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THE TWO-LOOP RENORMALIZATION OF THE YUKAWA SECTOR FOR AN ARBITRARY RENORMALIZABLE FIELD THEORY

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The two-loop Yukawa coupling constant renormalization and the mass renormalization of the fermion for an arbitrary renormalizable field theory, including vector, scalar and of course fermion fields, has been calculated. This has been done in two different regularization schemes, i.e. dimensional regularization and dimensional reduction. Both calculations were done in the conventional way, and not with the use of the so-called background-field method. Only the reduction scheme is shown to preserve supersymmetry up to this level.

Furthermore a simple example of a gauge theory is given to show that in general it is not possible to give an explicit transformation of the coupling constants in such a way that all β-functions become the same for the two regularization schemes, which proves that dimensional reduction, treated this way, is not a good regularization scheme.

1. Introduction

The two-loop β-functions are relevant to physical processes (such as the lifetime of the proton – if it decays at all), where they give corrections to the known one-loop predictions for these processes. In a former article [1] we gave the two-loop β-functions for the scalar potential and the gauge coupling constant(s) (the latter already known of course [2]) by means of the background-field method, for the lagrangian, which will be defined in sect. 2.

For the renormalization of the fermion mass and the Yukawa coupling constant(s) the background-field method (in contrast with the scalar potential) is not of much use. The diagrams to be calculated, as will be discussed in sect. 3, are of the same order of difficulty, or even worse.

This article is arranged as follows.

First in sect. 2 we define the most general renormalizable lagrangian with some definitions used throughout the whole article. Then, in sect. 3, we argue why the background-field method is of no use whatsoever for diagrams with outgoing fermions. In sect. 4 we introduce a "massive calculation scheme", which can be used for dimensional regularization [3] as well as for dimensional reduction [4], only for practical reasons. Working with the computer program Schoonschip [5] it is very comfortable to reduce the number of integrals to be carried out as often as possible. In sect. 5 we determine the double poles from the renormalization group equations, with an extremely simplified equation which was derived in ref. [6], in order to get as many checks as possible on our final results. Sect. 6 is devoted to a short review of the two already mentioned regularization schemes. The way to compute the difference between the single poles will be explained in ref. [14].

Within the framework of the massive calculation scheme all integrals can be transformed into

$$\int\int \mathrm{d}_n p\, \mathrm{d}_n q \frac{1}{(p^2+m^2)^{\alpha}(q^2+m^2)^{\beta}\left((p-q)^2+m^2\right)^{\gamma}}, \tag{1.1}$$

because a $p \cdot q$ in the numerator can be written as

$$p \cdot q = \tfrac{1}{2}\left((p^2+m^2)+(q^2+m^2)-\left((p-q)^2+m^2\right)-m^2\right). \tag{1.2}$$

In the case of outgoing fermions a few more nasty numerators can occur:

$$\not{p}\not{q}, \tag{1.3}$$

when computing the pure two-loop three-point function and

$$\not{p} q_{\mu} \tag{1.4}$$

for the renormalization of the fermion wave function. Sect. 7 gives a method for both dimensional regularization as well as for dimensional reduction to get these integrals in the desired form (1.1). In sect. 8 we give the final results for the fermion mass and Yukawa coupling constant for both regularization schemes. Supersymmetric Yang-Mills ($N=4$) indeed yields a vanishing β-function as expected for dimensional reduction. In sect. 9 we use a toy model, already used in [1] and compute the two-loop β-functions for the Yukawa coupling for both schemes. Together with the results of ref. [1], where the β-functions of the scalar coupling and the gauge coupling were determined, we investigated whether a finite transformation of diverse couplings existed that could transform all β-functions, computed in one scheme, into the β-functions of the other scheme. The result was negative. Sect. 10 is devoted to conclusions and discussion. Appendix A gives all the relevant integrals, and appendix B gives a model used in sect. 5.

2. The lagrangian

For matter of convenience we first give the most general renormalizable field theory, with some definitions, used throughout the rest of the article:

$$\mathcal{L} = -\tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - \tfrac{1}{2}(D_\mu\phi_i)^2 - \bar{\psi}\gamma_\mu D_\mu \psi - \bar{\psi}W(\phi)\psi - V(\phi), \tag{2.1}$$

where

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu , \tag{2.2}$$

$$D_\mu \phi_i = \partial_\mu \phi_i + A^a_\mu (T^a \phi)_i , \tag{2.3}$$

$$D_\mu \psi = \partial_\mu \psi + A^a_\mu U^a \psi , \tag{2.4}$$

$$W(\phi) = S(\phi) + iP(\phi)\gamma_5 . \tag{2.5}$$

Furthermore we need

$$U^a = U^a_s + U^a_p \gamma_5 , \tag{2.6}$$

$$\hat{U}^a = U^a_s - U^a_p \gamma_5 , \tag{2.7}$$

$$\hat{W}(\phi) = S(\phi) - iP(\phi)\gamma_5 , \tag{2.8}$$

$$C^{ab} = f^{apq} f^{bpq} , \tag{2.9}$$

$$R^{ab} = -\mathrm{Tr}(T^a T^b) , \tag{2.10}$$

$$Q^{ab} = -\mathrm{Tr}(U^a U^b) . \tag{2.11}$$

In order for the theory to be renormalizable, S and P are at most linear in the scalar field, V may be maximal of order four. And, of course, no anomalies may occur. The gauge group need not necessarily be simple, and the scalar and spinors may occur in various reducible representations. Any number of gauge coupling constants may be absorbed in the structure constants f^{abc} and the representation generators T^a and U^a (we left them out for compactness of the notation). Extension to Majorana spinors has already been discussed in [1].

3. Why the background-field method is not used

In a former publication [1] we derived the full two-loop counter-lagrangian for an arbitrary (but of course renormalizable) field theory using the background-field method. However this did not include the renormalization of the fermion mass and the Yukawa coupling.

In this section we argue why it is no advantage at all to use this method to determine these renormalization coefficients instead of just doing ordinary perturbation theory (i.e. the conventional way).

There are two ways of using the method. First the method described in ref. [1], the second one is the method used by Abbott [7].

Let us first discuss the method of ref. [1]. (For all notations see [1].) One always assumed that the propagator of each particle was of the form

$$W_{ij} \cdot k^{-2}, \tag{3.1}$$

where W_{ij} could be any matrix (not containing k). The second assumption was that the vertex

$$B^{\mu}_{ijk} \phi_i \phi_j D^{\mu} \phi_k, \tag{3.2}$$

had the pleasant property that

$$D^{\mu} B^{\nu}_{ijk} = 0 \tag{3.3}$$

for all indices. This diminished the number of terms needed to be calculated in such a way that they were much fewer than the number of diagrams to be computed in the conventional way. In the case of ref. [1] there was always a transformation available for the fermion field (which cannot have a propagator of the desired form (3.1) without violating (3.3).

