manuscript to 1654. Van Schooten may have had these notes by Huygens at his disposal,

[66] "Qua ratione hoc latus rectum $\frac{oz}{a}$ et linea IN $\frac{amm}{oz}$ inventa sit explicare operaepraetium videtur." Huygens's letter is a reply to Van Schooten's request for ideas on improvement of the second edition. Huygens to Van Schooten, [29 October 1654], [Huygens, 1888, 304].

[67] [Huygens, 1920, 423-427].

and and he may have used them in the preparation of the 1659 edition. However, Van Schooten does not refer to Huygens in his comment CC.

Van Schooten's comment CCC in the 1659 edition consists of two parts. Below I will give a short overview of the content of the first part, which is a concise recapitulation of the Apollonian theory of conics. I will not discuss the second part dealing with inventions of Johannes Hudde in geometry.

Van Schooten started with an overview of the theory on conic sections "for those who do not have these books by Apollonius or other books on conic sections at hand".[68] Using a cone intersected by a plane, he first proved the fundamental properties of the three conic sections (parabola, hyperbola, ellipse) which Apollonius proved in *Conics* I: 11-13[69] in the form of equations (e.g., for the parabola $\frac{ccd}{ab}x = y^2$ where a, b, c, d are given segments in the cone); and he then deduced the theorems *Conics* I: 20,21.[70] He continued with the construction of a cone which intersects the plane in a conic section with given data (for example a parabola with given vertex, axis and latus rectum). Thus he solved what he calls "the first, second and third problems in the Conics"[71] (i.e., Conics I: 52-58).

Then he turned to the propositions about the hyperbola and its asymptotes which Apollonius proved in *Conics* II: 1-4, 8-14.[72] Unlike Apollonius, Van Schooten already constructed the asymptotes of a hyperbola in the three-dimensional figure containing the cone which intersects the plane in that hyperbola. Apollonius constructed the asymptotes in a two-dimensional figure containing a hyperbola without reference to a cone.

We have seen that the influence of Huygens in a number of Van Schooten's comments was important. In the years 1654–1656 Huygens provided Van Schooten with a large number of remarks, suggestions and clarifications to the 1649 edition of the *Geometria*, and Van Schooten relied to a large extent on these remarks. We give another example here.

In March 1655, Huygens wrote to Van Schooten that there is a problem which concerns a mistake in the original text by Descartes.[73] In the original text of the *Géométrie*, Descartes asserted that if m^2 and $\frac{p}{m}$ have the same sign, plus or minus, the expression $m^2 + ox + \frac{p}{m}x^2$ is a square and its square root can be extracted. Huygens now indicated that m^2 cannot have the minus sign. In Comment C in his 1649 edition (see page 124), Van Schooten had not noticed this.

Huygens indicated that the comment of Van Schooten might cause difficulties for the readers and he suggested to change the text. And this is indeed what Van Schooten did. In the 1659 edition, the reference to the minus sign disappeared from the translation as a result.[74] Van Schooten also adjusted his comment C in such a way that it agrees with

[68] "Quibus hi Apollonii libri, aut etiam aliquorum, qui de Conici scripserunt, non sunt ad manus", [Schooten, 1659c, 206].

[69] *Conics* I:11 indicates Book One of the *Conics*, proposition 13.

[70] [Schooten, 1649c, 207-213].

[71] [Schooten, 1649c, 215].

[72] [Schooten, 1649c, 216-220].

[73] Huygens to Van Schooten, 25 March 1655, [Huygens, 1888, 323-324].

[74] In the 1659 edition the translated passage reads: "ut inde radix extrahi potuisset, hoc est, ut, mm & $\frac{p}{m}xx$ signo + notatis", whereas the same passage in the 1649 edition reads: "ut inde radix extrahi potuisset, (hoc est, cùm mm & $\frac{p}{m}xx$ eodem signo + vel – adfecti sunt". See [Schooten, 1649c, 32] and [Schooten, 1659c, 28-29].

the altered translation. He did not include a notice that the original text of Descartes has been changed here. I believe that Van Schooten considered the incorrect minus sign as a minor mistake, so the change in the text was justified because it improved the text without changing the line of Descartes's argument.

It must be noted that Van Schooten did not follow all suggestions and remarks made by Huygens. Here is an example. We recall that in case $m^2+ox-\frac{p}{m}x^2$ is a square, the points C satisfying the equation $y=m-\frac{n}{z}x+\sqrt{m^2+ox-\frac{p}{m}x^2}$ are on a straight line. Descartes had noticed this in the *Géométrie*, but he had not given the explicit construction of this straight line. He only indicated that the construction is not more difficult than the construction of the line IL he had just shown before. In Huygens's opinion, the reader would benefit by learning the actual constructions for the cases in which $m^2+ox-\frac{p}{m}x^2$ is a square. In reply to Van Schooten's request for suggestions to the second edition of *Geometria*, Huygens provided the construction in detail.[75] These constructions would have fitted nicely in the comment C in which Van Schooten discusses the various cases for which the square root of $m^2+ox-\frac{p}{m}x^2$ can be extracted. Nevertheless, he did not include Huygens's construction.

[75] Huygens to Van Schooten, 29 October 1654, [Huygens, 1888, 304].

CHAPTER 6

Concluding remarks

The central theme in the previous three chapters has been the Pappus problem in four lines and the debate about this problem in the 1640s and 1650s. The debate took place between Dutch and French mathematicians and the main players were Descartes, Mersenne and Roberval since the publication of the *Géométrie* in 1637, Van Schooten since 1648, and his former student Huygens since 1656.

The debate on the Pappus problem is a story of mathematics and at the same time a story of the mathematicians and their personal relations. From a mathematical point of view, our analysis of the Pappus problem and its solution by Descartes has led to a new interpretation of the use of plus and minus signs in Section 3.3. This interpretation explains some minus signs in the *Géométrie* and in the Latin translations by Van Schooten, which were hitherto qualified as errors by Descartes or misprints. We now know that Descartes and Van Schooten used them deliberately and that they are related to their views on algebraic substitutions, which are different from the modern views.

In the debate on the Pappus problem, mathematical truth was not the only issue; personal relations between mathematicians also played an important role. Now it is time to connect the story of the previous chapters to Van Schooten. What does the Pappus debate reveal about Van Schooten in particular?

Van Schooten became involved in the debate in 1648, when he was preparing *Geometria (1649)*, and also in 1656, when he was preparing *Geometria (1659)*. Both cases follow a similar pattern: fellow mathematicians (in 1648 Mersenne and in 1656 Huygens) were aware of Van Schooten's *Geometria*-project and confronted him with criticisms on the solution by Descartes of the Pappus problem in the *Géométrie*. In both cases, the origin of the criticisms was Roberval. I have argued that in 1646 or earlier, Roberval had discovered that the solution of the four line Pappus problem consisted of two conic sections. Above we have seen that Descartes was unaware of this second conic section. Descartes believed that the solution of the problem consisted of one conic section only. Roberval also criticized other aspects of Descartes's solution of the Pappus problem in the *Géométrie*.

Van Schooten's reactions in 1648–49 as well as in 1656–1659 also follow the same pattern. He tried to reconcile the criticisms with Descartes's text in the *Géométrie*, although he knew that at least some of the criticisms were legitimate. Van Schooten acted as a loyal and uncritical follower of Descartes and as a propagator and guardian of Descartes's mathematical ideas.

An important difference between the situations in 1648-9 and 1656 is the fact that Descartes was present in Holland in 1648-9 but dead in 1656. In 1648 Van Schooten consulted Descartes in order to determine how to deal with the criticisms by Roberval with regard to one of the figures for the Pappus problem in the *Géométrie*. In order to refute the criticisms, Descartes provided Van Schooten with a note in Latin. In the note, Descartes tried to argue why the figure in question (figure 4.1) was correct, and why no changes to it were necessary in the Latin version. As we have seen above, the arguments of Descartes are not very convincing mathematically. It seems that Van Schooten did not call Descartes's arguments into question at all. He inserted the note in *Geometria (1649)* with only very minor changes, and without reference to Descartes, who did not want to publicize the fact that he cooperated with Van Schooten in the preparation of the Latin edition.

Van Schooten's comments on the Pappus problem in *Geometria (1649)* deal to a large extent with elementary algebraic manipulations. This is understandable because these manipulations had been introduced by Descartes and were not commonly understood at that time. Algebraic manipulations were also the main issue of other introductory texts on the *Géométrie*, like Haestrecht's *Le Recueil du Calcul pour l'intelligence de la Geometrie de Monsr. des Cartes* and the *Matheseos universalis* by Bartholin, which was based on Van Schooten's lectures.

In 1656, Van Schooten was again confronted with criticisms of Descartes's solution of the Pappus problem, and he displayed the same attitude even more prominently than before. This time he could no longer consult Descartes, who had passed away in the meantime. Van Schooten consistently acted as a guard-dog of Descartes's mathematical legacy. When Descartes's solution was criticized on the grounds that it consisted of one conic section only, Van Schooten first tried to deny the second conic section and refute the criticims without entering into the mathematical details. This position became indefensible once Huygens had sent to Van Schooten detailed computations of the second conic solution, of which Descartes had been unaware. Huygens made his computations by means of the analytical techniques that had been developed by Descartes himself.

Van Schooten now tried another approach and changed his discourse. He claimed that the second conic solution was implicitly contained in the *Géométrie*, and then adduced various reasons why Descartes did not mention it explicitly. Van Schooten tried to hide behind the text of Descartes by means of a strained interpretation of passages in the

Géométrie. The whole situation is best described in the words of Huygens who wrote in this connection that Van Schooten "always tries to excuse his master".

These words of Huygens are a witness of the asymmetric relation between Van Schooten and Descartes. Van Schooten held Descartes in very high esteem, and his admiration shines through in his correspondence with Descartes.[1] In his *Mathematische Oeffeningen*, Van Schooten wrote that Descartes was a giant in all diciplines.[2] This admiration stood in the way of a critical attitude of Van Schooten towards the writings of Descartes. Descartes, on the other hand, had a rather condescending attitude towards Van Schooten at times.

To return to the criticisms, Van Schooten added three new comments in the *Geometria (1659)* to the ones that he had already published in the *Geometria (1649)*. One of the new comments dealt with the second conic solution and was inspired by Huygens. In this comment Van Schooten admitted the existence of the second conic solution, albeit only halfheartedly. His major concern was to reconcile the second conic solution with the text of the *Géométrie* and with Descartes's program for solving geometrical problems in general. To incorporate the second conic solution in this program, Van Schooten introduced the new concept of *universal equation* associated to a geometric problem. This concept enabled Van Schooten to associate with each geometrical problem a single equation, which provides all possible geometric solutions to the problem. In this way Van Schooten wanted to show that the second conic section was, at least implicitly, contained in the *Géométrie*.

However, the concept of universal equation is nowhere to be found in Descartes's work, and the universal equation did not serve any purpose in actual problem solving. For the four line Pappus problem, the universal equation according to Van Schooten's ideas is of degree 512, and unmanageable for practical use. Thus, Van Schooten's universal equation is no more than a useless concept, which he introduced with the sole purpose of rescuing Descartes's original text in the *Géométrie*.

The debate on the Pappus problem also reveals the changes in the relation between Van Schooten and his former pupil Huygens. In earlier days, Van Schooten instructed Huygens in mathematics but by 1656 the roles had changed.[3] In 1656, it was Huygens who informed Van Schooten about the serious shortcomings in Descartes's exposition of the Pappus problem, and subsequently it was Huygens who explained the mathematics of the Pappus problem in all details to Van Schooten. Ultimately, it was Huygens who pushed Van Schooten to include at least some information on the second conic solution in *Geometria (1659)*.

The previous chapters have also revealed Van Schooten's rather poor mastering of the mathematics involved in the Pappus problem, and his somewhat passive attitude in this regard. By 1656, Van Schooten seems not to have made any serious effort in understanding the mathematical background of the remarks by Roberval. In the end, it was Huygens

[1] See especially the letter of Van Schooten to Descartes of 10 March 1648, at the end of which he thanked Descartes very politely and extensively. His attitude is very different from the way Descartes adressed Van Schooten and the way Van Schooten addressed other correspondents such as Huygens, [Descartes, 1903, 318-320].

[2] "Een overvlieger in alles", [Schooten, 1660a, 192].

[3] We have already seen that Huygens developed a critical attitude with respect to Descartes in the biography of Van Schooten, see page 2.7.

who had to spell out the second conic solution to Van Schooten. In these chapters we have seen Van Schooten defending the Cartesian mathematical heritage. He left it to other mathematicians, including his own students, to go beyond it.

Part III

Van Schooten and the Duytsche Mathematicque

Introduction

During much of his life, Frans van Schooten jr. was active in the Duytsche Mathematicque, the mathematical training programme in the vernacular which had been founded in Leiden in 1600. The programme had been designed in order to teach useful mathematical knowledge as part of the training of engineers for the army. Frans van Schooten sr. had been professor in the Duytsche Mathematicque from 1615 until his death in 1646. From 1635 onwards, Frans jr. replaced his father when the latter was ill.[4] In 1646 Van Schooten jr. was officially appointed professor in the Duytsche Mathematicque, and he held this position until his death. The Duytsche Mathematicque provided him with a daily routine of lecturing, guaranteed him a fixed income, and was a permanent ingredient of his active life. Studying the scholarly life of Van Schooten jr. without the Duytsche Mathematicque is like considering only one side of a coin.

The teaching of Frans van Schooten jr. in the Duytsche Mathematicque has not received much attention, for two reasons. In the first place, historians have focused on Van Schooten's scholarly contributions and in particular on his role in spreading and clarifying Descartes's mathematical ideas.[5] Van Schooten's activity as professor at the Duytsche Mathematicque does not concern the mathematical ideas of Descartes, and has thus been ignored.

In the second place, historians have focused on the early decades (1600–1620) of the Duytsche Mathematicque. In particular, they have studied the involvement of stadtholder Maurice (1567–1625) and his tutor Simon Stevin and have investigated the role of the Duytsche Mathematicque within the political and military context of the Dutch Republic, as the initial aim of the institition was to educate men to serve the country as engineers in the army. The fate of the Duytsche Mathematicque therefore has been connected to the military situation of the Dutch Republic was in. In the Peace of Munster of 1648, the Dutch republic was recognized as an independent state, and the war with Spain came to an end. This Peace of Munster has been considered as the beginning of the decline of the Duytsche Mathematicque, because as a consequence of the peace treaty the military expenses were drastically cut and political interest for military education waned. According to Krüger,

[4] [Molhuysen, 1918, 197].
[5] See for instance [Le Noir, 1974], [Hofmann, 1962], and [Maanen, 1987].

stagnation in the curriculum even began before 1648, and she states that the "mathematical content (...) of the programme hardly changed during sixty years [i.e., from 1620 to 1679, JGD.]".[6] Thus, historians have argued that lack of innovation and development in the curriculum led to stagnation and eventually to the closing down of the Duytsche Mathematicque after the death of Petrus van Schooten, because the teaching did not meet the needs of society anymore.[7] Below I will challenge this view by showing that Van Schooten jr. lectured on a broader variety of subjects than his predecessors in the first decades of the Duytsche Mathematicque, and that he continued to improve his teaching and adjust it to the needs of his audience.

In this and the following two chapters I will scrutinize the teaching activities of Frans van Schooten jr. on the basis of unpublished manuscript material. Van Schooten was the only professor at the Duytsche Mathematicque, with no direct colleagues and no direct supervisor, so he probably had much freedom in shaping the courses. I will investigate the mathematical content of his courses in order to obtain insight in his role as a professor and instructor and in the choices he made in his teaching. The following questions will be discussed: What topics did he consider worthwhile to lecture on? In what way did he organize his teaching and structure his lectures? What was the level of his courses? What developments can be seen in his teaching, if any? In a broader framework, the following chapters will provide some new perspective on the contents and the level of mathematical education during the seventeenth century, and on the variety of the subjects that were taught.

I will begin by discussing the way in which the Duytsche Mathematicque was founded and its relation with the university in the next section 7.1. The available information on the teaching activities of Frans van Schooten jr. is to a large extent based on manuscripts related to his lectures. In section 7.2 I will introduce these manuscripts and other sources and give a general outline of the structure of Van Schooten's courses. The two subsequent chapters are devoted to the mathematical contents of the lectures. Chapter 8 deals with Van Schooten's teaching of arithmetic and fortification, two subjects that belonged to the core of the curriculum of the Duytsche Mathematicque since the beginning in 1600. The next chapter 9 focuses on algebra and logarithms, two subjects on which Van Schooten lectured but which were not initially on the curriculum. I will analyze the content of Van Schooten's courses and compare it with contemporary authors in order to discover characteristic aspects of his teaching. Chapter 10, the final chapter of this part, takes up the observations of the previous chapters in order to draw the broader picture of Van Schooten as a teacher.

[6] [Krüger, 2010, 159].

[7] Besides Krüger, this view on stagnation at the Duytsche Mathematicque is also found in [Schäfer, 2001, 296-297], [Goudeau, 2005, 92-93], and [Winter, 1988, 21]. Van den Heuvel stresses the continuity in the lectures of three Van Schooten professors, thus implicitly indicating that throughout no major changes were at stake in the curriculum, [Heuvel, 2006, 112-113].

CHAPTER 7

The Duytsche Mathematicque

7.1 The Duytsche Mathematicque and Leiden University

The Duytsche Mathematicque had a peculiar position in the periphery of Leiden University. The programme was created to fill a need in society, more specifically in the military. In order to fully understand the context of the lectures of Frans van Schooten, who was the third generation professor at the Duytsche Mathematicque, a closer look at the institution itself is required.

7.1.1 The foundation of the Duytsche Mathematicque

The Duytsche Mathematicque was founded in 1600 as the result of an initative of Maurice of Orange. He urged the board of curators and burgomasters[1] of Leiden University to organize a programme in Dutch aimed at training engineers for the military. Maurice recommended the mathematician and fencing-master Ludolph van Ceulen and the surveyor

[1] This council consisted of three curators, nominated by the States of Holland, together with the four burgomasters of the city of Leiden. In general, one curator was a representative of the knighthood, and one curator was a burgomaster in Delft. The background of the third curator varied over time. The board of curators and burgomasters was the main governing body of the university, and among its responsibilities was the determination of the salaries. For a detailed account of the governance of the university see [Sluijter, 2004, 17-61] and [Otterspeer, 2000, 75-103].

Simon van Merwen as professors.[2] The board of curators was receptive to this initiative of Maurice. One of the curators at the time was Jan Cornets de Groot, burgomaster of Delft, who had a lively interest in mathematics and natural sciences. De Groot was acquainted with Stevin and Ludolph van Ceulen and presumably an advocate of the proposed mathematical education.[3]

The initiative of Maurice has to be considered within the context of his more general reform of the military. His efforts, together with those of his cousin William Louis (1560–1620), led to a better trained army. The military historian Van Nimwegen considers the impact of these reforms so profound and marked that the term 'tactical revolution' is appropriate.[4] The reforms included a gradual professionalization of the men engaged in the construction and inspection of fortresses and other defensive works, the so-called engineers. During campaigns, these engineers were involved in the organization of the siege of a city. The authorities issued an increasing number of instructions and placards which specified the tasks and duties of these engineers and thus reflect their increasing importance.[5]

The increasing demand for engineers led on the short term to a shortage, and the States General had difficulties in hiring capable men.[6] In 1593, the lack had become so urgent that Maurice urged the Frisian stadtholder William Louis to send Frisian engineers to the siege of Geertruidenberg.[7] As a result, the Council of State insisted on the recruitment of new engineers.[8]

Maurice had a keen interest in engineering matters. He often discussed the plans for fortification projects with the relevant engineer before submitting these plans to the States General or the Council of State.[9] Maurice also discussed such projects with his tutor Simon Stevin. Their collaboration in mathematics is documented in a series of works which Stevin published under the title *Wisconstighe gedachtenissen (Mathematical memoirs)*[10]. The foundation of the Duytsche Mathematicque is another product of their collaboration.

At the request of Maurice, Simon Stevin drew up a document on the desired curriculum of the Duytsche Mathematicque. The document was printed in Leiden in 1600 by university printer Jan Paedts Jacobsz, and it is known as Stevin's Instruction.[11] This instruction

[2] In the historical literature Van Ceulen is mostly known for his calculation of decimal places of π, see [Katscher, 1979]. On Van Ceulen and his network see [Wepster, 2010], and for aspects of his mathematical work see [Wreede, 2010] and [Hogendijk, 2010]. Van Merwen had served the city of Leiden as burgomaster and treasurer and he was involved in the planning of town expansion of Leiden in 1594, see [Taverne, 1978, 188-189]; a manuscript in his handwriting is kept in Leiden Archief, AB 8399.

[3] De Groot performed experiments, in particular drop tests, in Delft together with Simon Stevin, [Stevin, 1586a, 66] and he translated parts of the Latin version of the *Measurement of a Circle* by Archimedes for Ludolph van Ceulen. On mathematicians in De Groot's network see [Houtzager, 1994].

[4] [Nimwegen, 2006, 83-102].

[5] [Scholten, 1989, 14].

[6] [Scholten, 1989, 14-15].

[7] [Nimwegen, 2006, 121].

[8] [Schäfer, 2001, 130].

[9] [Scholten, 1989, 15].

[10] [Stevin, 1608].

[11] [Stevin, 1600]. A handwritten draft of the printed text is Leiden Archief, Library, LB 36949. A version

specifies the following content and order of the curriculum:

- Arithmetic: addition, subtraction, multiplication, division and the rule of three; all in whole numbers, fractions and decimal fractions.
- Surveying on paper: area computations in decimal fractions; measurement of the area of rectilinear and curvilinear figures by dividing the figure into triangles; volume calculations of three-dimensional figures such as dykes and walls.
- Fieldwork: use of surveying instruments.
- Mapping on paper what has been measured in the field and setting out in the field a figure drawn on paper.
- Fortification: learning the fortification terminology by means of scale models; drawing plans for fortification works with four, five or more bastions; determining the dimensions of the different parts of a fortress; staking-off a plan in the field.

Once the students had mastered this final level, they were ready for an internship in the army. They were encouraged to return to Leiden in the winter for additional lessons of which Stevin did not specify the contents. As the title of Stevin's Instruction refers to exercise in engineering and other mathematical arts[12], it is likely that he foresaw additional lessons on a broad range of subjects from the realm of mixed mathematics. Stevin excluded pure geometry from the curriculum by his statement that "one will not learn to find other lines by means of some given lines".[13] He also omitted the theory and applications of conic sections.

Due to a lack of sources, hardly anything is known about how the first professors interpreted Stevin's instruction in their actual teaching. In 1610 the first professors Van Ceulen and Van Merwen died, and their former student Frans van Schooten sr. continued the teaching at the Duytsche Mathematicque. A short summary of the content of the courses in 1611–1615 was composed by Van Schooten sr. This document agrees with Stevin's programme, but root extraction had been added to the course in arithmetic, and the preparatory course on surveying included lessons in elementary geometry of a general nature.[14] The first document providing detailed information on teaching practice is a manuscript written by Frans van Schooten sr. in 1622.[15] Van Schooten sr. had been appointed professor at the Duytsche Mathematicque in 1615,[16] and he used this manuscript

of the same text, in slightly different spelling, which is found in the University Archives, was published in [Molhuysen, 1913, 390*-391*]. Paedts was appointed university printer in 1600, see [Hoftijzer, 2008, 86].

[12] "Tot oeffeninghe van het Ingenieurschap ende andere mathematische consten", [Stevin, 1600, 1].

[13] "Man niet en sal leeren deur eenige ghegeven linien ander linien te vinden", [Stevin, 1600, 2].

[14] UBL, Archief van Curatoren 1574–1815, inv. nr. 42/2.

[15] UBL, BPL 1013. The manuscript is described in the inventory of mathematical manuscripts of Leiden University, [Maanen, 1987, 200-203]

[16] His appointment is documented in UBL, Archief van Curatoren 1574–1815, inv. nr. 20, f. 354v. As a student, Van Schooten sr. attended the lectures of Van Ceulen and Van Merwen and he served as Van Ceulen's assistant in the Duytsche Mathematicque and in Van Ceulen's private arithmetic school, Groene Hart Archieven, Archief van de stad Gouda 1311–1815, inv. nr. 2806 and UBL, Archief van Curatoren 1574–1815, inv. nr. 42/2. After Van Merwen and and Van Ceulen passed away in 1610, Van Schooten sr. took over their lessons in the Duytsche

in his teaching. The content of the manuscript shows that Van Schooten sr. followed the instruction of Stevin on the whole, but he added root extraction and elementary lessons in geometry, in agreement with the summary of 1611–1615. Moreover, the use of trigonometric tables had become part of the curriculum at the latest in 1622. Throughout his course, Van Schooten sr. put much emphasis on surveying, and most of the geometric problems were situated in a surveying rather than a military context.[17]

7.1.2 The position of the Duytsche Mathematicque within the university

The Duytsche Mathematicque had a peculiar position within Leiden university. Dutch was chosen as a language of instruction as the intended audience was composed of men who did not master Latin, the lingua franca of the academic community. The same holds for the first professors: Ludolf van Ceulen is known to have mastered Latin poorly.[18]

The audience of the Duytsche Mathematicque consisted of people with various backgrounds. In a letter to Constantijn Huygens in 1646, Van Schooten jr. characterized his audience as university students and craftsmen. He claimed that some of these craftsmen had pursued a career as engineer, surveyor or schoolmaster after attending the Duytsche Mathematicque.[19] That craftsmen such as bricklayers and carpenters attended the lectures in the 1650s is attested by the Leiden resident De Parival and the Swedish student Olof Rudbeck.[20] The presence of such craftsmen in the lectures dates back to the beginning of the seventeenth century. In 1611–1612, the candidature of Frans van Schooten sr. for the vacant chair of professor of the Duytsche Mathematicque received the support of a considerable number of craftsmen.[21]

The craftsmen who attended the lectures in the Duytsche Mathematicque were not considered members of the academic community. Students and members of the academic community were people who had registered in the *album studiosorum* of the university, who enjoyed some privileges, such as exemption on certain taxes, tolls and town obligations, and who were within the jurisdiction of the university.[22] The membership of the academic community was not open to men practicing a profession in the city. As a consequence, they were excluded from the academic privileges. The exclusion served the purpose of preventing tax evasion by Leiden citizens, who could otherwise easily evade taxes

Mathematicque and in 1615 he was finally appointed as professor. In the years 1611–1614, several payments were made to Van Schooten for his lessons in the Duytsche Mathematicque, UBL, Archief van Curatoren 1574–1815, inv. nr. 20, f. 388r.-v.

[17] [Krüger, 2010, 151, 156] and [Taverne, 1978, 66, 77].

[18] [Wepster, 2010, 64, 66 and 69]. Jan Cornets de Groot translated mathematical treatises for Van Ceulen from Greek ("the Archimedean tongue") into Dutch, see [Ceulen, 1596, voor-reden].

[19] [Huygens, 1915, 279].

[20] [Parival, 1661, 188-189] and [Kallinen, 2006, 118-119].

[21] This is evident from the signatures under three petitions from the years 1611–1612 requesting the appointment of Frans van Schooten sr. One-third of the persons who signed the petition were craftsmen, with occupations such as carpenter, bricklayer and mason. Their number may have been higher, as less than half of the signatories mentioned their occupation. UBL, Archief van Curatoren, 1574–1815, inv. nr. 42/2.

[22] For a detailed account of the privileges, see [Zoeteman, 2011, 33-48] and [Otterspeer, 2000, 117-137].

by registering as students.[23] In the first years of the existence of the Duytsche Mathematicque, its auditors requested the same privileges as ordinary students, but these requests were denied.[24]

As a substantial part of the auditors was not considered member of the academic community, the question arises whether the position of the professors of the Duytsche Mathematicque differed from that of their colleagues at the university. The professorate of Leiden university consisted of two classes: ordinary professors and extraordinary professors. The extraordinary professors were refused the membership of the senate, had to teach elementary courses at unpopular hours, and received a lower pay. The distinction between ordinary and extraordinary professors reflected the hierarchy of the professorate.[25]

The professors of the Duytsche Mathematicque belonged to the category of extraordinary professors. They were not members of the senate, as Van Schooten explicitly mentioned to Constantijn Huygens in 1646.[26] Just like the other professors, the professors of the Duytsche Mathematicque were appointed by the council of curators and burgomasters. They were also paid from the same funds as the other ordinary and extraordinary professors. Their yearly remuneration is listed in table 7.1. In the period 1600–1680, an ordinary professor earned on average 1120 guilders per year, whereas an extraordinary professor received only 516 guilders per year on average. The average pay of an extraordinary professor varied to a considerable extent over the years: from 300 guilders in 1630 to 875 guilders in 1660.[27] Each of the professors of the Duytsche Mathematicque received a salary close to the average for an extraordinary professor at the university at the end of his career. That Van Schooten jr. received the best pay of all professors at the Duytsche Mathematicque was probably due to his scholarly work.

The difference in remuneration is also observed when comparing the professors at the Duytsche Mathematicque with the ordinary professors of mathematics, see table 7.1. During the period 1600–1667, Leiden University had three ordinary professors of mathematics: Rudolph Snellius (1546–1613), Willebrord Snellius (1580–1626) and Jacob Golius (1596–1667). The relatively high salary of Golius is explained by the fact that he had a dual position as professor of mathematics and Eastern languages from 1629 onwards. In addition, Golius was five times the rector magnificus of Leiden University.[28] The two Snellii were extraordinary professors before they were promoted to ordinary professors. Remarkably, their promotions of 1601 and 1615 respectively coincide with the appointment of professors at the Duytsche Mathematicque. It has been suggested that the promotion of Rudolph Snellius was related to the establishment of the Duytsche Mathematicque and to the development at Franeker University where a a full professor of mathematics had been

[23] [Zoeteman, 2011, 57-59].

[24] [Winter, 1988, 18-19] and UBL, Archief van Curatoren, 1574–1815, inv. nr. 42/2.

[25] [Sluijter, 2004, 118-119 and Bijlage 2, 296-298].

[26] [Huygens, 1915, 279]. The accounts of the meetings of the senate (i.e., the assembly of ordinary professors) also show that these meetings were not attended by the professors of the Duytsche Mathematicque.

[27] The average pay is based on the data provided in [Sluijter, 2004, 296-298].

[28] Golius was rector magnificus in the years 1642, 1651, 1656, 1657 (after the death of Arnold Vinnius) and 1665, [Molhuysen, 1918, 329, 409, 446, 459, 519]. The increase in Golius's remuneration of 1656 and 1665 coincides with the rectorate.

Professor	Year	Remuneration in guilders per year
Duytsche Mathematicque		
Ludolph van Ceulen and	1600	400 each
Simon van Merwen	1603	450 each
Frans van Schooten sr.	1615	350
	1627	400
	1632	400 + 100 additional pay
Frans van Schooten jr.	1646	400
	1647	400 + 100 additional pay
	1653	400 + 250 additional pay
	1658	400 + 300 additional pay
Petrus van Schooten	1661	200
	1663	400
	1666	500
Ordinary professors in mathematics		
Rudolph Snellius	1601	400
	1602	500
Willebrord Snellius	1615	400
	1616	500
	1618	600
	1620	600 + 200 additional pay
Jacob Golius	1629	600
	1631	600 + 100 additional pay
	1635	600 + 400 additional pay
	1656	600 + 700 additional pay
	1665	800 + 700 additional pay

Table 7.1 – Remuneration of the professors at the Duytsche Mathematicque and the ordinary professors of mathematics. The data of the Duytsche Mathematicque are based on [Molhuysen, 1913, 42, 47, and 579], [Molhuysen, 1916, 134, 170, and 304], [Molhuysen, 1918, 8, 173, 190, and 204] and UBL, Archief van Curatoren, 1574–1815, inv. nr. 22-25. The data on Snellius and Golius is based on [Wreede, 2007, 49 and 75-77], [Molhuysen, 1913, 142], [Molhuysen, 1916, 121, 146, 162, and 196], and [Molhuysen, 1918, 116 and 200].

appointed. It may be that the board of curators and burgomasters felt that they could not lag behind and had to appoint a full, ordinary, professor as well. The promotion of the Snellii is also understood from a desire to make a clear distinction between the professors lecturing to the proper students in Latin, and those lecturing in the veracular to illiterate men.[29]

Another difference between the Duytsche Mathematicque and other types of academic instruction was the graduation. Leiden University had the right to confer doctoral degrees, but such a degree could not be obtained by an auditor of the Duytsche Mathematicque. Discussions about degrees or final examinations of the Duytsche Mathematicque were always related to the admission procedure of surveyors at the Court of Holland.[30] Obtaining a degree in the Duytsche Mathematicque only mattered for those intending to practice the profession of surveyor.

The peculiar position of the Duytsche Mathematicque is also reflected by the longstanding absence of its lectures from the university schedules, the so called *series lectionum*. These schedules were issued from 1587 onwards for each semester on 1 March (summer) and 1 October (winter) and listed the subjects which the university professors were supposed to read. Most of the *series lectionum* were lost because they were usually thrown away after the end of the semester, but the surviving items give an insight in the organization of the teaching at Leiden University.[31] For more than fifty years, the lectures of the Duytsche Mathematicque were not included in the *series lectionum*, a fact indicating that they were not considered part of the academic curriculum. On 30 October 1653 the senate finally decided to include in the *series lectionum* the lectures by Van Schooten jr. in the vernacular.[32] The reason for this change of policy is unknown; Frans van Schooten's academic reputation may have cleared the way for including his name on the official schedules.

7.1.3 The closing down of the Duytsche Mathematicque

Petrus van Schooten succeeded Frans van Schooten as professor at the Duytsche Mathematicque in 1661. When Petrus died in 1679, the board of curators and burgomasters did not appoint a successor, leaving the chair vacant. On 8 May 1681, the Duytsche Mathematicque was officially closed by the board of curators and burgomasters. Their arguments were twofold: they considered the Duytsche Mathematicque to be superfluous because

[29] [Berkel, 1988, 157-161] and [Wreede, 2007, 39]. De Wreede noticed the connection between the appointment of Rudolph Snellius to ordinary professor with the appointments of Van Ceulen and Van Merwen as extraordinary professors, but she did not make the same connection regarding the promotion of Willebrord Snellius to ordinary professor and the appointment of Frans van Schooten sr. at the Duytsche Mathematicque in 1615.

[30] Such discussions occurred in 1602 and 1646–1648. For a detailed discussion on the examination at the Duytsche Mathematicque, see [Winter, 1988, 19-20, 23-25] and [Hoefer, 1928, 208-215]. The role of Duytsche Mathematicque in the examination of surveyors has been investigated in [Muller and Zandvliet, 1987, 25-26 and 150-154].

[31] In the period 1600–1670, the series lectionum from seventeen years have been preserved. These are the series lectionum of the years 1604, 1624, 1631, 1654, 1657–1668 and 1670, [Otterspeer, 2000, 228], [Ahsmann, 1990, 579-587] and [Molhuysen, 1916, 27*, 31*, 68*, 71*, 75*, 90*-95*, 118*, 140*, 150*, 167*, 174*-177*, 187*-194*, 202*, 207*-2011*, 215*, 224*-226*, and 232*].

[32] [Molhuysen, 1918, 71].

the original reason for its creation (namely the education and training of engineers for the army) had been irrelevant for some decades. Secondly, since the university had severe financial problems, closing the Duytsche Mathematicque would relieve the burden of salaries a little. In the same meeting, the burgomasters and curators decided to stop the growth of the university professors by limiting their number to 15, and by announcing a hiring freeze.[33] Thus, the vacancy of the chair of the Duytsche Mathematicque in combination with financial shortages of the university put an end to the lecturing tradition on practically oriented mathematics in the vernacular. In 1684 a revival of the Duytsche Mathematicque under the supervision of Van Schooten's former student Theodoor Craanen was considered, but this did not materialize.[34] Ultimately, the Duytsche Mathematicque was reestablished in the eighteenth century with an emphasis on fortification. The last decades of the seventeenth century saw a substantial change in the theory and practice of fortification, which resulted in the so-called New Dutch Fortification system. Thus Van Schooten jr. seems to have had little or no influence on the fortification teaching in the eighteenth-century Duytsche Mathematicque.[35]

7.2 Sources and cycles

Provided with an overview of the Duytsche Mathematicque as an institution, we will now take a closer look at the actual teaching of Frans van Schooten jr. Two types of sources are available: the above-mentioned *series lectionum* for the period after 1653, and manuscripts of some of the lectures. Both types of sources will be introduced below. On the basis of these sources, I will sketch a panorama of the lectures in the curriculum. We will see that Van Schooten organized his lectures in a cyclic way.

7.2.1 The sources: the *series lectionum* and the manuscripts

The public lectures at the university were organized according to predetermined rules. The lectures were on four weekdays: Monday, Tuesday, Thursday and Friday, while Wednesday and Saturday were reserved for other scholarly activities such as disputations and private lessons. On each of the public lecturing days, the professors lectured at the same time. The subjects of the courses were mentioned in the *series lectionum*, but as a historical source, these *series lectionum* have to be approached with caution. They represent what the professors intended to teach, not what they effectively taught. A major issue was the neglect of teaching obligations by the professors. The absence of the professors was especially frequent during the 1640s and 1650s, leading to increasing complaints by students. The curators did not have an adequate response to this phenomenon.[36]

[33] [Molhuysen, 1918, 364-366]. On the detoriating financial situation of the university in the period 1650–1700 and the hiring freeze of 1681 see [Sluijter, 2004, 127-128 and 248-254].

[34] At that time, Craanen was a professor of medicine in Leiden who was widely known for his Cartesian views. He had given private lectures on algebra and physics before, and had a special interest in mathematics, see [Molhuysen, 1918, 279] and [Molhuysen, 1920, 30].

[35] [Nimwegen, 2006, 329-331]. Hardly anything is known about the curriculum of the eighteenth-century Duytsche Mathematicque.

[36] [Otterspeer, 2000, 232-233].

The libraries of Groningen and Leiden University possess several manuscripts which are related to the Duytsche Mathematicque. These manuscripts were written by the three mathematicians of the Van Schooten family: Frans sr., Frans jr., and Petrus, and they shed considerable light on the teaching practice in the Duytsche Mathematicque. The Leiden manuscripts are described by Jan van Maanen,[37] and an inventory of the Groningen manuscripts is found in this thesis in Appendix A.

Five manuscripts in the Groningen collection are directly related to the Duytsche Mathematicque during Frans van Schooten jr.'s professorship.[38] One of these manuscripts was written by Frans jr.[39], and the other four were written by Petrus.[40] These four manuscripts are lecture notes of courses taught by Frans jr. Petrus matriculated on 11 January 1652 at the age of seventeen at Leiden University in order to study mathematics,[41] and he attended a fair part of the courses taught by his half brother Frans van Schooten jr. Petrus made very accurate lecture notes of these courses.[42] He also compared his lecture notes with the previous lectures by his half-brother, of which no manuscripts have survived. Thus, Petrus's notes provide very detailed information about the content of the courses in the Duytsche Mathematicque. They make it possible for us to determine which subjects were taught, and they give insights in the developments in the courses over time and they will be a valuable source in the rest of my thesis.

7.2.2 Cycles: repetition and structure

Table 7.2 gives an overview of the courses during the professorship of Frans van Schooten jr., based on the information in the *series lectionum* and the manuscripts.[43] Of several courses, a description in the *series lectionum* as well as lecture notes have been preserved.[44] Comparison of the *series lectionum* with the manuscripts reveals that Van Schooten closely followed the subjects that had been announced. This observation shows the reliability of the *series lectionum* as a source for the teaching of Frans van Schooten jr.

Table 7.2 has several gaps, due to the absence of *series lectionum* and manuscripts. This lack of sources is particularly serious for the first years 1647–1652 of Van Schooten's

[37] [Maanen, 1987]. I would like to thank Jan van Maanen, who kindly put his unpublished notes on the Groningen manuscripts at my disposal.

[38] These are the manuscripts UBG, Hs 435, Hs 436, Hs 437, Hs 441 and Hs 444.

[39] Namely UBG, Hs 435

[40] Namely UBG, Hs 436, Hs 437, Hs 441 and Hs 444.

[41] UBL, Archief van Senaat en Faculteiten, inv. nr. 10, p. 327.

[42] Frans van Schooten mentioned the lecture notes of Petrus in [Schooten, 1660b, 543]. As a consequence, I disagree with the opinion of Reinders that Petrus took over the lecturing in the Duytsche Mathematicque during the professorship of Frans so that Frans could devote himself completely to his scholarly activities, [Reinders, 1995, 264-265].

[43] The *series lectionum* are extant for the following semesters: summer 1654, winter 1654, summer 1656, summer 1657, winter 1657, summer 1658, winter 1658, summer 1659, winter 1659 and summer 1660; all series except summer 1656 are published in [Molhuysen, 1918, 27*, 31*, 68*, 71*, 90*, 91*-92*, 94*-95*, 118*-119*]; the series of summer 1656 is found in UBL, Archief van Senaat en Faculteiten, 1575–1877, inv. nr. 290, f. 88r. See footnote 38 for the manuscripts which I consulted.

[44] This is the case for the following semesters: summer 1657, winter 1657, winter 1658, summer 1659, winter 1659.

year	summer	winter
1646	arithmetic	arithmetic
1647-1651	[unknown]	[unknown]
1652	[unknown]	arithmetic [Hs 437]
1653	algebra [Hs 437]	[unknown]
1654	practical geometry, solid geometry and parts of Euclid and Archimedes	practical geometry, solid geometry and works by Euclid and Archimedes
1655	logarithms [Hs 441]	logarithms [Hs 435, 441]
1656	solid geometry and works by Euclid and Archimedes	fortification [Hs 441]
1657	fortification [Hs 441]	fortification
1658	sundials	spherical triangles [Hs 444]
1659	arithmetic [Hs 436]	algebra
1660	irrational magnitudes [Hs 437]	–

Table 7.2 – Subjects taught in the Duytsche Mathematicque, based on the *series lectionum* and the Groningen manuscripts. If lecture notes of the course are available, the shelf number of the manuscript appears in brackets.

professorship, when Petrus had not yet entered the university. A little bit of information on these gaps can be gleaned from other sources.

In 1660, Van Schooten's treatise on perspective entitled *Tractaet der perspective* (Treatise on perspective) was published in Amsterdam.[45] With this work, Van Schooten intended to present the mathematical principles of the theory of perspective in a clear way in the vernacular. In the last paragraph of the treatise, Van Schooten says:

> Furthermore, regarding the practice of perspective, which we have discussed several times at length in our public lessons, together with the above fundaments and other matters belonging to it...[46]

These public lessons must be Van Schooten's lectures in the Duytsche Mathematicque, and I conclude that perspective was part of the curriculum, although we do not know exactly when Van Schooten lectured on the subject.

The situation of fortification is similar. In his manuscript on fortification, Petrus refers several times to minor differences between Van Schooten's course in 1656–1657 and what

[45] [Schooten, 1660b]. The work was the second supplement to the *Mathematische oeffeningen*, [Schooten, 1660a], the first supplement being Christiaan Huygens's *Van rekeningh in spelen van geluck* (On calculation in games of chance), [Huygens, 1660]. The pagination of both supplements follows the pagination of the main text. Van Schooten's treatise on perspective has a separate title page.

[46] "Vorders wat aen-belangt de Practyck der Perspective / dewelcke wy beneffens de voorsz Fondamenten met andre dingen daer toe gehoorig in onse Publijcque Lessen tot meer malen wyt-loopig verhandelt hebben (...)." [Schooten, 1660b, 543].

he calls the first version of the course.[47] Therefore Van Schooten must have lectured on fortification in the period between 1647 and 1652. Thus he must have taught a similar course on fortification two different times. Table 7.2 shows the repetition of other subjects as well: Van Schooten taught algebra in 1653 and in 1659, and during the years 1646, 1652 and 1659 he lectured on arithmetic.[48] A comparison of the content of the various courses in arithmetic shows that these courses share some characteristics. This similarity and the repetition of the courses on fortification and perspective indicate that Van Schooten organized his teaching in the form of a cycle of courses. A detailed investigation of the manuscripts reveals that one cycle consisted of 13 semesters. The first cycle started in the spring of 1646, when Van Schooten had just been appointed, and ended with the summer semester of 1652. The second cycle started with the winter semester of 1652, and lasted until the winter semester of 1658. The summer semester of 1659 was the start of the last cycle. The cycles probably contained roughly the same subjects. All three cycles started with a course in arithmetic.

The repetitive nature of Van Schooten's teaching had been briefly noticed by Van Maanen,[49] but my interpretation is different in the following sense. Van Maanen states that Van Schooten jr. had nearly finished his second cycle at the time of his death in 1660, but the manuscript material which I consulted shows that he had already started his third cycle in the summer of 1659.

The structure of Stevin's instruction of 1600 is still visible in Van Schooten's educational cycles: each cycle begins with arithmetic, and practical geometry is treated before fortification. Van Schooten's lectures on practical geometry dealt with surveying and included elementary geometry as well as an introduction to trigonometry and the use of trigonometric tables.[50] To Stevin's trinity of arithmetic, geometry and fortification, Van Schooten added other topics. He considered algebra as an important subject, and he placed it immediately after arithmetic and before geometry in the curriculum. The topics of sundials and spherical triangles belonged to the realm of mixed mathematics, and logarithms were also relevant, because they could be used to simplify some of the tedious calculations in spherical triangles.

This overview of Van Schooten's courses also shows what was not included in the curriculum of the Duytsche Mathematicque. It is important to say a word on what was missing in the curriculum, because this will help us to understand what the Duytsche

[47] For instance, "Mijn broeder voor de eerste reise de fortificatie inde les leesende heeft dees fyguer aldus uytgewrogt" ("When my brother lectured on fortification the first time, he worked out this figure in the following way") and '‘Mijn boeder heeft de eerste reyse de fortificatie leesende de sijde gestelt van 90 Roeden'‘ (‘When my brother lectured on fortification the first time, he supposed the side to be 90 rods'), UBG, Hs 441, f. 118r. and f. 125r.

[48] Notes made by Petrus of the arithmetic courses of 1652 and 1659 exist, see UBG HS 437 (1652) and 436 (1659). Because the year 1646 in used in one of the numerical examples in the lecture notes of 1652, I conclude that the course in arithmetic was also taught in 1646; see UBG Hs 437 f. 2r.

[49] Van Maanen states the repetitive nature in a "stelling" in his thesis and thus does not discuss the evidence in detail, see [Maanen, 1987, Stelling 1].

[50] That trigonometry was treated in the course can be deduced from the fact that knowledge of trigonometry and trigonometric tables is assumed in later courses in the cycle.

Mathematicque was. The absence of navigation is noteworthy, because it was a prominent subject in mixed mathematics and besides that it was a prominent issue for the active seagoing and trading Dutch. Hortensius (1605–1639), professor of mathematics at the Amsterdam Athenaeum, stressed the importance of mathematical knowledge for navigation in his inaugural lecture.[51] In the Duytsche Mathematicque, some reference to geography and navigation was made in the course on spherical trigonometry, and the globes in the library were used for teaching purposes[52], but navigation as a subject on its own was not part of the curriculum. This absence can be attributed to the geographical situation of Leiden, which had no direct access to the sea and no harbour. The audience for courses on navigation, when not at sea, resided in harbour cities like Amsterdam and Rotterdam, and mathematical classes in navigation flourished in these cities.[53]

My investigation of Van Schooten's public teaching activities shows that Cartesian geometry was not taught by Van Schooten in the Duytsche Mathematicque. However, in the literature it is often claimed that Van Schooten introduced Cartesian geometry in the curriculum of the university in his lectures.[54] This claim is unfounded. At Leiden University, Van Schooten was appointed as an extraordinary professor of the Duytsche Mathematicque, and the only lectures he gave at the university were those at the Duytsche Mathematicque. Cartesian geometry was not part of these lectures. Jacob Golius, the ordinary professor of mathematics, took care of the ordinary lectures in mathematics aimed at the proper students of the university.

The only occasions where Van Schooten taught Cartesian geometry were his private lectures. These private lectures fell outside the responsibility of the university and as such were not a formal part of the curriculum. At Leiden University, it was common for professors to organize private lectures for their students in which certain subjects were studied more profoundly. These lectures offered the professors the possibility of extra revenues, as the students had to pay a fee. There was no formal supervision on these lectures on behalf of the university. During the seventeenth century, the private lectures gradually gained importance. As a side-effect, a competition developed between public and private lessons. This competition made the board of the university announce that private lessons should be organized at times when no public lectures were scheduled, preferably on Wednesdays and Saturdays.[55]

In his private lectures Van Schooten discussed Cartesian geometry as well as algebra in Cartesian notation. A document on such an introductory course in the form of private lectures is the treatise *Principia matheseos universalis, seu introductio ad geometriae methodum Renati Des Cartes*. This treatise was written by Van Schooten's student Erasmus Bartholin on the basis of lecture notes taken in the years before 1650.[56] Van Schooten's contacts with men like Hudde and Van Heuraet resulted from such private lectures. Van

[51] See [Imhausen and Remmert, 2006, 126-129].

[52] [Hulshoff-Pol, 1975, 417-417].

[53] On mathematical education in Amsterdam and its link with navigation, see the thesis of Tim Nicolaije, which will appear in the near future.

[54] For this claim, see for instance [Maronne, 2007, 26] or [Sasaki, 2003, 394].

[55] [Otterspeer, 2000, 231-233 and 310].

[56] [Bartholin, 1651].

Schooten encouraged these men to explore the new possibilities of the Cartesian approach to geometry.

CHAPTER *8*

Traces of Stevin: arithmetic and fortification

Arithmetic, practical geometry (surveying) and fortification were the subjects which belonged to the curriculum of the Duytsche Mathematicque since the beginning in 1600. Unfortunately, no source material has been preserved on Van Schooten's lectures in practical geometry. In this chapter I will discuss the content of his courses in arithmetic and fortification. My analysis will reveal how Van Schooten structured his lectures, what topics he considered to be of value for his students, and what the mathematical level of his lectures was. The investigation of these questions will help to understand the developments which Van Schooten initiated in his teaching. For arithmetic as well as fortification I also discuss the work of contemporaries in those fields, in order to compare the content of Van Schooten's lectures to that of his contemporaries. I first discuss the lectures in arithmetic in section 8.1 and then turn to fortification in section 8.2.

8.1 Arithmetic

As shown in the previous chapter, Van Schooten organized his teaching in cycles of courses. Each educational cycle began with a course in arithmetic to make the students familiar with the Hindu-Arabic numerals, the basic arithmetical operations and some elementary arithmetical rules. In the present section, I will compare Van Schooten's courses with the tradition of arithmetical treatises in the vernacular and with the teaching of arithmetic in universities. This will make it possible to identify the characteristics of Van Schooten's courses in the Duytsche Mathematicque, as compared to the regular arithmetic instruction of the time.

Arithmetic was already mentioned in Stevin's instruction of 1600. Stevin prescribed that a student at the Duytsche Mathematicque should learn

> arithmetic or counting, and surveying, but of each subject only as much as is needed for the usual practical engineering. (...) In counting: the four species[1] in whole numbers, in fractions and in decimal fractions will be taught, together with the rule of three in each of these number [types].[2]

Stevin thus envisaged a basic course in arithmetic, to train the student in the elementary arithmetical operations and the rule of three, in whole numbers, in the traditional fractions, and in decimal fractions as well. Stevin had also promoted the use of decimal fractions in his treatise *De Thiende (The tenth)*.[3] The arithmetical skills should only be taught in so far as they were necessary for ordinary practical engineering purposes. Frans van Schooten jr. widened the scope of arithmetic teaching, by adding new topics and placing the course in a commercial trading context.

8.1.1 Arithmetic instruction in the vernacular

In the Low Countries, arithmetic teaching developed in the sixteenth century, and was initially based on a tradition which emerged in the Italian Renaissance. This tradition was characterized by the embedding of arithmetical problems in a commercial context. During the Italian Renaissance, economic growth and an increase of trade required arithmetical skills on the part of many people involved in commercial and financial activities. The sons of Italian merchants attended the newly established arithmetic schools in order to obtain the right skills for a career in trade and commerce.[4] The increasing interest in arithmetic is reflected in a growing number of textbooks on the subject. In the books as well as the schools, the vernacular Italian was used as language of instruction instead of Latin. Through contacts between Italian and German merchants, the arithmetical teaching tradition spread over the Alps and also reached the Low Countries.[5] Men teaching arithmetic in the vernacular were known as reckoning masters.[6]

The tradition is represented by various sixteenth-century treatises in Dutch on arithmetic. These treatises are remarkably similar, showing that there was to a large extent consensus among reckoning masters on the content of an introductory course in arithmetic.[7] The tradition continued in the seventeenth century in texts such as *De Cijfferinghe*

---

[1] The four species consisted of addition, subtraction, multiplication and division. Note that Stevin did not include numeration in the species.

[2] "Hier toe sal men leeren de Arithmeticque ofte het tellen ende het landtmeten: maer alleenlijck van elck soo veel als tot het dadelijck gemeyne Ingenieurschap noodich is (...) In de telling sullen gheleert worden de vier specien int geheel ghetal int ghebroken ghetal ende in thiende-tal: mitsgaders den Regel van Dryen in elck der selve ghetalen," [Stevin, 1600, 1-2].

[3] [Stevin, 1585a].

[4] [Kool, 1999, 21-23] and [Swetz, 1992, 365-368].

[5] [Kool, 1999, 22, 24]

[6] The Dutch term is *rekenmeester*, compare the German *Rechenmeister*.

[7] [Kool, 1999, 56].

(Arithmetic) of Willem Bartjens, which appeared in 1604. The various revisions of this work were the leading arithmetical textbooks in the Netherlands until the eighteenth century.[8]

The general structure of a Dutch arithmetic textbook of the sixteenth or seventeenth centuries is as follows. The first part deals with the notation that is used in arithmetic and the basic arithmetical operations, which were called "species". There was no complete agreement among the reckoning masters on the number of "species" . Most authors included numeration (the pronunciation of numbers), addition, subtraction, multiplication and division. In some cases, doubling, halving and root extraction were also considered as "species".[9]

The second part of the textbook was a collection of arithmetical rules, of which the "rule of three" was most important. This rule was used to solve problems of the following kind in modern notation. For three given numbers a, b and c, a fourth one d has to be found in such a way that $a : b = c : d$. The general solution $d = \frac{(b \times c)}{a}$ was usually introduced in a needlessly complicated way.[10] This and other arithmetical rules were presented for the solution of a variety of problems, and it was understood that each type of problem required its own rule. The rules were usually introduced in a narrative way, in the form of a recipe, and in the context of concrete numerical examples. From these examples, the student had to figure out by himself how the particular rule had to be employed in other instances. The student was not supposed to understand the underlying mathematical ideas and the relationships between the rules; he only had to identify the appropriate rule for the problem.

The rules were named after the kind of problems, and as a consequence we find a bewildering proliferation of arithmetical rules in the textbooks.[11] The basic operations or "species" together with a multitude of arithmetical rules were the major ingredients of sixteenth- and seventeenth century textbooks. Some authors touched upon additional subjects such as root extraction, summation of finite series, calculations with proportions, arithmetic of the counting board, and some algebra.

8.1.2 Arithmetic teaching at universities

In the Middle Ages, mathematics was taught at universities because it belonged to the seven liberal arts. These arts consisted of the trivium, which was devoted to grammar, logic and rhetoric, and the quadrivium, dealing with the study of quantity, and consisting of arithmetic, geometry, music and astronomy. At medieval and early-modern universities, the students spent the first years on the liberal arts, before they continued to the higher faculties of theology, law or medicine. The teaching of mathematics thus belonged to the preparatory curriculum of the universities.

[8] [Bartjens, 2004, 77].
[9] [Kool, 1999, 56, 60]
[10] [Kool, 1999, 132].
[11] To make order in this chaos, Kool distinguished three categories of arithmetical rules, according to the underlying mathematical principle, [Kool, 1999, 136].

The contents of ancient and medieval arithmetic was not the same as that of arithmetic in the modern sense of the word. Ancient Greek scholars distinguished between arithmetic, dealing with the theory of numbers, and logistics, dealing with the art of counting and calculating.[12] Lectures on arithmetic at medieval and early modern universities focused on the investigation of integer numbers and some of their (very elementary) relationships, but not on practical computation, so the university teaching was very different from that in the commercial arithmetic schools led by the reckoning masters. The treatise *De Institutione Arithmetica* (Introduction to Arithmetic) of Boethius (ca. 480–524 or 525) often served as a textbook in the university lectures, and it remained a popular work during much of the sixteenth century. Boethius discussed even and odd numbers, perfect numbers and a few elementary properties of integers.[13]

During the sixteenth and seventeenth century, the university teaching of arithmetic changed as the result of developments in astronomy and in the general approach of scholars towards knowledge. The field of astronomy received gradually more attention and required competence in practical calculations on the part of its students. The predominantly scholastic teaching at universities was challenged by the influence of humanism, which entailed a renewed interest in classical scholars and their texts, and placed more emphasis on the utility of knowledge.[14] In the teaching of arithmetic, these developments led to a more practical approach, which included the hitherto neglected calculations in Hindu-Arabic numerals. In 1540, the humanist Gemma Frisius (1508–1555) wrote a textbook on arithmetic entitled *Arithmeticae practicae methodus facilis* (Easy method of practical arithmetic),[15] on the performance of actual computations. He may have used this book in his own lectures at Louvain university.[16] Around the same time, the French humanist Petrus Ramus (1515–1572) unfolded his views on education, and presented a reformed curriculum with a prominent place for mathematics. Ramus authored several mathematical textbooks and he produced successive editions of his *Arithmetica*, in which he gradually moved away from the ancient Greek arithmetical heritage.[17]

Both books found their way into academic teaching in Europe, as can be seen in the following examples. In the beginning of the seventeenth century, the *Arithmeticae practicae* of Frisius served as the main textbook for the lectures on arithmetic at the Lutheran University of Helmstedt, where mathematics was taught with a practical orientation, with due attention to engineering and mathematical instruments.[18] The textbook by Ramus and its reworked versions served as instruction manuals at several institutions. An example is the University of Basel, where Christianus Urstitius (1544–1588) wrote a popular

[12] On the distinction between arithmetic and logistics in Greek mathematics see [Heath, 1981, 13-16].

