A Model Study of the Relation Between Northern Hemisphere Glaciation and Precipitation Rates

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Abstract:

In order to study the relation between ice cover and precipitation rates in the Northern Hemisphere, we have forced a zonal dynamical climate model by prescribing the degree of glaciation as a lower boundary condition. The model is a modified version of the one developed by SALTZMAN and VERNEKAR (1971). It includes a hydrological cycle and computes a zonally symmetric solution for summer and winter.

We have found that if ice sheets grow to the south, the precipitation rate in the subtropics and the polar basin decreases, but in the latitude belt where the continental ice sheets are located it increases. This effect is due to en enhanced activity of baroclinic waves along the southern edge of the ice cover and a consequent increase of the poleward water vapour flux. If the ice cover extends south of 50 °N, the average precipitation rate over the ice sheet starts to decrease. These results are in agreement with the 'observational' fact that during full glacials the Northern Hemisphere continental ice sheets extend to about 50 °N.

Zusammenfassung: Eine Modell-Studie der Beziehung zwischen der Vereisung der Nordhalbkugel und der Niederschlagsintensität

Um die Beziehung zwischen Eisbedeckung und Niederschlagsintensität auf der Nordhalbkugel zu studieren wurde ein zonales dynamisches Klimamodell benutzt, bei dem der Grad der Vereisung als eine der unteren Randbedingungen vorgegeben war. Das Modell ist eine modifizierte Version desjenigen, das von SALTZMAN und VERNEKAR (1971) entwickelt wurde. Es enthält einen hydrologischen Zyklus und berechnet eine zonal symmetrische Lösung für Sommer und Winter.

Wir fanden, daß, wenn die Eisbedeckung südwärts wächst, der Niederschlag in den Subtropen und in Polnähe abnimmt, aber in den Breiten des kontinentalen Eises zunimmt. Dieser Effekt rührt her von der zunehmenden Aktivität der baroklinen Wellen entlang dem südlichen Rand der Eisbedeckung und einer daraus folgenden Zunahme des polwärts gerichteten Wasserdampf-Transportes. Wenn die Eisbedeckung sich weiter südlich als 50 °N ausdehnt, beginnt eine Abnahme der Niederschlagsintensität über dem Eis. Diese Ergebnisse hängen zusammen mit der „Beobachtungs-Tatsache", daß im Kern der Eiszeiten das Eis auf der Nordhalbkugel sich bis etwa 50 °N ausdehnte.

Résumé: Une Etude de Modele de la Relation Entre la Glaciation de l'Hemisphere Nord et les Taux de Precipitations

En vue d'étudier la relation entre la couverture de glace et les taux de précipitations dans l'hémisphère nord, on utilise un modèle dynamique zonal de climat où est prescrit, comme condition à la limite inférieure, le degré de glaciation. Le modèle est une version modifiée de celui de SALTZMAN et VERNEKAR (1971). Il comporte un cycle hydrologique et on calcule une solution symétrique zonale pour l'été et l'hiver.

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On trouve que si la glace s'étend vers le sud, le taux de précipitation décroît dans la zone subtropicale et près du pôle, mais augmente aux latitudes des glaces continentals. Cet effet est dû à une activité accrue des ondes baroclines le long de la limite sud de la couverture glaciaire et à l'augmentation du flux de vapeur d'eau vers le pôle qui en résulte. Si la couverture glaciaire s'étend au sud de 50 °N, alors commence une diminution du taux moyen de précipitation sur la glace. Ces résultats concordent avec le «fait d'observation» suivant lequel la glace s'étend jusqu'à environ 50 °N, lorsque la période glaciaire atteint son plein développement.

1 Introduction

One of the major problems in studies on the growth and decay of the Northern Hemisphere ice sheets is the parameterization of the ice accumulation rate (also referred to as mass balance). The difficulty lies in estimating precipitation rates in other climates. Due to the profound influence of extensive ice cover on the hydrological cycle, a substantial change in precipitation is expected if the degree of glaciation on earth changes.

