Generic Dictionaries

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Why is it easier to write programs?

- Programs that you could only write in an untyped style have types now.
- Some programs are for free.
- Some programs are simple adjustments of library functions.

Introduction

Ralf has introduced the concept of a generic (or polytypic, type-indexed) function: a function that can be instantiated on all Haskell data types to obtain data specific functionality.

Over the last few years we have worked on generic programming and **Generic H∀SKELL**,see

```
http://www.generic-haskell.org/
```

The development of **Generic H∀SKELL** has to a large extent been example driven. In my lectures I will talk about three larger examples:

- Generic dictionaries
- XComprez, a compressor for XML documents
- The Zipper

Applications of generic programming

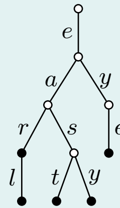
Applications of generic programming are found in several fields:

- Haskell's deriving construct: equality, read/show, ...
- Compiler tools: debuggers, garbage collectors, tracers.
- XML tools: editors, compressors, database tools.
- Typed term processing: rewriting, unification, pattern matching.
- Tree traversals.
- . . .

These fields have in common that structure drives the application. The programs are 'syntax-directed'.

Introduction Generic Dictionaries

A trie is a search tree scheme that employs the structure of a search key to organize information. For example, the set of strings $\{ear, earl, east, easy, eye\}$ can be represented by the following tree:



Overview

In this talk I will

- Define a module for generic dictionaries.
- Introduce type-indexed data types and kind-indexed kinds.
- Introduce default cases and constructor cases.
- Introduce dependencies and generic abstractions.

Dictionaries on strings

The data type of strings and dictionaries on strings can be defined as follows:

```
data String           =  Nil | Cons Char String

data FMap_String v  =
  Trie_String (Maybe v) (DictChar (FMap_String v))
```

where DictChar is a dictionary on characters. Here is the example:

```
eDict   :: FMap_String ()
eDict   =  Trie_String Nothing cDict1

cDict1  :: DictChar (FMap_String ())
cDict1  =  [('e',eDict1)]

eDict1  =  Trie_String Nothing cDict2
cDict2  =  [('a',eDict2),('y',eDict3)]
```

Looking up strings

Here is how you look up a string value.

```
lookup_String  ::  String -> FMap_String v -> Maybe v
lookup_String Nil        (Trie_String tn tc)  =  tn
lookup_String (Cons c s) (Trie_String tn tc)  =
  lookup_Char c tc  >>=  \t -> lookup_String s t

lookup_Char  ::  Char -> DictChar v -> Maybe v
```

Looking up characters

We implement a dictionary for characters as an ordered association list:

```
type DictChar v = [(Char,v)]

lookup_Char  ::  Char -> DictChar v -> Maybe v
lookup_Char c []                           =  Nothing
lookup_Char c ((c',v):xs) | c <  c'   =  Nothing
                          | c == c'   =  Just v
                          | c >  c'   =  lookup_Char c xs
```

Dictionaries on trees

A dictionary on trees is a value of the following data type.

```
data Tree a             =  Leaf | Node (Tree a) a (Tree a)

data FMap_Tree fma v  =
  Trie_Tree (Maybe v) (FMap_Tree fma (fma (FMap_Tree fma v)))
```

The `lookup` function on tree dictionaries is defined as follows.

```
lookup_Tree  ::  (forall v.a -> fma v -> Maybe v) ->
                 Tree a -> FMap_Tree fma w -> Maybe w
lookup_Tree lua Leaf           (Trie_Tree tl tn)  =  tl
lookup_Tree lua (Node l m r) (Trie_Tree tl tn)  =
  lookup_Tree lua l tn  >>=  \tm ->
  lua m tm              >>=  \tr ->
  lookup_Tree lua r tr
```

A module for generic dictionaries

We want to have a function `FMap` that takes a data type and returns the dictionary type for that data type, together with functions for looking up in a dictionary, and manipulating a dictionary.

Furthermore, we want to have a number of functions on the generated dictionary data type

```
type FMap{| t |} ...

lookup{|t|}  ::  t -> FMap{| t |} v -> v
empty{|t|}   ::  FMap{| t |} v
single{|t|}  ::  (t,v) -> FMap{| t |} v
insert{|t|}  ::  (v -> v -> v) -> (t,v) ->
                 FMap{| t |} v -> FMap{| t |} v
delete{|t|}  ::  t -> FMap{| t |} v -> FMap{| t |} v
...
```

A type-indexed trie type

Tries are based on the following isomorphisms, also known as the laws of exponentials.

$$\begin{aligned} 1 \to_{\text{fin}} V &\cong V \\ (T_1 + T_2) \to_{\text{fin}} V &\cong (T_1 \to_{\text{fin}} V) \times (T_2 \to_{\text{fin}} V) \\ (T_1 \times T_2) \to_{\text{fin}} V &\cong T_1 \to_{\text{fin}} (T_2 \to_{\text{fin}} V) \end{aligned}$$

In **Generic H∀SKELL**, a type-indexed data type for dictionaries is defined as follows.

```
type FMap{| Unit |}         v  =  FMUnit (Maybe v)
type FMap{| Char |}         v  =  FMChar (DictChar v)
type FMap{| :+:  |} fma fmb v  =  FMEither (fma v,fmb v)
type FMap{| :*:  |} fma fmb v  =  FMProd (fma (fmb v))
```

Type-indexed data types

A type-indexed data type is a data type that is constructed in a generic way from an argument data type. Type-indexed data types are used in almost all applications given at the beginning of this lecture.

