

The irreflexivity of Brouwer's philosophy*

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Abstract

I argue that Brouwer's general philosophy cannot account for itself, and, a fortiori, cannot lend justification to mathematical principles derived from it. Thus it cannot ground intuitionism, the job Brouwer had intended it to do. The strategy is to ask whether that philosophy actually allows for the kind of knowledge that such an account of itself would amount to.

Brouwer tried to go 'from philosophy to mathematics' and grounded his intuitionistic mathematics in a more general philosophy.¹ This background philosophy can be characterized as a transcendental one. That is, it purports to explain how a non-mundane subject builds up its world in consciousness. It is a radical transcendental philosophy in that this 'world' does not contain just physical objects but everything, including abstract objects and the mundane subject (the subject as part of the world). From the empirical point of view, such a non-mundane subject is an idealized one. Like fellow transcendentalists Kant and Husserl, Brouwer sought to account for mathematics by referring to structural features of acts of this idealized subject [18].

Because of its solipsistic tendencies (but see [17]) and its mystical characteristics, this part of Brouwer's thought is generally dismissed. In fact, more sense can be made of it than is commonly assumed (see Van Dalen's overview mentioned in footnote 1); however the claim I want to defend here is that, even if we grant Brouwer these features, his background philosophy could still not function as a basis for intuitionist mathematics. This philosophy, taken at face value, is not able to reflect on itself. It cannot thematize itself and, a fortiori, it can neither account for itself, nor be self-critical. Therefore it cannot do as the grounding for mathematics that Brouwer wanted it to be. For what is its

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¹ Recently, Van Dalen gave an overview [14], and Van Stigt presented a translation of Brouwer's seminal paper 'Leven, Kunst en Mystiek' [13, 20].

grounding force, if it cannot be distinguished from mere conjectures that would certainly strike us as less solid than mathematical knowledge?

There are different ways of interpreting my claim.

For example, one may be of the opinion that Brouwer's own philosophy is actually irrelevant to the viability of intuitionism in mathematics. Then what follows can be regarded as providing one precise argument in support of that opinion. Moreover, it is an argument that does not depend on an evaluation of solipsism or mysticism.

Or one may rather see it as further encouraging attempts at close reconstruction of Brouwer's ideas within more acceptable frameworks, for example along the lines of the phenomenological approaches of Weyl, Becker, and Heyting (e.g. [21], [1], [15]).

In any case, it is simply a historical point: Brouwer's philosophy was, as a matter of principle, not able to deliver what he wanted from it in mathematics.

Related to this is that, even if one denounces intuitionism or prefers an independent, less 'obscure' argument for it such as Dummett's, that doesn't make precise evaluation of Brouwer's own views unnecessary. Every philosophy deserves to be judged on its own merits.

Before embarking on the argument, let me address the following objection to even undertaking it. 'Brouwer thinks that mathematics is justified to the extent that it conforms with intuition and with certain properties of intuition that establish limitations on what we can know. What more could be needed? Why is reflexivity needed?'

This objection already grants Brouwer precisely that which is under discussion. How did Brouwer come to know these properties? Was he justified in ascribing these properties to intuition? Specifically, could the justification that he himself came up with actually do its job? I will argue that the answer is no, because his background philosophy is not reflexive, as explained above. The reflexivity does not come in only after the mathematics has been vindicated along the lines mentioned in the objection, it bears on that vindication itself.

The force of my argument would not be clear if all philosophies could be accused of this kind of irreflexivity; but clearly that is not the case. For example, nothing in naturalism stands in the way of naturalist talk about naturalism, and nothing in phenomenology hinders phenomenologists in reflecting on and refining phenomenology (in fact, reflexivity is one of its basic features [16, §§77–79]).

The following argument I set forth against the background of Brouwer's last and most detailed philosophical exposition, 'Consciousness, philosophy, and mathematics' [9] (abbreviated CPM). The precise meaning of the terms used will be made clear as we go along.

