Mathematical potential of special education students

This research was carried out in the context of the Dutch Interuniversity Centre for Educational Research.

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MATHEMATICAL POTENTIAL
OF SPECIAL EDUCATION STUDENTS

REKEN-WISKUNDIG POTENTIEEL
VAN SPECIAAL BASISONDERWIJS LEERLINGEN

(met een samenvatting in het Nederlands)

Proefschrift

ter verkrijging van de graad van doctor aan de Universiteit Utrecht op gezag van
de rector magnificus, prof. dr. G.J. van der Zwaan, ingevolge het besluit van
het college voor promoties in het openbaar te verdedigen
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door
Marjolijn Charlotte Peltenburg

geboren op 28 september 1980
te 's-Gravenhage

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For Barrie and Franka

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Introduction

Introduction

1 Background of the PhD study

There are many doubts about the abilities of weak performing students in mathematics. Are low achieving students in mathematics able to flexibly solve calculation problems by adapting their solution method to the characteristics of a task? Or should we teach them a fixed solution method so that they are assured of an answer? Based on their test performance, doubts about what these students are able to achieve are not surprising. For example, special education (SE) students often have a considerable developmental delay in mathematics compared to their peers in mainstream schools. In particular, the topic of subtraction is a very difficult one for many SE students. Therefore, much time is spent on the learning and teaching of subtraction, meaning that topics such as multiplication and division, ratio, fractions and percentage are postponed or not taught at all in SE (Kraemer, Van der Schoot, & Van Rijn, 2009; Van den Heuvel-Panhuizen, 1991; Woodward & Montague, 2002).

Lots of research on weak performing students in mathematics has focused on the type of instruction that students benefit the most from (see e.g., Allsopp, Lovin, Green, & Savage-Davis, 2003; Kroesbergen & Van Luit, 2005; Woodward & Brown, 2006; Timmermans, 2005; Milo, 2003; Vaughn & Linan-Thompson, 2003; Van Luit, 1987). These research studies are often motivated by ideas on teaching practices that accommodate the students' special needs or limited cognitive abilities. For example, Milo (2003) argued that SE students often experience great difficulties in structuring their learning process, and therefore questioned whether realistic instruction, characterized by students' own contributions in the learning and teaching process, is the most advantageous method for these students to learn mathematics. Other research (e.g., Timmermans, 2005) in SE was also inspired by the thought that low performing students may be less able to construct their own knowledge in mathematical domains, hypothesizing that these students profit more from a direct instruction approach in which they are taught to use a limited number of specific, proven solution methods. This latter approach is a dominant view with respect to the teaching of weak performing students in mathematics (Bottge, 2001). For example, in the KNAW report 'Mathematics education in primary school' [Rekenonderwijs op de basisschool] (2009), it is explicitly stated that these students seem to benefit less from a free form of instruction and more from a steering role by the teacher.

However, the assumption that it is not desirable or perhaps impossible to build further upon the (informal) solution methods that weak performing students generate themselves, has some serious consequences. Firstly, restricting students' developmental space can lead to an attenuation of the richness of the mathematics presented to them. Secondly, taking students by the hand may reinforce the assumption that they are not able to come up with mathematical ideas of their own.

The PhD study described in this thesis was set up to gain more insight in SE students' abilities in mathematics. In particular, the study focuses on revealing SE students' mathematical potential by means of new technological tools. The study builds upon previous work in SE. To begin with, the study is a sequel to a study of Van den Heuvel-Panhuizen (1991) aimed at investigating the ability of mildly mentally retarded students[1] in solving ratio problems. Because the topic of ratio was not included in the mathematics curriculum at the schools that these students attended, a particular type of assessment was employed that did not require any prior teaching of procedures or notations. The ratio problems were presented in meaningful contexts. The test results and the experiences gained from the testing supported the idea that the topic of ratio had been undeservedly omitted from the mathematics curriculum in SE. In general, the results from this study gave rise to reconsidering the presumed limitations of students in SE.

Furthermore, the current study has its roots in *Speciaal Rekenen*, a project (Boswinkel & Moerlands, 2003) that was launched in 2001 at the request of the Ministry of Education with the purpose to support SE schools in implementing a new generation of realistic mathematics textbook series. Characteristic of the approach to mathematics education in the Speciaal Rekenen project was the use of meaningful contexts as a starting point for triggering students' informal mathematical knowledge and insights. By not directly aspiring to more formal levels of doing mathematics, the scope of students' mathematical abilities was considerably extended. This approach to teaching mathematics was operationalized in several small-scale lesson experiments. Similar to the previously described pilot study, one of the lesson experiments focused on a topic that is generally not dealt with in SE, i.e., combinatorics (Abels, Peltenburg, & Verbruggen, 2008). Nevertheless, the students involved in the lesson experiment appeared to be quite able to solve elementary combinatorics problems by following a systematic approach, or by creating a multiplication rule to determine a total number of possible combinations. These and other experiences within the Speciaal Rekenen project revealed a glimpse of SE students' so far hidden potential.

The purpose of the present PhD study was to extend our knowledge of the mathematical potential of SE students in the Netherlands. In particular, the study was designed to enrich our understanding of SE students' performance and strategies in solving subtraction problems up to 100 as well as in carrying out elementary combinatorics problems, and to characterize possibilities for revealing SE students' potential by means of assessment. Furthermore, the study was aimed at investigating teacher perceptions of SE students' mathematical potential.

2. Making the most out of SE students' mathematical potential

2.1 Dynamic assessment approaches

Students are frequently subjected to standardized ability tests during their school career. These tests refer to administering a test in a structured manner for the purpose of obtaining a score (Venn, 2012). Characteristic for standardized assessments is that they reflect the student's present developmental level and previous learning, but do not offer any information about the student's potential, cognitive modifiability, or future development (Allal & Pelgrims Ducrey, 2000; Van den Heuvel-Panhuizen & Becker, 2003). Because of the focus on the student's current level of performance, rather than on the student's ability to respond to learning experiences, traditional assessment approaches may not reveal the student's full range of abilities. However, dynamic assessment (DA) represents an alternative or supplemental approach to these traditional assessments. A key feature of DA is providing the student some form of assistance with the intention to influence performance and hence reveal students' potential for change (Campione, 1989; Van den Heuvel-Panhuizen, 1996).

DA has its roots in Vygotsky's model of cognitive development (Vygotsky, 1986). Within this model, the student's knowledge develops in social interactions with more capable others. These experiences are culturally mediated and gradually become internalized as higher cognitive functions. According to Vygotsky (1978), learning takes place in the 'Zone of Proximal Development' (ZPD). A student acquiring new information initially requires help from another, more capable person, but eventually is able to have greater responsibility for the activity as the knowledge becomes internalized. The ZPD is famously described as the difference between what individuals are capable of doing independently and what becomes possible when working with others (Poehner & Van Compernolle, 2011). In DA, the goal is to determine the size of the ZPD. That is, the aim is to establish the amount of change that can be induced during interactions with the examiner during the assessment process (Gutiérrez-Clellen & Peña, 2001).

Nowadays, DA is used as an 'umbrella term to describe a heterogeneous range of approaches' (Elliott, 2003, p. 16) encompassing a variety of measurement techniques and instruments by several labels, e.g., learning potential assessment (e.g., Budoff, 1987a; 1987b), mediated learning (e.g., Feuerstein, Rand, & Hoffman, 1979), mediated assessment (e.g., Bransford, Delclos, Vye, Burns, & Hasselbring, 1987), learning tests (Guthke, 1992), testing-the-limits (Carlson & Wiedl, 1992), and assisted learning and transfer by graduated prompts (e.g., Campione, Brown, Ferrara, Jones, & Steinberg, 1985). All these approaches have in common that they include elements of teaching, in the form of examiner intervention, tutoring, coaching, or mediation, integrated in the assessment sequence as a means to obtain a more complete picture of the student's actual cognitive abilities (Allal & Pelgrims Ducrey, 2000; Caffrey, Fuchs, & Fuchs, 2008).

In general, DA approaches are seen as more sensitive measures for minority populations compared to traditional assessment approaches (Poehner, 2008). For example, in Israel, Feuerstein, Rand and Hoffman (1979) developed the *Learning Potential Assessment Device* (LPAD), which was used to determine and modify the low functioning of immigrant students. Feuerstein and his colleagues argued these students had acquired their own culture but, due to the discrepancy between the dominant culture and their own, they often struggled to connect the ways of thinking and the representations learned at home with those presented in the school setting. As a result, an extra burden was placed on the students' cognitive functioning. The LPAD was developed to establish how much a student was able to improve by offering assistance during the assessment. By using this test instrument and providing appropriate learning opportunities the hidden capacities of culturally different or deprived students were revealed.

Another example involves the use of DA for SE students. In his work, Budoff (1987a; 1987b) aimed at uncovering SE students' strengths and weaknesses and providing estimates of their potential and recommendations for appropriate special, remedial, or mainstream instruction. In his studies, Budoff (1987a) applied a learning potential test-train-retest assessment procedure and suggested that a substantial part of low-achieving students in special classes classified as educable mentally retarded should actually be considered as educationally handicapped students (p. 77) since they performed, after a training session, at a similar level to that of low-achieving students in regular classes. In general, what DA researchers as Feuerstein and Budoff have in common is that they share a concern that low performing students' potential is often underestimated and, as a consequence, low teacher expectations and assignment to special schooling may act in a self-fulfilling fashion (Elliott, 2003).

Although most DA research is aimed at assessing abstract, domain-general skills (Elliot, 2003), there are also DA studies that reflect particular subject domains, such as mathematics. For example, in the Netherlands, Hamers, De Koning and Ruijssenaars (1997) examined processes of classification and seriation, and Resing (2000) studied analogical reasoning. However, examples of DA studies with a more school curriculum-based mathematical content, such as addition and subtraction up to 100, are scarce (e.g., Allsop et al., 2008).

In sum, there is great variability across DA methods and applications, including its use in SE. Yet, little information is available concerning which DA-inspired assessment approaches may be suited to reveal SE students' potential with respect to the school subject domains of *mathematics*. In the next section the role of models and contexts as means to increase the accessibility of calculation problems up to 100 is briefly addressed.

2.2 Role of models and contexts with respect to calculation

Within current ideas about mathematics education, it is generally considered important to provide students with powerful tools in the form of models to gradually bridge the distance between the student's current informal operations and the intended formal reasoning (Menne, 2001a; Treffers, 1991; Van den Heuvel-Panhuizen, 2001). With respect to the domain of calculation up to 100 the two main models are group models and line models, which both can be a support in students' solution process in solving addition and subtraction problems up to 100. However, these models differ in how they represent numbers, i.e., group models represent numbers in an autonomous way by quantities which are structured in tens and ones, whereas line models represent numbers in the number-word sequence. This essential difference in the representation of numbers strongly influences the way in which operations are carried out: Groups models, such as rods of ten and blocks of one, mainly facilitate the splitting strategy in which groups of tens are combined separately from groups of ones, whereas line models such as the empty number line support the use of the stringing strategy, where students begin with one number in the problem and then move up or down the number sequence (Van den Heuvel-Panhuizen, 2008).

Another typical characteristic of the empty number line is that this model keeps a record of the steps in the solution process. This means that students can visually keep track of the actions they have carried out so far, which may help them in planning the next step. Research by Booth and Siegler (2008) has indicated that it is this particular feature of the number line which allows visual representations of mathematical relations that is particularly useful for promoting students' arithmetic learning.

Several Dutch researchers implemented and evaluated experimental instructional programmes for mental addition and subtraction with numbers up to 100, based on the empty number line as a flexible representational tool (Blöte, Klein, & Beishuizen, 2000; Klein, Beishuizen, & Treffers, 1998; Menne, 2001a; 2001b). They demonstrated that the empty number line can indeed provide a powerful and insightful model for addition and subtraction up to 100 for primary school students, including the lower achievers in mathematics. Moreover, these studies investigated students' solution methods when solving a special type of subtraction problems, i.e., small-difference problems (e.g., 61 – 59) or *nearly disappearing sums* as Menne (2001a) calls them. Because of the small difference to be bridged between the numbers involved in the problem, the *indirect addition* (IA) or *adding-on* procedure is generally considered to be particularly suited for solving these problems.

Research indicates that primary school students are not inclined to spontaneously apply the IA procedure (Blöte et al., 2000; Selter, 2001; Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009a), not even when they have been taught to use this procedure (De Smedt, Torbeyns, Stassens, Ghesquière, & Verschaffel, 2010; Torbeyns, De Smedt, Ghesquière, &

Verschaffel, 2009b). However, other studies (Blöte, et al., 2000; Klein et al., 1998, Menne, 2001a; Veltman & Treffers, 1995) did demonstrate that students of different mathematical ability levels can learn to flexibly apply IA on small-difference problems.

To assist students in using the IA procedure Van den Heuvel-Panhuizen and Treffers (2009) stress the suitability of the empty number line as a supporting model. These authors refer to Freudenthal (1983, p. 107) who advocated what he called using the “geometrical concreteness of the number line” in which the two solution methods connected to the two interpretations of subtraction can be observed: subtraction as taking away and subtraction as adding on.

In short, models like the empty number line can be of assistive support in calculation up to 100 and may support the use of particular strategies and procedures. Apart from the role of models, there are indications that the format in which a problem is presented, that is, as a bare number problem or a context problem, may also influence students’ ways of solving mathematics problems (Carpenter & Moser, 1984; De Corte & Verschaffel, 1987; Fuson, 1992). For example, Van den Heuvel-Panhuizen (1996) found substantial discrepancies in performance scores of second grade students who solved subtraction items with crossing the ten presented as bare number problems and as context problems. In one of the items, which was illustrated with a picture of two boys comparing their height, students were asked to determine the difference in heights. In contrast to the bare subtraction problems this context problem yielded a much higher performance score. It was suggested that instead of using the often time-consuming and error-sensitive taking-away procedure, which the students had been taught for solving subtraction problems, they had probably applied the IA procedure that was elicited by the context. Moreover, the presence of the minus sign in bare numbers problems was assumed to emphasize the taking away action (c.f., De Smedt et al., 2010; Verschaffel, Greer, & De Corte, 2007).

Basically, the context in which a problem is presented can be considered as a task characteristic, either referring to the words and pictures that help students to understand the problem or to the situation or event in which the task is presented (Borasi, 1986; Van den Heuvel-Panhuizen, 2005). It is important not to confuse context problems with word problems. In contrast to context problems, the situation in which a word problem is presented is often exchangeable without modifying the problem in its essence. Moreover, word problems often do not offer the opportunity for students to develop a mathematical model or concept (De Lange, 1987; Van den Heuvel-Panhuizen, 1996).

Van den Heuvel-Panhuizen (1996; 2005) distinguishes three main roles of contexts when assessing students’ mathematical understanding: Contexts can (1) enhance the accessibility of problems, (2), contribute to the transparency and elasticity of problems, and (3) suggest solution strategies to students. These roles of context problems can be helpful in offering students ample opportunities for demonstrating their mathematical abilities.

Scherer (2009) underlines the importance of offering context problems to low achievers in mathematics. She argues that these problems allow students to use their own notations and independently developed strategies which helps them to realize that they can solve mathematics problems with their own ideas. This approach contrasts with the concept of small steps and of same steps, which is still a common form of instruction and support for low achievers in mathematics (Scherer, 2003).

To conclude, a large amount of research has been carried out on how students solve addition and subtraction problems up to 100 and the kind of support that can help them in solving these problems. For example, some of the studies addressed focused on the assistive role of models within instructional settings. However, little is known about the role of models as auxiliary tools for assessment purposes. This PhD study aimed to find out whether and how enriching test items into a dynamic assessment format through using mathematical models can support SE students' solution processes and performances to obtain a more complete image of their true mathematical abilities regarding subtraction up to 100. Moreover, in addition to the role of models, other possible influences like the role of contexts and the numbers involved in subtraction problems were investigated to learn more about SE students' ability to flexibly adapt their solution method to the characteristics of the problems presented to them. In the study, special attention goes to the role of Information and Communication Technology (ICT) in assessing the mathematical abilities of SE students.

2.3 Role of ICT in the context of the learning and assessment of mathematics

According to the Principles and Standards of the National Council of Teachers of Mathematics (2000) ICT increases opportunities for the learning of mathematics. It is stated that using ICT can help students to make more sense of the mathematics they are learning, by providing more efficient ways to represent, analyze, and communicate their mathematical ideas. Moreover, NCMT stresses that ICT should support the learning of *all* learners of mathematics.

The use of ICT is in fact frequently recommended for its potentiality and flexibility to meet the instructional needs of students with learning difficulties (e.g., Education Week, 2011; Hasselbring & Glaser, 2000; Shamir & Margalit, 2011; Woodward & Rieth, 1997). Apart from the role of ICT as an instrument to offer a variety of individualized mathematics instructions to address students' special learning needs, ICT can serve unique mathe-didactical purposes as well. That is, ICT environments make it possible to present tasks to students in a way that cannot be done using a conventional paper-and-pencil format (Van den Heuvel-Panhuizen, Kolovou, & Peltenburg, 2011). As Clements (2000) points out, the dynamic and interactive character of ICT environments allows the development of more complex skills such as mathematical problems solving. For example, Kolovou, Van den Heuvel-Panhuizen, Bakker and Elia (2008) revealed that primary school students can be supported in solving non-routine mathematics problems in an ICT environment with appropriate tasks and dynamic interactive computer tools.

Besides assisting students in solving tasks with a high mathematical demand, ICT can make mathematics problems more accessible to students, as it has the possibility to incorporate particular features that can help students to better understand what the tasks are about. Such features may possess characteristics that their physical counterparts lack. For example, digital manipulatives can be offered in a manageable, clean, and flexible way (Clements, 1999).

Furthermore, from a teacher's or researcher's point-of-view, ICT can provide detailed information on student's cognitive processes which makes ICT environments a fruitful setting for investigating these processes. It is therefore not surprising that ICT-based tasks as opposed to conventional paper-and-pencil tasks have received growing interest from researchers for realizing high quality assessment (Burkhardt & Pead, 2003; Clements, 1998; Kolovou, Van den Heuvel-Panhuizen, Bakker, & Elia, 2008; Threlfall, Pool, Homer, & Swinnerton, 2007, Zenisky & Sireci, 2002), including assessment of low achievers in mathematics (Kumar & Wilson, 1997; Ryba, Selby, & Nolan, 1995; Singleton, 2004).

Several research studies have been carried out with a focus on the role of ICT in the teaching and learning of mathematics. Although meta-analyses (Kroesbergen & Van Luit; 2003; Seo & Pedrotty Bryant, 2009) with a specific focus on the influence of computer-assisted instruction (CAI) do not convincingly show effectiveness on the mathematics performance of low achievers, a review study carried out by Li and Ma (2010), covering 46 studies, revealed significant positive effects of ICT use on primary school students' mathematics achievement and particularly on that of low achievers. Moreover, Li and Ma found that studies that used non-standardized tests as measures of mathematics achievement reported larger effects of ICT than studies that used standardized tests.

For the aims of the present study, several ICT-based assessment environments were developed. These tests were meant to support and capture SE students' solution processes and performances in solving mathematics problems. What these assessment environments have in common is the availability of digital tools that can assist students in representing and organizing the problems involved.

3 Which students are meant by SE students in this thesis?

Learning difficulties in mathematics are investigated by different professional communities. As Sfard (2008) aptly remarks – depending on the research discipline – different views exist about the underlying causes of a mathematics learning difficulty. For example, a specialist in learning disabilities (LD) would consider deficient (meta)cognitive skills (e.g., Geary, 2004) or neurological functioning (Dehaene & Cohen, 1995) as main characteristics of learning difficulties in mathematics, whereas a mathematics educator would rather consider them in terms of misconceptions (e.g., Ernest, 1984), faulty schemes (e.g., Nesher & Herschkovitz, 1994) or buggy algorithms (Brown & Burton, 1978).

Together with different ideas on how mathematics learning difficulties should be framed, different terms are used for describing difficulties in the learning of mathematics: e.g., mathematical learning disorders (American Psychiatric Association, 1994; Rourke & Conway, 1997), mathematical learning disabilities (Dumont, 1994; Geary, 2004), mathematics learning retardation (Desoete & Roeyers, 2002), or dyscalculia (Ruijssenaars, Van Luit & Van Lieshout, 2004). Although the meaning of these terms differ to a certain extent, a general characteristic of students labeled with such terms is their underachievement for their age or grade level standards in mathematics. For the purposes of the present study underperforming students in mathematics are operationalized in terms of the current organizational structure of the Dutch educational system.

In the Netherlands, students who are four to approximately twelve years old may be educated in regular schools, in special primary schools ('speciaal basisonderwijsscholen') or in special schools ('scholen voor speciaal onderwijs'). About 95% of the 4-12-year-old students in the Netherlands attend regular schools, about 3% special primary schools and about 2% special schools (CBS, 2010).

Special primary schools are schools for students with mild learning difficulties and mild behavioral difficulties, whereas special schools include students with more severe limitations, e.g., physical handicaps, chronic illnesses, psychiatric disorders or severe social, emotional and behavioral problems. Instead of attending a special (primary) school, students may stay within a regular school and obtain a budget for extra support ('leerlinggebonden financiering'[2]), which should, for example, be spent on help by a remedial or peripatetic teacher. Regular schools do not include special classes, so students are either included in or excluded from regular school (Van der Veen, Smeets, & Derrikes, 2010). For a historical review of the Dutch special education school system, see Van Drenth (2005).

In this PhD study, the subjects of our research are students who are in special primary schools, and whose mathematical achievements on standardized mathematics tests are one to four years behind compared to their peers in regular education. In this thesis, they are referred to as SE students.

4 Structure of this thesis

The PhD thesis comprises a series of journal articles each addressing a different perspective of this research on SE students' potential in mathematics. Table 1 illustrates the structure of the thesis and shows the three main questions that have guided the research questions dealt with in each chapter.

Table 1
Structure of thesis

Chapter	
1	**Introduction**
2	**Main question 1** How can we reveal SE students' mathematical potential in solving subtraction problems up to 100?
3	
4	
5	
6	**Main question 2** How can we reveal SE students' mathematical potential in solving elementary combinatorics problems?
7	**Main question 3** How do teachers in SE think about their students' mathematical potential?
8	**Conclusion**

In SE, teachers often use standardized written tests to assess their students' mathematical understanding and computational skills. These tests do not allow the use of auxiliary resources and can therefore not provide information on what resources may help the students in solving the problems. To gather this information, an assessment instrument was developed in which items taken from a standardized test (see Appendix) were placed in an ICT environment including two different types of optional auxiliary tools that might help students when solving subtraction problems up to 100 that require crossing the ten. Two studies were carried out each focusing on a particular auxiliary tool for the same set of subtraction problems. In the first study, the ICT-based assessment environment contained a 100-board with digital manipulatives. In the second study, instead of the 100-board, a digital empty number line was included in the ICT environment. In both studies data were collected with the test items in the ICT version and in the standardized version.

Chapter 2 reports on the first study in which a comparison was made between SE students' performance in the standardized written version of the test items and the ICT version which included the digital manipulatives tool. The chapter addresses the following research question:

> *Can the ICT-based test items, including an optional auxiliary tool, reveal more about the students' potential with respect to subtraction than the standardized test items do?*

Chapter 3 further examines the results found in chapter 2 and includes both the data from the first study (i.e., manipulatives study) and the second study (i.e., empty number line study). The analysis focuses on the students' performance and tool use, and more particularly on how the students' use of the digital tools was related to the strategies they applied. This chapter addresses the following research questions:

How does the use of a digital tool influence students' performance in solving subtraction problems?

How does the use of a digital tool (manipulatives and empty number line) relate to students' application of strategies for solving subtraction problems?

Chapter 4 describes a study that is a continuation of the two previous studies, which revealed that SE students showed a higher performance in solving subtraction problems in an ICT-based dynamic assessment that included auxiliary tools than in the standardized test format. In the present study a secondary analysis is conducted from a cognitive load perspective, in which the features of the ICT-based assessment were related to the responses of the students. The chapter addresses the following research question:

What features of an ICT-based assessment can help to reveal SE students' mathematical potential in solving subtraction problems?

Chapter 5 investigates SE students' solution methods in doing subtraction. In particular, it is examined whether SE students can make use of indirect addition (adding on) alongside direct subtraction (taking away) for solving subtraction problems up to 100, and what conditions influence the students' use of the indirect addition procedure. The chapter addresses the following research question:

Can SE students flexibly solve subtraction problems up to 100 by adapting their solution method to characteristics of the problems presented to them?

How successful are SE students' in applying the indirect addition procedure?

Chapter 6 studies SE students' mathematical potential by exploring a domain that is new to them, i.e., combinatorics. In particular, SE students' performance and strategies in solving elementary combinatorics problems in a dynamic ICT-based assessment are investigated. The success rate and strategies of students in regular schools served as a reference. The chapter addresses the following research questions:

Do the success rate and strategy use of SE students in solving combinatorics problems differ from those of students in regular education?

Do the success rate and strategy use of SE students and students in regular education change over grades?

Chapter 7 takes a different perspective compared to the previous chapters. It surveys what perceptions primary school teachers in SE have of their students' mathematical potential and what possibilities they see to reveal this potential. The chapter addresses the following research questions:

What ideas do SE teachers have about the mathematical abilities of SE students?
Have SE teachers experienced the necessity to adapt their expectations about their students' development in mathematics?
What ideas do teachers have about revealing the mathematical potential of SE students?

Chapter 8 connects the findings from the series of studies on SE students' mathematical potential carried out in this PhD study and discusses what possible implications these findings have on the learning and teaching of mathematics in SE in the Netherlands so that SE students are provided with more opportunities for developing their mathematical potential, increasing their educational chances and future autonomy.

Notes

1. With the introduction of the Dutch *Primary Education Act* (Wet op het Primair Onderwijs) in 1998, the distinction between schools for mildly mentally retarded students ('Moeilijk Lerende Kinderen') and students with learning and behavioral difficulties (kinderen met 'Leer- en Opvoedingsmoeilijkheden') was removed. These two school types merged into special primary education ('speciaal basisonderwijs').
2. The *leerlinggebonden financiering* will disappear on 1 August 2014. Instead, *Passend onderwijs* ('suitable education') will be introduced. See for more background information http://www.rijksoverheid.nl.

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Chapter 2

Mathematical power of special educational needs pupils: An ICT-based dynamic assessment format to reveal weak pupils' learning potential

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Mathematical power of special educational needs pupils: An ICT-based dynamic assessment format to reveal weak pupils' learning potential

1 Introduction

As in regular primary education, it is crucial that teachers in special education who deal with pupils with learning difficulties (LD) in mathematics have a good understanding of their pupils' capabilities, i.e., their calculation skills. In the Netherlands, the CITO Monitoring Test for Mathematics (Janssen, Scheltens & Kraemer, 2005) is a frequently used assessment instrument for collecting information about the pupils' development. This instrument, which has the characteristics of a standardised test (e.g., no auxiliary resources are allowed), has been designed for pupils in regular primary education. Therefore, the question arises, whether this instrument gives full opportunities to pupils in special education with LD in mathematics to show what they are capable of.

The study reported here looks beyond the achievement scores that are to be found with a standardised test and intends to expose the pupils' competence that in standardized tests might remain below the surface. In order to do this, seven test items on subtraction in the number domain up to 100 which have been taken from the CITO Monitoring Test for Mathematics End Grade 2 are placed in an ICT environment and extended with a dynamic visual tool which the pupils can use when solving the problems. The research question investigated in this study is whether these ICT-based test items can reveal more about the LD pupils' potential with respect to subtraction than the standardised test items do.

2 Theoretical background

Before we describe how the study was carried out and what results we gained from it, we summarise some findings from research literature that guided the setup of our study. When investigating LD pupils' potential with respect to subtraction, we first needed to have a better image of the pupils' difficulties with subtraction problems and the role of manipulatives in overcoming obstacles when solving these problems. Special attention goes to what ICT-based virtual manipulatives have to offer in connection with dynamic assessment. Finally, we address the functions ICT had in our study: (1) offering the pupils an environment for doing mathematics; and (2) providing us the opportunity to trace the pupils' solution processes.

2.1 Subtraction with numbers up to 100

When pupils learn to calculate in the number domain up to 100, subtraction is mostly more difficult for them than addition (Kraemer, Van der Schoot & Engelen, 2000, p. 59), or, as Riccomini (2005) points out, subtraction is particularly problematic for many pupils and for pupils with LD the difficulties are 'even more troubling'.

Basically, there are three different strategies to solve subtraction problems of the type ‘a – b = ?’: (1) taking away b from a, (2) adding on from b until a is reached, or subtracting from a until b is reached, and (3) by tinkering, e.g., taking away from a a number that is different from b, but is easier to handle and then correct it afterward, or changing the problem into an easier problem by keeping the difference the same. Torbeyns, De Smedt, Stassens, Ghesquière, and Verschaffel (in press) call these three strategies respectively (1) direct subtraction, (2) indirect addition or indirect subtraction, and (3) subtraction by compensating.

Most difficult are the subtraction problems that require ‘borrowing’, which means that the value of the ones-digit of the subtrahend is larger than the ones-digit of the minuend (e.g., 62 – 58). Here many errors occur (Beishuizen, Van Putten & Van Mulken, 1997; Brown & Burton, 1978; Brown & VanLehn, 1982; Fiori & Zuccheri, 2005; Kraemer et al., 2000). A frequently made mistake in these problems is processing the digits in the reverse way (in the case of 62 – 58, subtracting 2 from 8 instead of 8 from 2). Some authors call this mistake the ‘smaller-from-larger’ bug (Ashlock, 2005; Beishuizen, 1993). Riccomini (2005) describes the ‘borrowing across zero’ error, where a pupil attempts to borrow from a zero (e.g., in the case of 102 – 37), and does not continue to borrow from the column to the left of the zero. This would appear to be an extension of the borrowing error previously described. Other errors that can occur, are ‘not carrying the tens’ (e.g., 62 – 58 = 14), ‘mistakenly subtracting all’ (e.g., 75 – 32 = 70 – 30 = 40, 40 – 5 = 35, 35 – 2 = 33) and ‘wrongly combining steps’ (e.g., 58 – 34 = 20, 8 + 4 = 12, together 20 – 12 = 8) (Beishuizen, 1993; Beishuizen et al., 1997; Woodward & Gersten, 1992).

2.2 Subtraction with manipulative materials

A common approach to assist pupils who have difficulties in operating with numbers is to provide them with manipulatives, such as blocks and counters (Driscoll, 1983; Sowell, 1989; Suydam, 1986). These manipulatives can make abstract ideas and symbols more meaningful and understandable for pupils.

Instead of using physically concrete materials, virtual ICT-based manipulatives can be used (Lee & Chen, 2008). Virtual manipulatives might also provide further advantages over physical manipulatives since they allow pupils to perform specific mathematical transformations on numbers easily. For example, whereas physical base-ten blocks must be exchanged (e.g., when subtracting, pupils may need to exchange one 10 for ten 1s), pupils can break a computer base-ten rod into ten 1s. Clements (2002) claims that such actions are more in line with the mental actions that we want pupils to learn. Thus, computer manipulatives can provide unique advantages such as offering a flexible and manageable manipulative, one that, for example, might ‘snap’ into position (Clements & McMillen, 1996).

2.3 Dynamic assessment

The standardised way of assessing requires an objective approach in which pupils are not allowed to receive any help, nor are they allowed to use manipulatives. In contrast to this static way of testing, where the test administrator must avoid giving any information that might be helpful, in 'dynamic assessment' (Campione, 1989; Feuerstein, 1979) a learning environment is created within the assessment situation, in which the pupil's reaction to a given learning task is observed and the assessment task suitably adjusted. Thus, a characteristic of this method of assessment is that it has a certain flexibility and sensibility, which makes it possible to reveal hidden competences of pupils. Comparative research conducted into static and dynamic assessment has shown that information about pupils that remains invisible in static assessment will come to light through dynamic assessment. Furthermore, weaker pupils in particular might benefit the most from this assessment method (Allsop et al., 2008; Van den Heuvel-Panhuizen, 1996).

2.4 An ICT environment to give pupils support to do mathematics

Several researchers have stressed the possibilities of computer-based assessment (CBA) for low achievers (Babbitt & Miller, 1996; Kumar & Wilson, 1997; Ryba, Selby & Nolan, 1995; Singleton, 2004; Woodward & Rieth, 1997). Bottge and Rueda (2006) developed and evaluated the effectiveness of a multimedia CBA compared to a paper-and-pencil assessment (PPA). The CBA measured the same concepts as the PPA, with the additional benefit of providing pupils access to information for solving the mathematical problem rather than having to recall the relevant information from the instructional period. The study showed that the CBA eliminated some of the cognitive demands for the low achieving pupils and thus enabled them to more fully demonstrate their understanding of the mathematical concepts they had learned. With respect to this reduction in cognitive demand, ICT can offer LD pupils a structured, stylised assessment environment in which they can easily keep track of their actions. Further, in comparison with 'real life' actions, such as moving beads on a string, the same actions in the ICT environment can require less concentration and cognitive demand (Kumar & Wilson).

2.5 An ICT environment as a window to the pupil's mind

In addition to advantages of reducing cognitive demand while working on a task, ICT can, as Singleton (2004) points out, help teachers gain a deeper understanding of key difficulties. Or, as Clements (1998) argues, the ICT environment offers 'windows to the mind' for many teachers and researchers. The potential of computer environments to provide insight into pupils' cognitive processes makes them a fruitful setting for research on how this learning takes place (Kolovou, Van den Heuvel-Panhuizen, Bakker & Elia, 2008). This is possible because ICT can register detailed information on pupils' strategies, and so provide a vehicle for orchestrating higher quality assessment (Woodward & Rieth, 1997). For example, capturing software enables us to record every command and statement pupils make while working in an ICT environment and thus provides insight into pupils' cognitive processes. This allows us to assess their strategies in more precise ways than can paper-and-pencil tasks (Kolovou et al., 2008; Van den Heuvel-Panhuizen, 2007).

