Investigating Indonesian students' difficulties in initial algebra

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Abstract

In the Trends in International Mathematics and Science Study (TIMSS) 2007, Indonesian students' achievement in the algebra domain was significantly below the international average. This fact leads to this study aiming at investigating Indonesian students' difficulties in algebra. To do so, an individual written test on algebra tasks was administered and followed by interviews. A sample of 33 grade seven Indonesian students worked on written test, and nineteen of them were interviewed afterwards. Data analysis revealed that mathematization, understanding algebraic expressions, applying arithmetic operations, understanding the different meanings of the equal sign, and understanding variables, constituted observed difficulties in both the written test and interview data. The consequence of these findings for future research is discussed.

Keywords: algebra, difficulties, Indonesia, linear equations, linear inequalities

1. Introduction

Algebra is a core topic within secondary school mathematics. Throughout the world, however, students experience difficulties in learning algebra (e.g., see Booth, 1988; Drijvers, 2003; Herscovics & Linchevski, 1994; Warren, 2003). Algebra has been increasingly recognized as a subject that is hard to learn (Stacey, Chick & Kendal, 2004). Although this is a worldwide phenomenon, the case of Indonesian algebra education deserves special attention, because Indonesian students showed low scores in the recent TIMSS 2007 study: their average score in the domain of algebra was 405, which was far below the international average of 500 and was significantly lower than the scores of students from other Southeast Asian countries, such as Thailand, Malaysia and Singapore, with students' average scores for algebra were 433, 454 and 579, respectively (Gonzales et al., 2008). This gives rise to the questions of why Indonesian students have such low algebra scores and why they seem to experience more difficulties than students in other countries. These are the issues addressed by the study presented here.

2. Difficulties in algebra and research question

What does existing research in algebra education tell us about students' learning difficulties in initial algebra? In literature, we identified five types of difficulties.

First, many studies shows that students often fail to add or subtract like algebraic terms and sometimes detach symbolic expressions from the operations (e.g., Herscovics & Linchevski, 1994; Linchevski, 1995). Also, students misapply commutative as well as associative properties when carrying out subtractions or divisions (Booth, 1988; Warren, 2003), and fail to use distributive property of a multiplication over an addition (Booth, 1988). All together we summarize these as difficulties in applying *arithmetic operations* (ARITH) both in numerical and algebraic expressions.

Concerning the literal symbols, research reveals that students have difficulties to distinguish a literal symbol as a variable that plays the role of a placeholder, a generalized number, an unknown, or a varying quantity (Booth, 1988; Herscovics & Linchevski, 1994). These difficulties are called difficulties in *understanding the notion of variable* (VAR).

In addition to the different views on literal symbols, students also have to see that an algebraic expression has a dual nature: as a calculation process and an algebraic object (Drijvers, 2003; Van Amerom, 2002), which is called the process-object duality (Sfard, 1991); the inability to switch between the process and the object view is called the process-product obstacle (Thomas & Tall, 1991). In the literature, the difficulties on algebraic expressions include the parsing obstacle, the expected answer obstacle, and the lack of closure obstacle. One characteristic of an ability to manipulate algebraic expressions technically as well as with insight that causes difficulties in algebra is the gestalt view on algebraic expressions (Arcavi, 1994). This concerns the ability to consider an algebraic expression as a whole and to foresee the effect of a manipulation strategy. These difficulties are called difficulties in *understanding algebraic expressions* (AE).

Another difficulty concerns the equal sign. In arithmetic, the equal sign often invites carrying out a calculation and writing down a numerical answer, whereas in algebra, it usually means 'is algebraically equivalent to' (Herscovics & Linchevski, 1994; Kieran, 1981). In this study, this difficulty is called difficulty in understanding the different meanings of the equal sign (EQS).

Finally, the difficulty in algebra concerns mathematization which consists of horizontal and vertical mathematization. The difficulty in horizontal mathematization concerns students' difficulty in going from the world of real phenomena to the world of symbols and vice versa (Treffers, 1987; Van den Heuvel-Panhuizen, 2003). The difficulty in vertical mathematization concerns students' difficulty in dealing with the process of moving within the symbolic world (Treffers, 1987; Van den Heuvel-Panhuizen, 2003). We call these difficulties as *mathematization difficulties* (MATH).

Taking as a point of departure the above difficulties and considering linear equations and inequalities in one variable as a central topic within the Indonesian initial algebra program (DEPDIKNAS, 2006), the research question of this study is:

> *What are Indonesian students' difficulties in initial algebra learning, particularly in solving linear equations and inequalities in one variable?*

3. Method

3.1 Sample

The subjects of this study were 33 Indonesian students who finished grade VII (13/14 year old), in which they studied linear equations and inequalities in one variable. The students came from three different schools, one a public school, and the two others religious schools.