It has been argued that during glacials the precipitation amounts are much smaller than at present. The potential effects of temperature dependent precipitation in creating climatic variations on the global scale (involving large changes in Northern Hemisphere ice cover) has recently been investigated by KÄLLEN et al. (1979). They find that a simple zero-dimensional climate model may exhibit internal oscillations if hemispheric temperature and snow accumulation are negatively correlated. Crucial to whether such oscillations occur is the strength of the ice-albedo-temperature feedback and the limits between which the accumulation of snow over the ice sheets varies. KÄLLEN et al. assume that a 10 K increase in hemispheric temperature (from 273 K to 283 K) creates a 400 % increase in accumulation.

At present, no evidence exists that accumulation of snow indeed varies so strongly with hemispheric temperature. Reliable proxy data are confined to lower latitudes. There is hardly any observational information on precipitation over the Laurentide and European Ice Sheets. On a global scale precipitation amounts will be less if atmospheric temperature is lower, simply because the ability of the air to contain water vapour is less. However, on a more regional scale, the picture will be modified substantially by atmospheric motions. Therefore, climate models including atmospheric dynamics and a hydrological cycle form the best tool to study the relation between precipitation rates and the degree of Northern Hemisphere glaciation.

Experiments with general circulation models (GCM's) of the atmosphere, using ice-age boundary conditions, have provided some valuable information on departure of ice-age precipitation patterns from present-day conditions. See HEATH (1979) for a review of these experiments. Each GCM produces an ice-age climate in which the global mean precipitation rate is lower than today, but on the regional scale substantial differences are present. A disadvantage of GCM experiments is that the signal-to-noise ratio is generally quite small, which makes it dangerous to draw definite conclusions from results based on integration over a short period. Most model integrations do not even cover a complete seasonal cycle.

A valuable addition to the information provided by GCM's may be obtained by experiments with zonal mean climate models in which the characteristics of the atmospheric circulation are computed. Such models have the advantage of requiring moderate computational resources, while they nevertheless have more internal freedom than the so-called energy balance climate models, which do not deal explicitly with atmospheric dynamics.

The dynamical zonal mean climate model developed by SALTZMAN and VERNEKAR (1971), hereafter referred to as the SV-model, includes a hydrological cycle and may therefore be used to study the latitudinal distribution of precipitation for various surface boundary conditions. We have conducted a number of experiments with this model. We varied the (prescribed) sea-ice boundary and the extent of the continental ice sheets *without changing the insolation* at the top of the atmosphere. This type of experiment is use-

ful, because it isolates the interaction between ice cover and climatic regime. Apart from this, the behaviour of the Northern Hemisphere cryosphere is highly nonlinear, and it seems likely that ice sheets were sometimes present during periods in which insolation was even above the long-term mean value (OERLEMANS, 1980).

In discussing the results of these experiments, we shall concentrate on the variation of the latitudinal distribution of precipitation as a result of changes in ice cover. The results will be discussed in the light of present ideas on the behaviour of the Northern Hemisphere continental ice sheets.

2 Brief Description of the SV-Model

The model we used in this study is a slightly modified version of that described in detail in SALTZMAN and VERNEKAR (1971). The modifications are:

(i) the feedback between water-vapour content and longwave radiation has been included,

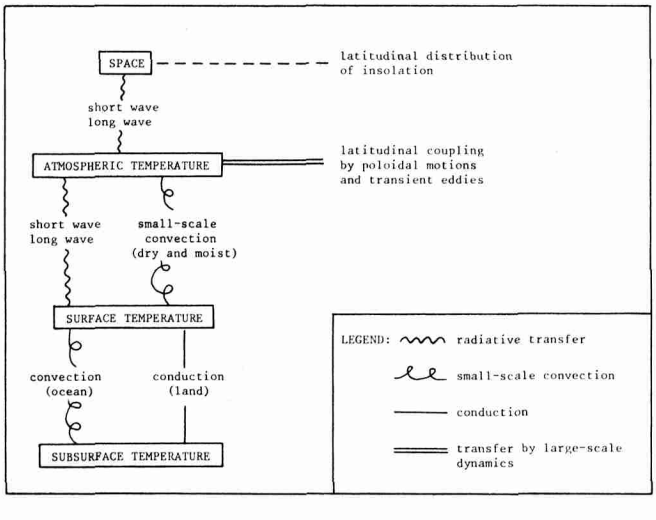
(ii) a number of empirical constants have been made independent of latitude.

