AIP Applied Physics Letters

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Citation: Appl. Phys. Lett. **99**, 262506 (2011); doi: 10.1063/1.3672207
View online: http://dx.doi.org/10.1063/1.3672207
View Table of Contents: http://apl.aip.org/resource/1/APPLAB/v99/i26
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Spin-transfer mechanism for magnon-drag thermopower

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(Received 19 September 2011; accepted 1 December 2011; published online 30 December 2011)

We point out a relation between the dissipative spin-transfer-torque parameter β and the contribution of magnon drag to the thermoelectric power in conducting ferromagnets. Using this result, we estimate β in iron at low temperatures, where magnon drag is believed to be the dominant contribution to the thermopower. Our results may be used to determine β from magnon-drag-thermopower experiments, or, conversely, to infer the strength of magnon drag via experiments on spin transfer. © *2011 American Institute of Physics*. [doi:10.1063/1.3672207]

A recurring theme in the field of spintronics is the interplay between electric and spin currents, and magnetization dynamics in conducting ferromagnets. This activity was initiated by the theoretical prediction of Slonczewski[1] and Berger[2] who showed that magnetic layers in nanopillars can be excited or even reversed by spin-polarized currents.[3] The underlying mechanism is dubbed spin transfer, as it involves the transfer of spin angular momentum from conduction electrons to magnetization. In part because of its promise for applications such as magnetic memories, spin transfer is now actively studied in the context of current-driven domain wall motion in magnetic nanowires.[4] As a result of these efforts, it is now understood[5] that there are at least two contribution to spin transfer in the long-wavelength limit, one reactive (sometimes called adiabatic)[6] and one dissipative.[7] This latter torque is parameterized by a dimensionless constant β, and the ratio of this constant to the Gilbert magnetization damping constant α is of crucial importance for the phenomenology of current-driven domain-wall motion.[8,9] Precise experimental determination of β from domain-wall experiments[10] or experiments on magnetic vortices[11] is, however, difficult.

A closely related development is the study of spin and charge currents induced by time-dependent magnetization, called spin pumping in layered systems,[12] and usually referred to as spin motive forces in magnetic textures.[13] The latter were observed in a very recent experiment on field-driven domain walls[14] and proposed for magnetic vortices.[15] Like spin transfer, spin motive forces have two contributions corresponding to the reciprocal of the reactive and dissipative spin-transfer torques.[16] In particular, the current induced by a time-dependent magnetization texture also depends on the parameter β.

In this letter, we show that β is determined by the thermoelectric power due to electron-magnon scattering, the so-called magnon-drag thermopower.[17] This result is derived by considering the electric current density induced by a time-dependent magnetization, with direction determined by the unit vector $\mathbf{m}(\vec{x},t)$, which is given by[13,16]

$$\vec{j} = -\frac{\hbar P\sigma}{2|e|}\left[\mathbf{m}\cdot\left(\frac{\partial \mathbf{m}}{\partial t}\times\frac{\partial \mathbf{m}}{\partial \vec{x}}\right) + \beta\frac{\partial \mathbf{m}}{\partial t}\cdot\frac{\partial \mathbf{m}}{\partial \vec{x}}\right], \qquad (1)$$

where $-|e|$ is the electron charge, σ is the electrical conductivity, and P is the current spin polarization. Although the above expression is usually considered for magnetization textures such as domain walls or magnetic vortices, it is straightforwardly evaluated for a magnetic configuration corresponding to a transport steady state of (Holstein-Primakoff) magnons. This results in[18]

$$\vec{j} = \beta\frac{\hbar^2\gamma P\sigma}{2|e|M_s D}\vec{j}_{Q,m}, \qquad (2)$$

where M_s is the saturation magnetization density, and γ is the (minus) gyromagnetic ratio. Equation (2) shows that a magnon heat current $\vec{j}_{Q,m}$ results in an electrical current $\vec{j}$. One assumption leading to the above result is that the energy of magnons with wave vector $\vec{k}$ is equal to $\hbar\omega_{\vec{k}} = Dk^2$, in terms of the spin stiffness D. This is a valid approximation for temperatures larger than the magnon gap, which is typically $\sim$1 K in metallic ferromagnets. In principle, we cannot exclude a contribution of order α to the right-hand side of Eq. (2), coming from the first term in Eq. (1). However, if $\alpha \ll \beta$, this correction would be small. (Experimental results on permalloy put β in the range $\geq\alpha$,[10,11] and $\beta \gg \alpha$ has been reported in some of these experiments.)