3 Method

3.1 Participants

In total, 37 pupils from two schools for special education in Utrecht, the Netherlands, participated in the study. Most of these pupils are of Dutch (43%) or Moroccan nationality (35%). The pupils' ages were between 8.9 and 12.11 (average age = 10.5 years). The mathematical level of these special-needs pupils was about end Grade 2. This means that they have a developmental delay in mathematics that ranged from 1 to 4 years, since 8 to 9 year olds are normally in Grade 3 and the 11 to 12 year olds are normally in Grade 6. The two schools used the same mathematics textbook series.

3.2 Data collection

Two types of performance scores were collected. First, in April-May 2008, performance data were collected by a Flash ICT assessment environment, especially developed for this study to function as a dynamic assessment environment. This ICT environment consists of seven subtraction problems in the number domain up to 100. The problems all require borrowing. The items were taken from the CITO Monitoring Test for Mathematics End Grade 2, but were redesigned for the ICT environment. The pupils worked for about 15 to 20 minutes individually in the ICT version of the test with one of the authors of this paper sitting next to them. In the background, Camtasia Studio software was running to record a screen video and an audio file of the pupil's working.

Second, performance data were collected from the CITO Monitoring Test for Mathematics End Grade 2. This data collection took place in June 2008. The pupils did the complete test, including the seven subtraction items. The reason for administering first the ICT version, followed by the standardised test version, is that we wanted to avoid a retest effect in the scores of the ICT version. Along with the two types of performances data, other data from the pupils were collected, such as age, sex, and nationality. Furthermore, we received allowance to extract information about the pupils' specific difficulties from their dossiers on developmental background.

3.3 ICT version of the test

The ICT version of the test contains scans from the seven items on subtraction with borrowing. Every item (consisting of text and a picture illustrating the context) is displayed on the screen and the accompanying text is spoken by the computer. The pupil can hear the problem again by clicking on a button, which shows a small ear. An important characteristic of the ICT environment is the flexible and natural way pupils can work on it.

For example, for filling in an answer to a problem, the pupil can drag one or more digits to the answer field. In case of a two-digit number, the pupil can easily change the positions of the digits if necessary. In this way, pupils can already fill in what they know. They are not obliged to first fill in the tens and then the ones, which can be a burden on their working memory.

Each test session started with filling in the pupil's name, grade, date of birth and the name of the school. The pupil was informed about the test procedure and that it was allowed to use the tool with virtual manipulatives. Before the pupil started with the test, he or she was given the opportunity to explore the tool.

3.4 A dynamic visual tool

The tool that is included in the ICT version of the test items offers the pupils a set of virtual manipulatives, consisting of a 100 board with a 10 by 10 grid and divided in four parts with a 5-5 structure. Next to the 100 board, there is a stock of counters, also structured in 5s. The pupils can select a number of counters, drag them to any place on the board, and rearrange or remove them. The tool can be used in any way that is helpful for the pupils. In particular, we expected that this tool would help pupils to overcome obstacles in solving subtraction problems that require borrowing. Our hypothesis was that, through an onscreen visual representation of the problem, the pupils would be less inclined to reverse the processing of the ones-digits. For example, in the case of 62 – 58, the tool can prompt them to find a solution for subtracting 8 from 2 by opening up the next ten.

4 Results

4.1 Comparison of scores in the two formats

Our observations included a total of 518 instances (37 pupils each did seven problems in two formats). A case is considered to be the combination of two responses, one for the standardised test format, and one for the ICT test format. Thus, Table 1 shows the results for the 259 cases. It shows that the percentage of correct answers was higher for the ICT version of the seven items (54%) than for the items in the standardised test format (34%). In 30% of the cases the pupils did not find a correct answer in the standardised test format, but their answer in the ICT version was correct. There were 36% of the cases which were incorrect in both formats.

Table 1
Cross tabulation of correct and incorrect answers in both test formats

		ICT test		Total
		Correct answer	Incorrect answer	
Standardized test	Correct answer	24% (61)	11% (28)	34% (89)
	Incorrect answer	30% (78)	36% (92)	66% (170)
Total		54% (139)	46% (120)	100% (259)

4.2 Tool use and pupil's performance

The choice of the pupils to make use of the tool in the ICT format matched their performance on the standardised test quite well. That is, tool use was higher when the pupils did not find a correct answer in the standardised test (49%), (see Table 2), than when the pupils found a correct answer (21%) in the standardised test format.

Table 2
Cross tabulation of correct and incorrect answers connected to tool use (n = 259)

		ICT version of test items				Total	
		Correct answer		Incorrect answer			
		Tool use		Tool use		Tool use	
Test items standardized test	Correct answer	Yes	18% (11)	Yes	29% (8)	Yes	21% (19)
		No	82% (50)	No	71% (20)	No	79% (70)
	Incorrect answer	Yes	70% (55)	Yes	32% (30)	Yes	49% (84)
		No	30% (23)	No	68% (62)	No	51% (86)
Total		Yes	47% (66)	Yes	31% (38)	Yes	40% (104)
		No	53% (73)	No	69% (82)	No	60% (155)

As Table 2 shows, using the tool does not, of course, guarantee that pupils find the correct answer. It also appeared that, incidentally, the tool worked counterproductively in some cases. In these eight cases the pupils answered the standardised test items correctly, but answered the ICT version of the items incorrectly, despite using the tool. In order to identify the pupils' strategies when using the ICT tool, we analysed all screen videos of the pupils' computer working. In the next section we give an overview of strategies applied in solving the problems that required borrowing.

4.3 Pupils' strategies and tool use in the ICT environment

Analysis of the screen videos of the cases ($n = 66$) in which the pupils found a correct answer and used the tool, showed that in 82% of the cases the pupils applied a taking-away strategy (the subtrahend is taken away from the minuend), in 9% an adding-on strategy (bridging the difference between the subtrahend and the minuend) and in 5% a comparing strategy (comparing the minuend and the subtrahend). In some cases, it was not clear how the pupil's work with the tool related to the answer that he or she had filled in. These cases were coded 'Unclear'; later on, we will illustrate this.

A surprising finding in our study was that applying one of the previously mentioned strategies did not necessarily mean that the pupils performed this strategy completely on the 100 board. In 38% of the cases in which the correct answer was found and the tool was used ($n = 66$), the tool use was incomplete; see Table 3.

Table 3

Percentage of computer operations with the tool in the cases ($n = 66$) in which the pupils found a correct answer and used the tool

Computer operations by category	%
Complete subtraction operation was carried out	62
Only the minuend was (partly) visualized	29
Only the subtrahend was (partly) visualized	3
Only the difference was (partly) visualized	3
Only the 100-board was opened (and a few counters were dragged to the board)	3
Total	66

4.4 Strategies, tool use and correct answers

Table 4 shows the number of cases using each of the strategies and which each led to the correct answer for the seven subtraction problems. Taking away was by far the most frequently used strategy for solving the problems (54 cases). When this strategy was applied, the pupil first represented the minuend by dragging counters from the stock to the board. Then, the subtrahend was taken away, either mentally or by dragging counters back to the stock, either on the left part or on the right. Finally, the pupil determined the remaining number of counters.

The adding-on strategy was applied when either the subtrahend was represented or nothing was represented on the board. There were six cases in which the adding-on strategy was used. It is remarkable that even in the case of 62 – 58, this strategy was not applied and in the case of 48 – 39 only once. Similar results were found in Torbeyns et al. (in press).

Table 4
Frequencies of strategies for subtraction problems with tool which led to the correct answer

Strategy	50–38	30–18	37–9	62–58	71–3	48–39	50–37	Total
Taking away	9	7	9	7	4	6	12	54
Adding on	2	1	2	0	0	1	0	6
Comparing	1	1	0	0	0	*1	0	3
Unclear	0	1	0	0	*2	0	0	3
Total	12	10	11	7	6	8	12	66

* Examples of this strategy are given in the text.

The strategy that is described as the comparing strategy is used three times by two of the pupils. For example, when Farah (for privacy reasons all names of the pupils have been changed) was solving the problem 48 – 39, she represented the minuend and the subtrahend by dragging counters to the board. Figure 1 shows that she is just dragging the nine 1s of 39. Then, she waited a couple of seconds and gave the correct answer.

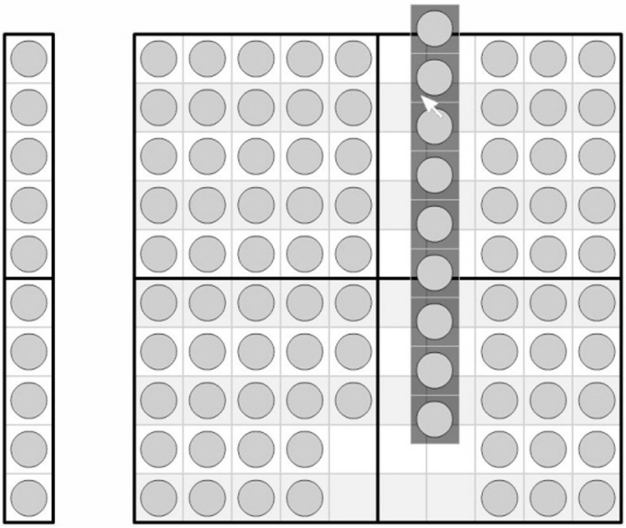

Fig. 1 Farah's representation of the minuend (48) and the subtrahend (39) on the 100 board

Table 4 also shows that in three instances it was unclear which strategy the pupils used. For example, when solving the problem 71 – 3, Saniya represented 70 counters on the board, which we considered as an incomplete representation of the minuend. Then Saniya thought for a couple of seconds and filled in the correct answer.

4.5 Strategies, tool use and incorrect answers

Table 5 shows the number of cases using each of the strategies and which each led to an incorrect answer for the seven subtraction problems. Taking away was the most frequent used strategy for solving the problems (15 cases). When this strategy was applied, the pupil first represented the minuend, then, the subtrahend was taken away.

Table 5

Frequencies of strategies for subtraction problems with tool which led to an incorrect answer

Strategy	50–38	30–18	37–9	62–58	71–3	48–39	50–37	Total
Taking away	3	0	3	*5	1	2	1	15
Adding on	0	1	1	0	0	*2	0	4
Comparing	1	0	0	0	0	0	0	1
Unclear	3	3	6	4	0	1	1	18
Total	7	4	10	9	1	5	2	38

* Examples of this strategy are given in the text.

Although it is not always clear how the pupils came up with their incorrect answer, as Table 5 shows, some cases provided us clear-cut information on their strategies and the errors they made. For example, when Mimoun solved the problem 62 – 58, he represented the minuend by dragging counters to the board. Then, he moved the cursor over the counters starting at the most right bar. He stopped when he arrived halfway to the left most column. He counted the five upper counters, and filled in five in the answer field. This strategy can be described as a taking-away strategy, because Mimoun took away 58 counters from 62 counters. Unfortunately a small counting error had crept in.

Another example is Ayman's strategy for solving the problem 48 – 39. Ayman used an adding-on strategy. He first tried to solve the problem mentally. He said: '39, 38, 39, 40, 41, 42, 43, 44, 45, wait, I want, wait, the board'. Then he opened the 100 board and selected two or three counters a time, which he dragged to the board. He counted out loud and probably to be sure he counted the counters on the board again, starting at 40, until he arrived at 48. Finally, he filled in his answer: 48. It is important to notice that, although Ayman did not find the correct answer, his strategy was in essence a good strategy for finding the answer.

5 Conclusions and discussion

The results of this study showed that the use of an ICT-based assessment including a dynamic tool had a positive effect on pupil scores. This effect was found, even though the pupils had never used the tool before. The fact that pupils with LD found ways in which they could benefit from using the tool, demonstrated convincingly their mathematical power.

In this study the pupils were stimulated to think about their learning process and their understanding of the subtraction problems involved in the test, because for each problem, they had to decide whether or not to use the aid tool. The availability of this tool can elicit pupils' reflection on their calculation skills and, consequently, effect that they become more active in their learning. Such a tool makes the assessment learning oriented (Wiliam, 2008). It is important to note that most pupils appeared to be quite capable of judging their mathematical proficiency and therefore could decide when they could benefit from tool use. In other words, choice of tool use was based upon well-considered understanding of one's own competences. This conclusion sharply contrasts with the lack of metacognitive skills that is generally attributed to LD pupils, and can be considered as yet another clue to the hidden learning potential of these pupils.

Furthermore, the study gave important information on the way the pupils used the tool. It was brought to the fore that finding the correct answer by using the tool does not necessarily mean that the pupils have to perform their strategy completely on the 100 board. Our results showed that 'partial-tool use' can also provide sufficient support to find the correct answer. This conclusion may be in contrast with the idea that in order to realise good understanding of number operations the materialised steps should be equivalent to the mental steps. Moreover, partial tool use could also be an indication of being in a transition phase between working on a concrete level and more abstract level.

The study showed two opportunities of ICT in assessing pupils with LD. First, ICT can provide unique advantages for offering pupils flexible and manageable manipulatives in a dynamic format by which they can overcome obstacles in solving subtraction problems with borrowing. Second, the ICT format makes it possible to examine pupils' actions and thinking processes in detail, which allows to assess their strategies in more precise ways than can paper-and-pencil formats.

Although this study showed that ICT tool use can help LD pupils overcome obstacles in solving subtraction problems, more research is necessary. So far, data have only been collected from 37 pupils. To have a more solid understanding of weak pupils' learning potential in mathematics, more pupils should be involved in future research as well as more mathematical domains. Another point is that it is not clear as to how the tool affected the pupils' strategies. Particularly, we do not know whether the 100 board encouraged or discouraged specific strategies. For this reason, our next step will be collecting data on pupils' strategies with and without tool use in the ICT environment. Other tools will be involved as well, as these could be more efficient and perhaps be more favourable for solving subtraction problems that require borrowing. Giving pupils opportunities to choose between tools, may provide even a better insight in the discrepancy in performance scores between the ICT version and the standardised test version.

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Chapter 3

ICT-based dynamic assessment to reveal special education students' potential in mathematics

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ICT-based dynamic assessment to reveal special education students' potential in mathematics

1 Introduction

When students in special education leave primary school, their mathematical level is generally one to four years behind the level of their peer group in regular schools. This implies a serious delay in the students' mathematical development. However, we have to realise that the students' mathematical level is generally assessed with standardized written tests. For example, in the Netherlands the CITO Monitoring Test for Mathematics (Janssen, Scheltens, & Kraemer, 2005) is a frequently used assessment instrument for collecting information about the students' development. It is characteristic of such standardised written tests that they do not allow the use of auxiliary resources. This implies that standardised written tests cannot provide information on what resources may help students in solving problems. As a consequence, part of the students' mathematical potential may remain invisible.

To collect this missing information, an information and communication technology (ICT)-based assessment instrument was developed that contains test items taken from a standardised test. The digitised items have been enriched with an optional auxiliary tool that students can use to solve them. In this paper, two sub-studies are presented, each focusing on a different auxiliary tool for the same set of test items. Both sub-studies investigate whether the ICT version of the test items, including an optional auxiliary tool, can reveal more about the students' potential with respect to subtraction than the standardised test items do.

2 Theoretical background

On the one hand, this research project on assessing special education students' potential in solving subtraction problems with crossing the ten is theoretically grounded on knowledge about how children solve calculations in the number domain up to 100, and on the other hand on what is known about applying dynamic assessment and the use of ICT in the context of mathematics education. Both the theoretical perspectives inspired the development of the dynamic auxiliary tools that were included in the ICT version of the test items.

2.1 Calculating up to 100: strategies and models

In general, subtraction in the number domain up to 100 is a difficult topic for students in special education. Based on the students' results at the end of primary special education, this topic can be considered a final goal, rather than an intermediate goal for learning calculation (Kraemer, Van der Schoot, & Engelen, 2000; Kraemer, Van der Schoot, & Van

Rijn, 2009). As subtraction up to 100 is a real obstacle in special education, we have chosen this topic as the focus of our study.

There are three basic strategies for calculating up to 100: splitting, stringing and varying (Van den Heuvel-Panhuizen, 2001; see similar classifications used by other authors as well: Beishuizen, 1993; Fuson et al., 1997; Klein, Beishuizen, & Treffers, 1998; Torbeyns, Verschaffel, & Ghesquière, 2006). Using a splitting strategy means that a problem is solved by splitting both numbers in tens and ones. These tens and ones are processed separately. Characteristic of a stringing strategy is that the first number is kept as a whole number and that the second number is added or subtracted in parts. A varying strategy is a kind of stringing, but not in a straightforward way. It implies flexible processing of numbers, based on number relationships and properties of operations, for example, by making use of nearby round numbers (e.g., $59 + 28 = [60 - 1] + [30 - 2] = 60 + 30 - 3$) or converting a problem to an easier problem but keeping the difference the same (e.g., $57 - 29 = 58 - 30$) (Van den Heuvel-Panhuizen, 2001).

Together with these different strategies, different models can be distinguished to support these strategies. In fact, each type of strategy matches with a particular type of model (Van den Heuvel-Panhuizen, 2008). Line models such as the empty number line are suitable models to support a stringing strategy. Groups models such as rods of ten and blocks of one are more adequate to represent a splitting strategy.

The three above-mentioned strategies for solving calculation problems all have in common that they describe how one deals with the numbers involved: They are either processed as whole numbers (in the case of the stringing and varying strategies), or they are split decimally and processed as tens and ones (in the case of the splitting strategy). Instead of focusing on how the numbers are processed, one can also describe calculations from the perspective of how an operation is carried out. Several procedures are possible. For subtraction problems, Torbeyns et al. (2009) distinguish: (1) direct subtraction; and (2) indirect addition or subtraction. We prefer a classification which includes three different ways of carrying out the subtraction operation. A subtraction problem like $a - b = ?$ can be solved by: (1) taking away b from a (direct subtraction); (2) adding on from b until a is reached ($b + ? = a$) (indirect addition or adding on) or subtracting from a until b is reached ($a - ? = b$) (indirect subtraction); and (3) taking away from a, a number that is different from b but is easier to handle and then correct it afterward ($a - [b + n] = ? - n$) or changing the problem into an easier problem by keeping the difference the same ($[a + n] - [b + n] = ?$) (subtraction by compensating).[1] If a direct subtraction procedure is used, it is possible to apply either a splitting or a stringing strategy. For indirect addition or subtraction, a stringing or varying strategy can be used. Subtraction by compensating goes together with varying.

In cases where students have to solve subtraction problems in which the ones digit of the subtrahend (the second number in a subtraction) is larger than that of the minuend (the first number in a subtraction), they can do this in different ways that clearly have a different success rate. The most error-sensitive approach is solving these problems by a direct subtraction procedure together with a splitting strategy, where one has to ‘borrow’ from the tens. For example, $62 - 58 = \_$; $60 - 50 = 10$, $12 - 8 = 4$. A frequent mistake in these problems is reversing the digits (in this case, subtracting 2 from 8 instead of 8 from 2). Some authors call this mistake the ‘smaller-from-larger’ bug (Beishuizen, 1993). A much less error-sensitive approach is solving these problems by an indirect addition procedure together with a stringing strategy. For example, $62 - 58 = \_$; $58 + 2 + 2 = 62$; so, the difference is 4. Using an indirect addition procedure is, however, not very common in primary special education in the Netherlands.

The two different auxiliary tools that were developed as part of the ICT-based assessment instrument are based upon the models that match the two basic strategies for subtraction, that is stringing and splitting. To support a stringing strategy, a digital empty number line was developed. To support a splitting strategy, a tool with digital manipulatives was developed.

As stated above, applying a splitting strategy is generally seen as an error-sensitive approach for solving subtraction problems with crossing the ten. However, in special education it is very common for students to learn to solve calculation problems up to 100 with support from manipulatives, such as rods and blocks. Therefore, we decided not only to develop a tool based on a line model but also one that is based on a group model. However, instead of only providing the students with the manipulatives, we added a 100-board with 10×10 and 5×5 structures to give them more of a hold when using the material. For more details about the tools, we refer to sub-section ‘Assessment instruments’.

2.2 ICT-based dynamic assessment

The standardised way of assessing requires an approach in which students are not allowed to receive any help from the person who administers the test, nor are the students allowed to use auxiliary resources. In contrast to this static way of testing, ‘dynamic assessment’ (Campione, 1989; Feuerstein, 1979) creates an environment which concentrates on gaining insight into the students’ potential, that is investigating whether a child is able to solve a problem with some help. Vygotsky (1934/1978) therefore makes a distinction between a child’s actual developmental level and his or her potential developmental level as revealed by working with an adult or with peers who are more able than the child. Since dynamic assessment is aimed at opening students’ zone of proximal development, it could provide better opportunities for observing and assessing students’ solution processes and their ways of tackling difficulties. To exploit these possibilities, we see an important role for ICT, since it can offer teachers and researchers ‘windows to the mind’ of students, as Clements

(1998) points out. The potential of computer environments to register detailed information on students' strategies can provide a vehicle for getting a higher quality assessment (Woodward & Rieth, 1997). For example, capturing software enables us to record a screen video of student work in an ICT environment, allowing their strategies to be assessed in more precise ways than can paper-and-pencil tasks. In this way, Barmby et al. (2009) collected a rich amount of qualitative audiovisual data on children's use of the array representation for solving multiplication problems using Camtasia Studio software.

In addition to the advantages of capturing students' solution processes, ICT can make problems more accessible for students. Several researchers have stressed the possibilities of ICT-based assessment for low achievers (e.g., Babbitt & Miller, 1996; Kumar & Wilson, 1997; Woodward & Rieth, 1997). For example, Bottge et al. (2009) showed that ICT can eliminate some of the cognitive demands for low-achieving students, which enables them to more fully demonstrate their understanding of the mathematical concepts they have learned. With respect to this reduction of cognitive demand, positive results were also found in several studies on computer-based assessment in which students with disabilities were read aloud mathematics problems on the computer by means of a digital aid (Elbaum, 2007; Helwig, Rozek-Tedesco, & Tindal, 2002; Trotter, 2008). Moreover, Johnson and Green (2006) argue that problems presented on the computer may be less demanding compared to those presented on paper, because '[…] students may have a more positive attitude and in turn greater motivation to complete computer-based questions than paper-based questions' (p. 28).

A third advantage of ICT that we would like to mention here is that it can offer an optional auxiliary tool which the students can activate when they are solving the subtraction problems. As pointed out by Bottino and Chiappini (1998) such a digital tool may assist in cognitive structuring, that is providing a structure for thinking and acting (Tharp & Gallimore, 1991). Digital tools may also provide further advantages over concrete physical materials such as blocks. For example, whereas physical base-ten blocks must be exchanged (e.g., when subtracting, students may need to exchange one ten for ten ones), students can break a computer base-ten rod into ten ones (Clements, 2002). Thus, computer manipulatives can provide unique advantages such as offering flexible and manageable manipulatives, that, for example, might 'snap' into position (Clements, 2002; Clements & McMillen, 1996).

3 Research questions and hypotheses

The main research question is: Can an ICT-based dynamic assessment format provide students in special education with more opportunities to show their potential in solving subtraction problems compared to a standardised test format? This question is subdivided in the following sub-questions and accompanying hypotheses:

1a. Is there a difference in percentages of correct answers on the standardised test items and the ICT version of the test items including the manipulatives?
Hypothesis 1a: The students will attain a higher percentage of correct answers on the ICT version of the test items including the manipulatives in comparison to the standardised test items.
1b. Is there a difference in percentages of correct answers on the standardised test items and the ICT version of the test items including the empty number line?
Hypothesis 1b: The students will attain a higher percentage of correct answers on the ICT version of the test items including the empty number line in comparison to the standardised test items.
2a. What strategies are applied by the students when using the manipulatives in the ICT version of the test items?
Hypothesis 2a: The manipulatives encourage the students to use a splitting strategy.
2b. What strategies are applied by the students when using the empty number line in the ICT version of the test items?
Hypothesis 2b: The empty number line encourages the students to use a stringing or a varying strategy.

4 Method

4.1 Assessment instruments

Data were collected with two assessment instruments: the CITO Monitoring Test for Mathematics End Grade 2 (Janssen, Scheltens, & Kraemer, 2005) and an ICT-based assessment instrument.[2] The latter consists of seven subtraction problems with crossing the ten. These items were taken from the CITO Monitoring Test for Mathematics End Grade 2. Every item, consisting of text and a picture illustrating the context, is displayed on the screen and the accompanying text is read out by the computer. By clicking on a button, the student can repeat the spoken problem once or several times. There are two versions of the ICT-based assessment instrument, each containing a different auxiliary tool: one offers digital manipulatives and the other offers a digital empty number line.

The manipulatives tool consists of counters that can be placed on a 100-board with a 10×10 grid, which is structured in four parts with a 5 × 5 structure. The students can select a number of counters from the stock on the left-hand side of the 100-board. Then they can drag the counters to the board and rearrange or remove them (see Figure 1). We expect that through an on-screen visual representation of the subtraction operation that has to be carried out, the students will be less inclined to process the ones digits in the reverse way. For example, in the case of 62 − 58, the tool can provide a visual prompt to find a solution for subtracting 8 from 2 by opening up the next ten.

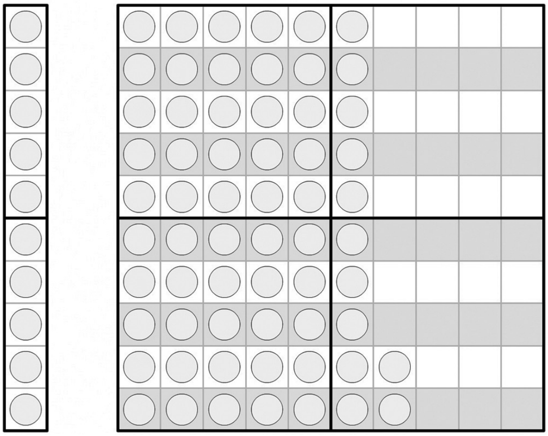

Fig 1. Manipulatives tool with counters and 100-board

The empty number line tool consists of a horizontal line on which the students can put markers and add number symbols and on which they can carry out operations by drawing jumps backward or forward (see Figure 2). The tool operates by touch screen technology. This tool can help the students in solving subtraction problems, since it can function as an aid to order the numbers involved in the problems and carry out the necessary operations. Moreover, when working on the number line the students can visually keep track of their actions. For example, in the case of 62 – 58, by putting the numbers 62 and 58 on the number line, it can become clear that these numbers are actually quite close to each other in the number sequence. This understanding can trigger the students to bridge the difference.

Fig 2. Empty number line tool

4.2 Participants and data collection

Two sub-studies were carried out in which the two auxiliary tools were investigated. Thirty-seven students from two schools for special education participated in the manipulatives study, and 43 students from two other schools for special education participated in the empty number line study. All four special schools are within the metropolitan area of Utrecht, the Netherlands. The students were 8 to 12 years old and their mathematical level was at about the end of Grade 2. This means that the students had a developmental delay in mathematics that ranged from one to four years, since eight- to nine-year-olds are usually in Grade 3 and 11- to 12-year-olds in Grade 6. In the Netherlands, about 3% of the children of primary school age are in special education schools, which is about 45,000 students (CBS, 2008, 2009). This percentage involves only the students who have learning difficulties, that is no students with physical disabilities are included.

In both sub-studies two types of performance scores were collected. First, data were collected with the ICT-based assessment instrument. The students worked individually in the ICT version of the test for about 15 minutes. In the background, Camtasia Studio software was running to record a screen video file of the student work. Five weeks later, data were collected with the CITO Monitoring Test for Mathematics End Grade 2. The students did the complete test, including the seven subtraction items. The reason for administering the ICT version first was to avoid a retest effect in the scores of the ICT version.

4.3 Data analysis

An analysis was conducted to compare the correctness of the students' answers in the standardised test format and the ICT-based test format. To test for significant differences between the scores in the two test formats, we used a *t*-test for paired samples both for the total score and for all individual items in the tests. We did this for the manipulatives study and the empty number line study separately. To make a comparison between the results from these two studies, we converted differences in *p*-values to Cohen's *d* effect sizes.

Furthermore, we analysed the students' procedures and strategies. These procedures and strategies are described in the section 'Calculating up to 100: Strategies and Models'. In the analysis we focused on the cases in which the students used the tool in the ICT environment. Their work was captured on screen video. All students' procedures and strategies were initially coded by the first author. In a second round they were coded by the second author. After discussion on the few cases of disagreement (<5%), full agreement was reached.

5 Results

5.1 Differences in proportions of correct answers

5.1.1 Influence of the ICT version with the manipulatives tool

In the manipulatives study, our observations included a total of 259 cases (37 students did seven problems each). A case covers two scores: one for the ICT test format and one for the standardised test format. Table 1 shows the percentages of correct and incorrect answers for both formats. The percentage of correct answers was higher for the ICT version of the seven items (54%) than for the items in the standardised test format (34%). This appears to be a significant difference in correct answers in favour of the ICT test format (t (36) = 3.67, $p < .01$, $d = .71$). This result confirms Hypothesis 1a.

Table 1
Cross tabulation of correct and incorrect answers in both test formats in manipulatives study

		ICT version of test items		Total
		Correct answer	Incorrect answer	
Test items standardized test	Correct answer	24% (61)	11% (28)	34% (89)
	Incorrect answer	30% (78)	36% (92)	66% (170)
Total		54% (139)	46% (120)	100% (259)

Table 2 displays for each item the proportion of correct answers in the two test formats in the manipulatives study. For five out of the seven items the difference between the two proportions was significantly larger than zero ($p < .05$).

Table 2
Percentage correct in two test formats in manipulatives study

Item	*p*-value				
	Standardized test format	ICT test format	Difference in *p*-value	*SE*	*t*
1	.22	.43	.22	.10	2.25*
2	.49	.54	.05	.09	0.57
3	.19	.38	.19	.09	2.02*
4	.30	.51	.22	.10	2.09*
5	.54	.73	.19	.12	1.64*
6	.41	.57	.16	.11	1.53
7	.27	.59	.32	.10	3.40**
M	.34	.54	.19	.05	3.67**
SD	.14	.11	.08		

* $p < .05$; ** $p < .01$

5.1.2 Influence of the ICT version with the empty number line tool

In the empty number line study, our observations included a total of 301 cases (43 students did seven problems each). As in the manipulatives study, a case covers two scores: one for the ICT test format and one for the standardised test format. Table 3 displays the results for the 301 cases. It appears that the percentage of correct answers was higher for the ICT version of the seven items (55%) than for the items in the standardised test format (36%). This is a significant difference in correct answers in favour of the ICT test format ($t\,(42) = 4.77$, $p < .01$, $d = .75$). This result confirms Hypothesis 1b.

Table 3
Cross tabulation of correct and incorrect answers in both test formats in empty number line study

		ICT version of test items				Total	
		Correct answer		Incorrect answer			
Standardized test	Correct answer	28%	(83)	9%	(26)	36%	(109)
	Incorrect answer	28%	(83)	36%	(109)	64%	(192)
Total		55%	(166)	45%	(135)	100%	(301)

Table 4 displays for each item the proportion of correct answers in the two test formats in the empty number line study. For four out of the seven items the difference between the two proportions was significantly larger than zero ($p < .01$).

Table 4
Percentage correct in two test formats in empty number line study

Item	*p*-value				
	Standardized test format	ICT test format	Difference in *p*-value	*SE*	*t*
1	.16	.42	.26	.09	2.89**
2	.33	.63	.30	.09	3.55**
3	.07	.47	.40	.08	5.24**
4	.37	.60	.23	.08	2.89**
5	.67	.72	.05	.10	0.47
6	.51	.49	–.02	.08	–0.27
7	.42	.53	.12	.08	1.40
M	.36	.55	.19	.04	4.77**
SD	.20	.11	.15		

* $p < .05$; ** $p < .01$

5.1.3 The two tools compared

Apparently, the ICT version of the test items including an optional auxiliary tool provided the students in both tool conditions with more opportunities to show their mathematical capabilities in solving subtraction problems as opposed to the standardized test items. Table 5 shows that the percentage of correct answers (54%) in the ICT version including the manipulatives and the percentage of correct answers (55%) in the ICT version including the empty number line are quite similar (see also Tables 1 and 3). At the same time, in both studies we found almost equal means and standard deviations in the standardised CITO test. The stability of the effect was also revealed by the similarities of the *d*-values in both studies.

Table 5
Effect of the ICT-based test format in both studies

Study		Standardized test		ICT test		
	n	*M*	*SD*	*M*	*SD*	*d*
Manipulatives	37	0.34	0.27	0.54	0.30	0.73
Empty number line	43	0.36	0.25	0.55	0.26	0.72

Remark: Effect size Cohen's *d* is calculated as the difference in mean scores divided by the pooled standard deviation of the standardized test

To estimate the effect of the ICT test format in both studies on an item level, we transformed the proportion differences (displayed in Tables 2 and 4) to *d*-values (by dividing these by the pooled standard deviation of the standardised test of .26). These *d*-values shown in Table 6 indicate that the ICT test format did not influence all items in the same way.

Table 6
Differences in effect of ICT test format between two studies per item

Item	*d*-value differences		
	Manipulatives study	Empty number line study	Difference in *d*-values
1	0.82	0.97	0.15
2	0.21	1.55	0.94
3	0.72	1.50	0.78
4	0.82	0.88	0.06
5	0.72	0.18	–0.54
6	0.62	–0.99	–0.71
7	1.23	0.44	0.79
M	0.73	0.72	–0.01
SD	0.31	0.56	0.70

In some items – such as Item 1 – the type of tool (manipulatives or empty number line) included in the ICT format did not really matter; in other items such as Items 2 and 5 the type of tool did matter. In Item 2 the students benefitted more from the ICT test format with the empty number line; in Item 5 the students benefitted more from the ICT test format with the manipulatives

Thus, although we did not find overall differences between the two ICT formats, we can conclude that on item level there are differences: The ICT version with the manipulatives seems to work differently on item level than the ICT version with the empty number line. This kind of differential item functioning in the two ICT formats should be further explored in tests with a larger number of items and in larger student samples.