A further discussion on these modification will soon be published (VERNEKAR, 1981).

We will now turn to a brief description of the SV-model, without discussing the technical details and mathematical formulation. Figure 1 presents an overview of the model structure in terms of heat balances. The various transfer mechanisms affecting the heat balances of the atmosphere, the surface layer, and the subsurface layer are indicated in the figure. Note that the latitudinal exchange of heat is accomplished by the large-scale transient eddies and poloidal motions. Heat transport by ocean currents is not explicitly modelled.

The model is formulated for the Northern Hemisphere geometry and computes steady-state solutions for summer and winter conditions. This is possible because a subsurface layer is included that damps the yearly cycle (as it does in reality). In the model the subsurface temperature for land equals the average of summer and winter surface temperatures, and hence it is a part of the solution. Ocean subsurface temperature is however prescribed from observations.

In computing the zonal mean heating of the atmosphere from below, the surface layer is treated separately for ocean and continent. Differences stem from different surface albedos, different water availability (wetness) and a different formulation of heat exchange with the subsurface layer.



- **Figure 1**
  Structure of the SV-model in terms of heat balances. For a detailed description of this model, see Saltzman and Vernekar (1971).

- **Bild 1**
  Struktur des SV-Modells ausgedrückt über die Wärmebilanzen. Eine ausführliche Beschreibung dieses Modells findet man bei Saltzman und Vernekar (1971).

The solutions are obtained using an iterative procedure. First, the solutions for the surface and mid-tropospheric temperature are computed by neglecting transport of heat by poloidal motions and effects of the convergence of water vapour by eddies and poloidal motions on the latent heat released by precipitation. The next step is to compute the zonal distributions of eddy transport of momentum and water vapour, using bulk properties of baroclinic and barotropic waves. The poloidal motions are computed such that the conservation of momentum is satisfied.

From these results the heat transport by poloidal motions and the effect of convergence of water vapour on the latent heat released by precipitation are derived. These effects are then included in the thermodynamic energy equation to compute new solutions for the surface and atmospheric temperature. This procedure is repeated until an equilibrium state is obtained.

The most relevant feature of the model to the present study is the hydrological cycle. Latitudinal transport of water vapour is accomplished by the poloidal motions and by the transient eddies. If we denote the mixing ratio by ϵ, and resolve this in a zonal mean part ϵ_0 and an eddy part ϵ', these transports can be denoted by $[\epsilon_0 v_0]$ and $[\overline{\epsilon' v'}]$, where the square brackets indicate a vertical mean over the depth of the atmosphere. The eddy flux is set proportional to $(\overline{v'^2})^{1/2} \partial\epsilon_0/\partial\varphi$, where φ is latitude. The vertical variation of ϵ is a prescribed function of pressure, and the relative humidity at the surface is taken constant (77 %). This implies that the mixing ratio only depends on surface temperature.

In a steady-state model, the rate of precipitation P may be expressed as the difference between evaporation E and the convergence of water vapour in the atmosphere, so

$$\mathrm{div}[\epsilon_0 v_0] + \mathrm{div}[\overline{\epsilon' v'}] - E + P = 0.$$

Evaporation depends on the vertical temperature gradient (stability) near the surface, and is proportional to the wetness w of the surface layer (w = 1 over sea; w = 0.8 over land; w = 0 over snow and ice).