To understand how the above result is related to magnon-drag thermopower, we consider the response of the system to electric field $\vec{E}$, magnon-temperature, and electron-phonon-temperature gradients, denoted by $\vec{\nabla}T_m$ and $\vec{\nabla}T_{e,p}$, respectively. Introducing two different temperatures for these subsystems is in the present case needed to make connection with the result in Eq. (2). We note that in theoretical discussions[19] of the spin-Seebeck effect,[20] such temperature differences are also invoked. The linear-response coefficients are determined by

$$\begin{pmatrix} \vec{j} \\ \vec{j}_Q \\ \vec{j}_{Q,m} \end{pmatrix} = \begin{pmatrix} \sigma & \sigma S_{e,p}T & \sigma S_m T \\ \sigma S_{e,p}T & \kappa'_{e,p}T & \zeta T \\ \sigma S_m T & \zeta T & \kappa'_m T \end{pmatrix} \begin{pmatrix} \vec{E} \\ -\dfrac{\vec{\nabla}T_{e,p}}{T} \\ -\dfrac{\vec{\nabla}T_m}{T} \end{pmatrix}, \qquad (3)$$

where $\vec{j}_Q$ is the heat current carried by electrons and phonons. In the above, the magnon-drag thermopower is denoted by S_m and the magnon heat conductivity at zero electric field

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by $\kappa_m^{'}$. The contribution of electrons and phonons to the thermopower is denoted by the Seebeck coefficient $S_{e,p}$, and their heat conductivity at zero field by $\kappa_{e,p}^{'}$. Drag effects between magnon heat currents and electron-phonon heat currents are denoted by ζ. Also note that we have used Onsager relations to eliminate the Peltier coefficients.

It is important to point out that disentangling heat currents in the above way only applies to weakly coupled situations. In case this is not possible, such that only the total Seebeck coefficient and thermal conductivity can be measured, our results below are applicable to the case that the thermal transport is dominated by magnons.

The result in Eq. (2) applies to the situation that the electron-phonon temperature gradient and electric field are zero. Taking $\vec{E} = \vec{\nabla} T_{e,p} = 0$ and $\vec{\nabla} T_m \neq 0$, we find a magnon heat current and a charge current that are proportional to $\vec{\nabla} T_m$ such that we have $\vec{j} = \vec{j}_{Q,m} \sigma S_m / \kappa_m^{'}$. We combine this with Eq. (2) to find our main result

$$\beta = \frac{2|e|M_s D}{\hbar^2 \gamma P} \frac{S_m}{\kappa_m^{'}}. \quad (4)$$

This result relates the spin-torque parameter β to the magnon-drag thermopower and the magnon heat conductivity at zero field $\kappa_m^{'}$. The magnon heat conductivity κ_m at zero electric current, defined by $j_{Q,m} = -\kappa_m \nabla T$ with $T_m = T_{e,p} = T$, in terms of the above transport coefficients, is given by $\kappa_m = \kappa_m^{'} + \zeta - \sigma S_m (S_{e,p} + S_m) T$. The last correction is small in most materials, except for very good thermoelectric materials. Assuming that the magnon-electron heat drag is small, i.e., $\zeta \ll \kappa_m^{'}$, we take $\kappa_m^{'} \approx \kappa_m$ in our estimates. Furthermore, we note that possible corrections of order α to the right-hand side of Eq. (2) that we mentioned previously would lead to similar corrections to the right-hand side of the above result and are again negligible provided $\beta \gg \alpha$.

We now estimate β using available experimental data on magnon-drag thermopower and magnon heat conductivity. In this order-of-magnitude estimate we take, for simplicity, $P = 1$, $\gamma = 2\mu_B/\hbar$ (μ_B is the Bohr magneton), and $M_s = \mu_B/a^3$ with $a \simeq 0.3$ nm a typical lattice constant. According to Blatt *et al.*,[21] the main contribution to the thermopower in iron at low temperatures is due to magnon drag and they give the result $S_m \approx 0.016\ (T/K)^{3/2}\ \mu$V/K. Hsu and Berger[22] find the value of $\kappa_m = 4.9 \times 10^{-2}$ W/K m for $Fe_{95}Si_5$ at 4 K (the iron is silicon doped to decrease the electronic contribution to the heat conductivity). Using a typical value $D = 4 \times 10^{-40}$ J m^2 (Ref. 22) for the spin stiffness, we find that $\beta \approx 0.1$ at 4 K. The main uncertainty in our estimate is the value of κ_m which is difficult to measure. Nonetheless, this value for β seems not unreasonable as room-temperature values for this parameter obtained from spin-transfer experiments usually find that $\beta \sim 0.1 - 0.01$ for permalloy.[10,11] We also point out that the $T^{3/2}$ temperature scaling of S_m would imply, according to Eq. (4), that $\kappa_m \propto T^{3/2}/\beta$. It can be shown, on the other hand, that $\kappa_m \propto T^{3/2}/\alpha$ (Ref. 23) within the Landau-Lifshitz-Gilbert phenomenology, suggesting the ratio β/α is insensitive to temperature, which, in turn, is supported by microscopic calculations.[9,25]