5.1.4 Students' awareness of their competence

In addition, it appears that the students from both studies were quite capable of judging their mathematical competence and therefore could decide whether the use of a tool could be beneficial. That is, the students who gave an incorrect answer on the standardised test had used the tool more frequently (49% in the manipulatives study and 60% in the empty number line study) than the students who gave a correct answer on the standardised test format (21% in the manipulatives study and 29% in the empty number line study). Note that the ICT version of the test items was administered in advance of the standardised written version of the test items.

5.2 Tool use and strategies

5.2.1 Influence of the manipulatives tool[3]

Table 7 shows the percentages of applied procedures in the cases that the students (n = 104) used the manipulatives tool, independent of the correctness of their answers. In two-thirds of these cases a direct subtraction procedure was applied. This procedure went together with a stringing strategy. In contrast with our expectations (Hypothesis 2a) the students did not perform a splitting strategy. An indirect addition or subtraction procedure was applied in only 10% of the cases. Even in the case of the problem 62 − 58, in which the numbers are relatively close to each other, this procedure was not applied, and in the case of 48 − 39 it was used only three times. In the cases that the students used an indirect addition or subtraction procedure, they all applied a stringing strategy. In the rare instances that a comparing procedure was applied (i.e. visually comparing the representation of the minuend and the subtrahend), it is not clear whether a stringing strategy was used. The cases in which it was unclear what procedure the students had applied often refer to cases in which the students came up with an incorrect answer. In fact, this occurred in 86% of the 'unclear' cases. This means that in the cases where the students found a correct answer it was often more clear how the students' work with the manipulatives related to their answer.

Table 7
Frequencies of applied procedures* for subtraction problems with the manipulatives (n = 104)

Strategy	%
Direct subtraction	66
Indirect addition or subtraction	10
Comparing the minuend and subtrahend	4
Unclear	20
Total	100

* Independent of the correctness of the answer

5.2.2 Influence of the number line tool

Table 8 shows the percentages of applied procedures in the cases that the students (n = 148) used the empty number line tool, independent of the correctness of their answers. The direct subtraction procedure was applied in more than a third of the cases. However, the indirect addition or subtraction procedure was also frequently applied, as it was used in more than a quarter of the cases. The indirect addition or subtraction procedure was frequently applied in cases where the numbers are relatively close to each other, that is 30 − 18 (in 7% of the cases) and 62 − 58 (in 5% of the cases), but most frequently in the cases where the numbers are not relatively close to each other, that is 37 − 9 (in 8% of the cases). As expected (see Hypothesis 2b) all procedures used by the students were applied through a stringing or varying strategy. As in the manipulatives study, the cases which were coded as unclear often refer to cases in which the students came up with an incorrect answer. In fact, this was the case in 76% of the 'unclear' cases.

Table 8
Frequencies of applied procedures* for subtraction problems with the empty number line (n = 148)

Strategy	%
Direct subtraction	39
Indirect addition or subtraction	28
Adding the minuend and subtrahend	5
Unclear	28
Total	100

* Independent of the correctness of the answer

6 Conclusions and discussion

6.1 Confirmation of hypotheses

From the experiences and results in this research project, we can conclude that an ICT based assessment including an optional dynamic auxiliary tool provides students in special education with more opportunities to show their mathematical capabilities in solving subtraction problems compared to a standardised assessment. In both sub-studies, the students attained a significantly higher percentage of correct answers in the ICT version of the subtraction items than in the standardised test items. This result was found even though the ICT-based test format preceded the standardised test format. Thus, any gain from retest effect would be in favour of the standardised test score, which implies that the 'true' effect size of the ICT version could be somewhat larger.

6.2 Unexpected results

Although both sub-studies generally confirmed our thoughts about the influence of the tools, there were some unexpected results as well. First, the match between the model and the strategy was not found in the students' responses. The students in the manipulatives tool condition did not apply the hypothesised splitting strategy. This could be a result of the fact that stringing is given more attention in the students' mathematics lessons than splitting. Another reason for not applying a splitting strategy could be that the students had to put the manipulatives on a 100-board with a 10 × 10 grid instead of on a neutral empty background on which the numbers can be presented as rods of ten and blocks of one. Finally, different than for an addition problem, for subtraction, decomposing and representing both numbers of a problem as tens and ones, which is characteristic for splitting, is not an obvious strategy.

A second unexpected finding was that in the number line tool condition, the procedure of indirect subtraction or addition was frequently applied, even though this procedure is not very common in primary special education. Apparently, the empty number line stimulated the students to bridge the difference between the numbers by making use of the inverse relation between addition and subtraction. So it could be argued that, from a mathematical point of view, working with the empty number line encouraged the students to apply more cognitive sophisticated strategies than working with the manipulatives.

A third unforeseen result was that the indirect procedure was most frequently used for solving a subtraction problem with a minuend and subtrahend which are not relatively close to each other. One explanation for this finding is that the context, in which the problem was presented, stimulated the students to use an indirect addition or subtraction procedure. As Van den Heuvel-Panhuizen and Treffers (2009) pointed out, contexts such as eating candy refer to direct subtraction, whereas contexts such as 'finding out how many pages still have to be read' may elicit an indirect addition or adding-on strategy. In fact, it was the latter type of context in which the particular subtraction problem in our study was presented. A second explanation for the frequent use of indirect procedures in problems with a large difference between the numbers is that in these cases the students more often used an empty number line which makes the strategy visible, while in the cases in which the numbers are relatively close to each other, the students are probably more inclined to mentally bridge the difference between the numbers.

6.3 Limitations of the study

The two sub-studies have shown that it is important to have adequate assessment instruments, which can open the students' zone of proximal development in order to obtain a good understanding of special education students' potential, albeit the studies also have some limitations. To begin with, we should keep in mind that the ICT test format and the standardised test format did not only differ in the availability of an auxiliary tool. The difference in test format (ICT-based or paper-based) as such could also have caused differences in performances. Several studies have indicated that it may not be naturally assumed that student work on a paper-and-pencil test is similar to that on a computer-based test (see e.g., Bennett et al., 2008; Clariana & Wallace, 2002; Johnson & Green, 2006). On the one hand, this difference in results between the two test formats can be a threat to the validity of the assessment. On the other hand, it can be argued that technologies included in a computer version can make a test more accessible for students (Van den Heuvel-Panhuizen & Peltenburg, 2011) and therefore result in a higher construct validity. For example, by implementing a read-aloud function, as is also shown in other studies (Elbaum, 2007; Helwig, Rozek-Tedesco, & Tindal, 2002; Trotter, 2008), children can concentrate on constructing meaning from text without the need to read it.

A possible limitation that is also related to the difference in test format concerns the lack of background information on the students' familiarity with working on the computer. Since the students' ages ranged between 8 and 12 years, it could be assumed that there were differences in familiarity with the computer, which may have affected the students' performance on the ICT-based test format. As Bennet et al. (2008) found '[…] some students may score better on a computer-based test compared to equally mathematically proficient peers simply because the former ones are more facile with computers' (p. 25).

A serious reason for treating our results with caution is that the length of the test differed between the two test formats. In the standardised test format, the students did the complete CITO Monitoring Test for Mathematics End Grade 2 (54 items administered on two successive days), whereas in the ICT test format they only did the seven items on subtraction with crossing the ten. Therefore, differences in performance on the two test formats could also be explained by factors such as tiredness, loss of concentration or lack of motivation on the standardised test format. Finally, both sub-studies have quite limited sample sizes (n = 37 and n = 43).

6.4 Further research

To attain a thorough understanding of special education students' potential in mathematics, we will extend our study to larger groups of students. Moreover, future investigations are required to refine our findings with other types of students. For example, Johnson and Green (2006) have shown that the use of on-screen mathematical tools may also help primary school students who are in regular education.

The sub-studies studies have yielded new insights in ICT-based dynamic assessment for special education students, but they also clearly point to further research in two directions. One direction involves the need to increase our mathe-didactical knowledge (see Van den Heuvel-Panhuizen & Treffers, 2009) on the domain of subtraction with crossing the ten. This includes further investigation of the relation between models and strategies. Therefore, the next step in our research project will be the revision of the manipulatives tool in such a way that it imposes fewer restrictions and offers better accessibility for students to show their mathematical capabilities. Moreover, our findings suggest the need for research on the influence of contexts and numbers on students' use of the indirect addition procedure.

The second direction includes the need to increase our knowledge on assessing students' work. To gain a more thorough insight into how the auxiliary tools affect the students' procedures and strategies, we will ask students to think aloud (see, e.g., Ericsson & Simon, 1993; Van Someren, Barnard, & Sandberg, 1994) during or after solving the problems, in a test format with and without an auxiliary tool. In addition, conducting post-hoc interviews could provide an adequate method to investigate students' reactions to and perceptions of ICT-based dynamic assessment.

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Notes

1. Torbeyns et al. (2009) mention subtraction by compensation only as another term for the varying strategy. Moreover, they see the splitting, stringing and varying strategies all as belonging to the direct subtraction class of procedures.
2. The ICT-based assessment instrument was programmed by Barrie Kersbergen, a software engineer at the Freudenthal Institute.

3. For a more detailed description of tool use and strategies in the manipulatives study, see Peltenburg, Van den Heuvel-Panhuizen, and Doig (2009).

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Chapter 4

A secondary analysis from a cognitive load perspective to understand why an ICT-based assessment environment helps special education students to solve mathematical problems

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A secondary analysis from a cognitive load perspective to understand why an ICT-based assessment environment helps special education students to solve mathematical problems

1 Two earlier studies as a start

1.1 Revealing special education students' mathematical abilities

Children in special education (SE) schools have a severe delay in their mathematical development compared to their peers in regular primary schools. In the Netherlands, at the end of special primary school, SE students' scores are between one to four years behind those of their peers in regular primary schools (Kraemer, Van der Schoot, & Engelen, 2000; Kraemer, Van der Schoot, & Van Rijn, 2009). A difficulty of these scores is that they are obtained from standardized paper-and-pencil tests, which do not allow children to use auxiliary resources. As a consequence, these tests may not be really appropriate to reveal possibly hidden abilities of SE students. Actually, this static way of testing does not give access to what Vygotsky (1978) called the students' zone of proximal development. This is a serious shortcoming when using test results for informed educational decision making. Therefore, we think it is important that assessment instruments are available that give insight in weak students' zone of proximal development in mathematics. In this matter we are following the footsteps of Feuerstein (1979) and Campione (1989) who developed the idea of dynamic assessment by means of which it can be investigated whether students are able to solve problems with some help and can be observed how this help is used. This makes dynamic assessment pre-eminently suitable for revealing the latent mathematical talents of SE students.

To exploit the dynamic approach to assessment, we see an important role for Information and Communication Technology (ICT). Firstly, ICT makes it possible to get access to the students' solution processes by registering detailed information on students' strategies (Woodward & Rieth, 1997). In this way, ICT offers teachers and researchers 'windows to the mind' of students (Clements, 1998). For example, Barmby, Harries, Higgins, and Suggate (2009) made screen videos of students' working in an ICT environment in which they used an array representation for solving multiplication problems.

Secondly, ICT can make problems more accessible for students. Bottge, Rueda, Kwon, Grant, and LaRoque (2009) showed that ICT can eliminate some of the cognitive demands for low achieving students, which enables them to more fully demonstrate their understanding of the mathematical concepts they have learned. With respect to this reduction of cognitive demand, positive results were also found in several studies on computer-based assessment in which students with disabilities were read aloud mathematics problems on the computer by means of a digital aid (Elbaum, 2007; Helwig, Rozek-Tedesco, & Tindal, 2002; Trotter, 2008).

A third advantage of ICT is that it can include optional auxiliary tools which may assist students in cognitive structuring, that is providing them with a structure for thinking and acting when solving problems (Bottino & Chiappini, 1998; Clements, 2002; Clements & McMillen, 1996). For example, digital base-ten blocks can provide flexible and manageable manipulatives which 'snap' into rods of five blocks, that might put students on the track of finding a solution by structuring instead of counting blocks one-by-one.

Based on the above, we see ICT-based dynamic assessment which includes auxiliary resources as a promising avenue to proceed for disclosing SE students' so far hidden mathematical abilities. To test this assumption, we carried out two studies in which SE students' performance in mathematics was assessed in two ways.

1.2 Methods of the two studies

In the first study (Peltenburg, Van den Heuvel-Panhuizen, & Doig, 2009), we assessed 37 children. The second study (Peltenburg & Van den Heuvel-Panhuizen, 2009) included 43 children (not the same as in the first study). The children were all attending a school for SE and were 8 to 12 years old. Their IQ scores were between 60 and 90.

They all had cognitive deficits, often in combination with having concentration and motivational problems. Their development delay in mathematics was between one to four years.

In each study, two different assessment formats were used. First, the children did an ICT-based assessment. This assessment was completely new for them. After the ICT-based assessment the children took the CITO Monitoring Test for Mathematics End Grade 2 (CITO-ME2) (Janssen, Scheltens, & Kraemer, 2005). All children in the two studies were familiar with the CITO tests. These tests are standardized paper-and-pencil tests that are widely used to measure students' mathematical performance in both regular and special primary education in the Netherlands. The test for end grade 2 consists of two parts which should be administered in two successive days. Part I contains problems that are read out by the teacher. In part II, the instruction text is included in the test sheet. The students have to read this instruction by themselves.

The focus in the studies was on subtraction problems up to 100 that require crossing the tens, i.e. problems in which the value of the ones-digit of the subtrahend is larger than the ones-digit of the minuend (e.g., 62 – 58). This type of problems was selected because it is generally recognized as especially difficult for weak students (e.g., Kraemer, Van der Schoot, & Engelen, 2000; Kraemer, Van der Schoot, & Van Rijn, 2009). A frequent mistake when solving this kind of subtraction problems is reverse processing of the digits (in the case of 62 – 58, subtracting 2 from 8 instead of 8 from 2).

The set of problems used in both studies and included in the ICT-based assessment was the complete collection of seven subtraction problems up to 100 with crossing the tens that is in the CITO-ME2 test. Four of the problems are in part I of the test and three problems are in part II, which include the instruction text. All except one, the problems are word problems. Figure 1 shows such a problem which belongs to part II of the test.

There are 48 comics of Spiroe. Jan already has 39 of them.
How many comics does he not have yet?

________ comics

Fig. 1 Spiroe problem in standardized paper-and-pencil test (CITO-ME2)

The ICT-based assessment was developed to offer students optimal opportunities to show their mathematical abilities. This means that the assessment has a number of auxiliary features which may give children better access to problems than regular standardized test do and may even help them to solve the problems. The features include: a read aloud function, a flexible answer field, and a mathematical auxiliary tool.

The read aloud function enables the children to not only read the problems, but to hear them as spoken text as well. The flexible answer field makes it possible that children can fill in the digits of an answer in the order they like. Moreover, with this answer field they can easily correct their answers. However, of all included features the optional mathematical auxiliary tool is the most important one. The tool can give students support in modeling the problem situations, carrying out operations at a concrete level and keeping track of solution processes. Because the students are free to use the tool or not, the ICT-based assessment has a dynamic, adaptable nature.

For the two studies we developed two different mathematical auxiliary tools, based on the two main models that are considered to be helpful when children solve calculations up to 100: the group model (i.e., manipulatives) and the line model (i.e., empty number line) (Van den Heuvel-Panhuizen, 2001). Both models are commonly used in Dutch mathematics textbook series and the children involved in the studies were familiar with these models.

The first study (Peltenburg, Van den Heuvel-Panhuizen, & Doig 2009), here after called the 'manipulatives study', included an optional digital manipulatives tool in the ICT-based assessment. This tool consists of a digital 100-board and a storage tray with counters. Any number of counters from 1 to 10 can be dragged in one movement to the board. The board features a 10 by 10 grid, with a 5-5 structure, which means that the board is divided in four parts. The students can use the tool to make a visual representation of the numbers involved by putting them on the board and can carry out the required operations by moving or removing the counters. They can work with the tool like they do with wooden manipulatives on their desk. Whether the students use the auxiliary tool or not is their own decision. If they want to use it, they can activate the tool by clicking the tool button (see Figure 2). When the button is clicked, the tool pops up (see Figure 3).

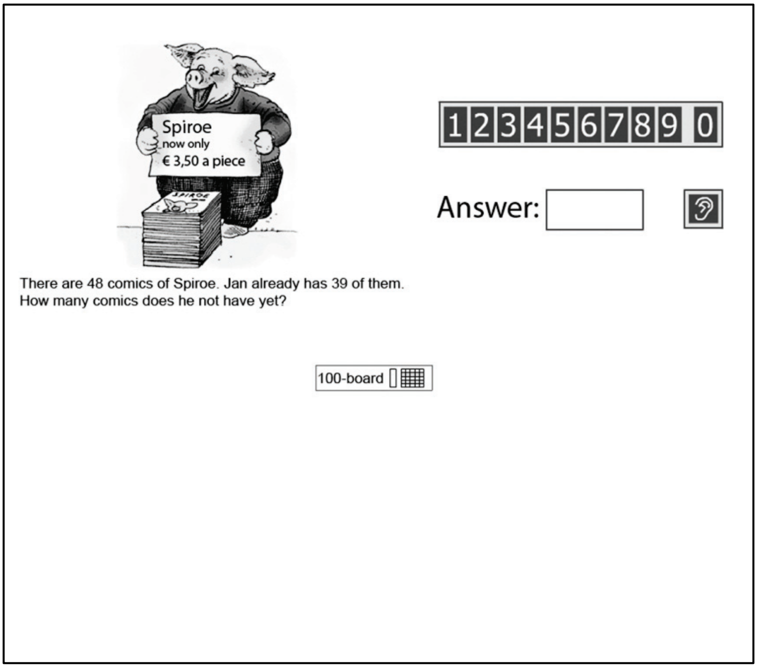

Fig. 2 Spiroe problem in ICT-based assessment with tool button for manipulatives tool

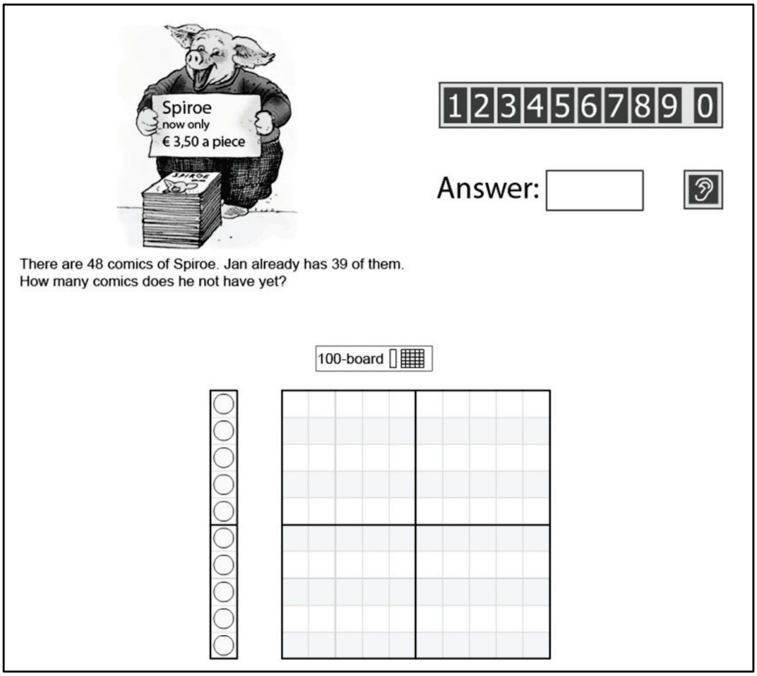

Fig. 3 Spiroe problem in ICT-based assessment with digital manipulatives tool

In the second study (Peltenburg & Van den Heuvel-Panhuizen, 2009), here after called the 'number line study', we included a digital empty number line as a mathematical auxiliary tool in the ICT-based assessment (Figure 4). This tool features a horizontal line, a pencil, an eraser, and a clear-all button. The students can use the pencil to carry out operations by putting markers and numbers on the number line and making jumps backwards and forwards. This means that, like in the manipulatives study, a flexible and easily manageable tool can be used. The students can work with the tool in the same way as with a pencil on a piece of scrap paper.

Fig. 4 Digital empty number line

Although the children in both studies had experience with working with manipulatives and number lines when solving calculations up to 100, the ICT-based environment with the digital version of these tools was new for them.

In both studies, data were collected on whether the students could solve the problems and how they solved them. The data collected with the paper-and-pencil test consisted only of the students' answers on paper. In the case of the ICT-based assessment capturing software was used to record screen videos of the students' working. In order not to place extra load on the students' working memory we did not ask them to explain their strategies or to think aloud when solving the problems.

1.3 Results from the two studies

A comparison of the data from the two test formats showed that the ICT-based assessment is a more appropriate instrument than the standardized test to reveal what the children are really capable of. In both studies, the children solved more problems correctly in the ICT-based assessment than in the standardized test (see Peltenburg, Van den Heuvel-Panhuizen, & Doig 2009; Peltenburg, Van den Heuvel-Panhuizen, & Robitzsch, 2010). To test for significant differences between the percentages of correct answers in the two test formats, we used a t-test for paired samples. We did this for the manipulatives study and the empty number line study separately.

In the manipulatives study, our observations included a total of 259 cases (37 students did seven problems each). A case covers two scores: one for the ICT test format and one for the standardized test format. Table 1 shows the percentages of correct and incorrect answers for all the cases in both formats. The percentage of correct answers was significantly higher ($t\ (36) = 3.67$, $p < .01$, $d = .71$) for the ICT version of the problems (54%) than for the problems in the standardized test format (34%).

Table 1
Cross tabulation of correct and incorrect answers in both test formats in manipulatives study

		ICT version of test items		Total
		Correct answer	Incorrect answer	
Test items standardized test	Correct answer	24% (61)	11% (28)	34% (89)
	Incorrect answer	30% (78)	36% (92)	66% (170)
Total		54% (139)	46% (120)	100% (259)

In the empty number line study, our observations included a total of 301 cases (43 students did seven problems each). As in the manipulatives study, a case covers two scores: one for the ICT test format and one for the standardized test format. Table 2 displays the results for the 301 cases. It appears that the percentage of correct answers was significantly higher ($t\,(42) = 4.77$, $p < .01$, $d = .75$) for the ICT version of the problems (55%) than for the problems in the standardized test format (36%).

Table 2
Cross tabulation of correct and incorrect answers in both test formats in empty number line study

		ICT version of test items				Total	
		Correct answer		Incorrect answer			
Standardized test	Correct answer	28%	(83)	9%	(26)	36%	(109)
	Incorrect answer	28%	(83)	36%	(109)	64%	(192)
Total		55%	(166)	45%	(135)	100%	(301)

2 Considering the ICT-based assessment environment and its results from a CLT perspective

The findings from these two earlier studies raised the question whether the features in the ICT-based assessment made that this assessment is more suited than the standardized paper-and-pencil test to reveal what students are capable of in mathematics. To find an answer to this question is the main purpose of the present study.

Because there is much evidence that SE students have working memory deficits (see, e.g., Gathercole & Pickering, 2001) the auxiliary features within the ICT-based environment could have influenced the students' ability to process information. This consideration led us to Cognitive Load Theory (CLT) and to take this perspective as a basis for identifying and understanding the features of the ICT-based environment that were effective in bringing the students to a higher level of performance.

Research inspired by CLT aims to develop instructional design guidelines that lead to instructional materials that foster students' learning by offering possibilities for reducing working memory load (Sweller, Van Merriënboer, & Paas, 1998). Although some researchers (Beddow, Kettler, & Elliott, 2008) have emphasized that CLT-based design principles can also be used for improving assessment, research in this area is scarce. Moreover, besides the fact that CLT is not often used explicitly to design and improve tests, studies on CLT have rarely been concerned with SE students. This is rather remarkable, because design guidelines based on CLT may be crucial especially for students with deficits in their working memory. In other words, it seems there would be much to gain for these students if instructional materials and tests were designed based on CLT principles. The result of such an approach to assessment may give SE students opportunities to show their competence.

Therefore, in the present study we adopted a CLT perspective and subjected data from the two earlier studies that we carried out to a secondary analysis. As Richey stated (1998), a secondary analysis can be a suitable methodology to enhance the usability of instructional technology research. The secondary analysis addressed in this study focuses on characteristics of the ICT-based assessment environment and the achieved results in the two test formats. The goal of this secondary analysis is to find out whether the results are in agreement with what CLT principles predict.

Research on CLT has shown that the quality of an instructional (and assessment) task can be improved by reducing extraneous (i.e., ineffective) load on working memory and increasing germane (effective) load (Sweller et al., 1998), for example by avoiding redundancy in the task presentation (Sweller, 2005) and allowing students to represent knowledge externally to off-load their working memory (Beers, Boshuizen, Kirschner, Gijselaers, & Westendorp, 2008). To test whether these principles were operational in the ICT-based assessment, we subjected the following features to a secondary analysis: (1) the read aloud function and the optional read aloud function; (2) the flexible answer field; (3) the optional mathematical tools (i.e., the digital manipulatives and the digital empty number line).

3 Results from secondary analysis from a CLT perspective

3.1 The read aloud function and the optional read aloud function

Many students in SE have reading difficulties which may influence their ability to understand mathematical problems in a paper-and-pencil test. This is especially true for word problems. To overcome these difficulties, it has been argued (Smolkin & Donovan, 2003) that struggling readers could benefit from hearing spoken text as a scaffold to eliminate the need to focus on decoding, allowing them to concentrate on constructing meaning from text. Moreover, giving students control over this function offers them the possibility to adapt the use of the read aloud options to their own needs (Reinking & Schreiner, 1985). Therefore, the ICT-based assessment environment was equipped with a read aloud function. This means that the visual presentation of every new test problem was accompanied by an oral presentation. After that, the students could listen to the text again by clicking a button depicting an ear. This optional read aloud function may make the text of the problems even more accessible for students (Beddow, Kettler, & Elliott, 2008; Elbaum, 2007; Helwig, Rozek-Tedesco, & Tindal, 2002; Trotter, 2008).

From the CLT perspective, it can be argued that presenting written text on the screen together with reading the text aloud is not recommended because the redundancy effect can lower the performance (Diao & Sweller, 2007; Mayer & Moreno, 2003). Processing the on-screen text and connecting it to the spoken text requires cognitive capacity that decreases the cognitive power that is left for solving the problems. What supports the presence of this mechanism is that we observed that the students paid attention to both sources of information.

Nevertheless, it is unclear whether, in the case of the read aloud functions, the redundancy effect will be shown in the results of the students. Because of their low reading ability, the read aloud function can help the students to ‘outsource’ their reading by listening to the spoken text. In other words, what might be a redundancy effect for capable students without reading difficulties could actually lower extraneous cognitive load for SE students. Consequently, the read aloud function would contribute to the students’ performance.

To find out whether and how the read aloud functions may have influenced student scores, we compared the percentage of correct problems that the students obtained in the ICT-based assessment with the percentage correct problems that they obtained in the standardized test. This comparison was based on the three problems from part II of the standardized test for which the written text was included in the test sheet. In the ICT-based assessment, these problems also have the text included and in addition they have the read aloud functions. To make the comparison as pure as possible, we excluded temporarily all problems for which the students used the mathematical tool in the ICT-based assessment (i.e., the digital manipulatives or the digital empty number line), as the use of these tools could have influenced the students’ scores as well.

Table 3 shows that 67 students completed one, two or all three problems in which they heard the spoken text once in addition to the written text (thus, these students did not use the optional read aloud function). The 67 students completed 132 problems in total. Applying a two-way repeated measures ANOVA using the difficulty of the three items as a fixed factor and the students’ ability as a random factor showed that the percentage of correct problems (61%) on the ICT-based assessment did not differ significantly from that (58%) on the standardized version of the test problems ($F\,(1,83.571) = .06$, $p = .798$). Table 3 also shows that 25 students used the optional read aloud function at least once (total of 29 cases). In this group, also no significant difference was found between the percentage of correct problems (55%) on the ICT-based assessment and the percentage of correct problems (59%) on the standardized version of the test problems ($F\,(1,24) = .009$; $p = .93$).

Table 3
Percentages of correct answers in standardized and ICT-based test format

Test format	Read aloud function	Optional read aloud function	Cases* (Students)	% Correct answers
ICT	Yes	No	132 (67)	61
Standardized	No	No	132 (67)**	58
ICT	Yes	Yes	29 (25)	55
Standardized	No	No	29 (25)**	59

* Total number of problems made by the students
** Same cases (and students) as selected from the ICT-based assessment

Although there is quite a number of students who used the optional read aloud function – which may suggest that there is a need to hear the text – we did not find evidence that it influenced the score of the students. Similarly, we did not find support for this when we compared the ICT-based assessment in which the problems were only read aloud once with the standardized test in which they were not read aloud. Therefore, we could not determine an impeding role of the read aloud function (redundancy effect) nor a compensating role. A tentative explanation for this might be that a possible positive effect of hearing spoken text compensating for SE students' low reading ability is cancelled out by a possible negative redundancy effect.

3.2 The flexible answer field

When students have to put a multi-digit number in an answer field on a computer or calculator, they generally have to do this by filling in the digits from left to right. This means that they have to start with the digit that stands for the largest value. In English, this way of writing down numbers is in agreement with the pronunciation of the numbers. However, in some other languages, including the Dutch language, the ones digit in a two-digit number is pronounced before the tens digit (e.g., in Dutch, 68 is verbalized as 'eight-and-sixty'). Consequently, some students may experience a discrepancy between the order of pronouncing a number and the order of writing or typing the digits of that number. Zuber, Pixner, Möller, and Nuerk (2009) found that this language-specific property of the number word system strongly influences primary school students' performance in writing two-digit numbers. For example, Dutch students may easily end up writing 86 instead of 68 if they write down first what they hear first. To write down 68, and especially to type 68 from left to right directly in the correct order would mean that the students have no choice but to change the pronunciation (and thinking) order into a writing order, which is rather hard for weak learners. To avoid this difficulty, a flexible answer field was included in the ICT-based assessment. This answer field contains a drag-and-drop answer box that allows students to fill in first the ones digit and then fill in the tens digit to the left of the ones digit (see Figure 5).

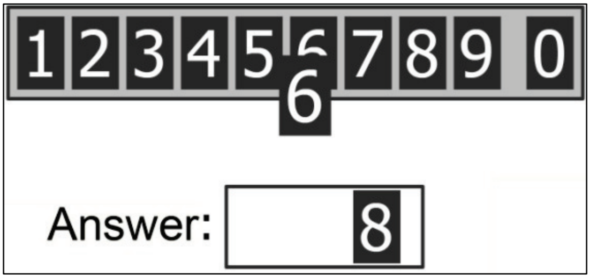

Fig. 5 Student is writing down 68 from right to left, filling in ones digit first

Moreover, the flexible answer field gives students who incorrectly filled in 86 instead of 68, the possibility to correct this by taking the 6 and 'pushing' the 8 aside (see Figure 6).

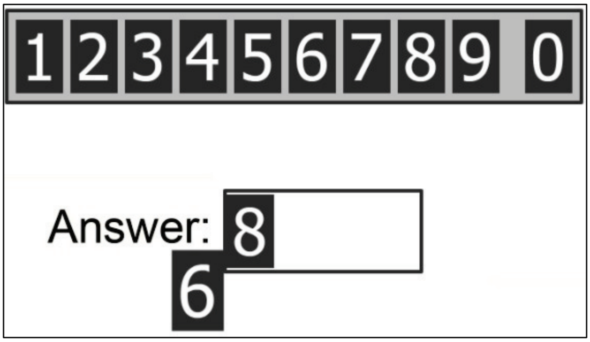

Fig. 6 Student is correcting the 86 into 68 by 'pushing' the 8 to the right

Finally, this flexible answer field offers the students the opportunity to 'park' the part of the answer they already have figured out.

Considering this flexible answer field from the perspective of CLT, it is clear that it offers ample opportunities to reduce the load on working memory for children who speak a language in which the pronunciation of numbers differs from the writing order. With the flexible answer field it is not necessary that students keep the answer in their working memory while at the same time mentally changing the pronounced order to adapt it to the writing order. Similarly, the easy way of pushing away numbers may reduce the children's working memory load because they can start filling in what they already know of the answer.

To investigate how the students used this flexible answer field while solving subtraction problems up to 100, we analyzed screen video data from their work in the ICT-based assessment environment. For this analysis, we only used the five test problems that had a two-digit answer. This resulted in 400 cases; 80 students did five problems each. Table 4 provides an overview of the relative frequencies of how the students filled in their answer.

Table 4
Relative frequency of type of use of flexible answer field

Type of use	Frequency (%) of use based on 400 cases (n = 80)
Student filled in the tens digit first	78
Student filled in the ones digit first	17
Student did not fill in an answer	3
Student only filled in one digit	2
Total	100

As Table 4 shows, in most cases the students preferred to fill in the tens digit first, followed by the ones digit. However, in 17% of the cases the students made use of the possibility to fill in the ones digit first. This type of use was most frequently applied (39%) in the problem which has 68 as an answer and less frequently (8% and 13% respectively) in the two cases where the answer is 12. This result may imply that the need for a flexible answer field increases when the numbers that have to be filled in become larger. A small two-digit number like 12 may function more as a visual Gestalt than a larger two-digit number like 68. Another related explanation could be that 12 is pronounced as one word 'twelve' and not as 'ten-two' (or as 'two-ten'). For both reasons it is plausible that writing down 12 may require less cognitive capacity than 68.

Compared to the option of filling in the ones digit first, the option of correcting the order of the digits after filling in both digits appeared to be less popular: in only 2% of the total cases did the students make use of this possibility. The option that we called 'parking', in which the students can write down that part of the answer they already know, was applied more frequently. By timing the students' work while filling-in their answer, we found that in 11% of the total cases the students paused for three or more seconds between filling-in the two digits.

3.3 The optional mathematical tools

From a didactical perspective, the two tools each have particular 'affordances' (Gibson, 1977) that prompt particular actions. The 100-board with the manipulatives will encourage the representation of numbers by decomposing them into tens and ones (in the case of 62 – 58, the students might put six tens and two ones on the board and then take away five tens followed by removing two ones and another six ones from the remaining ten). The digital number line will support keeping the starting number as a whole, positioning it somewhere on the number line and then applying a stringing strategy (making jumps on the number line) to move backward (62 ..., 12, 10, 4) or forward (58, 60, 62, ... 4) in order to find the answer. Although both tools can help students to solve the given subtraction problems, they differ in the structure they provide, the availability of processing traces, the representation of numbers, and the range of calculation procedures supported by the tool (see Table 5).