1. Brouwer's philosophy asserts that there are three (or four) kinds of consciousness: stillness, sensational, mathematical, and (perhaps) wisdom. (Premise)
2. But correctly making this assertion requires a kind of consciousness that

is itself neither stillness, nor sensational, nor mathematical, nor wisdom.
(Premise)

3. So Brouwer's philosophy cannot reflect on itself. (1,2)

To argue for the truth of premise 1, I will bring it out as a distinction that Brouwer stops just short of introducing explicitly, while clearly making it. I start from the first paragraph:

First of all an account should be rendered of the phases consciousness has to pass through in its transition from its deepest home to the exterior world in which we coorporate and seek mutual understanding. [9, p.1235]

Consciousness in its deepest home is the Urstate of consciousness. In this state, consciousness ‘seems to oscillate slowly, will-lessly, and reversibly between stillness and sensation’ [9, p.1235]. There is no sense of time in it. From this original state an ‘exodus of consciousness’ [9, p.1238] can start. This exodus happens in three phases, which Brouwer names the ‘naive’, the ‘isolated causal’, and the ‘social’ [9, p.1238].

In the naive phase the subject comes to distinguish between the present sensation and those of the past. During the following, isolated causal phase, out of sensations thus organised in time grows consciousness of things (among them individuals), and causal relations. The things make up the ‘exterior world of the subject’ [9, p.1236]. In the social phase, the end of the exodus, we become conscious of a world as shared with other individuals, with whom we can coorporate in our dealings with nature and things.

So Brouwer recognizes four states of consciousness, the deepest home and the three transitory phases. To all appearances, this division is meant as a proper one, i.e. exhaustive and consisting of mutually exclusive terms. Each transitory phase presupposes the previous ones and therefore we can speak of the distance of a given phase from the deepest home.

There is a second distinction relating to consciousness, one that Brouwer clearly uses but does not define explicitly. States of consciousness can be classified not only according to distance from the deepest home, as above, but also according to the basic structure of their content. The categories in this classification I call kinds of consciousness. As Brouwer clearly takes his fourfold division of states of consciousness to be exhaustive, a likewise exhaustive list of kinds of consciousness can be obtained by considering, for each state, which kinds he allows for. I think that there are three, and maybe four, which I call stillness, sensational, mathematical², and wisdom. (Except for ‘sensational’, for which he gives no alternative, these are terms used by Brouwer.)

²In Brouwer's philosophical expositions before CPM [2, 3, 7, 8], he has a somewhat wider concept of mathematics than the one described below. There, mathematics is the study of all exact thought; the Dutch word for mathematics, ‘wiskunde’, literally means ‘the art of that which is certain’. It comprises causal attention and mathematical abstraction; in CPM, Brouwer reserves ‘mathematics’ for the latter. The description of consciousness as a whole, however, seems no different across CPM and the earlier papers.

In the deepest home, as we saw above, there are two kinds of consciousness, stillness and the sensational. Note that in Brouwer's usage of the word, 'sensations' need not be sensory; for example, he speaks of 'sensations of vocation and of inspiration' [9, p.1236].

The naive phase is opened by the phenomenon of a move of time. 'By a move of time a present sensation gives way to another present sensation in such a way that consciousness retains the former one as past sensation' [9, p.1235]. Furthermore, by distinguishing between present and past sensation consciousness places itself at a distance from both and from stillness. Consciousness thus distanced Brouwer calls 'mind'. It is here that the subject-object distinction is introduced: mind, in the function of a subject, experiences both the past and present sensation as object. This 'two-ity phenomenon' can be iterated when yet another sensation becomes the present one, and so on, thus leading to 'a world of sensations of motley plurality'. Some sensations turn out to be inextricably bound to the subject; these are the 'egoic' sensations. All egoic sensations of an individual together are called the soul of this human being [9, p.1235]. The opposite of 'egoic' is 'estranged'. We will see in a moment that things in the exterior world are associated with such estranged sensations.