But what happens in the case of outgoing fermions? We set A^{cl}_{μ} equal to zero of course and the relevant piece of the lagrangian becomes, after splitting up the fields in their classical and quantum parts (for simplicity we took $W(\phi) = \phi$)

$$\mathcal{L}_{\mathrm{ferm}} = -\bar{\psi}^{\mathrm{qu}} \gamma_{\mu} \partial_{\mu} \psi^{\mathrm{qu}} - \bar{\psi}^{\mathrm{qu}} \phi^{\mathrm{cl}} \psi^{\mathrm{qu}} - \bar{\psi}^{\mathrm{cl}} \phi^{\mathrm{qu}} \psi^{\mathrm{qu}} - \bar{\psi}^{\mathrm{qu}} \phi^{\mathrm{qu}} \psi^{\mathrm{cl}} - \bar{\psi}^{\mathrm{qu}} \phi^{\mathrm{qu}} \psi^{\mathrm{qu}}. \tag{3.4}$$

In order to get the right propagator behaviour (3.1) we have to look for a transformation such as

$$\bar{\psi}^{\mathrm{qu}} \to \bar{\psi}^{\mathrm{qu}}, \tag{3.5}$$

$$\psi^{\mathrm{qu}} \to -\gamma_{\mu} \partial_{\mu} \psi^{\mathrm{qu}} + \text{anything}. \tag{3.6}$$

But whatever one chooses for "anything", the constraint (3.3) can never be fulfilled. The lagrangian becomes

$$\begin{aligned} \mathcal{L}_{\mathrm{ferm}} = {} & \bar{\psi}^{\mathrm{qu}} \partial^2 \psi^{\mathrm{qu}} + \bar{\psi}^{\mathrm{qu}} \phi^{\mathrm{cl}} \gamma_{\mu} \partial_{\mu} \psi^{\mathrm{qu}} + \bar{\psi}^{\mathrm{cl}} \phi^{\mathrm{qu}} \gamma_{\mu} \partial_{\mu} \psi^{\mathrm{qu}} \\ & - \bar{\psi}^{\mathrm{qu}} \phi^{\mathrm{qu}} \psi^{\mathrm{cl}} + \bar{\psi}^{\mathrm{qu}} \phi^{\mathrm{qu}} \gamma_{\mu} \partial_{\mu} \psi^{\mathrm{qu}} + \text{rest terms}, \end{aligned} \tag{3.7}$$

the rest terms coming from "anything". The reason that (3.3) cannot be met is that a covariant derivative has appeared which mixes two different kinds of particles (scalar-fermion in this case; the third term in (3.7), which cannot be eliminated by a suitable choice of "anything"). So, if we want to use the method of [1], the number of terms exceeds the number of diagrams, using the conventional way, and therefore this method is ruled out.

Abbott's approach [7] may at first sight be of some use (his method works with ordinary Feynman diagram calculations, but also after splitting up the relevant fields into quantum and classical parts). A closer examination however yields that the number of diagrams to be computed are exactly the same as in the conventional scheme. Using the gauge-fixing term as in ref. [1] (eq. (4.19)) could at first glance diminish the number of vertices, however the Faddeev-Popov ghost lagrangian yields more extra vertices than there were lost.

The conclusion of this section can only be that for the determination of the infinities in the Yukawa coupling constant and the fermion mass the conventional way of all needed diagrams must be preferred.

4. The "massive calculation scheme"

Working with the computer program Schoonschip [5] it is extremely useful that the number of needed integrals is as minimal as possible. That is why we chose a trick that we call the "massive calculation scheme". It can be used for dimensional regularization as well as for dimensional reduction.

The trick consists of two parts. First, make the diagram logarithmically divergent by differentiating with respect to one of the inflowing momenta. (Note that we also have to determine the wave function renormalization of the fermion (linearly divergent) and that of the scalar field (quadratically divergent), both up two loops, too.) Then put all external momenta equal to zero and give any particle (scalar, fermion and even the gauge field) an arbitrary but equal mass in its propagator. Any integral now can be transferred into the following form:

$$\frac{1}{\left[p^2+m^2\right]^{\alpha}\left[q^2+m^2\right]^{\beta}\left[(p-q)^2+m^2\right]^{\gamma}}, \tag{4.1}$$

where

$$\alpha+\beta+\gamma \geqslant 4 \tag{4.2}$$

is the only relevant case. (Note the $>$ sign; subdivergencies may occur.) Observe the fact that the numerator in (4.1) is one. But any dot product $p \cdot q$ can be eliminated by

$$p \cdot q = \tfrac{1}{2}\left(p^2+q^2-(p-q)^2\right). \tag{4.3}$$

And any sort of integral with $\not{p}\not{q}$ or $\not{p}q_\mu$ in the numerator can be expressed in terms of $p \cdot q$ or $p \cdot q\gamma_\mu$ respectively, but as these are dependent on the regularization scheme used they will be discussed later (sect. 7).

A table of all the identities and integrals needed for the full computation are depicted in appendix A.

Two questions are now to be asked about the validity of this calculation scheme.

(i) We break gauge invariance explicitly by giving the vector particle a mass too. Is this allowed?

(ii) By giving all particles the same mass, will it yield correct results? (Think of, for example, a term proportional to the ratio of the masses of the different kinds of particles, which is equal to one in the present case but could be any other number.)

Answer to the first question.

As long as no subdivergencies occur there are no problems. After having performed the first and hence finite integral one integral remains to be done:

$$\int \frac{1}{[q^2]^\alpha} \frac{1}{[q^2+m^2]^\beta} \mathrm{d}_n q, \tag{4.4}$$

if we have obeyed the rules. Of course only $\alpha+\beta=2$ or $(2+\frac{1}{2}\varepsilon)$ contributes. But as $1/(q^2+m^2)=1/q^2(1-m^2/(q^2+m^2)+\text{higher orders})$ the single pole of (4.4) becomes equal to the integral (4.4)

$$\int \frac{1}{[q^2+m^2]^{\alpha+\beta}} \mathrm{d}_n q. \tag{4.5}$$

(Remember all integrals were logarithmically divergent, so terms like

$$\int \frac{1}{[q^2+m^2]^{\alpha+\beta+1}} \mathrm{d}_n q \tag{4.6}$$

become finite, hence carry no poles.)