Table 5

Differences in didactical characteristics of the optional mathematical tools

100-board and manipulatives	Empty number line
Structure is provided	No structure provided
No processing traces are available	Processing traces are available
Making number values concrete	Positioning numbers
Favors direct subtraction	Favors direct subtraction and indirect procedures

From a CLT perspective, both tools can be considered as an aid for reducing extraneous load by offering the opportunity of knowledge externalization (Gathercole, Lamont, & Alloway, 2006). This means that both the 100-board and the number line tool allow students to make an external representation that they can work with instead of solving the problems mentally, which may offload working memory (Beers et al., 2008). Externally representing information with the tools involves creating an on-screen visual representation of the subtraction operation that has to be carried out. Compared to the 100-board, the number line tool has two advantages regarding knowledge externalization: (1) the number line leaves visual traces of the actions that have been carried out so far, which may lead to a further reduction of extraneous load, whereas no traces are left behind on the 100-board, i.e., removed counters are gone, and (2) the 100-board, although more concrete, requires more actions (counting, dragging and dropping) to represent a number, which may impose extraneous load, whereas representing a number on the empty number line only includes writing the number.

In addition to considering the tools as aids for reducing extraneous load by externally representing information, they can also be considered as aids for increasing germane cognitive load (Sweller et al., 1998). In fact, working with the tools may help students to gain a better understanding of carrying out subtraction problems. In the case of the 100-board, the students are more or less guided in following a taking-away procedure, i.e., taking away the subtrahend from the minuend. Compared to the 100-board the number line has less performance constraints (Beers et al., 2008), which means students are not so much guided towards one procedure. Therefore, the number line allows a larger variety of subtraction procedures, including both direct and indirect procedures. As a consequence, working with the number line forces students to come up with a subtraction procedure themselves, which may serve as an additional trigger for advanced ways of working, and as such increase germane cognitive load.

To investigate whether these CLT mechanisms are recognizable in the students' responses, we looked at the problems with the largest gain in correct answers in the ICT-based assessment compared to the standardized test. We chose these problems, because we expected that the described CLT mechanisms manifest themselves best in test problems where the students benefited the most from using the mathematical tools in the ICT-based assessment.

In the 100-board study, the students benefitted most from using the tool in two problems where they had to solve the subtractions 50 – 38 and 50 – 37. For both problems, 30% of the students (11 out of 37 students) had an incorrect answer in the standardized test and answered the test problem correctly in the ICT-based assessment. Based on this finding, it can be argued that representing a number on the 100-board imposes less extraneous load when the number only consists of tens.

The situation in the number line study was different. Here, the test problem with the largest gain was the subtraction 37 – 9. For this problem, 35% of the students (15 out of 43) had an incorrect answer in the standardized test and a correct answer in the ICT-based assessment. The context of this test problem prompts the use of an indirect procedure (8 out of 15 students applied this procedure). This means that the problem is solved not by taking away the 9 from the 37, but by bridging the difference by jumping from 9 to 37. This way of calculating the difference can be easily carried out on the number line, as shown by the work of Martin and Rosa (fictive names). They both applied an indirect procedure, which matches with the context in which the particular problem was presented; see also Torbeyns, Verschaffel, and Ghesquière (2006) for students' adaptive strategy choice. In general, indirect procedures appeared to be more frequently applied with the empty number line than with the manipulatives (see Peltenburg, Van den Heuvel-Panhuizen, & Robitzsch, 2010). Because students in the empty number line study made more use of the inverse relation between addition and subtraction in which they followed an indirect procedure, it could be argued that the empty number line encouraged the students to apply cognitively more sophisticated strategies than the manipulatives. The two examples of student work also demonstrate that the number line does hardly have any performance constraints, which makes it at the same time an accessible tool and one that stimulates sophisticated strategies.

Martin (see Figure 7) used an indirect addition procedure and solved the problem by starting with nine. Then he made three jumps of ten and after that he jumped back two ones to find the answer.

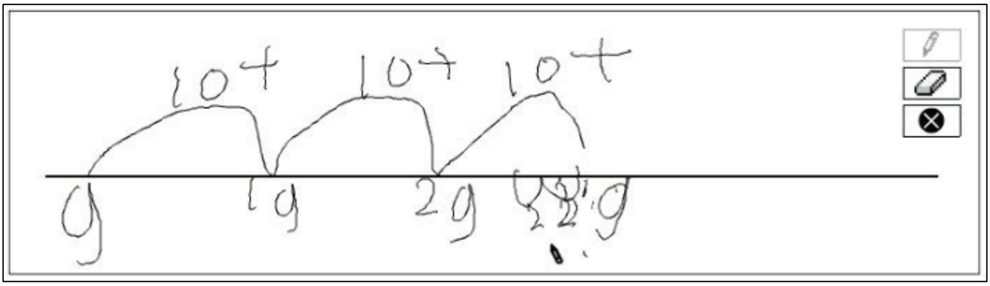

Fig. 7 Caption from screen video showing Martin's work when solving 37-9

Rosa (see Figure 8) used the number line in her own way. She applied an indirect subtraction procedure by jumping from 37 until she reached 9. However, instead of working from right to left she started with 37 on the left and counted backwards while jumping to the right.

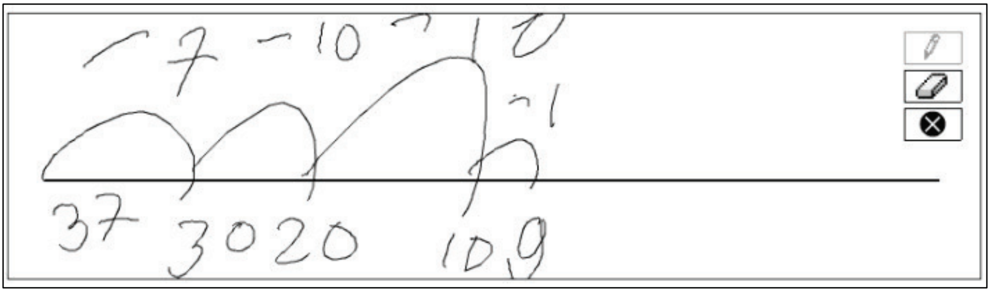

Fig. 8 Caption from screen video showing Rosa's work when solving 37 – 9.

These examples show how few performance constraints the number line has. This characteristic of the number line explains why embedding this tool in the ICT-based assessment may help SE students to solve mathematical problems.

4 Concluding remarks

In two earlier studies (Peltenburg & Van den Heuvel-Panhuizen, 2009; Peltenburg et al., 2009), it was found that an ICT-based assessment – compared to a standardized test – gave SE students a better opportunity to show their mathematical ability. By adopting a CLT perspective, we tried to find an a posteriori explanation for these findings. In our analysis we focused on the following significant features of the ICT-based assessment: the read aloud function, the flexible answer field, and optional mathematical auxiliary tools.

Concerning the read aloud function no evidence could be found for a redundancy effect, but the results made us aware of the fact that this effect may behave differently in SE students who lack particular abilities than in regular students. A possible positive effect of hearing spoken text that compensates for students' low reading ability could be cancelled out by the negative (redundancy) effect of getting an overload of information.

The results for the flexible answer field reveal that quite a number of students make use of its flexibility, which may imply that a flexible answer field could help a particular group of students to overcome obstacles and thus can reduce the load on working memory when writing down their answers.

Finally, the results for the mathematical tools show that such tools are not restricted to provide memory aids, but may also offer strategy support. This means that, due to a reduction of extraneous load, students can devote more cognitive resources to identifying or sequencing the solution steps necessary for achieving the goal, which may contribute to performance (i.e., germane load).

However, all above results should be considered with prudence. Our analysis was based on a small sample of students and we only had a limited amount of student variables available. More in particular, in our analysis we missed a direct measurement of the cognitive load for students, which is essential when building knowledge about the role of cognitive load in multimedia learning (Brünken, Plass, & Leutner, 2003). Despite these shortcomings, this secondary analysis from the CLT perspective has been a fruitful enterprise for us, since it gave us more insight in possible positive and negative effects of particular features of the ICT-based assessment environment.

Based on the findings regarding the mathematical tools one may ask whether it should be recommended to provide students with blocks or a number line as an auxiliary tool when they do a paper-and-pencil test. Of course, such a recommendation does not directly follow from our studies. To know more about this, data should be collected with a paper-and-pencil test including these auxiliary resources. However, we doubt whether we consider this as a next step in our study. Note that it was not without a reason that we chose to assess students' abilities in an ICT-based assessment environment. As mentioned before, ICT has some unique advantages, such as the possibility to register detailed information on students' steps in their solution process, which allows their strategies to be assessed in more precise ways than in a paper-and-pencil test.

The CLT perspective could help to design an assessment environment that makes this possible. However, this is not as obvious as one may think. Although multimedia learning is a well-established research domain in CLT, this is not the case for assessment design or, in general, for instructional design to be used in SE. Especially the latter is remarkable. Given the fact that SE students have learning difficulties and often deal with working memory deficits, we think CLT-inspired research can be of invaluable help for improving

SE students' learning and assessment environments. Moreover, testing CLT-based instructional design principles more systematically with SE students might be an interesting avenue to expand the CLT theory.

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Chapter 5

Special education students' use of indirect addition in solving subtraction problems up to 100 - A proof of the didactical potential of an ignored procedure

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Special education students' use of indirect addition in solving subtraction problems up to 100 - A proof of the didactical potential of an ignored procedure

1 Background

At the end of primary school, many special education (SE) students are considerably behind on the topic of subtraction with numbers up to 100 compared with their peers in regular education (Kraemer, Van der Schoot & Van Rijn, 2009). To support low-performing students and to give them confidence in carrying out subtraction problems, it is suggested, for example by the U.S. National Mathematics Advisory Panel (2008), that these students would benefit from being taught one prescribed way of solving calculations. This opinion is also expressed in the Netherlands.[1]

However, the idea of teaching only one method goes against the goal of developing numeracy in students. This goal implies that students should be able to choose a suitable method when solving number problems (Treffers, 1989; Van den Heuvel-Panhuizen, 2001; Warry, Galbraith, Carss, Grice, & Endean, 1992). Moreover, being numerate is also seen as a target for mathematically weaker students (e.g., Kilpatrick, Swafford, & Findell, 2001; NCTM, 2000; Verschaffel, Torbeyns, De Smedt, Luwel, & Van Dooren, 2007).

A further objection against the one-method approach is that if students would have to restrict themselves to only one way of solving problems, many problems would require an unnecessarily long solution path (see, e.g., Torbeyns, De Smedt, Stassens, Ghesquière, & Verschaffel, 2009). For example, to solve 62 − 58, first take away 50, resulting in 12; then take away 2, making 10; and finally, take away 6, for an answer of 4. This method, which requires a large number of taking-away steps, is more error sensitive than when students focus on the difference between the numbers and add on from the subtrahend until the minuend is reached. For 62 − 58, start with 58 and add 2, resulting in 60, and then add 2 more to reach 62, giving the difference of 4 as the answer. In this latter approach, students change the operation that is presented in the problem: subtraction is transformed into addition, which can bring computational advantages for them (see, e.g., Torbeyns, De Smedt et al., 2009).

Another difficulty with using prescribed methods is that they can lead to a "didactical ballast" for the students (Van den Heuvel-Panhuizen, 1986). Following a prescribed solution method can be a source of error for students because this method is not grounded in their own thinking, i.e., the ownership is completely on the side of the teacher or textbook author.

In sum, we can say that there are many disadvantages to teaching one fixed solution method for solving calculations. Moreover, studies (e.g., Torbeyns, De Smedt et al., 2009) have shown that flexible adaptation of the solution method can make problems easier for students. Therefore, one might decide to teach even students who are weak in mathematics the flexible use of solution methods. However, the critical point for making this decision is whether these students are able to operate in such a flexible way. Some studies (see, e.g., Milo, 2003; Timmermans, 2005) have indicated that SE students with learning difficulties in mathematics have trouble in choosing a solution method in a flexible way. In the study reported in this paper, we further investigate whether SE students are able to adapt their solution methods to the nature of the problems presented to them. The focus of the study is on subtraction up to 100. The students' solution methods and performances are assessed using an information and communication technology (ICT)-based test.

1.1 Strategies and procedures for solving addition and subtraction problems up to 100

Generally, three different types of strategies can be distinguished for solving addition and subtraction problems with numbers up to 100: splitting, stringing, and varying (Van den Heuvel-Panhuizen, 2001). Although researchers do not always use the same wording – for example, other expressions can be found in Klein, Beishuizen, & Treffers (1998) and Torbeyns, De Smedt et al. (2009) – there is broad agreement about the general meaning of these strategies. To use a splitting strategy, the subtraction problem is solved by decimally splitting both the minuend and the subtrahend and processing the tens and the ones separately (e.g., $54 - 31$ is calculated as $50 - 30$ and $4 - 1$, with 23 as the final answer). With a stringing strategy, the starting number, which could be either the minuend or subtrahend, is kept whole and the second number is added or subtracted in parts (e.g., $63 - 47$ is calculated as $63 - 40 = 23$, then $23 - 3$, and, finally, $20 - 4 = 16$). Applying a varying strategy requires a flexible processing of numbers that is based on known number relationships and properties of operations. For example, a number that is not the intended subtrahend but that is easier to handle may be taken away from the minuend, and afterwards, this "wrong number" is compensated (e.g., $77 - 29$ is calculated by $77 - 30 = 47$, and then $47 + 1$). Another example of a varying strategy is to change the subtraction into an easier problem by keeping the difference between the minuend and the subtrahend the same (e.g., $77 - 29$ is calculated by $78 - 30$).

Torbeyns, De Smedt et al. (2009) describe subtraction in a different way. They distinguish (1) direct subtraction (DS), which means taking away the subtrahend from the minuend; (2) indirect addition (IA), which means adding on from the subtrahend until the minuend is reached; and (3) indirect subtraction (IS), which means subtracting from the minuend until the subtrahend is reached.

According to Torbeyns, De Smedt et al. (2009), splitting, stringing, and varying belong to the class of DS procedures, whereas IA is considered as a separate class of procedures which do not fit the three strategies. However, we see this differently. Splitting, stringing, and varying can be considered strategies which all describe how we deal with the numbers involved (in splitting, both numbers are decimally decomposed in tens and ones; in stringing, one number is kept as a whole number; and in varying, one or both numbers are changed in order to get an easier problem). In contrast to these strategies, we can call DS, IA, and IS procedures which describe calculations from the perspective of how the operation is carried out. In fact, the strategies and the procedures complement each other. Together, they offer a complete framework for describing how students solve additions and subtractions up to 100.

Table 1 illustrates the strategies and procedures by prototypical examples of subtraction problems in which the numbers are likely to elicit particular strategies and procedures. The framework reflects how these are related. A DS procedure often goes together with splitting or stringing. For IA and IS, stringing is the most obvious strategy; although splitting can be applied as well. Finally, when a varying strategy is applied, multiple operations are required.

Table 1
Relation between procedures and strategies illustrated with problems

		Strategies Number perspective		
		Splitting	Stringing	Varying
Procedures Operation perspective	DS Direct Subtraction	**63–31=** * 60–30=30 3– 1= 2 30+ 2=32	**63–47=** 63–40=23 23– 3=20 20– 4=16	
	IA Indirect Addition	**67–52=** ** 50+10=60 2+ 5= 7 10+ 5=15	**62–58=** 58+ 2=60 60+ 2=62 2+ 2= 4	
	IS Indirect Subtraction	**67–52=** ** 60–10=50 7– 5= 2 10+ 5=15	**62–58 =** 62– 2=60 60– 2=58 2+ 2= 4	
	MO Multiple operations			**77–29 =** 77–30=47 or 78–30=48 47+ 1=48

* The problem can be solved by the following calculation steps. The description of these steps does not necessarily reflect how the problems are or should be notated by students. The students' use of materials and models is left out as well.
** These calculation steps are not very common to solve this problem; they are only given to explain this particular combination of procedure and strategy.

1.2 Solving subtraction problems with crossing the ten

Various studies (e.g., Beishuizen, Van Putten, & Van Mulken, 1997; Fiori & Zuccheri, 2005; Kraemer et al., 2009) have shown that SE students experience many difficulties in solving subtraction problems up to 100 that require crossing the ten, i.e., problems in which the ones digit of the subtrahend is larger than that of the minuend (e.g., 62 − 58). These problems can be solved in different ways that clearly have a different success rate. A highly error-sensitive approach is to solve such problems by a DS procedure together with a splitting strategy. SE students frequently apply this combination of DS and splitting, which often leads to the mistake of reversing the ones digits (Kraemer et al., 2009). In the case of 62 − 58, this means subtracting 2 from 8 instead of 8 from 2.

IA can be a good alternative for DS in problems that have a small difference between the subtrahend and minuend and which require crossing the ten. Several researchers (Beishuizen et al., 1997; Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009a; Van den Heuvel-Panhuizen, 2001) have shown the advantage of such a procedure. In small-difference subtractions, because of the small distance to be bridged, students can determine the difference relatively quickly and easily, while DS would take considerably more and/or more difficult steps (Torbeyns et al., 2009a). Students need to understand that addition and subtraction are inversely related in order to apply this change in direction and solve a subtraction problem flexibly, i.e. by addition instead of subtraction. This is an important understanding in the development of students' arithmetical competence, which can help them solve difficult subtraction problems.

1.3 Solving subtraction problems by indirect addition

Connected to the earlier described debate about whether or not teaching SE students one fixed method for solving number problems, there is also controversy on whether SE students are able to solve subtraction problems by applying IA. For example, a few recent intervention studies concluded that even students in regular primary education have difficulties using IA to solve subtraction problems (Torbeyns et al., 2009a; Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009b). Torbeyns et al. (2009a) found that students have great difficulty picking up the IA procedure, even when they have been taught IA. This was particularly true for low-achieving students. Situations where students have not been taught to use IA are even more disappointing. Several studies (Blöte, Van der Burg, & Klein, 2001; Klein et al., 1998; Torbeyns et al., 2009a, 2009b) suggested that in such situations, students will hardly apply this procedure. However, these studies are challenged by other intervention studies that support the claim that, already in the first grades of primary mathematics education, students with a wide range of mathematical abilities can learn to solve subtraction problems flexibly by applying IA (Blöte et al., 2001; Fuson & Willis, 1988; Klein et al., 1998; Menne, 2001). For example, the study by Klein et al. (1998) revealed that the early introduction of flexible strategies or procedures, such as IA, helped improve students' scores.

1.4 Conditions influencing students' procedure use

To get a thorough understanding of whether SE students can apply IA, we need to know which conditions influence students' procedure use. First of all, student characteristics, such as their general mathematical ability, age, and grade level (see Torbeyns et al., 2009b), are found to be of influence. Furthermore, teaching characteristics, for example, whether or not students have been taught a particular procedure, turned out to play a role, although not all researchers found this (see Section 1.3). A third source of influence is problem characteristics, including (a) the numbers involved and (b) the problem format (context problems or bare number problems).

1.4.1 Influence of numbers involved

Several studies (e.g., Blöte et al., 2001; Fuson & Willis, 1988; Klein et al., 1998; Menne, 2001; Torbeyns, De Smedt et al., 2009; Torbeyns, Ghesquière, & Verschaffel, 2009) have indicated that subtraction problems that require crossing the ten and that have a small difference between the minuend and subtrahend (e.g., $62 - 58$) may evoke the use of IA. However, IA could also be an efficient procedure for solving large-difference subtraction problems with a relatively small difference around the tens and requiring crossing the ten (Torbeyns et al., 2009a). For example, a problem like $82 - 29$ may be easily solved by IA (i.e., $29 + 1 = 30$, $30 + 50 = 80$, and $80 + 2 = 83$, so $1 + 50 + 2 = 53$). Finally, research suggested that small-difference problems that do not require crossing the ten (e.g., $47 - 43$) may also evoke the use of IA (Gravemeijer et al., 1993).

1.4.2 Influence of problem format

In subtraction, two didactical phenomenological interpretations can be distinguished: (1) subtraction as taking away and (2) as determining the difference. In the first interpretation, the matching operation is that of taking away the subtrahend from the minuend. However, in the second interpretation, the difference is determined by bridging the gap, which can be done in two ways: by adding on from the subtrahend until the minuend is reached and by decreasing the minuend until the subtrahend is reached. Both interpretations of subtraction need to be addressed if we want students to learn subtraction in a more complete way (Freudenthal, 1983; Müller & Wittmann, 1984; Van den Heuvel-Panhuizen & Treffers, 2009). To contribute to this broad understanding of subtraction, students should be given more than just bare number problems. Several studies (Klein et al., 1998; Torbeyns et al., 2009b; Blöte et al., 2001; Van den Heuvel-Panhuizen, 1996) revealed that bare number problems hardly evoke the use of IA, which can be explained by the presence of the minus sign that emphasizes the "taking-away" action (Van den Heuvel-Panhuizen, 1996). Context problems, on the contrary, lack this operation symbol and therefore open up both interpretations of subtraction (Van den Heuvel-Panhuizen, 2005). Moreover, the action described in the context of a problem may prompt the use of a particular procedure (Van den Heuvel-Panhuizen, 1996, 2005).

1.5 Research questions and hypotheses

Previous studies on weak-performing students' use of the indirect addition procedure in regular primary education have led to contradictory results, which may be due to not taking all relevant conditions into account in one study: the role of the problem format (context or bare number problems), the numbers involved, and the occurrence of prior instruction in IA. The present study is set up to include all these conditions. Moreover, the study addresses students in SE. The study has two foci: students' spontaneous use of IA, i.e., applying IA without being asked to use this procedure, and students' success rate. We formulated the following research questions:

1. Can SE students make spontaneous use of IA for solving subtraction problems up to 100, and which conditions influence the use of IA?
2. Does the use of IA help SE students solve subtraction problems up to 100 successfully, and under which conditions does IA use lead to successful problem solving?

Concerning the first research question, our general expectation is that SE students can make use of IA (Hypothesis 1a) and that they are more likely to apply IA:

- In small-difference subtraction problems with crossing the ten than in large difference problems with or without crossing the ten (influence of numbers involved, Hypothesis 1b)
- In context problems that reflect adding on than in context problems that reflect taking away or in bare number problems (influence of problem format, Hypothesis 1c)
- When having received instruction in IA than when not having received this instruction (influence of prior instruction, Hypothesis 1d)

With respect to the second research question, our general expectation is that applying IA results in a higher success rate than not applying IA (Hypothesis 2a) and that this is particularly true:

- When applying an IA procedure in combination with a stringing strategy rather than when applying a DS procedure together with a splitting strategy (Hypothesis 2b)
- In small-difference subtraction problems with crossing the ten rather than in large difference problems with or without crossing the ten (influence of numbers involved, Hypothesis 2c)
- When having received instruction in IA rather than when not having received this instruction (influence of prior instruction, Hypothesis 2d)

2 Method

2.1 Participants

In the Netherlands, subtraction up to 100 is mainly taught in the second grade of primary school. In total, 56 students from 14 second grade classes in three Dutch SE schools participated in the study. In the Netherlands, about 3% of the children of primary school age are in SE schools. This percentage involves only students who have learning difficulties; thus, no students with physical disabilities are included. The participating students (39 boys, 17 girls) were 8 to 12 years old, with a mean age of 10 years and 6 months (*SD* = 10.4 months). In regular education, 8- to 9-year-olds are in grade 3 and 11- to 12-year-olds in grade 6. This means that the students in our study were 1 to 4 years behind in mathematics compared with their peers in regular primary school. The students' mathematical ability level was established with the Cito Monitoring Test for Mathematics End Grade 2 (Janssen, Scheltens & Kraemer, 2005). The standardization of this test was based on a representative sample of Dutch second grade students in regular primary education whose average ability score was 56.4 (*SD* = 14.6). The ability scores of the students in our sample ranged from 32 to 56 with an average of 47.8 (*SD* = 6.8) which is a considerably lower score ($d = -.59$) than that of the students in regular primary school.

2.2 Materials

2.2.1 ICT-based test on subtraction problems

An ICT-based test[2] was developed in which item characteristics were varied systematically over 15 items (see Table 2). These characteristics include number characteristics and format characteristics.

The number characteristics refer to the size of the difference between the minuend and subtrahend (small means <7 or large means >11), whether the tens have to be crossed (e.g., 61 − 59) and whether or not the minuend and the subtrahend are close to a ten (<3). The format characteristics refer to whether or not the items are presented as a bare number problem (BN) or as a context problem. The latter can describe a taking-away situation (ConTA) or an adding-on situation (ConAO). Figure 1 shows an example of a ConAO item.

Fig. 1 Album item; the accompanying read aloud instruction is: "The album has space for 51 cards. 49 are already included. How many more cards can be added?"

In agreement with the aim of our study, all number and format characteristics were uniformly distributed over the test item positions (see Table 2). This means that the influence of item characteristics is not confounded by the position of the items. However, in order not to lose statistical power – given the small sample of students – we decided to present a fixed set of items to all students.

Table 2
Different types of subtraction items and number of items in ICT-based test on subtraction

Item no.	Number characteristics*	Format characteristics		
		Bare number problem (BN)	Context problem	
			Taking-away context (ConTA)	Adding-on context (ConAO)
1	A	x		
2	B		x	
3	C			x
4	D	x		
5	E		x	
6	A			x
7	B	x		
8	C		x	
9	D			x
10	E	x		
11	A		x	
12	B			x
13	C	x		
14	D		x	
15	E			x

* A= difference m and s<7; no crossing 10; closeness to 10 of m and s >3, e.g., 47 – 43 =
B= difference m and s<7; crossing 10; closeness to 10 of m and s <3, e.g., 61 – 59 =
C= difference m and s>11; crossing 10; closeness to 10 of m and s <3, e.g., 41 – 29 =
D= difference m and s>11; no crossing 10; closeness to 10 of m and s >3, e.g., 56 – 32 =
E= difference m and s>11; crossing 10; closeness to 10 of m and s >3, e.g., 52 – 36 =

The 15 items were displayed one per screen. The students could click to continue to the next item. The accompanying text was read out by the computer. By clicking on the ear button, the student could hear the spoken text again.

After a short introduction, the students worked individually on a touch-screen notebook.

Students were told that they were free to choose any solution method. After filling in an answer, they reported verbally how they found this answer. The students' on-screen work was recorded by Camtasia Studio software. All students and their parents gave their permission for collecting these records.

2.2.2 Psychometric properties of the test

We investigated the psychometric properties of the collection of 15 items with respect to IA use and success rate of the answers. The score for IA use was retrieved from a dichotomous division of the students' responses (IA used or IA not used) to the 15 items of the ICT-based test on subtraction.

The reliability of the scale for IA use was rather low ($\alpha = .45$). An exploratory factor analysis based on tetrachoric correlations showed that 30.8% of the total variance was explained by the first factor. The proportion of the first and second eigenvalues equaled 1.52, meaning that the first factor substantially dominated the second factor, which was confirmed by a factor analysis scree plot.

The reliability of the scale for success rate was moderate ($\alpha = .69$). An exploratory factor analysis indicated a first dominant factor which accounted for 37.8% of the total variance. The proportion of the first and the second eigenvalues equaled 2.79, which emphasizes the strength of unidimensionality. The factor analysis scree plot revealed a second, but weak, factor. To get a better understanding of the structure of the test, we carried out an oblique rotation of the factor loading matrix, which implies that items are forced to load on only one factor. For the IA use scale, the procedure revealed no clearly interpretable factor solution. For the success rate scale, it was found that one factor was mainly characterized by items of categories B and C (items that have the minuend and the subtrahend close to a ten) and the other factor to items of categories A, D, and E (items that do not have the minuend and the subtrahend close to a ten).

2.2.3 Online teacher questionnaire

To collect data about the students' prior instruction on subtraction problems, we asked their mathematics teachers which procedures they had taught their students for solving these problems. An online questionnaire was developed for collecting these data. The link for the questionnaire was sent by email to the 14 teachers of the students. The teachers received the questionnaire shortly after their students were administered the ICT-based test. All 14 teachers filled in and submitted the questionnaire.

Apart from a few general questions about the teachers' background, the questionnaire contained a specific question on the topic of "subtraction up to 100" to collect data on the procedures (DS or IA) they had taught their students for solving subtractions up to 100.

2.3 Analysis

The students' responses were classified on the basis of the screen videos, which captured the students' answers to the test items and their verbal reports. The students' answers were coded as correct or incorrect. The verbal reports were used to classify the students' strategies and procedures. The strategies were coded as splitting, stringing, or varying and the procedures as DS, IA, IS, or MO (see Table 1). In addition, when the student knew the answer to a problem by heart, we assigned the code KF (Known fact); when the student did not come up with an answer to a problem, the code NR (No response) was used; and when the student decided to erroneously add up the two numbers in a problem, it was coded as Ad (Addition). The responses were coded by two raters independently. There were only a few cases of disagreement (<5%). After discussing these cases, full agreement was reached.

The information about prior instruction in IA was derived from the online teacher questionnaire. In addition, we analyzed the mathematics textbook series used by the teachers on the frequency of tasks in grades 1 and 2 that address the inverse relation between addition and subtraction in solving calculations up to 100.

Item responses of students (procedures, strategies, and success rate) were collected at case level, and the cases (students × items) are on the one hand nested within students (who are in turn nested within teachers) and on the other hand nested within items. This structure enabled the use of cross-classified multilevel models with predictors at case, item, student, and teacher levels. We estimated the models in WinBUGS (Lunn, Thomas, Best, & Spiegelhalter, 2000). Because of the dichotomous nature of the dependent variables, we made use of multilevel logistic regression models.

In addition to the cross-classified multilevel analyses, we also used logistic regression models in which neither student, teacher, nor item effects were included. We did this by making use of generalized estimating equations (GEE). In this approach, standard errors of regression coefficients are adjusted as a result of the cross-classified data structure (Halekoh, Højsgaard, & Yan, 2006).

3 Results

3.1 Frequencies of procedures and strategies

The data analysis was based on all cases in which the students gave an answer to a particular item. Of the 840 possible cases (56 students doing all 15 items each), 72 cases were missing. This resulted in 768 cases to be analyzed. Table 3 gives an overview of the applied procedures in combination with the applied strategies at the level of the cases (the student's responses). DS was used in 63% of the total cases and went together almost equally often with a stringing or a splitting strategy. IA was used in 34% of the total cases of answered items and was often applied in combination with a stringing strategy.

Table 3
Cross tabulation of frequencies of procedures and strategies for 768 cases

		Procedure						
		Direct subtraction (DS)	Indirect addition (IA	Indirect subtraction (IS)	Multiple operations (MO)	Known fact (KF)	Addition instead of subtraction (Ad)	Total
		Cases (%)	*Cases (%)*	*Cases (%)*	*Cases (%)*	*Cases (%)*	*Cases (%)*	*Cases (%)*
Strategy	Splitting	260 (86)	33 (11)	-	-	-	9 (3)	302 (100)
	Stringing	220 (48)	227 (49)	9 (2)	-	-	7 (2)	463 (100)
	Varying	-	-	-	2 (100)	-	-	2 (100)
	Known fact	-	-	-	-	1 (100)	-	1 (100)
Total		480 (63)	260 (34)	9 (1)	2 (0)	1(0)	16 (2)	768 (100)

3.2 SE students' spontaneous IA use

Of the 15 subtraction problems, the total number of times the students applied IA to solve an item ranged from 0 to 8 items ($M = 4.6$, $SD = 1.9$).

3.2.1 Different conditions and IA use

Numbers involved Figure 2 shows that IA was most frequently applied in small-difference problems without and with crossing the ten (A and B, respectively). DS appeared to be the most popular procedure in large-difference problems (D and E), even in large-difference problems that have the minuend and subtrahend both close to a ten (C). The more frequent use of IA in A and B than in C, D, and E appeared to be significant in a GEE logistic regression ($b = 1.24$, $SE = .16$, $p < .05$).

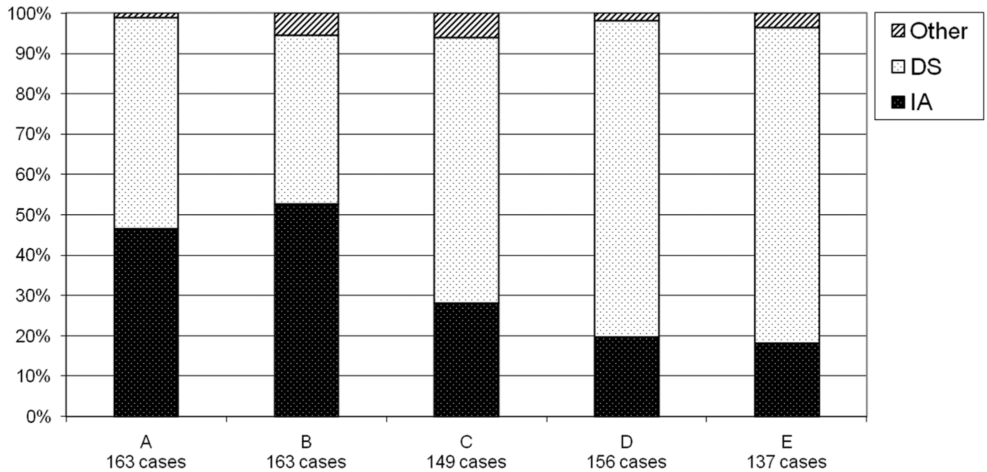

Fig. 2 Percentage of procedure use related to number characteristics of the items

Problem format Figure 3 shows that IA mainly appeared in the items with an adding-on context (ConAO) and that DS was most often used in items with a taking-away context (ConTA). Moreover, when solving BN problems, the students preferred DS. A GEE logistic regression showed a significant difference in IA use between context and bare number problems ($b = 2.38$, $SE = 0.25$, $p < .05$).