Mathematics seems to have its origins in the naive phase as well, as it

comes into being, when the two-ity created by a move of time is divested of all quality by the subject, and when the remaining empty form of the common substratum of all two-ities, the basic intuition of mathematics, is left to an unlimited unfolding. [9, p.1237]

In the isolated causal phase, Brouwer distinguishes types of entities that we can be conscious of, i.e., causal sequences, things (in particular, individuals), and objects in general. These are all identified with certain iterative complexes of sensations (sequences of sensations that occur repeatedly), differing in their additional properties. Iterative complexes come in two varieties, those whose elements can be permuted in time and those whose elements cannot (this may be a matter of gradation).

The first variety Brouwer does not name as a whole. Some of them, those that are completely estranged from the subject, he calls 'things'; one may suggest the use of 'object complexes' for the whole variety.

The second variety of iterative complexes are the causal sequences. Their defining characteristic is that these are sequences such that 'if one of its elements occurs, all following elements are expected to occur likewise, in the right order of succession' [9, p.1235]. As an illustration of the difference between causal sequences and object complexes: you cannot switch the cutting of an onion and your subsequent crying without breaking the relation between the two, but you can look at the different sides of the onion in any temporal order you want.

The social phase begins when causal attention detects acts of the other individuals. This opens the way to organized cooperation of a group of individuals. A prominent form of this is scientific thinking, 'which in an economical and efficient way catalogues extensive groups of cooperative causal sequences' [9,

p.1237]. However, the kinds of consciousness involved here are still just the sensational and the mathematical, and no new ones are found.

It is not easy to determine what Brouwer means by ‘wisdom’, or even whether it is a kind of consciousness or not. The sharpest characterization is not in CPM, but in ‘De onbetrouwbaarheid der logische principes’ [4] (The unreliability of the logical principles) of forty years earlier, where it is said that ‘wisdom abolishes the discernment between the subject and something different’ [11, p.108].³ But it seems the same is meant in the later text, as there it is stated several times that wisdom is not found in causal thinking, for which that separation is a necessary condition. The following are two examples of such statements.

Searching for *wisdom*, we may find it in knowing that causal thinking and acting is non-beautiful and hard to justify, and that in the long run it brings disappointment. [9, p.1240; original emphasis]

If the delusion of causality could be thrown off, nature, gradually resuming her right, would be (except for her bondage to destiny) generous and forgiving to a mankind decausalized and subsiding to more modest and harmonious proportions. [9, p.1242]

It is not clear to me to what state of consciousness wisdom belongs. On the one hand, it seems that this is the deepest home. For example, at the end of a passage where he describes how ‘there may be wisdom in a patient tending towards reversible liberation from participation in cooperative trade and from intercourse presupposing plurality of mind’, he writes that ‘perhaps at the end of the journey the deepest home vaguely beckons’ [9, p.1242]. Then wisdom would be a third kind of consciousness in the deepest home, for I take it that it is neither a sensation nor stillness. (Also, ‘in wisdom [...] the perception of time is no longer admitted’ [11, p.108].⁴) This leaves only the deepest home, because the other states require the move of time to come into being.

On the other hand, Brouwer often speaks of wisdom as consisting of thoughts that can be expressed, for example when he cites passages from the Bhagavad-Gita [9, p.1241–2] (see also the quote below, on p.6). And the place for thoughts that can be expressed can hardly be the deepest home.

To resolve this unclarity, I suggest that the word ‘wisdom’ as Brouwer uses it is ambiguous. It may mean ‘pure wisdom’, which somehow resides in the state of the deepest home, but which is of yet another kind than stillness or sensation; and it may mean ‘practical wisdom’, which is any knowledge in another state that points to, or tends to, the way back to the deepest home. But as we will see in the discussion of premise 2, for the present argument it will suffice to know that one of the defining characteristics of wisdom, whatever that may be in the final analysis, is its dissociation from causality.