3.2 Data collection

First, students were asked to solve a set of algebra tasks with paper and pencil individually for thirty minutes. Next, individual interviews were conducted and videotaped. Each interview took about 15–20 minutes. Out of the 33 participating students who did the written test (ten, thirteen and ten students from the first, second and third schools, respectively), nineteen (six, eight and five, from the three schools, respectively) were interviewed afterwards. They were selected as their written work contained unobservable difficulties. During the semi-structured interviews, students were encouraged to explain their solutions of the written tasks.

3.3 Tasks

The tasks were problems on the topic of linear equations and inequalities in one variable that stem from three sources: Indonesian mathematics textbooks for grade VII, the TIMSS 2003 released items on algebra (IEA, 2003), and the PISA 2006 released items on algebra (OECD, 2006). In total, there were sixteen tasks divided evenly into four sets, consisting of tasks 1-4, 5-8, 9-12 and 13-16, respectively. Each student was randomly assigned one of the four sets of tasks for the written test.

3.4 Analysis of the student data

The data include student written work, interview video registrations, and interview field notes. A case in this study is a student's written work on one single test item or a video clip which covers the interview on one single task. The data were analyzed in three steps. First, the data were clipped into cases as units of analysis. Next, the categorization described in Section 2 was used as an initial lens, and was elaborated through its uses in the initial analysis which came down to an analytical framework. Finally, this framework was applied once more through coding the dataset. To establish an inter-observer reliability, a second coder analyzed 20% of the cases. With a Cohen's Kappa 0.95, the agreement between the first author and the second coder was found to be almost perfect.

4. Results

Table 1 summarizes the result of the data analysis from both the written test and the interviews.

Table 1
Observed difficulties in written test and interviews

		Frequencies and percentages of observed difficulties		
Category	**Subcategory**	**Written test of all students (total 33 x 4 = 132 cases)**	**Written test of interviewed students, only 19 x 4 = 76 cases)**	**Interviews (19 x 4 = 76 cases)**
ARITH				
1	(i)	4	2	3
	(ii)	1	1	3
2	(i)	4	1	1
3	(i)	3	1	1
	(ii)	1	1	1
	(iii)	4	3	3
	Total	**17 (13%)**	**9 (12%)**	**12 (16%)**
VAR				
4	(i)	2	2	3
	(ii)	1	0	0
	Total	**3 (2%)**	**2 (3%)**	**3 (4%)**
AE				
5	(i)	5	3	5
6	(i)	2	2	2
7	(i)	4	0	2
8	(i)	9	4	5
	(ii)	7	5	10
	Total	**27 (21%)**	**14 (18%)**	**24 (32%)**
EQS				
9	(i)	4	4	4
	(ii)	0	0	3
	Total	**4 (3%)**	**4 (5%)**	**7 (9%)**
MATH				
10	(i)	2	2	3
	(ii)	32	18	19
	(iii)	6	1	4
11	(i)	0	0	1
	(ii)	0	0	1
	(iii)	0	0	2
	(iv)	1	0	0
	Total	**41 (31%)**	**21 (28%)**	**30 (40%)**

4.1 Findings from the written test

The analysis revealed that mathematization difficulties (MATH) were the most frequent in written test including all students. The second category in frequency concerned difficulties with algebraic expressions (AE). The same two categories had also the highest frequency in the written test data of the interviewed students. The other categories of difficulties on arithmetic (ARITH), the equal sign (EQS) and the variable (VAR) were observed less frequently, respectively. To illustrate the written test results, we present an example in Figure 1.

Figure 1 shows a written work with MATH and ARITH difficulties. For the former, the student could not formulate the word problem into the inequality $12(x + 2) \leq 180$,

instead he reformulated it as $2 + x = 180$. This is categorized as MATH, and as a problem of setting up an inequality from the given word problem in particular. If $2 + x = 180$ were a correct reformulation of the given task, the next step should have been $x = 180 - 2 = 178$ instead of $x = \frac{180}{2} = 90$. This mistake is categorized in the ARITH category, in the subcategory of an incorrect use of the additive inverse property in solving an equation.