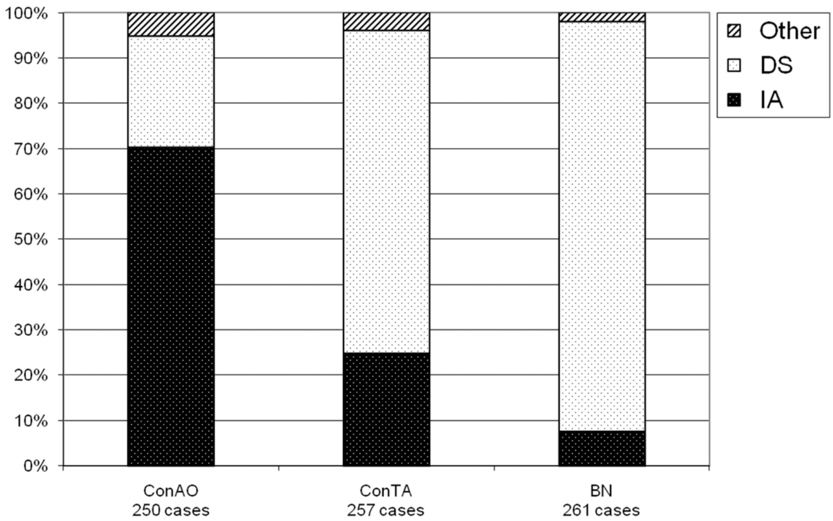

Fig. 3 Percentage of procedure use related to the problem format

Prior instruction The teachers' responses to the online questionnaire revealed that two different textbook series were used in the 14 classes. Although these textbook series each contain some missing addend problems, they do not explicitly address the inverse relation between addition and subtraction. Because teachers could have paid attention to IA without it being addressed in the textbook series, we also asked them which procedures they taught their students for solving subtraction problems. Their answers made it clear that all teachers taught DS. Only three teachers responded that they taught both DS and IA. Therefore, the students of these three teachers, 16 in total, were taught both procedures. These 16 students applied IA in 29% of the total of 209 cases (16 students answered 15 items each, minus 31 missing cases). The other 40 students who were not taught IA applied this procedure in 36% of the total of 559 cases (40 students answered 15 items each, minus 41 missing cases).

3.2.2 Multilevel analysis with IA use as the dependent variable

To examine the influence of the different conditions on IA use, we carried out a multilevel analysis in which we specified a cross-classified multilevel model containing an empty model 0 and a model 1 with predictors (see Table 4).

Table 3
Multilevel logistic regression model with IA use as dependent variable

	Model 0		Model 1		
Fixed Part	*b*	*SE*	*b*	*SE*	
Intercept	-1.47	.82	–1.05	1.73	
Item level					
Numbers involved A			–0.45	1.06	
Numbers involved C			–1.73	1.11	
Numbers involved D			–3.35*	1.26	
Numbers involved E			–2.98*	1.17	
Problem format ConAO			4.74*	0.93	
Problem format ConTA			1.42	0.99	
Student level					
Cito ability score			–0.01	0.03	
Male student			0.44	0.41	
Teacher level					
IA taught			–0.44	0.55	
Random Part	*SD*	*SE*	*SD*	*SE*	R^2 (Model 0–Model 1)
Item level	2.83	.72	1.19	0.53	0.83
Student level	.93	.19	0.99	0.19	0^
Teacher level	.41	.28	0.44	0.33	0^

* $p < .05$

^ R^2 was negative; therefore, it was set to 0
The intercept of the fixed part with respect to:
- Item level corresponds to Numbers involved B and Problem format Bare Number (Model 1)
- Student level corresponds to Average Cito score and Female student (Model 1)
- Teacher level corresponds to Not IA taught (Model 1)

In model 0, only random effects of items, students, and teachers are specified. The intercept represents the average use of IA transformed onto the logit scale of the multilevel logistic regression model. The intercept ($b = -1.47$, $SE = .82$) is smaller than zero, which implies that IA is applied in less than half of the cases.

The large *SD* of the random item effect ($SD = 2.83$) compared with the student effect ($SD = .93$) indicates that IA use is mainly an item characteristic. This means that the application of IA is elicited by the nature of an item rather than by the specific preference of a student. Thus, students seemed to apply IA in a flexible, item-specific way.

The *SD* of the teacher component (*SD* = .41) is also small compared with the *SD* at the item level. Nevertheless, it should be noted that there is a substantial variation between teachers whose instruction might have consequences for students' IA use.

In model 1, the *numbers involved* and the *problem format* are included as predictors at the item level. Here, all categories except a reference category of these variables are dummy coded (1=item possesses the property, 0=item does not possess this property). The regression coefficients of categories A, C, D, and E of the predictor *numbers involved* represent their contrast with the reference category B. In Table 4, the negative regression coefficients for numbers involved categories A, C, D, and E indicate that the frequency of IA use for the items belonging to these categories is smaller than for items in category B. However, we only observed a significant difference for numbers involved categories D ($b = -3.35$, $SE = 1.26$, $p < .05$) and E ($b = -2.98$, $SE = 1.17$, $p < .05$) and not for numbers involved categories A ($b = -0.45$, $SE = 1.06$, $p > .05$) and C ($b = -1.73$, $SE = 1.11$, $p > .05$). That the regression coefficient of A is close to zero indicates that in items like $47 - 43$ (category A) and $61 - 59$ (category B), IA is equally frequently used. For C, the regression coefficient suggests that students applied IA less frequently in category C than in B, but more often than in categories D and E.

With respect to the problem format, the regression coefficients of the categories of the predictor *problem format* (ConAO and ConTA) represent their contrast with the category BN problems. We found that IA was significantly more often applied for items that involve a context problem that reflects adding on (ConAO) than for BN problems ($b = 4.74$, $SE = 0.93$, $p < .05$). Such a significant difference was not found between items that involve context problems that reflect taking away (ConTA) and BN problems ($b = 1.42$, $SE = .99$, $p > .05$). To investigate whether there is a difference in IA use between the context problem types ConAO and ConTA, we created a new variable which is defined by the difference of the regression coefficients of the two context problem types. Based on the WinBUGS output, we computed the distribution of this new variable, which revealed that IA use occurred significantly more often for ConAO than for ConTA items ($b = 3.33$, $SE = .86$, $p < .05$). When examining whether there is a difference in IA use between context problems (ConAO and ConTA) and BN problems, we found that IA was significantly more used in context problems ($b = 3.08$, $SE = .86$, $p < .05$).

The *SD* of the item effect in model 1 ($SD = 1.19$) was substantially smaller than the corresponding *SD* in model 0 ($SD = 2.83$). This means that a large amount of item variance in IA use is explained by the item predictors *numbers involved* and the *problem format*. The explained variance at the item level ($R^2_{\text{Item}} = 0.83$) corresponds to the reduction of variance from model 0 to model 1. At the student level, neither *gender* ($b = .44$, $SE = .41$, $p > .05$) nor the *Cito ability score* ($b = -.01$, $SE = .03$, $p > .05$) turned out to be a significant predictor for IA use. At the teacher level, we found that despite the variation between the teachers, the variable *IA taught* ($b = -.44$, $SE = .55$, $p > .05$) is not significant. Both for teacher and student levels, a small increase of *SD* is observed in model 1 compared with model 0.

As shown in Table 3, not using IA almost always implies the use of DS. Therefore, an indication of the regression coefficients for DS can be found by multiplying the regression coefficients b in Table 4 by −1. The *SD*s for all three levels will hardly change.

3.3 SE students' success rate in IA

Of the 15 subtraction problems, the students solved between 1 and 14 items correctly ($M = 7.7$, $SD = 3.5$). In 68% of the 260 cases in which IA was applied and in 51% of the 480 cases in which DS was applied, the students' answers were correct. The higher success rate when using IA appeared to be significant in a GEE logistic regression model ($b = .82$, $SE = .17$, $p < .05$).

3.3.1 Different conditions and success rate in IA

Number involved Figure 4 shows that for items in category B, students' success rate when using IA is 87%, whereas it is 39% when using DS. This positive difference of 48 percentage points in success rate between applying IA and DS deviates from the negative difference of 11 and 4 percentage points found in the categories D and E, respectively. This difference in success rate between IA and DS for the different categories of numbers involved appeared to be significant in a GEE logistic regression model ($b = 2.64$, $SE = .56$, $p < .05$).

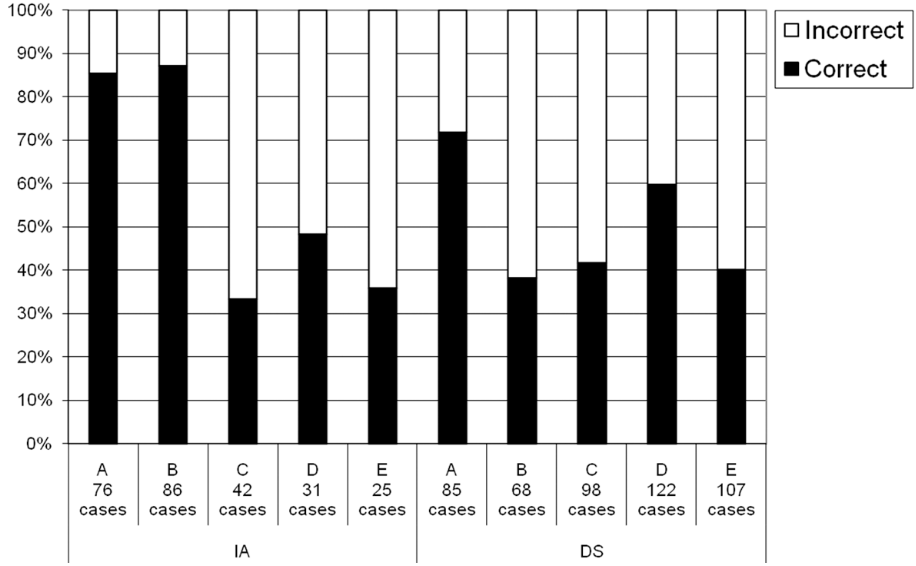

Fig. 4 Percentage of correct answers related to number characteristics of the items

Prior instruction The students who had received IA instruction correctly solved 77% of the 61 total cases for which they used IA. The students who did not receive IA instruction correctly solved 67% of the 199 total cases in which they applied IA. The difference in these percentages did not appear to be significant in a GEE logistic regression ($b = .55$, $SE = .34$, $p > .05$).

3.3.2 Multilevel analysis with success rate as the dependent variable

To examine the influence of the conditions on success rate, we carried out a multilevel analysis in which we specified a cross-classified multilevel model containing model 0 and model 1 (see Table 5).

Table 4

Multilevel logistic regression model with success rate as dependent variable

	Model 0		Model 1		
Fixed Part	*b*	*SE*	*b*	*SE*	
Intercept	0.36	0.38	0.14	0.58	
Case level					
IA use			–0.40	0.52	
Stringing use			0.72*	0.28	
IA use and Stringing use			1.17*	0.55	
Item level					
Numbers involved A			0.89	0.56	
Numbers involved C			–1.39*	0.52	
Numbers involved D			–0.24	0.55	
Numbers involved E			–1.30*	0.55	
Problem format ConAO			0.36	0.43	
Problem format ConTA			0.05	0.45	
Student level					
Cito ability score			0.12*	0.03	
Male student			–0.20	0.39	
Teacher level					
IA taught			–0.09	0.44	
Random Part	*SD*	*SE*	*SD*	*SE*	R^2 (Model 0–Model 1)
Item level	1.12	0.29	0.53	0.25	0.78
Student level	1.20	0.18	1.07	0.18	0.20
Teacher level	0.31	0.24	0.26	0.20	0.30

* $p < .05$

In model 0, the *SD* of the random student effects (*SD* = 1.20) is larger than the *SD* of the random item effects (*SD* = 1.12), which indicates that correctly solving an item is more student-related than item-related. In addition, the *SD* of the random teacher effect (*SD* = .31) is quite small compared with the *SD* at the item level.

In model 1, several predictors at the case, item, student, and teacher levels are included. At the case level, the predictors *strategy use* and *procedure use* are included to investigate their influence on the success rate. Because our focus is on the IA procedure, which was mostly combined with a stringing strategy, we used IA and stringing use as the dummy variables. Although there is a positive relation between IA use and success rate, IA use did not significantly predict success rate ($b = -.40$, $SE = .52$, $p > .05$). The use of the stringing strategy increases the success rate significantly ($b = .72$, $SE = .28$, $p < .05$). However, the best predictor of a correct answer is the combination of IA and stringing ($b = 1.17$, $SE = .55$, $p < .05$). This finding was obtained even after controlling for all the other predictors at the item, student, and teacher levels.

At the item level, the predictors *numbers involved* and *problem format* are included. Items belonging to the numbers involved categories C ($b = -1.39$, $SE = .52$, $p < .05$) and E ($b = -1.30$, $SE = .55$, $p < .05$) are significantly more difficult than items of category B. Concerning the problem format, we found that both types of context problems (ConAO and ConTA) did not significantly differ from the BN problems ($b = .36$, $SE = .43$, $p > .05$ and $b = .05$, $SE = .45$, $p > .05$, respectively).

At the student level, it appeared that students' success rate is positively related to the *Cito ability score* for mathematics ($b = .12$, $SE = .03$, $p < .05$); however, *gender* is not ($b = -.20$, $SE = .39$, $p > .05$). Finally, at the teacher level, we found that *IA taught* is not a significant predictor of success rate ($b = -.09$, $SE = .44$, $p > .05$).

Using the *SD*s at the item level in model 0 and model 1, we found that the item difficulties are largely explained ($R^2_{\text{Item}} = .78$) by the item predictors. The explained variance at the student level ($R^2_{\text{Student}} = .20$) is smaller. Apparently, other student characteristics besides the two included in model 1 are responsible for the variance at the student level. The explained variance at the teacher level ($R^2_{\text{Teacher}} = .30$) is also less than on the item level.

To investigate whether the success rate in the case of IA use differed for the different numbers involved, we specified an additional multilevel regression model including the predictors *IA use*, the categories of *numbers involved* (which are also used in model 1), and the interactions of IA use with each of these categories. As in model 1, category B served as a reference category. For all interactions of IA use with numbers involved categories, we found significant negative regression coefficients. This means that IA use is most successful when it is applied in small-difference problems with crossing the ten (category

B) compared with all the other categories of numbers involved (A vs. B: $b = -1.58$, $SE = .67$, $p < .05$; C vs. B: $b = -3.07$, $SE = .67$, $p < .05$; D vs. B: $b = -3.14$, $SE = .71$, $p < .05$; E vs. B: $b = -2.59$, $SE = .74$, $p < .05$).

4 Conclusions and discussion

Our study showed that SE students can indeed make use of IA when solving subtraction problems (Hypothesis 1a). The main prompt for using IA turned out to be the item characteristics. Students used IA in a rather flexible item-specific way. With respect to the numbers involved, we found that students mainly used IA in small-difference problems with crossing the ten (Hypothesis 1b). With regard to the problem format, our study revealed that students most frequently applied IA in context problems that reflect adding on (Hypothesis 1c). However, contrary to what we stated in Hypothesis 1d, students did not apply IA more often when having received instruction in IA.

Our study showed that the SE students were quite successful in solving subtraction problems when using IA (Hypothesis 2a), but the results from the two types of applied analyses were not univocal. In the GEE regression, IA use was found to significantly influence success rate, whereas in the multilevel regression (in which—in contrast to the GEE approach—the student's general ability of solving the subtraction problems in the test is included as a random effect), IA use was not a significant predictor for success rate. Because students were free to choose their solution method, the use of IA might be related to their general ability to solve test items correctly. This explains why the GEE approach and the multilevel approach lead to different results (see also Molenberghs & Verbeke, 2004).

Furthermore, solving the test items by applying IA together with stringing appeared to be more successful than applying DS together with splitting (Hypothesis 2b). This finding emphasizes the importance of examining procedures (IA use or DS use) as well as strategies (splitting, stringing, and varying) when investigating students' ability to solve number problems.

Regarding the numbers involved, we found that in small-difference problems with crossing the ten, students were more successful when applying IA (Hypothesis 2c). Again, for prior instruction, we did not find an effect of IA use on success rate (Hypothesis 2d).

In sum, our study has revealed that: SE students (1) are able to use IA spontaneously, (2) are rather flexible in applying IA to solve subtraction problems, and (3) are quite successful when solving subtraction problems by IA. These outcomes contrast with some research findings described in Section 1.3 which suggested that weak students have difficulties in applying IA to solve subtraction problems. Our findings made it clear that sensitive assessment tools are needed to reveal students' ability. In our case, test items designed with a particular format and number characteristics enabled us to make SE students' ability to use IA visible. Furthermore, our findings have consequences not only for assessment but also for teaching mathematics to SE students, and in particular teaching them subtraction problems. Restricting this teaching to the straightforward taking-away procedure underestimates SE students' mathematical abilities and does not offer the best environment for them to develop a deep understanding of different subtraction approaches.

Although the present study confirmed to a large degree our hypotheses about SE students use of IA to solve subtraction problems up to 100, our results should be handled with care. First of all, our study was limited in the number of students and schools. A second drawback of our study was that we did not carry out a detailed inventory of the students' prior instruction in IA, i.e., we only asked whether the students had been taught a particular procedure and not how it was taught. This lack of information on the quality of the instruction might explain why no influence was found of prior instruction on the students' success rate. Finally, the test we used for this study has some shortcomings. Not only did we have no more than a small number of items but we also offered these items to every student in the same order. The latter means that our results could be flawed as a result of order effects in the items. Although we tried to minimize the order effect of the item characteristics by distributing them uniformly over the test item positions, it cannot be guaranteed that the particular sequence of the item characteristics did not influence the outcomes of our study. Nevertheless, our findings showed that SE students were able to use IA. Providing evidence for this was the main goal of the study.

Notes

1. For example, in a summary of Timmermans' (2005) thesis published on the NWO web site (Netherlands Organization for Scientific Research; retrieved from http://www.nwo.nl/nwohome.nsf/pages/NWOP_6HKFLE), it is stated: "Weak performing students in arithmetic […] attain better results with the traditional approach in which they learn to solve number problems in one particular way" (translated from Dutch by the authors of this paper). Another example is based on Milo's (2003) thesis, of which the NWO web site (retrieved from http://www.nwo.nl/nwohome.nsf/pages/NWOP_5LEJNJ) also clearly states that "pupils at special schools for primary education can best learn arithmetic using one specific strategy."
2. The ICT-based test was developed by the first two authors of this article and programmed by Barrie Kersbergen, a software developer at the Freudenthal Institute. The complete test consists of two parts of 15 items each. In the second part, the items feature a digital number line as an auxiliary tool, whereas the first 15 items do not contain a number line tool. To ensure that the investigation of IA use was as pure as possible, we restricted the present analysis to the data collected with the first 15 items of the test.

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Chapter 6

Special education students' performance in solving elementary combinatorics problems in a dynamic ICT-based assessment

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Special education students' performance in solving elementary combinatorics problems in a dynamic ICT-based assessment

1 Introduction

Most research on supporting special education (SE) students in mathematics focuses on basic mathematical operations like addition and subtraction (Kroesbergen & Van Luit, 2003), with less attention for higher-order thinking processes. This is no surprise as, compared to their peers in regular education (RE), SE students are often behind in their mathematical development (Kraemer, Van der Schoot, & Van Rijn, 2009). Thus, the more advanced topics in primary school mathematics are considered out of reach of special education students. Some studies, though, have shown that low achieving students in mathematics may have a higher mathematical potential than is generally assumed. For example, SE students turned out to have proficiency in interpreting tables and constructing graphs (Bottge, Rueda, Serlin, Hung, & Kwon, 2007), using diagrams (Van Garderen, 2007), and solving ratio problems (Van den Heuvel-Panhuizen, 1996). Even more unexpected was the observation in a pilot study that SE students successfully solved a combinatorics problem with a 3 *x* 3 structure by systematically combining different colored items (Van den Heuvel-Panhuizen & Peltenburg, 2008). Their systematic approach was a surprise as they had not worked on combinatorics in school before. The aim of the present study was to further investigate SE students' performance in solving combinatorics problems. The strategies and success rate of students in RE served as reference data. The study was carried out in the Netherlands.

1.1 Revealing SE students' mathematical potential

An approach to reveal SE students' potential is to present them with mathematical content beyond the regular curriculum, particularly if it requires higher-order skills. As Zohar and Dori (2003) stated, teachers often see higher-order thinking tasks as difficult and highly demanding and therefore do not present such tasks to students they think will find these tasks hard and frustrating. These good intentions lead to a vicious cycle: those students whose thinking skills need to be developed receive less opportunity to do so. We started this study to break this vicious cycle, choosing the domain of combinatorics – which clearly appeals to higher-order thinking, and is not a part of the regular curriculum in SE in the Netherlands. By presenting combinatorics problems in a familiar context and using a dynamic ICT-based assessment environment we aimed to give SE students the opportunity to show their problem solving skills in this domain.

1.2 Combinatorics in primary school

Combinatorics is the domain of mathematics that involves systematic listing and counting (NCTM, 2009), based on the so-called 'fundamental counting principle' (DeGuire, 1991).

This principle describes how to determine the total possible choices when combining groups of items. If you can choose one item from a group of a choices, and another from a group of *b* choices, then the total number of two-item choices is *a* x *b*. The principle can also be viewed in terms of the Cartesian product of two given sets, *a* and *b*, which is the set formed by the combinations produced by pairing each member of a in turn with each member of *b* (English, 2005).

Several mathematics didacticians favor integrating combinatorics in the school mathematics curriculum at all grade levels (e.g., English, 1993; Feijs, Munk, & Uittenbogaard, 2009; Kapur, 1970; Kenny & Hirsch, 1991; NCTM, 2000). An important justification for teaching elementary combinatorics at primary school is that it can help students to develop their reasoning skills, e.g., making conjectures, generalizing and thinking systematically (e.g., Kapur, 1970, English, 2005; Piaget & Inhelder, 1975). Moreover, research has shown that students at primary school age can deal with elementary combinatorics problems. By embedding such problems in rich and meaningful contexts, regular primary school students were found to be able to tackle these problems unassisted (English, 1993; 2005; Maher, Powell, & Uptegrove, 2010).

However, recommendations to incorporate combinatorics in the primary school mathematics curriculum are often ignored (English, 1993). In the Netherlands, this is also the case, particularly in primary SE (Dutch Ministry of Education, 2006). The curriculum in primary SE mainly covers the four main operations (addition, subtraction, multiplication, and division) supplemented with tasks dealing with measurement, money, time and the calendar (Kraemer et al., 2009).

1.3 Research into solving combinatorics problems in primary school

English' extensive work (e.g., English, 1993; 1996; 2005) on combinatorics has found that regular primary school students are quite successful in solving two- (*XxY*) and three- (*XxYxZ*) dimensional combinatorics problems with success rates between approximately 50% and 80%. It was shown that older students (10- to 12-year olds) outperformed the younger ones (7- to 9-year olds) and that, in general, students were not so successful in solving three-dimensional problems (English, 1993).

Furthermore, English' studies showed that primary school students can use increasingly sophisticated solution strategies for identifying all possible combinations of two- and three-dimensional combinatorics problems. In line with the findings of Piaget and Inhelder (1975), she discovered that these strategies evolve in three stages. According to English (1996), the first or "non-planning stage" comprises random, trial-and-error approaches with no global planning components. Piaget and Inhelder (1975) called this is the "empirical combinations stage". English (1996) called the next stage the "transitional stage"; students try to find combinations in a systematic way, but do not succeed in doing so. Piaget and

Inhelder (1975) described this stage as “in search of a system” to generate all possible combinations, and it is followed by the final stage in which students “discover a system”. According to English (1996), in the third stage students construct the “odometer strategy”, which involves keeping one item constant and systematically finding all possible combinations with that item. After that, a new “constant” item is chosen and the same pattern of finding combinations is repeated. If students are close to this strategy, but deviate slightly from the pattern, English (1996) called this “almost odometer strategies”. Implicit in the use of an (almost) odometer strategy, according to Polaki (2002), is the student’s ability to use a crude form of the multiplication rule in order to figure out when the set of possible outcomes is complete.

1.4 An ICT-based environment to assess student performance in solving combinatorics problems

To let students show their ability in dealing with combinatorics problems, we developed an ICT-based assessment environment. Research (Peltenburg, Van den Heuvel-Panhuizen, & Robitzsch, 2010) has proven that such an environment is well suited to reveal SE students’ mathematical potential. ICT-based assessment may incorporate features that increase the accessibility of mathematics problems and can help students to better understand what the problems are about. For example, interactive, dynamic tools such as digital manipulatives and a digital number line (Peltenburg et al., 2010) can be added, letting students carry out specific mathematical operations on the screen (Sarama & Clements, 2006). As Bottino and Chiappini (1998) pointed out, digital tools may support cognitive structuring, that is providing a structure for thinking and acting (Tharp & Gallimore, 1991) because they direct students to focus on the mathematics in the assessment environment (Moyer-Packenham, Salkind & Bolyard, 2008). These tools give visual feedback to students’ input, showing them the results of their actions.

Moreover, digital tools may provide unique advantages over concrete physical materials (Clements & McMillen, 1996), for example by offering flexible and manageable manipulatives that are available in an unlimited quantity, can be easily stacked, moved, and can be quickly rearranged or removed.

The role of digital manipulatives was found to be particularly important in combinatorics. Abramovich and Pieper (1997) discovered that the mathematical visualization provided by movable colored disks yielded significant improvement in the basic combinatorial notions of secondary students. Because SE students often have problems with visualization as an effective strategy to represent problems (Van Garderen, 2007), using digital manipulatives might be particularly important to them for solving combinatorics problems. However, it is also emphasized (Kolloffel, Eysink, & De Jong, 2010; Tarlow, 2008) that students should be given sufficient opportunities to develop personal representations as well to develop meaningful ideas in combinatorics. So, our ICT-based environment should not only offer students ready-made structures but should also give them the freedom to make their own representations.

1.5 Research questions

This study builds on the work of English (1993) who investigated students' success rate and strategies in solving two- and three-dimensional combinatorics problems in RE. The main goal of the current study is to do this investigation with SE students and to examine whether their success rate and strategies in solving two- and three-dimensional combinatorics problems differ from those of students in RE. Considering SE students' delay in the development of their general mathematical ability compared to RE students (Kraemer et al., 2009), one may wonder whether contrasting SE and RE students' performances in solving combinatorics problems would be fruitful. However, by designing accessible tasks in a meaningful context and presented in a dynamic ICT environment we aimed to give both group of students the opportunity to demonstrate their abilities in combinatorics.

In the study four research questions were addressed.

1. Does the success rate of SE students in solving combinatorics problems differ from the success rate of RE students?
2. Does the success rate of SE and RE students change over grades?
3. Does the strategy use of SE students in solving combinatorics problems differ from the strategy use of RE students
4. Does the strategy use of SE and RE students change over grades?

As SE students generally have lower scores in mathematics than students in RE (Kraemer et al., 2009), we expected that SE students' success rate in solving combinatorics problems would be lower than that of RE students (*Hypothesis 1*). Based on English's (1993) findings, we conjectured for both SE and RE students that the success rate would increase over the grades (*Hypothesis 2*).

Regarding strategy use, earlier research has found that low achievers in mathematics generally have difficulties to approach novel mathematics problems structurally and to transfer their acquired strategies from one task situation to another (e.g., Ruijssenaars, Van Luit, & Van Lieshout, 2004). Therefore, we expected that SE students would use a systematic strategy less often than RE students (*Hypothesis 3*). However, because of students' general growing mathematical insights per grade level (Kraemer et al., 2009) we hypothesized that in both groups of students the use of systematic strategies would increase over grades (*Hypothesis 4*).

2 Method

2.1 Participants

A total of 84 students from five SE schools[1] and 76 from five RE schools participated in the study. For each SE school a RE school within the same area was recruited. To enable a comparison of mathematics competence levels of SE and RE students we asked the teachers of the participating schools to choose randomly four students who scored near the 50th percentile on the CITO LOVS Mathematics tests (Janssen, Scheltens, & Kraemer, 2005-2008) for M2, M3, M4, and M5 (the mid-grade tests for respectively grades 2 to 5). The LOVS Mathematics test is frequently used in the Netherlands and mainly contains items on calculation; no items on combinatorics are included. Table 1 shows that for each mid-grade test, the average test scores of the SE students were slightly lower than those of the RE students as confirmed by the small negative effect sizes *d*.

Table 1
CITO LOVS mathematics scores of SE and RE students

	CITO LOVS mathematics score										
	SE					RE					
LOVS test	*N*	*M*	*SD*	*Min*	*Max*	*N*	*M*	*SD*	*Min*	*Max*	*d**
M2	19	47.7	4.1	38	53	20	49.8	4.5	41	56	-0.14
M3	22	67.5	5.5	53	80	20	71.0	4.5	63	78	-0.24
M4	20	82.0	3.8	76	91	19	85.6	4.3	79	93	-0.25
M5	23	97.2	7.7	83	119	17	100.3	5.4	90	107	-0.24

*Cohen's *d* was calculated by using the standard deviation of the CITO reference sample in regular education (Janssen, Verhelst, Engelen, & Scheltens, 2010).

The students in RE were 7-11 year old, with a mean age of 9 years and 5 months (*SD* = 1,3) and the SE students were 8-13 years old, with a mean age of 11 years and one month (*SD* = 1,1). Figure 1 shows the relation between student age and mathematical level as measured by the CITO LOVS test. At the M2 and M3 levels, the SE students were about two years older than the RE students. At the M4 level the difference in ages decreased and at the M5 level the SE students were only about one year older than the RE students.

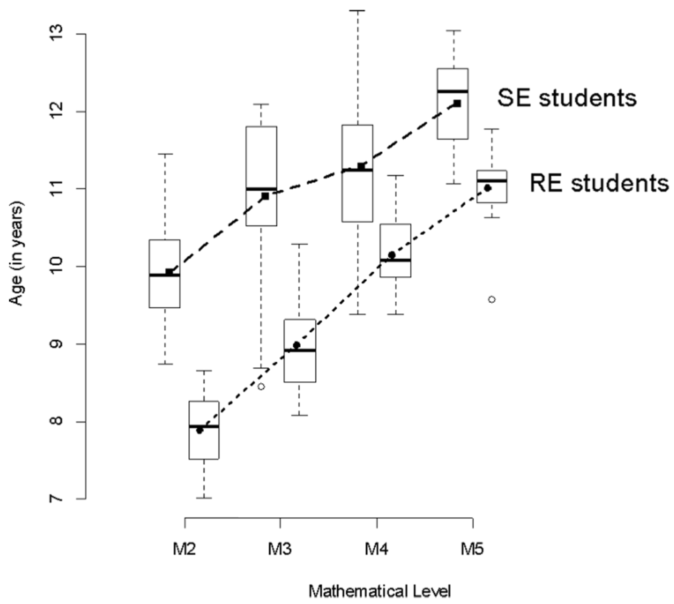

Fig. 1 Age-related increase in mathematical level as indicated by the CITO LOVS tests for SE and RE students

2.2 Data collection

To collect our data, we developed an ICT-based assessment environment[2] in which students had to find all possible outfits by combining clothing items. To make the different outfits students had an infinite supply of puppets that could be dressed with different types of clothing items in different colors. A drag-and-drop function enabled moving both puppets and clothing items to an empty field, where the student could dress the puppets and rearrange or remove them. Figure 2 shows a screen shot of the assessment environment. The ICT environment also offered a drawing tool, with a pencil, an eraser and an erase-all option (see upper right part of the screenshot in Figure 2). Figure 3 shows all possible combination for three differently colored t-shirts and three differently colored skirts.

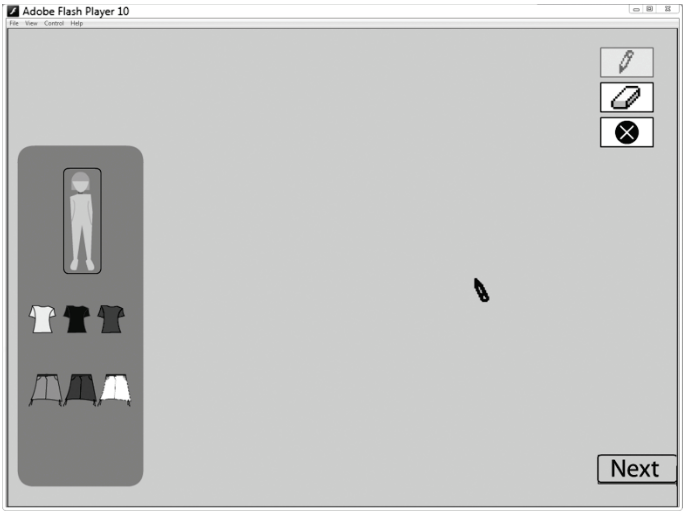

Fig. 2 Screenshot displaying the supply of little puppets and clothing items

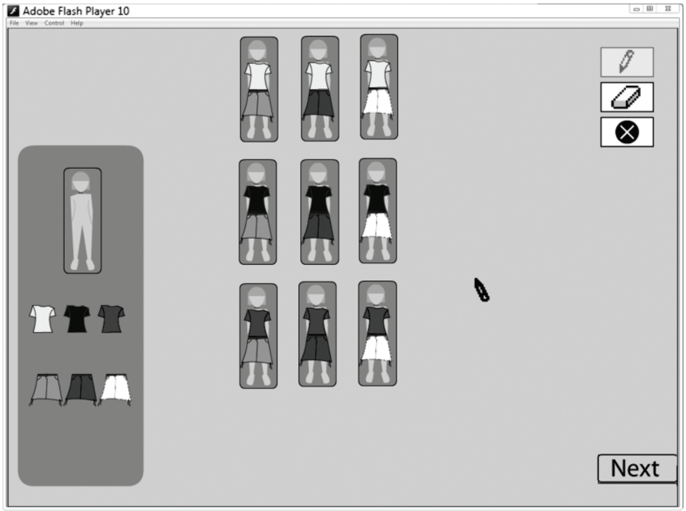

Fig. 3 Screenshot displaying all possible combinations

To assess the students' performance in solving combinatorics problems we developed a series of six combinatorics tasks in the ICT environment. In the first three tasks students had to find all possible combinations of two clothing items (t-shirts and skirts) in two to three different colors (i.e., tasks with a *2 x 3*, *3 x 2*, and *3 x 3* structure respectively). In the last three tasks three clothing items (t-shirts, skirts and shoes) had to be combined (i.e., tasks with a *2 x 2 x 2*, *2 x 2 x 2*, and *2 x 3 x 2* structure respectively).