In case subdivergencies exist we have to be more careful. Suppose that, after the first integration the rest is of the form

$$\int \frac{1}{\varepsilon} \frac{1}{[q^2]^{\alpha+\varepsilon/2}} \frac{1}{[q^2+m^2]^\beta} \mathrm{d}_n q, \tag{4.7}$$

or

$$\int \frac{1}{\varepsilon} \frac{1}{[q^2]^\alpha} \frac{1}{[q^2+m^2]^{\beta+\varepsilon/2}} \mathrm{d}_n q, \tag{4.8}$$

with the corresponding counter-diagram:

$$\int -\frac{1}{\varepsilon} \frac{1}{[q^2]^\alpha} \frac{1}{[q^2+m^2]^\beta} \mathrm{d}_n q, \tag{4.9}$$

whereas we take

$$\int \frac{1}{\varepsilon}\left[\frac{1}{\left(q^2+m^2\right)^{\alpha+\beta+\varepsilon/2}}-\frac{1}{\left(q^2+m^2\right)^{\alpha+\beta}}\right]\mathrm{d}_n q. \tag{4.10}$$

α is an integer $\geqslant 0$, β an integer >0 (note that $\beta=0$ is impossible for there will always be a scalar or fermion propagator in a diagram we computed).

Moreover if $\alpha+\beta>2$ then ((4.7)+(4.9)), ((4.8)+(4.9)) and (4.10) become finite. For example

$$(4.10)=\int\left(\frac{1}{2}\frac{\ln\left(q^2+m^2\right)}{\left(q^2+m^2\right)^{\alpha+\beta}}+\mathrm{O}(\varepsilon)\right)\mathrm{d}_n q \tag{4.11}$$

obviously is (mere power counting).

So the only problem is $\alpha+\beta=2$ (with $\beta>0$). Apart from the trivial factors of π, (4.10) becomes [3]

$$(4.10)=\frac{1}{\varepsilon}\left(\frac{\Gamma(\varepsilon)}{\Gamma(2+\frac{1}{2}\varepsilon)}-\Gamma(\tfrac{1}{2}\varepsilon)\right)=-\frac{1}{\varepsilon^2}-\frac{1}{2\varepsilon}. \tag{4.12}$$

We have to consider four cases, ((4.7)+(4.9)) and ((4.8)+(4.9)) denoted as I_1 and I_2, for $\alpha=1$, $\beta=1$ and $\alpha=0$, $\beta=2$.

First the trivial one: $I_2(\alpha=0,\ \beta=2)\equiv(4.10)$. Then

$$\begin{aligned}
I_1(\alpha=0,\ \beta=2)&=\frac{1}{\varepsilon}\int \mathrm{d}_n q\,\frac{1}{\left[q^2+m^2\right]^2}\left(\frac{q^2}{\left[q^2\right]^{1+\varepsilon/2}}-1\right)\\
&=\frac{1}{\varepsilon}\int\int_0^1 \mathrm{d}_n q\,\mathrm{d}x\,\frac{x(1-x)^{\varepsilon/2}q^2}{\left[q^2+m^2x\right]^{3+\varepsilon/2}}\,\frac{\Gamma(3+\frac{1}{2}\varepsilon)}{\Gamma(2)\Gamma(1+\frac{1}{2}\varepsilon)}-\frac{2}{\varepsilon^2}\\
&=\frac{1}{\varepsilon}\int_0^1 \mathrm{d}x\,(1-x)^{\varepsilon/2}\frac{\Gamma(\varepsilon)}{\Gamma(3+\frac{1}{2}\varepsilon)}\tfrac{1}{2}(4-\varepsilon)x^{1-\varepsilon}\frac{\Gamma(3+\frac{1}{2}\varepsilon)}{\Gamma(2)\Gamma(1+\frac{1}{2}\varepsilon)}-\frac{2}{\varepsilon^2}\\
&=\frac{1}{\varepsilon^2}\int_0^1 \mathrm{d}x\,(1-x)^{\varepsilon/2}x^{1-\varepsilon}-\frac{2}{\varepsilon^2}\\
&=\frac{1}{\varepsilon^2}\,\frac{\Gamma(1+\frac{1}{2}\varepsilon)\Gamma(2-\varepsilon)}{\Gamma(3-\frac{1}{2}\varepsilon)}(2-\tfrac{1}{2}\varepsilon)-\frac{2}{\varepsilon^2}\\
&=\frac{1}{\varepsilon^2}\,\frac{(1-\varepsilon)}{(1-\frac{1}{2}\varepsilon)}-\frac{2}{\varepsilon^2}=-\frac{1}{\varepsilon^2}-\frac{1}{2\varepsilon}.
\end{aligned} \tag{4.13}$$

We leave it to the reader that for I_1 and I_2 in the case $a = b = 1$ the same answer results.

So, up to two loops, all relevant integrals yield the same infinite parts, whether we give the vector particle a mass or not. The important conclusion is that, hence, up to this level renormalizability is *not* violated.

Answer to the second question.

As shown above renormalizability is not violated. Thus as we know from renormalization theory ratios of masses *cannot* occur in infinities of wave function and vertex renormalization (q.e.d.).

As an illustrative example for the two preceding questions, consider the following diagram (fig. 1) with its counter-diagram (fig. 2) for the pure massless case as well as with the use of "massive calculation scheme". (Integration over internal momenta is always assumed; the dashed line is a scalar field.)

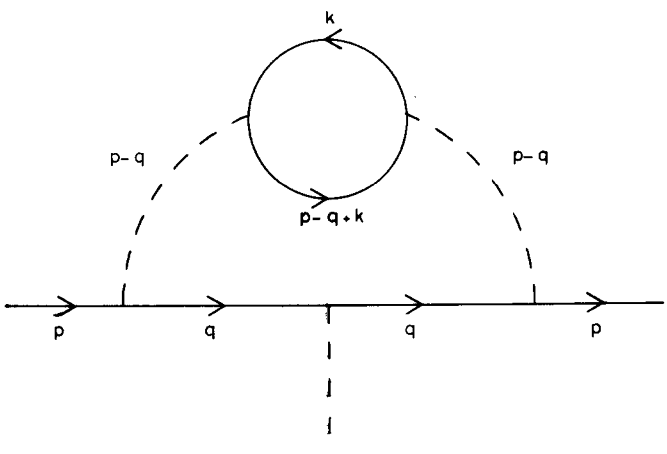


Fig. 1.