The treatment of the hydrological cycle sketched above permits a number of feedback loops to act in the model. To illustrate this, let us consider a situation in which the polar ice cap extends. The evaporation over the ice reduces to zero because the water availability w is zero. Due to the increased surface albedo the surface temperature decreases and hence reduces the precipitable water in the atmosphere. Both these processes tend to reduce precipitation over the region. But at the same time the gradient of humidity and temperature near the edge of the ice cap increase. This enhances the northward flux of water vapour and hence the precipitation over the ice cap. An accompanied strengthening of the westerlies over the ice boundary may enforce this effect.

The only test with which a climate model can be validated in some detail is to simulate the present climate. A discussion on the performance of the SV-model in such a test can be found in SALTZMAN and VERNEKAR (1971). In broad sense, the present climate is simulated quite well, but some shortcomings also exist:

(i) the midlatitude jet is too wide, in particular in winter (but of correct strength and location),
(ii) surface temperatures in the midlatitudes are somewhat high in summer and low in winter,
(iii) eddy activity is too small north of 70 °N.

In spite of these shortcomings, we believe that the model has adequate physics to yields realistic responses to changes in the surface boundary conditions. The SV-model was used earlier to calculate the response to the surface boundary conditions that prevailed during the last ice age (SALTZMAN and VERNEKAR, 1975). The results were by and large in agreement with similar studies based on GCM's (e. g. GATES, 1976).

We calculated the model response to different surface boundary conditions without changing the insolation. In the first experiment, we computed equilibrium states for a southward continental ice extent to 68 °N, 58 °N, 48 °N and 38 °N (same for summer and winter). In the second experiment, continental ice extent was varied in the same way, but now sea ice cover was adjusted (Table 1). In prescribing the extent of sea ice some ambiguity cannot be avoided, but for the present sensitivity experiments this is not too serious.

■ **Table 1** A listing of the experiments with adjustment of the sea-ice extent.

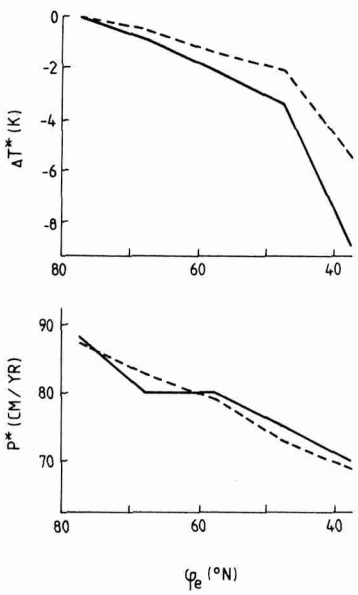
■ **Tabelle 1** Liste der Experimente mit Anpassung der Ausdehnung des See-Eises.

continental ice up to	sea ice up to	
	summer	winter
68 °N	73 °N	58 °N
58 °N	68 °N	53 °N
48 °N	63 °N	48 °N
38 °N	58 °N	43 °N

3 Hemispheric Statistics

Figure 2 shows the variations of summer and winter precipitation rates and surface temperature as a function of φ_e, the latitude at which the southern edge of the continental ice sheet is located. Values shown are hemispheric mean values (denoted by *), and apply to the experiment in which sea ice was adjusted according to Table 1. The drop in T* is given with respect to a control run, displayed in the figure by φ_e = 78 °N, which serves as the present climate.

Apparently, T* drops off more rapidly if φ_e decreases. For a very high degree of glaciation (φ_e = 38 °N, which probably never occurred), this drop amounts to 9 K in winter, and 5.5 K in summer. Conditions during the last glacial maximum (18000 yr BC) roughly correspond to φ_e = 50 °N, and in this case the present model predicts temperature drops of 3 and 2 K, respectively. Comparison with GCM experiments reveals that these drops are rather small: with ice-age boundary conditions, the GFDL model produces a climate in which T* for the summer halfyear is 5.8 K lower than today. GATES (1976), using the RAND model, finds a 4.9 K drop in July temperature, and experiments with the NCAR model (WILLIAMS et al., 1974) yielded even lower temperatures. The discripancy stems mainly from the fact that in the present experiment we used present-day ocean subsurface temperatures. If ocean subsurface temperatures are



● **Figure 2**
Hemispheric temperature drop (ΔT^*) and precipitation rate (P*) as a function of φ_e (latitude of the southern edge of the continental ice sheet). Dashed lines: summer; solid lines: winter. Results apply to the experiment with adjusted sea ice.