The students completed the tasks individually on a touch screen notebook with a stylus pen, interacting directly with the images on the computer screen. During the assessment the first author was sitting next to the student, who was first asked to simply dress two puppets (c.f., English, 1993). Guided by a few questions, these dressing activities were meant to familiarize the students with the drag-and-drop function, the drawing tool and the language used in the problems (e.g., the meaning of 'different outfits'). After this familiarization stage, the students were set the first task. Each task was displayed one per screen, and students could click to continue to the next task. For each task, the researcher asked the students how many different outfits were possible with the available clothing items and how they found their answer. The students' on-screen work and their verbal comments were recorded by Camtasia Studio software. Both students and parents had given their permission to make these records.

2.3 Data analysis

Coding. Apart from coding the correctness of the students' responses, we converted their on-screen work into tree diagrams schematizing the identified combinations (c.f., English, 1993). These tree diagrams provide an overview of the combinations that were successfully formed by the students. Based on the tree diagrams, two raters independently coded the students' work as systematic, semi-systematic or non-systematic (98% agreement in coding; *Cohen's kappa* = .97). A systematic strategy was defined by the use of a cyclic pattern, a constant item, or both. See Figure 4.

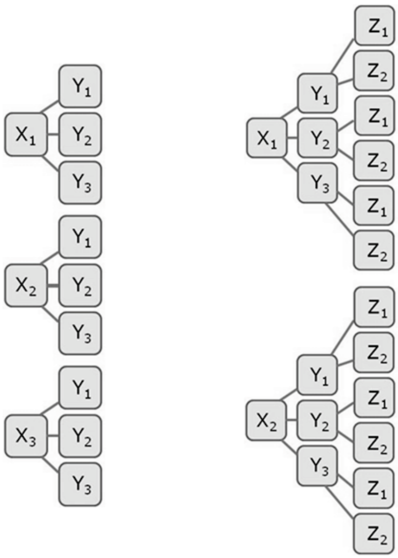

Fig. 4 Example tree diagram that reflects a systematic approach for both a two-dimensional problem (*3 x 3*) (left) and a three-dimensional problem (*2 x 3 x 2*) (right). Note that X_1-X_3 refer to tops, Y_1-Y_3 to skirts and Z_1-Z_2 to pairs of shoes

A semi-systematic strategy was characterized by using a constant item or a cyclic procedure in a non-consistent or non-exhaustive manner[3], whereas the complete absence of a systematic approach was classified as non-systematic.

Using sample weights. As the number of students per school type differed per mid-grade level (see Table 1), we used a weighting procedure giving a weighted sample size of 20 students for all the combinations of the mid-grade levels and school types. For example, each of the nineteen mid-grade level M2 SE students had a sample weight of 20/19 = 1.053. All results in the following section are based on analyses using sample weights.

Analyses of variance. To investigate differences between SE and RE students, variance analyses were carried out at student level. We did this with both success rate and strategy use as the dependent variable. In both analyses we specified three different models; respectively containing mathematical level and school type (Model 1), age and school type (Model 2), and mathematical level, age and school type (Model 3). All models treated mathematical level and age as linear predictors. In preparing the analysis of variance for strategy use, we calculated a score for each student reflecting the degree of systematic strategy use. Student scores were obtained in two steps. At the case level, we attributed 1 point to the use of a systematic strategy, 0.5 point to a semi-systematic strategy, and 0 points to a non-systematic strategy. Then, the mean score for solving the series of six combinatorics tasks in the test was calculated for each student, resulting in the interval-scaled variable strategy use.

Reliability of the series of tasks. Although the assessment contained only six combinatorics tasks we found a fairly high internal consistency for both the scale for success rate (*Cronbach's alpha* = .80) and that for strategy use (*Cronbach's alpha* = .84).

3 Results

3.1 Success rate in combinatorics tasks

3.1.1 Success rate in SE and RE

The SE students correctly solved the combinatorics tasks in 56% of all cases (students *x* tasks). For the RE students this applied for 57% of all cases. The difference in success rate between the two school types was not significant; t (158) = .26, p = .79.

3.1.2 Success rate per mathematical level

Success rate in solving the combinatorics tasks appeared to be positively related to the students' mathematical level for both SE and RE students (see Figure 5).

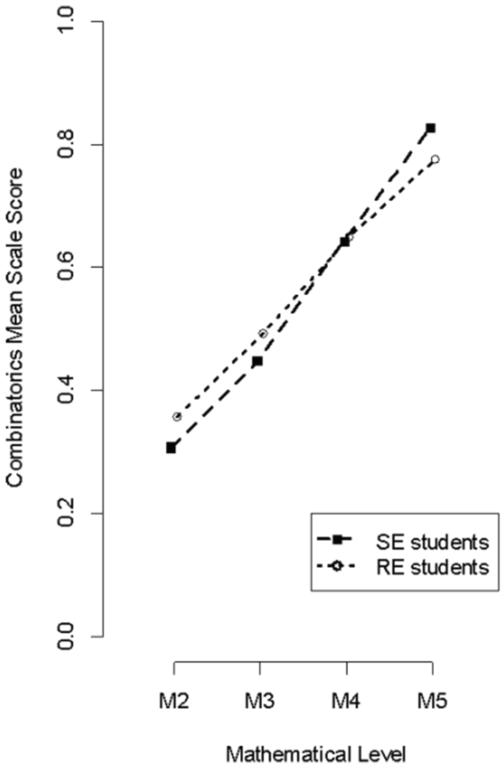

Fig. 5 Relation between percentage success rate on the combinatorics test and CITO LOVS mathematical level for SE and RE students

At the M2 and M3 level SE students solved slightly less combinatorics problems correctly than RE students at the same mathematical level. However, at the M4 level SE students reached the same mean success rate as RE students and at the M5 level SE students even solved slightly more combinatorics problems correctly than RE students. To further investigate differences between SE and RE students with respect to their success rate, we carried out an analysis of variance. The results are presented in Table 2.

Table 2
Results of analysis of variance of success rates from different models with Age, Mathematical level and School Type as predictors

		Model 1 (Mathematical level, School type)			Model 2 (Age, School type)			Model 3 (Mathematical level, Age, School type)		
	df	*F*	*P*	η^2	*F*	*p*	η^2	*F*	*p*	η^2
Math level	1	76.38	.00	.325				16.80	.00	.072
Age	1				36.37	.00	.189	.00	.96	.000
School type	1	1.06	.31	.005	.09	.76	.000	.30	.59	.001
Math level*School type	1	.94	.33	.004				1.17	.28	.005
Age*School type	1				.03	.87	.000	.63	.43	.003
R^2		.34			.19			.34		

Mathematical level was found to be a significant predictor (F (1,156) = 76.38, p < .01, η^2 = .325) in Model 1, while this was not the case for school type (F (1,156) = 1.06, p = .31, η^2 = .005). However, the interaction effect of school type and mathematical level that was suggested in Figure 2, did not appear to be significant either (F (1,156) = .94, p = .33, η^2 = .004). This means that we found similar success rates for SE and RE students at each mathematical level. In Model 2, age was found to be a significant predictor (F (1,156) = 36.37, p < .01, η^2 = .189). As in Model 1, the main effect of school type was not significant, which was also the case for the interaction of age and school type. Finally Model 3 included all three predictors. Only mathematical level appeared to be significant (F (3,154) = 16.80, p < .01, η^2 = .072). In contrast to Model 2, age was found to be non-significant. In sum, in all three models school type did not appear to be a significant predictor after controlling for mathematical level and age. This means that no differences in success rate were found between SE and RE students.

3.2 Strategy use in solving combinatorics tasks

3.2.1 Strategy use in SE and RE

Table 3 shows students' strategy use in both SE and RE. Generally, frequencies of the different types of strategy use differed no more than four percentage points between SE and RE students. In fact, no significant differences were found between the two groups of students in use of systematic, semi-systematic and non-systematic strategies (*Phi* = .051, Chi^2 = 2.485, *df* = 2, p = .29).

Table 3
Cross tabulation of frequencies strategies per school type for 960 cases

		Strategy type (Cases, %)			
		Systematic	Semi-systematic	Non-systematic	Total
School type	SE	216 (45)	183 (38)	81 (17)	480 (100)
	RE	236 (49)	178 (37)	66 (14)	480 (100)
	Total	452 (47)	361 (38)	147 (15)	960 (100)

3.2.2 Strategy use per mathematical level
Figure 6 represents students' strategy use per mathematical level for both SE and RE. It shows use of systematic strategies increasing per mid-grade level in both school types, while non-systematic strategies decreased.

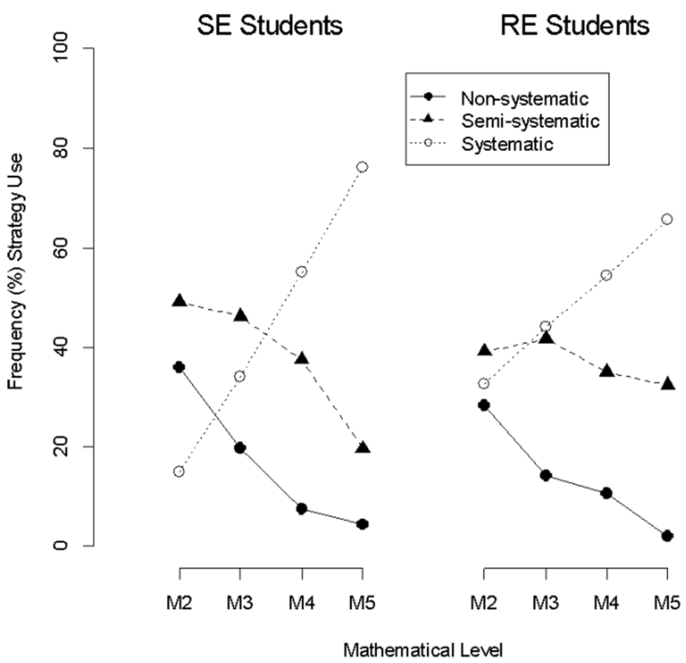

Fig. 6 Relation between percentage strategy use on the combinatorics test and CITO LOVS mathematical level for SE and RE students

At the M2 and M3 level SE students applied less systematic strategies than RE students. However, at the M4 level SE students reached the same percentage of systematic strategies as RE students. Moreover, at the M5 level SE students applied a systematic strategy more often than RE students. To further investigate differences between SE and RE students regarding their strategy use, we carried out an analysis of variance of which the results are presented in Table 4.

Table 4
Results of analysis of variance of strategy use from different models with Age, Mathematical level and School Type as predictors

		Model 1 (Mathematical level, School type)			Model 2 (Age, School type)			Model 3 (Mathematical level, Age, School type)		
	df	*F*	*P*	η^2	*F*	*p*	η^2	*F*	*p*	η^2
Math level	1	63.89	.00	.283				23.94	.00	.106
Age	1				23.26	.00	.130	1.79	.18	.008
School type	1	4.36	.04	.019	.91	.34	.005	.05	.82	.000
Math level*School type	1	3.44	.07	.015				.55	.46	.002
Age*School type	1				.27	.60	.002	.01	.93	.000
R^2		.31			.13			.32		

In agreement with Figure 6, Model 1 shows that mathematical level, school type and their interaction play a predicting role in use of systematic strategies. Mathematical level ($F\,(1,156) = 63.89$, $p = .00$, $\eta^2 = .283$) clearly appears to be a significant predictor, with school type ($F\,(1,156) = 4.36$, $p = .04$, $\eta^2 = .019$) and the interaction of mathematical level and school type ($F\,(1,156) = 3.44$, $p = .07$, $\eta^2 = .015$) both close to the .05 level of significance. From the results of Model 2 it can be concluded that age is a significant predictor ($F\,(1,156) = 23.26$, $p < .01$, $\eta^2 = .13$) while school type and the interaction between age and school type are not. Finally, in Model 3 only mathematical level appears significant ($F\,(1,156) = 23.94$, $p = .00$, $\eta^2 = .106$).

Strategy use and success rate for SE and RE. Tables 5 and 6 show students' strategy use in relation to the correctness of their responses per school type. It was found for both SE and RE students that systematic strategies mainly led to a correct response, while using non-systematic strategies most often led to an incorrect one. For both SE ($Phi = .740$, $Chi^2 = 262.551$, $df = 2$, $p < .01$) and RE ($Phi = .773$, $Chi^2 = 286.868$, $df = 2$, $p < .01$) a positive significant association between strategy use and success rate was found.

Table 5
Cross tabulation of frequencies of correct and incorrect answers per strategy type for SE students

SE		Strategy type (Cases, %)			
		Systematic	Semi-systematic	Non-systematic	Total
Student gave correct answer	Yes	207 (96)	33 (18)	27 (33)	267 (56)
	No	9 (4)	150 (82)	54 (67)	213 (44)
	Total	216 (100)	183 (100)	81 (100)	480 (100)

Table 6
Cross tabulation of frequencies of correct and incorrect answers per strategy type for RE students

RE		Strategy type (Cases, %)			
		Systematic	Semi-systematic	Non-systematic	Total
Student gave correct answer	Yes	226 (96)	37 (21)	10 (15)	273 (57)
	No	10 (4)	141 (79)	56 (85)	207 (43)
	Total	236 (100)	178 (100)	66 (100)	480 (100)

4 Conclusions and discussion

In this study, primary school students worked on a topic that was not part of their mathematics curriculum. It was found that SE students solved combinatorics problems equally successfully as RE students. We did not expect this, and thus had to reject Hypothesis 1. We also found that a significant and similar growth in success rates occurred in both school types for increasing mathematical levels. This finding is in agreement with Hypothesis 2. On average SE students applied a systematic strategy equally often as RE students. This result rejects Hypothesis 3. Finally, we discovered that in both school types a significant increase in the use of systematic strategies occurred, confirming Hypothesis 4.

Additionally, we found different patterns for strategy use across mathematical levels M2 to M5 by SE and RE students. At the M2 and M3 levels, SE students applied a systematic strategy less often than RE students, while at the M5 level SE students applied a systematic strategy more often. In fact, SE students had a larger growth in their use of systematic strategies. A possible explanation lies in the different teaching practice in SE and RE. While direct instruction (DI) is popular in SE (Bottge, 2001), this is not so much the case in RE. Characteristic of DI is its systematic step-by-step approach requiring student's mastery at each step. This emphasis on a structured approach, which students at higher grade levels have experienced for longer, might have caused SE students at the M5 level to apply a systematic strategy so often.

Surprisingly, this more frequent use of systematic strategies was not linked with a higher success rate than that of RE students. This could be explained by the lower working memory capacity of SE students compared to RE students (Kroesbergen & Van Luit, 2003). Perhaps the SE students in our study compensated for possible memory deficits by applying a systematic strategy to keep track of their work, while the RE students could follow a semi-systematic strategy to achieve the same performance.

Another finding is that both SE and RE students very much enjoyed reasoning about possible solutions for the combinatorics problems. For example, they made utterances about being unfamiliar with such problems and being eager to work on them more often. Of course, student attitude to solving combinatorics problems was not the topic of this study. It would, however, be interesting to investigate whether enriching the primary school curriculum with elementary combinatorics would affect how students perceive mathematics as a subject domain.

A further observation was that all students, in RE and SE, used the ICT-based assessment environment in a natural and self-evident way, with no difficulties in using the digital manipulatives. For example, they easily dragged and dropped the puppets or clothing items to (re)organize combinations. This observation strongly illustrates that digital manipulatives can be a powerful tool for eliciting mathematical problem solving.

Of course, this study also has its limitations. We used only a few combinatorics tasks and only of a particular type. Moreover, the selection of students can be criticized. Although we asked the teachers to choose four students at a particular mathematics level at random, there could be some bias because of teacher choice. Another limitation of the study is that we only had a one-shot data collection. Our results would have been more robust with a repeated measurement. Despite these limitations, the findings of our study convincingly demonstrated the mathematical power of SE students in the domain of elementary combinatorics. Consequently, we would like to recommend investigating the enrichment of the mathematics program in SE (c.f., Bottge, 1999), in particular by including activities related to elementary combinatorics. ICT environments such as we developed for this study could be of great value for this.

Notes

1. In the Netherlands, about 3% of children of primary school age are in SE schools. This percentage involves students with mild learning difficulties; no students with physical disabilities are included.

2. The ICT-based assessment environment was developed by the first two authors of this article and programmed by Barrie Kersbergen, a software developer at the Freudenthal Institute.
3. Our classification differs slightly from that of English (1993, 1996). The main reason for our adjustments is that, in our dataset of student work keeping one item constant (e.g., item X1) did not necessarily go together with systematically varying an item of another type (e.g., Y1, Y2 and Y3), – the so-called odometer strategy – which is considered a prerequisite in English' classification for coding a student's work as 'most sophisticated'. Because of this mismatch, we redefined the category of most sophisticated strategies (in our classification called 'systematic strategies') by approaches characterized by the use of a cyclic pattern, a constant item, or both.

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Chapter 7

Teacher perceptions of the mathematical potential of students in special education in the Netherlands

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Teacher perceptions of the mathematical potential of students in special education in the Netherlands

1 Importance of high teacher expectations

Over the last decades, it has become more and more clear that teacher expectations about what their students will accomplish in school are a determining factor in how students actually develop. This awareness of the crucial role of teacher expectations on student achievement originates in research by Rosenthal and Jacobson (1968), who were the first to describe the so-called ‘Pygmalion effect’. In short, this self-fulfilling mechanism, recently also called ‘expectancy effect’ (McGrew & Evans, 2003), means that teachers’ ideas about what their students’ can achieve in a particular subject – in other words, the teachers’ thoughts about their students’ potential – determine how teachers treat their students. This mechanism works twofold. Firstly, high expectations will make teachers ask their students more challenging questions, thus creating a learning environment that contributes to their students’ academic development (Babad, 1993; Good, 1987). Secondly, teachers who have high expectations will express to their students that they have confidence in their students’ capabilities, which increases students’ self-concept and motivation, and consequently positively affects students’ achievement (Marsh & Hattie, 1996; Möller & Köller, 2001). In other words, using this expectancy effect means that positive teacher expectations can be employed as a key for achieving better learning outcomes (Beswick, 2008; Jepma & Meijnen, 2004; Jussim & Harber, 2005; Schoen, Cebulla, Finn, & Fi, 2003). Because high expectations are a crucial requirement for this key to be effective, we wonder whether it can also be used for students with special needs and, in particular, in the case these students go to special education (SE) schools, which is the current situation in the Netherlands (see more about it in the Method section). Teachers in SE schools might too often experience in their daily classroom practice that their students have difficulties in learning mathematics, and therefore will not have high expectations. The aim of this study is to gather data about SE teacher expectations and to investigate their perceptions of the mathematical potential of SE students.

2 What is already known about teacher expectations of weak performing students?

Research (Babad, 1993; Brophy, 1983; Good, 1987) has shown that teachers generally have lower expectations of low-achieving students than of high achievers. These low expectations are also reflected in how teachers think about teaching low achievers. For example, a study by Zohar, Degani, and Vaaknin (2001) revealed that teachers believe that the cognitive demands of tasks requiring higher order thinking are beyond the capabilities of low-achieving students and that these students would become frustrated by such tasks.

Moreover, having low expectations of low achievers may result in not judging their work on its own merit or in providing them fewer opportunities to learn (Good 1987).

Regarding the subject domain of mathematics, Beswick (2008) found that teachers opt for a less varied and challenging curriculum for students they consider weak in mathematics than for students of whom they have higher expectations. For students with learning difficulties, teacher focus is moved towards training basic skills rather than to understanding and insight (Beswick, 2008). The results of Beswick's study are in line with Jungbluth's (2003) findings that inequality in learning opportunities within schools is mainly due to teacher assumptions of student abilities. Similarly, Jepma and Meijnen (2004) have shown that differences in learning outcomes between weak performing students in regular education and students in SE are linked to the different achievement standards teachers have set for these students.

How teachers think about the academic possibilities of their students is often connected to what they see as the causes of their students' success or failure in school (Cooper and Burger, 1980). Teachers attribute students' weak performance to various factors. They mentioned for example student factors, such as cognitive limitations, learning disabilities (Soodak & Podell, 1994), behavioural, motivation and concentration problems and lack of learning skills and self-confidence (Cooper & Burger, 1980). In addition, teachers blame home factors for causing learning problems. These include, for example, problems with upbringing, problems caused by divorces and lack of parental involvement (Soodak & Podell, 1994). Finally, teachers also think of teacher and teaching factors (Silva & Morgado, 2004). Among other things, these refer to an inadequate curriculum or a lack of individual attention for students (Soodak & Podell, 1994) and a heavy work load of teachers (Westwood, 1995).

3 Evidence for unused potential and possibilities for revealing it

Although low expectations about the mathematical potential of low achievers in mathematics do exist, there are also indications that SE students might be able to achieve better in mathematics than is generally assumed. For example, in the Netherlands, the Schools' Inspectorate (2007a) reports that in SE not all student talents are developed and that students' capabilities are not fully acknowledged by their teachers. According to the Schools' Inspectorate (2007b, 55) in SE, "all too often the actual achievement of the student is taken as a reference and not the student's potential development".

Evidence for the unused potential of low achievers came, for example, from a study by Woodward and Brown (2006), who revealed that students who were labelled weak in mathematics accomplished higher results the more the curriculum drew on their research skills. Furthermore, these authors found that including more research activities in the curriculum also had a positive effect on the students' attitudes towards mathematics. Other evidence for unused potential emerged from a study by Bottge and others (2007), who found that students who are weak in mathematics can be successful in mathematical problem solving when hints are available. What this research and other intervention studies (Bottge, 2001; Van Garderen, 2007) have in common is that they show that presenting low achievers mathematical content that goes beyond the commonly taught curricular topics, such as addition and subtraction, can be an effective approach to reveal the students' potential.

In addition to intervention studies, alternative assessment approaches can also be useful to reveal low achievers' mathematical potential; for example, by offering hints and including opportunities to adapt the tasks to the students' ability level (Peltenburg, Van den Heuvel-Panhuizen, and Robitzsch 2010), using worked examples (Van Gog & Rummel, 2010) or read aloud options (Helwig, Rozek-Tedesco, & Tindal, 2002). By these measures, which enhance the accessibility of the assessment tasks, the students' zone of proximal development (Vygotsky, 1978) can be opened. This type of assessment, which differs from using standardised tests, is also known as dynamic assessment (e.g. Campione, 1989). Whereas standardized assessment requires an approach in which students are not allowed to receive any help or use auxiliary resources, dynamic assessment creates an environment that concentrates on gaining insight into the students' potential by investigating whether the student is able to solve a problem with some help.

The idea behind dynamic assessment sharply contrasts with the notion of a fixed ability which originates from early psychometric theories of IQ and intelligence testing (Hart, Dixon, Drummond, & McIntyre, 2004; Poehner, 2008). Fixed ability thinking is based on the assumption that each young person is born with a given amount of intellectual power. Such a static view on student abilities may impose limits on the learning and teaching of students, in particular, the low achievers (Hart et al., 2004). Dynamic assessment researchers reject the idea of inherent ability and are aimed towards disclosing the possibilities of students to reach a higher achievement level than the current one. In other words, dynamic assessment is used to go beyond the actual performance and reveals the students' potential (Campione, 1989; Van den Heuvel-Panhuizen, 1996).

In several studies (e.g. Bosma & Resing, 2012), dynamic assessment has proven to be successful in disclosing the potential of SE students. For example, discrepancies between test scores emerged from research in subtraction with 'crossing the ten' (Peltenburg, Van den Heuvel-Panhuizen, & Robitzsch, 2010), which is generally recognised as a rather difficult topic for weak performing students. This research brought to the fore that SE students performed better on an ICT-based dynamic test, which allowed students to use optional auxiliary tools than when they took a standardised test. Similar to the earlier described intervention studies, it is also possible to include mathematical domains within the assessment that are beyond the regular primary school curriculum, like elementary combinatorics.1 A study about this topic (Peltenburg, Van den Heuvel-Panhuizen, & Robitzsch, 2012) showed that SE students could successfully find all possible combinations of different coloured clothing items. Especially, it was a surprise that the students could solve this combinatorics problem by systematically combining the clothing items even though they had not worked on combinatorics in school before.

4 Research questions

As indicated before, research has shown that teachers' expectations of their students' potential can contribute to better learning outcomes. However, it is unclear whether teacher expectations can also make this expectancy effect occur in SE. Necessary for this is that SE teachers have high expectations of their students' possible achievement levels. With respect to SE students in the Netherlands, at least the Schools' Inspectorate (2007a, 2007b) emphasises that there is unused potential in SE. However, we do not know how Dutch SE teachers think about this. Therefore, we formulated the following research questions:

1. What ideas do SE teachers have about the mathematical abilities of SE students? Do SE teachers agree that there is an unused potential in SE?
2. Have SE teachers experienced the necessity to adapt their expectations about their students' possible development in mathematics? If so, for what reason?
3. What ideas do teachers have about revealing the mathematical potential of SE students?

Based on the studies described above (e.g. Beswick, 2008; Jungbluth, 2003), we assume that the dominant view of SE teachers will be that there is no underutilization of mathematical potential in SE students (*Hypothesis 1*).

Although it could be argued that teachers are pre-eminently in a position to recognize students' potential, one may wonder whether this really happens. Teachers in SE are often overwhelmed by merely achieving the targets of the existing programme (Kraemer, Van der Schoot, & Van Rijn 2009). Therefore, many teachers will probably not try out new topics or new didactical approaches, which have shown evidence of unused talent in a research setting. Nor will there be much opportunity to develop and try out alternative testing methods alongside the required standardised test administration. As a result, there are not many opportunities for students to demonstrate their so far hidden potential. Therefore, we expect that it will be difficult for teachers to refer to experiences that necessitated them to raise their expectations about their students' development in mathematics (*Hypothesis 2*).

Research has shown that it is possible to reveal students' potential by using alternative methods of assessment (e.g. Campione, 1989) or by presenting students mathematical content that is not generally part of the regular curriculum for students who are weak in mathematics (e.g. Bottge, 2001). However, since both approaches are rather remote from daily practice in SE, we do not expect teachers to mention these ideas (*Hypothesis 3*).

5 Method

5.1 Data collection

The teachers involved in this study are all teachers in primary SE. In the Netherlands, this type of education (called 'speciaal basisonderwijs') is organised in separate schools, which are meant for students between 4- and 13-years-old with mild special educational needs. Students with special educational needs do not form a clearly defined group. In general, these students are characterised by a low intelligence (60 < IQ< 85), a low ability of understanding and more than half a year of insufficient progress on two or more of the four main subject areas (technical reading, reading comprehension, mathematics and oral fluency), despite extra help. In addition, regarding their social emotional development, SE students often have concentration or behavioural problems, poor self-confidence or anxiety to fail, which may result in psychosomatic difficulties. Students with more specific needs, for example, students who are blind or deaf, students who have severe mental handicaps and students who have a long-term disease or psychiatric problems are not included in primary SE schools; these students go to other types of SE.

Most of the students in primary SE have started their education in regular primary school. If in such a school the student's special educational needs are difficult to meet, it is usually the initiative of the regular school teacher, in consultation with the school principal, school support service and parents, to refer the student for assessment to a regionally operating assessment team which decides if the student is eligible for primary SE. However, instead of referring students to primary SE, regular primary schools may include these students and obtain a budget for additional support, which is often spent on appointing a so-called internal support coordinator (intern begeleider); in UK terminology called a special educational needs coordinator (Pijl & Van den Bos, 2001). There are no special classes in regular schools, so students are either included in or excluded from regular classes (Van der Veen, Smeets, & Derriks, 2010). Compared to their normal-achieving peers in regular primary education, SE students often have a developmental delay in mathematics of about 1–4 years (Kraemer et al., 2009). Therefore, similar to other school subjects, the mathematics programme in SE is tailored to the cognitive and social emotional needs of SE students and is given by specialised teachers. In addition to their education as a regular primary school teacher, teachers in primary SE have completed one or two years of schooling in the field of education and care for students with special educational needs. In the Netherlands, about 3%, or about 45,000 students of children of primary school age is in a school for primary SE (Central Bureau of Statistics [CBS], 2010).

To answer our research questions, we sent an email containing a link to an online questionnaire to 298 Dutch schools for SE. This number covers 95% of the total number of schools for primary SE in the Netherlands. The email addresses were either obtained from the Central Institute for Finances of the Ministry of Education in the Netherlands (CFI) or the schools' websites. In total, 84 completed questionnaires were returned. Although the response percentages of online questionnaires are known to vary a lot (Cook, Heath, & Thompson, 2000), a response of 28% corresponds with what is usual for digital questionnaires (Sheehan, 2001; Shih & Fan, 2008).

A non-response study was carried out to be sure that participation was not biased by a particular view on the investigated topic. An a-select sample of 20 schools that did not react was approached by telephone and was asked for their reason not responding to the questionnaire. All schools stated lack of time as the reason for non-response.

5.2 Group of respondents

Because most of the email addresses were general addresses, not linked to a person, some of the people who answered were not teachers. The group respondents all worked in primary SE and consisted of 52 teachers and 32 other staff members, such as internal coaches, school leaders and other people working in school. Because teachers are in direct contact with students more often than other school staff, we decided to only include the teachers in our analysis. Most teachers were quite experienced; on average they worked in SE for 16 years (*SD* = 11.2). The average age of the students with whom the teachers worked with varied from 9 years (*SD* = 1.9) to 11.5 years (*SD* = 1.7).

5.3 Questionnaire

The online questionnaire that was developed for this study, started with a number of general questions about the respondent's position in school and his or her work experience. These questions were formulated with predefined answer categories as much as possible. Then, three open questions followed about the respondent's views and ideas about the mathematical potential of SE students (Figure 1). By restricting the amount of open questions and providing plenty of space, our aim was to invite respondents to elaborate on their answers.

In the first of the open questions, the teachers had to react to a particular statement about the idea that there is unused potential in SE. The second question surveyed the respondents' experiences that may have led to adjusting their expectations of SE students' potential. To trigger a reaction from the respondents, this question was illustrated with a classroom experience showing that SE students could solve an elementary combinatorics problem. The third question asked the respondents in what way they think the mathematical potential of SE students can be revealed.

Q #	Topic	Phrasing in questionnaire
1	View on unused potential	Q: What is your opinion about the following statement? *Statement*: **There is unused potential in primary special education**. When it comes to underutilisation of talent, we often think of gifted students. However, unused potential also exists in primary special education. Students in primary special education can achieve more with respect to mathematics than they are generally credited for. `I agree / disagree with this statement, because ...`
2	Indications for adjusting expectations	*An experience from classroom as an introduction to the next question*: In an upper-grade classroom of a school for primary special education, student were handed out a printed sheet containing a number of flags each consisting of three parts. The task presented to the students was to create as many different flags in three different colours. Each colour should only occur once in a flag. The students soon discovered that in order to find all combinations, a systematic approach is needed. Moreover, a few students appeared to be able to create a rule in order to determine how many different flags in total can be made. Q: Did you ever have experiences like the above, which led you to adjust (upwards or downwards) your expectations about the ability of special education students? If yes, please describe your experience briefly.
3	Ideas on revealing unused potential	Q: Do you have ideas about how to determine special education students' potential in mathematics? If yes, please describe your ideas.

Note: By using the cursor in the text box of Q1, the teacher could remove 'agree' or 'disagree' and then could complete the sentence. The text boxes become larger as one writes.

Fig. 1 Open questions in questionnaire (translated from Dutch by authors)

5.4 Data analysis

The answers to the questionnaire were entered into the software programme Atlas ti. To carry out the coding, a classification scheme (see Figure 2) was developed in which eventually all answers of the teachers fit. The scheme was developed in three coding rounds.

As Figure 2 shows, a number of categories were distinguished for each question, which were further specified in subcategories for questions 1 and 3.

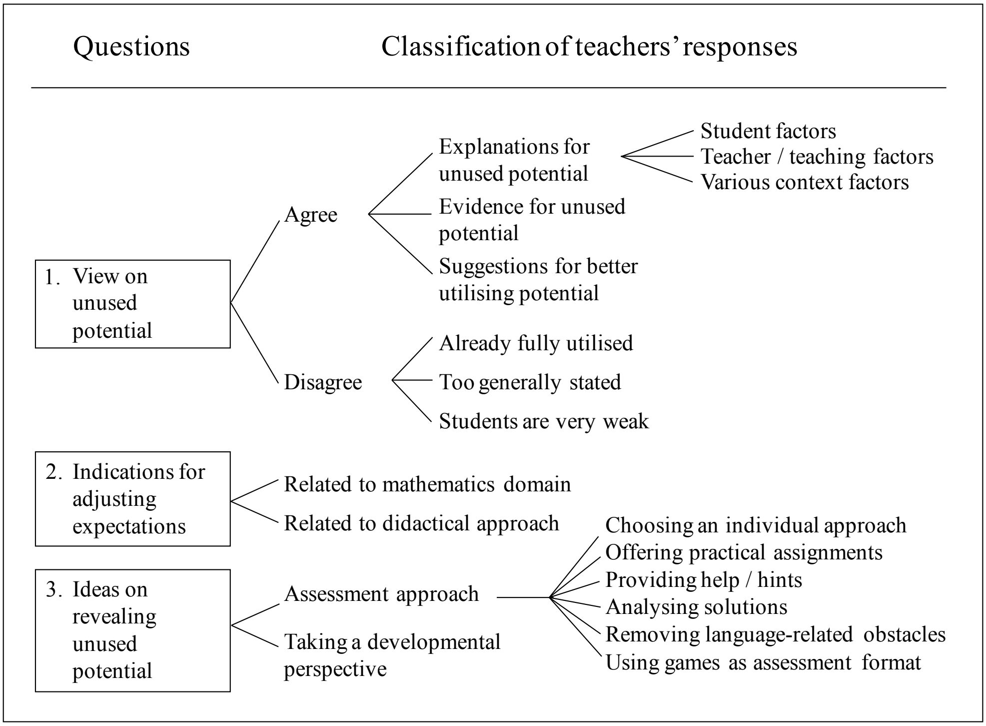

Fig. 2 Classification scheme for the teachers' answers

The final coding was based on the classification scheme and was carried out by the first author and an independent rater. For each of the three open questions a high agreement in coding was found (94% agreement for question 1, *Cohen's* $\kappa = .93$; 96% agreement for question 2, *Cohen's* $\kappa = .96$; 96% agreement in question 3, *Cohen's* $\kappa = .95$).