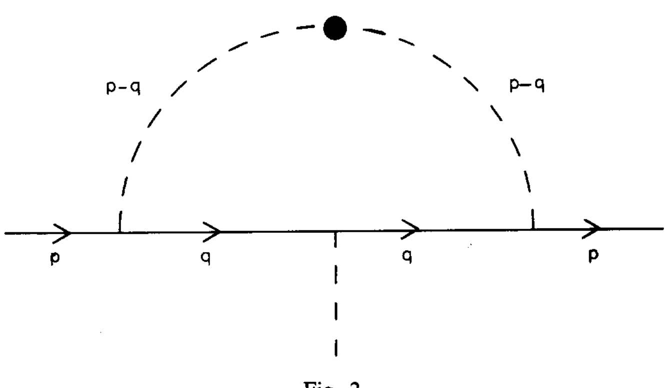
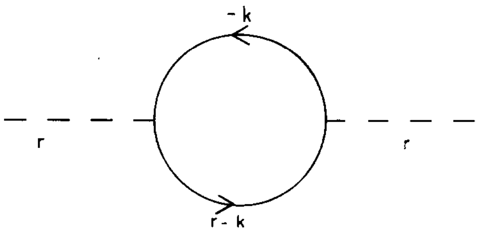


Fig. 2.

4.1. MASSLESS CASE



$$= -\frac{1}{k^2}\frac{1}{(r-k)^2}\mathrm{Tr}[(i\not{k})(-i(\not{r}-\not{k})]$$

$$= \frac{4r\cdot k - 4k^2}{k^2\cdot(r-k)^2} = \int_0^1 \frac{2r^2\mathrm{d}x}{(k^2-2r\cdot k\cdot x + r^2x)^2} = \frac{8r^{2-\varepsilon}}{\varepsilon}\int_0^1 \mathrm{d}x\, x^{-\varepsilon/2}(1-x)^{-\varepsilon/2}$$

$$= \frac{8}{\varepsilon}(r^2)^{1-\varepsilon/2}(1+\varepsilon), \tag{4.14}$$

and thus

$$= -\frac{8}{\varepsilon}r^2. \tag{4.15}$$

$$\text{fig. } 1 = -\frac{8}{\varepsilon}\frac{(1+\varepsilon)}{\left[(p-q)^2\right]^{1+\varepsilon/2}q^2} = -\frac{16}{\varepsilon^2}\left(1+\tfrac{5}{2}\varepsilon\right), \tag{4.16}$$

$$\text{fig. } 2 = \frac{8}{\varepsilon}\frac{1}{(p-q)^2q^2} = \frac{32}{\varepsilon^2}(1+\varepsilon). \tag{4.17}$$

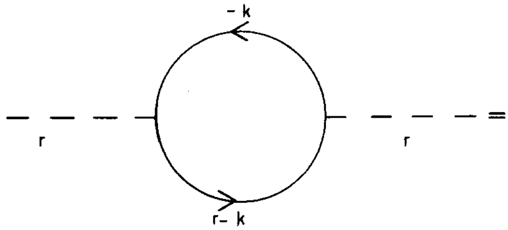
So

$$\text{fig. } 1 + \text{fig. } 2 = \frac{16}{\varepsilon^2} - \frac{8}{\varepsilon}. \tag{4.18}$$

4.2. MASSIVE CALCULATION SCHEME

We use the massive calculation scheme and break gauge invariance by using the fermion propagator

$$\frac{-i\not{p}}{p^2+m^2} \quad \text{instead of} \quad \frac{-i\not{p}+m}{p^2+m^2}. \tag{4.19}$$

P. P. 

$$= \frac{4r\cdot k - 4k^2}{[k^2+m^2]\left[(r-k)^2+m^2\right]}$$

$$= \frac{2r^2 - 2(k^2+m^2-m^2) - 2\left((r-k)^2+m^2-m^2\right)}{[k^2+m^2]\left[(r-k)^2+m^2\right]}$$

$$= \frac{-2}{(r-k)^2+m^2} + \frac{-2}{k^2+m^2} + \frac{2r^2+4m^2}{[k^2+m]\left[(r-k)^2+m^2\right]}$$

$$= \frac{1}{\varepsilon}(8r^2+32m^2). \tag{4.20}$$

So

$$= -\frac{1}{\varepsilon}(8r^2+32m^2). \tag{4.21}$$

Note we make extensive use of appendix A.

$$\text{fig. } 2 = \frac{1}{\varepsilon}\frac{\left(8(r^2+m^2)+24m^2\right)(r^2+m^2-m^2)}{(r^2+m^2)^4}$$

$$= \frac{1}{\varepsilon}\left(8W_1(2)+16W_1(3)-24W_1(4)\right) = \frac{32}{\varepsilon^2}+\frac{8}{\varepsilon}, \tag{4.22}$$

$$\text{fig. } 1 = \frac{-r^2(4r\cdot k-4k^2)}{[k^2+m^2]\left[(r-k)^2+m^2\right][r^2+m^2]^4}$$

$$= 4W_2(0,3,1) - 4W_2(0,4,1) - 2W_2(2,1,1) + 2W_2(4,1,1)$$

$$= -\frac{16}{\varepsilon^2} - \frac{16}{\varepsilon}. \tag{4.23}$$

The two diagrams add up to the same answer as in the massless case (4.8), although the single poles of each figure differ among each other.

5. Determination of the double poles of the Yukawa coupling constant and fermion mass renormalization

In ref. [6] 't Hooft derived a relation between the higher-order poles in terms of the poles one order lower and the one-loop poles. We quote his result (5.11). Let the counterterms of the kth coupling constant be of the form $\Sigma a_\nu^k (1/\varepsilon)^\nu$. Then

$$\sum_l a_{\nu+1,l}^k \rho(l)\lambda_{\mathrm{R}}^l - \rho(k) a_{\nu+1}^k = -\sigma(k) a_\nu^k + \sum_l a_{\nu,l}^k \Big(\sigma(l)\lambda_{\mathrm{R}}^l - \rho(l) a_1^l + \sum_m a_{1,m}^l \rho(m)\lambda_{\mathrm{R}}^m \Big), \tag{5.1}$$

where the dimension of the kth coupling constant, D^k, is

$$D^k = \sigma(k) - \varepsilon \cdot \rho(k); \tag{5.2}$$

$$a_{\nu,l}^k \equiv (\partial/\partial\lambda_{\mathrm{R}}^l) a_\nu^k(\lambda_{\mathrm{R}}). \tag{5.3}$$

As ε in (5.2) is arbitrary it is easy to see that (5.1) decouples into separate equations, one with only σ in it and another with only ρ:

$$\sigma(k) a_\nu^k = \sum_l a_{\nu,l}^k \sigma(l) \lambda_{\mathrm{R}}^l, \tag{5.4}$$

$$\sum_m a_{\nu+1,m}^k \rho(m)\lambda_{\mathrm{R}}^m - a_{\nu+1}^k \rho(k) = \sum_l a_{\nu,l}^k \times \Big(\sum_m a_{1,m}^l \rho(m) \lambda_{\mathrm{R}}^m - a_1^l \rho(l) \Big). \tag{5.5}$$

Formula (5.4) is self-consistent. (The naive idea that β-functions of masses are purely multiplicative is only true in special cases such as in $\lambda\varphi^4$ theory, but not in general. Even at the one-loop level, β-functions can even contain ratios of masses. See appendix B.)