● **Bild 2**
Hemisphärische Temperaturabnahme (ΔT^*) und Niederschlagsintensität (P*) als Funktionen von φ_e = Breite des südlichen Randes der kontinentalen Vereisung. Gestrichelt: Sommer; ausgezogen: Winter. Die Ergebnisse gelten für das Experiment mit angepaßter Ausdehnung des See-Eises.

adjusted, as was done in SALTZMAN and VERNEKAR (1975), the SV-model gives results similar to those of the GCM experiments.

With regard to Figure 2 it should be noted that the fact that winter temperature decreases more strongly with ice-sheet size than summer temperature should not be taken as a general result, but rather be considered as a consequence of the choice of sea-ice extent.

Let us now turn to the hemispheric mean precipitation rate P*. Figure 2 shows that P* steadily decreases if the degree of glaciation increases; this result applies to summer and winter. As we shall see later, changes in P* are small compared to local changes in P and thus represent a small net effect of a number of large changes at various latitudes. For φ_e = 50 °N, the SV model gives a 15 to 20 % decrease in precipitation rate (compared to the value of P* in the control run). For comparison, the GFDL model (MANABE and HAHN, 1977) predicts a 30 % decrease of P* in summer.

As noted already in the introduction, a decrease in P* has to be expected if surface temperature lowers, at least on the global scale. Of much more interest is the latitudinal variation of changes in P. We turn to this in the next section.

So far we did not show any results of the experiments in which the extent of continental ice was varied without changing sea ice cover. The reason for this is that in this experiment changes appeared to be very similar to those just described, except for the magnitude of the changes in P* and T*, which was about 25 % of the values shown in Figure 2. In the next section some results of the experiments with fixed sea ice will be shown.

4 Latitudinal Changes in the Rate of Precipitation

At the edge of the ice cover a strong latitudinal gradient in the thermal forcing of the atmosphere will be present, and we may thus expect that in this region large changes in the distribution of P will be initiated.

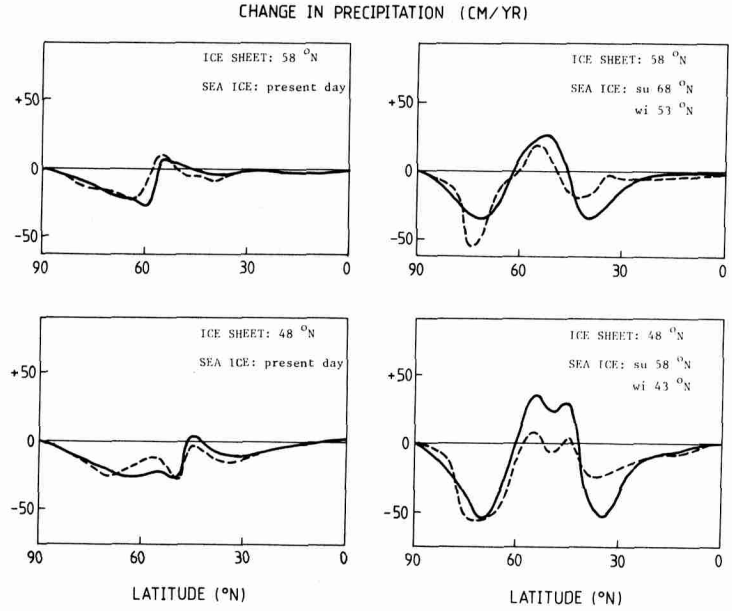
Figure 3 shows changes in P with respect to the control run as a function of latitude, for four degrees of glaciation: a continental ice sheet up to 58 °N with and without adjusted sea-ice extent; and a continental ice sheet up to 48 °N, also with and without adjusted sea ice.