³The original reads: ‘wijsheid heeft de splitsing op in subject en iets anders’. [4, p.3]

⁴The original reads: ‘in wijsheid [wordt] de verschijning van den tijd niet langer aanvaard’. [4, p.3]

In sum, we have found that Brouwer recognizes three (or four) kinds of consciousness: stillness, sensational, mathematical and (perhaps) wisdom. This establishes premise 1.

For the truth of premise 2, I consider each of these four kinds and ask whether the assertion could have its origin in consciousness of that kind.

First, that the assertion cannot have its origin in stillness follows immediately from the meaning of the latter term. In stillness, there is no activity at all.

Secondly, the assertion in question clearly is not purely mathematical; it does not deal with purely mathematical entities, i.e. entities created starting from the empty two-ity.

Thirdly, the assertion cannot be one of pure wisdom either. Introducing a threefold (or fourfold) distinction requires a mathematical operation. But in wisdom there is no place for a *mathematical* understanding, and one cannot coherently make an assertion while at the same time denying something that is presupposed by that assertion. Mathematics depends on the subject-object distinction, while in wisdom precisely that distinction is rejected. That is why Brouwer can write: ‘in wisdom, which abolishes the discernment between the subject and something different, there is no mathematical understanding’ [11, p.108].⁵

One may suggest that the assertion, although not part of pure wisdom for the reason just given, could be an approximation to it. Then the assertion would perhaps keep most of its value (i.e. as practical wisdom). In a sense, Brouwer allows for such approximations, namely, statements of (applied) mathematics that express wisdom in a one-sided or distorted way. This is implied by a characterization of mysticism that he gives:⁶

Perhaps the greatest merit of *mysticism* is its use of language independent of mathematical systems of human collusion, independent also of the direct animal emotions of fear and desire. If it expresses itself in such a way that these two kinds of representations cannot be detected, then the contemplative thoughts – whose mathematical restriction appears as the only live element in the mathematical system – may perhaps again come through without obscurity, since there is no mathematical system that distorts them.⁷

⁵The original reads ‘In wijsheid, die de splitsing opheft in subject en iets anders, is geen wiskundig intelligeeren’[4, p.3; original emphasis].

⁶Here I take it that for Brouwer, ‘wisdom’ and ‘mysticism’ are synonyms. Should it turn out that they are not, that would not weaken my criticism of Brouwer’s philosophy, as I employ this identification to see if it yields a more charitable reading of Brouwer.

⁷My modification of Van Stigt’s translation [19, p.398]; original emphasis. The original reads:

En misschien is de beste qualificatie van *mystiek* een gebruik van de taal, onafhankelijk van de wiskundige systemen der verstandhouding, maar ook onafhankelijk van directe dierlijke aandoeningen van vrees of begeerte. Kleedt zij zich zodanig in, dat het lezen van voorstellingen van de beide zooeven genoemde groepen, onmogelijk is, dan kunnen misschien die contemplatieve gedachten, waarvan de in het wiskundig systeem levende, de wiskundige vereenzijdigingen zijn, weer ongetroebeerd doorbreken, daar er geen wiskundig systeem is, dat ze verwringt. [12,

But it is hard to see how this possibility could be of any help here. If, as we just concluded, introducing distinctions runs counter to the fundamental nature of wisdom, how then could adding a distinction still lead to an approximation of wisdom?

Fourthly, it cannot be a case of causal consciousness. The reason is that causal attention only organizes sensations and does not add anything qualitatively different to them; a fortiori, consciousness itself is inaccessible to causal attention. To see the validity of this claim, consider the following argument (the numbering indicates that it deals with premise 2 of the argument above):

- 2.1 Sensations are individuated in time. (Premise)
- 2.2 Consciousness is prior to the move of time. (Premise)
- 2.3 Consciousness cannot be individuated in time. (2.2)
- 2.4 Hence consciousness is not a sensation. (2.1,2.3)
- 2.5 Consciousness is neither a sensation nor a complex of sensations. (2.4)
- 2.6 Consciousness is inaccessible to causal attention. (2.5)

In the whole argument, ‘consciousness’ refers to consciousness as such, and not to any of its particular kinds or stages. By premise 2.2 I mean that consciousness can exist without the move of time, but not vice versa. The priority is ontological.