The answers to question 1 about the view on unused potential fell into two categories. The respondents either agreed with the statement or rejected it. Those who *agreed* did this together with either giving an explanation of why there is unused potential in SE or providing evidence for it based on the respondent's own experiences or offering ideas for better using SE students' potential. The respondents who gave an explanation referred to student factors, teacher/teaching factors or various context factors. Respondents who *disagreed* with the statement offered three different arguments: the students' potential is *already fully utilised*, the statement is *too generally formulated* or *SE students are very weak*.

The answers to question 2 about the respondents' experiences that necessitated adjusting their expectations were either related to a *specific domain of mathematics* or to a *specific didactical approach*.

Two categories were distinguished in the answers to question 3, which deals with determining the potential of SE students. Ideas were either related to *specific assessment approaches* or to *formulating a 'developmental perspective'*. The respondents' views on specific assessment approaches were further specified into the following subcategories: *individual testing with a diagnostic character*, *using practical assignments*, *including auxiliary tools in tests*, *analysing students' solutions*, *avoiding mathematical word problems*, and *using games as a form of assessment*.

6 Results

6.1 Ideas about the existence of unused potential in SE

6.1.1 Agreement on the idea that there is unused mathematical potential in SE
In total, 31 teachers (60%) agreed that there is unused mathematical potential in SE. These teachers clarified their view by giving an explanation for this underutilization (referring to a student, a teacher/teaching or a context factor), by offering some form of proof or by giving a suggestion for better utilising SE students' potential.

Explanations for unused potential (n = 22)
Out of the 22 explanations that were given for the existence of unused potential in SE, only four teachers mentioned explanations that related to student factors (see Table 1). These teachers indicated that behavioural problems, lack of concentration and memory deficits in students are obstacles in developing their potential. One of the teachers explained:

> "Behavioural problems make it impossible to really get the students to do what they are actually able to."[1]

The teacher who referred to memory deficits as a cause for unused potential wrote that SE students find it difficult to remember calculation strategies, which often leaves them stuck on a low level.

Compared to the student factors, more frequently explanations were given related to the teachers themselves. There were seven teachers who mentioned teacher/teaching factors. The one that was mentioned most often was that teachers have low expectations of students. One of the teachers explained it as follows:

> "I think that many SE teachers concentrate on making the child feel safe and comfortable in school, and this is often combined with not imposing high demands."

This explanation is also reflected in another teacher's description:

> "It is generally assumed that students in SE are not capable of all that much. Therefore, teachers are not aimed at high performance achievement of SE students."

Table 1
Types of explanations for existence of unused potential

Explanation referring to	Short description of explanation	*Frequency*	*Total (%)*
Student factors	Behavioural and concentration problems lead to underachievement	3	4 (12%)
	Memory deficits as an obstacle for developing potential	1	
Teacher/teaching factors	Low expectations by teachers	5	7 (30%)
	Lingering too long over a particular topic	2	
Various context factors	Organisational problems	8	11 (58%)
	Poor connection of the text books to students' needs	2	
	Inhibiting role of standardised tests	1	
Total			22 (100%)

Another type of explanation that was given in the teacher-related explanations category was lingering too long over a particular topic. A teacher answered that she tends to let students practice longer on subjects that they have not yet mastered, or mastered imperfectly, resulting in postponing the next step. Another teacher reported that she also sticks to certain topics too long, while students should actually be offered the opportunity to move on.

The most common explanations, mentioned by 11 teachers, referred to various context factors. Of this group, eight teachers mentioned difficulties with classroom organisation. These teachers pointed out differences within the ability levels of students in their class, which were hard to cope with because of lack of time, money and resources. A teacher described that there was not enough time to offer students individual guidance:

> "Many children need more, and more varied, explanations, and need to rehearse with a teacher or assistant. We simply do not have the money for it."

Another context factor that was pointed out by two teachers referred to the quality of the mathematics textbooks. It was said that these do not connect to the knowledge and interests of SE students. Finally, one teacher mentioned the influence of standardised testing as a context factor. According to this teacher, mathematics teaching in SE is holding too much to standardised testing, and this holds back students. She goes on to say:

> "In addition, this prevents mathematics being taught at a higher level using the right aids."

Evidence for the existence of unused potential (n = 7)

Out of the 31 teachers who agreed the statement, seven teachers offered some kind of proof for the existence of unused potential. Of this group, three teachers described their experiences that students were able to do more when they received suitable instruction, either individually or in small groups, or when offered more practical activities. For example, a teacher wrote that SE students can and do move further in the curriculum. She stated:

> "I experienced that by making mathematics more attractive the students get more involved and achieve better results. I have seen how important it is to give students the opportunity to solve a problem on their own."

The other four teachers stated that students in their schools are working on activities that belong to the programme of grades 5 or 6 in regular primary school, activities that are clearly beyond what is generally seen as achievable in SE. A teacher wrote:

> "I have seen that SE students have often learnt many ways to 'survive' and adopt solutions from their environment; they are creative in how they approach a mathematics problem and therefore they can accomplish a lot."

Suggestions to better utilise potential (n = 2)

Out of the 31 teachers who agreed the statement, two teachers suggested ways to better utilise the potential of SE students and raised the matter of changing current mathematics education. According to them, teaching should focus more on the individual students and should be made more challenging, for example, by offering them practical assignments or not solely looking at measurable results, but also at how students cooperate during their mathematics lessons.

6.1.2 Disagreement on the idea that unused potential exists in SE

The 21 teachers (40%) who did not think that there is unused potential in SE, named three reasons for their view. A first argument was that the potential is already fully utilised. This argument was given by 10 teachers. They made it clear that their school already does all that can be done to 'fully achieve the students' potential' including putting more effort into challenging students, using better testing procedures and going through the curriculum faster. A second argument to reject the statement was the nature of the statement. According to nine teachers, the statement was too general. They made it clear that unused potential may exist, but certainly not for all students. The third argument that was put forward by two teachers really implied a rejection of the idea of unused potential in SE. These teachers disagreed with the statement, because according to them SE students are very weak in mathematics, as was expressed by the following reaction:

> "Our students do not have any number sense. They often do not have a clue."

6.2 Experiences that necessitated teachers to raise their expectations

In total, 32 teachers (62%) indicated they had to positively adjust their expectations of the potential of SE students as a result of particular experiences: 19 teachers had positive experiences related to a mathematics domain and 13 teachers with a didactical approach. Of the teachers who did not experience the need to adjust their expectations, 11 stated they did not have such experiences and the other nine teachers left the question unanswered.

6.2.1 Positive experiences related to a mathematics domain (n = 19)

Of the 19 teachers who had positive experiences related to a mathematics domain, seven teachers gave an example related to the domain of combinatorics. These teachers might have been inspired by the example from the questionnaire. On the one hand, mentioning this example may be a form of socially desirable behaviour. On the other hand, this topic is so far removed from the SE mathematics curriculum that it is unlikely that teachers who did not really experience this in their own classroom practice were influenced by this example in the questionnaire. Moreover, these teachers did not only mention the topic of combinatorics, they all illustrated their answer. A teacher explained:

> "Each school year I give my students a combinatorics task […]. For example, I ask them how many different paths there are to the forest […]. Some of them need more help than others. However, there are always a few students who come up with a systematic approach!"

Also, two teachers mentioned positive experiences with fractions and two teachers referred to the use of graphs. For example, a teacher wrote that SE students appeared to be able to collect some data and represent these in a graph. These data handling activities were performed in the context of the *Grote Rekendag* (Big Mathematics Day), a special day in the Netherlands on which primary school students of all grades can work on a set of rich mathematics problems focused on one particular topic in school. The teacher stated that the student work was above her expectations.

In addition, more common mathematics domains were also brought up. The domain of measurement and geometry, including working with shapes and symmetry was mentioned by six teachers. Furthermore, one teacher provided an experience in which a student made a calculation with money and very systematically determined what products could be bought for so many euros. Finally, one teacher mentioned positive results relating to calculation up to 100. To his surprise, a student who was working in the number domain up to 20 could calculate problems up to 100 with the aid of the empty number line.

6.2.2 Positive experiences related to a didactical approach (n = 13)

Of the 13 teachers who had positive experiences related to a didactical approach, four teachers changed their mind on the basis of a specific approach to assessment, i.e. giving students more opportunities by letting them use scrap paper or by allowing them to take more time to do a test. With respect to the latter, a teacher explained his experience as follows:

> "Many students have difficulties with standardised test formats. Therefore, I give them permission to take more time to do the test and to write down their solution method or to make drawings as a support. Taking the test may take some time under these adjusted conditions, but it helps them to pass it."

Additionally, two teachers had expectation-raising experiences with offering their students practical assignments, such as giving change to a customer. According to these teachers, these practical activities challenge the students and connect to their interests, and as a result students are quicker to pick up the topic than when they work in their textbook. Another two teachers also made discoveries about SE students' capabilities by letting students work together. Furthermore, two teachers had positive experiences by offering students games. For example, a teacher stated that she had seen students use surprising solution methods in mathematics games like Rush Hour (a traffic jam puzzle). Finally, two teachers implemented a step-by-step approach in their teaching and one teacher used concrete materials.

6.3 Ideas on revealing the mathematical potential of SE students

The third question asked for ideas about how to reveal the mathematical potential of SE students. In total, 23 teachers (44%) either gave no answer or explicitly stated that they did not have any suggestions. The other 29 teachers (56%) did have ideas about revealing potential. Of this group, six teachers mentioned formulating a 'developmental perspective', which means that teachers set their expectations and goals for each student and 23 teachers gave ideas related to specific assessment approaches.

6.3.1 Assessment ideas (n = 23)
The teachers who suggested specific assessment approaches, showed a diversity of ideas. Table 2 gives an overview of the suggestions that were made.

Table 2
Assessment ideas for revealing SE students' potential.

Assessment approach	*Frequency*	*Percentage*
Individual testing with a diagnostic character	7	30
Using practical assignments	4	17
Including auxiliary tools in tests	4	17
Analysing students' solutions	3	13
Avoiding mathematical word problems	3	13
Deploying games	2	9
Total	23	100

Individual testing with a diagnostic character was mentioned by seven teachers. They referred to 'learning interviews', 'diagnostic interviews' and 'diagnostic teaching'. Moreover, they were clear that focusing on individual students may reveal more information about student potential. What these teachers' responses have in common is the aspect of zooming in on students' thinking and learning processes. For example, a teacher stated:

> "As a starting point, one can use the tests belonging to the mathematics textbook series or the monitoring tests. In addition, it is important to observe the students' solution method, which gives better insight in his or her true potential."

The use of practical assignments was emphasised by four teachers. They expected that SE students would have better opportunities to show their abilities if assessment includes more so-called 'do-assignments', like paying in a shop or planning a journey by train. Such a practical approach also emerged earlier in the teachers' responses to the questions 1 and 2. The idea of including auxiliary tools in tests was given by four teachers. They mentioned providing hints, allowing students to use multiplication cards and calculators. Within this category, the use of so-called 'learnability tests' was mentioned as well. The teacher who used this terminology explained that students should be offered worked examples as a support to solve problems on their own. Then, there were three teachers who suggested not just looking at students' test scores, but also to analyse their ways of solving the problems. These teachers argued for a qualitative analysis of student work, allowing a better insight into their strong points. Furthermore, three other teachers saw opportunities to establish the mathematical potential of SE students by avoiding word problems in assessment. These teachers made it clear that they see the low technical reading level of SE students as an obstacle for successfully solving wordy problems. The solution offered by these teachers was to offer short, clearly formulated problems and reading aloud the problems. Finally, two teachers suggested assessing students by using games, since, according to them formal assessment does not always give good insight into students' abilities. For example, a teacher illustrated her answer as follows:

> "Many students get anxious during a test. As long as they do not realise it is a test, they are fine, for example by using a game format. Besides, they like games a lot, and I think they can learn a lot from them as well."

6.4 Conclusions and discussion

In this study, we explored perceptions among SE teachers about the mathematical potential of SE students. We also investigated what ideas SE teachers have about establishing the potential of students and the experiences they have with this.

An initial conclusion that can be drawn from the teachers' answers is that, contrary to our expectations (see *Hypothesis 1*), many SE teachers believed that there is unused mathematical potential in SE students: 60% agreed with this statement and 40% disagreed. To correctly interpret this result, we must take into account that about half of the teachers who rejected the statement did so because, according to them, in their schools student potential is already being fully utilised. They provided examples and, similar to the teachers who agreed with the statement, they were able to describe experiences that made them raise their expectations.

Another remarkable result is that the teachers were rather aware of their own contribution to the existence of unused potential. In particular, having low expectations by teachers was explicitly mentioned as an inhibiting factor for the development of student potential. The latter is indeed in agreement with research outcomes (Jungbluth, 2003), but it was rather a surprise to find that the SE teachers were aware of their own role in this process. Regarding context factors as an explanation for unused potential, problems with classroom organisation were most frequently mentioned as a context factor that hinders the developmental possibilities of SE students.

The robustness of our findings on the teachers' views on unused mathematical potential in SE students was evidenced by the number of different arguments teachers gave from their own teaching practice, in particular the many examples of experiences that caused teachers to raise their expectations about the potential of SE students. An important point that we can infer from this is that these teachers are probably likely to notice (unexpected) developmental possibilities or discoveries by SE students.

In contradiction to our expectations (see *Hypothesis 2*), a large number of teachers (62%) described experiences that opened their eyes to the developmental potential of SE students. On the one hand, these experiences related to mathematical domains that are characteristic for the curriculum of the upper part of primary school or to activities that can be described as 'problem solving' tasks. On the other hand, the experiences emerged from specific didactical approaches, particularly from a practically oriented approach to mathematics teaching and alternative ways of assessment.

Although there were fewer teachers (56%) who had ideas about how to establish the potential of SE students, we had not expected (see *Hypothesis 3*) that many of these ideas would stress the importance of setting expectations and goals for students (cf. Poppes et al. 2002), and present approaches that resemble forms of dynamic assessment.

To conclude, the present study revealed three striking outcomes:
- Many SE teachers believe that there is unused mathematical potential in SE students.
- SE teachers can give empirical support from their own teaching practice for the idea that SE students are more able than it is generally assumed.
- SE teachers have clear ideas about how to reveal the potential of SE students which largely reflect a preference for new assessment approaches.

These results are quite surprising, especially when considering the fact that the teachers in our study worked with students who are diagnosed and labelled as SE students and attend separate schools for students with special educational needs. Apparently, the SE teachers were able to look beyond the SE label and keep an open mind for their students' mathematical abilities. It is uncertain, however, how teachers in regular education who work with weak performing students in mathematics consider their students' mathematical potential. Investigating possible differences in views and expectations between teachers who work with students labeled as SE students and teachers in regular education who work with students who do not explicitly carry this label would be relevant for further research.

Because the results of this study are based on the responses of teachers who volunteered to fill in the online questionnaire, the sample of teachers might not be representative for all SE teachers in the Netherlands. Moreover, the sample of teachers was rather small. Also, we did not have information on the specific (mathematical) learning difficulties of the students that the teachers worked with. Finally, the study did not take into account personal details of the teachers, such as their age, sex, attended professional development and their approach to teaching mathematics. The above mentioned limitations warrant to be cautious in interpreting our findings. They cannot be generalised to the entire population of SE teachers in the Netherlands and the same is true for teachers in other countries. Another possible shortcoming of our study is the online format of questioning, which did not give us the opportunity to ask additional questions that would have given us a more in-depth understanding of the teachers' thinking about the mathematical abilities of SE students. Despite the restricted format, the teachers were very rich in their answers and gave us a good picture of their ideas, including helpful assessment approaches. This richness of the examples the teachers gave to explain their views can be considered as an indication of the reliability of our findings. The teachers did not just say 'I agree' or 'I disagree', but based on their answer from their own arguments and experiences.

The most important message from this study is that SE teachers are rather positive about SE students' potential, and do not shy away from stretching their own limits (in trying new didactical approaches) and those of their students (in trying new mathematical topics). We consider it as our task to help them to achieve this. An initial step in this direction is to provide SE teachers with assessment instruments that help to reveal their students' potential. Moreover, on the basis of our findings, we suggest that professional development programmes designed for teachers of SE students in mathematics should make use of teachers' positive perceptions based on experiences from their own classroom practice. Considering that teachers do have concrete ideas about disclosing their students' potential, we think that discussing these ideas with and among teachers would be a fruitful avenue to bring them into practice.

Notes

1. Combinatorics is the domain of mathematics that involves systematic listing and counting and is based on the so-called 'fundamental counting principle'. This principle describes how to determine the total possible choices when combining groups of items. If you can choose one item from a group of a choices, and another from a group of b choices, then the total number of two-item choices is $a \times b$.
2. The teacher quotes used in the text were translated from Dutch to English by the authors of this paper. In addition, a translator checked the authors' translations, and made corrections if necessary.

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Chapter 8

Conclusion

Conclusion

1 Gains from the study

The aim of this PhD study was to gain insight in SE students' mathematical potential and in how teachers in SE think of their students' potential. In order to investigate SE students' potential we applied two different assessment approaches. One approach is making a difficult topic, i.e., subtraction up to 100, more accessible for SE students. The other assessment approach is characterized by presenting mathematical content that is beyond the regular primary school curriculum in SE i.e., elementary combinatorics. In the study, we elaborated on both these assessment approaches. In addition, we made an inventory of teacher perceptions of SE students' mathematical potential.

1.1 Making a difficult topic more accessible

1.1.1 ICT-based dynamic assessment of subtraction up to 100 with crossing the ten

To increase the accessibility of subtraction problems up to 100 that require 'crossing the ten' we developed an assessment in which subtraction items taken from a standardized test were placed in an ICT environment. The digitized items were enriched with an optional auxiliary tool that students could use for solving the problems. To find out whether SE students could benefit from using an auxiliary tool in solving subtraction problems, two studies were carried out each focusing on a particular auxiliary tool for the same set of subtraction problems. Data were collected with the test items in the ICT version and the standardized version. In total, 79 SE students with a mathematical ability level of end grade 2 participated.

From the experiences and results in the two studies, we concluded that an ICT-based assessment including an optional auxiliary tool provides SE students with more opportunities to show their mathematical abilities in solving subtraction problems compared to a standardized assessment. In both studies, the students attained a significantly higher percentage of correct answers in the ICT version of the subtraction items than in the standardized test items. This result was found even though the ICT-based test format preceded the standardized test format. Thus, any gain from retest effect would have been in favor of the standardized test score. Another result related to the students' use of the aid tool. As the students had to decide for each problem whether or not to use the tool, they were stimulated to think about their learning process and their understanding of the subtraction problems included in the test. Thus, the availability of the optional auxiliary tool could elicit reflection from the students on their calculation skills, and consequently effect that they would become more active in their learning. In short, such a tool can make the assessment learning oriented (Wiliam, 2008). A surprising finding was that most students were indeed quite capable of judging their mathematical proficiency and were therefore able to decide when they could benefit from tool use. In other words, choice of tool use was based upon a well-considered understanding of one's own competences. This result contrasts with the lack of meta-cognitive skills that is generally attributed to low performing students in mathematics (e.g., Desoete, Roeyers, & Buysse, 2001; Montague & Applegate, 1993), and can be viewed as another clue to the hidden potential of SE students.

To conclude, an ICT-based assessment environment that includes an optional auxiliary tool which students can activate on demand can help them to overcome obstacles in solving calculation problems. Furthermore, our results showed that such an ICT environment can help to examine students' solution process in much detail.

1.1.2 Influence of particular features of the ICT-based assessment

In a secondary analysis we aimed to learn more about the influence of particular features of the ICT-based assessment environment on the results that we found in the two studies. Therefore, we subjected significant features of the ICT-based assessment to a secondary analysis from a cognitive load perspective. Next to the mathematical auxiliary tools, these features included a read aloud function and a flexible answer field.

With respect to the read aloud function no evidence could be found for a redundancy effect, but the results made us aware of the fact that this effect may behave differently in SE students who lack particular abilities than in regular education students. Apparently, for SE students a possible negative (redundancy) effect of being presented the same information in different modes is cancelled out by the positive effect of hearing spoken text that compensates for students' low reading ability. Concerning the flexible answer field we found that in about a fifth of the total cases students had made use of its flexibility, which may indicate that this feature helps students to reduce working memory load when writing down two-digit numbers. Finally, regarding the mathematical auxiliary tools it was found that such tools do not solely provide memory aids, but may also offer support to students in sequencing their solution steps. However, as we missed a direct measurement of students' cognitive load these results should be handled with care.

We concluded that design and research based on a cognitive load theory perspective could be helpful for improving SE students' learning and assessment environments.

1.1.3 Influence of item characteristics on students' procedure use

In yet another study we investigated whether SE students are able to apply the indirect addition procedure instead of the standard direct subtraction procedure for solving subtraction problems up to 100. Therefore, we developed a new set of ICT-based test items in which we systematically varied the item characteristics. These characteristics included both number and format characteristics. By incorporating a range of item characteristics we aimed to find out under which conditions SE students are inclined to apply indirect addition instead of direct subtraction. As in the previous studies, special attention went to the accessibility of the test items, e.g., formulating the text as simply as possible and reading the text out loud by the computer. In total, 56 students with a mathematical level of end grade 2 participated in the study.

The findings of the study showed that SE students can indeed make use of indirect addition when solving subtraction problems. With respect to the numbers involved in the problems, we found that students mainly used indirect addition in small-difference problems with crossing the ten. With respect to the problem format, students most frequently applied indirect addition in context problems that reflect adding on. The students did not apply indirect addition more often after receiving instruction in this procedure. Regarding the students' success rate, it was found that the students solved more subtraction problems correctly when using indirect addition instead of direct subtraction. Furthermore, solving the test items by applying indirect addition together with stringing appeared to be more successful than applying direct subtraction together with splitting. This finding emphasized the importance of examining procedures (indirect addition use or direct subtraction use) as well as strategies (splitting, stringing, and varying) when investigating students' ability to solve number problems.

In sum, the study revealed that SE students (1) are able to use the indirect addition procedure spontaneously; (2) are flexible in applying the indirect addition procedure; and (3) are rather successful when solving subtraction problems by indirect addition. These conclusions made it clear that – apart from the accessibility of test items – their sensitivity needs to be high as well in order to reveal SE students' flexible procedure use. In our study, this sensitivity is reflected in the design of test items with particular format and number characteristics that enabled detecting SE students' ability to apply the indirect addition procedure.

1.2 Presenting mathematical content beyond the regular curriculum

Instead of increasing the accessibility of a difficult topic like subtraction, a different approach was applied as well to reveal SE students' potential. The latter approach involved the assessment of a mathematical topic outside the regular primary school mathematics curriculum, i.e., elementary combinatorics. Similar to the previous studies, we developed an ICT-based assessment environment for the current study. This ICT-based test contained six items on elementary combinatorics. By presenting combinatorics problems in a familiar context and using a dynamic ICT-based assessment environment we aimed to give SE students the opportunity to show their problem solving skills in this domain. A total of 84 students from five SE schools and 76 students from five regular education (RE) schools participated in the study. For each participating school, four students were randomly chosen who scored near the 50th percentile on the CITO LOVS Mathematics tests (Janssen, Scheltens, & Kraemer, 2005-2008) for M2, M3, M4, and M5 (the mid-grade tests for respectively grades 2 to 5).

The results of the study showed that the SE students are able to solve combinatorics problems equally successfully as the RE students. We also found that a significant and similar growth in success rates occurred in both school types for increasing mathematical levels. With respect to the use of strategies, it appeared that on average the SE students applied a systematic strategy equally often as the RE students and that a significant increase in the use of systematic strategies occurred in both school types. Additionally, we found that the SE students showed more growth in their use of systematic strategies across mathematical levels M2 to M5 by SE compared to the RE students. Another outcome was that both the SE and RE students very much enjoyed reasoning about possible solutions for the combinatorics problems. A final observation was that all participating students used the ICT-based assessment environment in a natural and self-evident way, with no difficulties in operating with the digital manipulatives.

This study in the domain of combinatorics revealed that a dynamic ICT-based assessment can offer suitable opportunities in disclosing SE students' hidden mathematical potential. Apparently, SE students have a higher mathematical potential than is generally assumed as they appear to be able to successfully work on more advanced topics, such as elementary combinatorics, which are generally considered out of reach of SE students.

1.3 Teacher perceptions of SE students' mathematical potential

In a final study, it was investigated how teachers in SE think about their students' mathematical potential. To collect data about teacher perceptions we sent an email containing a link to an online questionnaire to 298 Dutch schools for SE (covering 95% of the total number of schools for primary SE in the Netherlands). The group respondents all worked in primary SE and consisted of 52 teachers and 32 other staff members. Because teachers are in direct contact with students more often than other school staff, only the teachers were included in the analysis.

The questionnaire contained three open questions. The first question asked the teachers to give their opinion about the existence of unused mathematical potential in SE. It was found that the majority of SE teachers believed that there is unused mathematical potential in SE students. In addition, about half of the teachers who rejected the statement did so because, according to them, in their schools student potential is already being fully utilized. They provided examples and, similar to the teachers who agreed with the statement, they were able to describe experiences that made them raise their expectations. The second question surveyed the respondents' experiences that may have led to adjusting their expectations of SE students' potential. In answer to this question, more than half of the teachers described experiences that opened their eyes to the developmental potential of SE students. On the one hand, these experiences related to mathematical domains that are characteristic for the curriculum of the upper part of primary school or to activities that can be described as 'problem solving' tasks. On the other hand, the experiences emerged from specific didactical approaches, particularly from a practically oriented approach to mathematics teaching and alternative ways of assessment. Finally, the third question asked the respondents how they think that the mathematical potential of SE students can be revealed. Again, more than half of the teachers appeared to have ideas about how to establish the potential of SE students. Many of these ideas stress the importance of setting expectations and goals for students, and present approaches that resemble forms of dynamic assessment.

We concluded that (1) many SE teachers believe that there is unused mathematical potential in SE students, (2) that the teachers can give empirical support from their own teaching practice for the idea that SE students are more able than it is generally assumed, and (3) that the teachers have clear ideas about how to reveal the potential of SE students, which largely reflect a preference for new assessment approaches.

2 Suggestions for further research

This PhD thesis describes a series of small-scale assessment experiments investigating SE students' mathematical potential. To attain a more thorough understanding of SE students' potential, it is necessary to extend our study to other grade levels and larger groups of students. Moreover, further investigations are required to refine our findings with other types of students. For example, Johnson and Green (2006) have shown that the use of on-screen mathematical tools may also help primary school students who are in regular education.

In our experiments we focused solely on one topic within the existing school curriculum, namely subtraction up to 100, and we explored only one topic outside the curriculum, namely combinatorics. Therefore, our conclusions are restricted to these two mathematical domains. The investigation of other mathematical domains, like fractions, might be a promising avenue to gain new insight into SE students' potential. The domain of fractions is generally known as difficult and is often postponed or not addressed at all in the SE curriculum (Kraemer, Van der Schoot, & Van Rijn, 2009). However, small scale lesson experiments within the *Speciaal Rekenen* project indicated that SE students' everyday experiences can be a solid starting point for building the required knowledge and language for reasoning about fractions (Abels, Goderie, Verbruggen, & Verschure, 2009).

In the assessment experiments in this PhD study the focus was on individual students' performance and solution process within an ICT assessment environment. It is crucial, however, to also pay attention to the complexities and challenges that arise when a computer assessment environment is used in a classroom setting (Kolovou, 2011). In particular, it is important to examine the usability and accessibility of newly available assessment instruments and data for educational practice.

A final suggestion in continuation of this thesis addresses teacher perceptions of their students' potential. In our study, we made a first inventory of how teachers in SE think of their students' potential. These perceptions could be further examined, for example, by carrying out in-depth interviews which allow asking further questions and which give teachers the opportunity to elaborate on their answers. Moreover, it is our intention to continue our research by investigating whether an ICT-based dynamic assessment approach can provide teachers with renewed insight in their students' mathematical potential. Next, our aim is to examine whether a possible positive change in perceptions could cause a change in student achievement.

3 How can we better recognize and utilize SE students' mathematical potential?

There is a human and societal obligation to take responsibility for students' competencies and stimulate the attainment of their full development. Not only is this utilization of human capital necessary for the economic performance of countries (e.g., Webbink, 2005), it is also a fundamental human right for students personally to receive 'good education' that furthers their academic, professional and social development (e.g., UNICEF/UNESCO, 2007). In the Netherlands, both the Onderwijsraad [Advisory Council for Education] (2007) and the Inspectie van het Onderwijs [Schools Inspectorate] (2007a) emphasize the importance of recognizing students' talent and not leaving it unused. According to the Inspectie van het Onderwijs (2007b) the latter is particularly relevant to SE.

In line with current political aims to better recognize and utilize the qualities of *all* students, the conclusions of our study indicate that the climate for raising mathematical performance in SE may be favorable.

The previous raises the following question:

> *In what ways could the gains of the current study contribute to helping SE students achieve their full mathematical potential?*

The study described in this thesis has shown that an alternative assessment format can shed new light on SE students' mathematical abilities. By increasing the accessibility of a difficult topic like subtraction or exploring mathematical content beyond the regular curriculum, we could disclose the mathematical potential of SE students. A next step would be to give teachers in SE the opportunity to work with assessment environments like the ones developed in this study. However, such an implementation requires more than just making available new assessment instruments.

First, we should keep in mind that within the current political agenda of improving primary school students' performance ('*Kwaliteitsagenda PO*') the use of norm-referenced tests is considered a key success factor (see e.g., Janssens, 2008). However, one major shortcoming of these 'static' tests is that they do not provide information about the students' *zone of proximal development*, and hence do not offer specific instructions for dealing with students who have learning difficulties (Campione, 1989; Van den Heuvel-Panhuizen, 1996). Consequently, a strong emphasis on the use of standardized tests and related accountability policies, like in the USA (see e.g., Venn, 2012), may have negative effects on curriculum, instruction, the percentage of students excluded from the tests, and student dropout rates. (e.g., Hong & Youngs, 2008; De Wolf & Janssens, 2007). Thus, to avoid these harmful effects and really work on the improvement of student performance in SE, a broader view towards the meaning and role of assessment is required.

Second, implementing new assessment instruments to better recognize and utilize SE students' potential is a true educational change, which, like any other educational innovation, requires sustained support for those involved in trying out new practices and learning new theory. More specifically, such a change asks for a real shift in both thinking and operating towards what students *may* be able to accomplish – rather than what they *cannot*.

Based on the above, an implication for policy makers and curriculum developers, indirectly derived from the studies carried out in the context of this thesis, is to broaden the dominant view on assessment of SE students and to provide their teachers with assessment instruments and accompanying in-service training to help them gain more insight in their students' potential. Supporting teachers with the use of assessment formats that help them to disclose their students' mathematical potential, could provide SE students better chances for school success.

Apart from possible consequences for current assessment practice in SE, the gains of this study also cause some reflection on mathematics education in SE. Although the latter was not the topic of the study, the assessment outcomes that we obtained do indicate that the mathematics curriculum in SE and its teaching methods may not sufficiently mirror SE students' potential. We will elaborate on this statement by referring to the study on SE students' use of the indirect addition procedure and the study on combinatorics.

When investigating SE students' use of indirect addition, most teachers involved appeared to have restricted their teaching of subtraction to the straightforward direct subtraction procedure. Based on our research findings, we think that teaching solely taking away underestimates SE mathematical abilities and does not offer the best environment for these students to develop a deep understanding of different subtraction approaches. In line with Freudenthal (1983) we think that both interpretations of subtraction – subtraction as taking away and as determining the difference – need to be addressed if we want students to learn subtraction in a more complete way. To support this two-way traffic of the direct subtraction and indirect addition procedure the model of the number line is particularly suitable (Van den Heuvel-Panhuizen & Treffers, 2009), as was confirmed in our study with the digital empty number line as an auxiliary tool.

Apart from establishing a better understanding of subtraction, another reason for paying more attention to the indirect addition procedure in SE is to confirm to students that this is a correct procedure for solving subtraction problems. During our assessment sessions it happened more than once that students verbally reported how they applied the indirect addition procedure by making comments like “Actually I cheated a bit, because instead of taking away this number from that number, I added on by starting at this number.” Comments like these illustrate that SE students may not be completely comfortable with applying a procedure that differs from the one taught in school. It is important that students realize that indirect addition is a legitimate procedure for subtraction and that, in some number problems, applying this procedure might even be more efficient than taking away.

Further, although our results indicated that students in SE haven already proven to be quite proficient in applying the indirect addition procedure, we think that giving students the opportunity to learn more about this procedure in their mathematics lessons might enable them to achieve even better results or develop their potential with respect to subtraction in a more extensive way. To achieve this, it is important that teachers realize that they have the power to cleverly choose particular problems. By intentionally including problems with specific characteristics, teachers can steer the learning process of their students. In the context of the study on students’ procedure use this meant including particular number and format characteristics in subtraction problems which can prompt students to use the indirect addition procedure.

When investigating how students in SE would react to elementary combinatorics problems it was found that – in contrast to their generally assumed limited abilities – the students involved were quite able to solve these problems. This finding illustrates that the topic of combinatorics is undeservedly omitted from the mathematics curriculum in SE. In our view, enriching the curriculum with combinatorics problems can promote SE students’ problem solving skills and enable them to discover important mathematical principles. Moreover, by presenting problems with a problem solving character, SE students’ possible limited perception of mathematics as only involving arithmetic could be extended.