Besides finishing the example, I will also discuss the 'type' of type-indexed data types.

Type-indexed data types are just as important as type-indexed functions!

Kind-indexed kinds

What is the type of a type-indexed data type? The type of a data type is a kind. But a type-indexed data type depends on the kind of the input type. So a type-indexed data type has a kind-indexed kind!

```
type FMap{| t :: k |}  ::  FMAP{[ k ]}

FMAP{[  *   ]}  =  * -> *
FMAP{[ k->l ]}  =  FMAP{[ k ]} -> FMAP{[ l ]}
```

Function lookup

The generic function `lookup` is defined as follows:

```
lookup{| t :: k |}  ::  Lookup{[ k ]} t

type Lookup{[  *   ]} t v  =  t -> FMap{| t |} v -> Maybe v
type Lookup{[ k->l ]} t v  =
  forall a.Lookup{[ k ]} a v -> Lookup{[ l ]} (t a) v

lookup{| Unit |}            Unit     (FMUnit v)           =  v
lookup{| Char |}            c        (FMChar t)           =
  lookup_Char c t
lookup{| :+:  |} lua lub (Inl a) (FMEither (t1,t2))  =
  lua a t1
lookup{| :+:  |} lua lub (Inr b) (FMEither (t1,t2))  =
  lub b t2
lookup{| :*:  |} lua lub (a:*:b) (FMProd t)          =
  lua a t  >>=  \t' -> lub b t'
```

Default cases

Suppose I have a more efficient lookup function for characters `lookup_Char_eff`. Then I can reuse `lookup` by means of a *default case*:

```
lookup'{| t :: k |}    :: Lookup{[ k ]} t

lookup'{| Char |} c t  =  lookup_Char_efficient c t
lookup'{|   a  |}      =  lookup{| a |}
```

Constructor cases

Suppose I want to use the efficient lookup function only for one occurrence of the type `Char` in my data type. Then I can use a *constructor case*:

```
data Example = C1 Char | C2 Char | ....

lookup'{| t :: k |}                :: Lookup{[ k ]} t

lookup'{| case C1 |} (C1 c) t  =  lookup_Char_efficient c t
lookup'{|   a     |}           =  lookup{| a |}
```

Function empty

The generic function empty is defined as follows:

```
empty{| t :: k |}  ::  Empty{[ k ]} t

type Empty{[  *   ]} t v  =  FMap{| t |} v
type Empty{[ k->l ]} t v  =
  forall a.Empty{[ k ]} a v -> Empty{[ l ]} (t a) v

empty{| Unit |}            =  FMUnit Nothing
empty{| Char |}            =  FMChar []
empty{| :+:  |} ea eb      =  FMEither (ea,eb)
empty{| :*:  |} ea eb      =  FMProd ea
```

Function `single`: dependencies

The generic function `single` depends on the function `empty`:

```
dependency single <- single empty

type Single{[ * ]}     t  =  (t,v) -> FMap{| t |} v
type Single{[ k->l ]} t  =
  forall a.Single{[k]} a -> Empty{[k]} a -> Single{[l]} (t a)

single{| t :: k |}                          :: Single{[ k ]} t
single{| Unit |}               (Unit,v)      =  FMUnit (Just v)
single{| Char |}               (c,v)         =  FMChar [(c,v)]
single{| :+: |} sA eA sB eB (Inl a,v)      =
  FMEither (sA (a,v),eB)
single{| :+: |} sA eA sB eB (Inr b,v)      =
  FMEither (eA,sB (b,v))
single{| :*: |} sA eA sB eB (a :*: b,v)   =
  FMProd (sA (a,sB (b,v)))
```

Function `insert`: generic abstractions

The generic function `insert` can be defined as a generic abstraction of `single` and `merge`:

```
insert{| t :: * |}         :: (v -> v -> v) -> (t,v) ->
                                 FMap{| t |} v -> FMap{| t |} v
insert{| t |} c (x,v) d  =
  merge{| t |} c (single{| t |} (x,v)) d
```

The other functions on generic dictionaries are implemented in a similar fashion.

Exercise

The definition of function `insert` as a generic abstraction of `single` and `merge` limits its application to data types of kind $*$. Define `insert` as a type-indexed function with a kind-indexed type.

Conclusions

I have shown:

- How you can define a module for generic dictionaries.
- How you can define type-indexed data types in **Generic H∀SKELL**.
- How type-indexed data types are translated by **Generic H∀SKELL** to Haskell.

Next lectures

- XComprez
- The Zipper