Formula (5.5) can be simplified considerably. Consider first the case of dimensionless coupling constants ($\sigma = 0$, $\rho \neq 0$). For any renormalizable theory ρ can be made equal to one-half. So ρ can be removed from eq. (5.5). Now consider a_ν^k only up to all leading order in ε. Then a_ν^k only contain a string of coupling constants of $(2\nu+1)$th power. This means for the leading terms

$$(2\nu+1) a_\nu^k = \sum_l a_{\nu,l}^k \lambda_{\mathrm{R}}^l, \tag{5.6}$$

or (5.5) converts into

$$(\nu+1) a_{\nu+1}^k = \sum_l a_{\nu,l}^k a_1^l. \tag{5.7}$$

For the mass renormalization

$$\rho(\text{mass}) = 0\,. \tag{5.8}$$

So if k refers to a mass parameter then a_ν^k will be a string of 2ν coupling constants. So then the r.h.s. of eq. (5.5) converts into once the r.h.s. of (5.6) if l stands for a coupling constant plus once again the r.h.s. of (5.6) if l stands for a mass term (which *must* be there and remember that $\rho(l) = 0$). The l.h.s. of (5.5) becomes $2(\nu+1)a_\nu^k$, so (5.6) and (5.7) are still valid for masses.

There is one snag however. As shown in appendix B there are possible mass terms with a ρ unequal to zero. However, after a detailed and lengthy analysis we have been able to show that (5.7) remains valid for the cases where such a mass term appears in (5.5).

So we see that for the two-loop double poles we only need to know the one-loop infinities. These can be found in ref. [8].

$$32\pi^2\varepsilon\Delta W^{(1)} = \left(W_i\hat{W}_iW + \text{h.c.}\right) + 4\left(W_i\hat{W}W_i\right) + 6\left(WU^2 + \text{h.c.}\right) + 4W_i\text{Tr}\left[\overline{M}\cdot M_i\right]. \tag{5.9}$$

Here and in the rest of the article the notation of ref. [1] and appendix C is used, where

$$\overline{M} = \left(\frac{\partial}{\partial\phi_i}M\right)\cdot\phi_i\,. \tag{5.10}$$

The last term in (5.9) causes a difference in behaviour of the Yukawa coupling constant versus the fermion mass under renormalization.

Note that:

(i) the double poles of W will not directly depend on the representation of the scalar field (i.e. the matrices T have been eliminated);

(ii) the scalar potential does not occur in (5.9). As a consequence of the last observation we only need the one-loop renormalization of the gauge coupling constant, which is given by

$$32\pi^2\varepsilon\Delta\mathcal{L}_{\text{gauge}} = G_{\mu\nu}^aG_{\mu\nu}^b\left(-\tfrac{11}{3}C^{ab} + \tfrac{1}{6}R^{ab} + \tfrac{4}{3}Q^{ab}\right). \tag{5.11}$$

Now everything is set to determine the two-loop counterterms proportional to $1/\varepsilon^2$. As an illustration let us compute the pure WU^4 double poles. The only relevant part in (5.9) is

$$(32\pi^2\varepsilon)^{-1}\times 6\times\left(WU^2 + \hat{U}^2W\right), \tag{5.12}$$

giving

$$a_2 = (32\pi^2\varepsilon)^{-2} \times \tfrac{1}{2} \times 36(W\cdot U^2\cdot U^2 + \hat{U}^2WU^2 + \hat{U}^2WU^2 + \hat{U}^2\hat{U}^2W)$$
$$= (32\pi^2\varepsilon)^{-2}(18(W\cdot U^2\cdot U^2 + \text{h.c.}) + 36(\hat{U}^2WU^2)). \tag{5.13}$$

A complete and straightforward calculation for the double poles in W gives

$$\begin{aligned}
&(32\pi^2\varepsilon)^2\,\Delta W^{(2)}_{\text{double poles}}\\
&= \tfrac{3}{2}(W_i\hat{W}_iW_j\hat{W}_jW + \text{h.c.}) + 4(W_i\hat{W}_iW_j\hat{W}W_j + \text{h.c.})\\
&\quad + (W_i\hat{W}_iW\hat{W}_jW_j) + 4(W_i\hat{W}_jW_i\hat{W}_jW + \text{h.c.})\\
&\quad + 8(W_i\hat{W}_jW_i\hat{W}W_j + \text{h.c.}) + (W_i\hat{W}_jW_j\hat{W}_iW + \text{h.c.})\\
&\quad + 4(W_i\hat{W}_jW_j\hat{W}W_i + \text{h.c.}) + 8(W_i\hat{W}_jW\hat{W}_jW_i)\\
&\quad + 8\,\mathrm{Tr}[M_i\cdot M_iM_j\cdot\overline{M}](W_j) + 16\,\mathrm{Tr}[M_i\cdot M_jM_i\cdot\overline{M}](W_j)\\
&\quad - 8\,\mathrm{Tr}[L_i\cdot L_iL_j\cdot\overline{L}](W_j) - 16\,\mathrm{Tr}[L_i\cdot L_jL_i\cdot\overline{L}](W_j)\\
&\quad + 4\,\mathrm{Tr}[M_i\cdot M_j](W_i\hat{W}_jW + \text{h.c.}) + 16\,\mathrm{Tr}[M_i\cdot M_j](W_i\hat{W}W_j)\\
&\quad + 24\,\mathrm{Tr}[M_i\cdot M_j]\,\mathrm{Tr}[M_j\cdot\overline{M}](W_i) + 4\,\mathrm{Tr}[M_i\cdot\overline{M}](W_i\hat{W}_jW_j + \text{h.c.})\\
&\quad + 16\,\mathrm{Tr}[M_i\cdot\overline{M}](W_j\hat{W}_iW_j) + 48[M_i\cdot\overline{M}U^2 + L_i\cdot\overline{L}X^2](W_i)\\
&\quad + 24\,\mathrm{Tr}[M_i\cdot\overline{M}](W_iU^2 + \text{h.c.}) + 12(W\hat{W}_iW_iU^2 + \text{h.c.}) + 6(W_i\hat{W}_iWU^2 + \text{h.c.})\\
&\quad + 24(W_i\hat{W}W_iU^2 + \text{h.c.}) + 24(W_i\hat{W}U^2W_i + \text{h.c.}) + 6(W\hat{W}_iU^2W_i + \text{h.c.})\\
&\quad + 36(\hat{U}^2WU^2) + 18(WU^2U^2 + \text{h.c.})\\
&\quad + (C^{ab}(-44) + R^{ab}(2) + Q^{ab}(16))(WU^aU^b + \text{h.c.}).
\end{aligned} \tag{5.14}$$

(The double poles for the scalar potential in ref. [1] were checked in the same way.)