[13] [Boethius, 1983, 11-12, 69].

[14] [Wreede, 2007, 22-23] and [Otterspeer, 2000, 31-33]. On the influence of humanism on the universitary education see [Grafton and Jardine, 1986].

[15] [Frisius, 1540].

[16] [Smeur, 1960, 21].

[17] There is a vast amount of literature on Petrus Ramus. For an overview of the literature up to 2000 see [Sharratt, 1972], [Sharratt, 1987], and [Sharratt, 2000]. The standard work on Ramus and this mathematics still is [Verdonk, 1966]; more recent work on Ramus and mathematics includes [Pantin, 2004] and [Loget, 2004]. For Ramus's influence on the educational institutions in the German countries including the Dutch Republic see [Hotson, 2007].

[18] [Omodeo, 2011, 6].

arithmetical textbook based on the *Arithmetica* of Ramus.[19] At Leiden university, the influence of Ramus was embodied in the successive professors Rudolf Snellius (1546–1613) and Willebrord Snellius (1580–1626). Rudolph and his son Willebrord wrote commentaries [Snellius, 1596] and [Ramus and Snellius, 1613] on Ramus's textbook of arithmetic, to be used in their own arithmetic teaching.[20]

That these developments did not happen at all places with the same intensity is shown by the case of the Englishman Samuel Pepys (1633–1703), who was president of the Royal Society from December 1684 until Februari 1686. After attending grammar school and St Paul's School in London, he went to Cambridge university in 1650 and graduated in 1654.[21] The lectures on the liberal arts did not provide him with the necessary knowledge to perform calculations by himself, and he learned to calculate at the age of 29 by the instruction of a private tutor.[22] By the mid-seventeenth century, it was still possible to graduate and have an academic reputation without competence in arithmetic.

Thus, the teaching of arithmetic at universities in the early modern period shows a wide variety. The quality of instruction depended heavily on the mathematical skills of the teacher. As arithmetic was part of the preparatory courses, the subject was sometimes taught by junior lecturers or by professors whose main qualifications were in fields other than mathematics. Although the interest in practical calculations was increasing, university teaching of arithmetic remained in general more philosophical and theoretical than the practical instruction in the vernacular by reckoning masters outside the universities.

8.1.3 Structure of Van Schooten's lectures on arithmetic

During the period of his professorship, Van Schooten lectured three times on arithmetic, in 1646, 1652 and 1659. Of his first lecture series of 1646, no manuscript has been preserved, but lecture notes of the courses of 1652 and 1659 are extant. In the 1652 manuscript, the number "1646" is used several times in numerical examples. These examples may well date back to the course of 1646, so we can assume that the 1652 course resembled the 1646 course to some extent.[23] Table 8.1 displays the structure of the courses of 1652 and 1659, which agrees with the standard Dutch arithmetic treatises of the sixteenth and seventeenth century. Thus, Van Schooten's courses are divided into a first part on arithmetical operations, and a second part dealing with arithmetical rules.

The course of 1659 was on the whole a revised version of the 1652 course,[24] but Van

[19] [Hotson, 2007, 22].

[20] [Wreede, 2007, 44, 71, 291].

[21] See the lemma on Pepys in the *Oxford dictionary of national biographies*, `http://www.oxforddnb.com/view/article/21906?docPos=7` Retrieved 8 August 2012.

[22] [Andersen and Bos, 2006, 698]. Pepys describes his instruction in arithmetic in his diary, see `http://www.pepysdiary.com/archive/1662/07` or [Pepys et al., 1995]. In 1665 Pepys was elected fellow of the Royal Society. The lessons in arithmetic and mathematics increased his interest in the subject, and he became an advocate of the use of mathematics in education. In 1673 he was involved with the establishment of the Royal Mathematical School at Christ's Hospital, an institution aimed at training young boys in the necessary mathematics for the purpose of navigation, [Plumley, 1976].

[23] The *number* 1646 is mentioned on UBG, Hs 437, f. 1r and the *year* 1646 is mentioned on f. 2r.

[24] There is a large overlap in the arithmetical problems of the two courses. For instance, in 1652 Van Schooten explained the rule of three by means of eleven problems, and in 1659 he used ten of these eleven problems, plus

1652	**1659**
basic operations: numeration, addition, subtraction, multiplication, division	*basic operations*: numeration, addition, subtraction, multiplication, division
basic operations in measures, coins and weights	basic operations in measures, coins and weights
fractions: reduction, basic operations and decimal fractions	*rules*: rule of three
rules: rule of three, converse rule of three, double rule of three	*fractions*: reduction and basic operations
sequences: arithmetical and geometrical	*rules*: rule of three in fractions, converse rule of three, rule of company, rule of allegation, rule of virgins, rule of false position
combinatorics	
rules: rule of company, rule of allegation, rule of false position	
root extraction	
rule of interest and construction of tables of interest	
rules: rule of profit and loss, rule of exchange, rule of change, rule of gold and silver, rule of virgins	

Table 8.1 – The structure of the arithmetic courses of 1652 and 1659.

Schooten treated fewer operations, fewer arithmetical rules and fewer additional topics. Thus, root extraction, sequences and combinatorics were omitted from the 1659 programme. He also presented some of the material in a different order, structuring the treatment of the arithmetical rules according to mathematical principles. In the second part of the section on fractions, he first discussed the rule of three, and its two mathematically equivalents, the "rule of company" and the "rule of allegation". Then he turned to rules based on other mathematical principles such as the "rule of virgins" and the rule of false position. In 1652 the order of the arithmetical rules had been less organized.

8.1.4 Numeral systems

Both courses began with an extensive introduction to the two numeral systems that were used at the time: Hindu-Arabic numerals and Roman numerals. In addition to this, Van

three new problems in his explanation. See UBG, Hs 437, f. 30v. 32r. and Hs 436, f. 52-57. The situation is similar for the other operations and rules.

Schooten also discussed the use of the counting board. Van Schooten began his discussion of the Hindu-Arabic numerical system with the ten number symbols. He put special attention on the concept of zero and the positional system, which had to be understood in order to perform calculations. The concept of zero was absent in the Roman numeral system and in the arithmetic methods on the counting board.

Van Schooten also explained the pronunciation of numbers, and of very large numbers in particular. The most common manner was to divide the whole number into groups of three digits from right to left. The number 509 117 543 200 should be pronounced as "fivehundred and nine thousand million and hundred seventeen million and five hundred fourty-three thousand and two hundred".[25] In the 1659 course, Van Schooten also discussed two other ways of pronouncing numbers, one based on ten thousands and one on hundreds. The manner of ten thousands was inspired by the ancient Greek numeral system. In this method, the numeral was divided in groups of four digits from right to left. Thus, the number 1 235 072 951 was pronounced as "Twelve tenthousand times tenthousand three thousand five hundred and seven times tenthousand two thousand nine hundred fifty one." [26]

Roman numerals were the second system to be explained. Van Schooten mentioned that these numerals were useful for indicating the year of publication in a book, but not for actual calculations.[27]

The third system was based on the counting board, which had been used for representation of numbers and calculations before the introduction of Hindu-Arabic numerals. A counting board consisted of a series of horizontal lines representing 10^n units, for $n = 0, 1, \ldots$. Coins were used to represent the numbers on the board (see figure 8.1), so the number 237 could be represented by seven coins on the line for units, three coins on the line for 10 units and two coins on the line for 100 units. Some users of the counting boards introduced additional lines for $5 \cdot 10^n$ between the lines for 10^n and 10^{n+1}, so that fewer coins were needed to express a number. These additional lines were also used by Van Schooten. Calculations on the counting board were common throughout the seventeenth century. At the end of the seventeenth century, special coins for the counting board were still manufactured in the Southern Netherlands.[28]

Van Schooten treated the four basic arithmetical operations in only two of the three numeral systems. He first showed how the operation could be done on paper using the Hindu-Arabic numerals and then how the same operation was performed on the counting board.

Both the Hindu-Arabic numeral system and the counting board had their advantages and disadvantages. The use of a counting board did not require a high level of literacy nor was any writing material needed. One could perform the computations on a table

[25] "Vijfhondert en negen duijsent millioenen hondert seventien millioenen vijhondert drieenveertich duijsent twee hondert"UBG, HS 437, f. 1r.

[26] "twaelf tienduijsent mael tienduijsent drie duijsent vijfhondert en seven tien mael duijsent twee duijsent negenhondert een en vijftich", UBG, Hs 436, 5

[27] UBG, Hs 436, p. 7.

[28] [Barnard, 1916, 90].

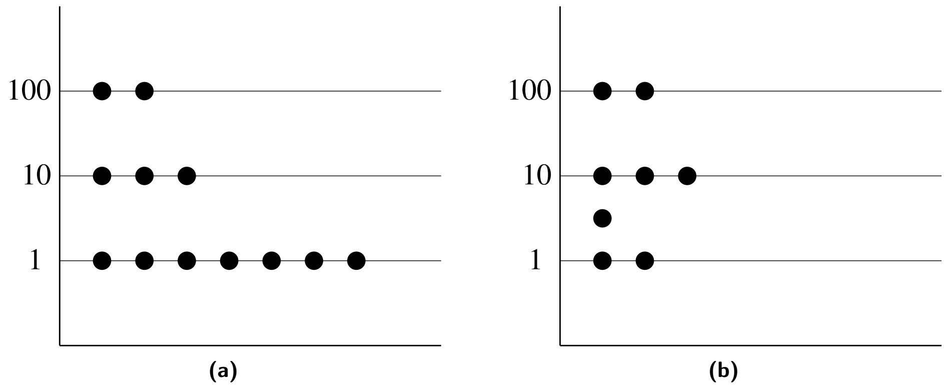


Figure 8.1 – The number 237 on a counting board (a) and a counting board with additional lines (b).

and communicate the final outcome by just pronouncing it. Sometimes the outcome was recorded in Roman numerals. The operations on the counting board were very illustrative. Addition meant physically adding coins to the counting board, whereas addition in Hindu-Arabic numerals was done by means of an abstract algorithm. The counting board did not require a symbol for the abstract concept of zero.

On the other hand, the more difficult operations of multiplication, division and root extraction were easier to perform by means of the Hindu-Arabic numerals than on the counting board. This was especially the case when large numbers were involved. The outcome of an arithmetical operation in Hindu-Arabic numerals was gradually written down in the course of the calculation process, whereas the user of the counting board had to write the outcome separately. In the Hindu-Arabic system, it was possible to check the final answer because the steps of the computation are recorded on paper or on a slate. Van Schooten provided various methods for the verification, like casting out threes or nines, or performing the operation in a different order. For users of a counting board, it was not possible to verify the outcome, since the calculation was a process in which coins were added or (re)moved. Finally, the use of Hindu-Arabic numbers stimulated the ability of abstract reasoning which facilitated the study of other mathematical subjects.[29]

In his arithmetic courses, Van Schooten discussed some of the above-mentioned differences between Hindu-Arabic numerals and counting board arithmetic with his students. In his view, calculating on a counting board was more illustrative and required less knowledge, even for the more difficult operations as multiplication. For instance, in order to perform efficient multiplications in the Hindu-Arabic numerals, one had to make an effort and master the tables of multiplication by heart. For Van Schooten, these efforts were outweighed by the advantage that the outcome of a computation in the Hindu-Arabic numerals can be checked. Therefore he preferred the Hindu-Arabic numbers over the counting board.[30]

[29] [Kool, 1999, 30-34].
[30] UBG, Hs 436, 31.

$$\text{original measure} \xrightarrow{\text{transformation}} \text{decimal fraction} \xrightarrow{\text{arithmetical operation}} \text{outcome in decimal fractions} \xrightarrow{\text{interpretation}} \text{final outcome}$$

Figure 8.2 – Use of decimal fractions.

Once the student was familiar with the basic arithmetical operations, he had to apply theses rules to practical problems involving measures, coins, and weights. In the early modern period the metric system had not yet been introduced, and a great variety of units of weights and measurement and local monetary systems were used. Each unit was subdivided in its own way; for example, one "pound Flemish" was equal to 20 "shilling"; 1 "shilling" equalled 12 "grooten" and one "groote" consisted of 24 "mites". Van Schooten gave an overview of the current measures, coins and weights, and explained how to calculate with them.

He then used measures, coins and weights in order to introduce the concept of fractions. In modern terms he explained that n shilling are the $\frac{n}{20}$-th part of one pound Flemish. By embedding the concept in a familiar context, he made it easy for his students to grasp the idea of a fraction. He then discussed the idea of a greatest common divisor (gcd) and least common multiple (lcm) of natural numbers. He used these concepts for simplifying fractions (using the gcd) and reducing fractions to the same denominator (using the lcm). Then he treated the four arithmetical operations on fractions in the same order as the operations on integers.

8.1.5 Decimal fractions

In his 1652 course, Van Schooten discussed the general idea of decimal fractions and computations in decimal fractions. Van Schooten's decimal notation is an interesting combination of Stevins notation and the modern notation using a decimal mark. Stevin wrote the number 123.456 as 123 ⓪ 4 ① 5 ② 6 ③; the numbers in circles specify the position of the successive digits after the decimal mark. Van Schooten wrote a decimal mark between the whole and fractional part, usually a comma, and he put Stevin's ⓝ above the decimal, as follows: $12\overset{⓪}{3},\overset{①}{4}\overset{②}{5}\overset{③}{6}$.

Van Schooten illustrated the use of decimal fractions by problems, most of which concern currency or surveying. He used decimal fractions in order to simplify the computations, and his general procedure consists of three steps which are represented schematically in figure 8.2. First, the original data had to be converted into decimal fractions. Then the necessary computations were performed, and the final step consisted of the interpretation of the result. There is an interesting difference between currency problems on one hand and surveying problems on the other hand.

In currency problems, the result in decimal fractions was converted back to the original coinage. An example is the following problem: If one pays to 124 men 16 stuivers each, how much is the total cost? Van Schooten multiplied 124 by 0.8, obtained the product 99.2

in decimal fractions, and converted the result to 99 guilders and 4 stuivers.[31] This example is easy, but in general the conversion could create a new and cumbersome problem. In such cases, decimal fractions did not facilitate the calculation.

In surveying problems, the final outcome in decimal fractions was not converted back to the original units of measurement, but interpreted in a new decimal measurement system. I will now illustrate this practice by an example from Van Schooten's lectures. Van Schooten used the Rhenish measurement system in which the Rhenish rod (ca. 3.7 meters) was the fundamental unit of measurement. One Rhenish rod contained 12 feet, each feet was 12 thumbs (inches), and each thumb equaled 12 "greijn" (grain). Van Schooten converted a given length in Rhenish rods into decimal rods, as in the following example: a length of 13 Rhenish rods, 5 feet and 8 thumbs became $13 + \frac{5}{12} + \frac{8}{144} \approx 13.437$ rods.[32] This decimal expression was to be pronounced as "13 rods, 4 feet, 3 thumbs and 3 grains". Note that these feet, thumbs and grains are not the original Rhenish feet, thumbs and grains as mentioned above, but *decimal* units of measurement. Thus two measurement systems were in use: the traditionally subdivided Rhenish rod, and the decimally subdivided Rhenish rod.[33] The decimal system was apparently used to simplify computations.

Van Schooten's 1652 lectures suggest the following phenomenon, which had also been observed by Beckers and Kool.[34] By the middle of the seventeenth century, the decimally subdivided rod was commonly used by surveyors, but decimal fractions were not used in financial computations. This phenomenon can be explained as follows. In surveying, the Rhenish rod was the standard unit of measurement in Holland, and its decimal subdivisions were well understood inside the closed community of surveyors. In commerce, merchants dealt with a variety of local coins and weights, and no standard currency existed. The introduction of decimal fractions would have implied that for all the different local currencies and units of weight, a decimal subdivision would have to be introduced, so confusion could easily arise. Moreover, many merchants were not well versed in decimal computations. These two reasons explain why decimal fractions were not introduced in commerce during the seventeenth century.

In his revised arithmetic course of 1659, Van Schooten omitted the decimal fractions altogether. The following explanation suggests itself. In the course of 1652, prospective surveyors were an important part of the audience, but in 1659, Van Schooten faced a more general public for whom decimal fractions would not be useful in their daily activities. Therefore he decided to transfer the subject to his course on practical geometry or surveying, the fields in which decimal fractions were used.[35]

---

[31] UBG, Hs 437, f. 28r.

[32] This example is taken from UBG, Hs 437, f. 26r. The ≈ sign is mine; Van Schooten gives three decimals but does not mention the fact that his result is an approximation.

[33] Already in 1600 a decimal measurement system was explained by the surveyors Sems and Dou, who did not use Stevin's notation [Sems and Dou, 1600, 2]. The simultaneous use of traditional Rhenish rods and decimally subdivided Rhenish rods was common in the eighteenth century as well, see for instance [Bordus and Stammetz, 1758, 391].

[34] [Bartjens, 2004, 50].

[35] The manuscript on the fortification course contains a note referring to the ordinary use of decimal fraction by surveyors: "keetten van tienen ordinaris by de landtmeetters" (chain of tens (as) ordinarily (used) by surveyors). Such a course could be a natural occasion to explain the conversion of calculated lengths of parts of fortresses

8.1.6 Arithmetical rules

In the 1652 course, Van Schooten taught many arithmetic rules, but in 1659 the number of these rules was substantially reduced. Some of the rules in the 1652 course are mathematically equivalent to the rule of three, but used in a particular context. An example is the rule of exchange, which was used to solve problems like the following:

> A merchant from Rotterdam is in London and he has to exchange 1020 pound Flemish into pound sterling. The rate is 30 shilling for a pound Flemish. How much pound sterling does he obtain?[36]

A mathematical equivalent of the rule of three was used in the solution but in a slightly different wording, and the new rule was named the rule of exchange. In 1652 Van Schooten discussed four other rules of this type, which were called the rule of interest, the rule of profit and loss, the rule of overland and the rule of gold and silver. In the 1659 course these rules had all vanished.

In his 1659 course Van Schooten adduced the following argument for reducing the number of rules. After the course in arithmetic, he planned to lecture on algebra, which provided a general method for the solution of arithmetical as well as geometrical problems. Because all remaining problems of arithmetic could also be solved by algebra,[37] there was no need to learn an abundance of arithmetical rules by heart.[38]

Two arithmetical rules kept a prominent place in the 1659 course. The rule of three was highly esteemed by authors of arithmetical textbooks of the sixteenth century, and Van Schooten agreed: without the rule of three, hardly any problem in arithmetic could be solved.[39] He pointed out its great use in geometry, astronomy, fortification, and commercial arithmetic, and added some further problems in a commercial context which could be solved by the rule of three.[40] Here, Van Schooten explicitly aimed at an audience with a commercial background. The other prominent rule was the rule of false position (regula falsi). This rule could be used in problems which are in modern terms equivalent to a linear equation or a system of linear equations. The use of the rule of false position however did not require any knowledge of equations. The rule specifies how the solution can be found by first assuming two arbitrary different numbers. In modern notation, the rule boils down to the following. Let our problem be equivalent to solving $f(x) = 0$ for a function $f(x) = ax + b$, and suppose that x_1 and x_2 are arbitrary numbers. Assuming $f(x_1) \neq 0$ and $f(x_2) \neq 0$ in modern notation, hence x_1 and x_2 are incorrect ("false") solutions of the

into decimal fractions, so that the fortress could be laid out in the field using the surveyors chain. UBG, Hs 441, f. 111v.

[36] Free translation of the question of UBG, Hs 437, f. 89r.

[37] "Dewijl ons voornemen is vervolgens de Algebra te verhandelen door welcke niet alleen alle Arithmetice maer oock Geometrice questien sonder onderscheyt op eene wijs cunnen ontbonden worden door welcke wij dan de voorgaende en alle de resteerende questien in arithmetica vallen sullen leeren ontbinden." UBG, Hs 436, 137.

[38] In the 1659 course, only two rules directly based on the rule of three were kept in the course: the rule of company and the rule of allegation. These rules had become so common that they were part of the standard corpus in arithmetic which every student had to learn.

[39] "Sonder deselve [i.e., the rule of three] geen questie inde Arithmetica bijnae can worden opgelost". UBG, Hs 436, 52.

[40] UBG, Hs 436, 55-57.

problem. Then the correct solution x can be found from

$$x = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}.$$

Van Schooten justified the prominent position of the two rules in the following way:

> This rule [i.e., regula falsi] together with the previous [i.e., rule of three] are the most essential and extraordinary [rules] of arithmetic, among which all the remaining rules are included, which abound among arithmeticians like [the rule] of interest, of exchange, of overland and of gold and silver, by which arithmetic is made harder than it ought to be.[41]

Thus Van Schooten was aware of the difficulties which were created for the students by the large amount of equivalent arithmetical rules. Students had to memorize each rule and in each problem they had to decide what rule to use. As the number of rules increased, arithmetic became more cumbersome.

Van Schooten correctly stated that all problems of arithmetic leading to "a first equation" could be solved by the use of the rule of false position.[42] This rule could be used to solve a wide range of arithmetical problems, even problems for which no particular arithmetical rule had been designed. Van Schooten therefore called the rule of false position "the little algebra", because the rule could be used for solving many problems, just like algebra.[43]

There exists another similarity between the rule of false position and algebra. In the rule of false position, one operates with the values x_1 and x_2 as if they were solutions of the problem. In algebra, one also assumes solutions and calculates with them as if they were known. By calling the rule of false position "the little algebra", Van Schooten made a connection to his course on algebra, which followed the course on arithmetic.

Just like the arithmetical textbooks of the time, Van Schooten's course contained a large number of problems which served as examples and explanations. As in the textbooks, Van Schooten placed most of the problems in a context of commercial activities: barter, exchange of money, settlement of costs of transportation, partition of profits in commercial partnerships, determination of discounted values etc. Only a few arithmetical problems in the course are related to surveying or military or other engineering activities. Van Schooten mentions the "extensive use in trade and daily human affairs" of the rule of

[41] "Dees reegel valt tsamen de voorgaende sijnde nootsaeckelickste en bijsonderste der Arithmetica onder welcke alle de resteerende reegels derselven begreepen zijn welcke noch veele getelt worden bij de Arithmetici als van Intrest, wissel, overlandt, van gout en silver & waerdoor d'Arithmetica swaerder gemaeckt wort als behoort", UBG, Hs 436, 137.

[42] By "a first equation" Van Schooten meant an equation of the first degree.

[43] "Reegel Falci (...) alsoo genaemt omdat deselve uut een Valsch (...) getal leert vinden het waer begeerde getal (...) waerom deselve oock de kleine algebra genoemt wort om dat door deselve als door algebra veelderhande questien op een wijs cunnen ontbonden worden welcke door geen ander reegel der Arithmetica cunnen werden opgelost te weetten alle diegeene welcke niet hooger en loopen als de 1ste aequatie", UBG, Hs 436, 130.

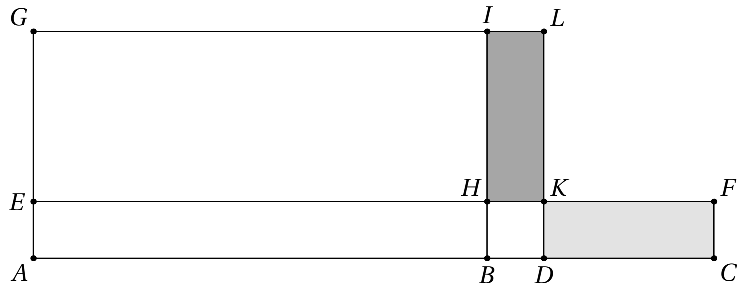


Figure 8.3 – Geometrical interpretation of the rule of allegation.

three, and he gave this as the reason why he added some problems "concerning trade".[44] His emphasis on the use of arithmetic for merchants and related occupations indicates that his lectures were at least partly addressed to an audience engaged in trade and commerce. Thus the students of the Duytsche Mathematicque did not only consist of men who planned to be become an engineer or a surveyor. Van Schooten had a broader audience in mind, and his course in arithmetic must have competed with the courses that were offered by reckoning masters in the city.

In the sixteenth-century and seventeenth-century arithmetical textbooks, little or no attention was paid to the reasons why an arithmetical rule yields the correct solution to a problem.[45] Van Schooten's course distinguished itself from this common practice by geometrical proofs of arithmetical rules, to some extent in the style of Euclid's *Elements*. I will use his proof of the rule of allegation as an illustration.

The rule of allegation was used for problems of the following kind. A merchant has a stock of various kinds of the same commodity, and he wants to mix these kinds and sell the mixture for a fixed predetermined price. Typical commodities were wine, beer, spices and grain. The rule of allegation for two commodities is as follows in modern notation: given are two commodities with prices p_1 and p_2 per unit respectively, and a given price p per unit with $p_1 < p < p_2$. How should the two products be mixed in order that the mixture is worth p per unit? The rule of allegation states that one should compose the mixture in the ratio of $p_2 - p$ units of the first commodity to $p - p_1$ units of the second commodity. Arithmetically, the rule can be easily proved by determining the price per unit of the mixture: $\frac{p_1(p_2-p)+p_2(p-p_1)}{(p_2-p)+(p-p_1)} = p$. If more than two commodities are involved, the rule can easily be extended.[46] Van Schooten gave his proof for the case of a numerical example. For sake of clarity, general quantities p, p_1 and p_2 will be used in my paraphrase of the proof instead of Van Schooten's concrete numbers.

[44] "naedemael de reegel van drieen groot gebruijck heeft inde koophandel, daegelickxen ommeganck der menschen en gansche mathesis soo sullen tselve aenwijsen daer toe voorstellende noch eenige andere questien," UBG, Hs 436, 55. "volgen eenig exempelen welcke de Coopmanschap aengaen", UBG Hs 436 56.

[45] [Kool, 1999, 220].

[46] If the number of commodities exceeds two, one mixes two commodities i and j with $p_i < p$ and $p_j > p$ at the time, until all the commodities have been used.

Geometrical interpretation (figure 8.3)

1. Choose a line segment which will serve as the unit.

2. Draw a line segment $AB = p_1$ units and extend it to D and C such that $AD = p$ units and $AC = p_2$ units.

3. Draw the line segment AE perpendicular to AB in such a way that $AE = BD = p - p_1$, and extend it to G in such a way that $EG = DC = p_2 - p$. Then $AG = BC = p_2 - p_1$.

4. Multiply AC by the line segment $AE = BD$ to obtain the rectangle $ACFE$. Multiply AB by the line segment $EG = CD$ to obtain the rectangle $EHIG$.

5. Multiply AD by the line segment $AG = BC$ to obtain the rectangle $ADLG$.

6. Together the two rectangles $ACFE$ and $EHIG$ form a figure $IGACFH$. The claim is that the area of this figure equals the area of the rectangle $ADLG$. This is the geometrical equivalent of the arithmetical identity that $p_1(p_2 - p) + p_2(p - p_1) = (p_2 - p_1)p$.

Proof

It is sufficient to show that the rectangle $DCFK$ is equal to the rectangle $HKLI$. But this is clear from the construction of the figure, because $KD = AE = BD = HK$ and $DC = EG = LK$.[47]

The proof resembles the proofs in sixteenth-century Italian works on algebra. The Italian algebraists used Euclidean geometric proofs as their main method for legitimizing mathematical knowledge.[48]

Van Schooten presented two arguments for using a geometrical proof of an arithmetical rule. The first argument appeals to the clarity of geometrical demonstrations:

> Because the geometrical demonstration is more apparent and clearer than the arithmetical [demonstration].[49]

It is worthwhile to take a closer look at the exact words of Van Schooten which I have translated as *apparent* and *clearer*. The Dutch words are *oogenschijnlijck* and *klaer*. The word *oogenschijnlijck*, literally meaning 'visible before the eyes', refers to the visual power of figures and is best translated by apparent. The Dutch *klaer* emphasizes the clarity of geometrical figures.

The combination *oogenschijnlijck* and *klaer* reminds us of the words *clairement et distinctement* which Descartes used in the *Discours de la méthode*. Descartes claims that

[47] Since $BC = BI$ and $BD = BH$, figure $CFKLIH$ is a gnomon in the terminology of Euclid's *Elements*. Probably for didactical reasons, Van Schooten does not refer to *Elements* I, prop. 43, where Euclid proves the equality of the rectangles $DCFK$ and $HKLI$.

[48] [Corry, 2008, 132].

[49] "Omdat de geometrice demonstratie oogenschijnlicker en klaerder is als de Arithmetice." UBG, Hs 436, 117.

the first precept of his method was to never accept anything to be true if it is not perceived clearly and distinctly.[50] In mathematics, this meant to him that straight lines were the principal objects in mathematics, since these were the most simple and distinct of all mathematical objects.[51] Is it by accident that Van Schooten, who was very familiar with Descartes's writings and ideas on mathematics, used similar words as Descartes in the justification of a geometrical argument? I believe that Van Schooten used the ideas of Descartes as a motivation to include geometrical proofs.

Van Schooten's second motivation for the use of a geometrical argument emphasizes the generality of geometrical proofs:

> also because the geometrical demonstrations are more general, (...) as these cannot only be applied to rational numbers only, but also to irrational numbers, since one can also represent irrational numbers by line segments.[52]

8.2 Fortification

Fortification held the most prominent place in the programme of the Duytsche Mathematicque. Van Schooten jr. devoted a quarter of his educational cycle to fortification and he lectured on the subject during three semesters in 1656 and 1657. In the instruction of 1600, Stevin also listed fortification as the main aim of the curriculum of the Duytsche Mathematicque. Thus it will be worthwhile to discuss Van Schooten's lectures on fortification in some detail. My account is structured as follows.

I begin by sketching the general background and the development of fortification in the Dutch Republic in the early seventeenth century. The following two sections deal with Van Schooten's courses on fortification in general, and with the structure of his lectures on defensive fortification, of which lecture notes survive. This will be followed by a detailed account of his design of regular fortresses (in section 8.2.4) and of irregular fortresses (in section 8.2.5). My investigation will focus on the mathematical principles. These will be used in the final section as a framework in order to identify influences by other authors on the work of Van Schooten jr. It will turn out that his theories on fortification have new and original features.

8.2.1 Fortification in the Dutch Republic

The Dutch art of fortification was influenced by developments in Italy, where a new fortification system had been designed in the Renaissance in order to withstand the increasing power of firearms. A new defence system was necessary because the mediaeval fortifications with high and tight walls were no longer effective. By the end of the sixteenth century, the Italian way of fortification, which was known as the "trace italienne," had been adapted

[50] [Descartes, 1637a, 20].
[51] [Descartes, 1637a, 21-22].
[52] "Meede om dat de Geometrice demonstratien generaelder syn (...) want deselve niet allen op rationaal maer oock op surde getallen cunnen toegepast worden dewijl men meede surde ghetallen door linien uutbeelden can", UBG, Hs 436, 117.

to the local circumstances in the Netherlands, both by the Spaniards and by the rebels.[53] The resulting system is known in the literature as the old Dutch fortification system (as opposed to the new Dutch fortification system which became popular after 1673). The old system was characterized by the use of earth instead of stone, resulting in massive earthen walls and earthen bastions. The use of earth had two advantages. A fortress of earth could be built faster than a fortress in stone. Earth was available at the construction spot itself, and stone had to be shipped from distant locations so stone as a building material was far more expensive. Once the fortress had been built, an earthen fortress was more costly than a stone fortress because it required more maintenance.

The walls and bastions in the old Dutch system were relatively low compared to medieval fortresses. The defence was based on preventing attacking forces to reach the walls. On the walls, bastions were erected in order to have a good view of the attacking army. The distance between the bastions was determined by the range of a musket, the most commonly used gun at the time. In the old Dutch system, the range of a musket (ca. 230 m) was taken as a maximum distance between adjacent bastions. These bastions were closer together than in fortresses in the Italian system.

The design of the fortress was further accommodated to the specific Dutch circumstances. The wet spongy soil of the Low Countries facilitated the construction of a wet ditch surrounding the entire wall of the fortress. The ditch surrounded a covered way, which could be used by the defending forces to organize a sally. The covered way was protected by the glacis, an artificial slope constructed in such a way that the attacking forces were under shot during their approach of the fortress. The glacis also made it impossible for the attacking forces to have a clear shot on the wall of the fortress. Thus the attacking forces could not easily fire a cannon at the wall in order to make a breach. The fortress had additional outer works to prevent the enemy from approaching.[54]

Fortification however was not only a military affair, but also an expression of intellectual and aesthetic ideas. The aesthetics of a fortress was judged by the same arguments that were used in civil architecture regarding the symmetry and proportions of a building. Mathematical principles on symmetry and proportion were used in the design of a fortress and they provided the art of fortification with a solid basis and an intellectual status. Thus, fortification developed into a science which was considered as part of "mixed mathematics," and which was based on specific geometrical principles. Charles van den Heuvel identifies the fortress as a "mathematical and cultural knowledge system".[55]

By the second half of the seventeenth century, knowledge of fortification was no longer restricted to military engineers alone. The mathematical foundations of fortification made the subject into a socially accepted part of the education of merchants, noblemen and regents. This is illustrated by the examples of the merchant Joost Crommelingh and of Christiaan Huygens, son of the diplomat Constantijn Huygens. The two young men were

[53] For a more detailed discussion on the introduction of Italian ideas and practices on fortification in the Low Countries, see [Heuvel, 1991] and [Westra, 1992].

[54] [Nimwegen, 2006, 116] and [Duffy, 1979, 90-92]

[55] [Heuvel, 2006].

instructed in fortification by Samuel Kechel and Frans van Schooten jr. respectively.[56] Of course there was a tension between fortification as an art at the drawing table, and the practice of fortification in the field. The architect at the drawing table could design ideal fortresses, whereas the military engineer in the army had to take into account local and practical conditions such as the landscape, the budget reserved for the project, and the availability of men to complete the work.

Throughout the seventeenth century, military engineers with knowledge of fortification were hired by authorities on various levels in the Dutch Republic. Engineers were working by order of the States General, the provincial States or local authorities such as city councils. In the cities, engineers were often engaged in major infrastructural works, including the planning of city extensions.[57] Besides these public engineers there were also engineers in the service of private persons. The stadtholders Maurice and William Louis had engineers on their private payroll.

The number of engineers in the service of the States General has been reconstructed by Schäfer. The average number was 10 during the period 1587–1597 and increased to 21 at the end of the sixteenth century. Throughout the seventeenth century twenty engineers were employed on average, with peaks in 1629, due to a Spanish offensive, and in 1672–1673, the beginning of the Franco-Dutch War. The Seven Years' Truce (1609–1621) did not affect the number of engineers. In the period 1648–1665, which roughly coincided with the professorship of Van Schooten jr., the average number of engineers dropped to thirteen. Schäfer relates the decrease to the Peace of Munster. Already in 1648 several fortification works were ordered to be dismantled.[58] Dutch engineers were not only active in the Republic, but also across the frontiers, primarily in Scandinavia, Germany and the Baltic area.[59]

The question arises to what extent these engineers were trained at the Duytsche Mathematicque. As no registration of auditors has survived, there is only sparse information on the identity of the men attending the lectures. Frans van Schooten jr. stated that some of his auditors had served the King of Sweden as engineers. This must have happened in the beginning of the second decade of the seventeenth century, when King Gustav II Adolf actively recruited in the Republic.[60] Van Schooten further mentioned three men who pursued a career in engineering after having attended the Duytsche Mathematicque: Genesis Paen, Gerard van Bellecum and Jean Gillot.[61] Genesis Paen became inspector of the fortification works in Holland, and in the period 1649–1679 he served as an examinator for surveyors who requested to be admitted in Holland.[62] Gerard van Bellecum was admitted as a surveyor in 1628 in Holland. In 1641 he registered as a student of mathematics in Leiden, where he attended lectures in the Duytsche Mathematicque as well. In 1643 he was appointed engineer in the army, but only for a short time. Already in 1645 he was

[56] [Heuvel, 2006, 113-115].

[57] [Westra, 2010, 17]. On the involvement of engineers in city planning, see [Taverne, 1978].

[58] [Schäfer, 2001, 126-135].

[59] For an overview of Dutch engineers abroad, see [Westra, 2010, 69-77]. For the role of Dutch engineers in Denmark, see [Roding, 1991] and for Sweden see [Ahlberg, 2005] and [Eimer, 1961].

[60] [Ahlberg, 2005, 260 and 317].

[61] Frans van Schooten jr. to Constantijn Huygens, 4 February 1646, [Huygens, 1915, 279].

[62] [Muller and Zandvliet, 1987, 157].

involved in the design of the fortification works of Lippstadt, and from 1652 on he was employed by Frederick William, Elector of Brandenburg.[63] Jean Gillot (1613 or 1614–1657) was a mathematician with a profound knowledge of Cartesian geometry. He worked as a private teacher in the second half of the 1630s before he left for Portugal in 1651 to serve as an engineer in the Portuguese army. He died during the Siege of Olivenca in 1657.[64]

In the modern literature, it is often assumed that all or most Dutch engineers, especially those active outside the Republic, had attended the Duytsche Mathematicque.[65] This assumption however overestimates the importance of the Duytsche Mathematicque in the teaching of fortification. Although some successful engineers did attend the Duytsche Mathematicque, there is no evidence suggesting a monopoly of the Duytsche Mathematicque, neither in the training of military engineers, nor in the training of surveyors.

8.2.2 Fortification and Van Schooten

Frans van Schooten jr. lectured on fortification during two cycles. The first series of lectures took place somewhere during the years 1647–1652, and the second series were read in three subsequent semesters from winter 1656 to winter 1657. The titles of these courses on the *series lectionum* reveal that they were about defensive and offensive fortification. The former is the fortification of cities and strategic places such as river banks, whereas the latter deals with the techniques used during sieges to capture cities and fortresses. The contents of a large number of lectures in the second cycle are documented by Petrus van Schooten. His manuscript consists of notes made during the lectures, collated with additional notes made afterwards,[66] and concern defensive fortification only.

Van Schooten jr. spread his ideas on fortification also in private lectures. Among his students was Christiaan Huygens, whose manuscript on fortification has been preserved.[67] This manuscript dates back to the early years which Huygens spent in Leiden. Another manuscript related to the fortification lectures by Van Schooten jr. has been preserved in the Tresoar library in Leeuwarden (shelf number 668 Hs). The content of this Leeuwarden manuscript is similar to that of the Huygens manuscript and to the lecture notes written by Petrus. All three manuscripts have the same structure, the same principles are used in the design of fortresses and the same examples are used throughout. The similarity between the manuscripts is illustrated by figures 8.4 and 8.5, in which the fortification of a rhombus with sides of 34.2 rods and angles of $65°30'$ and $114°30'$ is shown.[68]

We do not know who wrote the Leeuwarden manuscript. The manuscript gives details on fortification works that were completed around Groenlo in 1627, so it must have been

[63] [Westra, 2010, 47 and 73].

[64] On the life of Jean Gillot see [Witkam, 1967] and [Witkam, 1969].

[65] See especially [Eimer, 1961] and [Taverne, 1978]. Schäfer estimates the role of the Duytsche Mathematicque highly, mainly on the basis of the study of Eimer [Schäfer, 2001, 587-588].

[66] UBG, Hs 441. For more information on this manuscript, I refer the reader to appendix A.

[67] UBG, HUG 16.

[68] In figure 8.4, Petrus made a writing error in the length of the capital lines at *B* and *D*, where he he wrote 1218 2 instead of the correct value 1281 2. He however calculated the length in a correct way, as is clear form his calculations, see UBG, Hs 441, f. 119v./42.

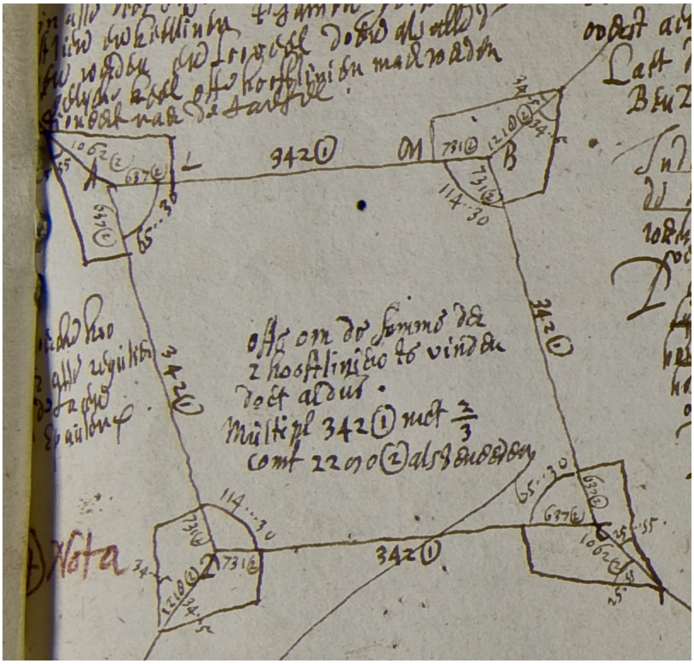


Figure 8.4 – Fortification of a rhombus in Ms. UBG, Hs 441, f. 120r.

written after 1627.[69] The writer of the manuscript remarks that the profiles of Groenlo "have been noted by Frans van Schoten Professor of the University of Leiden during the siege of Groenlo".[70] Thus the Leeuwarden manuscript must be somehow connected to Van Schooten sr., but it is not completely clear in what way. The methods in the Leeuwarden manuscript differ strikingly from the methods that were prescribed by Van Schooten sr. in his 1622 course on fortification, so the manuscript is in all probability related to the teaching activity of Van Schooten jr.[71] Recall that Van Schooten jr. replaced his father during illness from 1635 onwards.

An interesting feature of the Leeuwarden manuscript is a section on offensive fortification. Such a section is found neither in the lecture notes by Petrus nor in Huygens's manuscript. As the parts on defensive fortification in the three manuscripts are clearly based on the same kind of course, it is tempting to consider the section on offensive fortification in the Leeuwarden manuscript as the product of teaching by Van Schooten jr. The teaching at the Duytsche Mathematicque comprised offensive fortification as well.

My discussion of the fortification teaching of Van Schooten jr. will mainly be based on

[69] Tresoar, 668 Hs, f. 191. The manuscript is briefly described in [Sluis, 2007, 90].

[70] "sijnde aengeteeckent van Frans van Schoten Professor der Stadt [doorgehaald] Universiteijt tot Leijden in de belegeringe voor Groll", Tresoar, 668 Hs, 93.

[71] See section 8.2.6 for more details on the differences between the courses on fortification by Van Schooten sr. and jr.

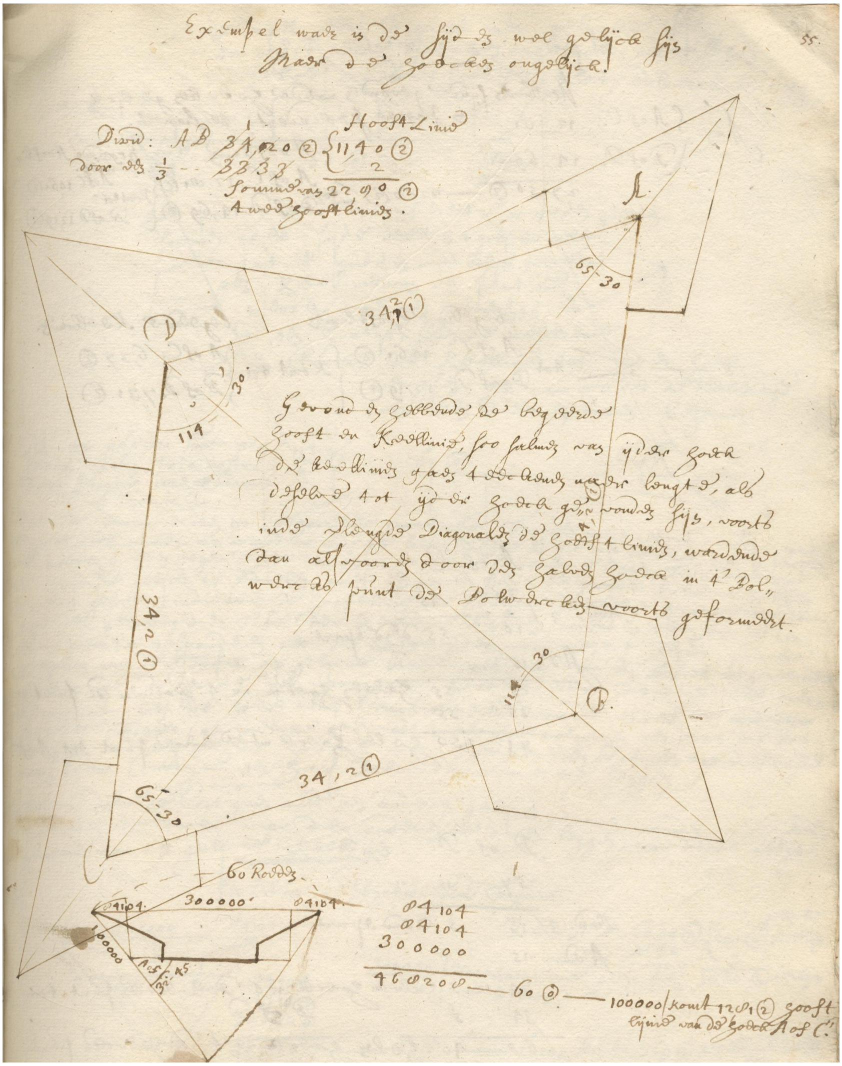


Figure 8.5 – Fortification of a rhombus in Ms. Tresoar, 668 HS, 55.

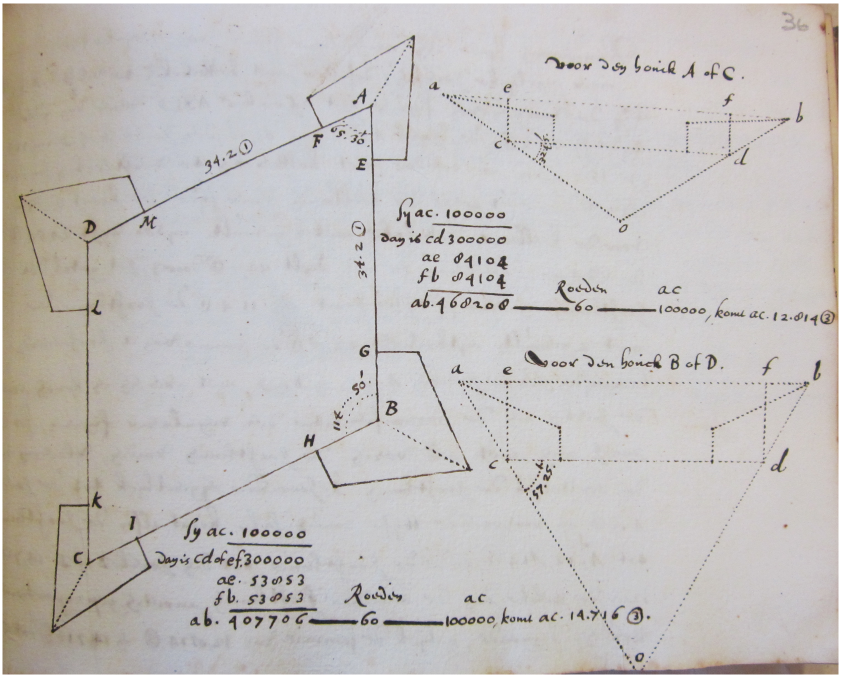


Figure 8.6 – Fortification of a rhombus in UBL, HUG 16, f. 36r.

the lecture notes by Petrus. These notes were taken in class and therefore they are our most reliable and detailed witness of Van Schooten's lectures.

Whereas Van Schooten sr. was involved in the actual construction of fortification works, there is no evidence that Van Schooten jr. took part in practical work on fortification projects, apart from him assisting his father in the fortification project in Utrecht in 1629 when he was a teenager.[72] Also, no maps of his hand are known. However, Van Schooten jr. had good contacts with men serving in the army. The Groningen manuscript reveals that he discussed matters on fortification with Genesis Paen, inspector of fortification works in Holland, and with George Gleser, who was employed in the army and who was on friendly terms with Frederick Henry.[73]

An investigation of the course on fortification by Van Schooten jr. is all the more interesting because he intended to publish a treatise on the subject under the title "De optima muniendi ratione" (About the best way of fortification). This publication project was never completed. Van Schooten mentioned his plans to the English mathematician John Pell (1611–1685), who was at the time professor of mathematics at the Illustre School in Breda (Netherlands). In November 1648, Pell informed his English correspondent Sir Charles Cavendish about the planned treatise by Van Schooten jr.[74] In August 1649 Pell wrote to

[72] See footnote 35 on page 22 above.

[73] UBG, Hs 441, f. f. 110r./59 and f. 114r./51.

[74] Pell to Cavendish, 4 November 1648, [Malcolm and Stedall, 2005, 517]. Pell's letter is in English, and because

Cavendish that Van Schooten had postponed the printing of "De optima muniendi ratione" because he gave priority to the publication of his reconstruction of *De locis planis* of Apollonius of Perga, after the printing of the *Geometria* had been finished.[75]

In Petrus's lecture notes one finds several references in Latin to a treatise on fortification.[76] These references are written in red ink, and because they are the only words in the manuscript which appear in red, they may have been added some time after the manuscript had been completed. I have not been able to identify any extant treatise on fortification which agrees with the references. Because the references are in Latin while the rest of Petrus's manuscript is in Dutch, the unidentified treatise may well have been in Latin. So a very likely candidate is Van Schooten's unpublished treatise "De optima muniendi ratione", which probably circulated in manuscript form. It is likely that Petrus added these references after Frans jr. had passed away and Petrus had the unpublished treatise in his possession.

8.2.3 Structure of the course of Van Schooten jr.

We now turn to Van Schooten's 1656 course on defensive fortification, as documented in the lecture notes by Petrus. The course had a clear structure, and the beginning is remarkably similar to Stevin's *De sterctenbouwing (The art of fortification)*. Just like Stevin, Van Schooten started with a list of numbered definitions of various parts of a fortress and of some terminology used in fortification. He stressed the connection to Stevin's work by quoting some of its definitions before he continued with his own definitions.[77]

Van Schooten then discussed in a nutshell the development of fortification over the past centuries, and he described how ongoing developments in artillery changed the way cities were fortified. This discussion led him finally to thirteen principles to be employed in fortification. These principles include the following: the defence of a fortress should be organized in such a way that it is equally strong from all sides; a fortress is a rectilinear figure; since the range of a musket equals 60 rhenish rods (ca. 200 m), the bastions of a fortress should be placed at a maximum distance of 60 rods from one another.[78]

In order to understand the contents of Van Schooten's course, we have to familiarize ourselves with the technical terminology of early modern fortification. For an overview of the relevant definitions we first turn to figure 8.7. The figure displays a fortress which is based on the underlying square $CDTU$. The square has to be fortified on its four vertices by four bastions $EGAV$, $FHBW$, etc. Points A and B are called the *salient* and are located on the extensions of the lines OC, OD etc., where O is the center of the square. Lines GE, HF etc. are perpendicular to the sides CD etc. of the square. From A we draw the four dashed straight lines AB, AF, AG and $ACOT$ as in the figure. Line AG extended intersects

he gives the title of the treatise in Latin, Van Schooten probably wrote the treatise in Latin.

[75] Pell to Cavendish, 31 August 1649, [Malcolm and Stedall, 2005, 538]. The reconstruction eventually saw the light in 1657 as Book III of Van Schooten's *Exercitationes mathematicae*, [Schooten, 1657].

[76] See for instance 'cap 14 p 127, cap 15 p 137', UBG, Hs 441, f. 137r./7.s

[77] UBG, Hs 441, f. 140r./1–136r./9.

[78] UBG, Hs 441, f. 132r./17–131r./19. Van Schooten's shooting distance of 60 Rhenish rods agrees with the values of his contemporaries Freitag and Marolois, and with Van Nimwegen's recent study on the army, [Nimwegen, 2006, 116].

EF at point N and BH extended at point R, and line BH extended intersects CD at point Q. The technical terms for the different line segments and angles are listed in table 8.2.

I will now give a general outline of Van Schooten's course on defensive fortification; the details will follow in the subsequent sections. The design of a fortress according to Van Schooten consisted of two elements: a plan of the basic shape of the fortress, and a profile giving in detail the heights and widths of walls, wet ditches etc. Van Schooten's design of the plan was based on an underlying regular polygon analogous to the square $CDTU$ in figure 8.7. He assumed that the *salients* (A, B etc. in 8.7) are 60 Rhenish rods distant from one another and then prescribed the way in which the bastions should be constructed. From these assumptions he obtained an ideal fortress, which I will call the *standard fortress* because it served as a reference model for the construction of larger or smaller fortresses, and even for fortresses based on irregular polygons. Van Schooten treated the sizes and angles of standard fortresses based on regular polygons with number of vertices from four to twelve. In the next section I will discuss the design and calculations in detail, to show how mathematics was used in designing a fortress. Subsequently, Van Schooten explained how one should stake out these designs in the field at the planned location of the fortification.

After his extensive discussion of the standard fortress, Van Schooten turned to the fortification of irregular polygons, gradually increasing the irregularity. He first discussed quadrilaterals with equal angles and different sides (rectangles), then quadrilaterals with different angles and equal sides (rhombi), and finally polygons with different angles and/or different sides. The main task for the engineer was to fortify each vertex of the polygon by means of a bastion in such a way that the bastions are (as much as possible) equally efficient, so that the resulting fortress is equally strong at every vertex. In fortresses based on irregular polygons, the distances between the adjacent *salients* also had to be calculated to make sure that they did not exceed the maximal distance of 60 Rhenish rods.

Once the student had mastered the plan of a fortress, it was time to discuss the profile. Van Schooten gave the basic measures for the various parts of the profile, such as the depth and width of the wet ditch, the height of the walls and the parapet (a defence of earth to conceal troops from the enemy's observation, raised on the top of the wall), etc. This part of the course did not involve volume calculations, because its purpose was the design of a fortress and not its actual construction in the field. Of course the earth dug out for the wet ditches should be used for the walls, so the volumes of wet ditches and walls should be determined before starting the construction to make sure that enough earth was available. The final part of Van Schooten's course was dedicated to the actual construction of fortresses, taking into account local circumstances such as existing fortification works and the presence of waterways.

8.2.4 Basic mathematical principles of fortification: the standard fortress

Van Schooten based the design of a standard fortress on four rules. In general, a fortress based on a regular polygon is completely determined once the following information is

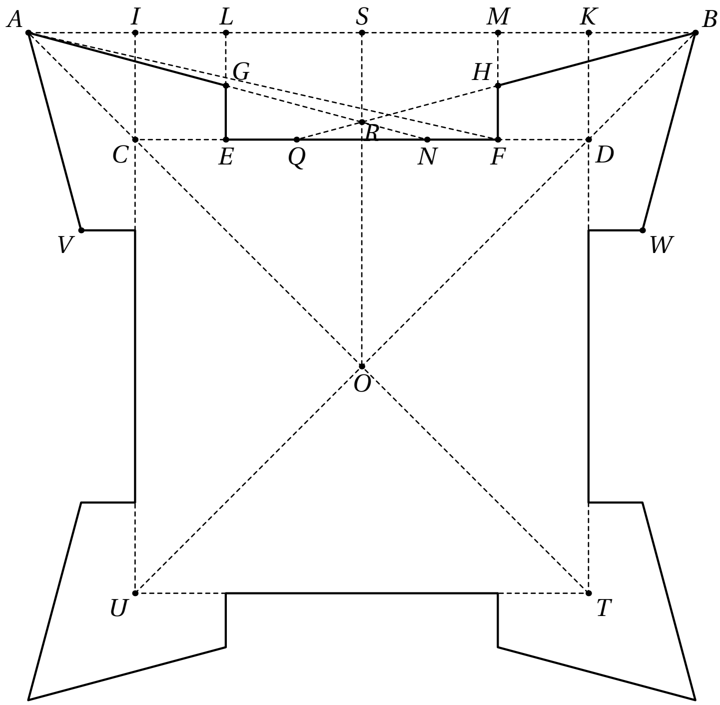


Figure 8.7 – Fortress.

line	name	angle	name
CD	side of the polygon	$\angle GAV$	angle of the bastion
EF	curtain	$\angle DOC$	interior angle of the fortress
AG	face	$\angle DCU$	vertex angle
GE	flank	$\angle GEF$	angle of the flank
EQ	second flank	$\angle EGA$	angle of the shoulder
CE	gorge line	$\angle ENA$	inward flanking angle
AC	capital line	$\angle ARB$	outward flanking angle
AN	flanking or stringent line		
AF	fixed line of defence or longest line of defence		

Table 8.2 – The lines and angles of a fortress.

available:

- The length of a side of the regular polygon and the number of its sides.
- The position where the flank meets the curtain (i.e., point E in figure 8.7).
- The position where the flank meets the face (i.e., point G in figure 8.7).
- The position of the salient (i.e., point A in figure 8.7).

As already mentioned before, Van Schooten assumed that the size of the polygon is such that the distance between two adjacent *salients* is exactly 60 rods. In order to determine the position and size of the bastion, Van Schooten prescribed the following rules:[79]

1. The length of the gorge line CE is one-fifth of the side of the polygon CD.
2. The length of the capital line AC is one-third of the side of the polygon CD.
3. The flank GE is perpendicular to the curtain EF. This rule was typical of the Old Dutch fortification system.
4. The angle of the bastion $\angle GAV$ is $\frac{1}{3}\varphi + 30°$, where $\varphi = \angle DCU$ is the vertex angle of the regular n-gon. The notation φ is mine.

On the basis of these four rules, Van Schooten computed the relevant lengths and angles in his standard fortress, and he then presented this information in tables for polygons with four to twelve sides. The tables also served for the construction of fortresses of other sizes, with the distance between the *salients* not equal to 60 Rhenish rods. In such cases the angles were the same and the lengths of the sides had to be scaled to the right dimensions. In case the fortress was larger, additional bastions had to be built on the curtain in order to make sure that the maximum distance between the adjacent *salients* was at most 60 rods. Such additional bastions were called flat bastions, because they were constructed on the (rectilinear) curtain and not on a vertex. The standard dimensions of a flat bastion were also incorporated in the tables.