From the figure it is obvious that changes in P depend very strongly on latitude. For a high degree of glaciation (lower right panel in Figure 3) changes in P have an order of magnitude of 50 cm/yr, which is quite substantial. It is also evident that sea ice has a large effect, in fact, it appears to be larger than that of the continental ice cover. This results from the larger difference in surface albedo, wetness parameter and subsurface heat flux between open and frozen ocean.

In all experiments patterns of change in P are rather similar for summer and winter. This applies in particular to the experiments in which sea ice was not adjusted. This has to be expected, because here prescribed surface conditions are the same for summer and winter.

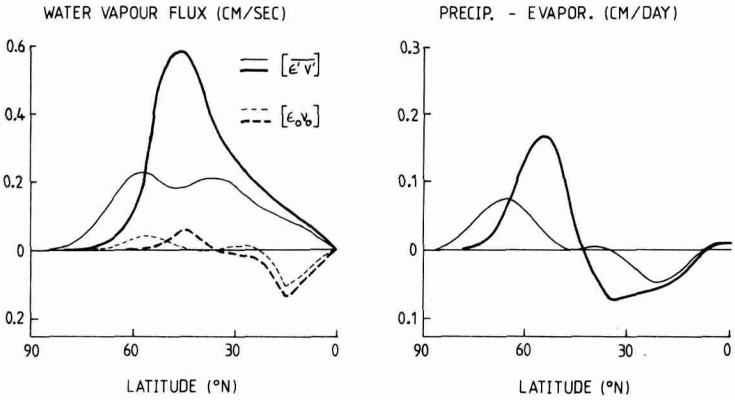
With sea ice fixed, precipitation decreases north of the southern edge of the continental ice sheet; south of the edge, changes in P are rather small. In the more realistic experiment in which sea ice is adjusted, very interesting effects take place. Around the southern edge of the ice cover, a zone with substantially more precipitation occurs, in particular in winter. This zone is so wide that it covers a large part of the continental ice sheet. We shall discuss this point in the next section. On the other hand, P is much less over the Arctic basin and the subtropics. This picture suggests that increasing eddy activity plays an important role in creating the changes in P. Let us consider this point in more detail.

Figure 4 shows changes in the annual hydrological cycle of the atmosphere. The left panel gives the poleward flux of water vapour by transient eddies and poloidal motions, the right panel the water balance of the surface (= P − E). The thin lines correspond to the control run, the heavy lines to the case with φ_e = 48 °N and adjusted sea ice (lower right in Figure 3).



- **Figure 3**
  The latitudinal distribution of the change in precipitation rate, for various surface boundary conditions. Dashed lines: summer; solid lines: winter.

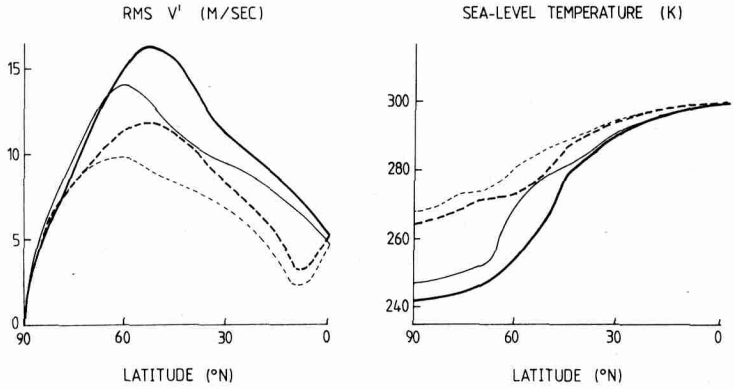
- **Bild 3**
  Breiten-Verteilung der Änderung der Niederschlagsintensität für verschiedene Oberflächen-Rand-Bedingungen. Gestrichelt: Sommer; ausgezogen: Winter.



- **Figure 4**
  Annual water vapour fluxes and water balance of the surface for the control run (thin lines) and for the run with $\varphi_e = 48\,°N$ (heavy lines).