6. Dimensional regularization versus dimensional reduction

Of course everyone is familiar with the two regularizations chemes used in practice, the original one first introduced by ’t Hooft and Veltman [3], dimensional regularization, and the so-called regularization by dimensional reduction, introduced by Siegel [4]. Whereas the first one was invented to have a suitable way for renormalizing gauge theories (it guaranteed that all Ward identities were preserved, whereas an ordinary cut-off would ruin them), the second scheme was proposed somewhere after the birth of supersymmetries [9]. In supersymmetries the number of degrees of freedom of the scalar particles must coincide with those of the fermions in the lagrangian. And as dimensional regularization prescribes: perform all the momentum integrations and the γ-matrix algebra in $4-\varepsilon$ dimensions.

It is obvious that dimensional regularization cannot preserve supersymmetry when gauge particles (or particles with higher spin of course) occur in the theory.

The prescription of dimensional reduction is: Perform all γ-matrix algebra in four dimensions first (so that the gauge particle has still exactly four components), then perform the integrations in $(4-\varepsilon)$ dimensions.

We used these two ways of handling all the diagrams in order to get the results, which will be presented in sect. 8.

7. The last hurdle

There is one last minor problem to be discussed. In diagrams without outgoing fermions it is not always possible to arrive at an integral of the desired form (appendix A).

7.1. THREE-POINT FUNCTION

$$\not{p}\not{q} \tag{7.1}$$

can occur in the numerator.

As any diagram of the form

$$\frac{p_\mu q_\nu}{[p^2+m^2]^\alpha [q^2+m^2]^\beta [(p-q)^2+m^2]^\gamma} \tag{7.2}$$

must be proportional to $\delta_{\mu\nu}$ (7.1) gives for the dimensional regularization

$$\gamma_\mu\gamma_\nu p_\mu q_\nu = \gamma_\mu\gamma_\nu \frac{\delta_{\mu\nu}}{n} \cdot p \cdot q = p \cdot q, \tag{7.3}$$

and for dimensional reduction

$$\gamma_\mu\gamma_\nu p_\mu q_\nu = \gamma_\mu\gamma_\nu \frac{\delta_{\mu\nu}}{n} \cdot p \cdot q = \frac{4}{n} p \cdot q. \tag{7.4}$$

7.2. FERMION WAVE RENORMALIZATION

Now

$$\gamma_\mu p_\mu q_\nu = \gamma_\mu \delta_{\mu\nu} \cdot \frac{1}{n} p \cdot q = \frac{\gamma_\nu}{n} p \cdot q \tag{7.5}$$

for both schemes.

Transformation (1.2) now can make all integrals of the desired form.

8. The final results

The total renormalization of W is given by

$$\Delta W^{(2)}_{\text{total}} = \Delta W^{(2)} + \frac{\delta \Delta W^{(1)}}{\delta \phi} \Delta\phi^{(1)} + \frac{\delta W}{\delta \phi} \Delta\phi^{(2)} + \left(\Delta W^{(1)} \Delta\psi^{(1)} + \Delta\bar{\psi}^{(1)} \Delta W^{(1)}\right) + \Delta\bar{\psi}^{(1)} W \Delta\psi^{(1)} + \left(W\Delta\psi^{(2)} + \Delta\bar{\psi}^{(2)} W\right) + \left(\Delta\bar{\psi}^{(1)} \Delta\phi^{(1)} \frac{\delta W}{\delta\phi} + \frac{\delta W}{\delta\phi} \Delta\phi^{(1)} \Delta\psi^{(1)}\right), \quad (8.1)$$

where the superscripts (1) and (2) stand respectively for one- and two-loop renormalization of W, ϕ, ψ and $\bar{\psi}$.

$$\begin{aligned}
\Delta W^{(2)}_{\text{total}} &= W^{(2)}_{\text{double poles}} + \varepsilon\Big(h_1(W_i\hat{W}_jW_j\hat{W}_iW + \text{h.c.}) + h_2(W_i\hat{W}_jW_j\hat{W}W_i + \text{h.c.}) \\
&+ h_3(W_i\hat{W}_jW\hat{W}_jW_i) + h_4\left(\text{Tr}[M_i\cdot M_iM_j\cdot\overline{M}] - \text{Tr}[L_i\cdot L_iL_j\cdot\overline{L}]\right)(W_j) \\
&+ h_5\left(\text{Tr}[M_i\cdot M_jM_i\cdot\overline{M}] - \text{Tr}[L_i\cdot L_jL_i\cdot\overline{L}]\right)(W_j) + h_6\text{Tr}[M_i\cdot M_j](W_i\hat{W}_jW + \text{h.c.}) \\
&+ h_7\text{Tr}[M_i\cdot M_j](W_i\hat{W}W_j) + h_8\left(\text{Tr}[M_i\cdot\overline{M}U^2] + \text{Tr}[L_i\cdot\overline{L}X^2]\right)(W_i) \\
&+ h_9(W\hat{W}_iW_iU^2 + \text{h.c.}) + h_{10}(W_i\hat{W}_iWU^2 + \text{h.c.}) + h_{11}(W_i\hat{W}W_iU^2 + \text{h.c.}) \\
&+ h_{12}(W_i\hat{W}U^2W_i + \text{h.c.}) + h_{13}(W\hat{W}_iU^2W_i + \text{h.c.}) + h_{14}(\hat{U}^2WU^2) \\
&+ h_{15}(WU^2U^2 + \text{h.c.}) + (h_{16}C^{ab} + h_{17}R^{ab} + h_{18}Q^{ab})(WU^aU^b + \text{h.c.}) \\
&+ h_{19}T^2_{ij}(W_i\hat{W}W_j) + h_{20}(T^2\phi)_i(W_j\hat{W}_jW_i + \text{h.c.}) + h_{21}T^2_{ij}(W_i\hat{W}_jW + \text{h.c.}) \\
&+ h_{22}(T^a\phi)_iT^a_{jk}(W_i\hat{W}_jW_k + \text{h.c.}) + h_{23}(T^2\phi)_i\text{Tr}[M_i\cdot M_j](W_j) \\
&+ h_{24}(T^2\phi)_i(W_j\hat{W}_iW_j) + h_{25}(W_i\hat{W}_jW\hat{W}_iW_j) + h_{26}V_{ijk}(W_i\hat{W}_jW_k) \\
&+ h_{27}V_{ijk}V_{ijkl}(W_l) + (h_{28}C^{ab} + h_{29}R^{ab} + h_{30}Q^{ab})(T^aT^b\phi)_i(W_i) \\
&+ h_{31}(T^2\phi)_i(W^iU^2 + \text{h.c.}) + h_{32}(T^2T^2\phi)_i(W_i)\Big).
\end{aligned} \quad (8.2)$$