Before Van Schooten jr., fortification tables had already been printed by the Italian mathematician Alghisi[80] in the second half of the sixteenth century and also by Freitag and Marolois in the first half of the seventeenth century. These tables were different from the ones of Van Schooten, and below I will discuss these and other differences in more detail. Fortification tables appear not only in books and manuscripts but also on instruments, for example on rulers used by military architects.[81]

In his lectures, Van Schooten calculated the values in tables 8.3 and 8.4 only for the case of a regular pentagonal fortress. The values for the other polygons could be obtained by similar methods, so they appear in the tables without further explanation.[82] I will discuss Van Schooten's calculations of the *inward flanking angle* and the length of the *capital*

[79] HS 108, f. 132v., 130r.

[80] [Heuvel, 2006, 100].

[81] See the internet exhibition "The geometry of war, 1500–1750" of the Museum of the History of Science in Oxford, `www.mhs.ox.ac.uk/geometry/summary.htm`, retrieved 12 July 2012, for example item 66.

[82] The table presents approximate values for the regular polygon with 7 and 11 sides.

line and the *face*. These calculations offer an insight in the arithmetical, geometrical and trigonometrical skills which Van Schooten assumed on the part of his students. Because the design of regular fortresses was based on the tables of the angles and sides of standard fortresses, the students had to be able to scale a design to the right size by means of the necessary arithmetical operations.

Determination of angles

Van Schooten expressed the required angles in terms of the angle of the bastion of a fortress, which in turn depends on the vertex angle of the underlying polygon. He used the standard pentagonal fortresses as an example,[83] but in order to illustrate the generality of his method, I use the modern notation $\varphi = 180° - \frac{360°}{n}$ for the vertex angle of a regular n-gon. Then, by definition, the angle of the bastion equals $\frac{\varphi}{3} + 30°$. For the inward flanking angle ENA in figure 8.7 we then find:

$$\begin{aligned}
\angle ENA &= 180° - \angle CAG - \angle ACE && (8.1)\\
&= 180° - \frac{1}{2}(\frac{\varphi}{3} + 30°) - (\frac{1}{2}(360° - \varphi)) && (8.2)\\
&= \frac{1}{3}\varphi - 15°. && (8.3)
\end{aligned}$$

The other angles are found in similar ways, so I only mention the results in my general notation, from which Van Schooten's results can be found by substituting $\varphi = 108°$:

$$\begin{aligned}
\angle AGE &= \frac{1}{3}\varphi + 75°, && (8.4)\\
\angle FNA &= 195° - \frac{1}{3}\varphi, && (8.5)\\
\angle EGN &= 105° - \frac{1}{3}\varphi, && (8.6)\\
\angle ARB &= 210° - \frac{2}{3}\varphi, && (8.7)\\
\angle ACE &= 180° - \frac{1}{2}\varphi, && (8.8)\\
\angle ACI &= 90° - \frac{1}{2}\varphi. && (8.9)
\end{aligned}$$

Calculations of lengths

We now turn to Van Schooten's computation of the relevant lengths for the standard pentagonal fortress.

Van Schooten used seventeenth-century trigonometry, which differs from modern trigonometry in the definition of the basic trigonometric functions. In the seventeenth century, the sine, tangent and secant were considered to be line segments related to a circle with centre O and a fixed radius OA in most cases not equal to 1, see figure 8.8. From

[83] HS 108, f. 130v.

n-gon	distance salient	flank	second flank	face
4	60	4.851	6.563	18.393
5	60	5.073	12.648	18.282
6	60	5.297	15.642	18.206
7	60	5.297	15.642	18.206
8	60	5.682	18.842	18.081
9	60	5.847	19.736	18.014
10	60	5.980	20.741	17.975
11	60	6.096	21.200	17.931
12	60	6.218	21.822	17.883
flat bastion	60	16.9705	28.000	16.970

Table 8.3 – The lengths of standard fortresses in Rhenish rods.

n-gon	angle at centre	vertex angle	angle of the bastion	inward flanking angle	outward flanking angle
4	90°	90°	60°	15°	150°
5	72°	108°	66°	21°	138°
6	60°	120°	70°	25°	130°
7	51°24’	128°36’	72°48’	28°	124°
8	45°	135°	75°	30°	120°
9	40°	140°	76°42’	$31°40'$	$116\,°40'$
10	36°	144°	78°	33°	114°
11	32°42’	147°18’	79°6’	34°	112°
12	30°	150°	80°	35°	110°
flat bastion	-	180°	90°	45°	90°

Table 8.4 – The angles of standard fortresses.

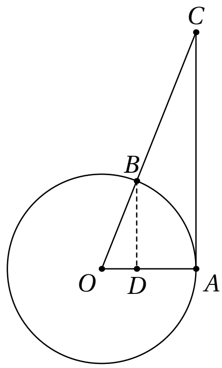


Figure 8.8 – Trigonometry.

the point B on a given angle AOB (or a given arc AB), drop a perpendicular BD onto AO, and draw a line through A perpendicular to OA, to meet OB extended at C. The sine of $\angle AOB$ is defined as the line segment BD, the tangent of $\angle AOB$ is the line segment AC and the secant of $\angle AOB$ is the line segment OC.

In order to avoid confusion between the modern functions and the seventeenth-century quantities, the following notation will be used. For any angle $\theta = \angle AOB$, the seventeenth-century sine in a circle with radius r will be written as $\mathrm{Sin}(\theta)$. We have $\mathrm{Sin}(\theta) = BD = BO \cdot \frac{BD}{BO} = r\sin(\theta)$. The notation Sec (with capital) will be used for the seventeenth century secant, which is also r times the modern secant.

Seventeenth-century trigonometrical tables were computed for r a power of 10, usually larger than 10 000, and Van Schooten used a table for $r = 100\ 000$, with the values of the trigonometrical quantities rounded to integers. The seventeenth-century applications of these tables often involved two similar triangles: a triangle taken from the problem at stake and a second triangle with one side equal to r. In the context of fortification, the authors (such as Van Schooten) often considered two similar fortresses: the standard fortress and a second, 'artificial' fortress in which one suitable length was equal to 100 000. If the length of a segment in the standard fortress is AB, the corresponding length in the artificial fortress will be indicated as $\widehat{AB}$.

I will now illustrate Van Schooten's methods by his computations of the length of the capital line AC and the face AG, see figure 8.7. Van Schooten uses the Sine in the former and the Secant in the latter.[84]

For the capital line AC, the artificial fortress is chosen similar to the standard fortress such that $\widehat{AC}$ is equal to 100 000. In a pentagonal fortress, $\angle ACI = 36°$, and Van Schooten finds $\mathrm{Sin}(36°) = \widehat{AI} = 58\ 779$ from the trigonometric table. From the rules for the construction of the fortress it follows that $\widehat{CD} = 3\widehat{AC} = 300\ 000$ and thus $\widehat{AB} = 2\widehat{AI} + \widehat{CD} = 417\ 558$.

[84] This account is based on UBG, Hs 441 and Tresoar, 668 Hs. A summary of the Van Schooten method is given in print in [Reyher and Alefeld, 1668, Caput V, I modus]. This work is a disputation by a German student, presided by the Kiel professor of mathematics Samuel Reyher, who was a former student of Frans van Schooten. On Reyher see [Schönbeck, 2007].

In the standard fortress however, we have $AB = 60$ rods. Hence, by similar triangles

$$\widehat{AB} : AB = \widehat{AC} : AC, \tag{8.10}$$

$$417\ 558 : 60 = 100\ 000 : AC, \tag{8.11}$$

and thus by the rule of three $AC = 14.3692$ rods. In modern terms we have $AB = 2AI + CD$, $AB = 60$, $AI = AC\sin(36°)$ and $CD = 3AC$, so

$$60 = AC(2\sin(36°) + 3). \tag{8.12}$$

Van Schooten computes $AL = 17.0676$ rods in a somewhat similar way which we will not discuss in detail. He then finds the length of the face AG as follows. The inward flanking angle $\angle ENA = 21°$, so also $\angle GAL = 21°$. Van Schooten considers an artificial fortress such that $\widehat{AL} = 100\ 000$. In the triangle $\widehat{ALG}$ he finds $\widehat{AG} = \mathrm{Sec}(\angle GAL) = \mathrm{Sec}(21°) = 107\ 114$ from the table. By the similar triangles $\widehat{ALG}$ and ALG we have:

$$\widehat{AL} : AL = \widehat{AG} : AG, \tag{8.13}$$

$$100\ 000 : 17.0676 = 107\ 114 : AG, \tag{8.14}$$

and thus $AG = 18.2818$ rods.

Van Schooten computes the lengths to four decimals, but since one Rhenish rod is approximately equivalent to 3.767 meters, this accuracy is of no practical use in fortification. Van Schooten argued that the accuracy was necessary because the values could be used in later computations. He explained to his students:

> We calculate this in the fourth [decimal] because it is the basis of the next operation, therefore it has to be calculated more accurately than needed in practice, for a little error produces a large difference in what follows.[85]

Staking off in the field

Once the design of a fortress had been completed, the fortress had to be staked off in the field. Van Schooten discussed this activity in his lectures, but we do not know whether his course included field work outside the lecture hall.

Van Schooten linked the staking out of a design in the field to the drawing of a design on paper. Ideally, the two tasks should be carried out by the same method. On paper, the drawing had to be performed by means of two instruments, a protractor and a marked ruler. A protractor is an instrument consisting of a rotating ruler attached to the centre of a (semi)circle provided with a scale in degrees. The instrument was used for drawing angles of given magnitude. In the field, the protractor was replaced by the astrolabe and the marked ruler by the surveyors' chain. Van Schooten distinguished constructions in pure

[85] "Wy reeckenen dit op de 4de want dewyl dit de gront is der volgende wercking moet dit accuraeter uutgereckent worden als wel in de practyc van node is want een weynich missende brengt het veel verschil in tvolgende." UBG, Hs 441, f. 129v.

geometry from constructions in practical geometry. He explained that in pure geometry, the vertices of a regular polygon were found by intersections of straight lines and circles, using a straight ruler and a pair of compasses.[86] For constructions in "mixed mathematics," such as fortification, other instruments than a straight (unmarked) ruler and a pair of compasses were to be used in setting out a figure on paper.[87]

8.2.5 Fortification of irregular convex polygons

Although the theory of fortification favors the regular polygon as a basic figure, it was often impossible in practice to design a fortress in this way. Local conditions had to be taken into consideration, such as existing fortification works or the presence of a city or buildings. Therefore an engineer should also be able to design a fortress on the basis of an irregular polygon. On each vertex of the irregular polygon, a bastion had to be erected, and its size and shape had to be determined. Again, the bastion had to be designed in such a way that the fortress had equal strength at all sides.

Van Schooten devotes a considerable part of his course to what he calls irregular fortresses, that is to say, fortified irregular convex polygons. He discusses three different types of irregular polygons:

- Quadrilaterals with different sides and equal angles, that is, rectangles
- Quadrilaterals with equal sides and different angles, that is, rhombi.
- Other polygons with different sides and/or angles.

For each of these types, Van Schooten explains the fortification process in detail. The construction is in essence an adaptation of the basic principles for the fortification of regular polygons.

As in the case of a regular fortress, Van Schooten prescribes that the angle of the bastion in an irregular fortress is $\frac{1}{3}\varphi_i + 30°$, where φ_i indicates the vertex angle of the underlying polygon at the vertex on which the bastion has to be constructed. In irregular fortresses, different bastions can therefore have different shapes. Van Schooten emphasizes that small angles need small bastions, and larger angles need larger bastions. He rejected the view of some of his contemporaries that all bastions must be of equal size and shape, with equal capital lines, gorge lines and angles at the vertex.[88]

Van Schooten then presented rules for the determination of the curtain, the capital lines and the gorge lines of irregular fortresses. These rules are to some extent inspired by the principles which he used for regular fortresses. In a regular fortress, each curtain is three-fifths of the side of the regular polygon, and each capital line is one-third of the length of the side. For irregular fortresses, Van Schooten prescribed that all curtains should have the same length, equal to three-fifths of the average length of the sides of the polygon.

[86] For Van Schooten it must have been a fact of experience that for regular polygons with 7, 9 and 11 sides, no exact construction by compass and ruler existed. That such a construction cannot exist was proved in the nineteenth century.

[87] UBG, Hs 441, f.129r./27.

[88] UBG, Hs 441, f. 125v./30.

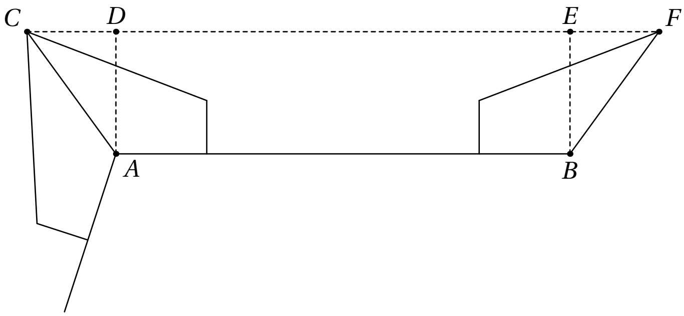


Figure 8.9 – Determination of CD.

If additional flat bastions are required to keep the bastions at a maximum distance of 60 rods from one another, the expression for the length of a curtain is adjusted as will be detailed below. In Van Schooten's irregular fortresses, the sum of the lengths of all capital lines must be equal to one-third of the sum of the sides of the polygon. Van Schooten let the length of the capital line in a vertex also depend on the vertex angle φ_i.

I now present the details of Van Schooten's method. He discussed the three above-mentioned categories of irregular fortresses one after the other, but because the method is similar, I will discuss all three types at once. For sake of clarity, I introduce the following modern notation which Van Schooten did not use in his course. Suppose we have an irregular convex n-gon. We label the subsequent vertices i, with $1 \leq i \leq n$. We indicate the angle at vertex i as φ_i and the side from vertex i to vertex $i+1$ as s_i, while s_n is the side between the vertices n and 1. Let k_i be the length of the capital line at vertex i, and for the moment I will use g_i for the length of the gorge line adjacent to vertex i.

Van Schooten's method is contained in the following step-by-step plan.[89]

1. Set the length of the curtain to $c = \frac{3}{5(n+f)} \sum_{i=1}^{n} s_i$ with f the number of additional flat bastions. (Flat bastions are designed as in regular fortresses).

2. Determine of the length of each capital line k_i in two steps.

   (a) First determine for each vertex the length of a capital line $\tilde{k}_i$ in a hypothetical "standard" fortress with vertex angle φ_i, and distance of 60 rods between the points of the adjacent bastions, that is, points C and F in figure 8.9. Note that φ_i does not need to be equal to an angle of a regular polygon. The method has already been explained above in connection with a pentagonal fortress ($\varphi_i = 108°$). The general formula becomes $60 = \tilde{k}_i(2\sin(90° - \frac{1}{2}\varphi_i) + 3)$. See figure 8.9 where $CF = 60, AC = \tilde{k}_i, \angle A = \varphi_i$ and AC bisects the exterior angle at A while AD is perpendicular to AB, so $\angle CAD = 90^o - \frac{1}{2}\varphi_i$.

[89] I have reconstructed this step-by-step plan from UBG Hs 441, f. 125v./30–f. 117v./50. The method is also explained in Tresoar, 668 Hs, 53-62.

Van Schooten finds $\tilde{k}_i$ from the trigonometrical table and the identity

$$\left(2\,\mathrm{Cos}\left(\frac{1}{2}\varphi_i\right)+300\ 000\right):60=100\ 000:\tilde{k}_i.$$

(b) Van Schooten wants the sum of all capital lines to be equal to one-third of the circumference of the polygon $\sum_{i=1}^{n} s_i$. He computes $\sum_{i=1}^{n}\tilde{k}_i$ and then obtains the actual capital lines k_i from the $\tilde{k}_i$ by scaling by a constant factor:

$$\sum_{i=1}^{n}\tilde{k}_i:\frac{1}{3}\sum_{i=1}^{n}s_i=\tilde{k}_i:k_i.$$

3. Now construct the gorge lines on the sides of the polygon. If the polygon is a rectangle or a rhombus: bisect each side, and from the midpoint of the side pace out half the length of the curtain to each side. The gorge is the segment between the endpoints of the curtain and the vertices of the underlying rectangle or rhombus. Thus for $1\le i\le n-1$ we have $g_i+c+g_{i+1}=s_i$, and for $i=n$ we have $g_n+c+g_1=s_n$. The angle of the bastion equals $\frac{1}{3}\varphi_i+30°$, as remarked above. From these data, we can now complete the bastions for rectangles and rhombuses.

   For other polygons the construction is more complicated, and consists of the following steps. We assume that there are no flat bastions.

   (a) Determine the quantity $\tilde{g}_i=\frac{3}{5}k_i$ which one could call "average gorge line at vertex i".

   (b) Because the two gorge lines of the same bastion can be different, I introduce the notation g_{i+} for the length of the gorge line from vertex i towards the next vertex $i+1$ if $i<n$ or towards the vertex 1 if $i=n$. Let g_{i-} be the length of the gorge line in vertex i towards the previous vertex $i-1$ if $i>1$ or towards vertex n if $i=1$. Starting in vertex 1, follow the algorithm:

      i. Put $g_{1+}=\tilde{g}_1$, and $g_{2-}=s_1-c-g_{1+}$.
      ii. For $i=2,3,\dots,n-1$ put $g_{i+}=2\tilde{g}_i-g_{i-}$ and $g_{(i+1)-}=s_i-c-g_{i+}$
      iii. Finally, put $g_{n+}=2\tilde{g}_n-g_{n-}$ and $g_{1-}=s_n-c-g_{n+}$.

   We note that by combining these rules we obtain $g_{1-}=\sum_{i=1}^{n}s_i-nc-2\sum_{i=2}^{n}\tilde{g}_i-g_{1+}$, so $g_{1-}+g_{1+}=\sum_{i=1}^{n}s_i-nc-2\sum_{i=1}^{n}\tilde{g}_i+2\tilde{g}_1$. Since $\sum_{i=1}^{n}\tilde{g}_i=\frac{3}{5}\sum_{i=1}^{n}k_i=\frac{3}{5}\frac{1}{3}\sum_{i=1}^{n}s_i=\frac{1}{5}\sum_{i=1}^{n}s_i$ and $nc=\frac{3}{5}\sum_{i=1}^{n}s_i$, we conclude $g_{1-}+g_{1+}=2\tilde{g}_1$, just like $g_{i-}+g_{i+}=2\tilde{g}_i$ which was assumed for $i>1$.

4. Construct the bastions, see figure 8.10. Suppose point A is vertex i of the underlying polygon, φ_i is the vertex angle at A, and AB and AC are the gorge lines on the sides of the polygon. This means $AB=g_{i-}$ and $AC=g_{i+}$, so $AB+AC=2\tilde{g}_i$. If $g_{i-}=\tilde{g}_i$, the bastion is constructed as in the case of a regular fortress, so we assume $g_{i-}\neq\tilde{g}_i$.

   (a) From B and C, pace out a line segment along the side towards A with length $\tilde{g}_i$. Thus obtain the points E and D.

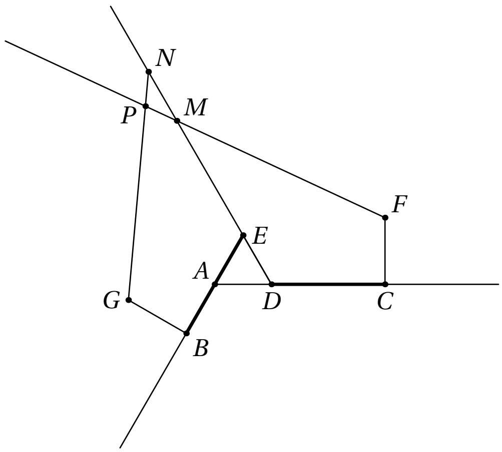


Figure 8.10 – Van Schooten's construction of a bastion on an irregular polygon.

(b) Draw a straight line through E and D and determine on its extension the points M and N such that $DM = k_i$ and $EN = k_i$. By construction $AE = AD$ and thus the triangle ADE is isosceles and $\angle ADE = \frac{1}{2}\angle BAD$. The line ED is parallel to the line which bisects the exterior angle A, which line would be the capital line in the case of a regular fortress.

(c) From M, construct a line MF such that $\angle DMF = \frac{1}{6}\varphi_i + 15°$, that is half the angle of the bastion. Point F is the intersection with the perpendicular through C to side AC.

(d) From N, construct a line NG such that $\angle ENG = \frac{1}{6}\varphi_i + 15°$, that is half the angle of the bastion. Point G is the intersection with the perpendicular through B to side AB.

(e) The salient P is found as the intersection of FM and GN, extended if necessary. The bastion is now determined by B, G, P, F and C.

In figure 8.10, figures $MDCF$ and $NEBG$ are congruent, but the bastion is not symmetric, whereas in regular fortresses, the capital line is always an axis of symmetry of the bastion. Van Schooten's method only works for polygons in which the sides are longer than the calculated length c of the curtain, that is to say that each side has to be longer than one third of the average length of all sides.

8.2.6 Comparison of Van Schooten's methods with contemporary authors

The place of fortification within the curriculum of the Duytsche Mathematicque has been discussed by several modern historians, who have stressed the similarities between the lectures of the different teachers. Van Maanen and Krüger emphasized the fundamental influence of the writings of Frans van Schooten sr. on the curriculum. Van Maanen argued that Petrus van Schooten based his lectures on the works of his father, and thus he suggested a continuity from Frans sr. via Frans jr. to Petrus.[90] Van Winter stated that innovation and change were absent in the teaching of fortification in the Duytsche Mathematicque. He called the lectures in fortification conservative, fully within the framework of Stevin's theory of fortification.[91] Taverne on the other hand argued that books by other authors such as Marolois and Freitag were influential in the Duytsche Mathematicque throughout the seventeenth century, but that its professors did not take their own initiatives to innovate of the curriculum.[92] To sum up, the recent historical literature suggests a continuity in the fortification teaching, which eventually led to stagnation and contributed to the closing down of the Duytsche Mathematicque. However, none of the above mentioned scholars seems to have investigated the mathematical details in the seventeenth century manuscripts and printed works on fortification which they mention. Therefore I will now compare the mathematical content of the lectures of Van Schooten jr. with the ideas of some of his contemporaries on fortification.

From the variety of authors of printed works and manuscripts on fortification, I have selected six men who may have influenced Van Schooten. My selection criteria have been that the author was mentioned in Van Schooten's lectures or that he was present in Leiden at some point during his life. In his lectures, Van Schooten makes explicit references to Simon Stevin, Samuel Marolois and Adam Freitag, and to his father Frans van Schooten sr. Finally, I will also consider two men who were involved in private teaching in Leiden, namely Nicolaus Goldmann and Christiaan Otterus. Goldmann lectured on mixed mathematics and architecture in roughly the same time period when Van Schooten was professor in Leiden, and Otterus was one of Van Schooten's teachers during his youth. I will compare Van Schooten's constructions of regular fortresses (compare section 8.2.4) with those of all of these authors, except for Otterus because we do not know his ideas on regular fortresses. In addition, I will say a word on how Van Schooten's method of irregular fortresses compares to those of Marolois, Freitag, Goldmann and Otterus.

Simon Stevin

In 1594, Simon Stevin published a work on fortification, entitled *De sterctenbouwing*.[93] He structured his treatise in seven chapters of which the first chapter is an introduction in the terminology of fortification. A small part of this first chapter was adopted by Van Schooten.[94] In the second and third chapter Stevin deals with the theoretical form and the practical lay-out of an ideal fortress based on a regular hexagon. Then he treats the design of fortresses based on other regular polygons. The last three chapters are devoted to the main principles of his theory of fortification, and to deviations from his general model for regular and irregular fortresses.

[90] [Krüger, 2010, 159] and [Maanen, 1987, 78].
[91] [Winter, 1988, 21].
[92] [Taverne, 1978, 64, 66].
[93] [Stevin, 1594]. A facsimile with English translation was printed in [Stevin, 1964].
[94] UBG, Hs 441, f. 140r.

There are similarities between the content of *De sterctenbouwing* of 1594 and the instruction which Stevin wrote for the Duytsche Mathematicque in 1600. Both stress the importance of drawing plans of the fortifications before starting the field work. In his treatise Stevin mentions three different drawings: the plan, the profile and the solid drawing.[95] The first two drawings were two-dimensional projections of the fortress, the third was a perspective drawing of the fortress. Moreover, both treatise and instruction emphasize the importance of wooden or earthen models, to show what the fortress would look like.[96] These models could be especially useful in the decision making process, because all men involved in the fortification project could get an clear idea of the result. Models of the different possible fortresses also facilitated the discussion of the different options and the local conditions. It is known that the maps and profiles indeed played an important role in the bureaucracy, in particular for estimating of the costs of a fortification project and in the planning and logistics of the project.[97] In his instruction of 1600, Stevin did not explicitly refer to his earlier treatise *De sterctenbouwing*, but he probably had a course in fortification in mind similar to the content of that treatise.

Stevin's design of a fortress differs strikingly from the design of Van Schooten, but also from the designs of Marolois and Freitag as we will see below. In *De sterctenbouwing*, the Italian influence is evident, and Stevin's fortresses do not have the full characteristics of the old Dutch fortification system: the flanks are retracted, but perpendicular to the curtains, and the face of a bastion exceeds the flanks, see figure 8.11. The result is a protruding bastion which is called an oreillon ("ear").

For the construction of a fortress, Stevin specified the lengths of a side of the underlying polygon, the length of the gorge line, and the position of the outermost point of the oreillon. He did not provide numerical values of these lengths in tabular form, and he did not specify any angles. In 1617, Stevin published another treatise concerning fortification, the *Nieuwe maniere van Sterctebou door spilsluysen (New manner of fortification by means of pivotted sluice-locks)*.[98] In this work, the fortresses display some typical characteristics of the old Dutch fortification system; the retracted flanks and the oreillons had disappeared.

Samuel Marolois

In 1615, Samuel Marolois published a work on fortification entitled *Fortification ou architecture militaire* in French with the printer Henrick Hondius in The Hague.[99] The treatise also appeared as part of his *Mathematical Works*,[100] which also included his *Geometrie, contenant la theorie, et practicque dicelle necessaire a la fortification*,[101] on the mathematics necessary for fortification. A second French edition of Marolois's work on fortification

[95] "Grondtteyckeninghe, verheven teyckening, ende lichamelicke teyckening", [Stevin, 1594, 1].
[96] [Molhuysen, 1913, 390*-391*] and [Stevin, 1594, 7].
[97] [Schäfer, 2001, 131].
[98] [Stevin, 1617].
[99] [Marolois, 1615].
[100] [Marolois and Vredeman de Vries, 1615].
[101] This work was also published separately as [Marolois, 1616].

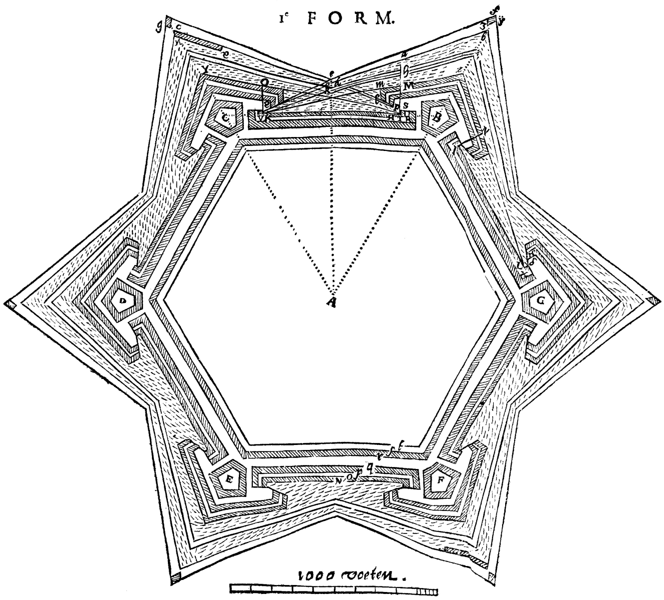


Figure 8.11 – Hexagonal fortress by Stevin, [Stevin, 1594, 18].

came off the press in 1617.[102] Throughout the seventeenth century, the book remained in print and it enjoyed a wide circulation.[103]

Just like the other authors, Marolois took a regular polygon as the basis of his theory of fortification. He used the following principles in designing a fortress:[104]

1. The flank is perpendicular to the curtain.

2. The angle of the bastion is equal to the minimum of 90° and $\frac{1}{2}\varphi + 15°$ or to the minimum of 90° and $\frac{2}{3}\varphi$ with φ the vertex angle of the regular n-gon.

[102] This edition was printed by Jan Janssonius in Amsterdam. It is known that Hendrick Hondius collaborated with Janssonius in the printing of books. Hondius was largely a printer of illustrations and Jansonius was better equipped for the printing of books, see [Orenstein, 1996, 54 and 109] and [Marolois, 1617].

[103] In 1627, a Dutch translation with additional notes by Albert Girard, mathematician and engineer, was published in [Marolois et al., 1627b]. In 1628, two French versions appeared. The first one was a reprint of Marolois's text with comments by Albert Girard, [Marolois et al., 1628a]. This version was published by Jan Janssonius, who was also responsible for the 1617 edition. The second 1628 edition was printed by Willem Jansz. Blaeu from a text that had been revised by Frans van Schooten sr., [Marolois and van Schoten, 1628]. All other editions were published by the Janssonius printing house. After 1628, French editions appeared in 1638 and 1662 ([Marolois and Girard, 1638b], [Marolois and Girard, 1662]), and Dutch editions in 1630, 1651 and 1662 ([Marolois and Girard, 1630], [Marolois, 1651], [Marolois et al., 1662]). The book was translated into English in 1631, ([Marolois et al., 1631]), in Latin in 1633 with a reprint in 1644 ([Marolois and Girard, 1633a], [Marolois and Girard, 1644]) and in German in 1627 and 1638 ([Marolois et al., 1627c] and [Marolois and Girard, 1638b]).

[104] [Marolois and Girard, 1630, 128-129] and [Marolois et al., 1662, 23-24].

3. The length of the face is to the length of the curtain as 2 : 3.

4. The angle GCE in figure 8.7 is equal to 40°.

5. The curtain has a length of 36 Rhenish rods.

Marolois presented these principles as suitable for practical use. In the theoretical part of his treatise, he claimed that the ratio between the face and the curtain should depend on the number of sides of the regular polygon.[105] But because he found it too tedious to use these different ratios, and because the results differ only slightly, he decided to use the ratio 2:3 for all regular polygons.

These principles are not completely identical to the ones that were used by Van Schooten jr. Both Marolois and Van Schooten used the old-Dutch fortification system, in which the fortress is based on a regular polygon and the flanks are perpendicular to the curtains. Just like Van Schooten, Marolois also used the angle of the bastion, but unlike Van Schooten. Marolois presented two different angles to the engineer to choose from, and he did not prefer one above the other. The principles of Marolois produced a standard fortress in which the distance between two bastions much exceeded the 60 rods which Van Schooten considered as the maximum. For Marolois, it was not the distance between two bastions that mattered, but rather the length of the defence line AGF (see figure 8.7), which should not exceed 60 rods.

For the fortification of irregular polygons, Marolois discussed one method with symmetric bastions and a method which produces irregular bastions. He explained the two methods not in an abstract setting for a general polygon, but only in concrete examples. In both methods, symmetry plays a distinctive role. In the first method the bastions are symmetric in the capital line, and Marolois scaled the size of these symmetric bastions by means of tables for bastions in regular fortresses. The second method yields irregular bastions, but involves symmetry, because the faces and flanks on the same curtain have to be identical. As a consequence, the halves of two adjacent bastions are symmetric in the axis through the midpoint of the curtain and perpendicular to it.[106]

Frans van Schooten sr.

Van Schooten sr. had been active in practical fortification projects during his professorship. He was present in army camps at the sieges of Bergen op Zoom (1622) and Groenlo

[105] Marolois considered the perfect ratio between face and curtain to be 4 : 5 for a square and a regular pentagon, 3 : 4 for a regular hexagon and 2 : 3 for regular polygons with 7 or more sides. His argument for using these different ratios is that the length of the defence line AF should be less than the reach of a musket, which he takes as 60 rods.

[106] [Marolois et al., 1627c, 63-67] and [Marolois et al., 1662, 54-58].

(1627), and participated in fortification projects in Utrecht (1629).[107] Van Schooten's presence during the sieges is documented by the maps which he made on the scene.[108]

The ideas on fortification of Van Schooten sr. are documented in a manuscript dating back to circa 1622, which is currently in the library of Leiden University.[109] The contents of the manuscript show some similarities with the way in which Frans van Schooten jr. organized his lessons in fortification. In the Leiden manuscript, the course starts with a lengthy list of definitions used in fortification, and much attention is given to staking off a fortress in the field, if the center of the polygon is accessible, and also if the center is inaccessible. However, there are striking differences in the design of a standard fortress. Van Schooten sr. used the following principles:

1. The flank is perpendicular to the curtain.
2. The angle of the bastion is equal to the minimum of 90° and $\frac{1}{2}\varphi + 15°$ with φ the vertex angle of the underlying regular n-gon.
3. The face is $\frac{3}{10}$ or $\frac{1}{4}$ times the distance between adjacent salients.
4. The capital line is $\frac{1}{3}$ of the side of the polygon.
5. The distances between the points of adjacent bastions is 60 rods.

The angle of the bastion agrees with one of Marolois's values, and Van Schooten sr. referred to Marolois.[110] He was familiar with Marolois's work, and participated in the preparation of a new edition of Marolois's *Fortification ou architecture militaire*, as his name appears on the title page of one of the 1628 reprints.[111]

The manuscript of Van Schooten sr. contains many well-drawn and richly produced drawings of plans of fortresses, but he did not pay much attention to the calculation of the lengths of parts of a fortress, and there are no tables for the sizes of standard regular fortresses.

Adam Freitag

In 1631, at the early age of 23, Adam Freitag published a work on fortification entitled *Architectura militaris nova et aucta, oder Newe vermehrte Fortification* (New and enlarged

[107] In Utrecht, Van Schooten was assisted by the Leiden surveyors Joris Gerstecoorn and Claes Slabbinck. By 1629, Gerstecoorn and Slabbinck had been acquainted with Van Schooten for almost twenty years, and both signed petitions requesting that Van Schooten sr. be appointed professor in the Duytsche Mathematicque in 1611/1612, see UBL, Archief van Curatoren, 1574–1815, inv. nr. 42/2. On the project in Utrecht, see [Taverne, 1978, 244-245] and [Bordes, 1856, 116].

[108] The map of Bergen op Zoom was published in [Grotius, 1629] and later in Blaeu's atlasses and it was also printed by Blaeu's collegue Janssonius, see [Koeman, 1967, 74, 280] and [Krogt, 2000, 527]. A map of Groenlo is kept in Bibliothèque nationale de France, département Cartes et plans, CPL GE DD-2987 (4728). A similar map, attributed to the engineer Theodoor Niels, was published in [Grotius, 1629]. On the relation between the two maps of Groenlo, see [Pluijm, 2006, 104-106].

[109] UBL, BPL 1013; the manuscript is described in [Maanen, 1987, 200-203].

[110] UBL, BPL 1013, f. 140v.

[111] [Marolois et al., 1628b]. This is the only edition which was published at the printing house of Willem Jansz Blaeu; all other editions and translations were printed by Jan Janssonius. On the role of the printer and bookseller Willem Blaeu in the infrastructure of knowledge, see the thesis of Van Netten, [Netten, 2012].

military architecture or new enlarged fortification) in Leiden with Elzevier.[112] The work was reprinted twice, in 1635 and in 1642, and French translations appeared in 1635 and 1657.[113] Freitag originated from the city of Torun (Poland), arrived in the Dutch Republic in the 1620s and joined the army. The mathematical instruction which he had received in Poland together with his practical experience in the field resulted in 1631 in his book on fortification. After the siege of Maastricht of 1632, he returned to Torun were he served as an engineer.[114]

Freitag also used a regular polygon as basic figure in his theory of fortification. In order to completely determine a fortress, he used the following principles:

1. The flank is perpendicular to the curtain.
2. The length of the face is to the length of the curtain as 2 : 3.
3. The curtain has a length of 36 Rhenish rods.
4. There are two possibilities for the angle of the bastion and the length of the flank.

Freitag's two possibilities for the angle of the bastion and the length of the flank will now be rendered in modern notation. Let n be the number of vertices of the underlying regular polygon and φ its vertex angle. In the first option, the angle of the bastion is the minimum of 90° and $\frac{1}{2}\varphi + 20°$ and the flank is the minimum of 12 and $n+2$ Rhenish rods. In the second option, the angle of the bastion is the minimum of 90° and $\frac{1}{2}\varphi + 15°$ and the flank is the minimum of 12 and $n+4$ rods. In this case, the angle of the bastion agrees with Marolois's first rule.[115] Also the length of the curtain and the ratio between curtain and face agree with Marolois's principles.

Freitag's design of an irregular fortress was based on the design of an regular fortress. In an irregular fortress, each bastion was symmetric, and was adapted to the results which he computed for regular polygons. The engineer had to look in the tables to find a regular polygon with approximately the same vertex angle and side, and thus construct the corresponding bastion on the irregular polygon. The method thus produced symmetric bastions of different size for each vertex. Freitag briefly discussed an alternative method yielding irregular bastions, but he considered this method to be inferior to his first method, mainly because the irregularity of the bastions complicated the defence of a bastion. In contrast with Van Schooten jr., Freitag thus preferred symmetric bastions in irregular fortresses.[116]

Nicolaus Goldmann

Nicolaus Goldmann, orginally from Breslau, took up his residence in Leiden in 1639. He had been in Leiden before and had registered as a student of law at Leiden university in

112 [Freitag, 1631].
113 The German versions are [Freitag, 1631], [Freitag, 1635a] and [Freitag, 1657]; the French editions are [Freitag, 1635b], and [Freitag, 1642].
114 [Notizen, 1825, 324].
115 [Freitag, 1635b, 18].
116 [Freitag, 1635b, 93-97].

1632. His interests were also in the field of mixed mathematics and architecture, and from the early 1640s onwards he was active as a private teacher. In 1643, Goldmann published a treatise entiteld *Elementorum architecturae militaris liber IV* (Four books on the elements of military architecture) at the Elzevier printing house in Leiden. A French translation of this work appeared in 1645 under the title *La nouvelle fortification*.[117]

Goldmann's regular fortresses were based on the following principles:

1. The flank is perpendicular to the curtain.
2. The length of the face is to the length of the curtain as 1 : 2.
3. The angle of the bastion is equal to $\frac{1}{2}\varphi + 15°$ with φ the vertex angle of the regular polygon.
4. The relation between the flank and the curtain depends on the number of vertices of the polygon in the following way:
   (a) In a square, the length of the flank is $\frac{1}{8}c$ with c the length of the curtain.
   (b) In an n-gon with $n > 4$, the length of the flank is the minimum of $\frac{1}{6}c + \frac{c}{48}(n-5)$ and $\frac{1}{4}c$.
5. The curtain has a length of 480 Rhenish feet.

This unit of measurement is remarkable, because all other authors use the Rhenish rod as standard unit. Goldman subdivided the Rhenish foot into smaller parts using a decimal subdivision: one foot is equivalent to 10 'primes', or 100 'secondes' or 1000 'tierces'. The subdivision facilitated the use of decimal fractions, and throughout his treatise, Goldmann calculated in decimal fractions up to the third decimal. He explained to the reader that he has chosen the foot in order to avoid confusion:

> It is something causing confusion, when finding out that a foot stands for two different lengths in the same work, that is to say the foot of the surveyors and the foot of Rhineland: this is why we never use rods but always Rhenish feet.[118]

This quotation is puzzling, until one realizes that the Rhenish rod was subdivided into feet in two different ways. The usual subdivision of the rod was into 12 feet but the surveyors also used a decimally divided rod in their calculations (see page 190 above). Goldmann made clear to his reader that his foot is one-twelfth of the Rhenish rod.

Goldmann also discussed the fortification of irregular polygons which have at least one axis of symmetry. He designed symmetric bastions, whose size is analogous to the size of bastions in regular fortresses.[119]

[117] [Goldmann, 1643] and [Goldmann, 1645]. Goudeau discusses both treatises in [Goudeau, 2005, 69-99] without paying attention to the technical mathematical content.
[118] [Goldmann, 1645, Proeme].
[119] [Goldmann, 1645, 30-43].

Christiaan Otterus

Otterus (1598–1660) was one of the teachers of Van Schooten jr., and details on his biography can be found in Section 2.1.1 of this thesis. Otterus wrote several manuscripts on mathematics, which were kept in the city library of Königsbergen (nowadays Kaliningrad, Russia), near his native Ragnit. The catalogue of 1909 describes four manuscripts by Otterus, of which three deal with fortification and one with geometry, perspective and architecture. One of the fortification manuscripts consisted of lecture notes of a course on fortification read in Leiden from 13 August until 30 October 1637.[120] What has become of these manuscripts after the turmoil of World War II remains unclear. Parts of the collection of the city library of Königsbergen have been located in libraries in Poland (primarily in Warsaw and Torun), Lithuania, and Russia (Moscow and Saint Petersburg), but a complete overview is still missing. To my knowledge, the fate of Otterus's manuscripts is unknown.[121]

During his lifetime, Otterus published a treatise entitled *Specimen problematum hercotectonico-geometricorum quo ut fortificationis (vulgo ita dictae) modi universalis ita sectionis rationalis linearum vestigium exhibetur* (Example of geometrical fence-building problems, in which manners of universal fortification (as it is commonly called) as well as a trace of the rational section of lines are displayed.) in which he discussed 22 problems related to fortification.[122] His solutions were classical Greek geometrical constructions not involving the use of numbers and trigonometry. Although he presented his work in the context of fortification, it differed strikingly from the standard literature on the subject. His general ideas on fortification were published in 1763 by F.J. Buck.[123] This treatise deals with the ideas of Otterus on fortification and its history, but not with the mathematical principles of fortification.

Otterus's printed works thus show a lively interest in fortification and mathematics, but they do not inform us about his ideas on the fortification of regular and irregular polygons. But some of his views can be gleaned from a note by Van Schooten jr. on the construction of a bastion in an irregular polygon by Otterus.[124]

Van Schooten's note only contains the following information. We are given (I use modernized notation): a vertex A of the polygon, and the two adjacent sides, the gorge line AB on one of these sides, the length k of the capital line, the length f of the flanks and a line segment a with $a < AB$. It remains unknown how Otterus determined AB, k, f and a. The construction of the bastion is as follows (see figure 8.12):

1. On the gorge line AB, choose D such that $BD = a$.

2. Find C on the other side of the vertex A such that $DC=DB$ by drawing a circle with center D and radius BD.

[120] [Seraphim, 1909, 20-21].
[121] On the whereabouts of the special collections of the city library of Königsbergen after World War II, see [Komorowski, 1980], [Garber, 1995], [Walter, 2004], [Walter, 2008] and [Stüben, 2007, 645-650].
[122] [Otterus, 1646].
[123] [Otterus and Buck, 1763].
[124] Van Schooten wrote 'a Mr Ottero' at the construction of the bastion, UBG, Hs 108, f. 25v.

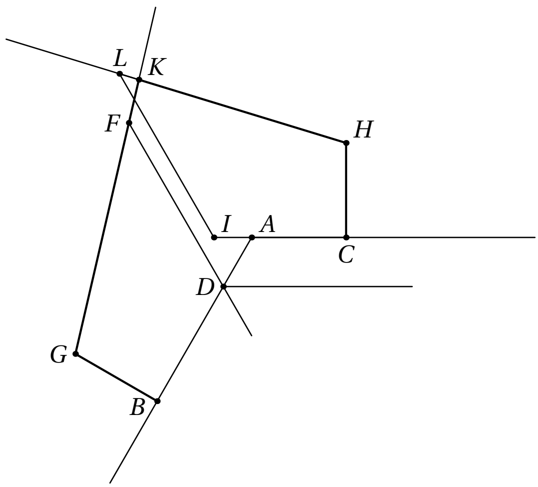


Figure 8.12 – Otterus's construction of the bastion in an irregular polygon.

3. Draw a line DE through D, parallel to AC.

4. Bisect the angle $\angle BDE$. On the bisector, choose F such that $FD = k$.

5. Draw the flanks $GB = CH = f$ perpendicular to the curtain and draw FG.

6. Find I on CA such that $CI = BD$ and draw line IL parallel to FD such that $IL = FD = k$.

7. Join L and H, and extend FG to intersect LH at K. The bastion is determined by B, G, K, H and C.

The construction is somewhat similar to the construction of an irregular bastion by Van Schooten jr. but by no means identical to it. Van Schooten's construction starts with two different gorge lines AB, AC rather than one, and it is not based on the length of the flank f but on the angles between lines DF, FG and IL, LH.

Concluding remarks on the comparison

The ideas on fortification of Van Schooten jr. and the six other authors share some important characteristics. Marolois, Freitag, Goldmann and Van Schooten jr. all structured their discussion around standard fortresses of a specific size. These served as reference models for other fortresses of different sizes or shapes. The exact definition of the standard fortress however differed from one author to another, but all gave standard fortresses based on various regular n-gons, mostly for n from four to ten. Most authors included extensive tables, specifying the lengths of relevant parts of the standard fortress. Freitag even gives these lengths for regular polygons with 24, 36 and 72 sides. This was of no practical use but shows Freitag's mathematical competence. Most authors also computed the lengths of the various parts of the standard fortresses by means of trigonometric tables.

It may not be surprising that Stevin's *De sterctenbouwing* of 1594, which is the oldest work that we have considered, differs the most from the general pattern. Stevin's fortresses with the peculiar oreillons did not make it into the seventeenth century, and his design of the plan of a fortress differed from the designs that were used by the later authors. Stevin did not use a standard fortress as a model for all other fortresses, nor did he provide extensive tables. Instead, he discussed some particular types of fortresses, in which the plan and the design of the inner works, with their differences in level etc. are all taken into account at the same time. The work of Stevin was important because it made fortification known in the Netherlands. In the following decades, other authors continued the work on fortification and gradually changed and improved the basic ideas.

The works which we have discussed illustrate the development from the specific to the general case. In 1594, Stevin discussed in great detail the plan of a fortress based on the regular hexagon, but he paid little attention to the other regular polygons. Marolois already gave more attention to the general case, although particular examples still play an important role. With Freitag and also Van Schooten sr., fortification becomes an art, following prescribed rules. This tendency continues in the work of later authors.

My investigation of the principles of the design of a regular fortress shows that each author used his own procedure. This is not surprising for printed sources, because an author who publishes a work on fortification would want to distinguish himself from other published works on the subject. Apparently, authors could distinguish themselves by means of the mathematical principles of the construction of the regular fortress. In table 8.5 I have listed the basic mathematical parameters which our authors used in the fortification of a regular polygon.[125] The polygon refers to the underlying polygon and the "distance bastions" means the distance between two adjacent *salients*. Freitag and Goldmann can be considered as variants on Marolois because they share three out of four parameters with him. Marolois used as his fourth parameter the angle $\angle GCE$ in figure 8.7, so he had to use the sine rule for the determination of lengths. Instead of the angle, Freitag and Goldmann used the length of the flank.

The argument of distinguishing oneself also works for lecturers on fortification. The methods of both Van Schooten sr. and jr. are different from those of Stevin, Marolois, Freitag and Goldmann. Van Schooten sr. used one of the values of the angle of the bastion that had already been given by Marolois, but his other basic parameters are not the same as those of Freitag, Marolois and Goldmann. The same holds for Van Schooten jr. Thus the method of Van Schooten jr. was not based on the published works of Marolois, Freitag or Goldmann. Instead, he seems to have taken the ideas of his father as a starting point and to have changed them according to his own insights.

The fortification of irregular polygons also marks a clear difference between the

[125] Stevin and Otterus are omitted from the table. Stevin's method differs from those of the other authors, and there is not enough information available on Otterus's construction of a regular fortress.

Parameter	Van Schooten jr.	Marolois	Van Schooten sr.	Freitag	Goldmann
Angle of bastion	x	x	x	x	x
face : curtain		x		x	x
face : distance bastions			x		
capital : side polygon	x		x		
flank : curtain					x
gorge : side polygon	x				
angle *GCE*		x			
length of flank				x	
length of curtain		x		x	x
distance bastions	x		x		

Table 8.5 – Parameters used in the fortification regular fortress.

method of Van Schooten jr. and the methods known from printed works. Whereas Marolois, Freitag and Goldmann attach great importance to symmetric bastions,[126] Van Schooten jr. abandons the idea of symmetry in the case of irregular figures. His approach was inspired by the method of his teacher Christiaan Otterus for the fortification of irregular polygons, and again he adapted Otterus's method to his own views.

The comparison of the mathematics behind the designs in fortification thus shows that Van Schooten jr. relied neither on the methods of Stevin, nor on the textbooks by Marolois or Freitag, nor on Goldmann. Instead, he developed his own ideas on the mathematical principles of fortification. It is very likely that he wanted to to convey his ideas to a wider audience in a printed treatise "De optima muniendi ratione". However, this work never saw the light.

[126] The differences between the methods propagated by Marolois, Freitag and Goldmann for the fortification of irregular polygons are so small that they are hardly visible on a drawing.

CHAPTER 9

Altering the curriculum: logarithms and algebra

During the professorship of Van Schooten, new topics emerged in the curriculum of the Duytsche Mathematicque, such as logarithms, spherical trigonometry, algebra and perspective. The presence of these new topics show that the curriculum at the Duytsche Mathematicque was not static but changed over time, and that the professor could determine the topics of the curriculum to a large extent.

The extant manuscripts enable a reconstruction of the lectures on logarithms and algebra, which will be the subject of this chapter. Just as in the previous chapter, we will study the way in which Van Schooten structured his courses, which topics he considered important for his students, and the level of his lectures. Before discussing the courses on logarithms and algebra in detail, I will make some brief comments on the course on perspective, which was also a new subject in the curriculum.

The lecture series on perspective consisted of two parts. The first part contained the basic mathematical principles on which the theory of perspective was built. This part was published in 1660 in the *Tractaet der perspective*[1], a treatise on perspective that was added to the *Mathematische Oeffeningen*. The basic principles were to a large extent based on the ideas of Simon Stevin.[2] The second part concerned the way in which these principles

[1] [Schooten, 1660b].

[2] Stevin published his views on perpective in [Stevin, 1605].

were used in the practice of drawing floors, stairs, buildings, fortresses, and sundials. This part was not included in the printed treatise, but lecture notes made by several auditors circulated.[3]

The fact that Van Schooten used the same theoretical principles as Stevin does not mean that his printed treatise was a copy of Stevin's text. Van Schooten improved on Stevin's ideas and he omitted some of Stevin's mathematical observations, thus making it easier for the reader to grasp the essential features. Van Schooten's treatise was also influenced by Guidobaldo del Monte.[4]

9.1 Logarithms

The first traces of logarithms in the curriculum of the Duytsche Mathematicque date back to 1655, the year in which Frans van Schooten jr. lectured on this subject. The earliest works on logarithms were published in the second decade of the seventeenth century, but logarithms are not mentioned in the extant manuscripts written by Frans van Schooten sr.[5] Frans jr. was introduced to logarithms by his tutor Christiaan Otterus in the period before 1643.[6] Thus it is likely that Van Schooten jr. was the first to give lectures on logarithms in the Duytsche Mathematicque. Before turning to these lectures, we will give a brief overview of the history of logarithms in the early seventeenth century.

9.1.1 Development of logarithms

Logarithms were developed in order to simplify the tedious multiplications and divisions of large numbers which were used in astronomy and trigonometry. From the sixteenth century onwards, mathematicians tried to find techniques for simplifying these calculations. By the end of the sixteenth century, a technique called prosthaphaeresis was used for multiplication and division. The main idea was to convert a multiplication into an addition or subtraction using trigonometrical identities of the following kind in modern notation: $\sin p \cos q = \frac{1}{2}\sin(p+q) + \frac{1}{2}\sin(p-q)$. This method reduced the amount of work and also prevented calculation errors.[7]

At the same time, the English mathematician John Napier (1550–1617) was working on a different method to simplify multiplications of trigonometrical quantities. The results of his work appeared in 1614 in *Mirifici logarithmorum canonis descriptio* (Description of the wonderful canon of logarithms, from now on: *Descriptio*) and posthumously in 1619 in *Mirifici logarithmorum canonis constructio* (Construction of the wonderful canon of logarithms, from now on: *Constructio*). The *Descriptio* includes a table of logarithms and

[3] [Schooten, 1660b, 543]. Petrus van Schooten was one of the auditors who made notes of the course, but these notes are lost today. Petrus lecured on perspective during the years 1663–1664 and 1670–1674. See [Molhuysen, 1918, 176*, 177*, 187*, 190*].

[4] [Andersen, 2007, 319-321]. For a detailed discussion on the relation between Stevin's and Van Schooten's ideas on perspective, see [Andersen, 1990].

[5] These manuscripts are preserved in the UBG and UBL.

[6] Notes by Frans jr. on the logarithmic form of the law of sines from a course of Otterus are found in UBG, Hs 108, f. 92r.-93r.

[7] Van Schooten jr. was familiar with prosthaphaeresis, see UBG, Hs 108, f. 84r.-91v.

a manual for using the table, and the method for computing the tables is explained in *Constructio*.[8].

Napier's concept of logarithms differs from the modern concept in various ways. In modern terms, Napier computed for an argument x a quantity $\mathrm{Nap}\log(\mathrm{Sin}(x))$, which can be shown to be approximately $-10^7 \log_e(\sin(x))$, where $\log_e$ indicates the modern natural logarithm. Recall that the sine at that time, in my notation $\mathrm{Sin}(x)$, was the length of a line segment in a circle with a fixed radius, which Napier took as 10^7; thus $\mathrm{Sin}(x) = 10^7 \sin x$. Napier defined his logarithms by means of a complicated kinematical model.[9] He put the value of the logarithm of the "total sine" (i.e., $\mathrm{Nap}\log\mathrm{Sin}(90°) = \mathrm{Nap}\log(10^7)$) equal to zero and let the value of the logarithm increase as the sine decreased.

Napiers tables were developed for use in trigonometry and turned out to be not very convenient for other purposes. He realized the inconvenience and he suggested in his *Constructio* to develop a different kind of logarithms, based on a geometrical progression with ratio 10, and with the logarithm of 1 equal to zero.

It was eventually Henry Briggs (1561–1630) who developed these logarithms. In 1617 he published in *Logarithmorum chilias prima* (The first thousand logarithms)[10] the first table of logarithms in base 10 for the integers 1 up to 1000, to 14 decimal places. In 1624 he printed in *Arithmetica logarithmica* (Arithmetic of logarithms) the logarithms of the integers from 1 to 20,000 and 90,001 to 100,000, also to 14 places.[11] The gap between 20,000 and from 90,000 was filled in 1628 by the Dutchman Adriaen Vlacq, who calculated the missing logarithms together with Ezechiel de Decker.[12]

The logarithms of Briggs, which I will denote as Brilog, differ in two ways from Napiers logarithms. First, Briggs took $\mathrm{Bri}\log(1) = 0$ and $\mathrm{Bri}\log(10) = 1$, so his logarithm agrees with our modern logarithms in base ten. There is also a conceptual difference because Briggs considered the logarithms of numbers, whereas Napier calculated the logarithms of trigonometrical quantities. In 1633, trigonometrical logarithms based on the Briggsian logarithms were published by Briggs and Vlacq separately.[13] In his trigonometrical logarithms, Briggs divided one degree into one hundred parts, whereas Vlacq adhered to the

[8] [Napier, 1614] and [Napier and Briggs, 1619].

[9] The model involved two lines. The first line ℓ_1 was extended to infinity and the other line ℓ_2 was a line segment AB of given length 10^7, which is the radius of the circle. On the first line ℓ_1, the point α travels at a constant speed. On the second line ℓ_2, the point β moves geometrically, that is, the distances traversed at equal time intervals $[nt,(n+1)t]$ form a geometrical sequence. The initial velocity of point β equals the velocity of point α. If after a certain time interval the point α arrives at position a and β at position b, Napier defined the distance a travelled by point α to be the logarithm of line segment bB, that is the remaining distance to be travelled by β. For a more detailed investigation of Napier's kinematical model, see [Braunmühl, 1971, 5-6], [Goldstine, 1977, 2-13] and [Roegel, 2011a, 4-6].

[10] [Briggs, 1617].

[11] [Briggs, 1624].

[12] During their project on the calculation of the missing logarithms, Vlacq and De Decker became at odds with one another. De Decker was the first to publish the tables in 1627 in [Decker, 1627], but Vlacq managed to keep this work of De Decker from being spread and he published the tables in 1628 under his own name in [Vlacq, 1628b] and [Vlacq, 1628a]. On the relation between De Decker and Vlacq see [Haaften, 1928], [Poelje, 2005] and [Roegel, 2011b].

[13] The work of Briggs was published posthumously. It is remarkable that both works were published by the same publisher in Gouda, the Netherlands.

classical division of one degree into 60 minutes.[14]

9.1.2 Van Schooten's lectures on logarithms

Van Schooten jr. devoted two semesters to logarithms in 1655: the first semester to Napierian logarithms, and the second semester to Briggsian logarithms.[15]

Van Schooten based his lectures on Napierian logarithms on Napier's *Descriptio* and *Constructio* but he introduced the logarithms in a different way. He did not use Napier's kinematic model but a discrete model instead, in which he compared an arithmetical and a geometrical progression.[16] Van Schooten speaks of logarithms as *numbers*, whereas Napier defines them in terms of *line segments*.[17] In order to explain the Napierian logarithms, Van Schooten then turned to the geometrical construction of a geometrical progression on a given line segment. He performed this construction on the radius of a circle, and in this way he connected the concept of logarithms to trigonometry. He let the value of the sine correspond to a term in the geometrical progression, and determined the logarithm from the corresponding term in an arithmetical progression. He continued with the logarithm of the secant and tangent, using the logarithms of the sine and the main arithmetical properties of the logarithms.

Van Schooten then discussed Napier's original works explaining large parts of the *Descriptio*, which is a manual for using logarithms. He closely followed Napier's discussion in chapters IV and V of Book I, and chapter I and II of Book II. These chapters deal with the use of the tables and the logarithms in computations which often occur in arithmetic and in plane triangles. Van Schooten carefully explained the same examples that had been used by Napier in his treatise.[18]

In order to deepen the students' understanding of the subject, Van Schooten elucidated the main ideas of the computation of Napier's tables, closely following Napier's own explanations in the *Constructio* of 1619.[19] Napier's method was very intricate, but by an ingenious approach he managed to keep the rounding errors to a minimum,[20] and Van Schooten carefully explained the whole method.