Step 2.3 is a specification of premise 2.2. For specific varieties that arise after the introduction of the move of time, the step need not be valid; but here it is, as the argument is one about consciousness as such. The inference from 2.1 and 2.3 to 2.4 is licensed by the general principle that two objects that cannot be individuated by the same principle cannot be of the same kind. From step 2.4 we get to 2.5 by another general principle, namely that if objects of kind K are individuated in time, then so are complexes of objects of kind K. In this case, complexes of sensations would be the result of activity of the subject. Finally, we get 2.6 from 2.5 by the definition of causal attention as the ‘free-will-phenomenon’ that ‘performs identifications of different sensations and of different complexes of sensations’ [9, p.1235].

The textual evidence for premises 2.1 and 2.2 is not hard to find in CPM. Let me cite its second paragraph in full (the first one was quoted above, p.3). I split it in two:

- (1) *Consciousness* in its deepest home seems to oscillate slowly, will-lessly, and reversibly between stillness and sensation. And it seems that only the status of sensation allows the initial phenomenon of the said transition. This initial phenomenon is a *move of time*.

(2) By a move of time a present sensation gives way to another present sensation in such a way that consciousness retains the former one as a past sensation, and moreover, through this distinction between present and past, recedes from both and from stillness, and becomes *mind*. [9, p.1235; emphases in the original]

According to the second part, it is by a move of time that one present sensation makes place for another. The former is retained in the present one as ‘past’. A further move of time iterates the process, which can be pictured like this:

1
2 (1)
3 (2 (1))
⋮

So when sensation 2 occurs, sensation 1 is retained as past; then a move of time gives way to sensation 3 as present, retaining 2 as past, which retained 1 as past. And so on. The structure of time is a nested one, so with each sensation a different moment in time is associated. Conversely, this association serves to individuate sensations; this yields premise 2.1. (The picture is meant to indicate a principle of structure, and not to suggest perfect memory.)

Regarding premise 2.2, the first part of the quoted paragraph says that consciousness can exist without having sensations, but that the status of sensation is a necessary condition for a move of time to occur.⁸ Hence a move of time depends for its existence on consciousness, but not vice versa. Thus consciousness is (ontologically) prior to the move of time. (For a remark on the word ‘seems’, see p.10.)

This conclusion is corroborated by Brouwer’s statement further on that one can always ‘easily [realize] temporary refluence to the deepest home leaving aside naivety, through the free-will-phenomenon of detachment-concentration’ [9, p.1238]; recall from p.4 that by ‘naivety’ Brouwer means the phase of consciousness where the move of time occurs. An objection here may be that the word ‘temporary’ suggests that permanent refluence is not possible and that this indicates that somehow consciousness cannot exist entirely independent from time. However, I think that ‘temporary’ was put in to justify the ‘easily’, and not to suggest that permanent refluence is impossible in principle. Such an addition does not appear in the earlier formulation of this idea in the first of the two 1928 Vienna lectures:

Everyone can have the inner experience, that he can at will dream himself to be without time awareness and without the separation of the *I* and the world of perceptions, or bring about this latter separation by his own effort.⁹

⁸The ambiguity in Brouwer’s use of ‘sensation’ in the whole paragraph between 1. sensation as a state of consciousness and 2. sensation as content of consciousness is harmless, I think, as being in the status of sensation is equivalent to having sensation(s) in the second sense.