TABLE 1
Coefficients of (8.2) for both regularization schemes

	red.	reg.		red.	reg.		red.	reg.		red.	reg.
h_1	$-\frac{1}{4}$	$-\frac{1}{4}$	h_9	$\frac{1}{2}$	$\frac{7}{2}$	h_{17}	$-\frac{7}{6}$	$-\frac{11}{6}$	h_{25}	4	4
h_2	-2	-2	h_{10}	-4	-6	h_{18}	$-\frac{16}{3}$	$-\frac{20}{3}$	h_{26}	-4	-4
h_3	-4	-4	h_{11}	-6	-6	h_{19}	-4	-12	h_{27}	$\frac{1}{6}$	$\frac{1}{6}$
h_4	-6	-6	h_{12}	-14	-10	h_{20}	4	6	h_{28}	$-\frac{19}{6}$	$-\frac{49}{2}$
h_5	-4	-4	h_{13}	$-\frac{9}{2}$	$-\frac{11}{2}$	h_{21}	-3	-3	h_{29}	$-\frac{5}{6}$	$\frac{1}{2}$
h_6	-3	-3	h_{14}	-8	0	h_{22}	4	12	h_{30}	$\frac{4}{3}$	4
h_7	-8	-8	h_{15}	-3	-3	h_{23}	8	0	h_{31}	16	12
h_8	-28	-20	h_{16}	$\frac{59}{3}$	$\frac{97}{3}$	h_{24}	24	24	h_{32}	-21	-21

Of course we made use of the two important identities, coming from gauge invariance, in order to reduce the terms as much as possible:

$$[U_s^a, S_i] - i\{U_p^a, P_i\} = S_j T_{ji}^a, \tag{8.3}$$

$$[U_s^a, P_i] + i\{U_p^a, S_i\} = P_j T_{ji}^a. \tag{8.4}$$

And as expected [11] the single poles differ for both regularization schemes. A straightforward calculation yields that the β-function for g in the Yukawa coupling of supersymmetric SO(4) Yang-Mills [12] vanishes when we use the coefficients from the reduction scheme but not when we use those of the regularization scheme. So dimensional reduction seems the only candidate in preserving supersymmetry.

9. A toy model

As a toy model we use the one already introduced in [1]:

$$\mathcal{L} = -\tfrac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a - \bar{\psi}^a\gamma_\mu D_\mu\psi^a - \tfrac{1}{2}(D_\mu\phi^a)^2 - \tfrac{1}{8}\lambda[\phi^2]^2 - iyf^{abc}\bar{\psi}^a\phi^b\psi^c, \tag{9.1}$$

where the gauge group is SU(2).

Using (5.9) and (8.2) for both regularization schemes, we got for the β-functions of the Yukawa coupling

$$\beta_{\text{reg}}(y) = \frac{1}{(4\pi)^2}(+8y^3 - 12yg^2) + \frac{1}{(4\pi)^2}(-43y^5 + 92y^3g^2 - 5y^3\lambda - \tfrac{160}{3}yg^4 + \tfrac{5}{8}y\lambda^2), \tag{9.2}$$

$$\beta_{\text{reg}}(y) = \frac{1}{(4\pi)^2}(+8y^3 - 12yg^2) + \frac{1}{(4\pi)^4}(-43y^5 + 92y^3g^2 - 5y^3\lambda - \tfrac{136}{3}yg^4 + \tfrac{5}{8}y\lambda^2). \tag{9.3}$$

As the β-functions for the two other coupling constants are also known for both regularization schemes [1], we tried to find a finite transformation for the three coupling constants

$$\bar{\lambda}_i = \lambda_i + f^i(g, y, \lambda) \tag{9.4}$$

to transform the three β-functions in one regularization scheme simultaneously into the β-function of the other scheme. We proved that already for such a relatively simple theory as (9.1) this transformation does not exist. A good regularization scheme is required to be the equivalent to others (such as dimensional regularization or Pauli-Villars) via a substitution in $\mathcal{L}$ at lower orders. This clash will be discussed in ref. [14].

10. Discussion and conclusions

Using a massive calculation method we computed the two-loop infinities in the Yukawa sector for a completely general renormalizable field theory. This was done with two different regularization schemes, dimensional regularization [3] and dimensional reduction [4] and as expected the two results for the single poles differ. The final result (8.2) yields immediately the β-function of the Yukawa coupling constant as it is proportional to the residues of the single poles in (8.2). For the β-function of the mass of the fermion some more work has to be done because it is inevitable that infinite constant shifts have to be carried out in the scalar field in order to get rid of infinite linear terms in the counter-lagrangian (appendix B). Furthermore we showed that in general it is not possible to transform all β-functions in a theory, computed in both regularization schemes, into each other simultaneously by a finite redefinition of the coupling constants involved. We therefore conclude that dimensional reduction in the way proposed by Siegel [4] is inconsistent.

Appendix A

IDENTITIES AND INTEGRALS USED

Denoting $W_2(\alpha, \beta, \gamma)$ (which is of course completely symmetric in its indices) and $W_1(\alpha)$ by

$$W_2(\alpha, \beta, \gamma) = \iint \frac{(32\pi^2)^2}{[p^2+m^2]^\alpha [q^2+m^2]^\beta [(p-q)^2+m^2]^\gamma} \, d_n p \, d_n q, \tag{A.1}$$

$$W_1(\alpha) = \int \frac{32\pi^2}{[p^2+m^2]^\alpha} \, d_n p, \tag{A.2}$$

the only identities and integrals needed are the following. Identities:

$$\begin{aligned} W_2(-2, \alpha, \beta) = {} & W_1(\alpha-2) W_1(\beta) + W_1(\alpha) W_1(\beta-2) \\ & + (2 + \tfrac{1}{4}\varepsilon) W_1(\alpha) W_1(\beta) + (3 + \tfrac{1}{4}\varepsilon)\big(W_1(\alpha-1) W_1(\beta-1) \\ & - W_1(\alpha-1) W_1(\beta) - W_1(\alpha) W_1(\beta-1)\big), \end{aligned} \tag{A.3}$$