---

[14] On the construction of logarithmic tables during the seventeenth century, see the work of Denis Roegel at `locomat.loria.fr`. The Locomat project aims at "making available a number of interesting and/or important historical tables, and facilitating the study of the original tables by historians of mathematics". The part completed by 21 May 2012 provides a fruitful analysis of the construction and the use of the tables.

[15] My reconstruction of the courses is based on two manuscripts of 1655, one written by Frans van Schooten jr. on the Briggsian logarithms (UBG, Hs 435) and the second one written by Petrus van Schooten while attending his brother's lectures (UBG, Hs 441). For more detailed information on these manuscripts, see appendix A.

[16] UBG, Hs 441, f. 49r.-v.

[17] Van Schooten's definition of a logarithm was inspired by the way Briggs defined the logarithm in [Briggs, 1624, 1] and Van Schooten used the same examples as Briggs, see UBG, Hs 441, f. 49v.

[18] Folio's 57v.-68v. of UBG Hs 441 correspond to Chapter IV, V of Book I and Chapter I and II of Book II of [Napier, 1614].

[19] The folii 69r.-80r. correspond with the pages 9 to 32 of [Napier and Briggs, 1619].

[20] For a detailed discussion of Napier's construction of the tables, I refer the reader to the insightful study [Roegel, 2011a] or to [Goldstine, 1977, 2-13].

In the second semester, Van Schooten based his lectures mainly on the first seven chapters of Briggs's *Arithmetica logarithmica* of 1624, with additional examples from Briggs's *Lucubrationes*, which had been printed as an appendix to Napier's *Constructio*.[21] Van Schooten first discussed the main differences between Napierian an Briggsian logarithms, and he emphasized that if $x_1 > x_2$ then $\text{Nap}\log(x_1) < \text{Nap}\log(x_2)$ and $\text{Bri}\log(x_1) > \text{Bri}\log(x_2)$.

Van Schooten devoted a fair part of the course to the computation of logarithms. He first treated a method that had been outlined in an appendix to Napier's *Constructio* and that had been worked out by Briggs.[22] Van Schooten used this method to compute the logarithms of 2 and 7. He then treated a second method based on successive root extraction. Following Briggs, Van Schooten used this method for the determination of the logarithms of 2 and 6. By means of the fundamental properties of the logarithm, he then computed the logarithms of $5 = 10/2$ and $3 = 6/2$.

Van Schooten considered this to be sufficient for his students, and therefore he did not discuss the subsequent chapters of the *Arithmetica logarithmica*, in which Briggs explained more sophisticated methods for the computation of logarithms.[23] It was not Van Schooten's aim to recompute the tables,[24] but he wanted to give his students some insight in the background of the Briggsian logarithms. Van Schooten then turned to a simple interpolation method for finding the logarithm of a number which was not included in the table. At the time of his lectures, Vlacq's tables of the logarithms of the integers from 1 to 100,000 were generally available. Interpolation methods were needed for numbers above 100,000 and numbers with a fractional part. In both cases, Van Schooten prescribed linear interpolation, although he knew that the result was somewhat less than the actual value of the logarithm.[25] In most of the computations involving logarithms, the results of linear interpolation were sufficiently accurate for practical purposes.

In this part of the course Van Schooten used an interesting notation for the terms of a geometrical progression with first term 1 and ratio a, namely $1, a, a^2, a^3, \ldots$. Briggs did not use algebraic notation but indicated a geometric progression in words or by means of a numerical example. Van Schooten's notation shows the Cartesian influence in his use of the letter a for an undetermined quantity and in the exponential notation for repeated multiplication.

Van Schooten's elaborate treatment of Napierian logarithms is surprising because they were soon replaced by the Briggsian logarithms. Vlacq's tables of the Briggsian logarithms had been published in 1628, and were the basis of almost all tables of logarithms that

[21] In *Lucubrationes*, Briggs explores his first ideas on a logarithm such that $\log(1) = 0$, [Napier and Briggs, 1619, 42-49].

[22] The method was outlined in the second Appendix of [Napier and Briggs, 1619, 40-41], in which he gave his first ideas on a definition of base-10 logarithms and determined the logarithm of 2. Briggs developed this idea and gave the details of the calculations in [Briggs, 1624, 7-9].

[23] For a description of these methods, see [Roegel, 2011c] or [Goldstine, 1977, 13-20].

[24] "Genoech achtende de manier daer van in t'kort verklaert te hebben, voorbijgaende de verdere cortichheden, dieder om deselve met lichticheijt en behendicheijt te vinden, wijders stonden aen te wijsen: nademael ons oochmerck niet is, die op nieuws te construeren", UBG Hs 425, f. 16r.

[25] UBG, Hs 435, f. 20v.

appeared between 1628 and the twentieth century.[26] Van Schooten stated that Briggsian logarithms were more accurate and easier to use than Napierian logarithms[27] and he encouraged the students to buy a copy of the Briggsian tables. He recommended in particular the tables in octavo that had been issued in 1651 by the Leiden printer Philips de Croy. These tables were based on Vlacq's tables and contained the logarithms from 1 to 10,000 and trigonometrical logarithmic tables.[28] It is unclear why Van Schooten treated the Napierian logarithms in so much detail. His motivation may have been to give a complete overview of the development of the logarithms.

At the end of our discussion of Van Schooten's courses on logarithms, we may well ask to what extent this knowledge was used in other courses in the Duytsche Mathematicque. An investigation of the relevant manuscripts reveals that logarithms were indeed used in the course on spherical triangles. Van Schooten does not explicitly mention what logarithm table he used, but comparison of the logarithm values that he used with the Napierian and Briggsian tables reveals that he used the Briggsian tables of trigonometrical logarithms.[29]

A second possible application of logarithms was in surveying, where area and volume calculations often involved multiplication of large numbers. Surveying was taught as part of the course on practical geometry, of which no lecture notes have been found. Because the lectures on practical geometry were scheduled *before* the lectures on logarithms in Van Schooten's educational cycle,[30] it is highly unlikely that logarithms were used in the standard course on surveying. We can support this by the following argument. The Duytsche Mathematicque was used by some aspirant surveyors as a preparation for the examination of surveyors, but since the examinations of surveyors at the Court of Holland contain no traces of the use of logarithms, they were not considered to be part of the standard knowledge a surveyor should possess. This is in line with the observation by Muller and Zandvliet that the practice of surveying did not change in the two centuries after 1600.[31]

Van Schooten's courses on logarithms were quite advanced. He devoted a large amount of time to theoretical backgrounds and the computation of logarithm tables. Only after this theoretical treatment he discussed the use and usefulness of logarithms in the solution of practical problems. Van Schooten's theoretical emphasis is in striking contrast with the initial aim of the Duytsche Mathematicque as formulated by Stevin in 1600. As we have seen above, Stevin explicitly stated that the teaching should be restricted to what an engineer needed in practice.

---

26 [Roegel, 2011b, 15].
27 UBG, Hs 435, f. 1r.
28 UBG, Hs 435, f. 18v. and [Vlacq, 1651].
29 UBG, Hs 444.
30 See for the order of the courses table 7.2 on page 176.
31 [Muller and Zandvliet, 1987, 38]. The first surveying books in which tables of logarithms appear were published in the last quarter of the eighteenth century.

9.2 Algebra

During the professorship of Frans van Schooten jr., algebra was an undisputed part of the curriculum of the Duytsche Mathematicque, and it was the second subject taught in his educational cycle. The fact that he placed the course in algebra immediately after the course on arithmetic and before the course of geometry shows that he attached much importance to the subject.

When the Duytsche Mathematicque was founded in 1600, algebra was not part of the curriculum. Simon Stevin was familiar with algebra[32] but because it was the purpose of the Duytsche Mathematicque to educate engineers within a short time, he did not place algebra on the program.[33]

It is not known when algebra became part of the curriculum of the Duytsche Mathematicque. A manuscript in the hand of Frans van Schooten sr. dating from ca. 1623 treats algebra.[34] This manuscript is clearly an instructional text, but it does not follow that algebra was taught in the Duytsche Mathematicque under Van Schooten sr., because he may have used the manuscript for private teaching. However this may be, I have compared this manuscript by Van Schooten sr. with the 1653 course on algebra by Van Schooten jr., using the lecture notes made by Petrus van Schooten.[35] The comparison reveals that Van Schooten jr. in 1653 took a large amount of problems from his father's 1623 manuscript,[36] although the 1653 course on algebra was more profound and elaborate.

9.2.1 Notation

Throughout the 1653 course in algebra, Van Schooten jr. used cossic symbols to indicate unknown quantities. These symbols had been in use for more than a century. They were developed in the fifteenth century and became widespread in the sixteenth century through the works of Christian Rudolff and Michael Stifel.[37] The cossic system used different symbols for the unknown, its square and its cube, but no algebraic notation was used for known quantities, which were always represented by concrete numerical values.

Van Schooten's choice for cossic symbols was remarkable and unexpected in hindsight, because he was well aware of the further improvements in algebraic notation in the sixteenth and early seventeenth century. One such development was the notational system of François Viète,[38] who used letter symbols not only for unknowns but also for known

[32] In his *Arithmetique*, Stevin discussed algebra, in his own symbolic notation for unknown quantities.

[33] Perhaps algebra belonged to the "deeper matters" ("diepsinnigher stoffen") which Stevin mentioned in his instruction as possible subjects of study for students who had finished the Duytsche Mathematicque.

[34] The manuscript is UBG Hs 443; for more details see page 289 of Appendix A.

[35] The 1653 course is documented in UBG Hs 437.

[36] I give the pages on which the same problems are found by means of pairs (x, y) with x the folio of UBG Hs 437 (Van Schooten sr.) and y the folio of UBG Hs 443 (Van Schooten jr.): (151v., 29r.), (151v., 29v.), (154v., 31v.), (161r., 54r.), (161v., 53v.) and (169r., 56v.). This list is not exhaustive.

[37] In particular the works [Rudolff, 1525], [Stifel, 1544] and [Stifel, 1553]. See [Reich, 2003, 196] for a discussion of the history of cossic symbols and [Heeffer, 2008b] and [Heeffer, 2009] for developments in algebraic thought and notations in the period before Viète.

[38] Viète introduced his notation in [Viète, 1591].

0	1	2	3	4	5	6	7	8	9	10	11
	𝔵	𝔷	𝔠	𝔷𝔷	ß	𝔷𝔠	Bß	𝔷𝔷𝔷	𝔠𝔠	𝔷ß	Cß
1	2	4	8	16	32	64	128	256	512	1024	2048

Table 9.1 – Introduction of cossic symbols. Based on UBG Hs 437 f. 119r.

but indeterminate quantities, and who interpreted the science of algebra as manipulation of abstract magnitudes.[39] For operations, Viète used words rather than symbols, and therefore his equations are closer to the verbal tradition than to symbolic formulas. For instance, Viète wrote "*B* in *A* quadratum plus *D* plano in *A* aequari *Z* solido" for the equation which could be written in modern notation as "$BA^2 + DA = Z$",[40] where the consonants *B*, *D* and *Z* indicate a known line segment, area and solid respectively, and the vowel *A* is the unknown line segment. The development culminated in the notational system of Descartes, which is essentially the one still in use today. Just like Viète,[41] Descartes used letters not only for unknown quantities but also for known indeterminate quantities. In general, he took x, y or z for the former category and $a, b, c, ...$ for the latter. Unlike Viète, Descartes also used symbols for operations, and he introduced the now familiar exponential notation, as in x^3. Frans van Schooten jr. was the editor of Viètes collected works and the translator of Descartes's *Géométrie*, so he was more than anybody else familiar with the notational systems of Viète and Descartes.

Van Schooten introduced the cossic symbols to his pupils in the same way as Clavius did in his *Algebra* of 1608,[42] by means of a correspondence between the symbols and a geometrical progression, see table 9.1. The numbers of the first row $(0, 1, 2, \dots)$ are the exponents of the powers, indicating the number of times a quantity is multiplied by itself.[43] The second row displays the unknown and its powers in cossic symbols. The unknown was represented by 𝔵 and was called "cosa" (German: coss) or "root". The notation for the square of the unknown was 𝔷, from "zenso", and the cube of the unknown was indicated as 𝔠 and called "cubo". The third row displays the powers of 2. The table illustrates that if 𝔵 = 2 then 𝔠 = 8 and vice versa.

For unknowns raised to a power n, with n not equal to a prime number, the cossic symbol is composed of the cossic symbols for the unknown to the power p_i for the prime factors p_i of n. For instance, the notations for the fourth and sixth power of the unknown were 𝔷𝔷 and 𝔷𝔠. For powers with prime exponents, new symbols were introduced. The fifth power of the unknown was denoted by ß and called "sursolidum". The following prime powers were denoted by successive capital letters and the symbol ß, thus: Bß

[39]On the significance of Vìtee's notation and the connection with his interpretation of algebra see [Bos, 2001, 146-154] and [Mahoney, 1980].

[40]This example is taken from *Isagoge* in the edition of [Viète, 1646, 9].

[41]We have to bear in mind that Descartes denied having read Viète before the publication of the *Géométrie* and that Descartes's geometrical interpretation of arithmetical operations differs from Viètes ideas, see [Bos, 2001, 297-301].

[42][Clavius, 1608].

[43]For $a = b^n$, a is the power and n is the exponent. Exponents are sometimes referred to as powers, saying a equals b raised to the n-th power.

for the unknown to the 7th power, Cß for the unknown to the 11th power, and so on.

According to Van Schooten, the cossic symbols of higher powers were composed from the symbols of lower powers for the following reason:

> Because one cannot imagine another solid in the *rerum natura* [i.e., the nature of things], therefore the subsequent signs [i.e., the unknown to the power four or higher] all have their origin in the three preceding and are composed of them.[44]

Van Schooten here appeals to the classical geometrical interpretation of the unknown and its powers. As the dimension of space is three, geometrical objects perceived by humans have dimension three at most. If the unknown algebraic quantity represents a line segment, the second power of the unknown can be represented as a square and the third power by a cube, which are well-defined geometrical objects. Thus the first three powers of the unknown were denoted by distinct symbols (i.e ℛ, ℨ and ℭ) whose names refer to the geometrical interpretation. Van Schooten makes this connection explicit by drawing a line segment, a square and a cube in his discussion of the powers.[45] For unknowns to the power four of higher, the geometrical interpretation could not be continued, and therefore it was preferable not to introduce new distinct symbols for these unknowns, but to use combinations of the previous symbols instead. Van Schooten does not explain why the sursolidum ß and its variants were used for the unknown raised to the pth power with p a prime equal to 5 or larger.

In modern (Cartesian) notation, multiplication (or division) of powers of the unknown can easily be achieved by adding (or subtracting) exponents: $x^n x^m = x^{n+m}$; $x^n/x^m = x^{n-m}$. In cossic notation the procedure was more laborious and less clear. First one had to determine the exponents corresponding to the cossic symbols. Then the exponent of the product or quotient had to be determined by addition or subtraction, and finally the corresponding cossic symbol had to be looked up in the table, unless one knew the symbol by heart. In the Cartesian notation, the exponent is directly encoded in the notation. Nevertheless, Van Schooten chose to adhere to cossic symbols in his course. The only notational influence of Descartes in the course is the Cartesian equality symbol. In the manuscript of Van Schooten sr. no equality symbol occurs but the equalities are written out in words.

Van Schooten jr. mentions two other notational systems in his 1653 course, namely the system of Stevin and the notation attributed to Diophantus in the Bachet edition.[46] He did not say a word on the Cartesian notation. This silence is remarkable because he preferred the Cartesian notation over the cossic notation in his published work, and he even promoted the Cartesian notation.[47]

---

[44] "Dewijl nu niet in rerum natura van soliden een ander kan geimagineert worden, soo ist dat de volgende teijckens alle haer oospronck hebben uut de 3 voorgaende ende daer uut gecomposeert sijn." UBG, Hs 437, f. 119r.

[45] UBG, Hs 437, f. 118v.

[46] [Diophantus of Alexandria and Bachet, 1621].

[47] In Van Schooten's printed work, the algebraic notation is usually Cartesian, although he sometimes used other systems. In the Appendix to the *Organica Conicarum* (1646) on the solution of third degree equations, he used four different systems: the notations of Descartes, Stevin, Bachet's version of Diophantos, and the cossic

Van Schooten may have used the cossic symbolism in his course because Cartesian notation was not common at all in the community of mathematical practitioners in the mid-seventeenth century. It took some time until the first Dutch introductions to algebra in Cartesian notation appeared. In 1661, Gerard Kinckhuysen published his *Algebra* and in 1672 the *Beginselen van de algebra* (Principles of algebra) of Abraham de Graaf came of the press.[48] Both authors used the Cartesian notation using x and y for the unknowns and exponents for the powers of the unknowns. Around the same time, treatises using the cossic symbols still appeared: in 1661 Anthoni Smyters's *Arithmetica* was reprinted and 1663 saw the first edition of Brassers's introduction to cossic algebra, which was reprinted in 1672.[49]

Van Schooten may also have had a didactic reason for the use of cossic symbols. He had been familiar with Descartes's notation from the beginning, and he had seen that many mathematicians found it difficult to manipulate letter symbols in the early decades after the publication of the *Géométrie*. Part of these difficulties may have been caused by the fact that these letter symbols were also used for writing words. With cossic symbols one did not have this problem, because the cossic symbols were different from the number and letter symbols already known to students. The difference probably made it easier for the students to accept the cossic symbols. In the traditional literature, these symbols were referred to as numbers, namely "cossic numbers."

9.2.2 Structure and content

Van Schooten lectured on algebra during the summer semester of 1653, the winter semester of 1659 and the summer semester of 1660.[50] The structure of both courses is displayed in table 9.2. The course of 1653 was divided into two parts. The first part dealt with the cossic symbols and the operations on "single cossic numbers" and "composed cossic numbers", that is in modern terms on monomial[51] and polynomial expressions. The second part treated the concept of an equation, the classification of equations, and reduction of equations to standard forms. In the tradition of cossic algebra, this part was amply illustrated by problems. The 1660 course had the same structure as the 1653 course, but Van Schooten made some adjustments. This time, the second part on equations included only linear equations in one unknown and systems of linear equations. He also included a third part on the theory of surd or irrational numbers. In modern terms, these numbers are described as the irrational roots of quadratic and cubic equations with rational coefficients. Van Schooten described surds as numbers whose value one "cannot express exactly, but

system [Schooten, 1646, 98-109]. The Bachet notation also pops up in one problem in the fifth Book of the *Exercitationum mathematicarum libri quinque*, probably because this notation was also used in a letter of Nicolaas Huberts van Persijn to which Van Schooten makes reference in his discussion of the problem. [Schooten, 1657, 435-436] and [Schooten, 1660a, 405]. Van Schooten infrequently used cossic notation in his own Dutch notes in handwriting, see UBG, Hs 108, f. 49v.-50r.

[48] [Kinckhuysen, 1661] and [Graaf, 1672].

[49] [Smyters, 1661], [Brasser, 1663] and [Brasser, 1672].

[50] It is possible that he also lectured on algebra in his first teaching cycle in 1646–1652, but no documents have survived.

[51] In modern notation: ax^n with $n \in \mathbb{N}$.

	summer 1653	winter 1659- summer 1660
Part 1	Notation and symbols	Notation and symbols
	Species of single cossic numbers	Species of single cossic numbers
	Species of composed cossic numbers	Species of composed cossic numbers
Part 2	Classification of equations	Classification of equations
	Reduction of equations	Reduction of equations
	Problems in several unknowns, both determined and undetermined problems	Problems in several unknowns, both determined and undetermined problems
	Diophantine problems involving square numbers	
	Quadratic equations	
Part 3		Species of surd numbers
		Binomial and residual numbers

Table 9.2 – Structure of the course on algebra. Based on UBG Hs 437.

only geometrically".[52] He could not finish this third part on surds as he passed away in May 1660.

Van Schooten introduced the arithmetical operations on cossic symbols in a way similar to the corresponding operations in his course on arithmetic. This is not surpring because he considered the monomial or polynomial expressions in cossic symbols as "cossic numbers".[53] In the cossic tradition, monomials and polynomials were considered as numbers of the same category as whole numbers and fractions. A cossic number had one determinate, but as yet unknown, numerical value.[54] In the discussion of operations on cossic numbers, Van Schooten presented the extraction of a square root and a cube root, operations which he had not discussed in his arithmetic course on numbers.

Van Schooten then discussed the relationship between algebra on the one hand, and geometry and arithmetic on the other hand. He presented algebra as a universal method for solving all "hard and apparently quite unfeasible problems".[55] He was enthusiastic about the generality of algebra: algebra could be used in all mathematical disciplines because it could be used to solve all problems involving quantity.[56] The algebraic method for the solution of problems is also general, because algebra in problem solving meant

52 "Wiens waerden men niet volcoomen, als alleen naer den meetkonst uytdrucken kan." UBG, HS 437, f. 210v.

53 See for instance UBG, Hs 437, f. 121r. for the monimals and f. 124v. for polynomials.

54 [Heeffer, 2008a, 118-119].

55 "De Algebra alle moeijlijcke ende gans ondoenlick schijnende questien leert ombinden [sic] ende beantwoorden", UBG, HS 437, f. 159r.

56 "Sulcx wij dan sien dat de algebra geapplicceert can worden tot alle soorten van questien, somtijts beteijckene $1\mathcal{X}$ een getal somtijts een linij somtijts mannen, al soo dat de algebra streckt sich over alle wanneer maer alleen enichsins een quantiteijt geconsidereert wort en streckt daerom sich oover de gantsche mathesis." UBG, Hs 437, inserted folio between f. 146v. and 147r.

performing the same procedure for each problem, no matter whether the problem was of arithmetical, geometrical or other background. Van Schooten stated that by the use of algebra “all problems are considered to be one”.[57]

In the algebraic method for problem solving, Van Schooten distinguished two stages. The first stage consisted of the introduction of an equation corresponding to the problem. In this stage, the unknown[58] had to be identified and the relation between the given quantities and the unknown quantity had to be expressed in an equation. The second stage was the solution of the equation leading to the solution of the problem. First, the equation had to be reduced to a standard form by one or more manipulations. Permitted manipulations were addition or subtraction of the same quantity from both sides of the equation, removing fractions by multiplication, removing multiples by division, removing surds by squaring both sides, and removing squares or third powers by taking the square or cube root.

Van Schooten considered the first stage (i.e., the determination of the equation) as the hardest part in the algebraic method for problem solving. There were no standard procedures for deriving an equation from the problem, so a student could only learn by experience to transform a problem into an algebraic equation. Once the equation had been found, the mathematician had the problem “in his power”, in the words of Van Schooten.[59] The equation could now be reduced to a standard form by means of well-defined rules, and the problem could be solved.

The algebraic manipulation of the equation was performed on an abstract level and did not take the (geometric) context of the problem into account. An equation was considered as a full representation of the problem, containing all information needed for the solution.

In all geometrical problems which Van Schooten treated in his algebra course, he assigned numbers to line segments and/or areas.[60] The problems required the determination of a line segment with a certain property, and Van Schooten took the length of this segment as algebraic unknown. The problem was considered solved once the length (i.e., a number) was determined.[61] For Van Schooten this meant that in geometrical problems, the figure was not needed anymore once the equation had been established. By using algebra, one “unravels” oneself from the geometrical figure. Once the problem had

[57]“Door dit middel [i.e., algebra] alle questien voor een questie acht.” UBG, Hs 437, inserted folio between f. 146v. and 147r.

[58]Or several unknowns. Van Schooten discussed various problems in several unknowns, see UBG, Hs 437, 161r-174v.

[59]“Want een aequatie gevonden hebbende, heeft men de questie in sijn macht”, UBG, HS 437, inserted folio between f. 146v. and 147r.

[60]Van Schooten’s practice of accepting numbers in geometrical problems agreed with the tradition in practical geometry and surveying in the sixteenth and seventeenth centuries. The ancient Greek mathematicians in the time of Euclid made a clear distinction between arithmetic and geometry and did not assign a number to an arbitrary line segment. The use of numbers in geometry posed difficulties for early modern scholars who adhered to the classical Greek legacy. For a discussion on the use of numbers in geometry in this period, see [Bos, 2001, 119-158] and [Wreede, 2007, 179-213].

[61]This contrasts with the practice in pure geometry, where a construction of the solution had to be given in order to solve the problem. Van Schooten’s course on algebra did not contain any geometrical construction of a solution of a geometrical problem.

been translated into an algebraic equation, the manipulations on the equation can be performed regardless of the geometrical figures.[62]

In arithmetical problems, algebra could also be used with profit, because it made the knowledge of arithmetical rules superfluous. We have seen above that the standard introductory courses in arithmetic in the early modern period treated an abundancy of arithmetical rules, with a separate rule for each particular type of arithmetical problem. Because all these problems could be solved by means of algebra, there was no need anymore to learn all these rules by heart, and Van Schooten considered this a great benefit. Moreover, the arithmetical rules themselves could even be found by algebra.[63] Thus Van Schooten considered algebra as the fundament of all arithmetical rules.[64] Algebra had the following advantage as well. In problems which are nowadays called indeterminate systems of equations, algebra was capable of finding all solutions, whereas the arithmetical rules only gave one solution of the problem. Van Schooten illustrated the difference by means of the following problem of allegation.[65] Just like above, I use the notation p_1, p_2 and p_3 where Van Schooten uses concrete numbers;[66] the notations A, B and the cossic symbols are due to Van Schooten.

A brewer has three kinds of beer. The cost of these kinds is p_1, p_2 and p_3 guilders per cask respectively. The brewer wants to compose a mixture of these types which costs p guilder per cask, with $p_1 < p < p_2 < p_3$. It is required to determine how the three kinds of beer should be mixed.

Using the classical rule of allegation, one would find that the mixture can be composed of $p_2 + p_3 - 2p$ casks of the first beer, $p - p_1$ casks of the second beer, and $p - p_1$ casks of the third beer. Note that the rule gives only one way of mixing the three kinds of beer. Using algebra, Van Schooten sets $1\mathcal{X}$, $1A$ and $1B$ for the unknown quantity of the first, second and third kind of beer.[67] As the mixture should costs p guilders per casks, the equation is

$$p_1\mathcal{X} + p_2 A + p_3 B = p(\mathcal{X} + A + B).$$

Solving for $\mathcal{X}$, Van Schooten obtains.

$$\mathcal{X} = \frac{(p_2 - p)A + (p_3 - p)B}{p - p_1}.$$

From this last equation, Van Schooten concludes that one can choose A and B freely[68] and therefore there are infinitely many ways of mixing the three beers, depending on the initial choice of A and B.

Van Schooten considered exercises in solving problems necessary in order to become competent in using algebra. Therefore he discussed a large number of problems with

[62] UBG, Hs 437, inserted folio between f. 145v. and f. 146r.

[63] Van Schooten gave algebraic demonstrations of the rule of three, the converse rule of three, the double rule of three and the rule of company with time, see UBG, Hs 437, f. 159r.-160v.

[64] UBG. Hs 437, f. 159r.

[65] I discussed the rule of allegation in detail on page 193.

[66] See. UBG, Hs 437, f. 174v.

[67] Van Schooten's notation of the second and third unknown follows the notation used by Stifel.

[68] "daer men $1A$ en $1B$ naer welgevallen neemt." UBG, Hs 437, inserted note between f. 174v. and 175r.

solutions. The course in 1653 included the quadratic equation and some Diophantine quadratic problems, but Van Schooten did not discuss the solution of the general equation of degree three and four in his algebra courses.

CHAPTER *10*

Concluding remarks on the Duytsche Mathematicque

In Van Schooten's time, the Duytsche Mathematicque had the same peculiar position within the structure of Leiden University as in the beginning in 1600. The language of instruction was the vernacular, and not Latin which was used in academia. Students of the Duytsche Mathematicque were not considered full members of the academic community as they were not registered as such and did not enjoy the academic privileges. Nevertheless, the position Van Schooten jr. as professor did not differ from that of the other extraordinary professors at the university, and his yearly pay was in line with that of his collegues. From the winter semester of 1653 onwards, his lectures in the vernacular were mentioned on the university schedules. Thus the Duytsche Mathematicque became more accepted during Van Schooten's professorship, probably as a result of his scholarly reputation. He was an esteemed mathematician and his Latin edition of Descartes's *Géométrie* was well received in the academic world.

As we have seen, Van Schooten organized the teaching programme of the Duytsche Mathematicque in cycles of thirteen semesters. In the second cycle (1652–1658), he reviewed his own lectures in the first cycle (1646–1652), and adapted them to what he thought best for his students. The general outline of the Duytsche Mathematicque as envisaged by Stevin in 1600 is still visible in the emphasis of Van Schooten's programme on arithmetic, practical geometry and fortification. However, Van Schooten adapted his teaching to his own views, and the programme was extended to include logarithms, sundials, algebra and perspective.

Van Schooten's teaching in the Duytsche Mathematicque reflects a tension between scholarly, academic mathematics on one hand and more practical oriented mathematics on the other. Van Schooten was a product of both traditions: his father had been active as an engineer in the army and had been present at various sieges during the Dutch Revolt, and his teacher Jacob Golius was a representative of the scholarly tradition.

The tension is most prominent in the treatment of algebra. Van Schooten gave algebra an important position in the curriculum as the second subject, after arithmetic, and even before practical geometry. In his lectures, Van Schooten stressed the importance and universal character of algebra, which could be used for solving all kinds of problems in arithmetic, geometry or other fields. Van Schooten's emphasis on the universal role of algebra reminds us of his activity in explaining the methods of Descartes, in which algebra and manipulation of symbols play an important role. Yet a striking aspect of Van Schooten's lectures on algebra is the almost complete absence of the new notation of Descartes. He took the sign for equality from Descartes, but for the rest he used the cossic symbols for the unknowns instead of the Cartesian x. Unlike Descartes, Van Schooten did not use a symbolic notation for known undetermined quantities at all. Instead he preferred to deal with concrete numerical examples, as was usual in the earlier algebraic tradition.

Van Schooten was not the pioneer who explained the new Cartesian notation in the vernacular to a general audience. This was left to Gerard Kinckhuysen, Abraham de Graaf, and other authors in the decades after Van Schooten's death. Van Schooten's traditional approach may have been motivated by more than one reason. In the literature on algebra in the vernacular, only the cossic symbolism was used, and Van Schooten may have wanted his lectures to agree with the literature that his audience could read. Van Schooten was also aware of the difficulties of the new system, perhaps more than anyone else. In the first decades after the publication of the *Géométrie*, it was a difficult challenge for mathematicians to work with the new symbolic language of Descartes. Hence, by using the cossic symbols Van Schooten may have wanted to avoid such difficulties for his students in the Duytsche Mathematicque.

The tension between the scholarly and the more practical approach is also visible in Van Schooten's lectures on fortification. At first sight, his course in fortification seems to have a practical orientation. For example, he teaches a procedure for staking off a fortress in the field by means of instruments, and he explains how local circumstances such as rivers should be taken into account in the design of a fortress. But a closer analysis of Van Schooten's mathematical design of a fortress reveals its complexity and theoretical nature. Van Schooten had developed his own method for the fortification of a regular polygon by adjusting the method of his father to his own views. He propagated this method in his lectures in the Duytsche Mathematicque as well as in private lectures to Christiaan Huygens. The method was different from that of other authorities on fortification such as Marolois, Goldmann and Freitag, and Van Schooten noted these differences to his students. Van Schooten's method for the fortification of irregular polygons is even more original, and strikingly different from the methods of Marolois, Freitag and Goldmann. Inspired by the ideas of his teacher Christiaan Otterus, Van Schooten developed a complicated algorithm for the construction of an irregular fortress with non-symmetric bastions. The algorithm is theoretically refined but of uncertain practical value. Van Schooten planned to publish his theories on fortification in a treatise in Latin with the title "De optima muniendi ratione". For unknown reasons the treatise never appeared, and no manuscript of it has come down to us.

A similar tension between the scholarly and the practical orientations can be found in Van Schooten's lectures on arithmetic. At first sight, his lectures were in the tradition of reckoning masters, with a strong emphasis on concrete examples in a mercantile context. Lectures in arithmetic in the same vein had been given in the Duytsche Mathematicque in the previous decades. But just as in the case of fortification, Van Schooten gave his own twist to the lectures. He reduced the abundance of arithmetical rules in the traditional arithmetical course, and paid more attention to rules that are generally applicable such as the regula falsi. This was of course for the benefit of the students. Moreover, Van Schooten put more emphasis on geometrical proofs of arithmetical rules than was common in the tradition of the reckoning masters. His use of geometrical figures was probably related to the ideas of Descartes, because Van Schooten used the Cartesian words "clear" and "distinct" in his lectures to emphasize the importance of figures. Here we see the more theoretical side of Van Schooten, who wanted to teach his students not only the rules for counting and calculating, but also the reasons why these rules are correct.

Van Schooten's lectures on logarithms were by far the most advanced and most scholarly part of the curriculum of the Duytsche Mathematicque. He spent a considerable amount of time on the theoretical background of logarithms and on the computation of the logarithmic tables, according to the methods of Briggs as well as Napier, although the latter was already out of date in his time. He sometimes followed the handbooks of Napier and Briggs closely, but deviated from them on several occasions.

Altogether, the investigations of Van Schooten's teaching show that he took the initiative to adapt the courses of the Duytsche Mathematicque, including the lectures by his father, to his own views and standards. The changes included new theorerical treatments of existing subjects (fortification), restructuring of existing courses (arithmetic) and the introduction of new topics in the curriculum (logarithms, perspective). The exclusively practical orientation of the Duytsche Mathematicque in the time of Simon Stevin also changed under Van Schooten to an approach where practice was firmly based on theory. Thus the programme of the Duytsche Mathematicque was not static, as historians have hitherto believed, but it showed important developments over time.

General conclusion

CHAPTER 11

General conclusion

In the previous chapters we have encountered Frans van Schooten's activities in early modern mathematics in various roles: as a translator and commentator of the work of Descartes, as a teacher instructing craftsmen in the vernacular, as a draughtsman, as a private teacher, as an editor preparing his own works and the works of others for publication, and as an author of his own treatises. We have also seen him operating in various circles and networks: maintaining relations with Descartes, exploring with university students the joys of Cartesian geometry in an academic setting, negotiating on the publication of mathematical treatises with printer-publishers, and lecturing in the vernacular to a non-academic audience at the Duytsche Mathematicque. It is now time to make some connections to the broader themes sketched in the introduction and to draw a final picture of Frans van Schooten.

Van Schooten and the themes of the introduction

Early modern mathematics

The young Frans van Schooten received his mathematical education in Leiden. He was introduced to the different ways mathematics was taught and practiced in the seventeenth century by three teachers. With his father, who was professor at the Duytsche Mathematicque, Van Schooten explored the world of the practical mathematics of fortification and surveying in the field. The scholar and professor Jacob Golius introduced him to the world of learning and to scholarly, academic mathematics. With Golius, Van Schooten studied the treasures of the Ancients as well as the more recent mathematics in the work of Viète. Van Schooten's third teacher, the German Christiaan Otterus, had a theoretical interest in practical subjects such as fortification, and he therefore formed a bridge between the two other teachers Van Schooten sr. and Golius. The interest in both theory and practice remains visible in later works by Van Schooten, especially in his *Exercitationum mathematicarum libri quinque*.

René Descartes was the fourth and most influential mentor of Van Schooten, with a decisive impact on his work and career. The two met for the first time in or just before 1636 and kept in touch until Descartes's death in 1650. The mental capacity of the French thinker and the clear way in which he was able to articulate his thoughts made a profound impression on Van Schooten, to such an extent that he eventually became one of the most stalwart disciples of Descartes in the field of Cartesian geometry. Van Schooten was interested in the technical mathematical aspects of Descartes's mathematical work only. He showed little or no concern with the foundational issues which were relevant to the *Géométrie*, nor with Descartes's philosophical work in general.

In 1646, when he became professor of the Duytsche Mathematicque, Van Schooten had an ambitious and diverse agenda for the upcoming years. He was putting the final touch on his new edition of the collected works of Viète and had almost finished his treatise on conic sections. For the next years he envisaged following in the footsteps of the humanists by restoring the lost work on plane loci by Apollonius. He also planned to put his views on fortification in print. At that time, the translation of the *Géométrie* of Descartes seems to not have been on his mind yet.

Soon after 1646, this project would take most of his time and delay his other projects for years. Descartes had originally published the *Géométrie* in 1637 as one of the three essays connected to his philosophical work *Discours de la méthode*. In 1644, the *Discours de la méthode* and the two other essays on optics and meteorology had appeared in a Latin edition, without the *Géométrie*. In 1649, Van Schooten's translation of the *Géométrie* established the treatise as a separate mathematical work on its own, rather than an appendix to the *Discours de la méthode*. After 1646 Van Schooten spent much of his energy in making the mathematical ideas of Descartes available to a learned public. We can gain an impression of his private scholarly lectures by the treatise *Principia matheseos universalis*, which is based on Van Schooten's lessons on symbolic algebra. These private lectures were intended to be an introduction to the *Géométrie*.

At the same time, Van Schooten was lecturing to an "illiterate" audience of craftsmen as professor at the Duytsche Mathematicque. Van Schooten taught the various subjects in cycles of a few years, and he reworked and refined his lectures each time. A striking feature of these lectures in the Duytsche Mathematicque is the fact that no textbooks were used. Thus, the lectures were not a recapitulation of the main ideas of one or more printed books, but carefully devised courses which had no precedents. Moreover, the lecture notes on the various subjects contained hardly any references to textbooks. The only extensive references to books made in Van Schooten's lectures are to the works on logarithms by Napier and Briggs, which are primarily theoretical treatises rather than textbooks. To sum up, Van Schooten's activities give an impression of the energetic ways in which early modern mathematics was taught and practiced in Leiden.

The Dutch Republic

The political and socioeconomic structures in the Dutch Republic influenced the way mathematicians shaped their careers. Career paths of mathematicians in the Republic differed from those of mathematicians in other European countries where the royal courts had a major role in the patronage of mathematicians. The best known example is Galileo Galilei, who became the court philosopher and protege of the Grand Duke of Tuscany. Biagioli has shown how Galilei actively delineated a plan aimed at a career at the court.[1] Van Schooten's teacher Otterus, who was originally from Ragnitz, Prussia, returned to Germany in 1647 to become the court mathematician of Frederick William, the Elector of Brandenburg. In the Republic, the virtual absence of a court culture and the presence of a burgher culture lessened the career possibilities for mathematicians considerably, as patronage was not common. Only Simon Stevin managed to obtain a position at the stadholderly court.

Due to the weak development of patronage, the university was a welcome possibility for employment for a mathematician, perhaps the best possibility which was available, and thus we can understand why Van Schooten put so much effort in obtaining his university position. His appointment as professor of the Duytsche Mathematicque in 1646 was the result of a well-considered strategy of the Van Schooten family which had its roots in the ten years before. The first step was to get Frans jr. accepted as a temporary substitute for his father as professor of the Duytsche Mathematicque. This was achieved by 1635. The second step was the well-organized lobby initiated by Van Schooten jr. directly after the death of his father. He utilized his network to put pressure on the men who made the final decision, namely the burgomasters and curators of the university. Van Schooten sought the support of Huygens and by the intermediary of Descartes, he had access to princess Elisabeth who was in contact with one of the curators. As a result, Van Schooten was successful in obtaining the position.

Dissemination of knowledge

Van Schooten was involved in the dissemination of knowledge at various levels: by the publication of books, by his lectures at the Duytsche Mathematicque and by the private lectures he held for his students. Leiden had a well-developed infrastructure for spreading knowledge in different ways to different audiences. The university attracted Dutch and foreign students, resulting in a permanent presence in the city of a population of young people who were eager to learn. A significant part of Van Schooten's knowledge transfer took place within the realm of the university. He gathered students around him with whom he studied topics in Cartesian geometry. Van Schooten's lectures at the Duytsche Mathematicque also contributed to the dissemination of mathematical knowledge to a different audience consisting of "illiterate" craftsmen and the like.

Leiden was a city of books,[2] with a good infrastructure for publication. Books were bought and read, written and published by the learned academic community, and books

[1] [Biagioli, 1994].
[2] [Bouwman, 2008].

were produced in the city. Leiden had several printer-publishers who targeted the academic community. The printing house of Elzevier is best known, but other printers such as Maire published academic works as well.

Van Schooten was already involved in book production at the age of twenty when he drew the figures for Descartes's essays. Van Schooten considered it very important to bring work into print and to publish ideas. Throughout his career, he helped other mathematicians, and also students of mathematics, to publish their work. His help went further than just encouragement. He facilitated the practical preparations by contacting printers, editing the work of his students and preparing the mathematical figures. The most important results of these efforts were the two editions of the *Geometria* (1649 and 1659-1661), which included treatises by Van Schooten's students.

Not all projects initiated by Van Schooten made it to the printing press in the way he wanted. For example, one of his publication projects was abandoned in 1653, namely the project of publishing a Latin translation of Descartes's *Dioptrique* by the Louvain professor Van Gutshoven together with a treatise on dioptrics by Huygens. Although the printer Maire was positive towards the project, the book was never printed. Another project which at first did not work out in the way Van Schooten wanted was the printing of the Dutch version of his own *Exercitationum mathematicarum libri quinque*. The printing house Elzevier published the Latin version, which was anticipated to have a commercial benefit, but declined to print a Dutch version due to the supposed lack of commercial interest.[3] Thus the Latin version was published first. The Dutch version only became available several years later, with the Amsterdam printer Van Goedesbergh, who had experience in the printing and distribution of mathematical treatises in the Dutch language.

Van Schooten and contrasts

My final characterisation of Van Schooten will be inspired by the different contrasts which he embodied in his person and his work. I opened this thesis with a quotation on the contrast between the erudite and learned professor Van Schooten in his robe, while he delivered the lectures at the Duytsche Mathematicque to craftsmen who had just left their shops and were in their working clothes. This is the most striking contrast in the activities of Van Schooten in our modern eyes, and the quotation shows that the contrast was felt in the seventeenth century as well. Yet, at the same time, I have not come across indications that Van Schooten was not accepted as a professor in academic circles because of his lecturing to an "illiterate" audience.[4] Instead, it seems that during Van Schooten's professorship the Duytsche Mathematicque became gradually more accepted within the structure of the university. Van Schooten's pay fit into the range of extraordinary professors and his lectures were included in the time-tables of the university.

Van Schooten embodied a tension between a desire to explore new ideas in mathematics on one hand, and a predilection for tradition and conservatism on the other hand. The

[3] On the relation between printers and scholars in the first half of the seventeenth century see [Netten, 2012].

[4] There are indications that the first professors at the Duytsche Mathematicque were not fully accepted within academia, see [Wreede, 2007, 29].

contrast between new, innovating ideas and conservatism can be observed in his activities at the Duytsche Mathematicque, besides the tension between a more theoretical and scholarly approach to mathematics and a more practical orientation.

His innovations at the Duytsche Mathematicque included the revision of existing subjects such as arithmetic and fortification and the introduction of new subjects in the curriculum such as logarithms and algebra. The basic elements from the beginning of the century were still visible, but he broadened the curriculum to include new topics such as perspective, gnomonics, logarithms and algebra. Van Schooten adapted the contents of the lectures at the Duytsche Mathematicque to his own views. He increased the role of theory in his lectures in order to simplify the courses for his students. In his arithmetic course, for instance, he stripped the course of the overwhelming number of traditional arithmetical rules which were essentially variants of the rule of three. The reduction of the number of useless rules made the life of his students easier. The introduction of algebra has to be considered an innovation as well and Van Schooten praised the subject in his lectures on arithmetic. He favoured an algebraic approach to problem solving in arithmetic and in geometry, because the algebraic method was general, and a powerful tool for problem solving, provided that the method was well understood and properly used.

Yet, the course of algebra also reveals the conservative side of Van Schooten. At first sight, his use of algebra might be viewed as a bridge between his scholarly work on Cartesian mathematics and his teaching activities at the Duytsche Mathematicque. However, this impression is misleading. Of course there was a difference in the level of algebraic manipulation, because Cartesian geometry required a more advanced level of algebra than the elementary problems that were treated in the Duytsche Mathematicque, and which often led to a linear or quadratic equation in only one variable. But more importantly, Van Schooten used different symbolic notations for his different audiences. In his work on Cartesian mathematics, directed at a scholarly audience, Van Schooten adhered to the Cartesian notation using letters, with x and y reserved for unknown quantities and the letters of the beginning of the alphabet for known but indeterminate quantities. In his algebra lessons at the Duytsche Mathematicque he did not employ the Cartesian notation at all, but instead he used the older cossic notation in which only the powers of the unknown are indicated by different symbols or symbol combinations, and no symbolism exists for known indeterminate quantities. This difference may seem striking to a modern mathematician, because the cossic notation was complicated, and the easier Cartesian notation using exponents (such as x^3) gives more insight in the subject. Thus, Van Schooten introduced a new subject, but the new subject was implemented according to traditional methods, in agreement with the traditional notation that was used in most early seventeenth century works on algebra in the vernacular.

A contrast between conservatism and innovation can also be observed in Van Schooten's attitude towards Cartesian geometry, and in the way in which he dealt with the legacy of Descartes. Van Schooten sympathized with the new mathematical ideas of the *Géométrie* and also with the person of Descartes, whom he greatly admired. As a propagator of Cartesian geometry Van Schooten contributed to the dissemination and understanding of the new ideas by means of his Latin editions of the *Géométrie* and his private teaching in Leiden. Yet Van Schooten identified himself to such an extent with the cause of Cartesian geometry as stated in the *Géométrie* that he was unable to critically reflect on the mathematical work of Descartes. Eventually Van Schooten became a frenetic guardian of Cartesian mathematical ideas, and in this regard his attitude can be qualified as conservative.

I have demonstrated this conservative side of Van Schooten in my case study of the Pappus problem. Descartes had given an incomplete solution, which consists of one conic section only, whereas the complete solution consists of two different conic sections. In the debates on the missing second conic solution in 1648-49 and in 1656-59, Van Schooten chose the side of Descartes. In the end, when Van Schooten could not deny the existence of the second conic solution anymore, he unconvincingly tried to reconcile the original text of the *Géométrie* with the second conic solution.

The same attitude of Van Schooten can also be observed in his relation with Huygens. Whenever Huygens wanted to start a critical investigation of the work of Descartes, Van Schooten tried to convince him to abandon the subject in advance. Such an investigation seemed useless in the eyes of Van Schooten, because the great mind Descartes, who reasoned so carefully, could not have erred.

On the other hand, Van Schooten also helped to pave the way beyond the *Géométrie* by the publication of works by his students, such as the letter by Van Heuraet[5] on the rectification of curves in the *Geometria (1659)*. This letter crosses the demarcation lines of geometry that had been defined by Descartes. Descartes believed that the ratio between a straight line and a curved line could not be known and this idea was fundamental in his classification of curves into geometrical and non-geometrical (mechanical) curves.[6] This led to the peculiar situation that in the *Geometria*, Van Schooten strictly adheres to the ideas of Descartes, while his students go beyond these ideas. Thus Van Schooten contributed to innovations in mathematics indirectly, by his publication of the work of his students.

[5] [Heuraet, 1659].
[6] [Bos, 1981, 314-315].

Appendices

APPENDIX A

Sixteen early-modern mathematical manuscripts of the University Library in Groningen

Sixteen mathematical manuscripts in Groningen have been attributed to the three Van Schooten mathematicians who were professors of the Duytsche Mathematicque in Leiden in the period 1615–1679. These sixteen manuscripts belong to the special collections of Groningen University Library (UBG) and have the current shelfmarks Hs 107 – Hs 112 and Hs 435 – Hs 444.[1]

The most recent description of these manuscripts is in the 1898 catalogue of manuscripts by Hajo Brugmans.[2] Just like other nineteenth century catalogues, this catalogue has some serious shortcomings and does not meet the present standards of manuscript description. For instance, Brugmans made a distinction between manuscripts written in

[1]Thirteen of these manuscripts are in the digital collection of University of Groningen Library. Hs 108-112, Hs 435-438 and Hs 441-444 are accessible via the portal on `http://facsimile.ub.rug.nl/cdm` (retrieved 15 September 2013).

[2]See [Brugmans, 1898, 42-43 and 243-244]. The copy in the reading room of UBG contains additional handwritten notes and is available on `http://facsimile.ub.rug.nl/cdm/compoundobject/collection/boeken/id/3197` (retrieved 4 August 2013). Hajo Brugmans (1868–1939) was the head of the manuscript department of UBG `http://www.historici.nl/Onderzoek/Projecten/BWN/lemmata/bwn1/brugmans` (retrieved 4 August 2013).

Latin and in Dutch, and put some manuscripts in the wrong category.[3] Moreover, Brugmans only provides a very general description of the topics treated in the manuscript, and in some cases his description does not agree with the contents of the manuscript because his knowledge of the history of mathematics was insufficient.[4] The shortcomings and inaccuracies of the 1898 catalogue can still be noticed today, because the information in the online catalogue of Groningen University Library is based on Brugmans.[5]

In this present appendix I will first discuss the provenance of the sixteen manuscripts, and show that they were once in the possession of the Baart de la Faille family. Furthermore I will provide an inventory of the sixteen manuscripts. For easy reference, I have included all references to published work in the main description instead of in footnotes. The inventory will reveal that at least two manuscripts, namely Hs 438 and Hs 440, cannot be directly linked to the three Van Schooten mathematicians. As Brugmans attributed all sixteen manuscripts to Van Schooten, the manuscripts have been known as "the Van Schooten collection", although a better name would have been "the Baart de la Faille collection", after the donor of the manuscripts.

A.1 Provenance

Acquisition by the university

In 1833, fifteen of the sixteen manuscripts were mentioned for the first time in the catalogue of the Groningen University Library by Van Eerde.[6] The previous catalogue of 1758 does not mention any of the manuscripts,[7] so the manuscripts must have been acquired by Groningen University between 1758 and 1833.

The information in the 1833 catalogue by Van Eerde has been listed in table A.1. His brief descriptions of the manuscripts allows for a partial identification.[8] The folio manuscript attributed to Frans van Schooten includes worked out questions taken from Diophantus, so this manuscript must be Hs 439.[9] Because two quarto manuscripts attributed to Petrus van Schooten treat Euclid's *Elements* and Guido Ubaldo del Monte's *Planisphaeriorum universalium theorica*,[10] these manuscripts can be identified as Hs 111 and Hs 107 respectively.

[3]Example: Hs 111 has a Latin title, but the main text is written in Dutch. Nevertheless it is categorized as a Latin manuscript [Brugmans, 1898, 43].

[4]For instance, he misinterpreted the title on the binding of Hs 443. The title mentions *De Cos Rekening* (*The calculation with cos*), but in [Brugmans, 1898, 244], Brugmans describes the content of the manuscript as "De Cosinus rekening" ("the cosine calculation"). In the early modern period, "cos reekening" meant algebra, and the word *cos* comes from the Italian *cosa*, meaning the "thing", or unknown quantity. In northern regions like Germany and the Netherlands, the art of algebra therefore became known as the "regula cos." Brugmans took the word *cos* in its nineteenth century meaning as cosine and he classified the manuscript as a text on trigonometry.

[5]Currently, 11 manuscripts (Hs 108 – Hs 112 and Hs 435 – Hs 438, Hs 440 and Hs 441) are included in the digital catalogue `http://catalogus.rug.nl` of Groningen University Library (retrieved 3 August 2013).

[6][Eerde, 1833, 299 and 306].

[7][Offerhaus, 1758].

[8]The identification is based on the information given in the inventory below.

[9]Van Eerde does not make clear whether "Frans van Schooten" designates the father (sr.) or the son (jr.).

[10][Monte, 1579].

size	author	description
in folio	Frans van Schooten	verschillende Stukken van Diophantus Alexandrinus uitgewerkt. *In charta*
in folio	Petrus van Schooten	Oplossing van verschillende Geometrische en Algebraische werkstukken, en andere wiskundige Verhandelingen, III deelen. *In charta.* Ex dono Viri Clar. Iacobi Baart de la Faille, Matheseos, Astronomiae et Physices Prof. Ord.
in quarto	Frans van Schooten	Onderscheidene verhandelingen over de Arithmetica, Algebra, Mathesis, Astronomie en Muzijk, IX deelen. *In charta*
in quarto	Petrus van Schooten	over de Elementa van Euclides, de Geographie van Varenius en over Guido Ubaldi e Marchionibus Montis Planisphaeriorum Universalium Theoria, II deelen. *In charta.* Ex dono Viri Clar. Iacoi [sic] Baart de la Faille, Matheseos et Physices Profess. Ordin.

Table A.1 – The description of the manuscripts in [Eerde, 1833, 299, 306].

The descriptions of the three folio manuscripts of Petrus van Schooten and the nine quarto manuscripts of Frans van Schooten are less detailed. Because of the size, the three manuscripts of Petrus can be identified as Hs 112, 442 and 443. Additional evidence for this identification is the fact that these three manuscripts are the only in folio manuscripts which are attributed to Petrus van Schooten in the 1898 catalogue. According to Van Eerde, the nine in quarto manuscripts by Frans van Schooten treat various topics in arithmetic, algebra, mathematics, astronomy and music. The manuscripts on astronomy and music can be identified as Hs 444 and Hs 108 respectively.

The remaining seven manuscripts cannot be completely identified. Nevertheless, all topics mentioned by Van Eerde are indeed treated in the extant Van Schooten manuscripts, so we may conclude that these seven manuscripts mentioned by Van Eerde are among the sixteen manuscripts nowadays in the collection.

Note that the 1833 catalogue thus contains a collection of fifteen Van Schooten manuscripts, whereas sixteen Van Schooten manuscripts are mentioned in the 1898 catalogue. I have not been able to explain the difference in number. One of the sixteen manuscripts may have been acquired in the period between 1833 and 1898, and may have been considered by Brugmans as part of the Van Schooten collection.

Previous owners of the manuscripts

The 1833 catalogue by Van Eerde informs us that the manuscripts attributed to Petrus were a donation of Jacob Baart de la Faille, professor of mathematics, astronomy and physics (see table A.1). A sticker nowadays present in Hs 442 confirms this donation. Van Eerde does not mention the provenance of the manuscripts attributed to Frans van Schooten.

The 1898 catalogue indicates that the manuscripts Hs 107-112 and Hs 435-444 were a gift of Jacob Baart de la Faille. For Hs 107-112 the catalogue adds that Baart de la Faille was professor of medicine.

The information in both catalogues does not completely agree. It is clear that someone called Jacob Baart de la Faille was involved in a donation of at least part of the manuscript collection, but it is unclear what his professional background was.

Between 1758 and 1833, the period in which the library acquired the manuscripts, two men called Jacob Baart de la Faille had connections with Groningen University. Jacob Baart de la Faille (1757–1823) was appointed professor of mathematics and physics in 1790, and he held this chair until his death in 1823. The second Jacob Baart de la Faille (1795–1867) was the son of the first. The son grew up in Groningen and he spent the rest of his life there. He enrolled as a student at the university in 1808, and became doctor of philosophy in 1812 and medical doctor in 1817. In 1832 he was appointed professor of medicine, and he passed away in 1867. In order to avoid confusion, I will refer to the father as Jacob the mathematician and to his son as Jacob the medical doctor. If we assume that the manuscripts were donated by one of the Baart de la Failles after they had established ties with Groningen University, the timespan in which the manuscripts were donated is reduced to the period 1790–1833.

Now the question arises which Jacob Baart de la Faille donated the manuscripts. Two arguments point in the direction of Jacob the mathematician. First, it is more likely that the 1833 catalogue is correct with regard to recent donations than a catalogue of over fifty years later. Secondly, because the manuscripts have a mathematical content, it is likely that they once were in the possession of Jacob the mathematician. Brugmans may have incorrectly identified the donor as Jacob the medical doctor because the medical doctor became a quite famous professor in Groningen and was better known than his father. Another possible scenario is the following: Jacob the doctor may have donated (part of) his father's manuscripts after his father had passed away. Anyway, it seems very plausible that the fifteen manuscripts were all in the possession of Jacob the mathematician, and were donated together to the university, maybe by intermediary of the son.

It is not known how Jacob the mathematician acquired the manuscripts. The books, maps, manuscripts and instruments of Frans sr. as well as the collection of Frans jr. were inherited by Petrus van Schooten.[11] After Petrus's death in 1679, the books and instruments were auctioned on 4 March 1680.[12] The fate of the manuscripts remains unknown until they show up in the possession of the Baart de la Failles.

A brief discussion of the life of Jacob the mathematician and the influence of his father on his education will show that both Jacob and his father are likely candidates for the acquisition of the manuscripts. Jacob the mathematician was born in The Hague in 1757. His father was also named Jacob Baart de la Faille (1716–1777), and I will refer to him as Jacob the teacher. Jacob the teacher was appointed as the first mathematics teacher at the Foundation of Renswoude in 1755. This Foundation was established by the will of Maria Duyst

[11] RAL, Heilige Geest- of Arme Wees- en Kinderhuis (HGW), toegang 519, inv. nr. 4591.

[12] The auction catalogue of the collection of Petrus van Schooten is [Haro, 1680].

van Voorhout (1662–1754), a wealthy lady who wanted her bequest to be employed for the education of poor boys. In her will she stated that the education should pay particular attention to technical skills, amongst which mathematics played an important role.[13]

Jacob the teacher was aware of the career possibilities offered by a good education. Therefore he was willing to spend his earnings on the best possible education for his son. Jacob the (young) mathematician received mathematical instruction from his father and obtained his doctorate degree at the age of seventeen at Leiden University.[14] He continued his studies in Utrecht and then went abroad to Paris. After the death of his father in 1777, Jacob had to provide his own income because his father had spent all his savings on the education of his son. Jacob tried to obtain the position of his father but he was refused the position, mainly because of his age, because he was even younger than some of the students he would have to instruct. He however did obtain the position of city lecturer in mathematics of The Hague. In 1790 Jacob the mathematician was appointed professor in Groningen, a chair he held until his death in 1823.[15]

The connections of Jacob the teacher with the Foundation of Renswoude and his efforts to provide a good mathematical education for his son, make him also a good candidate for having acquired the Van Schooten manuscripts. In that case they were inherited by Jacob the mathematician.