The transport coefficients in Eq. (3) determine the dissipation, which must be positive by the second law of thermodynamics. This imposes the condition that the determinant of the response matrix be positive, which is satisfied if

$$T\sigma\left(\frac{S_{e,p}^2}{\kappa_{e,p}^{'}} + \frac{S_m^2}{\kappa_m^{'}}\right) + \frac{\zeta}{\kappa_{e,p}^{'}\kappa_m^{'}}(\zeta - 2S_{e,p}S_m) \leq 1. \quad (5)$$

It is conventional to define $Z_{e,p}^{'} = \sigma S_{e,p}^2/\kappa_{e,p}^{'}$, $Z_m^{'} = \sigma S_m^2/\kappa_m^{'}$, so that this relation reads $Z_{e,p}^{'}T + Z_m^{'}T \leq 1$, where we assumed $\zeta \ll \kappa_m^{'}$ like before. Using Eq. (4), this condition imposes an upper bound on β

$$\beta \leq \frac{2|e|M_s D}{\hbar^2 \gamma P} \frac{1}{\sigma S_m T}(1 - Z_{e,p}^{'}T), \quad (6)$$

where σS_m and γP are assumed positive, which is typically the case. Using the same values as in our previous estimate and taking $Z_{e,p}^{'}T \ll 1$, we find that $\beta \lesssim 1$ at 4 K, using a value of $\sigma \approx 10^{11}$ Ω m at 4 K.[24] This result gives an upper bound for β for a material, which is particularly useful when looking for materials with large β. This is of particular interest for spintronics applications, since large β implies a large current-to-domain-wall coupling.

In conclusion, we have shown that the spin-transfer-torque parameter β is related to the ratio of the magnon-drag thermopower and the magnon heat conductivity. From an experimental point-of-view, this relation can be used to either determine β experimentally or to obtain information on the contribution of magnons to heat conduction and thermopower from experimental knowledge of β. From a theoretical point-of-view, the relation derived in this letter opens the way for methods to calculate β. The microscopic calculations in the literature usually focus on the contribution to β due to spin-dependent disorder scattering[9,25] or take into account scattering phenomenologically.[26] In future work, we intend to microscopically determine β by calculating the magnon-drag thermopower and magnon heat conductivity and then using the relation in Eq. (4). With respect to this, it is important to mention that the relation in Eq. (4) contains all contributions to β provided that $\beta \gg \alpha$. We expect that using this relation will be particularly useful in determining the temperature dependence of β.

This work was supported by Stichting voor Fundamenteel Onderzoek der Materie (FOM), the Netherlands Organization for Scientific Research (NWO), by the European Research Council (ERC) under the Seventh Framework Program (FP7), the Alfred P. Sloan Foundation, and by the NSF under Grant No. DMR-0840965.

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[18]It is obtained by evaluating $\langle \mathbf{m}\cdot(\partial\mathbf{m}/\partial t\times\partial\mathbf{m}/\partial\vec{x})+\beta\partial\mathbf{m}/\partial t\cdot\partial\mathbf{m}/\partial\vec{x}\rangle$ after linearizing $\mathbf{m}\simeq(\delta m_x,\delta m_y,1)$ and a Holstein-Primakoff transformation $\hat{b},\hat{b}^\dagger\rightarrow\sqrt{S/2\hbar}(\delta m_x\pm i\delta m_y)$ to canonical boson operators $\hat{b}$ and $\hat{b}^\dagger$, where S is the spin density. This yields (after normal ordering) $2\int d^3k\hbar\omega_{\vec{k}}\vec{k}\langle\hat{b}^\dagger_{\vec{k}}\hat{b}_{\vec{k}}\rangle/(2\pi)^3S$. Here, we assumed that the magnons can be treated semi-classically. For quadratic magnon dispersions $\hbar\omega_{\vec{k}}=Dk^2$, we can rewrite it in terms of the magnon heat current $\vec{j}_{Q,m}\equiv\int d^3k\hbar\omega_{\vec{k}}\vec{v}_{\vec{k}}\langle\hat{b}^\dagger_{\vec{k}}\hat{b}_{\vec{k}}\rangle/(2\pi)^3$ with $\vec{v}_{\vec{k}}=\partial\omega_{\vec{k}}/\partial\vec{k}$, obtaining the final result in Eq. (2) with $M_s=\gamma S$.

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