$$W_2(-1,\alpha,\beta) = W_1(\alpha)W_1(\beta-1) + W_1(\alpha-1)W_1(\beta) - W_1(\alpha)W_1(\beta), \quad \text{(A.4)}$$

$$W_2(0,\alpha,\beta) = W_1(\alpha)\cdot W_1(\beta). \quad \text{(A.5)}$$

Integrals:

$$W_1(1) = -4/\varepsilon - 2, \quad \text{(A.6)}$$

$$W_1(2) = 4/\varepsilon, \quad \text{(A.7)}$$

$$W_1(3) = 1, \quad \text{(A.8)}$$

$$W_1(4) = \tfrac{1}{3}, \quad \text{(A.9)}$$

$$W_1(5) = \tfrac{1}{6}. \quad \text{(A.10)}$$

$$W_2(2,1,1) = 8/\varepsilon^2 + 4/\varepsilon, \quad \text{(A.11)}$$

$$W_2(3,1,1) = 4/\varepsilon, \quad \text{(A.12)}$$

$$W_2(4,1,1) = 4/3\varepsilon, \quad \text{(A.13)}$$

$$W_2(5,1,1) = 2/3\varepsilon, \quad \text{(A.14)}$$

$$W_1(0) = 0, \quad \text{(A.15)}$$

Appendix B

EXAMPLE OF A THEORY WHERE THE MASS RENORMALIZATION IS NOT PURELY MULTIPLICATIVE

Consider the following lagrangian

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\mu\phi)^2 - \bar{\psi}\gamma_\mu\partial_\mu\psi - \bar{\psi}(y\phi + m)\psi - \tfrac{1}{2}M^2\phi^2 - \tfrac{1}{6}\kappa\phi^3 - \tfrac{1}{24}\lambda\phi^4. \quad \text{(B.1)}$$

Observe we added a ϕ^3 term, *needed* for renormalizability, as will be shown in the following. Moreover the κ in front of this term, which is a mass, has a ρ unequal to zero ($\rho = \frac{1}{2}$).

According to [8]

$$\begin{aligned} 32\pi^2\varepsilon\Delta &= \left(\tfrac{1}{2}\lambda\phi^2 + \kappa\phi + M^2\right)^2 + 4\left(\tfrac{1}{6}\lambda\phi^4 + \tfrac{1}{2}\kappa\phi^3 + M^2\phi^2\right)y^2 \\ &\quad - (\lambda\phi + m)^4 + \alpha\left(\tfrac{1}{6}\lambda\phi^3 + \tfrac{1}{2}\kappa\phi^2 + M^2\phi\right) \\ &= \text{const} + \phi\left(2\kappa M^2 - 16ym^3 + \alpha M^2\right) \\ &\quad + \phi^2\left(\lambda M^2 + \kappa^2 - 24y^2m^2 + 4M^2y^2 + \tfrac{1}{2}\alpha\kappa\right) \\ &\quad + \phi^3\left(\lambda\kappa - 16y^3m + 2\kappa y^2 + \tfrac{1}{6}\alpha\lambda\right) + \mathrm{O}(\phi^4). \end{aligned} \quad \text{(B.2)}$$

(Note a term proportional to ϕ^3 is present, so κ was needed in the original lagrangian.)

We added an infinite shift in ϕ:

$$\phi \to \phi + \frac{\alpha}{32\pi^2\varepsilon}\phi, \tag{B.3}$$

in order to get rid of the linear term in ΔV, which we obviously do not want.

Now α is fixed under this condition:

$$\alpha = -2\kappa + 16\frac{m^3}{M^2}. \tag{B.4}$$

After this shift the quadratic term in ϕ becomes

$$\beta(M^2) \propto a_1^{M^2} = \lambda M^2 - 24y^2m^2 + 4y^2M^2 + 8y\kappa\frac{m^3}{M^2}, \tag{B.5}$$

$$\beta(\kappa) \propto a_1^{\kappa} = \tfrac{2}{3}\lambda\kappa - 16y^3m + 2\kappa y^2 + \tfrac{8}{3}\lambda y\frac{m^3}{M^2}. \tag{B.6}$$

And the poles in one-loop for the fermion can also be determined from [8]

$$32\pi^2\varepsilon W = 6y^2((y+\alpha)+m) + 4y^3\phi, \tag{B.7}$$

so

$$\beta(m) \propto a_1^m = 6y^2m - 2\kappa y + 16y^2\frac{m^3}{M^2}. \tag{B.8}$$

Note that dimensionless coupling constants can never get factors of m_1^2/m_2^2 in their β-functions as they can never be affected by a translation of a scalar field.

For gauge theories nothing really changes as long as there is a scalar field that does not couple to any vector field. Take the only realistic theory we have so far, the standard model [13], there indeed we have such a particle, the Higgs field. But as one can see from (B.8) the effects will be extremely slight ($m \ll M$).

One might expect that the fact that now ratios of masses occur, seems to undermine our "massive calculation scheme", described in sect. 4. However β-functions are determined after an infinite shift in the scalar field, whereas the infinities are computed before them.

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References

[1] R. van Damme, Nucl. Phys. B227 (1983) 317
[2] D.R.T. Jones, Nucl. Phys. B87 (1975) 127
[3] G. 't Hooft and M. Veltman, Nucl. Phys. B44 (1972) 189
[4] W. Siegel, Phys. Lett. 84B (1979) 193
[5] M. Veltman, Program Schoonschip;
H. Strubbe, Comp. Phys. Comm. 8 (1974) 1
[6] G. 't Hooft, Nucl. Phys. B61 (1973) 455
[7] L. Abbott, Lectures at 21st Cracow School of Theoretical Physics, Cern preprint TH 3113 (1981)
[8] R. van Damme, Phys. Lett. 110B (1982) 239
[9] J. Wess and B. Zumino, Nucl. Phys. B70 (1974) 34;
A. Salam and J. Strathdee, Nucl. Phys. B76 (1974) 477
[10] D.M. Capper, D.R.T. Jones and P. van Nieuwenhuizen, Nucl. Phys. B167 (1980) 479
[11] A. Schellekens, PhD thesis (1981)
[12] F. Gliozzi, J. Scherk and D. Olive, Nucl. Phys. B122 (1977) 253;
L. Brink, J. Schwarz and J. Scherk, Nucl. Phys. B121 (1977) 77
[13] S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264
[14] R. van Damme and G. 't Hooft, in preparation