A.2 Methodology

For my inventory of the Groningen manuscripts, I have used as my point of departure Jan van Maanen's approach for describing a collection of seventeenth century mathematical manuscripts in Leiden.[16] I have compiled the following inventory for an audience of historians of mathematics, or more general historians of science, and therefore the emphasis will be laid on the contents of the manuscripts. Thus the present inventory is not and does not pretend to be a codicological study, although a concise physical description of each manuscript will be given. As has been noted by Jan van Maanen, an inventory of mathematical manuscripts should have a subject classification in order to facilitate the search for relevant manuscripts for the researcher in the history of mathematics.[17]

Below I list the various categories, together with a short description of the content of a category. I will slightly deviate from Van Maanen's method with respect to two items. As shown above, the manuscripts were acquired by the library at the same time[18], and all manuscripts were owned by Jacob Baart de la Faille the mathematician. Therefore there is not a separate category "provenance" in this inventory. Secondly, Van Maanen used a systematic category on the state of writing, in which he distinghished between a draft

[13] [Zuidervaart, 2012], [Gaemers, 2004].

[14] His thesis was on the so-called exhaustion method [La Faille, 1774]. According to Danny Beckers, this thesis was one of the very few original and novel work that Dutch mathematicians produced during the eighteenth century [Beckers, 2003, 31].

[15] [Gaemers, 2004, 177-180].

[16] [Maanen, 1987, 147-241, especially 149-156].

[17] [Maanen, 1987, 149].

[18] Maybe one manuscript was acquired after 1833 and before 1898, see above on page 257.

version and a final version. Because it is hard to make such a distinction, and because the choice can be very subjective, I decided to omit this general category. In particular cases where the writing can be identified as draft or final version, this information is added, and extremely dense writing will also be noted.

The rest of the categories will be based on Van Maanen's method. This means that an item of the inventory will be described in the following way:

current shelfmark

author The person who has conceived the text. If the author is also the scribe, we indicate this by "(autograph)" after the author's name; in that case no scribe is mentioned. Brackets around the name of an author indicate that the manuscript is attributed to the author on the basis of evidence which is strong but not absolutely conclusive.

scribe The person who has actually written the manuscript.

contents An indication of the contents. Further information is presented under the heading systematic classification.

language If more than one language is used, a note indicates how much is written in each language.

date If the date is mentioned in the manuscript, a reference is given in parentheses. In case the date is not explicitly mentioned, other methods have been used. In such a case the arguments leading to the date are presented in the section **evidence**. Sometimes the date is concealed in the text of the manuscript (in a mathematical problem for instance). Sometimes a *terminus post quem* (or less frequently a *terminus ante quem*) can be derived if the manuscript refers to persons or books.

origin The location where the text was written. In some cases, the origin is explicitly mentioned, in other cases the origin is derived from the content of the manuscript.

catalogue List of the catalogues in which the manuscript is described.

sec lit List of secondary literature in which the manuscript is mentioned.

physical description The information on the basis of the physical appearance of the text:

- The form in which the manuscript is preserved.
- The total number of leaves in the manuscript, including blank pages. Enclosed loose leaves are not taken into account, unless otherwise stated.
- The number of written pages. A page is counted as written page if it contains a written text or a drawn illustration. Pages with only drawn margins, and lacking any text or illustrations, are not considered to be written pages. Enclosed loose leaves are not taken into account, unless otherwise stated.

- Information about the foliation (every other page numbered) or pagination (every page numbered). If the foliation or pagination is not in Arabic numerals, this is explicitly mentioned.
- The dimensions of the paper.
- Notes on the state of the writing if appropriate.
- Information about the writing material.
- Information about illustrations.
- The date and the type of binding.
- The title on the spine. Several manuscripts bear a title written in a late eighteenth or nineteenth century hand.[19] This title is quoted, and the symbol "/" is used to indicate a line break. Intralinear additions are indicated by pointed brackets: <a>. Interlinear additions are indicated by ≤ a ≥ in case "a" is added under the writing line, and by $\leqslant$ a $\geqslant$ in case "a" is added above the writing line. For the readibility, crossed out parts of the text are underlined: $\underline{a}$. Abbreviations and omissions have been completed between square brackets. thus: [abc] if abc is missing from the title.

systematic classification My classification is based on the classification used by Jan van Maanen, see [Maanen, 1987, 214-216] in order to facilitate the researcher who is looking for manuscripts on a particular subject. An overview of all categories in the classification is found in section A.4.

type/purpose/function of the manuscript A short description of the nature of the manuscript, the reason why it was written and the (intended) function of the manuscript.

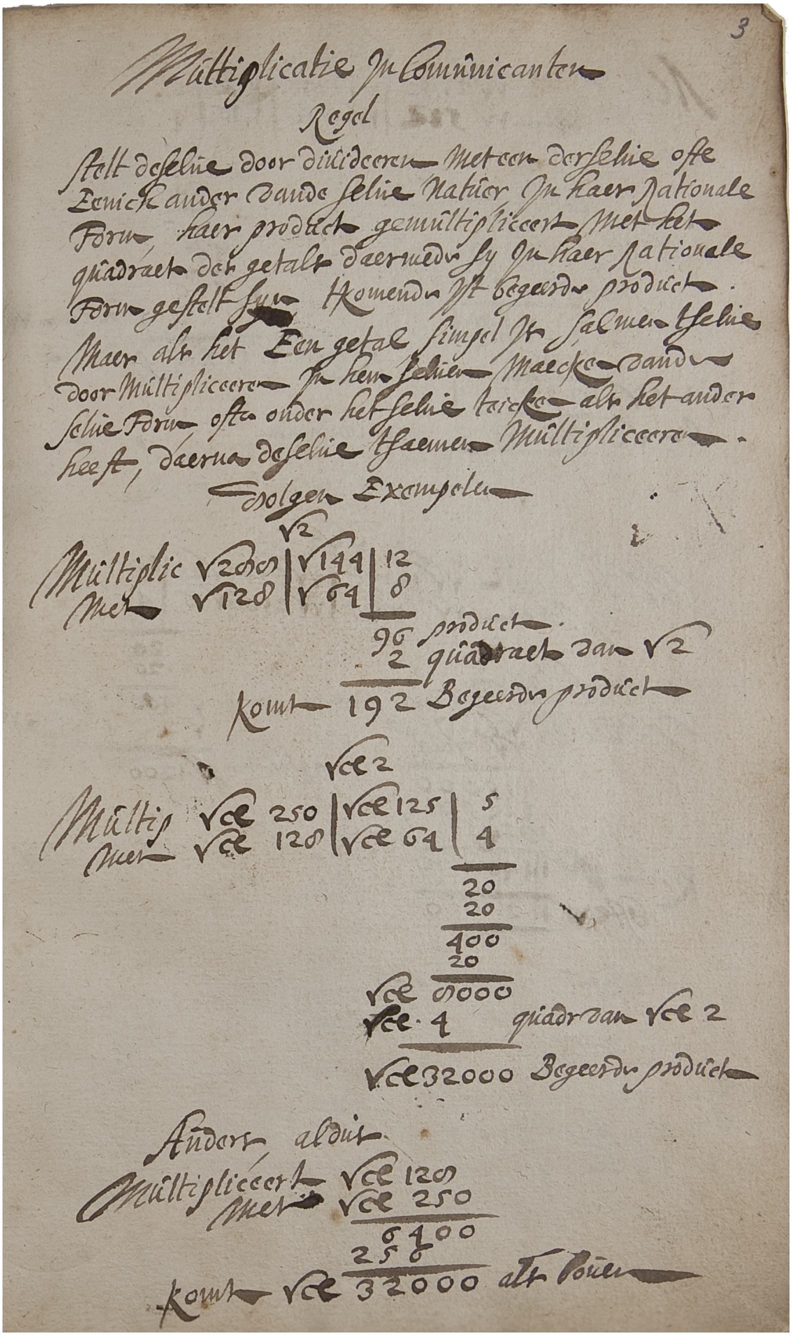
names mentioned The names of persons mentioned in the text.

evidence Arguments for statements made above. Statements supported by additional evidence are marked with [+].

notes Additional information on the manuscript which does not fit in the other categories.

The three mathematicians Frans sr., Frans jr. and Petrus van Schooten have distinctive handwriting. Samples of the handwriting are shown in figure A.1 (Frans van Schooten sr.), figure A.2 (Frans van Schooten jr.) and figure A.3 (Petrus van Schooten).

[19] These titles were already mentioned by Bierens de Haan in 1877, [Haan, 1878, 263-265]. From the account of Bierens de Haan I conclude that he had only seen Hs 108 by himself, for information on the other manuscripts he relied on the communications of an acquaintance. It is not known who added these titles to the manuscripts. Since the titles for some cases do not meet the content, it is likely that the titles were added by someone with feeble mathematical knowledge, probably a librarian or an auctioneer.



Multiplicatie In Communicanten
Regel
Stelt deselue door diuideeren met een derselue ofte
Eenich ander vande selue Natuer In haer Rationale
Form, haer product gemultipliceert met het
quadraet des getals daermede sy In haer Rationale
Form gestelt syn, komende Is t begeerde product.
Maer als het Een getal simpel Is salmen tselue
door Multipliceeren In hem seluen maecken vande
selue Form ofte onder het selue teicken als het ander
heeft, daerna deselue tsaemen Multipliceeren.
Volgen Exempelen

√2
Multiplic √288 | √144 | 12
Met √128 | √64 | 8
96 product.
2 quadraet van √2
Komt 192 Begeerde product

√ce 2
Multip √ce 250 | √ce 125 | 5
Met √ce 128 | √ce 64 | 4
20
20
400
20
√ce 8000
√ce 4 quadr van √ce 2
√ce 32000 Begeerde product

Anders aldus
Multipliceert √ce 128
Met √ce 250
6400
256
Komt √ce 32000 als vooren

Figure A.1 – Handwriting of Frans van Schooten sr., taken from UBG, Hs 443, f. 3r.

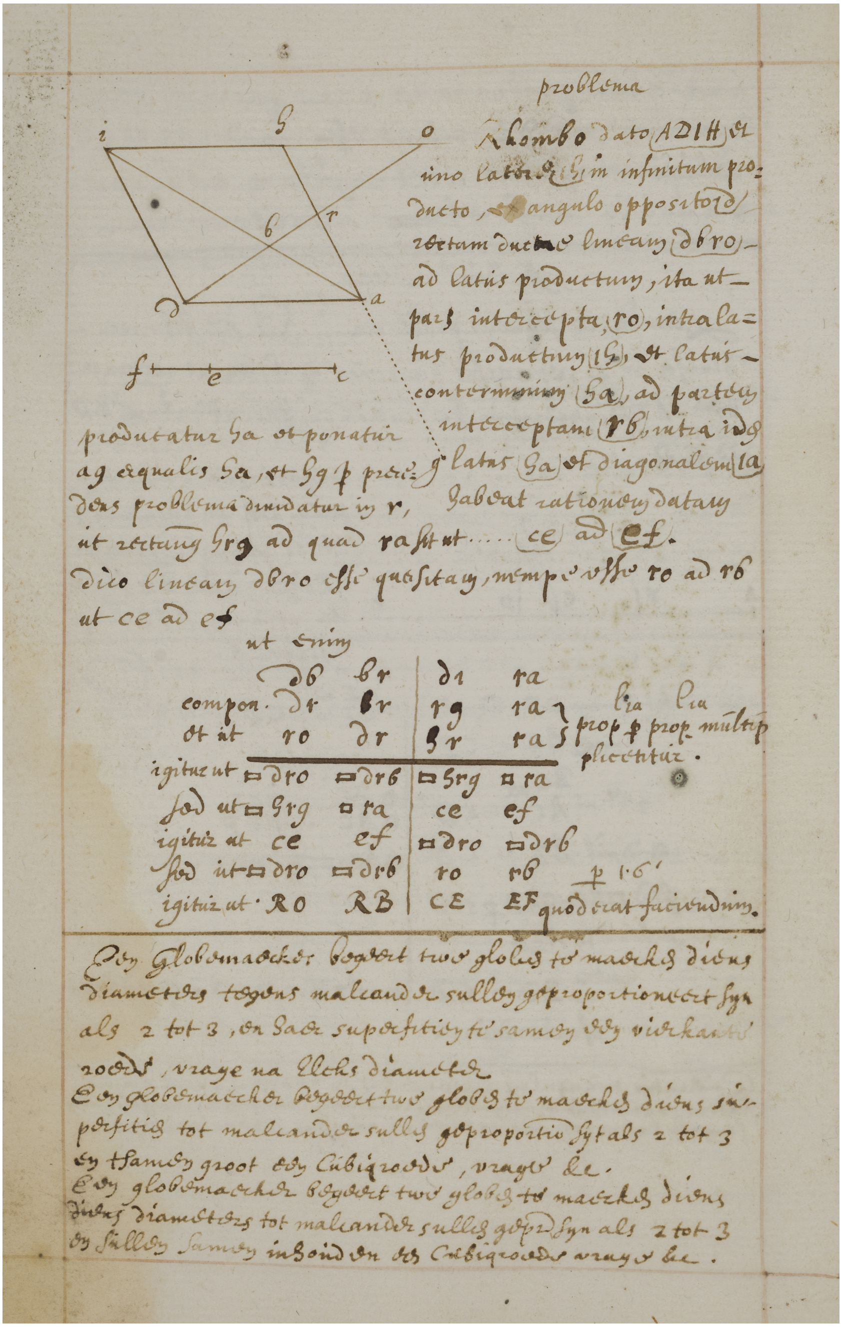
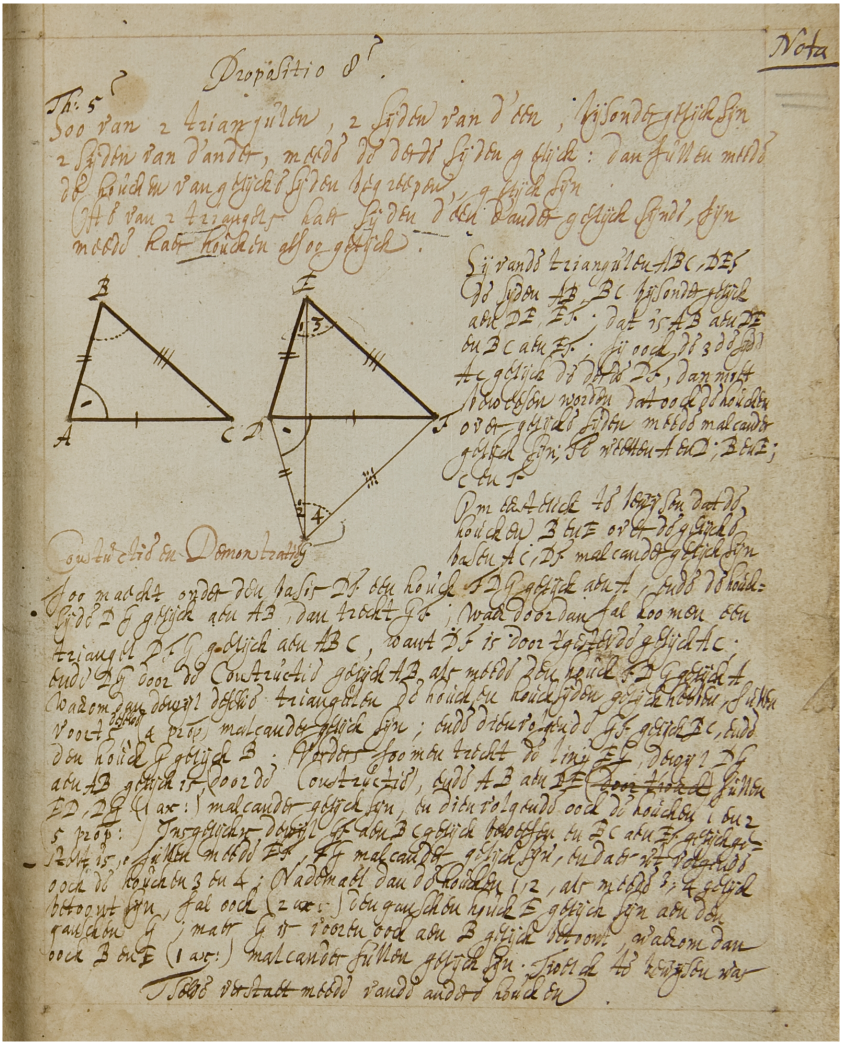


Figure A.2 – Handwriting of Frans van Schooten jr., taken from UBG, Hs 108, f. 12v.


Nota

Propositio 8.

Th: 5. Soo van 2 triangulen, 2 zijden van d'een, bijsonder gelijck zijn 2 zijden van d'ander, mede de derde zijden gelijck: dan sullen mede de houcken van gelijcke zijden begrepen, gelijck zijn.

Ofte van 2 triangels haer zijden d'een d'ander gelijck sijnde, sijn mede haer houcken alsoo gelijck.

Constructie en Demonstratie

Figure A.3 – Handwriting of Petrus van Schooten, taken from UBG, Hs 111, f. 16r.

A.3 Inventory

Hs 107

author Bernhard Varenius, Guidobaldo del Monte

scribe Petrus van Schooten (f. 62v.)

contents A selection taken from Bernhard Varenius's *Geographia generalis in qua affectiones generales telluris explicantur* [Varenius, 1650], and a copy of Guidobaldo del Monte's *Planisphaeriorum universalium theorica*, which was published in 1579 and reprinted in 1581, see [Monte, 1579] and [Monte, 1581].

language Latin

date 1655 (f. 2r. and f. 62v.) [+]

origin The Hague (f. 62v.)

catalogue mentioned in [Eerde, 1833], described in [Brugmans, 1898, 42]

sec lit none

physical description

- 1 bound volume
- 61 leaves
- 60 written pages
- nineteenth or twentieth century foliation in pencil
- paper 153x97 mm, [Brugmans, 1898, 42]
- black and red ink
- pen drawings
- original marbled paper binding, rebound in a modern cardboard binding
- no title on the spine; on original cover is a note attached with a description of the contents: *van Petrus van Schooten* $\leqslant$ *a^o 1661–1679 / Prof. te Leiden. Ao 1653 / verschillende verhandelingen / geschreven Ao 1655 / 1. Figurae, ceteraque quae desiderantus in libro 2 / Geographiae Bernhardi Varenii / 2. Guidi Ubaldi e Marcheonibus Montes / Planisphaeriorum Universalium Theoria / No 11*<5>

systematic classification
applications: astronomy, geography
geometry: dimension three
instruments: for astronomical purposes

type/purpose/function of the manuscript copy of printed text

names mentioned Apollonius, Frederico Commandino, Euclid, Gemma Frisius, Guidobaldo del Monte ("Ubaldi e Marchionibus"), Pliny the Elder, Ptolemy, Johannes Regiomontanus, Juan de Rojas, Petrus van Schooten, Willem van Straaten ("Strateni") [+], Theodosius, Bernhard Varenius

evidence The titles of the copied works are given on f. 2r. and f. 51r. The work of Del Monte was copied on 1 and 2 September 1655 (f. 62v.); the parts from Varenius were copied in 1655, but no month is known.

notes The pages f. 39r. and 39v. are glued together and two other pages are included in between.

A recipe for a medicine against fever formulated by "Strateni" is included on f. 1r. This "Strateni" is probably Willem van Straaten, medical doctor of the city of Utrecht and the first professor of medicine of Utrecht University, [Baumann, 1952].

Hs 108

author various authors (see the contents); some material has been conceived by Frans van Schooten himself

scribe Frans van Schooten jr. (f. 1r.) and Godefroy van Haestrecht [+]

contents Various notes on mathematical subjects, for a fair part copied from existing works, and for a small part his own work. Two documents are inserted in the binding, one at the beginning and one at the end. The first is a pamphlet entitled *Problema astronomicum* issued by Johan Stampioen in 1638 [Stampioen, 1638], with notes on it written by Frans van Schooten. The second is an autograph of Godefroy de Haestrecht which deals with page 378 of *Géométrie*.

The mathematical notes include notes on the mathematical work of Descartes (f. 1r.-v., f. 4v., f. 9r., f. 13r.-14r., f. 20r., f. 51v.-53r., f. 54v., f. 55v.-60v., f. 94r.)., various notes from lessons of Jacob Golius (f. 5r.-6v., f. 20r.-21r., f. 29r., f. 30r.-31r., f. 56r.) and Christiaan Otterus (f. 12r., f. 13r., f. 25v., f. 59r., f. 92r.-93r.). The copied fragments include parts from the following works: Viète's *Apollonius Gallus* [Viète, 1600] (f. 21v.-22r.); Archimedes's *On Spirals* (f. 26r.-29v.); Oughtred's *Arithmeticae in numeris et speciebus institutio: quae tum logisticae, tum analyticae, atque adeo totius mathematicae quasi clavis est* [Oughtred, 1631] (f. 31v.-41r.); Stifel's *Arithmetica integra* [Stifel, 1544] (f. 41r.-48v.); Bachet's edition of Diophantus's *Arithmetica* (f. 44v.-48r.); Descartes's *Compendium Musicae* (f. 60r.-83v.); work of Albert Girard (f. 95v.-96v.); Weland's *Strena Mathematica* [Weland, 1640] (f. 98r.-v.); Stampioen's *Algebra ofte nieuw stel-regel* [Stampioen, 1639] (f. 101v.-102r.); Simon Stevin's *Arithmetique*, [Stevin, 1585a] (f. 103r.); Francesco Maurolico *Arithmeticorum libri duo* [Maurolico, 1575], (f. 103v.).

language Latin, French, Dutch. The majority of the text is in Latin, some notes related to Descartes are in French, and a few notes are in Dutch.

date between 1632 and 1641 [+]

origin Leiden

catalogue [Eerde, 1833], [Brugmans, 1898, 42]

sec lit [Haan, 1878, 263-270], [Descartes, 1908, 635-647], [Wardhaugh, 2008, 23-34]

physical description

- 1 bound volume
- 103 leaves
- 206 written pages
- nineteenth or twentieth century foliation in pencil
- paper 225x150 mm [Brugmans, 1898, 42]
- writing in black ink, the margin is drawn in red ink, some notes in pencil in the *Compendium Musicae*
- pen-drawings of geometrical figures and of some instruments in ink, mostly very accurately drawn
- contemporary parchment binding
- the title on the spine almost perished, only the word *manuschript* is decipherable; a note is attached to the binding in late eighteenth or nineteenth century handwriting: *In dit manuscript van Franciscus a Schooten A.o 1632 5 Decemb / zijn de Volgende Stukken begrepen. / Demonstratio Constructionis 4 ovalium / oplossing der Cubische Aequatien door de Parabel en Hyperbola / Problemata uit de Lessen Prof. Golius en Otterlo / aanmerking opde beschrijving der Parabola en Hyperbola volgens descartes / [aanmerkingen] over verscheidene Problemata van Apollonius, Archimedes/ Clavis Mathematica/ Specimen Arithmet. et Algebraicum/ de grootste gemeene maat/ Van de gebrokene getallen/ de Genesi et Analysi Potestatum/ over de wortels eener Cubische Aequatie/ Verschillende Regelen om tebepalen, welke getallen quadraat of Cubicq Zijn/ Compendium Musicae/ Doctrina Prostaphaeretica/ Trigonometria Logerithmica [sic] / No. 11 <6>.*

systematic classification
algebra: equations, cossist
analytic geometry: equation of curves
applications: astronomy, fortification, music theory, optics
classical problem: Pappus problem
arithmetic: general, decimal system, elementary operations, fractions
combinatorics: general

conic sections: general, ellipse, hyperbola, parabola
curves: folium of Descartes, ovals, spirals, other
geometry: general, constructions, dimension two, dimension three, area, volume
instruments: for mathematical constructions
logarithms: theory
number theory: divisibility, Pythagorean (Heronian) triangles, squarefree numbers
sequences: arithmetical, geometrical
trigonometry: spherical

type/purpose/function of the manuscript survey of existing knowledge; research notes; student's notes; copy of parts of printed texts

names mentioned Apollonius, Archimedes, Claude Gaspar Bachet, Isaac Beeckman, Tycho Brahe, René Descartes, Diophantus, Jacob Golius, Albert Girard, Francesco Maurolico, Menaechmus, Christiaan Otterus, Pappus, Peter Roth, Francisco de Salinas, Frans van Schooten jr., Gioseffo Zarlino, Johan Stampioen the Younger, Simon Stevin, Michael Stifel, Woldeck Weland

evidence Godefroy van Haestrecht is identified as the scribe of a note attached to the binding by the content of the note. The note discusses algebraic manipulations of page 378 of the *Géométrie*. In a letter to Frans van Schooten, Descartes refered to this "note of Mr Haestrecht on page 378", on which Frans van Schooten jr. had sought Descartes's advice, [Descartes, 1898, 577-578].

The manuscript contains a date on f. 1r.: "Anno 1632 5 Decembris". At that time, Frans van Schooten jr. was 17 years of age. The manuscript contains however several references to works which were published after 1632. Therefore, we take 1632 as terminus post quem. The content of the manuscript suggests that the greater part of the manuscript was written before 1641, the year in which Van Schooten traveled to France. This tour of Van Schooten is to be considered the end of his formative years. Several indications point in this direction:

- The manuscript contains material from lessons of Jacob Golius, and from lessons of Christiaan Otterus. Otterus was active in Leiden as a private mathematics instructor in the years 1634–1638. In 1639 he left the Dutch Republic, only returning in 1658 when Van Schooten was an established scholar (on Otterus see also page 25). This suggests that these writings were made during Van Schooten's formative years before 1641.
- The most recent printed work mentioned in the actual manuscript is a work of 1640, written by Weland [Weland, 1640].
- The manuscript contains fragments of Descartes's *Géométrie* which differ slightly from the printed text of 1637 (for instance, f. 4v. and f. 14r.). Moreover, in these fragments, reference is made to folios and lines, whereas other fragments which are identical to the printed text refer to pages (for instance at the foot of f. 55v. and 59v.). This shows that Van Schooten had access to a manuscript or a proof of the *Géométrie* before the *Géométrie* was actually printed.

The notes with references to folios very likely date back to before the publication of the *Géométrie*.

- The manuscript does not contain any notes on work by French mathematicians other than Descartes. During his tour in France, Van Schooten met various French mathematicians, but this manuscript does not reflect any of these contacts.

The manuscript contains a version of Descartes *Compendium Musicae*. Wardhaugh stated that this version is a second generation copy derived from Constantijn Huygens's manuscript (Leiden, University Library, Ms Hug. 29a). The Huygens manuscript dates from 1635, hence Van Schooten's copy was made after 1635; Wardhaugh dates the Van Schooten copy in 1641, [Wardhaugh, 2008, 23-24].

The manuscript contains a pamphlet published by Johan Stampioen de Jonge in 1638 in which he challenged other mathematicians to solve a mathematical problem known in the literature as the *Problema Astronomicum*. Van Schooten published a solution of the proposed problem in *Geometria (1649)* and *Geometria (1659)*. For a discussion on the history, background and relevance of this problem as well as its solutions see [Maronne, 2007, 382-387 and 425-434].

Adam and Tannery wrongly attributed this manuscript to Van Schooten sr., see [Descartes, 1908, 635-647].

notes In some instances, cossic algebraic notation is used, especially in the Dutch parts of the text (f. 49v.-51r.).

Van Schooten marked his own inventions with “FaSi” or “FaS” which stands for “Franciscus a Schooten (invenit)”. These are found on f. 13v., f. 49r., f. 93v.-94r., and f. 96r.

Hs 109

author unknown [+]

scribe unknown [+]

contents *Matheseos universalis* (f. 1r-102v.), a treatise on the science of quantities, divided in four books, treating elementary operations (Book I), ratios and proportions (Book II), rationality and irrationality (Book III), equations (Book IV) and a final chapter with 17 questions and their solutions; four chapters of a treatise called *Arithmetica* (f. 103r.-108v.)

language Latin with some notes in Greek

date end 16th century or 17th century

origin unknown

catalogue [Eerde, 1833], [Brugmans, 1898, 42]

sec lit none

physical description

- 1 bound volume
- 115 leaves
- 106 written pages
- nineteenth or twentieth century foliation in pencil; the quires were numbered by the scribe
- paper 165x105 mm
- black and red ink
- pen-drawings of geometrical figures
- contemporary paper binding, which is bound in a later binding of cardboard covered with marbled paper
- no title on spine; a separate note is attached to the contemporary paper binding with the content in a late eighteenth or nineteenth century hand: *In dit Manuscript van F. à Schooten is begrepen het volgende / Matheseos universalis Lib. 1 / Ratio et proportio, quae et quotuplex Lib. 2 / de Summetria et Assummetria Lib. 3 / de Analysi Mathematica Lib. 4 / Pars altera, de Arithmetica / No* <u>*14*</u> *<10>*

systematic classification
algebra: mathesis universalis, equations, elementary operations
arithmetic: general

type/purpose/function of the manuscript problems with solutions; research text

names mentioned Aristotle

evidence The manuscript is undated and does not give the name of the scribe. Comparison of the handwriting with that of Frans van Schooten jr. of Hs 108 and Hs 110 reveals that Hs 109 was not written by Frans van Schooten. Especially the writting of the letters "r" and "p" in Hs 109 is different from Hs 108 and Hs 110: in Hs 109 the "r" and "p" are predominantly open, whereas in Hs 108 and Hs 110 they are predominantly closed.

The author of the text is unknown. At first sight, based on the title, Frans van Schooten jr. seems to be a good candidate, since a treatise entitled *Principia matheseos universalis* based on lectures by Frans van Schooten jr. was published by Erasmus Bartholin in 1651 [Bartholin, 1651]. A closer look reveals that the content of the manuscript is very different from the *Mathesis universalis* as published by Bartholin.

Hs 110

author Pierre de Fermat, Gilles Personne de Roberval and Marin Mersenne

scribe Frans van Schooten jr. [+]

contents various parts of treatises and letters of Fermat, one letter of Roberval, and one letter containing notes by Mersenne

language French and Latin. The fragments of the treatises are in Latin, except one in French; the letters are in French, except two in Latin.

date Winter 1642–1643 [+]

origin Paris

catalogue the manuscript was probably amongst the nine in quarto manuscripts of [Eerde, 1833]; description in [Brugmans, 1898, 42]

sec lit A detailed description of the unpublished letters of Fermat is found in [Waard, 1917] and in the introduction of [Fermat, 1922]. The works of Fermat, with reference to extant manuscripts including the Groningen copies, are described and dated in [Mahoney, 1994, 415-423].

physical description

- 1 bound volume
- 26 leaves
- 44 written pages
- nineteenth or twentieth century foliation in pencil
- paper 297x197 mm
- written in black ink, some additional notes were added in pencil
- pen and pencil drawings of geometrical figures; the pen drawings are mostly very careful and accurate, but some are sketchy
- contemporary paper binding
- no title on the spine; on the cover a note in late eighteenth or nineteenth century hand is attached with information on the content: *Continentur haec Mss A P. à Schooten et F. a Schooten Fratribus. / Ad locos Planos et Solidos A.D.F. / Appendix ad Isagonen Topicam, Continent Solutionem Problema tum Solidorum per Locos / De Tangentibus Linearum Curvarum / [De] Centro Gravitatis Parabolici Conoidis / de Tangentibus Linearum Curvarum / Extrait d'une Lettre du 15 Juin Anno 1636 au R.P. Mersenne &. / Propositio per 4 puncta Parabolam describere / Problema 10e Novb. 1642 invenire Cylindrum maximi ambitus in Sphaera / No. 4*

systematic classification
algebra: construction of equations
analytic geometry: general, coordinates, equations of curves
curves: cissoid, conchoid, cycloid, quadratrix, folium of Descartes, ovals, other
conic sections: ellipse, parabola, hyperbola
geometry: dimension two, dimension three
infinitesimal calculus: centre of gravity, maxima and minima, tangents and normals, quadrature

type/purpose/function of the manuscript survey of contemporary research results

names mentioned Apollonius, Archimedes, Aristaeus, Aristotle, Beaugrand, Descartes, Diocles, Diophantus, Etienne d'Espagnet ("Mr. Despagne"), Eutocius, Fermat, Mersenne, Nicomedes, Pappus, Mr Philon, Mr Prades, Roberval, Viète

evidence Comparison of the handwriting with Hs 108 reveals that Hs 110 was written by Frans van Schooten jr.

The manuscript contains letters and treatises of the period 1636–1642. The most recent part is a letter of Fermat to Roberval dated 10 December 1642. During the years 1641–1643, Van Schooten made a journey which led him amongst others to Paris where he copied documents owned by Mersenne.

notes The manuscript was listed as Hs 110 in the 1898 catalogue with the title *Petri et Francisci van Schooten fratrum, varia arithmetica*, which does not cover the content at all. Petrus van Schooten was not at all involved in the composition of the manuscript. In the 1910s, M.C. de Waard discovered that the manuscript contains parts of the works of Fermat and fragments of hitherto unknown letters. He published his findings first in [Waard, 1917] and later in a revised version in [Fermat, 1922]. An offprint of [Waard, 1917] is sticked to the binding of Hs 110.

Hs 111

author Euclid

scribe Petrus van Schooten [+]

contents Euclid's *Elements* in Dutch

language Dutch, with some minor remarks in Latin

date ca. 1660 [+]

origin Leiden

catalogue mentioned in [Eerde, 1833], described in [Brugmans, 1898, 43]

sec lit none

physical description

- 1 bound volume
- 143 leaves
- 154 written pages
- nineteenth or twentieth century foliation in pencil
- paper 210x163 mm
- black and red ink; the definitions and propositions are written in red, the explication and the proofs are written in black ink
- pen drawings of geometrical figures
- contemporary parchment binding
- the title on the spine has almost completely perished and is unreadable; on the cover a note in late eighteenth or nineteenth century hand is attached with information on the content: *Euclidis Elementa van Petrus van Schooten No. 9.*

systematic classification
geometry: selection from Euclid's *Elements*

type/purpose/function of the manuscript survey of existing knowledge; student notes

names mentioned Euclid

evidence Comparison of the writing with Hs 107 reveals that Petrus van Schooten wrote Hs 111. The date is based on the statement by Frans van Schooten jr. in his treatise on perpective of 1660 [Schooten, 1660b, 543], to the effect that Petrus was busy translating the books of Euclid in Dutch at the time.

Hs 112

author Theodorus Craanen or Petrus van Schooten [+]

scribe unknown

contents The manuscript consists of two parts. The first part (p. I–LXIII) is an elementary text on geometry, treating the main propositions of Euclid's Elements, transformation of figures and area computations. The second part of the manuscript (p. 1–383) concerns arithmetic, problem solving using algebra and analytical geometry.

language Latin

date after 1661, probably before 1687 [+]

origin the Netherlands, probably Leiden [+]

catalogue mentioned in [Eerde, 1833], described in [Brugmans, 1898, 43]

sec lit [Maanen, 1986], [Maanen, 1987, 211]

physical description

- 1 bound volume
- 231 leaves
- 380 written pages, of which 60 in the first part and 320 in the second
- The two parts of the manuscript have a separate pagination. The first 63 pages are paginated in pencil by Roman numerals, not written by the scribe but added later, probably in the nineteenth century. The second part is paginated and had a partial signature mark in ink. The signature mark is contemporary, and marks only the first 37 sections. The pagination is in ink, in an eighteenth century hand in which also the contents of pages 382-383 are written.
- paper 330x218 mm; the first part has a visible margin line; the second part does have wider margins, but without the margin line
- black ink
- pen drawings of geometrical figures
- contemporary parchment binding
- the title on the spine has almost completely perished and is unreadable; a note is attached to the binding which reads in an nineteenth or twentieth century hand: *Specimen Problematum Algebraicorum / Sectionumque, quae in Cono effici possunt. / Auctor. P. van Schooten / Mathem. Prof. Leidensi./ No 3 <2>*

systematic classification

algebra: general, equations, construction of equations, systems of equations
analytic geometry: coordinates, equations of curves, problem of Pappus
arithmetic: rule of three and similar rules, surds
conic sections: general, ellipse, hyperbola, parabola
curves: Cartesian parabola, cissoid, conchoid
geometry: general, area, dimension two, dimension three, constructions, selection from Euclid's *Elements*, transformation of figures, volume
infinitesimal calculus: centre of gravity, maxima and minima, tangents and normals
number theory: Diophantine problems, figurate numbers
perspective: general

type/purpose/function of the manuscript overview of existing knowledge; problems together with solutions; reworking of printed text

names mentioned Apollonius, Archimedes, Bachet, Van Berckel ('Berckelio'), 'Gerrit Gerritsen Brugman', Cardano, Cardinael ('Sybrant Hanssen'), Clavius, Craanen, Descartes, Dibuad, Diocles, Diophantus, Euclid, Fermat, Hartsinck ('Hartzingius'), Herigone, 'Heusdano', Christiaan Huygens, Jacques Lefèvre d'Étaples or Jacob Faber

Stapulensis, Christoffe Ludolffs, Pappus, Nicolaas Petri, Ptolemy, Pythagoras, Van Schooten jr., Van Schooten sr., Slath, Smyters, 'Strunckede', François Viète, Samuel de Vivere

evidence The second part of the manuscript shows a strong resemblance with three other manuscripts. Two are in Amsterdam (UBA, II A 28 and II A 39) and one in Leiden (UBL, BPL 1968), see for descriptions [Dold-Samplonius, 1968, 242-250] and [Maanen, 1987, 210-211] respectively. According to Jan van Maanen, the Groningen manuscript was the source from which the Amsterdam and Leiden manuscript were derived.

The Groningen manuscript refers to p. 167 of [Schooten, 1661b], thus the manuscript was written after 1661. As Van Maanen dates the Leiden manuscript in 1687, and regards the Groningen manuscript as its source, we conclude that the Groningen manuscript was written between 1661 and 1687.

The author of the manuscript is not mentioned in the manuscript itself, However, there are two likely candidates for the authorship. The first is Theodorus Craanen. The manuscript contains a considerable number of references to unpublished work of several seventeenth century mathematicians. Of them, Theodoor Craanen is most often mentioned, which suggests that he was involved in the manuscript. Craanen, being a Cartesianist, and mostly known for his work in medicine, also had a lively interest in mathematics. The authorship of Craanen also explains the references to Strunckede, Slath and De Vivere: all three were students in Duisburg when Craanen lectured on mathematics in that city. This Duisburg connection his hitherto not been remarked.

The second candidate is Petrus van Schooten. Van Maanen considered Petrus as the main candidate for the authorship, mainly based on the facts that this manuscript came in the possession of the University together with the other Van Schooten manuscripts, and that the style of the manuscript shows strong similarities with the style of [Schooten, 1661b], the text which Petrus edited after Frans van Schooten had passed away. Another argument pointing in the direction of Petrus van Schooten is the fact that the manuscript contains questions which are also found in the manuscripts Hs 437, Hs 442 and Hs 443, which are attributed to Van Schooten sr. and Petrus van Schooten. A detailed comparison of all questions in the manuscripts Hs 437, 442, 443 and 112 and the Amsterdam manuscripts UBA II A 28 and II A 39 and the Leiden manuscript UL BPL 1968 might reveal more information on the relation between these manuscripts.

notes The person referred to as 'Berckelio' is probably the 'Berkelij' mentioned by Huygens in his notes on the method of finding normals, [Huygens, 1910, 80] and who is mentioned in Huygens's correspondence [Huygens, 1888, 242, 246, 274]. There is no consensus on the identity of Van Berkel. According to some scholars, this is Abraham van Berkel (1639/1640–1686)([Huygens, 1888, 242] although others doubt this identification [Molhuysen and Blok, 1911, 310]. I suggest another identification:

'Berckelio' is the same person as the Petrus Berckelius (Pieter van Berkel) who held a disputation on physics with De Raey in 1651 [Berckelius, 1651].

The notation of the manuscript is in Cartesian style, using the Cartesian exponent notation and the Cartesian equal sign.

Hs 435

author Frans van Schooten jr. (autograph) [+]

contents a course on the construction of logarithmic tables of Henry Briggs and Adriaen Vlacq, and on the use of logarithms

language the main text is in Dutch, some remarks are added in Latin

date 1655–1656 [+]

origin Leiden

catalogue [Brugmans, 1898, 243]

sec lit none

physical description

- 1 bound volume
- 37 leaves
- 61 written pages
- foliation in pencil; not by the author; probably dating back to the nineteenth or early twentieth century
- paper 210x162 mm
- black ink, added remarks in same hand in red ink
- pen drawings of geometrical figures
- contemporary binding of marbled paper
- no title on the spine; on the binding a note is attached, with a title in late eighteenth or nineteenth century hand: *Lessen gehouden door F. van Schooten / 9 December 1655, over de Natuur / Constructie en Gebruik der Logarithmi- / sche tafelen van Briggs & Vlacq / No $\underline{7}$ <2>.*

systematic classification
logarithms: tables, theory

type/purpose/function of the manuscript teacher's notes; reworking of printed text; instructional text

names mentioned Briggs, Philips de Croij [f. 18v., printer in Leiden], Hérigone, Adriaen Vlacq, John Napier, Ptolemy

evidence Comparison of the handwriting with Hs 108 reveals that Frans van Schooten is the scribe of the manuscript.

At the inner side of the binding the author noted that he started to read the course on Thursday 9 December 1655.

The text was an instructional text for students at the Duytsche Mathematicque. The course was read during the winter semester of 1655 (see section 9.1 of this thesis).

notes The text contains references to the following works: John Napier, *Mirifici logarithmorum canonis descriptio, ejusdemque usus, in utraque trigonometria : ut etiam in omni logistica mathematica, amplissimi, facillimi, & expeditissimi explicatio* [Napier, 1614]; Henry Briggs, *Lucubrationes*, published in [Napier and Briggs, 1620]; Henry Briggs, *Arithmetica logarithmica sive Logarithmorum chiliades triginta* [Briggs, 1624]; Henry Briggs, Adriaen Vlacq, *Trigonometria artificialis sive magnus canon triangulorum logarithmicus ad radium 10000000000, et ad dena scrupula secunda* [Vlacq and Briggs, 1633], and Henry Briggs, *Trigonometria Britannica: sive de doctrina triangulorum libri duo* [Briggs and Gellibrand, 1633].

Hs 436

author Frans van Schooten jr. [+]

scribe Petrus van Schooten [+]

contents An introductory course in arithmetic

language Dutch, with some minor remarks in Latin

date 1659 [+]

origin Leiden

catalogue [Eerde, 1833], [Brugmans, 1898, 243]

sec lit none

physical description

- 1 bound volume
- 150 leaves
- 139 written
- contemporary pagination in ink
- paper 186x159 mm

- black and red-brown ink
- drawings of several geometrical figures in black ink
- contemporary parchment binding
- on the binding a note is attached with in a late eighteenth or nineteenth century hand a title: *Lessen van Franciscus van / Schooten behelzende eene / uitvoerige verhandeling over / de Arithmetica / No* <u>*8*</u> <*4*>; title on spine: *Arithmetica*, in seventeenth century hand

systematic classification
arithmetic: classic, classical measurement systems, decimal system, elementary operations, Roman numerals, rule of three and similar rules, counting board
number theory: divisibility (least common multiple, greatest common divisor)

type/purpose/function of the manuscript lecture notes taken by a student; problems together with solutions

names mentioned Coccejus, Frans van Schooten jr., Simon Stevin, Euclid, Reijchel (118)

evidence The scribe is identified by a comparison of the handwriting with Hs 107, and by the references made of the scribe to "my brother", who is Frans van Schooten jr.

The manuscript are the notes made by Petrus van Schooten during the arithmetic lecture of his half-brother Frans. On the content of the arithmetic lectures see section 8.1 of this thesis.

Petrus mentioned the date at several instances in the manuscript. On the title page of the manuscript Petrus noted that reading of the course started on 27 February 1659, and on the verso side of the title page he once more wrote the year 1659. Moreover, the year 1659 figures in a problem (p. 2).

notes In a problem (p. 50), it is stated that 1658 years, 92 days, 11 hours and 30 minutes have passed, which means that the current date was Thursday 3 April 1659. From this fragment I also conclude that Frans van Schooten jr. lectured between 11 and 12 o'clock. On the title page the name of the rector, Johannes Coccius, is mentioned, which indicates that these notes were indeed made during lectures at Leiden University. Thus, during the summer semester of 1659, Frans van Schooten lectured on arithmetic at the Duytsche Mathematicque.

In the manuscript, Van Schooten jr. stresses the use of arithmetic for merchants several times. A large amount of the problems are placed in a merchant context, which makes it plausible that the audience for this course consisted for a large extent of people with a mercantile background.

The course pays attention to the use of fractions, but does not introduce the notation by decimal fractions. This is remarkable because Stevin had stressed the use of decimal fraction in the curriculum of the Duytsche Mathematicque.

Hs 437

author Frans van Schooten jr. [+]

scribe Petrus van Schooten [+]

contents the manuscript contains notes on courses on arithmetic and on algebra taught by Frans van Schooten

language Dutch, with some notes in Latin

date the main text was written beteen 16 October 1652 (flyleaf) and 17 July 1653 (f. 210r.); after Frans van Schooten passed away in 1660, Petrus added additional notes to the manuscript and added a treatise on surds [+]

origin Leiden

catalogue [Brugmans, 1898, 243]

sec lit none

physical description

- 1 bound volume
- 273 leaves
- 408 written pages
- nineteenth or twentieth century foliation in pencil; the part on algebra (f. 118r–177r.) has a contemporary pagination in pen
- paper 204x157 mm
- the part on algebra has additional notes in a draft in the margins
- black and red ink
- geometrical figures in ink
- contemporary parchment binding
- the title on the spine has almost perished, readable is only *FRANCISCUS van SCHOOTEN*; on the binding a note is attached with a title in late eighteenth or nineteenth centry hand: *No 10 <5> / Kompleet Traktaat over de / Arithmetica, en een groot gedeelte / van de Algebra: Zijnde dit Stuk / het laatste, over het welk de prof. / Franciscus van Schooten gelezen / heeft; gestorven 30 Mei 1660; / opgevolgd door Zijnen Broeder / Petrus van Schooten./ NB. de aantekening op het <de> laatste / Bladzijden, is ingelascht door eenen / onbekenden, welke deze Manuscrip- / ten op de verkooping gekogt had.*

systematic classification

algebra: general, cossic notation, equations (1, 2, 3), systems of equations
applications: interest
arithmetic: general, cossist, decimal system, decimal fractions, elementary operations, extraction of roots (2nd degree), rule of three and similar rules, counting board
combinatorics: general
number theory: divisibility (least common multiple, greatest common divisor), perfect numbers, prime numbers, Pythagorean triangles
sequences: arithmetical, geometrical
series: arithmetical, geometrical
tables: interest

type/purpose/function of the manuscript instructional text; student's notes; problems together with solutions

names mentioned Archimedes, Boethius, Cardano, Clavius, Ludolph van Ceulen, Descartes, Diophantus, Euclid, Ghetaldi, Jordanus, Ptolemy, Willebrord Snellius, Smyters, Stevin, Viète, Vitruvius

evidence Petrus van Schooten is identified as the scribe by comparison of the handwriting with Hs 107 and by noticing that the scribe refers several times explicitly to "his brother" (for instance on f. 128r.-v. and 136r.), pointing at Frans van Schooten jr.

The manuscript contains the notes taken by Petrus van Schooten while he attended lectures of his half-brother Frans jr. On the content of the arithmetic lectures see section 8.1 of this thesis; for the algebra lectures consult section 9.2.

The lectures on arithmetic started on 16 October 1652 (flyleaf). After the course on arithmetic, Frans van Schooten jr. continued lecturing on algebra, which he finished on 17 July 1653 (f. 210r.).

notes In the manuscript the year 1646 is explicitly used in a calculation, which suggests that the course on arithmetic was also read in 1646. A note on f. 98r. confirms that the course was indeed also read between 13 September 1646 and 15 March 1647.

The manuscript contains a lot of notes by Petrus comparing the reading of the course in 1660–1661 with his notes of the 1652–1653 course. This provides detailed and valuable information on the differences between the course as taught in 1652–1653 and in 1660–1661. Petrus calls the lectures of 1660–1661 the "second time" ("tweede reyse" or "secunda vice", f. 128v. and 169r.).

In the algebra course, a cossic notation was used. This is remarkable, because Frans van Schooten had been made familiar with the Cartesian notation at a relatively young age.

Hs 438

author Abraham de Graaf [+]

scribe unknown [+]

contents an introductory text on the use of algebra in Cartesian notation, based on the writing of Abraham de Graaf [+]

language Dutch

date after 1676 [+]

origin the Netherlands

catalogue [Brugmans, 1898, 243]

sec lit none

physical description

- 1 bound volume
- 188 leaves
- 165 written pages
- nineteenth or twentieth century foliation in pencil
- paper 196x160 mm
- black ink
- pen drawings of geometrical figures
- contemporary binding of marbled paper
- no title on the spine; on the binding a separate note is attached with a title in late eighteenth or nineteenth century hand: *Verschillende uitgewerkte Stukken / van / FR. à Schooten / over de Algebra / No 8* <1>

systematic classification
algebra: general, equations (1, 2, 3), systems of equations

type/purpose/function of the manuscript survey of existing knowledge; abridgment of printed text; problems together with solutions

names mentioned Descartes, Euclid, Abraham de Graaf

evidence The manuscript contains references to two works by Abraham de Graaf. The first work is *De geheele mathesis of wiskonst, herstelt in zijn natuurlijke gedaante* (citations on f. 1r. and 6r.), which was published in 1676 and reprinted in 1694 and 1717, [Graaf, 1676], [Graaf, 1694], [Graaf, 1717]. The second work is *De beginselen van de algebra of stelkonst, volgens de manier van Renatus Des Cartes: verklaart met* *uytgelezene voorbeelden; zoo wel in de meetkonst, als in de rekenkonst* (citations on f. 9r., 56v. and 66v.) of 1672 [Graaf, 1672]. The earliest printing dates of these works imply that the manuscript was conceived in or after 1676. A comparison between the text of the manuscript and the two works shows that the whole manuscript was compiled on basis of the two books. The eighteen problems at the end of the manuscript are copied from *De beginselen van de algebra of stelkonst.*

A comparison with the handwriting of Petrus van Schooten of Hs 109 reveals that Petrus van Schooten was not the scribe of the manuscript. As the manuscript was written after 1676, it is not possible that the scribe of the manuscript was Frans van Schooten sr. or jr.

notes The Stadsarchief and Athenaeum Bibliotheek (SAB) of Deventer possesses a manuscript on algebra which is also based on the works of De Graaf (SAB: 100 B 1 KL). Further research is necessay to determine whether the Groningen and the Deventer manuscripts are related.

Hs 439

author Diophantus (in Bachet edition), Mauritius Zons, Hermann Friesenburch, Martin Wilkens, Tiado Fockens

scribe Tiado Fockens, with some notes in an unknown hand [+]

contents a Dutch translation of Bachet's Diophantus edition [Diophantus of Alexandria and Bachet, 1621], with the title *Claudij Gasparis Bachetij Sebusianij Uijtlegginge [de ses eerste] boeck[en] van Diophantes Alexandria* (*Bachet's explanation of the first six books Diophantus*) and *[de twee] Boeck[en] van die veelhoekige getallen* (*The two books on polygonal numbers*), followed by a treatise on Diophantine problems in cossic notation; the manuscript further contains extracts from works of Mauritius Zons, Martin Wilkens and Hermann Friesenburch, and a collection of problems, partly with solutions

language Dutch and Low German; some notes in Latin

date ca. 1665 [+]

origin The Hague or Groningen [+]

catalogue [Eerde, 1833] and [Brugmans, 1898, 244]

sec lit none

physical description

- 1 bound volume
- 126 leaves

- 204 written pages
- nineteenth or twentieth century foliation in pencil; the foliation includes enclosed leaves between f. 66v. and 74r. which are not bound into the volume
- paper 316x210 mm
- black and brown ink
- pen drawings of geometrical figures on f. 73r., 86r., and 98v.-101v.
- contemporary parchment binding
- the title on the spine has partly perished, only the name "Bachet" is readable; on the binding a note is attached with a title in late eighteenth or nineteenth century writing: *Verschillende stukken van Diophantes / Alexandrinus, uitgewerkt door / Fr. à Schooten. / No 2 <5>*

systematic classification
algebra: cossist
applications: astronomy
arithmetic: rule of three and similar rules, problems in a geometrical context
geometry: general, context for equations, dimension two
number theory: Diophantine problems, figurate numbers
sequences: arithmetical, geometrical
series: general, arithmetical
tables: figurate numbers

type/purpose/function of the manuscript survey of existing knowledge; problems together with solutions; reworking and translation of a classical text

names mentioned Christian Martin Anhaltin, Bachet, Ludolph van Ceulen, Bonaventura Constance, Diophantus, Euclid, Johann Faulhaber, Hermann Friesenburch ("friesenborch"), Tiado Fockens, Marco Froom, Nicolaas Huberts van Persijn, Martin Wilkens, Mauritius Zons

evidence Information about the scribe and the date is provided in the pages dealing with tables of figurate numbers (f. 87r.-94v.). On f. 87. is written "Uit Martin Wilkens eigen schrift hebbe ick dat geschreven Tiae fockens" (I wrote this from Martin Wilkens's own writing Tiae fockens) and on f. 93v. the date is mentioned: "Tiado fockens 15 die ianuari 1665". It is not known when the other parts of the manuscript were written and therefore the manuscript is dated ca. 1665.

The main text is written in the same hand by Tiado Fockens. The folios 41r.-58v. contain notes in the margin in another hand; this handwriting is also found on f. 124v-125v. The scribe of this second hand is unknown.

The manuscript originates from either Groningen or The Hague, both cities in the Dutch Republic. Several indications point to Groningen. As stated above, part of the manuscript was copied from Martin Wilkens's own writing. Wilkens was an arithmetic teacher in Groningen. Moreover, in several problems the name of the city of Groningen is mentioned (f. 100r. and 101r.). In the manuscript several Ost-frisian mathematicians are mentioned (Marco Froom, Christiaan Martini Anhaltin and Hermann Friesenburch), revealing contacts between the mathematical and schoolmaster's communities of Emden and Groningen. The reference to Tiado Fockens in [Ferguson and Fockens, 1667, iii, 208] is in favour of The Hague. This Tiado Fockens was living in The Hague by August 1667, had a keen interest in mathematics, and his interests overlap with the subjects of the manuscript. Tiado Fockens may have moved from Groningen to The Hague, but there is no further evidence for such a move.

notes In 1667 twelve problems solved by Tiado Fockens were published in [Ferguson and Fockens, 1667]. Just like Fockens, Ferguson had an interest in Diophantine problems and figurate numbers. A manuscript written by Ferguson also contains a Dutch translation of Bachet's Diophantus edition (signature UL, Ltk 968, described in [Maanen, 1987, 178-180]). Further investigation is needed in order to find out whether there is a connection between Fockens's and Ferguson's manuscript.

Hs 440

author Gielis van den Hoecke [+]

scribe unknown

contents a literal copy of f. 1r.-123r. of [Hoecke, 1537]

language Dutch

date 1558 (f. 48v.)

origin Northern or Southern Netherlands

catalogue [Eerde, 1833], [Brugmans, 1898, 244], [Jansen-Sieben, 1989]

sec lit [Kool, 1991] and [Kool, 1999, 256]

physical description

- 1 bound volume
- 98 leaves
- 178 written pages
- nineteenth or twentieth century foliation in pen
- paper 205x152 mm
- black ink

- illustrations in pen of a counting board and some simple geometrical figures
- contemporary parchment binding
- no title on the spine; on the binding a separate note is attached with a title in a nineteenth or twentieth-century hand: *F. Van Schooten / Verhandeling over de Arithmetica. / No 12 <7>*

systematic classification
algebra: equations (1,2), systems of equations
applications: astronomy, commerce
arithmetic: general, cossist, classical measurement systems, elementary operations, extraction of roots (2, 3), rule of three and similar rules, surds, counting board
geometry: context for equations
sequences: general, arithmetical, geometrical
series: general
trigonometry: general

type/purpose/function of the manuscript copy of a printed text; instructional text; problems together with solutions; extraction of roots

names mentioned because the manuscript has been identified as a literal copy of a printed text, no attempt has been made to list the names

evidence In [Kool, 1991], Marjolein Kool identified the text as a literal copy of [Hoecke, 1537].

Hs 441

author Frans van Schooten sr. and Frans van Schooten jr. [+]

scribe Frans van Schooten sr. (f. 1r-47r.), Petrus van Schooten (f. 49 r.-140r.) [+]

contents an introductory text on arithmetic (f. 1r.-47r.), two treatises on logarithms (f. 49r.-80r. and f. 81r.-84v.), problems on the division of figures (f.87r.-93r.), some notes on solid geometry and platonic solids (f. 93r. and f. 140v.-141v.) and a treatise on fortification (f. 94r.-140r.)

language French (f. 1r.-47r.) and Dutch (the rest of the manuscript); some notes in Latin

date the introductory text on arithmetic by Frans van Schooten sr. is from 1630; the parts written by Petrus van Schooten are from the years 1655 and 1656; the treatises on logarithms are from 1655 (f. 80r.), the notes on solid geometry are from April 1656 (f. 93r.) and the treatise on fortification was started in September 1656 (140r.) [+]

origin Leiden

catalogue [Eerde, 1833], [Brugmans, 1898, 244]

sec lit [Heuvel, 2006]

physical description

- 1 bound volume
- 143 leaves
- 272 written pages
- nineteenth or twentieth century foliation in pencil; the treatise on fortification has a contemporary pagination in pen; this treatise is written in reverse order such that the first page corresponds with f. 140r. of the foliation, and the final page with number 89 corresponds with f. 95r.
- paper 166x214 mm
- the writing in the fortification manuscript is very dense
- black, red ink; the red ink is used in the fortification manuscript for adding additional notes
- pen drawings of geometrical figures, plans of fortifications and sections of fortification works
- contemporary parchment binding
- the title on the spine has almost vanished, only the name "Briggium" is still readable; on the binding is a separate note with a title in late eighteenth or nineteenth century writing: *In dit Manuscript / van / Fr. A. Schooten, Professor inde Ma- / thesis te Leiden: gevoegd bij de Aante- / keningen van P. van Schooten in dato 1656 / mense April. Zijn begrepen de volgende / stukken. / Arithmetique ou l'art à Cyfrer in het Frans / Tabulae logarithmicae in het Nederduitsch / Deeling der Figuren uit evenwijdige Linien / tesamen gesteld / Fundamenta, quibus usus fuit Fr. A Schooten / Frater meus / Eene gehele Verhandeling over de Fortifi- / catie / No 13 <8>.*

systematic classification
applications: fortification, interest
arithmetic: general, classical measurement systems, elementary operations, rule of three and similar rules
geometry: area, volume, dimension two, dimension three
logarithms: tables, theory

type/purpose/function of the manuscript instructional text; problems together with solutions; lecture notes taken by a student

names mentioned Henry Briggs, Julius Caesar, Adam Freitag, Samuel Marolois, John Napier, Frans van Schooten jr., Petrus van Schooten, Simon Stevin

evidence The scribes have been identified by comparing the handwriting with autographs of Frans van Schooten sr. (one manuscript, UBL, BPL 1013, and petitions UBL, Archive Curators, inv. nr. 42) and with an autograph of Petrus van Schooten (Hs 107). Petrus refers to his half-brother in the treatise on fortification (f. 107v., 131r.).