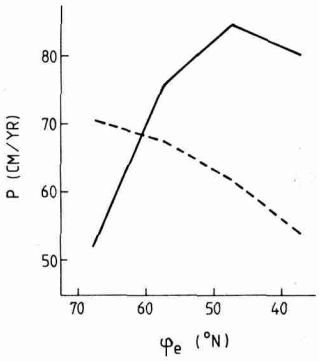
- **Bild 4**
  Jahreswerte des Wasserdampf-Flusses und Wasserbilanz der Oberfläche für den Vergleichs-Lauf ($\varphi_e = 78°$, gegenwärtiges Klima) (dünn) und für den Rechenlauf mit $\varphi_e = 48\,°N$ (dick).

By comparing the two cases the most striking feature is the tremendous increase in the water vapour flux by the transient eddies $[\overline{\epsilon' v'}]$ in the midlatitudes. Relative changes in $[\epsilon_0 v_0]$ are also substantial, but, except for $\varphi < 20\,°N$, this term plays a minor role in the water balance of the model atmosphere. The strong increase in $[\overline{\epsilon' v'}]$ is due to an enhancement of transient eddy activity near the southern edge of the ice cover, and to an increased meridional gradient of ϵ_0 in that region. This point is illustrated in Figure 5. The left panel shows the eddy activity, measured with the root-mean-square value of the eddy component of the meridional velocity (v'). Results are shown for the control run and the run with $\varphi_e = 48\,°N$. The right panel gives the corresponding distribution of surface temperature. Apparently, during glacial conditions the eddy activity increases in both summer and winter, and the same applies to the surface-temperature gradient. Both quantities reach their highest values in the vicinity of the edge of the ice cover, and this fully explains the very strong poleward flux of water vapour.



- **Figure 5**
  Eddy activity (left panel) and surface temperature (right panel) for the control run (thin lines) and the run with $\varphi_e = 48\,°N$ (heavy lines). Dashed lines: summer; solid lines: winter.

- **Bild 5**
  Aktivität der baroklinen Störungen (links) und Oberflächentemperatur (rechts) für den Vergleichs-Lauf (dünn) und den Rechenlauf mit $\varphi_e = 48\,°N$ (dick). Gestrichelt: Sommer; ausgezogen: Winter.



- **Figure 6**
  Precipitation rates averaged over the latitude belt over which the continental ice sheet extends, as a function of φ_e. Dashed line: summer; solid line: winter.

- **Bild 6**
  Niederschlagsintensität gemittelt über die Breitenzone, über die sich das kontinentale Eis ausdehnt, als Funktion von φ_e. Gestrichelt: Sommer; ausgezogen: Winter.

The larger values of P − E for $40 < \varphi < 65\,°N$, and the smaller values of this quantity in the subtropics (right panel in Figure 4) are a direct consequence of the increase in $[\overline{\epsilon' v'}]$.

5 Precipitation Rates Averaged Over the Ice Sheet Region

The most relevant question with regard to ice-sheet growth is how the precipitation over the ice sheet changes during its evolution. Since the present model does not distinguish between precipitation over land and over ocean, it cannot be used to give a reliable answer. To get a first impression, however, we may look at the zonal mean precipitation rates, averaged over the meridional extent of the ice sheet, i. e. over $73\,°N > \varphi > \varphi_e$. Figure 6 shows the result. Due to the fact that the ice sheet 'grows into the westerlies', the mean precipitation rate over the ice sheet appears to increase with increasing ice-sheet size. On the other hand, in summer the precipitation rate decreases if the ice sheet grows. Obviously, this is a situation that favours expansion of the ice sheet. Our results thus suggest that the feedback between ice sheet size and precipitation rate is positive, or at least not strongly negative as has been assumed frequently. If the ice sheet attains real ice-age size (φ_e around 50 °N), the positive feedback becomes a negative one. This picture suggests that the maximum size of the Northern Hemisphere ice sheets during glacials is partly determined by the interplay between ice-sheet size and atmospheric circulation.

6 Discussion

The results of the experiments discussed in this paper can be summarized as follows.