The previous mentioned suggestions for opening up the ways in which SE students are assessed and taught mathematics are intended to give SE students better opportunities to realize their mathematical potential. It is our job as educational researchers, developers and teachers to make sure that students in SE receive the right opportunities to fully develop this potential.

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Summary

Samenvatting

Acknowledgements

Curriculum vitae

Overview of publications related to this thesis

Overview of presentations related to this thesis

FIsme Scientific Library

ICO Dissertation Series

Appendix Test items

Summary

The aim of this PhD study was to investigate SE students' mathematical potential. Therefore, we developed and tried out new assessment approaches that are characterized by a certain flexibility and sensibility that make it possible to explore students' potential. An important reason for investigating new assessment approaches is that they can function as an eye opener for students' mathematical abilities. This is particularly relevant for students in SE, who are often behind in their mathematical development compared to peers in regular education, and for whom new assessment experiences could serve as an impulse for raising expectations on what they can achieve.

The study addressed three main questions:

1. How can we reveal SE students' mathematical potential in solving subtraction problems up to 100?

2. How can we reveal SE students' mathematical potential in solving elementary combinatorics problems?

3. How do teachers in SE think about their students' mathematical potential?

To find an answer to these main questions a series of studies was carried out and reported in chapters 2 to 7 of this thesis. First, we developed an ICT-based assessment environment containing optional auxiliary tools that SE students could use for solving subtraction problems. Both students' performance and solution process in the ICT environment were analyzed in relation to their tool use. Also, a secondary analysis of the data was carried out in which we adopted a cognitive load perspective. Next, a series of subtraction problems within an ICT environment was developed in order to asses SE students' ability to use the indirect addition procedure instead of the standard direct subtraction procedure. The following step was the development of yet another ICT assessment environment which contained a series of elementary combinatorics problems. Again, SE students' success rate and strategies were investigated, this time when dealing with a mathematical topic that was completely new to them. Finally, an online questionnaire was developed to collect data about how teachers view SE students' mathematical potential.

Chapters 2 and 3 report on two sub studies that investigated SE students' mathematical potential by means of an ICT-based assessment environment. The focus is on a topic that is generally recognized as rather difficult for weak performing students in mathematics: subtraction problems with 'crossing the ten'. These are problems in which the value of the ones-digit of the subtrahend is larger than that of the minuend (e.g., 62 – 58). A frequently made mistake in these problems is reversing the digits (in this case, subtracting 2 from 8 instead of 8 from 2). The students involved in the studies were 8 to 12 years old. They attended a school for SE. Their mathematical level was one to four years behind the level of their peer group in mainstream schools.

In SE, teachers often use standardized written tests to assess their students' mathematical understanding and computational skills. These tests do not allow the use of auxiliary resources and can therefore not provide information on what resources may help the students in solving the problems. To gather this information, an assessment instrument was developed and used in which seven subtraction items taken from a standardized test were placed in an ICT environment. The digitized items were enriched with an optional auxiliary tool that students could use for solving the problems. In the two sub studies described in chapters 2 and 3 it was investigated whether the ICT-based test items can reveal more about the students' potential with respect to subtraction than the standardized test items. The two studies each focus on a particular auxiliary tool for the same set of subtraction problems. These tools are based on the two main didactical models that can support calculation up to 100: a group model (such a the 100-board) and a line model (such as the empty number line). By providing students with a tool based on these models, we wanted to find out how the mathematical proficiency of SE students can be revealed. In both studies data were collected with the test items in the ICT version and in the standardized version.

In the one sub study, the ICT-based assessment environment includes a dynamic visual tool that provided students with digital manipulatives, consisting of counters that can be placed on a 100-board with a 10 by 10 grid. The students can select a number of counters, which are next to the 100-board, drag them to the board, and rearrange or remove them. We expected that through an on-screen visual representation of the operation to be carried out, the students would be less inclined to process the ones-digits in the reverse way. For example, in the case of 62 – 58, the tool can provide a visual prompt to find a solution for subtracting 8 from 2 by opening up the next ten. In total, 37 students from two schools for SE participated in this sub study.

In the other sub study, instead of the 100-board an empty number line was included in the ICT environment. This tool operates by touch-screen technology. The empty number line consists of a horizontal line, on which the students can put markers and add number symbols, and on which they can carry out operations by drawing jumps backward or forward. The tool can help the students in solving subtraction problems with borrowing, since it can function as an aid to order the numbers involved in the problems and carry out the necessary operations. Moreover, when working on the number line the students can visually keep track of their actions. For example, in the case of 62 – 58, by putting the numbers 62 and 58 on the number line it can become clear that these numbers are actually quite close to each other in the number sequence. This understanding can trigger the students to bridge the difference. Whether the students activated the tool and how they used it – e.g., the jumps they draw on it – were both considered as an indication of their level of competence in subtracting with crossing the ten. In this sub study, 43 students from two other schools for SE participated.

In both sub studies the students' scores in the ICT version of the test items were compared with their scores on the standardized written version of the test items. The results of both sub studies showed that the students could benefit from using an auxiliary tool. Consequently, the ICT-based dynamic assessment could help to reveal weak students' potential in subtraction with crossing the ten. In addition, the studies disclosed that in the manipulatives study there was a clear preference for direct subtraction procedures (taking away), whereas in the number line study indirect addition or subtraction procedures (e.g. adding on) were frequently used as well.

Chapter 4 examines further the data from the two previously described studies which revealed that SE students showed a higher performance in mathematics in an ICT-based dynamic assessment including auxiliary features than in the regular standardized paper-and-pencil test. In the study described in this chapter we tried to find an a posteriori explanation for this finding by adopting a cognitive load theory perspective. We conducted a secondary analysis in which we related the features of the ICT-based assessment to the responses of the students. The results of this analysis were varied. For some features we found evidence that they might have influenced the students' responses, for other features this was not clear. Nevertheless, the analysis gave us a better understanding of how we can develop assessment tools that offer weak performing students in mathematics opportunities to show what they are capable of.

Chapter 5 describes the study in which we examined SE students' use of indirect addition (subtraction by adding on) for solving two-digit subtraction problems. Fifty-six students (8- to 12-year-olds), with a mathematical level of end grade 2, participated in the study. They were given an ICT-based test on subtraction with different types of problems. Although most students had not been taught indirect addition for solving subtraction problems, they frequently applied this procedure spontaneously. The item characteristics were the main prompt for using indirect addition. With respect to the numbers involved in the problems, students mainly used indirect addition in small-difference problems with crossing the ten. With respect to the problem format, students most frequently applied indirect addition in context problems that reflect adding on. Furthermore, indirect addition was identified as a highly successful procedure for SE students, and the best predictor of a correct answer was found in combination with a stringing strategy.

Chapter 6 reports on a study aimed at revealing SE students' mathematical potential by means of a dynamic ICT-based assessment including a drag-and-drop facility and drawing tool. The study focused on the domain of elementary combinatorics, which is generally not taught in primary special education. Six combinatorics problems on finding all possible combinations of a number of different types of clothing items were presented on screen. Data were collected on students' success rate and strategy use in solving these items. The success rate and strategy use of students in regular education served as a reference. The total sample consisted of 84 students (8- to 13-year-olds) from SE and 76 students (7- to 11-year-olds) from regular education (RE). The students' mathematics ability ranged from halfway grade 2 to halfway grade 5. The data analysis showed that both with respect to success rate and strategy use SE students did not differ significantly from RE students. These findings suggest a hidden mathematical potential in SE students.

Chapter 7 describes the study on teacher perceptions of SE students' potential in mathematics. Research has shown that high expectations of teachers about their students' academic development have a positive influence on how these students actually develop. Therefore, when aiming to improve students' learning results it is essential to know how teachers think about their students' abilities. The study described in this chapter was meant to investigate what perceptions primary school teachers in SE have of their students' potential in mathematics and what possibilities they see to reveal this potential. Data were collected through an online questionnaire. Surprisingly, the responses showed that, although the teachers teach students with low achievement scores in mathematics, most of the teachers were positive about the mathematical potential of their students. The teachers often attributed unused potential to causes outside the student and they underpinned this view with observations from school practice. The most important message from this study is that SE teachers are rather positive about SE students' potential, and do not shy away from stretching their own limits (in trying new didactical approaches) and those of their students (in trying new mathematical topics). The chapter concludes with discussing the consequences of these findings for the professional development of teachers in SE.

Chapter 8 gives an overview of the conclusions of this PhD study and offers suggestions for further research and assessment practice in SE. The major conclusion of this study is that alternative assessment approaches may help to reveal SE students' mathematical potential. In answer to the first main question we found that (a) enriching assessment by incorporating an optional auxiliary tool in an ICT environment helps to reveal SE students' potential on a difficult topic like subtraction up to 100 with crossing the ten, and that (b) designing test items with particular item characteristics can disclose SE students' flexible solution methods when solving subtraction problems up to 100. In answer to the second main question we found that presenting elementary combinatorics problems in a dynamic ICT environment can uncover SE students' problem solving abilities. Finally, with respect to the third main question, an inventory of teacher perceptions indicated that SE teachers believe that there is unused mathematical potential in SE students. These conclusions indicate that – in line with the aims of current educational policy – the climate for raising mathematical performance in SE may be advantageous. Some recommendations are provided for better recognizing and utilizing SE students' mathematical potential.

Samenvatting

Het doel van het onderhavige promotieonderzoek was het onderzoeken van het reken-wiskundig potentieel van speciaal basisonderwijs (sbo) leerlingen. Daartoe hebben we nieuwe toetsaanpakken ontwikkeld en uitgeprobeerd welke gekenmerkt worden door een zekere flexibiliteit en gevoeligheid die het mogelijk maken het potentieel van leerlingen af te tasten. Een belangrijk argument voor het onderzoeken van nieuwe toetsvormen is dat ze als eyeopener kunnen dienen voor de reken-wiskundige mogelijkheden van leerlingen. Dit is vooral van belang voor sbo-leerlingen, die vaak achter zijn in hun reken-wiskundige ontwikkeling ten opzichte van leeftijdgenoten in het regulier basisonderwijs en voor wie nieuwe toetservaringen een impuls kunnen zijn om verwachtingen over wat ze kunnen positief te doen bijstellen.

In het onderzoek komen drie hoofdvragen aan de orde:

1. *Hoe kan het reken-wiskundig potentieel van sbo-leerlingen zichtbaar worden gemaakt op het gebied van het aftrekken tot 100?*
2. *Hoe kan het reken-wiskundig potentieel van sbo-leerlingn zichtbaar worden gemaakt op het gebied van elementaire combinatoriek?*
3. *Hoe denken leerkrachten in het sbo over het reken-wiskundig potentieel van hun leerlingen?*

Om een antwoord op deze vragen te vinden is een serie deelstudies uitgevoerd, waarvan achtereenvolgens verslag is gedaan in de hoofdstukken 2 tot en met 7 van dit proefschrift. Allereerst hebben we een ICT-toetsomgeving ontwikkeld met optionele hulptools die leerlingen konden inzetten tijdens het oplossen van aftrekopgaven. Zowel de prestaties als de oplossingsaanpakken van een groep sbo-leerlingen binnen de toetsomgeving zijn geanalyseerd in relatie tot hun toolgebruik. Ook is een secundaire analyse uitgevoerd van de gevonden data vanuit de cognitieve belastingstheorie. In een volgende deelstudie hebben we een serie aftrekopgaven in een ICT-toetsomgeving ontwikkeld waarmee we de bekwaamheid van sbo-leerlingen om, in plaats van de standaard afhaalprocedure, de aanvulprocedure te gebruiken. De volgende stap in het onderzoek was het ontwikkelen van een ICT-toetsomgeving waarin een serie elementaire combinatoriekopgaven is opgenomen. Opnieuw werden de prestaties en oplossingsaanpakken van sbo-leerlingen onderzocht, maar deze keer met betrekking tot een reken-wiskundig onderwerp dat volledig nieuw voor ze was. Tot slot hebben we een online vragenlijst opgezet waarmee gegevens zijn verzameld over hoe leerkrachten in het sbo denken over het reken-wiskundig potentieel van hun leerlingen.

Hoofdstukken 2 en 3 doen verslag van twee deelstudies waarin het reken-wiskundig potentieel van sbo-leerlingen is onderzocht middels een ICT-toetsomgeving. Beide deelstudies richten zich op een onderwerp dat in het algemeen wordt beschouwd als moeilijk voor rekenzwakke leerlingen, namelijk aftrekken tot 100 met tientaloverschrijding. Kenmerkend voor dit type opgaven is dat de waarde van de enen in de aftrekker groter is dan die van het aftrektal (bijv. 62 – 58). Een veelgemaakte fout in deze opgaven is het omdraaien van de enen (in de voorbeeldopgave, 2 afhalen van 8 in plaats van 8 van 2). De aan de deelstudies deelnemende leerlingen waren sbo-leerlingen in de leeftijd van 8 tot 12 jaar. Hun reken-wiskundig niveau lag een tot vier jaar achter ten opzichte van het niveau van hun leeftijdgenoten in het regulier basisonderwijs.

In het sbo worden vaak gestandaardiseerde, papieren toetsen gebruikt om het reken-wiskundig niveau en de rekenvaardigheden van leerlingen in kaart te brengen. Deze toetsen laten het gebruik van hulpmiddelen niet toe en kunnen daarom geen informatie bieden over het type hulp waarvan leerlingen kunnen profiteren bij het oplossen van de toetsopgaven. Om deze informatie te verzamelen, hebben we een ICT-toetsomgeving ontwikkeld waarin zeven aftrekopgaven uit een gestandaardiseerde toets zijn opgenomen. De gedigitaliseerde items zijn verrijkt met een optionele hulptool die leerlingen al naar gelang konden gebruiken voor het oplossen van de opgaven. In de twee deelstudies die beschreven worden in de hoofdstukken 2 en 3 is onderzocht of de ICT-toetsitems meer van het reken-wiskundig potentieel rond het aftrekken zichtbaar konden maken in vergelijking tot de gestandaardiseerde items.

De twee deelstudies richten zich elk op een bepaalde hulptool voor dezelfde set aftrekopgaven. Deze tools zijn gebaseerd op de twee belangrijkste didactische modellen ter ondersteuning van het rekenen tot 100: het groepjesmodel (zoals het 100-bord) en het lijnmodel (zoals de lege getallenlijn). Door leerlingen te laten beschikken over een tool gebaseerd op deze modellen, wilden we nagaan hoe de reken-wiskundige bekwaamheid van sbo-leerlingen zichtbaar kan worden gemaakt. In beide deelstudies werden gegevens verzameld met de toetsitems in de ICT-versie en de gestandaardiseerde versie.

In een van de twee deelstudies was binnen de ICT-toetsomgeving een dynamische, visuele tool opgenomen waarmee leerlingen digitale fiches op een 100-bord met een 10x10 structuur konden plaatsen. De leerlingen kunnen de fiches, die naast het bord liggen, selecteren, naar het bord slepen, herschikken of verwijderen. We verwachtten dat door een visuele representatie van de uit te voeren rekenoperatie op het computerscherm de leerlingen minder geneigd zouden zijn om de enen om te keren. Zo kan in het geval van de opgave 62 – 58 de tool een visuele prompt bieden om een oplossing te vinden om 8 van 2 af te trekken door een staafje van tien aan te breken. In totaal hebben 37 leerlingen van twee sbo-scholen aan deze deelstudie deelgenomen.

In de andere deelstudie was, in plaats van het 100-bord, een digitale lege getallenlijn opgenomen in de ICT-toetsomgeving. Deze tool kan op basis van touch screen technology worden bediend. De lege getallenlijn bestaat uit een horizontale lijn waarop leerlingen kunnen noteren en tekenen en waarop ze operaties kunnen uitvoeren door sprongen vooruit of achteruit te maken. De tool kan leerlingen helpen bij het oplossen van aftrekopgaven met tientaloverschrijding, omdat de tool hulp kan bieden bij het ordenen van de getallen in een opgave en bij het uitvoeren van de nodige rekenoperaties. Bovendien kunnen leerlingen wanneer ze op de lege getallenlijn werken zicht houden op hun reeds uitgevoerde handelingen. Zo kunnen leerlingen in de voorbeeldopgave 62 – 58 de getallen 62 en 58 op de getallenlijn plaatsen waardoor het duidelijk kan worden dat deze getallen zich dichtbij elkaar bevinden in de getallenrij. Dit inzicht kan leerlingen stimuleren om het verschil tussen de twee getallen te overbruggen. Of de leerlingen de tool inzetten en hoe ze de tool gebruikten – zoals de sprongen die ze op de getallenlijn maakten – werden beide gebruikt als indicatie voor hun vaardigheid in het aftrekken tot 100. Aan deze tweede deelstudie hebben 43 leerlingen van twee andere sbo-scholen deelgenomen.

In beide deelstudies werden de scores op de ICT-versie van de toetsitems vergeleken met de scores op de gestandaardiseerde versie van de toetsitems. De resultaten van beide deelstudies lieten zien dat leerlingen kunnen profiteren van het werken met een hulptool. Dit betekent dat de dynamische ICT-toetsomgeving kan bijdragen aan het zichtbaar maken van het potentieel van sbo-leerlingen op het gebied van het aftrekken met tientaloverschrijding. Daarnaast vonden we in de studie met het 100-bord dat de leerlingen een duidelijke voorkeur lieten zien voor het direct aftrekken (afhalen), terwijl in de getallenlijnstudie ook regelmatig indirecte optel- en aftrekprocedures (zoals aanvullen) werden gebruikt.

Hoofdstuk 4 beschrijft een verdere analyse van de gegevens uit de twee voorgaande deelstudies waaruit bleek dat sbo-leerlingen hoger scoorden in een dynamische ICT-toetsomgeving met hulptools dan op de gestandaardiseerde, papieren toets. In de studie in dit hoofdstuk hebben we geprobeerd om een a posteriori verklaring te vinden voor dit resultaat. Daartoe hebben we een secundaire analyse uitgevoerd waarin we kenmerken van de ICT-toetsomgeving hebben gerelateerd aan de antwoorden of handelingen van de leerlingen. De resultaten van deze analyse waren gevarieerd. Voor bepaalde kenmerken vonden we bewijs dat ze mogelijk de antwoorden of oplossingsaanpakken van de leerlingen hebben beïnvloed, terwijl dit voor andere kenmerken minder duidelijk was. Desalniettemin heeft de analyse ons meer inzicht geboden in hoe we toetsinstrumenten kunnen ontwikkelen die zwakpresterende leerlingen op het gebied van rekenen-wiskunde mogelijkheden kunnen geven om te laten zien wat ze kunnen.

Hoofdstuk 5 behandelt de studie waarin we het gebruik van indirect optellen (aftrekken door aanvullen) door sbo-leerlingen voor het oplossen van tweecijferige aftrekopgaven hebben onderzocht. In totaal hebben 56 leerlingen (8-tot-12-jarigen) met een reken-wiskundig niveau van eind groep 4 deelgenomen aan deze studie. De leerlingen hebben een ICT-toets gemaakt met verschillende typen aftrekopgaven. Hoewel de meeste leerlingen niet de aanvulprocedure hadden geleerd voor het oplossen van aftrekopgaven, gebruikten ze regelmatig spontaan deze procedure. De itemkenmerken vormden de belangrijkste prompt voor het gebruik van de aanvulprocedure. Ten aanzien van de getalskenmerken bleken de leerlingen het vaakst de aanvulprocedure gebruikten als de getallen in een opgave dichtbij elkaar liggen. Ten aanzien van het opgave format bleken de leerlingen het vaakst aan te vullen in contextopgaven waarin een aanvulsituatie werd geschetst. Verder bleek dat aanvullen een zeer succesvolle procedure was voor sbo-leerlingen en dat de beste voorspeller van een goed antwoord werd gevonden in combinatie met de rijgstrategie.

Hoofdstuk 6 rapporteert over een studie gericht op het zichtbaar maken van het reken-wiskundig potentieel van sbo-leerlingen middels het gebruik van een dynamische ICT-toetsomgeving waarin een *drag-and-drop* functie en een tekentool zijn opgenomen. De studie richtte zich op het onderwerp elementaire combinatoriek, dat over het algemeen niet wordt onderwezen in het sbo. In totaal werden zes combinatoriekopgaven over het vinden van alle mogelijke combinaties van verschillende kledingitems op het computerscherm gepresenteerd. Data werden verzameld over de successcore en het strategiegebruik van de leerlingen bij het oplossen van deze opgaven. De successcore en het strategiegebruik van leerlingen in het regulier basisonderwijs dienden als referentie. De totale onderzoeksgroep bestond uit 84 leerlingen (8- tot 13-jarigen) uit het sbo en 76 leerlingen (7- tot 11-jarigen) uit het regulier basisonderwijs. Hun reken-wiskundig niveau liep uiteen van halverwege groep 4 tot halverwege groep 7. Op basis van de geanalyseerde data bleek dat zowel met betrekking tot successcore als strategiegebruik de sbo-leerlingen niet significant verschilden van de leerlingen uit het regulier basisonderwijs. Deze bevindingen geven aan dat er sprake is van verborgen reken-wiskundig potentieel bij sbo-leerlingen.

Hoofdstuk 7 richt zich op de deelstudie waarin leerkrachtpercepties ten aanzien van het reken-wiskundig potentieel van sbo-leerlingen zijn onderzocht. Onderzoek heeft aangetoond dat hoge verwachtingen van leerkrachten over de onderwijsontwikkeling van hun leerlingen een positieve invloed heeft op hoe deze leerlingen zich werkelijk ontwikkelen. Dit betekent dat wanneer er wordt gestreefd naar het verbeteren van leerlingprestaties het van cruciaal belang is te weten hoe leerkrachten denken over de mogelijkheden van hun leerlingen. De studie die in dit hoofdstuk wordt beschreven had dan ook als doel na te gaan hoe leerkrachten in het sbo denken over het reken-wiskundig potentieel van hun leerlingen en welke mogelijkheden leerkrachten zien voor het zichtbaar maken van dit potentieel. De gegevens werden verzameld door middel van een online vragenlijst. Verassend genoeg bleek uit de leerkrachtantwoorden dat, hoewel de leerkrachten met leerlingen werken die lage resultaten behalen, de meesten van de leerkrachten positief waren over het reken-wiskundig potentieel van hun leerlingen. De studie gaf aan dat leerkrachten onbenut potentieel veelal toeschrijven aan factoren buiten de leerling. Bovendien konden de leerkrachten hun mening ondersteunen met observaties uit hun klassenpraktijk. De belangrijkste boodschap uit dit hoofdstuk is dat leerkrachten in het sbo er niet voor schuwen om hun eigen grenzen (als het gaat om het uitproberen van nieuwe didactische aanpakken) en die van hun leerlingen (als het gaat om het verkennen van nieuwe reken-wiskundige domeinen) te verleggen. Het artikel sluit af met aanbevelingen voor de professionele ontwikkeling van leerkrachten in het sbo.

Hoofdstuk 8 geeft een overzicht van de conclusies van dit dissertatieonderzoek en biedt suggesties voor verder onderzoek en de toetspraktijk in het sbo. De belangrijkste conclusie van dit onderzoek is dat alternatieve toetsaanpakken kunnen helpen bij het zichtbaar maken van het reken-wiskundig potentieel van sbo-leerlingen. Als antwoord op de eerste hoofdvraag hebben we gevonden dat (a) het verrijken van een toets door het aanreiken van optionele hulptools in een ICT-toetsomgeving kan helpen bij het zichtbaar maken van het reken-wiskundig potentieel van sbo-leerlingen rond een moeilijk onderwerp als aftrekken tot 100 met tientaloverschrijding, en dat (b) het ontwerpen van toetsitems met bepaalde itemkenmerken kan bijdragen aan het zichtbaar maken van het flexibel oplosgedrag van sbo-leerlingen bij het oplossen van aftrekopgaven tot 100. Als antwoord op de tweede hoofdvraag zijn we te weten gekomen dat het presenteren van elementaire combinatoriekopgaven in een dynamische ICT-toetsomgeving de probleemoplos-vaardigheden van sbo-leerlingen kan blootleggen. Tot slot, als antwoord op de derde hoofdvraag, is op basis van een inventarisatie van leerkrachtpercepties gebleken dat leerkrachten in het sbo veronderstellen dat er sprake is van onbenut reken-wiskundig potentieel in het sbo. Deze conclusies geven aan dat – in lijn met de doelstellingen van het huidige onderwijsbeleid – de omstandigheden voor het verbeteren van de reken-wiskundige prestaties in het sbo gunstig zijn. Het hoofdstuk wordt afgesloten met enkele aanbevelingen voor het beter herkennen en benutten van het reken-wiskundig potentieel van sbo-leerlingen.

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With much pleasure I look back to my work as the "teacher with the magic computer" as I have been called during my school visits to De Brug, De Cirkel, De Evenaar, De Kring, De Krullevaar, De Oostvogel, De Stuifheuvel, De Vijverhof, De Wenteltrap, Het Keerpunt, Het Klaverblad, Het Mozaïek, Het Zand, Johannes-Martinus, Luc Stevensschool, Merlijn, Nicolaasschool, Op Dreef, St. Maarten, Triangel, Van der Hoeveschool and Willem van der Velden. I would like to thank the teachers, school leaders and students of these schools for their participation to my PhD study and their hospitality when receiving me.

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My dear B, thank you. I dedicate this book to you and our little girl.

Curriculum Vitae

Marjolijn Peltenburg was born on September 28, 1980 in The Hague (the Netherlands) and completed her secondary schooling (‘Voorbereidend Wetenschappelijk Onderwijs’) in 1998 at the Emmaus College in Rotterdam. She studied Educational Sciences at Utrecht University and graduated with honors in 2004. During her master’s program, she specialized in learning with new media and educational development. As part of the master’s program Marjolijn carried out a traineeship at educational publisher Thieme Meulenhoff where she first got in touch with didactics of mathematics at the primary level. Her master’s thesis focused on special education students’ estimation strategies for solving number problems. The study took place in the *Speciaal Rekenen* project of the Freudenthal Institute at Utrecht University, where Marjolijn worked from 2004 to 2008 as an educational developer of mathematics in primary (special) education. At the same institute, she started her PhD candidacy in August 2008 in the *IMPULSE* project (Inquiring Mathematical Power and Unexploited Learning of Special Education students). Her PhD research was carried out under supervision of Prof. dr. Marja van den Heuvel-Panhuizen. During the research one of the IMPULSE articles (see chapter 3 of the present thesis) was nominated for best paper at the biannual pre conference of EARLI and another article (see chapter 5) was awarded as best paper at the annual conference for PhD students of the Dutch Interuniversity Centre for Educational Sciences (ICO). The ICO accepted Marjolijns PhD research by fulfilling all requirements of the ICO PhD education program.

Overview of publications related to this thesis

Peltenburg, M., & Van den Heuvel-Panhuizen, M. (2008). Mathematical power of special needs students. In M. van den Heuvel-Panhuizen & O. Köller (Eds), *Challenging Assessment – Book of Abstracts of the Fourth Biennial EARLI/Northrumbria Assessment Conference 2008* (p. 142). Berlin: IQB.

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37. Kortland, J. (2001). *A problem-posing approach to teaching decision making about the waste issue.*
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35. Margadant-van Arcken, M., & Van den Berg, C. (2000). *Natuur in pluralistisch perspectief – Theoretisch kader en voorbeeldlesmateriaal voor het omgaan met een veelheid aan natuurbeelden*.
34. Janssen, F. J. J. M. (1999). *Ontwerpend leren in het biologieonderwijs. Uitgewerkt en beproefd voor immunologie in het voortgezet onderwijs*.

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31. Beeftink, C. (2000). *Met het oog op integratie – Een studie over integratie van leerstof uit de natuurwetenschappelijke vakken in de tweede fase van het voortgezet onderwijs*.
30. Vollebregt, M. J. (1998). *A problem posing approach to teaching an initial particle model*.
29. Klein, A. S. (1998). *Flexibilization of mental arithmeticsstrategies on a different knowledge base – The empty number line in a realistic versus gradual program design*.
28. Genseberger, R. (1997). *Interessegeoriënteerd natuur- en scheikundeonderwijs – Een studie naar onderwijsontwikkeling op de Open Schoolgemeenschap Bijlmer*.
27. Kaper, W. H. (1997). *Thermodynamica leren onderwijzen*.
26. Gravemeijer, K. (1997). *The role of context and models in the development of mathematical strategies and procedures*.
25. Acampo, J. J. C. (1997). *Teaching electrochemical cells – A study on teachers' conceptions and teaching problems in secondary education*.
24. Reygel, P. C. F. (1997). *Het thema 'reproductie' in het schoolvak biologie*.
23. Roebertsen, H. (1996). *Integratie en toepassing van biologische kennis – Ontwikkeling en onderzoek van een curriculum rond het thema 'Lichaamsprocessen en Vergift'*.
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17. De Jong, O., Van Roon, P. H., & De Vos, W. (1995). *Perspectives on research in chemical education.*
16. Van Keulen, H. (1995). *Making sense – Simulation-of-research in organic chemistry education*.
15. Doorman, L. M., Drijvers, P., & Kindt, M. (1994). *De grafische rekenmachine in het wiskundeonderwijs*.

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8. Van Galen, F., Dolk, M., Feijs, E., & Jonker, V. (1991). *Interactieve video in de nascholing reken-wiskunde.*
7. Elzenga, H. E. (1991). *Kwaliteit van kwantiteit*.
6. Lijnse, P. L., Licht, P., De Vos, W., & Waarlo, A. J. (Eds.) (1990). *Relating macroscopic phenomena to microscopic particles: a central problem in secondary science education*.
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4. Vogelezang, M. J. (1990). *Een onverdeelbare eenheid*.
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Appendix Test items

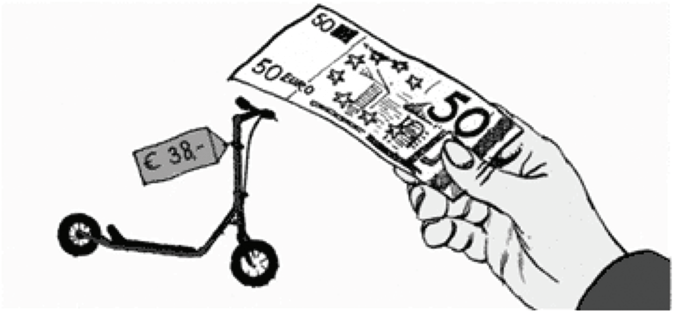
The test items on subtraction with ‘crossing the ten’ which are displayed on the following pages were taken from the CITO Monitoring Test for Mathematics End Grade 2 (Janssen, Scheltens, & Kraemer, 2005).

The items were used with permission from CITO and translated by the author of the thesis. Test items 1 to 4 belong to part I of the test and test items 5 to 7 belong to part II.

Reference

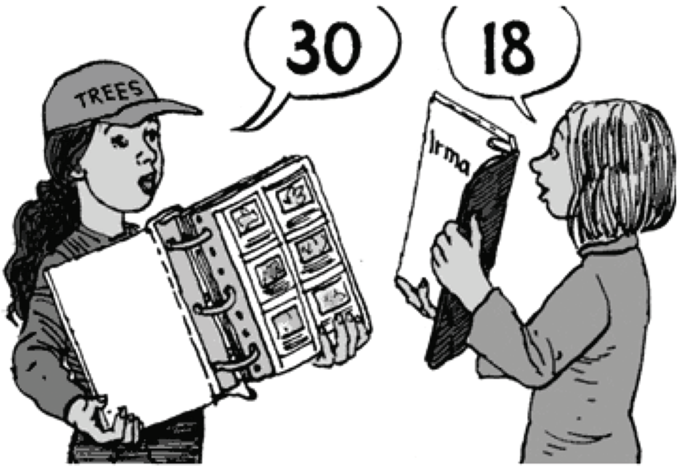
Janssen, J., Scheltens, F., & Kraemer, J. (2005). *Leerling- en onderwijsvolgsysteem. Rekenen-wiskunde groep 4. Handleiding* [Student and Education Monitoring System. Mathematics Grade 2. Teachers guide]. Arnhem: CITO.

1.



Sandra buys a scooter for 38 Euro.
She pays with a 50 Euro note.
How many Euros does she receive in return?

2.

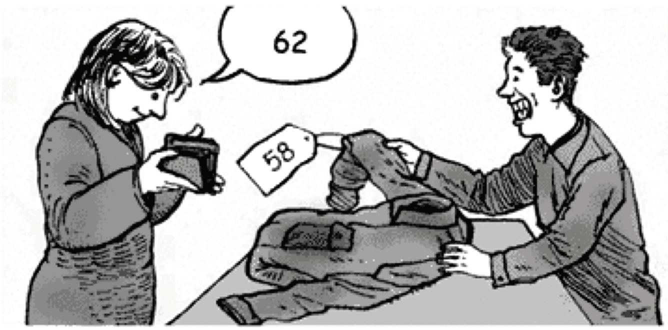


Trees has 30 pictures, Irma has 18.
How many pictures does Trees have more than Irma?

3.

Mirjam and her mother have their birthdays on the same day.
How many years older is mother than Mirjam?

4.



Michel's mother has 62 Euro.
She buys a jacket for 58 Euro.
How many Euros will she have left?

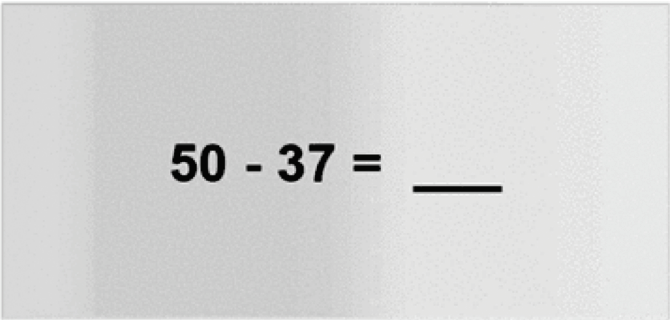
5.

Tessel scores 71 points in a computer game.
Wilma scores 3 points less. How many points did Wilma score?

6.

There are 48 Spiroe comics.
Jan already has 39.
How many comics does he not have yet?

7.



50 minus 37 equals …