The writings of Petrus van Schooten are elaborate notes taken in the classes of Frans van Schooten jr. These writings provide information on the teaching at the Duytsche Mathematicque during the 1650s.

The French text on arithmetic in the hand of Frans van Schooten sr. can be conjecturally dated because it contains the year 1630 in an example on f. 5r. This strongly suggests that the text was composed in the year 1630, and it is likely that the manuscript was also written in that year.

notes Petrus van Schooten used a booklet of his father for his own notes.

Petrus van Schooten added many additional notes to his fortification manuscript. These notes are inserted in the manuscript or attached to the pages.

There are similarities between the French treatise on arithmetic and the Dutch treatises of Hs 436 and Hs 437. These similarities show that Frans van Schooten jr. based his arithmetic course partly on the writings of his father Frans sr.

Hs 442

author Petrus van Schooten, Apollonius

scribe Petrus van Schooten and Frans van Schooten sr. [+]

contents an instructional text on the use of algebra in solving arithmetical and geometrical problems; an unbound quire on conic sections is included

language Dutch, with some notes in Latin

date the part written by Petrus van Schooten was written between 1659 and 1679; the text on conic sections by Frans van Schooten was written in the period 1600–1645, probably after 1637 [+]

origin Leiden

catalogue [Eerde, 1833], [Brugmans, 1898, 244]

sec lit none

physical description

- 1 bound volume, and six inserted unbounded folio's (f. 83r.-88v.)
- 147 leaves plus 6 inserted unbound folio's

- 146 written pages, plus 11 written pages
- nineteenth or twentieth century foliation in pencil; the foliation continues also on the inserted folio's written by Frans van Schooten sr.; the bound treatise written by Petrus van Schooten has a contemporary pagination in ink
- paper 338x236 mm
- very dense writing
- black and red ink; some notes added in pencil (f. 67r., 70v., 72r., 77v.)
- pen drawings of geometrical figures; an illustration of a tower on f. 35v., 41v., 43v., and 44r.
- contemporary parchment binding, partly covered with marbled paper
- no title on the spine; the title on the cover is in late eighteenth or nineteenth century writing and reads *Oplossing van verschillende Geometrische en / Algebraïsche werkstukken / door / P. van Schooten / No 1*

systematic classification
algebra: general, equations (up to 6th degree), construction of equations
analytic geometry: general, coordinates
conic sections: general, ellipse, hyperbola, parabola
geometry: general, constructions, context for equations, dimension two
instruments: for mathematical constructions, for surveying purposes
meta-mathematics: general
number theory: Diophantine problems
sequences: arithmetical, geometrical
trigonometry: general

type/purpose/function of the manuscript survey of existing knowledge; problems together with solutions; instructional text

names mentioned Apollonius, Aristaeus, De Beaune, Cardano, Cardinael, Clavius, Descartes, Diophantus, Euclid, Eutocius, Fermat, Ghetaldi, Golius, Hartsinck, Hudde, Huygens, Kinckhuysen, Mydorge, Ptolemy, Frans van Schooten jr. ("mijn broeder" or "frater"), Viète, Martin Wilkens, De Witt

evidence The scribes have been identified by comparing the handwriting with autographs of Frans van Schooten sr. (one manuscript, UBL, BPL 1013, and petitions UBL, Archive Curators, inv. nr. 42) and with an autograph of Petrus van Schooten (Hs 107).

The manuscript consists of a bound volume, written by Petrus van Schooten, in which six unbounded folio's (f. 83r.-88v.) written by Frans van Schooten sr. are inserted.

The manuscript is undated, but because it contains references to the second edition of *Geometria* of 1659–1661 (for instance f. 50r.), we can date the manuscript between 1659 and 1679 (the year in which Petrus died).

The inserted folios in the handwriting of Frans van Schooten sr. do not contain a date either. These folios contain a reference to Descartes's *Géométrie* (f. 85r.), but the colour of the ink of this reference suggests that it was added after the main text had been written. We conclude that the text was written in the seventeenth century before 1646 (Frans van Schooten sr. died on 11 December 1645) and that it was used by Van Schooten sr. after 1637.

notes The stamp on f. 3r. shows that this manuscript was donated by Jacob Baart de la Faille (1757–1823), professor of mathematics and physics at Groningen University.

Several pages show the marks of drawings of fortifications (f. 6r., 7r., 50r.), looking like carbon copies.

A fair part of the problems of Hs 442 is also found in Hs 437 and Hs 112. It seems that Frans van Schooten jr. used the same problems for his various courses and instructional texts.

Hs 443

author Frans van Schooten sr. (autograph), Petrus van Schooten (autograph) [+]

contents an introductory text on algebra in cossic symbols and the arithmetic of surds [+]

language Dutch

date part of the manuscript was written by Frans van Schooten sr. between 15 February 1623 and 14 February 1624; the rest was written by Petrus van Schooten between 1650 and 1679 [+]

origin Leiden

catalogue [Eerde, 1833], [Brugmans, 1898, 244]

sec lit [Maanen, 1993]

physical description

- 1 bound volume
- 81 leaves
- 149 written pages
- foliation in pencil, in nineteenth century hand
- paper 328x211 mm
- black ink
- pen drawings of geometrical figures

- nineteenth century cardboard binding, the flyleaf contains a watermark with the text “Pro patria eendracht maakt macht” and “V d L”
- no title on the spine; on the binding a note is attached with the title in a late eighteenth or nineteenth century hand: *De Cos Rekening, benevens een Verhandeling / over de Irrationale Grootheden / van / Petrus van Schooten / No 5* <*3*>.

systematic classification
algebra: general, equations (degree 1 and 2), systems of equations
arithmetic: cossist, decimal system, extraction of roots (up to degree five), surds, problems in a geometrical context
geometry: context for equations

type/purpose/function of the manuscript instructional text; problems together with solutions

names mentioned Clavius, Euclid

evidence The scribes have been identified by comparing the two handwritings with identified autographs of Frans van Schooten sr. (manuscript BPL 1013 from UBL and petitions from UBL, Archief van Curatoren, 1574–1815, inv. nr. 42) and of Petrus van Schooten (Hs 107).

The manuscript contains two separate texts. The first one (f. 2r.-63v. with the exception of f. 38r.-40v.) was written by Frans van Schooten sr. and deals with the elenmetary arithmetic of surds and algebra in cossic notation. The second text (f. 64r.-to 81r. and 38r.-40v., in this order) was written by Petrus van Schooten and is a treatise on surds, in particular binomials and residual numbers.

The manuscript is undated. The dating of the part written by Frans van Schooten sr. is based on the implicit information provided by the solution of a problem on f. 54r. In this problem, the age of a man, wife and child are to be determined. It turns out that the solutions for the ages of the man and the child correspond to the situation of the Van Schooten family from 15 February 1623 to 14 February 1624.[20] The date of the part written by Petrus is based on his active years. It is possible that this part was based on lectures of Frans jr., as is the case in other manuscripts, although there is no sound proof in the current manuscript other than that its content shows similarities with the content of the part on surds of Hs 437.

notes Several problems of the text by Frans van Schooten sr. also appear in the manuscripts Hs 437 and Hs 112. See for instance the problems on f. 41r. and 45v.

[20] The family consisted of the father Frans van Schooten sr., the mother Jannetgen Harmensdr. van Hogervorst and their son Frans jr. For a full account of the determination of the date of the manuscript, see [Maanen, 1993, 260-262].

Hs 444

author Frans van Schooten jr. [+]

scribe Petrus van Schooten [+]

contents text on spherical triangles and their use in astronomy and gnomonics

language Dutch, with some annotations in Latin

date September 1658 - 7 February 1659 [+]

origin Leiden

catalogue [Eerde, 1833], [Brugmans, 1898, 244]

sec lit [Heuvel, 2006]

physical description

- 1 cover, partly bound
- 86 leaves
- 170 written pages
- contemporary pagination in ink; the pagination runs from 3 to 16 and from 41 to 179; the pages 17 up to 40 are missing; between page 80 and 81 ten unfoliated folios are included
- paper 208x167 mm
- black ink and red-brown ink (see p. 59, 95)
- pen drawings of geometrical figures
- contemporary paper binding
- no title on the spine; on the binding is written a title in a nineteenth century hand *Astronomij Compleet / van F. Verschoote*; a seperate note is attached to the binding with a title in a late eighteenth or nineteenth-century handwriting: *Eene Volledige Verhandeling over / de spheerische Driehoeksmeting, van / Fr. van Schooten M. Prof. tot 7 Febr / 1659: door zijnen Broeder / bijwelke veele Astronomische / Problemata / No 3*

systematic classification
applications: astronomy, geography, gnomonics, navigation
instruments: sundials
logarithms: general
trigonometry: solution of triangles, spherical triangles

type/purpose/function of the manuscript instructional text; problems together with solutions; lecture notes taken by a student

names mentioned Henry Briggs, Julius Caesar, Ezechiel de Decker, Euclid, Edmund Gunter, Samuel Kechel, John Napier, Frans van Schooten jr., Willebrord Snellius, François Viète, Adriaen Vlacq

evidence The scribe explicitly mentions his name on p. 179: “Scripsit Petrus a Schooten in auditorio sui fratris”. Throughout the manuscript, Petrus van Schooten refers in notes to his half-brother Frans van Schooten jr.

The manuscript mentions the year 1659 (p. 116), and ends (p. 180) with a note indicating that Frans van Schooten finished his lectures on spherical triangles on 7 February 1659. According to the series lectionum of Leiden University, Frans van Schooten jr. read a course on spherical triangles during the autumn semester of 1658 [Molhuysen, 1918, 90*]. The manuscript was thus written between September 1658 and 7 February 1659 and shows the content of a course of the Duytsche Mathematicque. Petrus added many additional notes in the main text and in the margin. The exact dating of these notes is not known, but they were made in 1679 at the latest.

notes There are references to the following texts:

A text of Viète on triangles, probably the *Canon mathematicus* which was first printed in 1579 [Viète, 1579], and reprinted in 1609 [Viète, 1609].

A work of Snellius which was published by Maire after Snellius’s death (p. 82: “Dit bouck van Snellius is eerst nae sijn doot uutgecomen en bij le Maire gedruckt”). This work has been identified as [Snellius and Hortensius, 1627].

A.4 Index of subjects

An index on all the subjects mentioned in the section 'systematic classification'. The numbers mentioned are the shelfmarks of the manuscripts treating the subject.

A.5 Index of people

The number behind the names is the shelfmark of a manuscript. The index provides information on the authors, scribes and people mentioned in the manuscripts in the following way:

- a **bold** shelfmark indicates the authorship of the person
- an *italic* shelfmark indicates that the person was the scribe of the manuscript
- a non-bold and non-italics shelfmark indicates that the person was mentioned in the manuscript

Using this coding system, the shelfmark of a manuscript may occur several times for the same person, in case he has several roles for the same, like for instance being the author and the scribe.

A capital letter [B] indicates that a short biographical note is included. A biographical note is only included in case I collected new or additional information which is not available in common reference books and biographical dictionaries and in case an entry in common reference books is absent.

Biographical notes

Van Berckel/Berckelio Petrus Berckelius studied in Leiden in the early 1650s. On 4 June 1651, he disputed on Aristotelian physics [Berckelius, 1651]. He was also mentioned in the correspondence of Christiaan Huygens, see [Huygens, 1888, 242, 246, 274] and [Huygens, 1910, 80]. He might be the Petrus a Berckell who matriculated in April 1646 at age 12 at Leiden University. No further information about his life is known.

'Gerrit Gerritsen Brugman' Mathematician from Amsterdam, active in the second half of the seventeenth century. No additional information on him has been found.

Theodorus Craanen (ca. 1633 – ca. 1689) Born in Cologne around 1633, Craanen matriculated at Utrecht University in 1651 [Album, 1886, 27], where he studied under Henricus Regius. In 1655 he went to Leiden, where he was registered on 17 February 1655 as a student in philosophy and theology [Rieu, 1875, 293]. During his stay in Leiden he became acquainted with Van Schooten, and attended private lectures by Van Schooten in mathematics [Huygens, 1888, 509]. Craanen continued his studies at Duisburg University, where he registered as a student in medicine and philosophy on 6 November 1656. He obtained his doctorate in medicine on 4 May 1657, and in the same year he was appointed professor at the medical faculty in Duisburg. Besides his duties in medical courses, he also lectured on mathematics in the philosophical faculty [Roden and Jedin, 1968, 263]. In 1661 he was called to Nijmegen to become a professor of philosophy and medicine at the university and to become the medical doctor of the city. In 1670 he was appointed professor of philosophy in Leiden, and subregent of the Staten Collegium. Being a Cartesian, and promoting Cartesians thoughts, he came in conflict with professor Spanheim in 1673, which led to change of professorship: the curators of the university made him resign the chair of philosophy and take up medicine instead. During his time in Leiden, Craanen gave private lessons in mathematics. The Curators of Leiden University in 1684 considered a revival of the Duytsche Mathematicque in 1684 with Craanen as professor, but the plan was not carried out. According to Boerhaave, Craanen was more interested in geometry than in visiting the sick, [Luyendijk-Elshout, 1975, 298].

Bonaventura Constance A seventeenth century pharmacist living in Groningen. See Groninger Archieven, Volle Gerecht van de stad Groningen (toegang 1534), inv. nr. 91, 9 July 1623 and 11 August 1623; inv. nr. 3956 f. 21 (1642/1643); inv. nr. 3957 f. 126, f. 280 and f. 241 (1643/1644); inv. nr. 3959, f. 159 and f. 163 (1645/1646); inv. nr. 3961, f. 141 and 322 (1647); inv. nr. 3971, f. 106 (1654/1655).

Etienne d'Espagnet (ca. 1602 – after 1666) Etienne d'Espagnet was a son of the Bordeaux magistrate Jean d'Espagnet. He had an interest in science and in mathematics in particular, was on intimate terms with Fermat, and had good relations with the Bordeaux mathematician François de Verdus (1621–1675). He was a key figure in the scientific community of Bordeaux, attended the Bordeaux counterpart of an "Académie des sciences" and was involved in the nomination of the professor of mathematics at the Collège de Guyenne in 1665. See [Mahoney, 1994, 49], [Fermat, 1896, xv], [Hobbes and Malcolm, 1994, 911].

Tiado Fockens (before 1638 – after 1672) In 1667 twelve questions by Tiado Fockens were published in [Ferguson and Fockens, 1667]. The same source indicates that he was living in The Hague in 1667, and that he was Ferguson's master. Information from Hs 439 suggests that Tiado Fockens was originally from Groningen. No further information about him is known.

Herman Friesenburch (ca. 1582–1656) Friesenburg was a writing and arithmetic teacher living in Emden. During his lifetime, he published various works on the art of writing and on mathematical subjects like arithmetic and algebra. A biography is found in [Tiaden, 1787, 364-374].

Marco Froom He was a 'schrieb- und rechenmeister' (writing and arithmetic teacher) in Emden in 1638 [Friesenburch, 1638].

'Heusdano' Heusdano remains unidentified.

Gielis van der Hoecke (ca. 1505 – after 1637) Van der Hoecke was the author of an arithmetical textbook in Dutch [Hoecke, 1537]. In 1521 he matriculated at Leuven University. In 1535 he was living in Ghent, making a living as seller of almanacs and probably as arithmetic teacher [Bockstaele, 1985, 5-9].

Philon "Mr. Philon" was one of the correspondents of Fermat (1601 or 1607–1665), and he resided in the area of Bordeaux. Mahoney tentatively identifies him with the philologist François Philon who published a French translation of Virgil's *Aeneid* in 1640 [Mahoney, 1994, 49].

Pierre Prades (before 1605–1664) Pierre Prades held a degree in theology from the University of Toulouse. In 1625 he applied for the Ramus chair of mathematics at the Collège de France in Paris, but he was not appointed. After serving as a canon in Perigueux, he obtained the chair of mathematics at the Collège de Guyenne in Bordeaux in 1629, a position he held until his death in 1664. He corresponded with Fermat and was a member of the party of the ormes, a political movement opposed to monarchy in Bordeaux in the 1650s. See [Darnal, 1666, 31, 97], [Gaullieur, 1874, 429], [Bouchel and Bechefer, 1667, 917], [Sarrazin, 1996, 179].

'Mr. Reijchel' Reijchel remains unidentified.

Peter Roth (before 1590–1617) Roth worked as an reckonmaster in the German city of Nuremberg. In 1608 he published a treatise entitled *Arithmetica philosophica*, [Roth, 1608], in which he formulated an early form of the fundamental theorem of algebra. He maintained a correspondence with the German mathematician Johann Faulhaber, but this correspondence seems to be lost today [Schneider, 1999].

Hermann Slath (1641/1642–1678) Slath, original from Duisburg, matriculated on 11 April 1659 at Duisburg University at the age of 17. He obtained his doctorate in philosophy and medicine in Padua in 1665. He returned to Duisburg and became extraordinary professor of mathematics in 1667 and ordinary professor of mathematics one year later. He died in 1678. See [Wijnhoven et al., 2010], [Komorowski, 2001] and [Roden and Jedin, 1968, 272].

Willem van der Straaten (1593–1681) Van der Straaten was a medical doctor of the city of Utrecht and the first professor of medicine at Utrecht University in 1636. From 1646 onwards he was the personal physician of the stadholders Frederick Henry, William II and the prince William III [Baumann, 1952].

'Strunckede' It has not been possible to determine the identity of "Strunckede" indisputably. There are three candidates. The most likely candidate is Fridericus Wilhelmus a Strunckede who matriculated on 5 December 1658 at Duisburg University. There are no traces left of any particular interest in mathematics on his part, but his matriculation date of December 1658 is close to those of Samuel de Vivere (April 1657) and Hermann Slath (11 April 1659), two mathematicians who were also mentioned in manuscript Hs 112. The other two candidates are the brothers Henricus Johannes a Strunckede and Gothardus a Strunckede, who matriculated at Leiden University in March 1646. They are less likely because there is no evidence that one of these brothers had a particular interest in mathematics.

Samuel de Vivere (before 1643 – after 1662) De Vivere originated from Duisburg and matriculated as a student at Duisburg University in April 1657. In 1662 he disputed, with two other students, on Cartesians physics [Wijnhoven et al., 2010] and [Trevisani, 2011, 109].

Woldeck Weland (1614–1641) Originally from the German city of Verden, Weland matriculated in 1631 at the Academic Gymnasium in Hamburg where he was residing in the house of his professor Joachim Jungius. In 1633 he matriculated at the University of Rostock where he studied mathematics and medicine. He continued his studies in Leiden (matriculation 11 May 1635) and completed his education with a journey through England and France. He stayed in Oxford, Paris and Orléans and eventually returned to Hamburg in 1638. Around this time he started to work on a reconstruction of Apollonius's *De locis planis*, a project which his teacher Joachim Jungius had already started in the 1620s. In June 1639 he returned to Leiden in order to continue his studies in medicine. During his stay in Leiden he published a mathematical treatise entitled *Strena mathematica* [Weland, 1640]. In 1640 Weland returned to his hometown Verden where he died on 6 Mai 1641. After his death, his manuscripts on the reconstruction of *De locis planis* came in the hands of Johannes Müller, professor of mathematics at the Academic Gymnasium, who prepared the treatise for publication. As Müller died before he finished the work, the treatise remained unpublished until publication in [Elsner, 1988].

Martin Wilkens (before 1610 – between 1639 and 1669) Originally from Emden, he settled in Groningen as a schoolmaster, arithmetic master and precentor of the Martini church. He published various works on arithmetic and algebra, and one arithmetical text together with Herman Friesenburch. His instructional work on arithmetic was reprinted into the beginning of the eighteenth century.

Mauritius Zons (unknown – after 1602) Teacher of arithmetic in Cologne, who was alive in 1602 [Kästner, 1799, 129].

APPENDIX *B*

Apollonian theory of conic sections

In the construction of the solution of the four line Pappus problem, Descartes used the concepts and terminology of the theory of conic sections from the *Conics* of Apollonius of Perga (ca. 200 BC). For the reader not familiar with these concepts and terminology, the necessary elements of the Apollonian theory of conic sections are given in this appendix, without taking into account the complicated order in which Apollonius builds up his theory.[1]

Apollonius defines a conic section as the intersection of an (oblique) circular cone with a plane. In his study of conic sections he uses the concept of diameter and corresponding ordinates. Consider a set of parallel line segments AB, $A'B'$, $A''B''$, etc. whose endpoints are on the conic section. Then it turns out that the midpoints C, C', C'', etc. of these line segments are on a straight line. The straight line is called a *diameter* of the conic section and the halves of the segments AC, $A'C'$, etc. are called the *ordinates* corresponding to the diameter. If the diameter intersects the conic at some point M, the tangent to the conic at M is parallel to the ordinates which correspond to the diameter. The constant angle $\angle ACM = \angle A'C'M$ etc. between the diameter and its ordinates is called the angle of ordinates and is equal to the angle between the tangent at M and the diameter. If the angle of ordinates is a right angle, the diameter is called an axis.

First suppose that the conic is a parabola. Apollonius proves that the parabola has one

[1] For an overview of the structure of the *Conics*, see for example the introduction of Thomas Heath in [Apollonius, 2010].

axis, and that every straight line parallel to the axis is a diameter of the parabola. He also proves that for any diameter, the ordinates AC $A'C'$ etc. and the corresponding abscissas CM, $C'M'$ etc. satisfy a fundamental property, to the effect that the square of AC is equal to the rectangle contained by CM and a constant line segment p which does not depend on the position of the ordinate. In modern terms

$$AC^2 = p \cdot CM.$$

The line segment p is called the *latus rectum*, or "erect side;" the idea is that this is one side of a rectangle, whose other side is the abscissa CM, and whose area is equal to AC^2. The length of p is dependent on the diameter; if one considers two different diameters of the same parabola, the corresponding p are different. Putting $v = AC, u = CM$, the fundamental property takes the form of the equation

$$v^2 = pu.$$

Point M is called the *vertex* of the parabola corresponding to the diameter MCC'. Note that every point of the parabola is a vertex in the Apollonian sense.

We now consider the ellipse. Apollonius proves that an ellipse has two axes which are perpendicular to one another and which intersect at a point which he calls the center of the ellipse. Any straight line through the center is a diameter of the ellipse, and the corresponding ordinates are parallel to the tangents at the points of intersection M, M' of the diameter and the ellipse. Points M and M' are called the vertices corresponding to the diameter MM'. Again, every point of the ellipse is a vertex in the Apollonian sense, which differs from the modern sense to the effect that a vertex is the point of intersection of the ellipse with one of its axes. Apollonius also proves a fundamental property of the ellipse which can be expressed in different ways, such as the following. Let A be an arbitrary point on the ellipse and let AC be the ordinate through A which meets the diameter MM' at point C. Then

$$AC^2 : CM \cdot CM' = p : MM',$$

where again p is a constant segment called the *latus rectum*, and MM' is called the *latus transversum*. The lengths of p and MM' do not depend on A but they do depend on the choice of the diameter.

Putting $v = AC, u = CM, d = MM'$, the fundamental property of the ellipse takes the form

$$v^2 = \frac{p}{d} u(d - u).$$

For a circle, any diameter is an axis in the sense of Apollonius, and d and p are equal to the diameter of the circle.

For the hyperbola things are analogous to the ellipse. Apollonius considered the two branches of the hyperbola in the modern sense as different, "opposite" hyperbolas. The fundamental property is again

$$AC^2 : CM \cdot CM' = p : MM'$$

but now point M' is the intersection of the diameter with the opposite hyperbola. Putting $v = AC$, $u = CM$, $d = MM'$, the fundamental property takes the form

$$v^2 = \frac{p}{d}u(d+u).$$

Again point M is called a vertex of the hyperbola corresponding to the diameter through it; p is called the *latus rectum*, and MM' is called the *latus transversum*.

Sources

Consulted libraries

Amsterdam, Library of the University of Amsterdam (UBA)
Groningen, University of Groningen Library (UBG)
Leeuwarden, Tresoar
Leiden, Leiden University Library (UBL)
London, British Library
Paris, Bibliothèque nationale de France
The Hague, Koninklijke Bibliotheek
Utrecht, Utrecht University Library
`books.google.com`

Consulted archives

Antwerp, FelixArchief
Gouda, Streekarchief Midden-Holland

- Archief van de stad Gouda, 1311–1815

Groningen, RHC Groninger Archieven
Leiden, Leiden University Library

- Archief van Curatoren, 1574–1815
- Archief van Senaat en Faculteiten, 1575–1877

Leiden, Regionaal Archief Leiden

- Stadsarchief van Leiden (Stadsbestuur (SA II)), (1253) 1574–1816 (1897), toegang 501A
- Weeskamer van Leiden, (1343) 1397–1860 (1866), toegang 518
- Heilige Geest- of Arme Wees- en Kinderhuis (HGW), toegang 519
- Doop-, trouw- en begraafboeken Leiden, 1575–1811 (1859), toegang 1004

The Hague, Nationaal Archief

Consulted museum collections

Amsterdam, Rijksmuseum
Amsterdam, Het Scheepvaartmuseum, the National Maritime Museum
Leiden, Museum De Lakenhal
Leiden, Museum Boerhaave
Paris, Musée du Louvre
Utrecht, Geldmuseum[1]

[1] The Geldmuseum is closed as of 1 November 2013 due to cuts of the state subsidy. The collection of the Geldmuseum will be transferred to De Nederlandsche Bank in Amsterdam and to the Rijksmuseum voor Oudheden in Leiden. See `http://www.rijksoverheid.nl/nieuws/2013/10/09/collectie-geldmuseum-blijft-toegankelijk.html`, retrieved 24 December 2013.

Bibliography

[Ahlberg, 2005] Ahlberg, Nils (2005). *Stadsgrundningar och planförändringar: svensk stadsplanering 1521–1721.* Swedish University of Agricultural Sciences, Uppsala.

[Ahsmann, 1990] Ahsmann, Margreet (1990). *Collegia en colleges. Juridisch onderwijs aan de Leidse Universiteit 1575–1630, in het bijzonder het disputeren.* Wolters-Noordhoff/Egbert Forsten, Groningen.

[Album, 1886] (1886). *Album studiosorum Academiae Rheno-Traiectinae MDCXXXVI–MDCCCLXXXVI: Accedunt nomina curatorum et professorum per eadem secula.* Apud J. L. Beijers et J. van Boekhoven, Utrecht.

[Ames-Lewis, 1999] Ames-Lewis, Francis, editor (1999). *Sir Thomas Gresham and Gresham College: studies in the intellectual history of London in the sixteenth and seventeenth centuries.* Ashgate, Aldershot.

[Andersen, 1990] Andersen, Kirsti (1990). Stevin's theory of perspective: the origin of a Dutch academic approach to perspective. *Tractrix*, 2:25–62.

[Andersen, 2007] Andersen, Kirsti (2007). *The geometry of an art: the history of the mathematical theory of perspective from Alberti to Monge.* Springer, New York.

[Andersen and Bos, 2006] Andersen, Kirsti and Bos, Henk J. M. (2006). Pure mathematics. In Park, Katharine and Daston, Lorraine, editors, *Early Modern Science*, The Cambridge History of Science, pages 696–723. Cambridge University Press, Cambridge.

[Andriesse, 2005] Andriesse, C.D. (2005). *Huygens. The man behind the principle.* Cambridge University Press, Cambridge.

[Apollonius, 1990] Apollonius (1990). *Conics, books V to VII. The Arabic translation of the lost Greek original in the version of the Banū Mūsā.* Springer, New York. Translated, edited, and commentated by G.J. Toomer.

[Apollonius, 2010] Apollonius (2010). *Treatise on conic sections.* Lightning Source, Milton Keynes. Translated and edited by Thomas Heath, reprint of ed. Cambridge 1896.

[Atzema and Vermij, 1995] Atzema, A.J. and Vermij, R.H. (1995). *Specilla Circularia*: an unknown work by Johannes Hudde. *Studia Leibnitiana*, 27:104–121.

[Baarsen and Estié, 2000] Baarsen, R.J. and Estié, J. (2000). *The splendour of the Dutch interior 1600–1800.* Salomon Stodel Antiquités, Amsterdam.

[Barnard, 1916] Barnard, J.P. (1916). *The casting-counter and the countingboard.* Clarendon Press, Oxford.

[Bartholin, 1651] Bartholin, Erasmus (1651). *Principia matheseos universalis, seu introductio ad geometriae methodum Renati Des Cartes.* Ex Officina Elseviriorum, Leiden.

[Bartholin, 1661] Bartholin, Erasmus (1661). *Principia matheseos universalis, seu introductio ad geometriae methodum Renati Des Cartes.* Apud Ludovicum & Danielem Elzevirios, Amsterdam. Published in [Schooten, 1661a], vol. 2, 1–47.

[Bartjens, 2004] Bartjens, Willem (2004). *De cijfferinghe (1604): het rekenboek van de beroemde schoolmeester.* Verloren, Hilversum. Edited by Danny Beckers and Marjolein Kool.

[Baumann, 1952] Baumann, E.D. ([1952]). *Een lijfarts der Oranjes in de XVIIde eeuw.* Kemink en Zoon, Utrecht.

[Beaune, 1649] Beaune, Florimond de (1649). *In Geometriam Renati Des Cartes Notae Breves.* Ex Officina Ioannis Maire, Leiden. Published in [Schooten, 1649b], 119–161.

[Beaune, 1659] Beaune, Florimond de (1659). *In Geometriam Renati Des Cartes Notae Breves.* Apud Ludovicum & Danielem Elzevirios, Amsterdam. Published in [Schooten, 1661a], vol. 1, 107–142.

[Beaune, 1661a] Beaune, Florimond de (1661a). *De limitibus aequationum.* Apud Ludovicum & Danielem Elzevirios, Amsterdam. Posthumously edited by Erasmus Bartholin and published in [Schooten, 1661a], vol. 2, 117–152.

[Beaune, 1661b] Beaune, Florimond de (1661b). *De natura et constitutione aequationum.* Apud Ludovicum & Danielem Elzevirios, Amsterdam. Posthumously edited by Erasmus Bartholin and published in [Schooten, 1661a], vol. 2, 49–116.

[Beaune, 1975] Beaune, Florimond de (1975). *Doctrine de l'angle solide.* Vrin, Paris. Introduction and annotation by Pierre Costabel, transcription by Bernard Barbiche.

[Beckers, 2003] Beckers, Danny (2003). *"Het despotisme der Mathesis": opkomst van de propaedeutische functie van de wiskunde in Nederland, 1750–1850.* Verloren, Hilversum.

[Belgioioso, 1990] Belgioioso, Giulia, editor (1990). *Descartes, Il metodo e i saggi. Atti del Convegno per il 350e anniversario della pubblicazione del Discours de la Méthode e degli Essais.* Aremando Paoletti.

[Berckelius, 1651] Berckelius, Petrus (1651). Disputationum physicarum ad Problemata Aristotelis tertiae de origine motus, pars prior. Disputation held on 14 June 1651.

[Berg, 1998] Berg, J. van den, editor (1998). *Biografisch lexicon voor de geschiedenis van het Nederlands protestantisme*, volume 4. Kok, Kampen.

[Berkel, 1988] Berkel, Klaas van (1988). A note on Rudolf Snellius and the early history of mathematics in Leiden. In Hay, C., editor, *Mathematics from manuscript to print 1300–1600*, pages 156–161. Clarendon Press, Oxford.

[Berkel, 2010] Berkel, Klaas van (2010). The Dutch Republic. Laboratory of the Scientific Revolution. *BMGN - Low Countries Historical Review*, 125(2-3):81–105.

[Biagioli, 1994] Biagioli, Mario (1994). *Galileo, courtier. The practice of science in the culture of absolutism.* The University of Chicago Press, Chicago and London. first edition in 1993.

[Bockstaele, 1985] Bockstaele, P. (1985). Gielis van den Hoecke en zijn sonderlinghe boeck in dye edel conste arithmetica. *Academiae Analecta. Mededelingen van de Koninklijke Academie voor wetenschappen, letteren en schone kunsten van België. Klasse der wetenschappen*, 47:3–29.

[Boethius, 1983] Boethius (1983). *Boethian Number Theory: A Translation of the* De Institutione Arithmetica, volume 6 of *Studies in Classical Antiquity.* Rodopi, Amsterdam. Introduction and translation by Michael Masi.

[Bordes, 1856] Bordes, J.P. de (1856). *De verdediging van Nederland in 1629: eene bijdrage tot de geschiedenis van het vaderland.* J.G. Broese, Utrecht.

[Bordus and Stammetz, 1758] Bordus, Wilhelmus la and Stammetz, Joan Levinus (1758). *Groot en volledig woordenboek der wiskunde: sterrekunde, meetkunde, rekenkunde, tuigwerkkunde, burger-, scheeps- en krygsbouwkunde, gezichtkunde, water- en vuurwerkkunde, benevens andere nuttige kunsten en wetenschappen ....* Steven van Esveldt, Amsterdam.

[Bos, 1981] Bos, H.J.M. (1981). On the representation of curves in Descartes' Géométrie. *Archive for History of Exact Sciences*, 24:395–338.

[Bos, 1984] Bos, H.J.M. (1984). Arguments on motivation in the rise and decline of a mathematical theory; the "construction of equations", 1637–ca.1750. *Archive for History of Exact Sciences*, 30:331–380.

[Bos, 1997a] Bos, H.J.M. (1997a). The concept of construction and the representation of curves in seventeenth-century mathematics. In *Lectures in the history of mathematics*, pages 23–36. American Mathematical Society, Providence. First published in *Proceedings of the International Congress of Mathematicians, August 3-11, 1986 Berkeley, California, USA,* A. Gleason ed. (Providence, American Mathematical Society, 1987), pages 1629–1641.

[Bos, 2001] Bos, H.J.M. (2001). *Redefining geometrical exactness. Descartes' transformation of the early modern concept of construction.* Springer, New York.

[Bos, 1997b] Bos, Henk J.M. (1997b). The structure of Descartes' Géométrie. In *Lectures in the history of mathematics*, History of mathematics 7, pages 37–57. American Mathematical Society, Providence. Corrected reprint of edition 1993.

[Bos, 2003] Bos, Henk J.M. (2003). *The Correspondence of René Descartes 1643*, chapter Descartes, Elisabeth and Apollonius' Problem, pages 202–211. Quaestiones Infinitae. Zeno Institute of Philosophy, Utrecht.

[Bos, 1992] Bos, Henk J. M. (1992). Descartes, Pappus' Problem and the Cartesian Parabola: a conjecture. In Hartman, P.M. and Shapiro, Alan E., editors, *An investigation of difficult things — Essays on Newton and the History of the Exact Sciences*, pages 71–96. Cambridge University Press, Cambridge.

[Bos, 2009] Bos, Henk J. M. (2009). Descartes' attempt, in the *Regulae*, to base the certainty of algebra on mental vision – a conjectural reconstruction. In Clark Glymour, Wang Wei and Westerståhl, Dag, editors, *Logic, Methodology and Philosophy of Science: Proceedings of the Thirteenth International Congress*, pages 354–373. College Publications, King's College London, London.

[Bos, 2002] Bos, Jan Jacobus Frederik Maria (2002). *The correspondence between Descartes and Henricus Regius.* Zeno, The Leiden-Utrecht Research Institute of Philosophy, Utrecht.

[Bos, 1675] Bos, Lambert van den (1675). *Leeven en bedrijf van syn hoogheyt Willem Hendrik de Derde, Prince van Orangien, en Nassau, etc.* Jan Claesz. ten Hoorn, Amsterdam.

[Bouchel and Bechefer, 1667] Bouchel, Laurent and Bechefer, Jean (1667). *La bibliotheque, ou, thresor du droit françois: ou sont traittées les matieres civiles, criminelles, et beneficiales, tant reglées par les ordonnances & coustumes de France, que decidées par arrests de cours souveraines ....* Chez Iacques Dallin, Paris.

[Bouwman, 2008] Bouwman, André, editor (2008). *Stad van boeken: handschrift en druk in Leiden, 1260–2000.* Primavera Pers, Leiden.

[Bowen and Imhof, 2008] Bowen, Karen L. and Imhof, Dirk (2008). *Christopher Plantin and engraved book illustrations in sixteenth-century Europe.* Cambridge University Press, Cambridge.

[Boyer, 1956] Boyer, Carl B. (1956). *History of analytic geometry.* Yeshiva University, New York.

[Brasser, 1663] Brasser, J.R. (1663). *Regula Cos, of Algebra, zijnde de alder-konstrijcksten regel om het onbekende bekent te maken: ofte een Korte Onderwijsinge, waer in geleert werdt het uyttrecken der Wortelen, soo verre men begeeren mag ....* Gerrit van Goedesbergh, Amsterdam.

[Brasser, 1672] Brasser, J.R. (1672). *Regula Cos, of Algebra, zijnde de alder-konstrijcksten Regel om het onbekende bekent te maken: ofte een Korte Onderwijsinge, waer in geleert werdt het uyttrecken der Wortelen, soo verre men begeeren mag ....* Amsterdam.

[Braunmühl, 1971] Braunmühl, A. von (1971). *Vorlesungen über Geschichte der Trigonometrie*, volume 2. Sändig, Niederwalluf bei Wiesbaden. Reprint of first edition of 1903.

[Breugelmans, 2003] Breugelmans, Ronald (2003). *Fac et Spera. Joannes Maire, Publisher, Printer and Bookseller in Leiden, 1603–1657.* PhD thesis, Universiteit Leiden.

[Briggs, 1617] Briggs, Henry ([1617]). *Logarithmorum chilias prima.*

[Briggs, 1624] Briggs, Henry (1624). *Arithmetica logarithmica sive Logarithmorum chiliades triginta, pro numeris naturali serie crescentibus ab unitate ad 20,000: et a 90,000 ad 100,000.* Excudebat Gulielmus Iones, London.

[Briggs and Gellibrand, 1633] Briggs, Henry and Gellibrand, Henri (1633). *Trigonometria Britannica: sive De doctrina triangvlorum libri dvo.* Excudebat Petrus Rammasenius, Gouda.

[Brugmans, 1898] Brugmans, H. (1898). *Catalogus Codicum Manu Scriptorum Universitatis Groningana Bibliothecae.* J.B. Wolters, Groningen.

[Buck, 1764] Buck, Friedrich Johann (1764). *Lebens-Beschreibungen derer verstorbenen Preußischen Mathematiker überhaupt und des vor mehr denn hundert Jahren verstorbenen großen Preußischen Mathematikers P. Christian Otters insbesondere.* Joh. Heinr. Hartungs Erben und J.D. Zeise, Königsberg and Leipzig.

[Burnett, 2005] Burnett, D. Graham (2005). *Descartes and the hyperbolic quest: lens making machines and their significance in the seventeenth century.* American Philosophical Society, Philadelphia.

[Ceulen, 1596] Ceulen, Ludolph van (1596). *Vanden circkel. Daerin gheleert werdt te vinden de naeste proportie des circkels-diameter tegen synen omloop... noch de tafelen sinuum, tangentium ende secantium ... ten laetsten van interest ....* Jan Andriesz., Delft.

[Clavius, 1581] Clavius, Christophorus (1581). *Gnomonices libri octo, in qvibvs non solum horologiorum solariu, sed aliarum quoq(ue) rerum, quae ex gnomonis umbra cognosci possunt, descriptiones geometrice demonstrantur.* Apud Franciscvm Zanettvm, Rome.

[Clavius, 1608] Clavius, Christophorus (1608). *Algebra.* Apud Bartholomaeum Zannettum, Rome.

[Clucas, 1999] Clucas, Stephen (1999). 'No small force': natural philosophy and mathematics in Thomas Gresham's London. In Ames-Lewis, Francis, editor, *Sir Thomas Gresham and Gresham College: studies in the intellectual history of London in the sixteenth and seventeenth centuries*, pages 146–173. Ashgate, Aldershot [etc.].

[Corry, 2008] Corry, Leo (2008). The development of the idea of proof up to 1900. In Gowers, Tim, editor, *The Princeton Companion to Mathematics*, pages 129–142. Princeton University Press, Princeton.

[Costabel, 1988] Costabel, Pierre (1988). La réception de la *Géométrie* et les disciples d'Utrecht. In Méchoulan, Henry, editor, *Problématique et réception du discours de la méthode et des essais*, Histoire des idées et des doctrines, pages 59–63, Paris. Librairie Philosophique.

[Dahl, 1995] Dahl, Per (1995). *Svensk ingenjörskonst under stormaktstiden: Olof Rudbecks tekniska undervisning och praktiska verksamhet.* Uppsala Universitet, Institutionen för idé- och lärdomshistoria, Uppsala.

[Darnal, 1666] Darnal, Jean (1666). *Supplément des chroniques de la noble ville et cité de Bourdeaux.* Par Iac. Mongiron Millanges, Bordeaux.

[Dear, 2011] Dear, Peter (2011). Mixed mathematics. In Harrison, Peter, Numbers, Ronald L., and Shank, Michael H., editors, *Wrestling with nature: from omens to science*, pages 149–172. University of Chicago Press.

[Decker, 1626] Decker, Ezechiel de (1626). *Nieuwe Telkonst, inhoudende de logarithmi voor de ghetallen beginnende van 1 tot 10000, mitsg. de Tafel van Hoeckmaten ende Raecklijnen door het ghebruyck van Logarithmi, de Wortel zijnde van 10000,0000 deelen, gemaeckt van Edm. Guntero, Welcke getallen eerst ghevonden zij van Ioc. Nepero Heer van Merchistoun: ende 't gebruyc daer van is . . . aenghewesen door Ez. de Decker.* By Pieter Rammaseyn, Gouda.

[Decker, 1627] Decker, Ezechiel de (1627). *Tweede deel van de nieuwe tel-konst ofte Wonderlijcke konstighe tafel, inhoudende de logarithmi, voor de getallen van 1 af tot 100000 toe . . . .* Pieter Rammaseyn, Gouda.

[Decker and Smeur, 1964] Decker, Ezechiel de and Smeur, A.J.E.M. (1964). *Tweede deel van de nieuwe tel-konst, 1627: facsimile of the only copy extant.* De Graaf, Nieuwkoop. Facsimile of [Decker, 1627].

[Decker et al., 1626] Decker, Ezechiel de, Vlacq, Adriaen, Napier, John, and Stevin, Simon (1626). *Eerste deel vande Nieuwe Telkonst, inh. verscheyde manieren van rekenen, waer door seer licht kunnen volbracht worden de geometrische ende arithmetische questien . . . / eerst ghevonden van Joanne Nepero . . . , ende uyt het Latyn overgheset door Adrianum Vlacq; waer achter bygevoegt zijn eenig seer lichte manieren van rekenen tot den coophandel dienstigh . . . mitsgaders Nieuwe Tafels van Interesten . . . door Ezechiel de Decker . . . noch hier achter byggevoeght de Thiende van Symon Stevin . . . .* By Pieter Rammaseyn, Gouda.

[Descartes, 1637a] Descartes, René (1637a). *Discours de la méthode pour bien conduire sa raison et chercher la verité dans les sciences; plus la dioptrique les meteores et la geometrie qui sont des essais de cete methode.* Ian Maire, Leiden.

[Descartes, 1637b] Descartes, René (1637b). *La dioptrique.* Ian Maire, Leiden. Published in [Descartes, 1637a].

[Descartes, 1637c] Descartes, René (1637c). *La géométrie.* Ian Maire, Leiden. Published in [Descartes, 1637a], a facsimile print of this edition in [Descartes, 1954].

[Descartes, 1644a] Descartes, René (1644a). *Principia philosophiae.* Apud Ludovicum Elzevirium, Amsterdam.

[Descartes, 1644b] Descartes, René (1644b). *Renati Des Cartes Specimina philosophiae: seu Dissertatio de methodo recte regendae rationis, et veritatis in scientiis investigandae: dioptrice, et meteora / ex Gallico translata, et ab auctore perlecta, variisque in locis emendata.* Apud Ludovicum Elzevirium, Amsterdam.

[Descartes, 1649] Descartes, René (1649). *Geometria.* Ioannis Maire, Leiden. Translation of [Descartes, 1637c] by Frans van Schooten, published in [Schooten, 1649b].

[Descartes, 1656] Descartes, René (1656). *Specimina philosophiae: seu dissertatio de methodo Recte regendae rationis, & veritatis in scientiis investigandae: Dioptrice, et Meteora.* Apud Ludovicum & Danielem Elzevirios, Amsterdam.

[Descartes, 1659] Descartes, René (1659). *Renati des Cartes Geometriae.* Apud Ludovicum & Danielem Elzevirios, Amsterdam. Translation of [Descartes, 1637c] by Frans van Schooten, revised version of [Descartes, 1649], published in [Schooten, 1661a], vol 1, 1–106.

[Descartes, 1897] Descartes, René (1897). *Oeuvres de Descartes*, volume 1. Léopold Cerf, Paris. Edited by Charles Adam and Paul Tannery.

[Descartes, 1898] Descartes, René (1898). *Oeuvres de Descartes*, volume 2. Léopold Cerf, Paris. Edited by Charles Adam and Paul Tannery.

[Descartes, 1899] Descartes, René (1899). *Oeuvres de Descartes*, volume 3. Léopold Cerf, Paris. Edited by Charles Adam and Paul Tannery.

[Descartes, 1901] Descartes, René (1901). *Oeuvres de Descartes*, volume 4. Léopold Cerf, Paris. Edited by Charles Adam and Paul Tannery.

[Descartes, 1902] Descartes, René (1902). *Oeuvres de Descartes*, volume 6. Léopold Cerf, Paris. Edited by Charles Adam and Paul Tannery.

[Descartes, 1903] Descartes, René (1903). *Oeuvres de Descartes*, volume 5. Léopold Cerf, Paris. Edited by Charles Adam and Paul Tannery.

[Descartes, 1904a] Descartes, René (1904a). *Oeuvres de Descartes*, volume 7. Léopold Cerf, Paris. Edited by Charles Adam and Paul Tannery.

[Descartes, 1904b] Descartes, René (1904b). *Oeuvres de Descartes*, volume 9. Léopold Cerf, Paris. Edited by Charles Adam and Paul Tannery.

[Descartes, 1905] Descartes, René (1905). *Oeuvres de Descartes*, volume 8. Léopold Cerf, Paris. Edited by Charles Adam and Paul Tannery.

[Descartes, 1908] Descartes, René (1908). *Oeuvres de Descartes*, volume 10. Léopold Cerf, Paris. Edited by Charles Adam and Paul Tannery.

[Descartes, 1909] Descartes, René (1909). *Oeuvres de Descartes*, volume 11. Léopold Cerf, Paris. Edited by Charles Adam and Paul Tannery.

[Descartes, 1910] Descartes, René (1910). *Oeuvres de Descartes*, volume 12. Léopold Cerf, Paris. Edited by Charles Adam and Paul Tannery.

[Descartes, 1913] Descartes, René (1913). *Oeuvres de Descartes*, volume Supplément. Léopold Cerf, Paris. Edited by Charles Adam and Paul Tannery.

[Descartes, 1936] Descartes, René (1936). *Correspondance*, volume 1. Librairie Félix Alcan, Paris. Edited by Charles Adam and Gaston Milhaud.

[Descartes, 1939] Descartes, René (1939). *Correspondance*, volume 2. Librairie Félix Alcan, Paris. Edited by Charles Adam and Gaston Milhaud.

[Descartes, 1941] Descartes, René (1941). *Correspondance*, volume 3. Presses universitaires de France, Paris. Edited by Charles Adam and Gaston Milhaud.

[Descartes, 1947] Descartes, René (1947). *Correspondance*, volume 4. Presses universitaires de France, Paris. Edited by Charles Adam and Gaston Milhaud.

[Descartes, 1951] Descartes, René (1951). *Correspondance*, volume 5. Presses universitaires de France, Paris. Edited by Charles Adam and Gaston Milhaud.

[Descartes, 1954] Descartes, René (1954). *The geometry of René Descartes with a facsimile of the first edition.* Dover publications, New York. Translated from French and Latin by David Eugene Smith and Marcia L. Latham.

[Descartes, 1956] Descartes, René (1956). *Correspondance*, volume 6. Presses universitaires de France, Paris. Edited by Charles Adam and Gaston Milhaud.

[Descartes, 1960] Descartes, René (1960). *Correspondance*, volume 7. Presses universitaires de France, Paris. Edited by Charles Adam and Gaston Milhaud.

[Descartes, 1963] Descartes, René (1963). *Correspondance*, volume 8. Presses universitaires de France, Paris. Edited by Charles Adam and Gaston Milhaud.

[Descartes, 2003] Descartes, René (2003). *The correspondence of René Descartes 1643*. Zeno Institute for Philosophy, Utrecht. Edited by Theo Verbeek, Erik-Jan Bos and Jeroen van de Ven.

[Descartes, 2007] Descartes, René (2007). *Specimina philosophiae*. Zeno Institute for Philosophy, The Leiden-Utrecht Research Institute, Utrecht. Introduction and critical edition by Corinna Vermeulen.

[Descartes and Huygens, 1926] Descartes, Rene and Huygens, Constantijn (1926). *Correspondence of Descartes and Constantijn Huygens 1635–1647*. Clarendon Press, Oxford. Edited by Leon Roth.

[Dijksterhuis, 2004] Dijksterhuis, Fokko Jan (2004). *Lenses and waves: Christiaan Huygens and the mathematical science of optics in the seventeenth century*. Springer.

[Dijksterhuis, 2007] Dijksterhuis, Fokko Jan (2007). Constructive thinking. A case for dioptrics. In Roberts, Lisa, Schaffer, Simon, and Dear, Peter, editors, *The Mindful Hand: Inquiry and Invention from the Late Renaissance to Early Industrialisation*, pages 59–82. Koninklijke Nederlandse Akademie van Wetenschappen, Amsterdam.

[Dijksterhuis, 2011] Dijksterhuis, Fokko Jan (2011). Moving around the ellipse. Conic sections in Leiden, 1620–1660. In Dupré, Sven and Lüthy, Christoph, editors, *Silent Messengers: The Circulation of Material Objects of Knowledge in the Early Modern Low Countries*, pages 89–124. Lit, Berlin.

[Dijkstra, 2012] Dijkstra, Arjen (2012). *Between academics and idiots. A cutural history of mathematics in the Dutch province of Friesland (1600–1700)*. PhD thesis, Universiteit Twente.

[Diophantus of Alexandria and Bachet, 1621] Diophantus of Alexandria and Bachet, Claude Gaspar (1621). *Diophanti Alexandrini Arithmeticorvm libri sex, et de nvmeris mvltangvlis liber vnvs. Nunc primom Graecè & Latinè editi, atque absolutissimis commentariis illustrati*. Sumptibus Hieronymi Drovart, Paris.

[Dold-Samplonius, 1968] Dold-Samplonius, Yvonne (1968). Die Handschriften der Amsterdamer mathematischen Gesellschaft. *Janus, revue internationale de l'histoire des sciences, de la médicine, de la pharmacie et de la technique*, 60:241–303.

[Donkersloot-de Vrij, 1981] Donkersloot-de Vrij, Ypkje Marijke (1981). *Topografische kaarten van Nederland vóór 1750: handgetekende en gedrukte kaarten aanwezig in de Nederlandse rijksarchieven*. Wolters-Noordhoff, Groningen.

[Duffy, 1979] Duffy, Christopher (1979). *Siege warfare. The fortress in the early modern world 1494–1660*. Routledge & Kegan Paul, London and Henley.

[Eerde, 1833] Eerde, J.R. van (1833). *Catalogus librorum Bibliothecae Universitatis quae Groningae est . . .* . Oomkens, Groningen.

[Eijnatten, 2004] Eijnatten, Joris van (2004). Between practice and principle. Dutch ideas on censorship and press freedom, 1579–1795. *Redescriptions. Yearbook for political thought and conceptual history*, 8:85–113.

[Eimer, 1961] Eimer, Gerhard (1961). *Die Stadtplanung im Schwedischen Ostseereich, 1600–1715 mit Beiträgen zur Geschichte der Idealstadt*. Svenska Bokförlaget, Stockholm.

[Elsner, 1988] Elsner, Bernd (1988). *"Apollonius Saxonicus": die Restitution eines verlorenen Werkes des Apollonius von Perga durch Joachim Jungius, Woldeck Weland und Johannes Müller*. Vandenhoeck & Ruprecht, Göttingen.

[Epple et al., 2013] Epple, Moritz, Hoff Kjeldsen, Tinne, and Siegmund-Schultze, Reinhard (2013). From "mixed" to "applied" mathematics: tracing an important dimension of mathematics and its history. Technical Report 12/2013, Mathematisches Forschungsinstitut Oberwolfach.

[Eriksson, 2004] Eriksson, G. (2004). Olaus Rudbeck as scientist and professor of medicine. *Svensk medicinhistorisk tidskrift*, 8:39–44.

[Euclid, 1617] Euclid (1617). *De propositien van de XV boucken der elementen Euclidis*. Govert Basson, Leiden. Translated and edited by Frans van Schooten sr.

[Euclid, 1625] Euclid, Claude Hardy, Marinus Neapolitanus (1625). Ευκλειδου Δεδομενα. και Μαρινου φιλοσοφου Εις Δεδομενα Ευκλειδου υπομνεμα *Euclidis Data: opus ad veterum geometriae autorum Archimedis, Apollonij, Pappi, Eutocij, caterorumque non modo lectionem, sed ad geometricae quoque analyseos instaurationem plane necessarium, et à multis diu desideratum / Claudius Hardy Sebast. Fil. graece nunc primum ed., latine vertit scholiisque illustravit; adjectus est eadem bibliotheca Marini philosophi Commentarius Graecè & Latinè, quo Dati natura, Datorumque Euclideorum utilitates explicantur*. Impensis Melchioris Mondiere, Paris.

[Feingold, 1984] Feingold, Mordechai (1984). *The mathematicians' apprenticeship: science, universities and society in England, 1560–1640*. Cambridge University Press.

[Feingold, 1999] Feingold, Mordechai (1999). Gresham College and London practicioners: the nature of the English mathematical community. In Ames-Lewis, Francis, editor, *Sir Thomas Gresham and Gresham College: studies in the intellectual history of London in the sixteenth and seventeenth centuries*, pages 174–188. Ashgate, Aldershot.

[Ferguson and Fockens, 1667] Ferguson, Johan Jacob and Fockens, Tiado ([1667]). *Labyrinthus algebrae. Verdeelt in vijf Deelen, Verhandelende de Ontbindinge der ʒ, ℭ en ʒʒ vergelijkingen; Als mede de Verhandelinge van Alderhande Getallen, met haer oneyndelijcke AGGREGATEN. Mitsgaders verscheyde EXEMPELEN Tot verklaringe van alle 't voorsz. Door Joannem Jacobum Ferguson, Liefhebber der Mathesische Consten. Hier zijn noch by gevoegt twaelf questien, van Tiado Fockens*. Gedruckt voor den Autheur, en sijnde te bekomen by Johannes Tongerlo, en Jacob Scheltus, Den Haag.

[Fermat, 1891] Fermat, Pierre de (1891). *Oeuvres de Fermat*, volume 1: Oeuvres mathématiques diverses. Observations sur Diophante. Gauthier-Villars et fils, Paris. Edited by Paul Tannery and Charles Henry.

[Fermat, 1894] Fermat, Pierre de (1894). *Oeuvres de Fermat*, volume 2: Correspondance. Gauthier-Villars et fils, Paris. Edited by Paul Tannery and Charles Henry.

[Fermat, 1896] Fermat, Pierre de (1896). *Oeuvres de Fermat*, volume 3: Traductions 1. Des écrits et fragments Latins de Fermat; 2. De l'Inventum novum de Jacq. de Billy; 3. Du Commercium epistolicum de Wallis. Gauthier-Villars et fils, Paris. Edited by Paul Tannery.

[Fermat, 1912] Fermat, Pierre de (1912). *Oeuvres de Fermat*, volume 4: Compléments: Supplément à la correspondance; Appendice; Notes et tables. Gauthier-Villars, Paris. Edited by Charles Henry.

[Fermat, 1922] Fermat, Pierre de (1922). *Oeuvres de Fermat*, volume Supplément aux tomes 1-4: Documents inédits avec notices sur les nouveaux manuscrits. Gauthier-Villars et Cie, Paris. Edited by C. de Waard.

[Ferrier, 1980] Ferrier, Richard Delahire (1980). *Two exegetical treatises of François Viète translated, annotated and explained.* PhD thesis, Indiana University.

[Flaccus, 1608] Flaccus, Quintus Horatius (1608). *Q. Horatius Flaccus, cum erudito Laevini Torrentii commentario, nunc primum in lucem edito. Item Petri Nannii Alcmariani in artem poëticam.* Ex officina Plantiniana apud Joannem Moretum, Antwerp.

[François Viète, 1983] François Viète, translator T. Richard Witmer (1983). *The analytic art.* The Kent State University Press, Kent.

[Freitag, 1631] Freitag, Adamus (1631). *Architectvra militaris nova et aucta, oder Newe vermehrte Fortification.* B. [&] A. Elzeviers, Leiden.

[Freitag, 1635a] Freitag, Adam (1635a). *Architectvra militaris nova et aucta, oder Newe vermehrte fortification, von regular vestungen, von irregular vestungen vnd aussen wercken, von praxi offensiva vnd defensiva: auff die neweste Niederländische praxin gerichtet vnd beschrieben.* Bey Bonaventura vnd Abraham Elzeviers, Leiden.