Extension of the polar ice cap leads to an increased planetary albedo and hence to a drop in hemispheric temperature. This temperature drop causes a decrease in the hemispheric mean precipitation rate. Along the southern edge of the ice cover, the eddy activity strengthens considerably, which results in a stronger poleward flux of water vapour and an increased annual precipitation rate over the ice sheet. Precipitation over the polar basin decreases. For a further extension of the ice cover (edge south of 50 °N), the picture changes. Not only is there a drop of precipitation rates averaged over the hemisphere but also the precipitation over the continental ice sheet decreases.

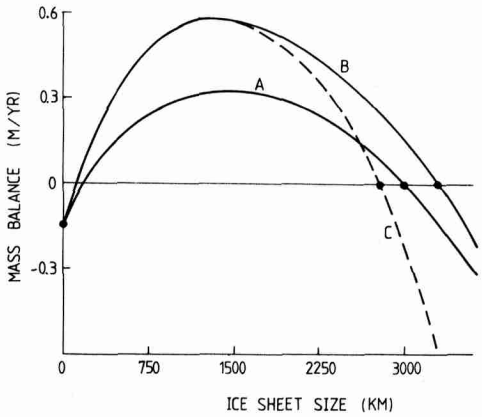
Model studies of the response of Northern Hemisphere ice sheets to climatic variations have shown that this response has a bi-stable character, which results from the dependence of ice accumulation rates on surface elevation and the fact that the Northern Hemisphere continents are bounded by the polar sea (WEERTMAN, 1961; OERLEMANS, 1981). It appears that for a wide range of climatic conditions two stable equilibrium states exist: no ice sheet, and a large ice sheet. This result follows when precipitation rates are kept constant (and melting rates decrease with surface elevation). The present results seem to enforce this bi-stable structure, because they suggest that the precipitation reaches a maximum for an ice sheet that is in between the stable states mentioned above.

Let us discuss this in more detail. Figure 7 illustrates this bi-stable structure schematically. Curve A shows, for the constant precipitation case, how the average mass balance M (or ice accumulation rate) of a Northern Hemisphere ice sheet depends on its size L. L is measured southwards from the northern edge of the continent (around 73 °N). If the climatic conditions are constant, the evolution of the ice sheet is given by an equation of the type (e. g. OERLEMANS, 1981)

$$k \frac{dL^{3/2}}{dt} = M(L)$$

where M only depends on the ice-sheet size L. Equilibrium states are found by solving $M(L) = 0$. An equilibrium state is stable if $\partial M/\partial L < 0$, and unstable if $\partial M/\partial L > 0$. Note that the state with $L = 0$ and $M < 0$ also represents a stable state. In Figure 7, stable equilibria are indicated by black spots.

If we now take into account the results of the present study, the curve M(L) has to be modified. Curve B shows the new situation (it has been drawn arbitrarily). Since $|\partial M/\partial L|$ is larger now, the ice sheet is more strongly forced to grow (or shrink) to an equilibrium state. We may thus state that the interaction



- **Figure 7**
  Schematic illustration of the bi-stable behaviour of Northern Hemisphere ice sheets. Black spots represent stable equilibria. See text for further explanation.

- **Bild 7**
  Schematische Illustration des bistabilen Verhaltens des nordhemisphärischen Eises. Die dicken Punkte stellen stabile Gleichgewichtszustände dar. Wegen weiterer Erklärungen, siehe Text.

between ice-sheet size and precipitation enforces the bi-stable character of the behaviour of the Northern Hemisphere ice sheets.

Finally, it should be stressed that we only studied the zonally symmetric response of the atmosphere to the changing surface conditions. In reality, large ice sheets on the Eurasian and American continents will create strong planetary wave in the atmosphere, which on their turn substantially affect the redistribution of heat and moisture. When the ice-sheet size becomes very large, it is likely that a 'drying out' effect will occur, i. e. that the central part of the ice sheet will not receive much snow anymore. Such a situation could be represented by the dashed curve in Figure 7 (curve C). It is obvious that more detailed model studies are required to study such effects.

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