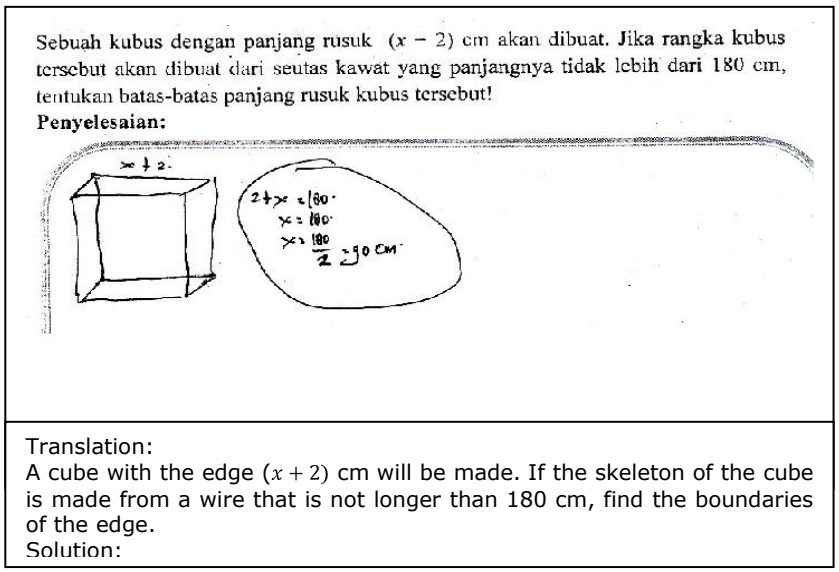

Figure 1. A student's written work with MATH and ARITH difficulties

4.2 Findings from the interviews

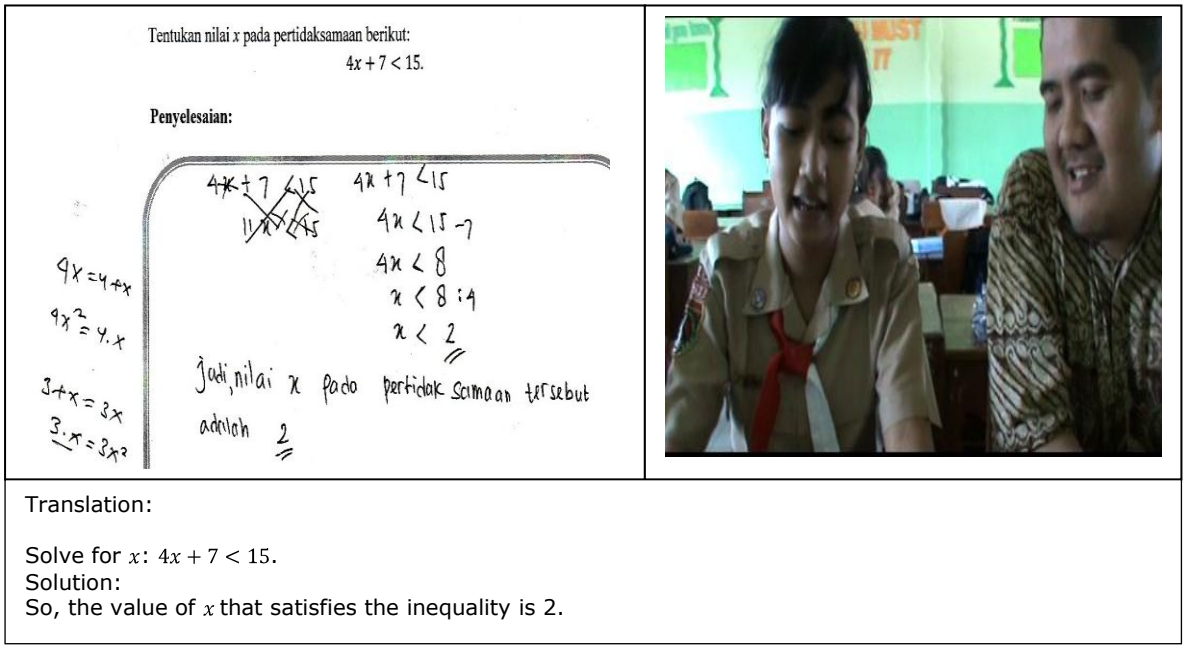

Figure 2. A student interview with AE and ARITH difficulties

The analysis of the data showed little differences between the total group of students on the written test and the interviewed students on the written test. In the interviews,

again, the mathematization (MATH) and the algebraic expression (AE) categories were the most frequently observed difficulties. However, in the interviews more difficulties were found than in the written test. This suggests that the interviews were a better assessment tool for revealing students' difficulties. To illustrate the interview results, we present an example (see Figure 2).

Although the student made an inappropriate conclusion, the written solution is true. Noticing the crossed part, the student seems to encounter the algebraic expression difficulty and the lack of closure obstacle in particular, as revealed in the interview transcript below.

I: Please let me know what you did!
S: [Reads her written solution]
I: What does $4x$ mean: does it mean $4 + x$ or 4 times x?
S: $4 + x$.
I: So, $4x$ is equal to $4 + x$. Is it what you mean?
S: Yes!
I: What is the meaning of $4x^2$
S: 4 times x.

The transcript revealed that the student encountered the algebraic expression (AE) difficulty and the parsing obstacle in particular, i.e., $4x$ perceived as $4 + x$ (conflicting the order of natural and algebraic languages). Furthermore, she calculated, for instance, 4 times x to be equal to $4x^2$, which falls into an ARITH difficulty and the inability to carry out multiplication of algebraic expressions.

5. Discussion

The above results raise several points to discuss. First, the result that the Mathematization category is the most frequently observed category may explain low performance of Indonesian students on studies such as TIMSS 2007 (36st position out of 48, see Gonzales et al., 2008) and PISA 2009 (61st position out of 65, see OECD, 2010). Second, this study addressed the rationale of why students encountered these difficulties only to a limited extent. We did not yet, however, address the rationale behind the difficulty. Thus, this study signals a future research direction. Third, for the purpose of this study, the framework which we set up has worked quite well. Still, it has only been applied to a small sample of students and to a specific case of initial algebra.

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