[Freitag, 1635b] Freitag, Adam (1635b). *L'architecture militaire, ou La fortification nouvelle, augmentée et enrichie de forteresses regulieres, irregulieres et de dehors; le tout à la practique moderne.* Chez les Elzeviers, Leiden.

[Freitag, 1642] Freitag, Adam (1642). *Architectura militaris nova et aucta, oder newe vermehrte Fortification, von regular Vestungen, von irregular Vestungen und Aussen wercken, von praxi offensiva und defensiva / auff die neweste Niederländische Praxin gerichtet und beschrieben.* Bonaventura und Abraham Elzevier, Leiden.

[Freitag, 1657] Freitag, Adam (1657). *Architecture militaire, ou La fortification nouvelle: augmentée et enrichie de forteresses regulieres, irregulieres et de dehors; le tout à la practique moderne.* 66, Paris.

[Frénicle de Bessy, 1657] Frénicle de Bessy, Bernard (1657). *Solutio duorum problematum circa numeros cubos & quadratos, quae tanquam insolubilia universis Europae mathematicis a clarissimo viro D. Fermat sunt proposita, & ad D. Cl. M. Laurenderium doctorem medicum transmissa. A D. B. F. D. B. inventa. Nec non. Alia duo problemata numerica a D. Cl. M. Laurenderio vicissim proposita, cum quibusdam solutionibus ab eodem D. F. D. B. datis. His accessit. Inquisitio in solutionem prioris problematis à D. Francisco à Schooten in academia Lugduno Batava matheseos professore datam. In qua continentur. Sex aliae solutiones prioris problematis terminis analiticis ab eodem D. B. F. sub forma problematis datae. Insuper et. Solutio alterius problematis ab eodem Cl. viro D. Fermat circa numeros unitate à quadrato deficientes propositi, cum ipsius solutionis constructione.* Apud Jacobum Langlois, Paris.

[Friesenborch and Wilken, 1624] Friesenborch, Hermann and Wilken, Martin (1624). *Arithmetica, dat is Ein nye und wol gegrundet Rekenboeck mit veelen schönen lustigen Exempeln und upgaven. Sampt einen lustigen cossischen und polygonal. Appendix.* Embden.

[Friesenburch, 1638] Friesenburch, Hermann (1638). *Geometrisches Lust-gärtlein, darauss man Radicem Quadratam vnd Cubicam extrahiren vnd lernen kan, mit angeh. Taf.* Helvicus Kallenbach, Emden.

[Frijhoff, 2002] Frijhoff, Willem (2002). *Embodied Belief: Ten Essays on Religious Culture in Dutch History.* Verloren, Hilversum.

[Frijhoff and Spies, 2004] Frijhoff, Willem and Spies, Marijke (2004). *1650 hard-won unity.* Van Gorcum, Assen.

[Frisius, 1540] Frisius, Gemma (1540). *Arithmeticae practicae methodus facilis.* Gregorius Bontius, Antwerp.

[Gaemers, 2004] Gaemers, C. (2004). *Nalatenschap als toekomst: de Fundatie van de Vrijvrouwe van Renswoude 1754–1810.* Walburg Pers, Zutphen.

[Galuzzi, 1990] Galuzzi, Massimo (1990). I marginalia di Newton alla seconda edizione latina della Geometria di Descartes e i problemi ad essi collegati. In *Descartes, Il metodo e i e saggi. Atti del Convegno per il 350e anniversario della pubblicazione del Discours de la Méthode e degli Essais*, pages 387–417. Aremando Paoletti.

[Garber, 1995] Garber, Klaus (1995). Königsberger Bücher in Polen, Litauen und Rußland. *Nordost-Archiv*, 4:29–61.

[Gaullieur, 1874] Gaullieur, Ernest (1874). *Histoire du Collége de Guyenne, d'après un grand nombre de documents inédits.* Sandoz et Fischbacher, Paris.

[Goldmann, 1643] Goldmann, Nicolaus (1643). *Elementorum architecturae militaris libri IV.* Ex off. Elseviriana, Leiden.

[Goldmann, 1645] Goldmann, Nicolaus (1645). *La nouvelle fortification.* Chez les Elseviers, Leiden.

[Goldstein, 2012] Goldstein, Carl (2012). *Print Culture in Early Modern France: Abraham Bosse and the Purposes of Print.* Cambridge University Press, Cambridge.

[Goldstine, 1977] Goldstine, Herman H. (1977). *A history of numerical analysis from the 16th through the 19th century.* Springer-Verlag, New York e.a.

[Golius, 1653] Golius, Jacobus (1653). *Lexicon Arabico-Latinum, contextum ex probatioribus Orientis lexicographis.* Bonaventurae et Abrahami Elseviriorum, Leiden.

[Goudeau, 2005] Goudeau, Jeroen (2005). *Nicolaus Goldmann (1611–1665) en de wiskundige architectuurwetenschap.* Philip Elchers, Groningen.

[Graaf, 1662] Graaf, Abraham de (1662). *Principia arithmeticae, theoreticae, & practicae: of De beginselen der telkunst, of rekenkunst; haar eigenschappen en toepassing, voornamelijk in de koophandel ....* Steven van Lier, Amsterdam.

[Graaf, 1672] Graaf, Abraham de (1672). *De beginselen van de algebra of stelkonst, volgens de manier van Renatus Des Cartes: verklaart met uytgelezene voorbeelden; zoo wel in de meetkonst, als in de rekenkonst.* Jan Rieuwertsz, Amsterdam.

[Graaf, 1676] Graaf, Abraham de (1676). *De geheele mathesis of wiskonst, herstelt in zijn natuurlijke gedaante.* Amsterdam.

[Graaf, 1694] Graaf, Abraham de (1694). *De geheele mathesis of wiskonst, herstelt in zijn natuurlijke gedaante.* Gedrukt, by Jacobus de Veer, voor Jan ten Hoorn, Amsterdam.

[Graaf, 1717] Graaf, Abraham de (1717). *De geheele mathesis of wiskonst, herstelt in zijn natuurlijke gedaante.* By Joannes Loots en Andries van Damme, Amsterdam.

[Graaff, 1966] Graaff, A.F. de (1966). Remonstranten uit Leiden en omgeving. *Rijnland*, pages 246–262.

[Grafton and Jardine, 1986] Grafton, Anthony and Jardine, Lisa (1986). *Humanism to the Humanities: Education and the liberal arts in Fifteenth and Sixteenth-Century Europe.* Duckworth, London.

[Grattan-Guinness, 2004] Grattan-Guinness, I. (2004). The mathematics of the past: distinguishing its history from our heritage. *Historia Mathematica*, 31:163–185.

[Grootendorst and de Witt, 1997] Grootendorst, A.W. and Witt, Jan de (1997). *Elementa curvarum linearum liber primus.* Stichting Mathematisch Centrum, Amsterdam.

[Grosholz, 1991a] Grosholz, Emily (1991a). Descartes' *Geometry* and the classical tradition. In Barker, Peter and Ariew, Roger, editors, *Revolution and continuity. Essays in the history and philosophy*, pages 183–196. The catholic university of America press, Washington D.C.

[Grosholz, 1991b] Grosholz, Emily R. (1991b). *Cartesian method and the problem of reduction.* Clarendon Press, Oxford.

[Grotius, 1629] Grotius, Hugo (1629). *Grollae obsidio cum annexis anni MDCXXVII.* Apud Guilielmum Blaeuw, Amsterdam.

[Haaften, 1928] Haaften, M. van (1928). Ce n'est pas Vlack, en 1628, mais De Decker, en 1627, qui a publié le premier, un table de logarithmes étendue et complète. *Nieuw Archief voor wiskunde*, 2/15:49–54.

[Haaften, 1941] Haaften, M. van (1941). De Decker, Vlacq en de oudste logarithmetafels. *De Verzekeringsbode*, 60:2–3, 7–8.

[Haan, 1878] Haan, D. Bierens de (1878). *Bouwstoffen voor de geschiedenis der wis- en natuurkundige wetenschappen in de Nederlanden.* [Amsterdam]. Offprint of the *Verslagen en Mededeelingen der Kon. Akademie van Wetenschappen*, Afd. Natuurk., 2e Reeks, Deel VIII, IX, X en XII.

[Haas, 1956] Haas, Karlheinz (1956). Die mathematischen Arbeiten von Johann Hudde. *Centaurus*, 4:235–284.

[Harmsen, 1978] Harmsen, Th. W. (1978). *De beknopte Lant-Meet-Konst: beschrijving van het leven en werk van de Dordtse landmeter Mattheus van Nispen (circa 1628–1717).* Delftse universitaire pers, Delft.

[Haro, 1680] Haro, Felix Lopez de (1680). *Catalogus rarissimorum librorum variarum disciplinarum, sed praecipue Mathematicorum & Astronomicorum, clarissimi & doctissimi viri Petrus à Schoooten.* Ex officina Felici Lopez, Leiden.

[Heath, 1981] Heath, Thomas Little (1981). *A History of Greek Mathematics: I. From Thales to Euclid*, volume 1. Dover Publications. Reprint of the 1921 edition of Clarendon Press, Oxford.

[Heath, 2006] Heath, Thomas Little (2006). *A History of Greek Mathematics: II. From Aristarchus to Diophantus*, volume 2. Elibron press. Reprint of the 1921 edition of Clarendon Press, Oxford.

[Heeffer, 2008a] Heeffer, Albrecht (2008a). A conceptual analysis of early Arabic algebra. In Rahman, Shahid, Street, Tony, and Tahiri, Hassan, editors, *The unity of science in the Arabic tradition: science, logic, epistemology and their interactions*, pages 89–128. Springer Verlag, w.p.

[Heeffer, 2008b] Heeffer, Albrecht (2008b). The emergence of symbolic algebra as a shift in predominant models. *Foundations of Science*, 13:149–161.

[Heeffer, 2009] Heeffer, Albrecht (2009). On the nature and origin of algebraic symbolism. In Kerckhove, B. van, editor, *New perspectives on mathematical practices. Essays in Philosophy and History of Mathematics*, pages 1–27, Singapore. World Scientific.

[Heuraet, 1659] Heuraet, Hendrick van (1659). *Epistola de transmutatione curvarum linearum in rectas.* Apud Ludovicum & Danielem Elzevirios, Amsterdam. Published in [Schooten, 1661a], vol. 1, 517–520.

[Heuvel, 1991] Heuvel, Charles van den (1991). *'Papiere bolwercken'. De introductie van de Italiaanse stede- en vestingbouw in de Nederlanden (1540–1609) en het gebruik van tekeningen.* Canaletto, Alphen aan den Rijn.

[Heuvel, 2006] Heuvel, Charles van den (2006). De vesting als mathematisch en cultureel kennis-systeem. Het onderwijs in de vestingbouw aan hovelingen, kooplui en ambachtslieden in de Hollandse Republiek (17e eeuw). *Scientiarum Historia*, 32:99–117.

[Hobbes and Malcolm, 1994] Hobbes, Thomas and Malcolm, Noel (1994). *The correspondence of Thomas Hobbes*, volume 2 1660–1679. Clarendon Press, Oxford.

[Hoecke, 1537] Hoecke, Gielis van den (1537). *Een sonderlinghe boeck in dye edel conste arithmetica: met veel schoone perfecte regulen als die numeracie vanden ghetale metten specien int gheheele ende int ghebroken ....* By mi S. Cock, Antwerp.

[Hoefer, 1928] Hoefer, F.A. (1928). Mededeelingen omtrent het onderwijs in de versterkingskunst aan onze hooge en illustre scholen. *Bijdragen voor vaderlandsche geschiedenis en oudheidkunde*, 7:205–242.

[Hofmann, 1944] Hofmann, J. E. (1944). Neues über Fermats zahlentheoretische Herausforderungen von 1657 (mit zwei bisher unbekannten Originalstücken Fermats). *Abhandlungen der Preussischen Akademie der Wissenschaften, Mathematische-Naturwissenschaftliche Klasse*, 1943-9.

[Hofmann, 1962] Hofmann, Joseph Ehrenfried (1962). *Frans van Schooten der Jüngere*. Franz Steiner Verlag, Wiesbaden.

[Hofmann, 1975] Hofmann, J. E. (1975). Schooten, Frans van. In Gillespie, Charles Coulston, editor, *Dictionary of Scientific Biography*, volume 12, pages 205–207. Scribner, New York.

[Hoftijzer, 2007] Hoftijzer, P.G. (2007). Leidse studentenbibliotheken in de zeventiende eeuw. In e.a., J.M.M. Hermans, editor, *De Franeker universiteitsbibliotheek in de zeventiende eeuw*, pages 135–153. Verloren, Hilversum.

[Hoftijzer, 2008] Hoftijzer, P.G. (2008). Het Leidse wonder. In *Jaarverslag 2007 van de Koninklijke Brill NV*, pages 73–96. E.J. Brill, Leiden.

[Hogendijk, 2010] Hogendijk, Jan P. (2010). The scholar and the fencing master: The exchanges between Joseph Justus Scaliger and Ludolph van Ceulen on the circle quadrature (1594–1596). *Historia Mathematica*, 37:345–375.

[Horace, 2008] Horace (2008). *The satires of Horace, translated by A.M. Juster*. University of Pennsylvania Press, Philadelphia.

[Hotson, 2007] Hotson, Howard (2007). *Commonplace Learning: Ramism and Its German Ramifications, 1543–1630*. Oxford University Press, Oxford.

[Houtzager, 1994] Houtzager, Hendrik Leonard (1994). Enkele wiskundigen uit de kring rond Jan Hugo Cornets de Groot. *Delfia Batavorum jaarboek*, pages 49–59.

[Hudde, 1659] Hudde, Johannes (1659). *Epistola duae, quarum altera de Aequationum Reductione, altera de Maximis & Minimis*. Apud Ludovicum & Danielem Elzevirios, Amsterdam. Published in [Schooten, 1661a], vol. 1, 401–516.

[Huisman, 2003] Huisman, Gerda C. (2003). Bibliothecae instructissimae. Geleerd boekenbezit in Groningen in de 17e en 18e eeuw. In jr., A.H. Huussen, editor, *Onderwijs en onderzoek. Studie en wetenschap aan de academie van Groningen in de 17e en 18e eeuw.*, pages 299–328. Verloren, Hilversum.

[Hulshoff-Pol, 1975] Hulshoff-Pol, Elfriede (1975). The library. In Scheurleer, Th.H. Lunsingh and Meyjes, G.H.M. Posthumus, editors, *Leiden university in the seventeenth century: an exchange of learning*, pages 393–459. Universitaire pers Leiden E.J. Brill, Leiden.

[Huygens, 1660] Huygens, Christiaan (1659–1660). *Van rekeningh in spelen van geluck.* Gerrit van Goedesbergh, Amsterdam. Published in [Schooten, 1660a], 485–500.

[Huygens, 1888] Huygens, Christiaan (1888). *Oeuvres complètes, Correspondance 1638–1656*, volume 1. Martinus Nijhoff, Den Haag. Edited by D. Bierens de Haan.

[Huygens, 1889] Huygens, Christiaan (1889). *Oeuvres complètes, Correspondance 1657–1659*, volume 2. Martinus Nijhoff, Den Haag. Edited by D. Bierens de Haan.

[Huygens, 1890] Huygens, Christiaan (1890). *Oeuvres complètes, Correspondance 1660–1661*, volume 3. Martinus Nijhoff, Den Haag. Edited by D. Bierens de Haan.

[Huygens, 1910] Huygens, Chrstiaan (1910). *Oeuvres complètes. Travaux de mathématiques pures 1652–1656*, volume 12. Martinus Nijhoff, Den Haag. Edited by D.J. Korteweg.

[Huygens, 1913] Huygens, Constantijn (1913). *De briefwisseling van Constantijn Huygens, (1608–1687)*, volume 2 (1634–1639) of *Rijks geschiedkundige publicatiën. Grote serie 24*. Martinus Nijhoff, Den Haag. Edited by J.A. Worp.

[Huygens, 1915] Huygens, Constantijn (1915). *De briefwisseling van Constantijn Huygens, (1608–1687)*, volume 4 (1644–1649) of *Rijks geschiedkundige publicatiën. Grote serie 24*. Martinus Nijhoff, Den Haag. Edited by J.A. Worp.

[Huygens, 1916] Huygens, Chrstiaan (1916). *Oeuvres complètes. Dioptrique 1653, 1666, 1685–1692. Fascicule I 1653, 1666*, volume 13. Martinus Nijhoff, Den Haag. Edited by D.J. Korteweg.

[Huygens, 1920] Huygens, Chrstiaan (1920). *Oeuvres complètes. Probabilités. Travaux de mathématiques pures 1655–1666*, volume 14. Martinus Nijhoff, Den Haag. Edited by D.J. Korteweg.

[Hynes, 2010] Hynes, Darren (2010). Parallel traditions in the image of Descartes: Iconography, intention, and interpretation. *The international history review*, 32:575–597.

[Imhausen and Remmert, 2006] Imhausen, Annette and Remmert, Volker (2006). The oration on the dignity and the usefulness of the mathematical sciences of Martinus Hortensius (Amsterdam 1634): Text, translation and commentary. *History of universities*, 21:71–150.

[Israel, 1995] Israel, Jonathan I. (1995). *The Dutch republic: its rise, greatness, and fall 1477–1806.* Clarendon Press, Oxford.

[Jansen-Sieben, 1989] Jansen-Sieben, R. (1989). *Repertorium van de Middelnederlandse Artes-literatuur.* HES, Utrecht.

[Johnston, 1996] Johnston, Stephen (1996). The identity of the mathematical practitioner in 16th-century England. In Hantsche, Irmgarde, editor, *Der "mathematicus": Zur Entwicklung und Bedeutung einer neuen Berufsgruppe in der Zeit Gerhard Mercators*, volume 4 of *Duisburger Mercator-Studien*, pages 93–120. Brockmeyer, Bochum.

[Jorink, 2008] Jorink, Eric (2008). *'Geef zicht aan de blinden': Constantijn Huygens, René Descartes en het Boek der Natuur.* Huygens Museum Hofwijck, Voorburg. Constantijn lezing 2007.

[Jullien, 1997] Jullien, Vincent (1997). *Descartes: la Géométrie de 1637.* Presses universitaires de France, Paris.

[Jullien, 1998] Jullien, Vincent (1998). Descartes-Roberval, une relation tumultueuse. *Revue d'histoire des sciences*, 51:363–372.

[Jungius and Avé-Lallemant, 1863] Jungius, Joachim and Avé-Lallemant, Robert C.B. (1863). *Dr. Joachim Jungius aus Lübeck Briefwechsel mit seinen Schülern und Freunden: ein Beitrag zur Kenntnis des grossen Jungius und der wissenschaftlichen wie socialen Zustände zur Zeit des dreissigjährigen Krieges.* F. Aschenfeldt, Lübeck.

[Juynboll, 1931] Juynboll, W.M.C. (1931). *Zeventiende-eeuwsche beoefenaars van het Arabisch in Nederland.* Kemink en Zoon, Utrecht.

[Kallinen, 2006] Kallinen, Maija (2006). Lectures and practices. the variety of mathematical and mechanical teaching at the University of Uppsala in the 17th century. In Feingold, M. and Navarro-Brotons, V., editors, *Universities and Science in the Early Modern Period*. Springer, Dordrecht.

[Kampen, 1826] Kampen, Nicolaas Godfried van (1826). *Beknopte geschiedenis der letteren en wetenschappen in de Nederlanden, van de vroegste tijden af, tot op het begin der negentiende eeuw*, volume 3. Weduwe J. Allart, Delft.

[Kästner, 1799] Kästner, Abraham Gotthelf (1799). *Geschichte der Mathematik: Seit der Wiederherstellung der Wissenschaften bis an das Ende des achtzehnten Jahrhunderts*, volume 3: Reine Mathematik, Analysis, praktische Geometrie, bis an Cartesius. Johan Goerg Rosenbusch, Göttingen.

[Katscher, 1979] Katscher, Friedrich (1979). Einige Entdeckungen über die Geschichte der Zahl Pi sowie Leben und Werk von Christoffer Dybvad und Ludolph van Ceulen. *Denkschriften Philosophisch-historische Klasse*, 116:85–129. Österreichische Akademie der Wissenschaften; Kommission für Geschichte der Mathematik und der Naturwissenschaften.

[Keblusek, 1997] Keblusek, Marika (1997). *Boeken in de hofstad: Haagse boekcultuur in de Gouden Eeuw.* Verloren, Hilversum.

[Kinckhuysen, 1661] Kinckhuysen, Gerard (1661). *Algebra ofte stel-konst, beschreven tot dienst van de leerlinghen.* Passchier van Wesbusch, Haarlem.

[Kingma, 2000] Kingma, J. (2000). Uitgaven met verstrekkende gevolgen. De Elzeviers als verspreiders van nieuwe denkbeelden. In Dongelmans, B.P.M., Hoftijzer, P.G., and Lankhorst, O.S., editors, *Boekverkopers van Europa: het 17de-eeuwse Nederlandse uitgevershuis Elzevier*. Walburg Pers, Zutphen.

[Kist, 1836] Kist, N.C. (1836). Aanteekeningen uit de synodale vergadering van Zuidholland van al het voorgevallene in de zaak en leer der remonstranten, 1619–1777. *Archief voor kerkelijke geschiedenis, inzonderheid van Nederland*, 7:1–331.

[Knappert, 1938] Knappert, L. (1938). Prof. Frans van Schooten jr. en Margrieta Wijnants maken hun testament. *Jaarboekje voor geschiedenis en oudheidkunde van Leiden en omstreken*, 30:164–170.

[Koeman, 1967] Koeman, C. (1967). *Atlantes Neerlandici: bibliography of terrestrial , maritime and celestial atlases and pilot books, published in the Netherlands up to 1880*, volume 1. Theatrum Orbis Terrarum, Amsterdam.

[Komorowski, 1980] Komorowski, Manfred (1980). Das Schicksal der Staats- und Universitätsbibliothek Königsberg. *Bibliothek Forschung und Praxis*, 4:139–154.

[Komorowski, 2001] Komorowski, Manfred (2001). Duisburger Studenten der frühen Neuzeit. Website. Paper given at the Institut für Niederrheinische Kulturgeschichte und Regionalentwicklung on 21 November 2001.

[Kool, 1991] Kool, Marjolein (1991). Op zoek naar de bron van een bijzonder arithmeticatraktaat. In Gerritsen, W.P., Gijsen, Annelies van, and Lie, Orlanda S.H., editors, *Een school spierinkjes. Kleine opstellen over Middelnederlandse artes-literatuur*, pages 104–106. Verloren, Hilversum.

[Kool, 1999] Kool, Marjolein (1999). *Die conste vanden getale. Een studie over Nederlandstalige rekenboeken uit de vijftiende en zestiende eeuw, met een glossarium van rekenkundige termen.* Verloren, Hilversum.

[Korteweg, 1894] Korteweg, D.J. ([1894]). *Het bloeitijdperk der wiskundige wetenschappen in Nederland.* Amsterdamsche Universitieit.

[Kowalewski, 1935] Kowalewski, Arnold (1935). Eine Descartes-Reliquie in Königsberg. *Kant-Studien*, 40:264–269.

[Kramm, 1861] Kramm, Christiaan (1861). *De levens en werken der Hollandsche en Vlaamsche kunstschilders, beeldhouwers, graveurs en bouwmeesters, van den vroegsten tot op onzen tijd*, volume 5: P-S. Diederichs, Amsterdam.

[Krogt, 2000] Krogt, P.C.J. van der (2000). *Koeman's Atlantes Neerlandici*, volume 2. HES, 't Goy-Houten.

[Krüger, 2010] Krüger, Jenneke (2010). Lessons from the early seventeenth century for mathematics curriculum design. *BSHM Bulletin*, 25:144–161.

[La Faille, 1774] La Faille, Jacob Baart de (1774). *Dissertatio philosophica inauguralis de methodo exhaustionis.* C. Van Hoogeveen, Leiden.

[Le Noir, 1974] Le Noir, Timothy (1974). *The social and intellectual roots of discovery in seventeenth century mathematics.* PhD thesis, Indiana University.

[Lipstorp, 1653] Lipstorp, Daniel (1653). *Specimina philosophiae Cartesianae: Quibus accedit ejusdem authoris Copernicus redivivus.* Apud Johannem & Danielem Elsevier, Leiden.

[Loget, 2004] Loget, F. (2004). La Ramée critique d'Euclide. Sur le *Prooemium mathematicum* (1567). *Archives Internationales d'Histoire des Sciences*, 54:3–28.

[Luyendijk-Elshout, 1975] Luyendijk-Elshout, Antonie M. (1975). The rise and fall of the mechanical philosophical school of Theodoor Craanen (1621–1690). In Lunsingh Scheurleer, Th. H. and Posthumus Meyjes, G.H.M., editors, *Leiden University in the seventeenth century. An exchange of learning*, pages 294–307. Universitaire pers Leiden/E.J. Brill, Leiden.

[Maanen, 1987] Maanen, J.A. van (1987). *Facets of seventeenth century mathematics in the Netherlands.* Elinkwijk BV, Utrecht.

[Maanen, 1993] Maanen, J. van (1993). The 'double-meaning' method for dating mathematical texts. In *vestigia mathematica: studies in medieval and early modern mathematics in honour of H.L.L. Busard*, pages 253–564. Rodopi, Amsterdam/Atlanta.

[Maanen, 2003] Maanen, Jan van (2003). Precursors of differentiation and integration. In Jahnke, Hans Niels, editor, *A history of analysis*, pages 41–72. American Mathematical Society & London Mathematical Society, Providence.

[Maanen, 1986] Maanen, Jan A. van (1986). Die Mathematik in den Niederländen im 17. Jahrhundert und ihre Rolle in der Entwicklungsgeschichte der Infinitesimalrechnung. In Heinekamp, Albert, editor, *300 Jahre "Nova Methodus" von G.W. Leibniz (1684–1984)*, Studia Leibnitiana, pages 1–13. Franz Steiner Verlag, Stuttgart.

[Maanen and Groenveld, 2003] Maanen, R.C.J. van and Groenveld, S., editors (2003). *Leiden: de geschiedenis van een Hollandse stad*, volume 2: 1574–1795. Stichting Geschiedschrijving Leiden, Leiden.

[Mahoney, 1980] Mahoney, M.S. (1980). The beginnings of algebraic thought in the seventeenth century. In Gaukroger, S., editor, *Descartes: Philosophy, Mathematics and Physics.* The Harvester Press/Barnes and Noble Books, Brighton/Totowa.

[Mahoney, 1994] Mahoney, Michael Sean (1994). *The mathematical career of Pierre de Fermat 1601–1665.* Princeton University Press, Princeton. Second edition.

[Malcolm and Stedall, 2005] Malcolm, Noel and Stedall, Jacqueline (2005). *John Pell (1611–1685) and his correspondence with Sir Charles Cavendish: the mental world of an early mathematician.* Oxford University Press, Oxford.

[Mancuso, 1992] Mancuso, Paolo (1992). Descartes's *Géométrie* and revolutions in mathematics. In Gillies, Donald, editor, *Revolutions in mathematics*, pages 83–116. Clarendon Press, Oxford.

[Marolois, 1615] Marolois, Samuel (1615). *Fortification ov architectvre militaire.* Ex off. H. HondiI, Den Haag.

[Marolois, 1616] Marolois, Samuel (1616). *Geometrie, contenant la theorie, et practicque dicelle necessaire a la fortification.* Ex officina Henrici Hondii & apud Ioannem Ianssonium, Den Haag and Arnhem.

[Marolois, 1617] Marolois, Samuel (1617). *Fortification ou Architectvre militaire tant offensive que defensive.* Apud Ioann: Ianssonium, Amsterdam. On the titlepage: Hagae Comitis: Ex. off. Henr. Hondius; French text.

[Marolois, 1651] Marolois, Samuel (1651). *Fortification ou architecture militaire tant offensive que deffensive / Samuel Marolois Sterckten-bouwingh of fortificatie.* Ian Ianssen, Amsterdam.

[Marolois et al., 1651] Marolois, Samuel, Dekker, Ezechiel de, and Girard, Albert (1651). *Opera mathematica ou oeuvres mathematiques traictans de geometrie, perspective, architecture, et fortification / par Samuel Marolois; de nouueau reueüe, augm. et corrigee par Albert Girard.* Chez Ian Ianssen, Amsterdam. Dutch text.

[Marolois and Girard, 1628a] Marolois, Samuel and Girard, Albert (1628a). *Opera mathematica ou Oeuvres mathématiques traictans de géométrie, perspective, architecture et fortification.* Chez Ian Ianssen, Amsterdam. Titlepage in French, text in Dutch.

[Marolois and Girard, 1628b] Marolois, Samuel and Girard, Albert (1628b). *Opera mathematica ou Oeuvres mathématiques traictans de géométrie, perspective, architecture et fortification.* Chez Ian Ianssen, Amsterdam.

[Marolois and Girard, 1630] Marolois, Samuel and Girard, Albert (1630). *Opera mathematica, ofte Wis-konstige wercken: handelende vande geometrie, perspective, architecture, en fortificatie / Sam. Marolois; Van nieuw oversien ende verb. door Alb. Girard.* By Jan Jansz.

[Marolois and Girard, 1633a] Marolois, Samuel and Girard, Albert (1633a). *Artis muniendi sive fortificationis pars prima (- secunda): . . . / Studio atque opera Alberti Girardi, recognita e multis notis illustrata.* Sumpt. ac typis Ioannis Ianssonii, Amsterdam.

[Marolois and Girard, 1633b] Marolois, Samuel and Girard, Albert (1633b). *Mathematicum opus absolutissimum: continens geometriae, fortificationis, architecturae et perspectivae theoreticae ac practicae regulas, demonstrationes et figuras perfectissimas . . . .* Sumpt. ac typis Ioannis Ianssonii, Amsterdam.

[Marolois and Girard, 1638a] Marolois, Samuel and Girard, Albert (1638a). *Fortification das ist Vestung-bauw, in Kriegs-zeiten / Forgestelt und beschrieben durch Samuel Marlois.* Bey Johan Jansson, Amsterdam.

[Marolois and Girard, 1638b] Marolois, Samuel and Girard, Albert (1638b). *Fortification ou architectvre militaire.* J. Ianssen, Amsterdam.

[Marolois and Girard, 1644] Marolois, Samuel and Girard, Albert (1644). *Samuelis Marolois Artis muniendi, sive Fortificatio / Studio atque opera Alb. Girardi recogn. et multis notis ill. Artis muniendi.* Sumpt[ibus] ac typ[is] Joann[is] Janssonii, Amsterdam.

[Marolois and Girard, 1662] Marolois, Samuel and Girard, Albert (1662). *Opera mathematica ou oeuvres mathematiques: traictans de geometrie, perspective, architecture, et fortification / par Samuel Marolois; nouveau rev., augm. et corr. par Albert Girard.* Chez Jan Janssen, Amsterdam.

[Marolois et al., 1662] Marolois, Samuel, Girard, Albert, and D., W. (1662). *Fortificatie, dat is sterckte bouwing: so wel tot offensive als defensive oorlogh: . . . oversien ende verbetert door Albert Girard; nu nieus uyt de Fransche in onse Nederlandsche tale over-geset, tot dienst van de liefhebbers der selve konst door W.D.* Jansz, Amsterdam.

[Marolois et al., 1627a] Marolois, Samuel, Girard, Albert, Decker, Ezechiel de, and D.W. (1627a). *Fortification ou Architectvre militaire tant offensive que deffensive.* Chez Jan Janssen, Amsterdam.

[Marolois et al., 1631] Marolois, Samuel, Girard, Albert, and Hexham, Henry (1631). *The art of fortification, or architecture militaire as vvell offensiue as defensiue, compiled & set forth, by Samuell Marolois revievved, augmented and corrected by Albert Girard mathematician: & translated out of French into English by Henry Hexam.* M. Iohn Iohnson, Amsterdam.

[Marolois et al., 1627b] Marolois, Samuel, Girard, Albert, and W.D. (1627b). *Fortificatie, dat is sterckte bouwing: so wel tot offensive als defensive oorlogh / Sam. Marolois; oversien en verbetert door Alb. Girard; nu nieus uyt de Fransche in onse Nederlandtsche Tale over-geset door W.D.* Chez Jan Janssen, Amsterdam.

[Marolois et al., 1627c] Marolois, Samuel, Girard, Albert, and W.D. (1627c). *Fortification, wie ein Ort nach der wahren und Fundamenthal-Kunst zubefestigen, anzugreiffen, zubestürmen oder auch wider allen feindlichen Gewalt und Anlauff zubeschirmen, vermehret, gebessert und erläutert durch Albert Gerhardt.* Chez Ian Ianssen, Amsterdam.

[Marolois et al., 1628a] Marolois, Samuel, Girard, Albert, and W.D. (1628a). *Fortificatie, dat is, sterckte bouwing: so wel tot offensive als defensive oorlogh / beschreven en voor-gestelt door Sam. Marolois; Oversien ende verb. door Alb. Girard; Nu nieus uyt de Fransche in onse Ned. tale over-geset door W.D.* By Jan Jansz, Amsterdam.

[Marolois and van Schoten, 1628] Marolois, Samuel and Schoten, Francois van (1628). *Fortification ou architecture militaire, tant offensive que deffensive.* Blaeu (Willem Janszoon), Amsterdam. Published in [Marolois et al., 1628b].

[Marolois et al., 1628b] Marolois, Samuel, Verbeeck, Théod., and Schooten, Francois van (1628b). *Oeuvres mathematiques de Samuel Marolois, traictant de la geometrie et fortification, reduictes en meilleurs ordre, et corrigees d'un nombre infiny de fautes escoulees aux impressions presendentes.* Chez Guill. Jansson Caesius (Willem Jansz. Blaeu), Amsterdam. Fortification revised by Frans van Schooten sr., Geometry revised by Théod. Verbeeck.

[Marolois and Vredeman de Vries, 1615] Marolois, Samuel and Vredeman de Vries, Johannes (1614–1615). *Opera mathematica ou Oeuvres mathématiques, traictans de géometrie, perspective, architecture et fortification / Samuel Marolois; ausquels sont aioints les fondements de la perspective et architecture de I. Vredeman Vriese, augm. et corr. par le mesme auteur.* H. Hondius, Den Haag.

[Maronne, 2006] Maronne, Sébastien (2006). Sur une lettre de Descartes à Schooten qu'on dit de 1639. *Revue d'histoire des mathématiques*, 12:199–248.

[Maronne, 2007] Maronne, Sébastien (2007). *La théorie des courbes et des équations dans la Géométrie cartésienne: 1637–1661.* PhD thesis, Université Paris 7 - Denis Diderot.

[Maurolico, 1575] Maurolico, Francesco (1575). *Arithmeticorum libri duo.* Apud Franciscum Senensem, Venice.

[Mersenne, 1972] Mersenne, Marin (1945–1972). *Correspondance du P. Marin Mersenne: religieux minime.* Bibliothèque des archives de philosophie. Presses Universitaires de France, Paris. Edited by Paul Tannery, Cornelis de Waard, Bernard Rochot and René Pintard.

[Meskens, 2009] Meskens, Ad (2009). *Wiskunde tussen Renaissance en Barok: aspecten van wiskunde-beoefening te Antwerpen 1550–1620.* Rondeel, Antwerpen. Reprint of work of 1994.

[Mesnard, 1991] Mesnard, Jean (1991). Sur le chemin de l'Académie des sciences: le cercle du mathématicien Claude Mylon (1654–1660). *Revue d'histoire des sciences*, 44:241–251.

[Miert, 2005] Miert, Dirk van (2005). *Illuster onderwijs: het Amsterdamse Athenaeum in de Gouden Eeuw, 1632–1704.* Bert Bakker.

[Molhuysen and Blok, 1911] Molhuysen, P.C. and Blok, P.J., editors (1911). *Nieuw Nederlandsch biografisch woordenboek*, volume 1. A.W. Sijthoff, Leiden.

[Molhuysen and Blok, 1914] Molhuysen, P.C. and Blok, P.J., editors (1914). *Nieuw Nederlandsch biografisch woordenboek*, volume 3. A.W. Sijthoff, Leiden.

[Molhuysen et al., 1927] Molhuysen, P.C., Blok, P.J., and Kossmann, F.H., editors (1927). *Nieuw Nederlandsch biografisch woordenboek*, volume 7. A.W. Sijthoff, Leiden.

[Molhuysen, 1913] Molhuysen, P. C. (1913). *Bronnen tot de geschiedenis der Leidsche universiteit*, volume 1. Martinus Nijhoff.

[Molhuysen, 1916] Molhuysen, P. C. (1916). *Bronnen tot de geschiedenis der Leidsche universiteit*, volume 2. Martinus Nijhoff.

[Molhuysen, 1918] Molhuysen, P. C. (1918). *Bronnen tot de geschiedenis der Leidsche universiteit*, volume 3. Martinus Nijhoff.

[Molhuysen, 1920] Molhuysen, P. C. (1920). *Bronnen tot de geschiedenis der Leidsche universiteit*, volume 4. Martinus Nijhoff.

[Monte, 1579] Monte, Guidobaldo del (1579). *Planisphaeriorum universalium theorica.* Apud Hieronymum Concordiam.

[Monte, 1581] Monte, Guidobaldo del (1581). *Planisphaeriorum universalium theorica.* Apud Maternum Cholinum, Cologne.

[Montucla, 1758] Montucla, J.-E. (1758). *Histoire des mathématiques.* Ch. Ant. Jombert, Paris.

[Mulder, 1990] Mulder, H.M. (1990). Pure, mixed and applied mathematics. the changing perception of mathematics through history. *Nieuw Archief voor Wiskunde*, 8:27–42.

[Müller, 1655] Müller (1655). *Disputatio mathematica de luna.* Ex Officina Francicsi Hackii, Leiden.

[Muller and Zandvliet, 1987] Muller, E. and Zandvliet, K., editors (1987). *Admissies als landmeter in Nederland voor 1811: bronnen voor de geschiedenis van de landmeetkunde en haar toepassing in administratie, architectuur, kartografie en vesting- en waterbouwkunde.* Canaletto, Alphen aan den Rijn.

[Müller, 1656] Müller, Johann (1656). *Theses mathematico-physicae.* Walterus, Heidelberg.

[Mydorge, 1631] Mydorge, Claude (1631). *Prodromi catoptricorum et dioptricorum sive Conicorum operis ad abdita radii reflexi et refracti mysteria praevii et facem praeferentis libri primus et secundus.* Ex typografia J. Dedin, Paris.

[Napier, 1614] Napier, John (1614). *Mirifici logarithmorum canonis descriptio: ejusque usus, in utraque trigonometria; ut etiam in omni logistica mathematica, amplissimi, facillimi, & expeditissimi explicatio.* Ex officina Andreae Hart, Edinburgh.

[Napier and Briggs, 1619] Napier, John and Briggs, Henry (1619). *Mirifici logarithmorum canonis constructio . . . una cum appendice de alia eaque praestantiore logarithmorum specie condenda quibus accessere propositiones ad triangula sphaerica faciliore calculo resolvenda una cum Annotationibus aliquot . . . Henrici Briggii in eas et memoratam appendicem, authore et inventore Joanne Nepero, ...* Excudebat Andreas Hart, Edinburgh.

[Napier and Briggs, 1620] Napier, John and Briggs, Henry (1620). *Mirifici logarithmorum canonis constructio, et eorum ad naturales ipsorum numeros habitudines: una cum appendice, de alia eaque praestantiore logarithmorum specie condenda, quibus accessere propositiones ad triangula sphaerica faciliore calculo resolvenda.* Apud Bartholomaeum Vincentium, Leiden.

[Netten, 2010] Netten, Djoeke van (2010). *Nicolaus Mulerius (1564–1630): een geleerde uit Groningen in de discussies van zijn tijd.* Barkhuis, Groningen.

[Netten, 2012] Netten, Djoeke van (2012). *Koopman in kennis: de uitgever Willem Jansz Blaeu (1571–1638) in de geleerde wereld van zijn tijd.* PhD thesis, Rijksuniversiteit Groningen.

[Newton, 1972] Newton, Isaac (1972). *The mathematical papers of Isaac Newton*, volume 5. University Press Cambridge, Cambridge.

[Newton and Whiteside, 2008] Newton, Isaac and Whiteside, D.T. (2008). *The Mathematical Papers of Isaac Newton,*, volume 1. Cambridge University Press, Cambridge. First edition published in 1967.

[Nierop, 2012] Nierop, Dirck Rembrantsz van (2012). *The correspondence of Dirck Rembrantsz van Nierop (1610–1682).* DWC Huygens ING, Den Haag. Edited by Marlise Rijks.

[Nieuwenhuis, 1840] Nieuwenhuis, G., editor (1840). *Aanhangsel op het algemeen woordenboek van kunsten en wetenschappen: in 8 deelen bewerkt door wijlen den Heer Gt. Nieuwenhuis, en te Zutphen van 1820–1829 in het licht verschenen.* J.F. Thieme, Nijmegen.

[Nimwegen, 2006] Nimwegen, Olaf van (2006). *'Dese landen crijchsvolck' Het Staatse leger en de militaire revoluties 1588–1688.* Bert Bakker, Amsterdam.

[Noack and Splett, 1997] Noack, Lothar and Splett, Jürgen (1997). *Bio-Bibliographien - brandenburgische Gelehrte der frühen Neuzeit. Berlin-Cölln 1640–1688.* Veröffentlichungen zur brandenburgischen Kulturgeschichte der frühen Neuzeit. Akademie Verlag, Berlin.

[Noordam, 1994] Noordam, D. J. (1994). *Geringde buffels en heren van stand: het patriciaat van Leiden, 1574–1700.* Verloren, Hilversum.

[Nordström, 1958] Nordström, Johan (1958). *Till Cartesius' ikonografi.* Swedish History of Science Society, Uppsala. Reprinted from Lychnos. Annual of the Swedish history of science society. Vol. 17. 1957–1958.

[Notizen, 1825] (1825). Notizen über die Befestigung der Stadt Thorn, vom Jahre 1590 bis zum Jahre 1650. *Zeitschrift für Kunst, Wissenschaft und Geschichte des Krieges*, Vierter Band:234–243 and 311–325.

[Offerhaus, 1758] Offerhaus, Leonardus (1758). *Catalogus librorum bibliothecae illustris ac almae universitatis illustrium et praepotentium Groningae et Omlandiae d.d. ordinum. Secundum seriem litterarum alphabeti digestus. Notitia librorum manuscriptorum auctus.* Typis Jacobi Bolt, Groningen.

[Ommen and Bos, 2003] Ommen, Kasper van and Bos, Henk (2003). De bibliotheca mathematica van Bierens de Haan. *Nieuw archief voor wiskunde*, 5/4:242–251.

[Omodeo, 2011] Omodeo, Pietro Daniel (2011). Sixteenth century professors of mathematics at the German university of Helmstedt: a case study on Renaissance scholarly work and networks. Technical Report preprint 417, Max Planck Institut für Wissenschaftsgeschichte, Berlin.

[Orenstein, 1996] Orenstein, Nadine M. (1996). *Hendrick Hondius and the business of prints in seventeenth-century Holland.* Sound & vision interactive, Rotterdam.

[Otegem, 2002] Otegem, Matthijs van (2002). *A bibliography of the works of Descartes (1637–1704).* PhD thesis, Utrecht University.

[Otterspeer, 2000] Otterspeer, Willem (2000). *Groepsportret met dame I. Het bolwerk van de vrijheid. De Leidse universiteit 1575–1672.* Bert Bakker, Amsterdam.

[Otterus, 1646] Otterus, Christiaan (1646). *Specimen problematum hercotectonico-geometricorum quo ut fortificationis (vulgo ita dictae) modi universalis ita sectionis rationalis linearum vestigium exhibetur.* Johannes Fabel, Amsterdam.

[Otterus and Buck, 1763] Otterus, Christiaan and Buck, F.J. (1763). *Christiani Otteri Ragnetani Principia Architecturae Militaris.* [Joh. Heinr. Hartungs Erben und J.D. Zeise], Königsberg. published in [Buck, 1764].

[Oughtred, 1631] Oughtred, William (1631). *Arithmeticae in numeris et speciebus institutio: quae tum logisticae, tum analyticae, atque adeo totius mathematicae quasi clavis est.* Apud Thomam Harperum, London.

[Palatinate and Descartes, 2007] Palatinate, Elisabeth of the and Descartes, René (2007). *The correspondence between Princess Elisabeth of Bohemia and René Descartes.* The other voice in early modern Europe. University of Chicago Press. Edited by Lisa Shapiro.

[Panhuysen, 2005] Panhuysen, Luc (2005). *De ware vrijheid: de levens van Johan en Cornelis de Witt.* Atlas, Amsterdam.

[Pantin, 2004] Pantin, I. (2004). Ramus et l'enseignement des mathématiques. In Meerhoff, K. and Magnien, editors, *Ramus et l'Université*, pages 71–86. Éditions Rue d'Ulm, Paris.

[Pappus, 1588] Pappus (1588). *Pappi Alexandrini Mathematicae collectiones a Federico Commandino Urbinate in Latinum conversae et commentariis illustratae.* Apud Hieronymum Concordiam, Pesaro. Translated edition by F. Commandino.

[Pappus, 1589] Pappus (1589). *Pappi Alexandrini mathematicae collectiones. A Federico Commandino Vrbinatae in latinum conuersae, & commentarijs illustratae.* Apud Franciscum de Franciscis Senensem, Venice.

[Pappus, 1602] Pappus (1602). *Federici Commandini vrbinatis mathematici celeberrimi exactissima commentaria in libros octo mathematicarvm collectionvm Pappi Alexandrini e Graeco in Latinvm a se accuratissimé conuersos.* Apud Hieronymum Concordiam, Pesaro.

[Pappus, 1660] Pappus (1660). *Pappi Alexandrini Mathematicae collectiones a Federico Commandino Urbinate in Latinum conversae et commentarijs illustratae, in hac nostra editione ab innumeris, quibus scatebant mendis & praecipue in Graeco contextu diligenter vindicatae.* Typographia H.H. Duccijs, Bologna. Translated edition by F. Commandino.

[Pappus, 1933] Pappus (1933). *La collection mathématique.* Descl'ee de Brouwer, Paris. Translated and edited by Paul Ver Eecke.

[Pappus, 1986] Pappus (1986). *Book 7 of the Collection*, volume 1. Springer-Verlag, New York etc. Translation and commentary by Alexander Jones.

[Parival, 1661] Parival, Jean Nicolas de (1660–1661). *De vermaecklijckheden van Hollandt: een werck deses lands lof uytbasuynend. Met een tractaet van de regeeringe end' een kort begrijp van 't gene aenmerckens waerdigst voorgevallen is, tot het jaer 1655.* by Samuel Imbrechts, Amsterdam.

[Pelinck, 1967] Pelinck, E. (1967). De functionarissen belast met de zorg voor de stadsbouwwerken te Leiden (1575–1850). *Jaarboekje voor de geschiedenis en oudheidkunde van Leiden en omstreken*, 59:59–76.

[Pell, 1647] Pell, John (1647). *Controversiae de vera Circuli mensura Anno MDCXLIV exortae inter Christianum Severini, Longomontanum, Cimbrum, Superiorum Mathematum in regia Danorum Academia Hauniensi, Professorum publicum et Ioannem Pellium, Coritano-Regnum, Anglum, Matheseos, In Illustri Amstelodamensium Gymnasio, Professorem publicum, Pars Prima.* Apud Ioannem Blaeu, Amsterdam.

[Pepys et al., 1995] Pepys, Samuel, Latham, Robert John Wilkinson, Matthews, William, and Armstrong, William Arthur (1995). *The diary of Samuel Pepys*, volume 3: 1662. HarperCollins, London.

[Pluijm, 2006] Pluijm, J.E. van der (2006). *De vestingstad Grol in de kaart gekeken: topografisch historische atlas van Groenlo.* Stadsmuseum Groenlo, Groenlo.

[Plumley, 1976] Plumley, N. (1976). The Royal Mathematical School within Christ's Hospital: The early years. – its aims and achievements. *Vistas in Astronomy*, 20:51–59.

[Po-Chia Hsia and Nierop, 2002] Po-Chia Hsia, R. and Nierop, Henk van, editors (2002). *Calvinism and religious toleration in the Dutch Golden Age.* Cambridge University Press, Cambridge.

[Poelje, 2005] Poelje, Otto E. van (2005). Adriaen Vlacq and Ezechiel de Decker: Dutch contributors to the early tables of Briggsian logarithms. *Journal of the Oughtred Society*, 14:30–40.

[Pouls, 2004] Pouls, H.C. (2004). *De landmeter Jan Pietersz. Dou en de Hollandse Cirkel.* Nederlandse Commissie voor Geodesie, Delft.

[Prak, 2005] Prak, Maarten (2005). *The Dutch Republic in the seventeenth century: the Golden Age.* Cambridge University Press, Cambridge.

[Ramus and Snellius, 1613] Ramus, Peter and Snellius, Willebrord (1613). *Arithmeticae libri duo, cum commentariis Wilebrordi Snellii R.F.* Ex Officina Plantiniana Raphelengii, Leiden.

[Rasterhoff, 2012] Rasterhoff, Clara (2012). *The Fabric of Creativity in the Dutch Republic. Painting and publishing as Cultural Industries, 1580–1800.* PhD thesis, Universiteit Utrecht.

[Reich, 2003] Reich, Karin (2003). The 'coss' tradition in algebra. In Grattan-Guinness, Ivor, editor, *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences*, volume 1, pages 192–200. John Hopkins University Press, Baltimore.

[Reid and Wilson, 2011] Reid, Steven J. and Wilson, Emma Annette, editors (2011). *Ramus, pedagogy and the liberal arts: Ramism in Britain and the wider world.* Ashgate, Farnham.

[Reidemeister and Peters, 1933] Reidemeister, Kurt and Peters, Theodor (1933). *Christian Otter: Mechanismen.* Niemeyer, Halle.

[Reinders, 1995] Reinders, H.R. (1995). Van de fortificatie. In Boersma, J.W. and Jörg, C.J.A., editors, *Ere-saluut: opstellen voor mr. G. Overdiep*, pages 251–270. Regio-Project Uitgevers, Groningen.

[Remmert, 2013] Remmert, Volker (2013). "Of a gardiner, and how he is to be qualified": landscape design and the early modern mathematical sciences. In *From "mixed" to "applied" mathematics: tracing an important dimension of mathematics and its history*, pages 673–676. Part of [Epple et al., 2013].

[René Descartes, 1692] René Descartes, Jan Hendrik Glazemaker (1692). *Brieven, aan veele hooggeagte personen, van verscheide ampten, geschreven: handelende van de over-natuur-kunde, natuur-kunde, zede-kunst, genees-kunst, wis-kunst, zang-kunst, en werk-daad, enz. Zijnde ten hoogsten dienstig, om de voorgaande werken des schrijvers te verligten, en duidelijker te doen verstaan.* Jan Claesz. ten Hoorn, Amsterdam.

[Reyher and Alefeld, 1668] Reyher, Samuel and Alefeld, Benedictus (1668). *De Munitionibus.* Reumannus, Kiel.

[Rieu, 1875] Rieu, Guilielmus du, editor (1875). *Album studiosorum Academiae Lugduno Batavae MDLXXV–MDCCCLXXV: accedunt nomina curatorum et professorum per eadem secula.* Apud Martinum Nijhoff, Den Haag.

[Roden and Jedin, 1968] Roden, Günter von and Jedin, Hubert (1968). *Die Universität Duisburg,* volume 12. Braun, Duisburg.

[Roding, 1991] Roding, Juliette Germaine (1991). *Christiaan IV van Denemarken (1588–1648): architectuur en stedebouw van een Luthers vorst.* Cantina Architectura, Alkmaar.

[Roegel, 2011a] Roegel, Denis (2011a). Napier's ideal construction of the logarithms. Technical report, LORIA, Nancy.

[Roegel, 2011b] Roegel, Denis (2011b). A reconstruction of De Decker-Vlacq's tables in the *Arithmetica logarithmica* (1628). Technical report, LORIA, Nancy.

[Roegel, 2011c] Roegel, Denis (2011c). A reconstruction of the tables of Briggs' *Arithmetica logarithmica* (1624). Technical report, LORIA, Nancy.

[Roomen, 1593] Roomen, Adriaan van (1593). *Ideae mathematicae pars prima, sive methodus polygonorum, qua laterum, perimetrorum & arearum cujuscunque polygoni investigandorum ratione exactissima & certissima; una cum circuli quadratura continetur.* Ioannem Keerbergium, Antwerp.

[Roomen, 1596] Roomen, Adriaan van (1596). *Problema Apolloniacum quo datis tribus circulis, quaeritur quartus eos contingens.* Fleischmann, Würzburg.

[Roth, 1608] Roth, Peter (1608). *Arithmetica Philosophica, oder schöne newe wolgegründte überauss künstliche Rechnung der Coss oder Algebrae.* Johann Lantzenberger, Nuremberg.

[Rowen, 1986] Rowen, Herbert H. (1986). *John de Witt statesman of the "True Freedom".* Cambridge University Press, Cambridge.

[Rudolff, 1525] Rudolff, Christoff (1525). *Behend vnnd Hubsch Rechnung durch die kunstreichen regeln Algebre so gemeincklich die Coss genennt werden.* Vuolfius Cephaleus Joanni Jung, Strasbourg.

[Ruse, 1654] Ruse, Henrick (1654). *Versterckte vesting, uytgevonden in velerley voorvallen, en geobserveert in dese laetste oorloogen, soo in de Vereenigde Nederlanden, als in Vranckrijck, Dvytsland, Italien, Dalmatien, Albanien, en die daer aengelegen landen.* By Joan Blaeu, Amsterdam.

[Saint-Vincent, 1647] Saint-Vincent, Gregorius de (1647). *Opus geometricum quadraturae circuli et sectionum coni decem libris comprehensum. Problema Austriacum plus ultra quadratura circuli.* Apud Joannem et Jacobum Meursios, Antwerp.

[Sarrazin, 1996] Sarrazin, Hélène (1996). *La Fronde en Gironde: l'Ormée, un mouvement révolutionnaire, 1648–1654.* Les Dossiers d'Aquitaine, Bordeaux.

[Sasaki, 2003] Sasaki, Chikara (2003). *Descartes's mathematical thought.* Boston studies in the philosophy of science. Kluwer Academic Publishers, Dordrecht.

[Savini, 2011] Savini, Massimiliano (2011). *Johannes Clauberg: Methodus Cartesiana et Ontologie.* Vrin, Paris.

[Schäfer, 2001] Schäfer, Christoph (2001). *'Krygsvernuftelingen'. Militäringenieuren und Fortification in den Vereinigten Niederlanden.* PhD thesis, Justus-Liebig Universität Gießen.

[Scher, 1997] Scher, Stephen K. (1997). *The proud Republic: Dutch medals of the Golden Age.* Frick Collection, New York.

[Scherf and Dekker, 1994] Scherf, Yvonne and Dekker, Rudolf M. (1994). *Reisverslagen van Noord-Nederlanders uit de zestiende tot begin negentiende eeuw: een chronologische lijst.* Verloren, Hilversum.

[Schmidt, 2005] Schmidt, J. (2005). Between author and library shelf: The intriguing history of some middle eastern manuscripts acquired by public collections in the Netherlands prior to 1800. In Hamilton, Alistair, Boogert, Maurits H. van den, and Westerweel, Bart, editors, *The Republic of Letters and the Levant*, pages 27–51. Brill, Leiden.

[Schneider, 1993] Schneider, Ivo (1993). *Johannes Faulhaber 1580–1635: Rechenmeister in einer Welt des Umbruchs.* Vita mathematica, Band 7. Birkhäuser, Basel.

[Schneider, 1999] Schneider, Ivo (1999). Peter Roth († April 1617). In Gebhardt, Rainer, editor, *Rechenbücher und mathematische Texte der frühen Neuzeit.* Adam-Ries-Bund e.V., Annaberg-Bucholz.

[Scholten, 1989] Scholten, F.W.J. (1989). *Militaire topografische kaarten en stadsplattegronden van Nederland 1579–1795.* Canaletto, Alphen aan den Rijn.

[Schönbeck, 2007] Schönbeck, Jürgen (2007). Samuel Reyher und sein *In Teutscher Sprache vorgestellter Euclides. NTM: international journal of history and ethics of natural sciences, technology and medicine*, 15:118–136.

[Schooten, 1646] Schooten, Frans van (1646). *De organica conicarum sectionum in plano descriptione, tractatus: cui subnexa est appendix de cubicarum aequationum resolutione.* Ex off. Elzeviriorum.

[Schooten, 1649a] Schooten, Frans van (1649a). Additamentum. In *In geometriam Renati Des Cartes commentarii*, pages 295–336. Ex Officina Ioannis Maire. Published in [Schooten, 1649b], 295–336.

[Schooten, 1649b] Schooten, Frans van, editor (1649b). *Geometria à Renato Des Cartes anno 1637 Gallicè edita; nunc autem cum notis Florimondi de Beaune ... in linguam Latinam versa & commentariis illustrata, Operâ atque studio Francisci à Schooten, ...* Ex Officina Ioannis Maire, Leiden.

[Schooten, 1649c] Schooten, Frans van (1649c). *In geometriam Renati Des Cartes commentarii.* Ex Officina Ioannis Maire, Leiden. Published in [Schooten, 1649b], 162–294.

[Schooten, 1657] Schooten, Frans van (1657). *Exercitationum mathematicarum libri quinque.* Ex officina Johannis Elsevirii, Leiden.

[Schooten, 1659a] Schooten, Frans van (1659a). Additamentum. In *In geometriam Renati Des Cartes commentarii*, pages 369–400. Apud Ludovicum & Danielem Elzevirios. Published in [Schooten, 1661a], vol. 1, 369–400.

[Schooten, 1659b] Schooten, Frans van (1659b). Appendix de cubicarum aequationum resolutione. In *In geometriam Renati Des Cartes commentarii*, pages 345–368. Apud Ludovicum & Danielem Elzevirios. Published in [Schooten, 1661a], vol. 1.

[Schooten, 1659c] Schooten, Frans van (1659c). *In geometriam Renati Des Cartes commentarii.* Apud Ludovicum & Danielem Elzevirios, Amsterdam. Published in [Schooten, 1661a], vol. 1, 143–400.