⁹‘Es kann jedermann die innere Erfahrung machen, daß man nach Willkür entweder sich

Likewise, one of the rejected passages of his thesis reads: ‘We can go even further and say that the creation of time as a matrix of moments is a free act of ourselves’.¹⁰

Both quotations state that time is a free creation of consciousness, which implies that consciousness can exist independently from time.

This concludes the subordinate argument, begun on p.7, for premise 2. It is able to stand by itself but is corroborated by the cited earlier passages in Brouwer’s writings. Moreover, it completes the main argument.

Finally, I have to address two objections.

The first is that Brouwer sometimes is self-critical. Let me discuss two examples.

One case concerns Cantor’s fundamental theorem. In a book review of 1914, Brouwer considered this theorem evidently true and as not even in need of a proof [5, p.79]. Only a few years later he rejects this view and now the same theorem is said to be false [6, p953n2]. Then the objection might be that this reversal of view would not be possible according to my argument.

Now I do not mean to rebut this objection by saying that such self-criticism, when it occurs, is not part of Brouwer’s official philosophy (in the sense that the official philosophy would not allow for it and that in such cases Brouwer does not practice what he preaches). It is rather that Brouwer’s philosophy, as mentioned in the introduction, describes an idealized subject. Various contingent factors (such as limitations of memory and attention) could influence the acts of an actual subject. It is such factors that Brouwer held responsible for his change of view. He commented that the said review was written at a stage where he had found the general principles of intuitionism, but was not yet completely clear about their consequences [6, p.953n2].

Another example of Brouwer’s self-criticism brings out the same point. In ‘Historical Background, principles and methods of intuitionism’ [10], he discusses choice sequences and restrictions on the freedom of choosing numbers. Then he adds a footnote:

In former publications I have sometimes admitted restrictions of freedom which regard also further restrictions of freedom. However this admission is not justified by close introspection. [10, p.142]

The root of the error, Brouwer says, is that the original act of introspection was not careful enough; but such a lack of care is a problem that only the empirical, not the idealized, subject can have.

What matters here is not so much whether the details of Brouwer’s diagnoses are correct. The point is that they aim at shortcomings of the empirical subject, not of the idealized subject of the transcendental philosophy. What Brouwer

ohne zeitliche Einstellung und ohne Trennung zwischen Ich und Anschauungswelt verträumen, oder die letztere Trennung aus eigener Kraft vollziehen [...] kann.’ [7, p.154]

¹⁰My modification of Van Stigt’s translation [19, p.401]. The original reads: ‘We kunnen nog verder gaan, en zeggen dat de schepping van den tijd als matrix van momenten, een vrije daad van onszelf is.’ [12, p.31].

finds fault with are not the basic properties of the idealized subject as stated in his philosophy, but with his own thinking about them.

The second objection is that Brouwer in CPM nevertheless speaks about ‘the joyful miracle of the self-revelation of consciousness’ [9, p.1238], which implies the presence of a self-reflexive structure in consciousness.

Now this may be meant as a metaphor, but that would take the sting out of the objection. So for the sake of argument, let me assume that it is to be taken literally. Then my reply would be that, in the way Brouwer understands it, this is not quite self-revelation. He says it is ‘apparent in egoic elements of the object found in forms and forces of nature’ [9, p.1238]. But egoic elements are sensations, and part of the subordinate argument just concluded is that consciousness itself is neither a sensation nor a complex of them. It follows that what can become apparent in egoic elements is not consciousness itself but, at best, consciousness as sensation given. (But it is hard to see how this consequence could be correctly asserted *within* Brouwer’s philosophy: doing so would require the access to consciousness that this philosophy, I argued, denies.) One might take the tentativeness in the first part of the quote on p.7, expressed by ‘seems’ (twice), as an indication that Brouwer may have had some notion of this (note that this tentativeness is absent in the description of other phases of consciousness in CPM). But that would imply a radical scepticism about consciousness itself, thus certainly making it impossible to ‘render an account of the phases of consciousness’, as Brouwer claims to do [9, p.1235].

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