[Schooten, 1660a] Schooten, Frans van (1659–1660a). *Mathematische oeffeningen, begrepen in vijf boecken: I Verhandeling van de vijftig Arithmetische en vijftig Geometrische Voorstellen. II Ontbinding der Simpele Meet-konstige Werck-stucken. III Apollonii Pergaei, herstelde Vlacke Plaetsen. IV Tuych-werckelycke beschryving der Kegel-sneden op een vlak. V Dertich Af-deelingen van gemengde stoffe.* Gerrit van Goedesbergh, Amsterdam.

[Schooten, 1661a] Schooten, Frans van, editor (1659–1661a). *Geometria à Renato Des Cartes anno 1637 Gallicè edita; postea autem una cum notis Florimondi de Beaune . . . Gallicè conscriptis in Latinam linguam versa & Commentariis illustrata, Operâ atque studio Francisci à Schooten, . . . Nunc demum ab eodem diligenter recognita, locupletioribus Commentariis instructa, multisque egregiis accessionibus, tam ad uberiorem explicationem, quàm ad ampliandam hujus Geometria excellentiam facientibus, exornata*, volume 1 and 2. Apud Ludovicum & Danielem Elzevirios, Amsterdam. Vol. 1 (1659), vol. 2 (1661).

[Schooten, 1660b] Schooten, Frans van (1660b). *Tractaet der perspective ofte schynbaere teyckenkonst, waer in de fondamenten derselve konst op het kortste verhandelt en betoont worden.* Gerrit van Goedesbergh, Amsterdam. Published in [Schooten, 1660a], 501–543.

[Schooten, 1660c] Schooten, Frans van (1660c). *Vierde bouck der mathematische oeffeningen, begrijpende de tuych-werckelycke beschryving der kegel-sneden op een vlack.* Gerrit van Goedesbergh, Amsterdam. Published in [Schooten, 1660a], 273–342.

[Schooten, 1661b] Schooten, Frans van (1661b). *Tractatus de concinnandis demonstrationibus geometricis ex calculo algebraïco.* Apud Ludovicum & Danielem Elzevirios, Amsterdam. Edited by Frans van Schooten and published in [Schooten, 1661a], vol. 2, 341–420.

[Schooten sr., 1627] Schooten sr., Frans van (1627). *Tabulae sinuum tangentium secantium, ad radium 10000000 met 't gebruyck der selve in rechtlinische triangulen.* Bij Willem J. Blauw, Amsterdam.

[Sems and Dou, 1600] Sems, Johan and Dou, Jan Pietersz (1600). *Practijck des lantmetens: leerende alle rechte ende cromsijdige landen, bosschen, boomgaerden ende ander velden meten, soo wel met behulp des quadrants, als sonder het selve . . . .* Jan Bouwensz., Leiden.

[Seraphim, 1909] Seraphim, A. (1909). *Handschriften-Katalog der Stadtbibliothek Königsberg i. Pr.* Beyer, Königsberg.

[Serfati, 1993] Serfati, Michel (1993). Les compas cartésiens. *Archives de philosophie*, 56:197–230.

[Shapin, 1998] Shapin, Steven (1998). *The scientific revolution.* University of Chicago Press, Chicago.

[Shapin and Schaffer, 1985] Shapin, Steven and Schaffer, Simon (1985). *Leviathan and the air-pump: Hobbes, Boyle, and the experimental life.* Princeton University Press, Princeton.

[Sharratt, 1972] Sharratt, Peter (1972). The present state of studies on Ramus. *Studi francesi*, 47-48:201–13.

[Sharratt, 1987] Sharratt, Peter (1987). Recent work on Peter Ramus (1970–1986). *Rhetorica*, 5:7–58.

[Sharratt, 2000] Sharratt, Peter (2000). Ramus 2000. *Rhetorica*, 18:399–455.

[Siegmund-Schultze, 2013] Siegmund-Schultze, Reinhard (2013). The establishment of the notion and the word 'applied mathematics' around 1800. In *From "mixed" to "applied" mathematics: tracing an important dimension of mathematics and its history*, pages 663–665. Part of [Epple et al., 2013].

[Sinke, 2001] Sinke, J. (2001). De verrassing op de vriendendag, driehonderd drie en zeventig jaar na dato. Hoog bezoek aan Het laag. *De Waterschans*, 31:26–28.

[Sluijter, 2004] Sluijter, Ronald (2004). *'Tot ciraet, vermeerderinge ende heerlyckmaeckinge der universiteyt'. Bestuur, instellingen, personeel en financiën van de Leidse universiteit 1575–1812*. Verloren, Hilversum.

[Sluis, 2007] Sluis, Jacob van (2007). *Inventaris van de handschriften van de voormalige Provinsjale Biblioteek fan Fryslân*. Tresoar, Leeuwarden.

[Smeur, 1960] Smeur, Alphons Johannes Emile Marie (1960). *De zestiende-eeuwse Nederlandse rekenboeken*. Martinus Nijhoff, Den Haag.

[Smit, 1940] Smit, J. (1939–1940). De weeskamers, haar taak, inrichting en werkwijze in het huidige gebied van Zuid-Holland. *Nederlandsch Archievenblad*, 47:86–114.

[Smyters, 1661] Smyters, Anthoni (1661). *Arithmetica: dat is de reken-konste*. Jac. van Leest, Amsterdam.

[Snellius, 1596] Snellius, Rudolph (1596). *In P. Rami Arithmeticam Rudolphi Snellii explicationes lectissimae: Lazari Schoneri, Bernh. Salignaci, & Christiani Urstisii, commentationibus passim locupletatae*. Ex officina typographica Ioannis Saurii, impensis haeredum Petri Fischeri, Frankfurt.

[Snellius, 1608] Snellius, Willebrord (1608). *Apollonius Batavus, seu, exsuscitata Apollonii Pergaei* Περὶ διωρισμένης τομῆς *Geometria*. Ioannes a Dorp, Prostant apud Iohannem Maire, Leiden.

[Snellius and Hortensius, 1627] Snellius, Willebrord and Hortensius, Martinus (1627). *Doctrinae triangulorum canonicae libri IV . . . in lucem editi a Mrt. Hortensio, qui istis problematum geodeticorum et sphaericorum tractatus singulos adjunxit*. Ex officina Ioannis Maire, Leiden.

[Stampioen, 1638] Stampioen, Johan ([1638]). Problema Astronomicum & Geometricum voorgestelt Door Iohan Stampioen de Jonghe Mathematicus, Residerende in 's Graven-Haghe, Aende Vytgevers van het Antwerpsch Vraeg-Stuck. [by Johan Stampioen], [Den Haag]. In University of Groningen Library, Hs 108.

[Stampioen, 1639] Stampioen, Johan (1639). *Algebra ofte nieuwe stel-regel, waer door alles ghevonden wordt inde wis-konst, wat vindtbaer is, noyt voor desen bekendt*. Sphaera Mundi [Johan Stampioen], Den Haag.

[Stedall, 2000] Stedall, Jacqueline A. (2000). Catching Proteus. the collaborations of Wallis and Brouncker. II number problems. *Notes and records of the Royal Society*, 54:317–331.

[Stevin, 1585a] Stevin, Simon (1585a). *De Thiende: Leerende door onghehoorde lichticheyt allen rekeningen onder den menschen noodig vallende, afveerdighen door heele ghetalen sonder ghebrokenen*. By Christoffel Plantijn, Leiden.

[Stevin, 1585b] Stevin, Simon (1585b). *L'arithmetique de Simon Stevin de Bruges: contenant les computations des nombres arithmetiques ou vulgaires . . . Ensemble les quatre premiers livres d'algebre de Diophante d'Alexandrie, maintenant premierement traduicts en François. Encore un livre particulier de la pratique d'arithmétique*. De l'imprimerie de Christophle Plantin, Leiden.

[Stevin, 1586a] Stevin, Simon (1586a). *Anhang van de weeghconst, inde welcke onder anderen weerleydt worden etlicke dwalinghen der wichtighe ghedaenten*. Christoffel Plantijn, Leiden. Appendix to [Stevin, 1586b].

[Stevin, 1586b] Stevin, Simon (1586b). *De beghinselen des waterwichts*. Christoffel Plantijn, Leiden.

[Stevin, 1594] Stevin, Simon (1594). *De sterctenbouwing*. Francoys van Ravelenghien, Leiden.

[Stevin, 1600] Stevin, Simon (1600). *Maniere ende ordre die sijn Extie verstaen heeft dat sal worden achtervolght int doen van de Nederduytsche Lessen in de Universiteyt tot Leyden tot onderrichtinghe ende bevorderinghe der ghene die hun ten dienste van den Lande sullen begheven tot oeffeninghe van het Ingenieurschap ende andere Mathematische consten.* Jan. Paedts Jacobsz., Leiden. Het Scheepvaartmuseum, the National Maritime Museum, Library, signature B-I-0073 (II,37).

[Stevin, 1605] Stevin, Simon (1605). *Derde Stuck der Wisconstighe Ghedachtenissen*, chapter Eerste bouck der deursichtighe vander verschaeuwing. Inde Druckerye van Ian Bouvvensz, Leiden.

[Stevin, 1608] Stevin, Simon (1605–1608). *Wisconstighe gedachtenissen, inhoudende 't ghene daer hem in gheoeffent heeft Maurits Prince van Oraengien . . . .* J. Bouwensz., Leiden.

[Stevin, 1617] Stevin, Simon (1617). *Nievvve maniere van sterctebov, door spilsluysen.* By Ian van Waesberghe, Rotterdam.

[Stevin, 1964] Stevin, Simon (1964). *Principal works of Simon Stevin: The art of war*, volume 4. Swets en Zeitlinger, Amsterdam. Translated and edited by W.H. Schukking.

[Stifel, 1544] Stifel, Michael (1544). *Arithmetica integra.* Apud Johan. Petreium, Nuremberg.

[Stifel, 1553] Stifel, Michael (1553). *Die Coss Christoffe Ludolffs mit schönen Exempeln der Coss.* Gedrückt durch Alexandrum Lutomyslensem, Köningsberg.

[Strien-Chardonneau, 2006] Strien-Chardonneau, Madeleine van (2006). Leiden, metropool der muzen en tuin van Holland: Leiden in Jean Nicolas Parivals 'Les délices de la Hollande'. *De zeventiende eeuw: cultuur in de Nederlanden in interdisciplinair perspectief*, pages 171–184.

[Stüben, 2007] Stüben, Jens (2007). Die historische Literaturregion Ostpreußen - Westpreußen - Danzig in der Forschung. Eine Auswahlbiographie. In Stüben, Jens, editor, *Ostpreußen, Westpreußen, Danzig: Eine historische Literaturlandschaft*, volume 30, pages 607–724. Oldenbourg Verlag, Oldenburg.

[Swetz, 1992] Swetz, Frank (1992). Fifteenth and sixteenth century arithmetic texts: What can we learn from them? *Science & Education*, 1:365–378.

[Taisbak, 2003] Taisbak, Christian Marinus (2003). *Dedomena. Euclid's Data or the importance of being given.* Museum Tusculanum Press, Copenhagen.

[Taton, 1962] Taton, Rene (1962). L'oeuvre de Pascal en géométrie projective. *Revue d'histoire des sciences et de leurs applications*, 15:197–252.

[Taverne, 1978] Taverne, Ed (1978). *In 't land van belofte: in de nieue stad. Ideaal en werkelijkheid van de stadsuitleg in de Republiek 1580–1680.* Schwartz, Maarssen.

[Taylor, 1970] Taylor, E.G.R. (1970). *The mathematical practitioners of Tudor & Stuart England.* Cambridge University Press, Cambridge.

[Tennulius, 1668] Tennulius, Samuel (1668). *Notae Damuelis tennulii in librum quartum Jamblichi Chalcidensis de Arithmetica Nicomachi introductione.* [Joh. Fridericum Hagium], Deventer. Published together with [Tennulius and Chalcidensis, 1668] in one volume.

[Tennulius and Chalcidensis, 1668] Tennulius, Samuel and Chalcidensis, Jamblichus (1668). *Jamblichus chalcidensis ex Coele-Syria In Nicomachi geraseni Arithmeticam introductionem, et De fato.* Prostant apud Joh. Fridericum Hagium, Arnhem.

[Th. H. Lunsingh Scheurleer and van Dissel, 1990] Th. H. Lunsingh Scheurleer, C. W. Fock and Dissel, A.J. van (1990). *Het Rapenburg: geschiedenis van een Leidse gracht*, volume 5: 's Gravensteyn. Rijksuniversiteit Leiden, Leiden.

[Tiaden, 1787] Tiaden, Enno Johann Heinrich (1787). *Das gelehrte Ost Friesland*, volume 2. Johann Hinrich Ludolph Borgeest, Aurich.

[Tideman, 1847] Tideman, Joannes (1847). *De Remonstrantsche broederschap: biographische naamlijst van hare professoren, predikanten en proponenten, met historische aanteekeningen omtrent hare kweekschool en gemeenten.* Erven F. Bohm, Haarlem.

[Tideman et al., 1905] Tideman, Joannes, Rogge, H.C., and Tideman, B. (1905). *De Remonstrantsche broederschap: biographische naamlijst van hare professoren, predikanten en proponenten, met historische aanteekeningen omtrent hare kweekschool en gemeenten.* Rogge, Amsterdam.

[Toomer, 1996] Toomer, G.J. (1996). *Eastern wisedome and learning: the study of Arabic in seventeenth-century England.* Clarendon Press, Oxford.

[Trevisani, 2011] Trevisani, Francesco (2011). *Descartes in Deutschland: Die Rezeption des Cartesianismus in den Hochschulen Nordwestdeutschlands*, volume 25 Naturwissenschaft - Philosophie - Geschichte. LIT Verlag, Münster. Translation of original Italian study of 1992.

[Vaeck, 1986] Vaeck, M. van (1986). Adriaen van de Venne: Sinne-Vonck op den Hollandtschen Turf (1634). *De zeventiende eeuw*, 2, deel 2:1–20.

[Varenius, 1650] Varenius, Bernhardus (1650). *Geographia generalis in qua affectiones generales telluris explicantur.* Ap. L. Elzevirium, Amsterdam.

[Vaughan, 1839] Vaughan, Robert (1839). *The protectorate of Oliver Cromwell, and the state of Europe during the early part of the reign of Louis XIV: Illustrated in a series of letters between Dr. John Pell, resident ambassador with the Swiss cantons, Sir Samuel Morland, Sir William Lockhart, Mr. Secretary Thurloe, and other distinguished men of the time*, volume 2. Colburn, London.

[Veilingcatalogus, 1882] (1882). *Veilingcatalogus, boeken van A. G. de Visser e.a., 22 november 1882.* Frederik Muller & Co., Amsterdam.

[Verdonk, 1966] Verdonk, J. J. (1966). *Petrus Ramus en de wiskunde.* Van Gorcum, Assen.

[Vérin, 1993] Vérin, Hélène (1993). *La gloire des ingénieurs: l'intelligence technique du XVIe au XVIIIe siècle.* Albin Michel, Paris.

[Vermij, 1995] Vermij, Rienk (1995). Bijdrage tot de bio-bibliografie van Johannes Hudde. *Gewina*, 18:25–35.

[Vermij, 2002] Vermij, Rienk (2002). *The Calvinist Copernicans: the reception of the new astronomy in the Dutch Republic, 1575–1750.* History of science and scholarship in the Netherlands. Koninklijke Nederlandse Akademie van Wetenschappen, Amsterdam.

[Vermij, 2007] Vermij, Rienk (2007). Varenius and the world of learning in the Dutch Republic. In Schuchard, Margret, editor, *Bernard Varenius (1622–1650)*, pages 99–115. Brill, Leiden & Boston.

[Viète, 1579] Viète, François (1579). *Canon mathematicus seu ad triangula cum appendicibus Canonion triangularum laterum rationalium. Universalium inspectionum ad canonem mathematicum liber singularis.* Apud Ioannum Mettayer, Paris.

[Viète, 1591] Viète, François (1591). *In artem analyticem Isagoge seorsim excussa ab opere restitutae mathematicae analyseos, seu, algebra nova.* Apud Jametium Mettayer, Tours.

[Viète, 1593] Viète, François (1593). *Francisci Vietae Supplementum geometricae. Ex opere restitutae mathematicae analyseos, seu Algembra nova.* Excudebat Iametius Mettayer, Tours. Reprinted in [Viète, 1646].

[Viète, 1595] Viète, François (1595). *Ad problema quod omnibus mathematicis totius orbis construendum proposuit Adrianus Romanus Francisci Vietae responsum.* Apud J. Mettayer, Paris. Reprinted in [Viète, 1646].

[Viète, 1600] Viète, François (1600). *Apollonius Gallus, seu Exsuscitata Apollonii Pergaei* περι επαφων *geometria.* Excudebat D. Le Clerc, Paris. Reprinted in [Viète, 1646], page reference to edition [Viète, 1646].

[Viète, 1609] Viète, François (1609). *Fran. Vietaei Libellorum supplicum in regia magistri insignisque mathematici varia opera mathematica: in quibus tractatur Canon mathematicus, seu Ad triangula. Item Canonion triangulorum laterum rationalium: vnà cum Vniuersalium inspectionum ad canonem mathematicum, libro singulari; quae quidem omnia illustrantur tabulis & appendicibus ad eodem authore recognitis.* Apud Bartholomeum Macaeum, Paris.

[Viète, 1615] Viète, François (1615). *De aequationum recognitione tractatus duo.* Ex Typographia Ioannis Laquehay, in Monte D. Hilarij, in Area Albretia, Paris. Reprinted in [Viète, 1646].

[Viète, 1646] Viète, François (1646). *Francisci Vietae Opera mathematica, in unum volumen congesta, ac recognita.* Ex officina Bonaventurae & Abrahami Elseviriorum, Leiden. Edited by Frans van Schooten.

[Viète and de Beaugrand, 1631] Viète, François and Beaugrand, Jean de (1631). *In artem analyticem isagoge. Eiusdem ad logisticem speciosam notae priores.* Apud Guillaume Baudry, Paris.

[Viète et al., 1986] Viète, François, Beaune, Florimond de, Girard, Albert, and Schmidt, Robert H. (1986). *The early theory of equations: on their nature and constitution: On the recognition of equations by François Viète from the 1615 Anderson edition of* De aequationum recognitione et emendatione*; A new discovery in algebra by Albert Girard; On the nature and constitution of equations by Florimond de Beaune.* Golden Hind Press, Annapolis.

[Vlacq, 1628a] Vlacq, Adriaen (1628a). *Arithmetica Logarithmica, Sive Logarithmorum Chiliades Centum, Pro Numeris naturali serie crescentibus ab Unitate ad 100000. Una Cum Canone Triangulorvm, Seu Tabula Artificialium Sinuum . . . .* Excudebat Petrus Rammasenius, Gouda.

[Vlacq, 1628b] Vlacq, Adriaen (1628b). *Arithmetique logarithmetique ou la construction et usage d'une table contenant les logarithmes de tous les nombres depuis l'unité jusques à 100000: et d'une autre table en laquelle sont comprins les logarithmes des sinus . . . .* Chez Pierre Rammasein, Gouda.

[Vlacq, 1633] Vlacq, Adriaen (1633). *Trigonometria artificialis sive magnus canon triangulorum logarithmicus ad radium 100000,00000, et ad dena scrupula secunda . . . .* Excudebat Petrus Rammasenius, Gouda.

[Vlacq, 1651] Vlacq, Adriaen (1651). *Tabulae sinuum, tangentium et secantium: et logarithmi sinuum, tangentium, et numerorum ab unitate ad 10.000. Cum methodo facillime illarum ope resolvendi omnes triangulos rectilineos et sphaericos, et plurimas quaestiones astronomicas.* Apud Philippum de Croy, Leiden.

[Vlacq and Briggs, 1633] Vlacq, Adriaen and Briggs, Henry (1633). *Trigonometria artificialis sive magnus canon triangulorum logarithmicus ad radium 10000000000, et ad dena scrupula secunda.* Ph. Rammasenius, Gouda.

[Vlieghe, 2001] Vlieghe, Hans (2001). The fine and decorative arts in Antwerp's golden age. In O'Brien, Patrick, editor, *Urban achievement in early modern Europe: golden ages in Antwerp, Amsterdam and London.* Cambridge University Press, Cambridge.

[Waard, 1917] Waard, C. de (1917). Lettres inédites de Fermat. *Mémoires de l'Academie des sciences, inscriptions et belles-lettres de Toulouse*, 11. 11e série. t. 5.

[Waessenaer, 1640] Waessenaer, Jacob van (1640). *Den on-wissen wis-konstenaer I.I. Stampioennius ontdeckt door sijne ongegronde weddinge ende mis-lucte solutien van sijne eygene questien.* Johannes Maire, Leiden.

[Wallis, 1658] Wallis, John, editor (1658). *Commercium epistolicum de quaestionibus quibusdam mathematicis nuper habitum: Inter nobilissimos viros Gulielmum Brouncker, Kenelmum Digby, D. Fermatium, D. Freniclum. Una cum Joh. Wallis, Franc. à Schooten.* A. Lichfield, impensis Tho. Robinson, Oxford.

[Wallis, 1685] Wallis, John (1685). *A treatise of algebra, both historical and practical: shewing the original, progress, and advancement thereof, from time to time, and by what steps it hath attained to the heighth at which it now is: with some additional treatises.* Printed by John Playford, for Richard Davis, London.

[Wallis et al., 2003] Wallis, John, Scriba, Christoph J., and Beeley, Philip (2003). *The correspondence of John Wallis (1618–1703).* Oxford University Press, Oxford.

[Walter, 2004] Walter, Axel E., editor (2004). *Königsberger Buch- und Bibliotheksgeschichte.* Aus Archiven, Bibliotheken und Museen Mittel- und Osteuropas. Böhlau Verlag, Cologne and Weimar.

[Walter, 2008] Walter, Axel E. (2008). Die Königsberger Universitätsbibliothek in der Frühen Neuzeit – Grundzüge ihrer Geschichte. Bibliothekgeschichtliche Quellen, Perspektiven der Forschung. In Marti, Hanspeter and Komorowski, Manfred, editors, *Die Universität Königsberg in der Frühen Neuzeit*, pages 264–305. Böhlau Verlag, Cologne and Weimar.

[Wardhaugh, 2008] Wardhaugh, Benjamin (2008). Musical logarithms in the seventeenth century: Descartes, Mercator, Newton. *Historia Mathematica*, 35:19–36.

[Warusfel, 2010] Warusfel, André (2010). *L'oeuvre mathématique de Descartes dans La Géométrie: de la résolution des équations algébriques à la naissance de la géométrie analytique.* PhD thesis, Paris Sorbonne- Paris IV.

[Weland, 1640] Weland, Woldeck (1640). *Strena Mathematica Sive Elegantiorum Problematum Triga.* Ex officini Wilhelmi Christiani, Leiden.

[Wepster, 2010] Wepster, Steven (2010). Ludolph van Ceulen in Hollandse kringen. *Nieuw archief voor wiskunde*, 11:63–69.

[Westra, 1992] Westra, Frans (1992). *Nederlandse ingenieurs en de fortificatiewerken in het eerste tijdperk van de Tachtigjarige Oorlog, 1573–1604.* Canaletto, Alphen aan den Rijn.

[Westra, 2010] Westra, Frans (2010). *Bouwers van sterke werken. Nederlandse ingenieurs in het tweede tijdperk van de Tachtigjarige Oorlog, 1605–1648.* Stichting Menno van Coehoorn, Utrecht.

[Wijnhoven et al., 2010] Wijnhoven, Joseph, Komorowski, Manfred, and Weibel, Julia (2007–2010). *Neuedition der Duisburger Universitätsmatrikel 1652–1818.*

[Wijsenbeek-Olthuis, 1995] Wijsenbeek-Olthuis, Th. F. (1995). Boedelinventarissen. In Instituut voor Nederlandse Geschiedenis, editor, *Broncommentaren*, volume 2, pages 1–73. Instituut voor Nederlandse geschiedenis, Den Haag.

[Wilken, 1622] Wilken, Marten (1622). *Flores Algebraici, dat ist Algebra oder Coss.* Helwich Kallenbach, Embden.

[Wilkens, 1630] Wilkens, Martin (1630). *Arithmetica, oft Reken-konst: verciert met veel schoone regulen ende exempelen, zeer dienstich voor alle kooplieden, ende vlytighe aenvangers deser konst, etc.* By Nathanael Rooman, Groningen.

[Wilkens, 1636] Wilkens, Mart. (1636). *Officina algebrae, waerinne door een miraculosische Inventie seer konstelyck gewrocht worden d'aequatien ofte v̄gelijkinge over de gesommeerde quadraeti cubi; zensizens; asursol: ende andere dierghel. ghetalen, enz. Item de vergelijckingen over de 1e, 2e, 3e, 4e, 5e en 6e corperlijcke ghetalen van pyramidalien. Voorders eene Extraordinaris konstelycke en Practicabele ontbindinghe sommigher alsulcker Polygonalische en Columnarische questien, enz. Ten besluyt noch 300 binomische als andere questien.* A. Eissens, Groningen.

[Wilkens, 1639] Wilkens, Marten (1639). *Aritmetica ofte reken-konst, verciert met veel schoone regulen ende exempelen, seer nut ende bequaem voor alle leerlingen ende vlytige aenvangers deser konst.* By de weduwe van Nathanael Reoman, Groningen. Second edition.

[Wilkens, 1669] Wilkens, Marten (1669). *Aritmetica ofte reken-konst, verciert met veel schoone regulen ende exempelen, seer nut ende bequaem voor alle leerlingen ende vlijtige aenvangers deser konst.* By Frans Bronchorst, ordinaris stadts boeck-drucker, Groningen. Die 4e ende laeste dr., ende nu verb., verm., en van veel druck-fouten gesuivert. Beschreven door Marten Wilkens Embdenaer, in sijn levent voorsanger in Martini Kercke tot Groningen.

[Wilkens, 1719] Wilkens, Marten (1719). *Aritmetica ofte reken-konst, verciert met veel schoone regulen ende exempelen, seer nut ende bequaem voor alle leerlingen ende vlijtige aenvangers deser konst.* By Jurjen Spandaw, Groningen.

[Willems, 1880] Willems, Alphonse (1880). *Les Elzevier: histoire et annales typographiques.* G.A. van Trigt, Bruxelles.

[Winter, 1988] Winter, P.J. van (1988). *Hoger beroepsonderwijs avant-la-lettre: bemoeiingen met de vorming van landmeters en ingenieurs bij de Nederlandse universiteiten van de 17e en 18e eeuw.* Noord-Hollandsche Uitgeversmaatschappij, Amsterdam.

[Witkam, 1967] Witkam, H.J. (1967). Jean Gillot, een Leids ingenieur. *Leids Jaarboekje*, 59:29–54.

[Witkam, 1969] Witkam, H.J. (1969). Jean Gillot, een Leids ingenieur (tweede deel). *Leids Jaarboekje*, 61:39–70.

[Witkam, 1980] Witkam, J.J. (1980). *Jacobus Golius (1596–1667) en zijn handschriften.* Brill, Leiden.

[Witkam, 2007] Witkam, Jan Just (2007). *Inventory of the oriental manuscripts of the library of the University of Leiden*, volume 1. Ter Lugt Press, Leiden.

[Witt, 1661] Witt, Johan de (1661). *Elementa curvarum linearum.* Apud Ludovicum & Danielem Elzevirios, Amsterdam. Edited by Frans van Schooten and published in [Schooten, 1661a], vol. 2, 153–340.

[Witt, 1919] Witt, Johan de (1919). *Brieven aan Johan de Witt, 1648–1660.* Müller, Amsterdam. Edited by Robert Fruin.

[Witt, 1922] Witt, Johan de (1922). *Brieven aan Johan de Witt, 1661–1672.* Müller, Amsterdam. Edited by Robert Fruin.

[Witt, 2000] Witt, Johan de (2000). *Jan de Witt's Elementa curvarum linearum, liber primus.* Springer, London. Translated, edited and commented by Albert W. Grootendorst.

[Witt, 2010] Witt, Johan de (2010). *Jan de Witt's Elementa curvarum linearum, liber secundus.* Springer, London. Translated, edited and commented by Albert W. Grootendorst, Jan Aarts, Miente Bakker and Reinie Erné.

[Worm, 1751] Worm, Ole (1751). *Olai Wormii et ad eum Doctorum Virorum Epistolae*, volume 2. Copenhagen.

[Wreede, 2007] Wreede, Liesbeth de (2007). *Willebrord Snellius (1580–1626), a humanist reshaping the mathematical sciences.* PhD thesis, Utrecht University.

[Wreede, 2010] Wreede, Liesbeth C. de (2010). A dialogue on the use of arithmetic in geometry: Van Ceulen's and Snellius's Fundamenta Arithmetica et Geometrica. *Historia Mathematica*, 37:376–402.

[Yoder, 2004] Yoder, Joella G. (2004). *Unrolling Time: Christiaan Huygens and the Mathematization of Nature.* Cambridge University Press, Cambridge.

[Yoder, 2013] Yoder, Joella G. (2013). *A Catalogue of the Manuscripts of Christiaan Huygens including a concordance with his "Oeuvres Complètes".* Brill, Leiden.

[Zandvliet, 1989] Zandvliet, Kees, editor (1989). *Prins Maurits' kaart van Rijnland en omliggend gebied door Floris Balthasar en zijn zoon Balthasar Florisz. van Berckenrode in 1614 getekend.* Canaletto, Alphen aan den Rijn.

[Zoeteman, 2011] Zoeteman, Martine (2011). *De studentenpopulatie van de Leidse universiteit, 1575–1812: 'een volk op zyn Siams gekleet eenige mylen van Den Haag woonende'.* Leiden University Press, Leiden.

[Zuidervaart, 2012] Zuidervaart, Huib (2012). "De grootste verzameling die men ergens vindt". De collectie wetenschappelijke instrumenten van de Haagse Fundatie van Renswoude 1755–1829. *Lessen. Periodiek van het Nationaal Onderwijsmuseum*, 7:10–13.

Samenvatting

Frans van Schooten jr. (1615–1660) was een van de meest invloedrijke wiskundigen uit de zeventiende eeuw in de Republiek der Zeven Verenigde Nederlanden. Hij verwierf internationale bekendheid door zijn Latijnse vertaling met commentaar van de *Géométrie* van René Descartes, een werk dat in 1637 in het Frans was verschenen als appendix van de *Discours de la méthode*. Dit proefschrift bestudeert Frans van Schooten jr. in de context van zijn eigen tijd. Van Schootens "life of learning in Leiden" omvatte zijn werk als geleerde, zijn privélessen en zijn openbare colleges aan de Duytsche Mathematicque, een opleiding verbonden aan de Universiteit Leiden.

Het proefschrift bestaat uit drie delen. Het eerste deel is een biografie van Van Schooten. Het tweede deel gaat in op de houding van Van Schooten ten opzichte van de wiskundige ideeën van Descartes aan de hand van een studie van een concreet meetkundig probleem. Het derde deel behandelt Van Schootens onderwijsactiviteiten aan de Duytsche Mathematicque.

Biografie

Hoofdstuk 1 bevat een uitgebreid overzicht van Van Schootens leven, op basis van gepubliceerde bronnen en ongepubliceerd archiefmateriaal. Hierbij wordt aandacht besteed aan de diverse netwerken waarin Van Schooten als wiskundige en geleerde functioneerde: zijn leermeesters, de wiskundigen met wie hij omging, de curatoren van de universiteit, de drukkers die zijn werk uitgaven, zijn familierelaties, en de studenten aan wie hij doceerde.

Frans van Schooten jr. werd geboren in Leiden in 1615, in hetzelfde jaar waarin zijn vader werd benoemd tot hoogleraar wiskunde aan de Duytsche Mathematicque. De familie Van Schooten was van Vlaamse komaf en had remonstrantse sympathieën, en de jonge Frans kreeg onderwijs van de remonstrant Jacobus Batelier. Zijn eerste wiskundeonderwijs kreeg Frans van Schooten jr. van zijn vader. Hierbij lag de nadruk op rekenen, praktisch georiënteerde meetkunde en vestingbouw.

Op 15 mei 1631 schreef hij zich in aan aan de Universiteit Leiden als wiskundestudent. Hij volgde colleges bij Jacob Golius, hoogleraar Oosterse talen en wiskunde. In deze colleges maakte hij kennis met de klassieke Griekse meetkunde waarin de nadruk lag op constructies en bewijzen, en ook met meer recent wiskundig werk van onder andere François Viète. Naast de openbare colleges van Golius volgde Van Schooten privélessen bij de uit Pruissen afkomstige wiskundige Christiaan Otterus. Het onderwijs van Otterus vormde de brug tussen de academische wiskunde van Golius en de meer praktisch georiënteerde wiskunde van Van Schootens vader.

De kennismaking met de Franse filosoof en wiskundige René Descartes was een belangrijke gebeurtenis in het leven van Van Schooten. De eerste ontmoeting vond voor of in de zomer van 1636 plaats, waarschijnlijk op initiatief van Jacob Golius. Descartes was toen op zoek naar een kundig illustrator voor het maken van de figuren in zijn te verschijnen *Discours de la méthode* (1637) en

Van Schooten was de ideale kandidaat omdat hij talent had voor wiskunde en voor tekenen. Van Schootens rol bleef niet beperkt tot die van illustrator. Gaandeweg de voorbereidingen ontpopte hij zich tot een discussiepartner die als een van de weinigen in de Republiek in staat was de wiskundige ideeën van Descartes te begrijpen en op waarde te schatten. Tot de dood van Descartes in 1650 onderhielden Van Schooten en Descartes regelmatig contact waarbij de wiskunde zelf het belangrijkste gespreksonderwerp was. Van Schooten toonde weinig tot geen interesse in Descartes' filosofische ideeën over wiskunde en ook niet in de rest van zijn filosofisch gedachtengoed. De scherpzinnigheid van Descartes maakte grote indruk op Van Schooten en maakte hem tot een van Descartes' trouwste volgelingen op het gebied van de wiskunde. Ook na de dood van Descartes bleef Van Schooten zeer toegewijd aan diens wiskundige erfenis, soms tot in het absurde toe.

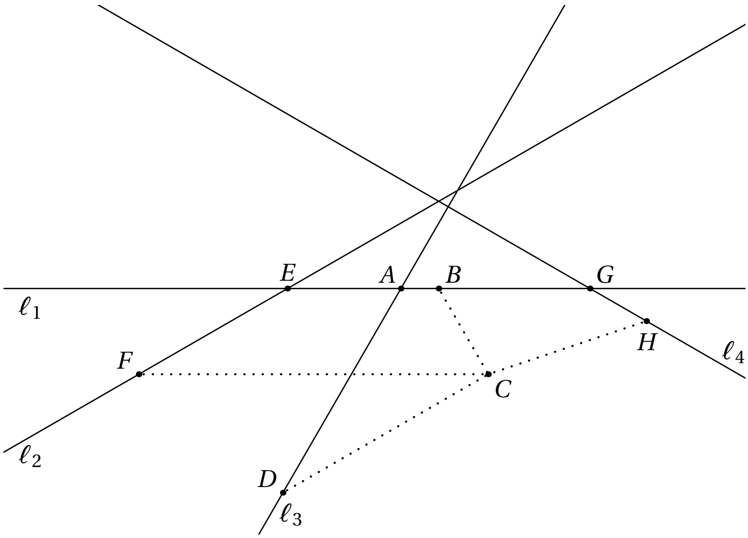
De reis naar Frankrijk, Engeland en Ierland van 1641–1643 markeert het einde van de opleiding van Van Schooten. In Frankrijk ontmoette hij de invloedrijke geleerde Marin Mersenne, die de spil vormde van de Franse wiskundige gemeenschap en contacten met vele wiskundigen uit Europa onderhield. Ook kwam hij in contact met wiskundigen zoals Gilles Personne de Roberval en Florimond de Beaune. In Parijs maakte Van Schooten transcripties van recente correspondentie tussen Mersenne, Pierre de Fermat, en Roberval. Op de hoogte van de nieuwste wiskundige ideeën keerde Van Schooten terug naar Leiden, de stad waar hij de rest van leven zou blijven wonen en werken.

In het jaar 1646 vestigde Van Schooten zich feitelijk als wiskundige. In dat jaar overleed zijn vader en onmiddellijk schakelde Van Schooten zijn netwerk in om tot zijn vaders opvolger benoemd te worden als hoogleraar aan de Duytsche Mathematicque. Dat deze benoeming rond kwam was het resultaat van een lobby die reeds tien jaar eerder was ingezet door zijn vader. In de Republiek waren door de relatief kleine rol van patronage de carrièremogelijkheden voor wiskundigen beperkt en daarom waren universitaire posities gewild. In 1646 publiceerde Van Schooten tevens zijn twee eerste boeken. Het eerste was een uitgave van het verzameld werk van François Viète uitgebracht bij Elzevier en opgedragen aan zijn leermeester Jacob Golius. Het tweede was een studie over kegelsneden die Van Schooten opdroeg aan de curatoren van de Leidse universiteit.

Van Schooten had in 1646 een ambitieus onderzoeksprogramma voor ogen. Hij wilde het verloren gegane werk over vlakke plaatsen van Apollonius van Perga (ca. 200 v. Chr.) reconstrueren en een verhandeling over vestingbouwkunde publiceren. Beide projecten werden echter op de lange baan geschoven toen Van Schooten zich intensief ging bezighouden met het bekend maken en uitleggen van de nieuwe wiskundige ideeën uit de *Géométrie* van Descartes. De eerste editie van Van Schootens Latijnse vertaling met commentaar verscheen in 1649 en was gericht op een internationaal academisch publiek dat zich grotendeels buiten de Leidse universitaire gemeenschap bevond. Door deze publicatie veranderde de *Géométrie* van een appendix van de *Discours de la méthode* in een op zichzelf staand boek *Geometria*. Van Schooten verzamelde studenten om zich heen aan wie hij privélessen gaf over de nieuwe Cartesiaanse wiskunde, en die hij aanzette tot het verkennen van de nieuwe mogelijkheden die hierdoor ontstonden. Veel van hun resultaten voegde hij toe aan de nieuwe tweedelige Latijnse editie *Geometria*, waarvan het eerste deel nog tijdens zijn leven in 1659 verscheen. Na het overlijden van Van Schooten op 30 mei 1660 zorgde zijn halfbroer Petrus ervoor dat het tweede deel werd gepubliceerd in 1661.

Van Schooten en de cartesiaanse meetkunde: het Pappusprobleem

De publicatie van de *Géométrie* door Descartes in 1637 is een mijlpaal in de geschiedenis van de wiskunde omdat dit werk het begin van de analytische meetkunde markeert. In het werk legt Descartes een verband tussen meetkundige krommen in het vlak en algebraïsche vergelijkingen in twee onbekenden. Dit was onderdeel van zijn systematische methode om meetkundige problemen op



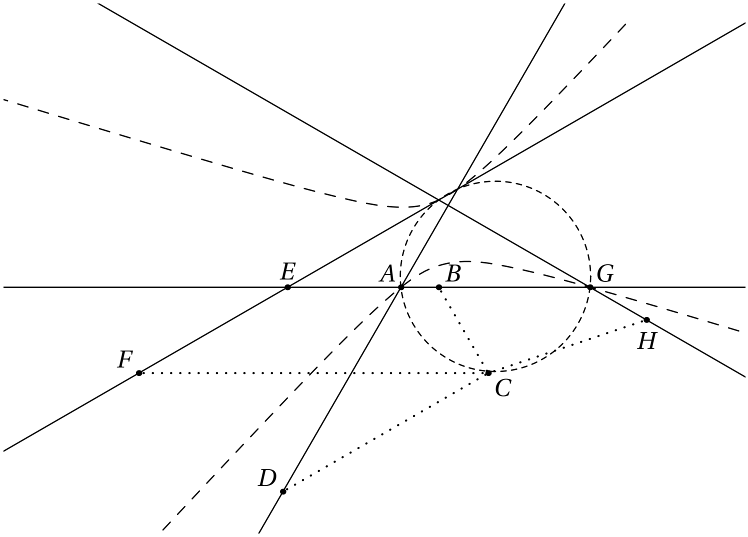
Figuur 1 – Het Pappusprobleem.

te lossen door gebruik te maken van algebraïsche technieken. Daarnaast is de *Géométrie* een belangrijk voor de geschiedenis van wiskundige notatie: zo worden hier voor het eerst de letters x en y gebruikt voor onbekenden in een algebraïsche vergelijking.

Het tweede deel van het proefschrift onderzoekt de houding van Van Schooten ten opzichte van de nieuwe wiskundige ideeën van Descartes aan de hand van een meetkundig probleem, genoemd naar de Griekse wiskundige Pappus van Alexandrië (ca. 300–350). Descartes gebruikte het Pappusprobleem om de kracht van zijn nieuwe methode aan te tonen en hierdoor kreeg het een prominente rol in de *Géométrie*. Echter, Descartes' eigen oplossing van het Pappusprobleem was incompleet en werd bekritiseerd door tijdgenoten met wie Van Schooten in contact stond. Tijdens het maken van de Latijnse versies van de *Géométrie* (verschenen in 1649 en 1659–1661) kwam Van Schooten in aanraking met deze kritieken en moest hij een manier vinden om hiermee om te gaan.

Hoofdstuk 3 geeft een uitgebreide bespreking van de oplossing door Descartes van een speciaal geval van het Pappusprobleem. Dit is noodzakelijk als voorbereiding om de kritieken door andere wiskundigen goed te kunnen begrijpen. Het gaat hierbij om het volgende meetkundige probleem (een geval van het "Pappusprobleem voor vier lijnen"): Gegeven zijn vier rechte lijnen in het vlak. Zie figuur 1 die precies overeenkomt met de figuur die Descartes in de *Géométrie* in detail onderzoekt: de vier gegeven lijnen zijn ℓ_1, ℓ_2, ℓ_3 en ℓ_4, en de snijpunten E, A en G zijn daardoor ook gegeven. We bekijken nu vanuit een willekeurig punt C de scheve afstanden naar elk van de vier lijnen; van elk van deze afstanden is vastgelegd in welke richting de afstand wordt gemeten. Zo is in figuur 1 de afstand van punt C naar ℓ_1 het gestippelde lijnstuk BC. Analoog zijn de afstanden tot de andere drie lijnen vanuit het punt C weergegeven als CF, CD en CH. Het Pappusprobleem bestaat eruit om alle punten C in het vlak te vinden die voldoen aan $CB \cdot CF = CD \cdot CH$. Oftewel, alle punten C in het vlak waarvoor geldt dat de afstand tot de eerste lijn vermenigvuldigd met de afstand tot tweede lijn gelijk is aan de afstand tot de derde lijn vermenigvuldigd met de afstand tot de vierde lijn.

De oplossing van dit probleem staat weergegeven in figuur 2. Hier zien we weer de vier lijnen, het punt C met de scheve afstanden tot de vier lijnen, en daarnaast een cirkel, en een hyperbool (deze



Figuur 2 – De oplossing van het Pappusprobleem.

bestaat uit twee losse stukken). Alle punten (zoals C) op de cirkel en alle punten op de hyperbool voldoen aan het gevraagde. De tekortkoming van Descartes' oplossing in de *Géométrie* bestond eruit dat hij enkel de punten op de cirkel had gevonden. Voor het algemene Pappusprobleem in vier lijnen vond Descartes de punten C op één kegelsnede (ellips, parabool of hyperbool) terwijl de volledige oplossing uit twee kegelsneden bestaat. De oplossing van Descartes was dus niet zozeer fout als wel incompleet.

Het onderzoek in dit proefschrift naar het oplossing van Pappusprobleem door Descartes en het commentaar daarop door Van Schooten heeft nieuwe inzichten gegeven in de manier waarop Descartes en Van Schooten algebraïsche notaties gebruikten. Deze wijkt sterk af van wat tegenwoordig gebruikelijk is en is zelfs verwarrend voor de hedendaagse lezer die bekend is met algebra. De nieuwe inzichten die in hoofdstuk 3 worden beschreven verklaren het gebruik van plus- en mintekens bij substituties en werpen nieuw licht op belangrijke algebraïsche formules uit de *Géométrie*. Tot dusver hebben historici van de wiskunde aangenomen dat er inconsistenties in de formules waren, als gevolg van vergissingen van Descartes of drukfouten. De nieuwe interpretatie van algebraïsche substituties verklaart deze ogenschijnlijke inconsistenties.

De volgende twee hoofdstukken behandelen het debat over het Pappusprobleem en de houding van Van Schooten in het debat. Hoofdstuk 4 gaat over de periode vanaf de publicatie van de *Géométrie* in 1637 tot de publicatie van de *Geometria* in 1649, en in hoofdstuk 5 staat de periode vanaf 1649 tot de verschijning van het eerste deel van de *Geometria* in 1659 centraal. Het debat over het Pappusprobleem is een verhaal over wiskunde waarin ook de onderlinge relaties tussen wiskundigen een rol spelen. Van Schooten werd betrokken bij het debat in 1648, toen hij bezig was met het schrijven van de eerste editie van de *Geometria*, en opnieuw in 1656, toen hij voorbereidingen trof voor de tweede editie van de *Geometria*. Beide keren verliepen volgens hetzelfde patroon: collega-wiskundigen (in 1648 Marin Mersenne en in 1656 Christiaan Huygens) waren op de hoogte van de plannen voor de *Geometria* en confronteerden Van Schooten met kritiek op de oplossing van het Pappusprobleem in de *Géométrie*. In beide gevallen was Roberval de bron van de kritiek.

Zowel in 1648 als in 1656 probeerde Van Schooten de kritiek in eerste instantie te pareren. In 1648 wendde Van Schooten zich hiervoor tot Descartes, en nam hij Descartes' (niet zeer overtuigende) verweer integraal op in de *Geometria (1649)*, zonder hierbij te vermelden dat dit verweer van Descartes zelf afkomstig was. In 1656 kon Van Schooten niet meer terugvallen op Descartes die in 1650 was overleden. Eerst probeerde Van Schooten de kritiek te weerleggen, maar deze tactiek bleek onhoudbaar toen zijn eigen leerling Christiaan Huygens hem het bestaan van de tweede kegelsnede in detail liet zien. Hierop wijzigde Van Schooten zijn tactiek en trachtte hij de kritiek te verenigen met een nieuwe en geforceerde interpretatie van de oorspronkelijk tekst van Descartes in de *Géométrie*.

Het debat over het Pappusprobleem laat zien dat Van Schooten zo onder de indruk was van Descartes dat hij niet in staat was om goed om te gaan met gerechtvaardigde kritieken op diens werk door andere wiskundigen.

Van Schooten en de Duytsche Mathematicque

Het derde deel van het proefschrift onderzoekt de onderwijsactiviteiten van Van Schooten aan de Duytsche Mathematicque, de onderwijsinstelling waaraan hij vanaf 1646 tot zijn dood in 1660 als hoogleraar was verbonden. De Duytsche Mathematicque was in 1600 opgericht op instigatie van stadhouder Maurits van Nassau en had als primair doel ingenieurs op te leiden voor het leger. Het lesprogramma was opgesteld door Maurits' mentor Simon Stevin, tevens kwartiermeester in het Staatse leger, en legde de nadruk op rekenen, landmeten en fortificatie. In de loop van tijd verbreedde het publiek zich naar toekomstige landmeters, ambachtslieden en kooplieden.

De Duytsche Mathematicque bevond zich aan de periferie van de Leidse universiteit. Het meest in het oog springende kenmerk was het gebruik van het Nederlands in de lessen, terwijl Latijn de gangbare taal was in de rest van de universiteit. Daarnaast werden de toehoorders van de lessen niet beschouwd als volwaardige studenten. Zij konden zich niet inschrijven als student aan de universiteit (een enkele uitzondering daargelaten) en waren geen lid van de universitaire gemeenschap. Voor de hoogleraren aan de Duytsche Mathematicque lag dit anders. Zij waren aangesteld onder vergelijkbare arbeidsvoorwaarden als andere buitengewoon hoogleraren ("professores extraordinarii") aan de Leidse universiteit.

Aan de hand van ongepubliceerde manuscripten wordt in dit proefschrift de onderwijsactiviteit van Van Schooten onderzocht. Tot dusver hebben historici aangenomen dat het onderwijsprogramma van de Duytsche Mathematicque statisch was, maar bestudering van de manuscripten levert een ander beeld. Van Schooten heeft het programma van de Duytsche Mathematicque naar eigen inzicht aangepast. Hij herzag bestaande cursussen zoals de cursus rekenkunde, voegde nieuwe theoretische onderbouwingen toe aan bestaande vakken zoals vestingbouwkunde, en plaatste nieuwe onderwerpen op het curriculum zoals algebra, logarithmen en perspectief.

In Van Schootens onderwijs aan de Duytsche Mathematicque is een combinatie te zien tussen geleerde, academische wiskunde enerzijds en meer praktisch georiënteerde wiskunde anderzijds. Deze tweevoudige achtergrond is zichtbaar in Van Schootens cursus over de rekenkunde. Op het eerste gezicht sluit deze aan bij de rekenmeestertraditie met veel aandacht voor concrete voorbeelden in een koopmanscontext. Van Schooten vernieuwde de cursus door de hoeveelheid traditionele rekenregels aanzienlijk te reduceren en meer aandacht te besteden aan algemeen geldige regels dan aan regels die alleen in speciale gevallen toepasbaar waren. Ook behandelde hij meetkundige bewijzen van rekenkundige regels.

De spanning tussen traditie en vernieuwing is ook te zien in de vakken die Van Schooten toevoegde aan het curriculum van de Duytsche Mathematicque. Een goed voorbeeld hiervan is algebra. Van Schooten beschouwde algebra in de geest van Descartes als een krachtige methode om rekenkundige en meetkundige problemen op te lossen. Echter, hij gaf zijn lessen in algebra binnen de traditionele kaders. Het meest in het oog springend aspect hierbij is dat hij aan de Duytsche Mathematicque de traditionele ("cossische") algebraïsche notatie gebruikte die in de zestiende en vroege zeventiende eeuw gangbaar was. In de Duytsche Mathematicque liet hij zijn toehoorders geen kennis maken met de nieuwe notatie die Descartes in de *Géométrie* had ingevoerd en de mogelijkheden die deze nieuwe notatie bood. Dit aspect bleef beperkt tot zijn privélessen.

De spanning tussen vernieuwing en traditie is ook zichtbaar in de manier waarop Van Schooten omging met de erfenis van de *Géométrie* van Descartes. Enerzijds had hij de wens om nieuwe wiskundige ideeën te verkennen, en hij stimuleerde zijn studenten op deze weg verder te gaan. Anderzijds was zijn hang naar traditie en behoudendheid duidelijk in de manier waarop hij reageerde op de kritiek op Descartes' *Géométrie*: hierin wilde hij tot elke prijs vermijden de autoriteit van Descartes te ondermijnen.

Het proefschrift bevat tenslotte een appendix waarin zestien zeventiende-eeuwse handschriften uit de collectie van de Universiteitsbibliotheek Groningen in detail worden beschreven. De meeste van deze handschriften zijn geschreven door een van de Van Schootens (Frans sr., Frans jr. of Petrus). Het blijkt dat enkele van deze handschriften die tot dusver aan een lid van de familie Van Schooten waren toegeschreven, niet in direct verband tot deze familie staan.

Woord van dank

Toen ik in september 2007 begon als aio aan het Mathematisch Instituut stond vast dat ik me zou gaan bezighouden met "een onderwerp" uit de wiskunde in de zeventiende eeuw in de Republiek. Dat onderwerp vond ik in de persoon van Frans van Schooten. De afgelopen jaren heb ik veel tijd met hem doorgebracht en ik moet bekennen dat hij een aangenaam gezelschap was. Daarnaast ben ik in de afgelopen jaren in contact gekomen met personen en instanties die direct of indirect hebben bijgedragen aan dit proefschrift en ik hecht eraan enkelen op deze plek te noemen.

Allereerst is er mijn promotor en dagelijks begeleider Jan P. Hogendijk, die ik dank omdat hij dit promotieavontuur met mij aandurfde. Hij gaf mij mij de vrijheid om mijn eigen invulling te geven aan het onderzoek. Dit was voor mij niet altijd de meest eenvoudige en wellicht ook niet de meest directe route naar afronding van het proefschrift, maar wel een zeer leerzaam pad. Er was altijd tijd om de voortgang, of in sommige gevallen juist het ontbreken daarvan, te bespreken. Na een gesprek met Jan had ik altijd nieuwe inzichten gekregen.

Een groot deel van het onderzoek heb ik verricht in studiezalen van verscheidene bibliotheken, archieven en andere instituten en dit was niet mogelijk geweest zonder de ondersteuning van bibliotheekmedewerkers die vele meters papier voor mij hebben aangesleept. In het bijzonder hecht ik aan een welgemeend woord van dank voor conservator Gerda Huisman en het bibliotheekpersoneel van de zaal Bijzondere Collecties in Universiteitsbibliotheek Groningen voor de behulpzaamheid in de studiezaal en bij het digitaliseren van de manuscripten uit de collectie.

Ik ben het A.F. Monnafonds en het Landelijk Werkcontact voor de Geschiedenis en Maatschappelijke Functie van de Wiskunde erkentelijk voor de financiële ondersteuning die de digitalisering van de manuscripten uit de collectie van de Universiteitsbibliotheek Groningen mogelijk heeft gemaakt en mijn onderzoek heeft gefaciliteerd.

In de groep geschiedenis van de wiskunde binnen het departement Wiskunde heb ik mij altijd op mijn plaats gevoeld. Ik bewaar goede herinneringen aan de wekelijkse lunch waar de laatste ontwikkelingen in het vakgebied ter tafel kamen en iedereen met een hart voor de geschiedenis van de wiskunde welkom was.

De verschillende versies van de hoofdstukken van dit proefschrift zijn gelezen door Viktor Blåsjö en Steven Wepster. Zij hebben mij voorzien van waardevolle suggesties, nauwgezette commentaren en kritische opmerkingen. Daarmee hebben zij een wezenlijke bijdrage geleverd aan de inhoud en de tekst.

Verder waren er collega's die ik niet dagelijks tegenkwam in de wandelgangen maar die op eigen wijze in verschillende stadia hebben bijgedragen aan de totstandkoming van dit proefschrift. Ik dank Kirsti Andersen en Henk Bos voor het delen van hun kennis over de wiskunde uit de zeventiende eeuw, hun gastvrijheid in de eenentwintigste eeuw en hun vriendelijke nieuwsgierigheid gecombineerd met uitdagende vragen die tot nadenken stemmen. Ik dank Henrik Kragh Sørensen, Jan van

Maanen, Sébastien Maronne en Liesbeth de Wreede voor plezierige en prikkelende discussies. Voor hulp bij het vertalen van Neolatijnse teksten kon ik terecht bij Jan Waszink, waarvoor ik hem hartelijk dank.

De ontmoetingen met andere jonge historici van de wiskunde leverden waardevolle discussies, nieuwe inzichten en bovenal inspiratie. Ik trof mijn collega's op nationaal niveau bij de GWAD en op internationaal niveau tijdens de jaarlijkse Novembertagung. Op beide podia kwam ik Jeanine Daems weer tegen, wiens inspirerende verhaal over wiskunde er in 2000 toe heeft bijgedragen dat ik wiskunde ging studeren. Ik ben bijzonder verheugd dat zij als paranimf zal optreden.

Er waren ook vrienden om me heen die mij ervoor behoedden alle tijd in het onderzoek en het proefschrift te steken. Zo zorgde de vrolijkheid van de blubjes voor ontspanning in Amsterdam en was Joeri bijzonder behulpzaam bij het het drukklaar maken van het manuscript. Verder bleef de band met de oud-huisgenoten in stand, ook toen ik het Leidse voor het Amsterdamse verruilde. Met Allard deelde ik een studentenhuis, de studie wiskunde, en nu ook de woonplaats Amsterdam. Ik vind het mooi dat hij als paranimf nu ook bij de promotie zal zijn.

Tenslotte dank ik mijn familie die de totstandkoming van dit project met betrokkenheid en vertrouwen zijn blijven volgen. Het is bijzonder fijn nu te mogen zeggen dat het proefschrift af is.

Mijn liefste dank gaat uit naar Wouter. Zonder zijn onvoorwaardelijke steun, onuitputtelijke geduld en concrete hulp had het afronden van dit boek nog veel langer op zich laten wachten. Ik kijk uit de toekomst waarin er nog zoveel meer te ontdekken valt.

Curriculum vitae

Jantien Dopper werd geboren op 7 maart 1981 in Veendam. In 1998 behaalde zij het vwo-diploma aan de Openbare Scholengemeenschap Winkler Prins te Veendam. Vervolgens bracht ze een schooljaar door in Frankrijk in het kader van een uitwisselingsprogramma. In dat jaar behaalde ze het baccalauréat économique et social met *mention bien* aan het Lycée Auguste-Chevalier in Domfront, Frankrijk. Daarna keerde ze in 1999 terug naar Nederland voor een studie politicologie aan de Universiteit Leiden. Na het behalen van haar propedeuse (*cum laude*) maakte ze de overstap naar de studies wiskunde en geschiedenis, beide aan de Universiteit Leiden. Voor haar afstudeeronderzoek wiskunde verbleef Jantien een half jaar aan het Laboratoire Informatique et Distribution (tegenwoordig Laboratoire Informatique de Grenoble) in Grenoble. In 2006 studeerde ze af in de wiskunde op een onderwerp uit de wachtrijtheorie en een jaar later rondde ze haar geschiedenisstudie af met een scriptie over het optreden van de stadhouder Schenck van Tautenburg in de periode 1521–1540 in de noordelijke gewesten van de Habsburgse Nederlanden.

In september 2007 begon ze aan haar promotieonderzoek op het gebied van de geschiedenis van de wiskunde aan het Mathematisch Instituut van de Universiteit Utrecht. Naast haar onderzoek verzorgde ze verschillende colleges en seminaria aan de Universiteit Utrecht en voor HOVO Leiden. Daarnaast was ze in 2009 co-organisator van de Novembertagung, een jaarlijks internationaal congres voor jonge historici in de wiskunde.

Sinds 2012 werkt Jantien als investment manager bij pensioenuitvoeringsorganisatie PGGM te Zeist.