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SHORT AND SWEET

An antisymmetric psychometric function on a logarithmic scale

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Abstract. This very brief report introduces a psychometric function, very suitable for psychophysical data that displays Weber-like behaviour, because it is antisymmetric on a logarithmic scale.

A psychometric curve describes the relationship between stimulus intensity and the probability of a particular observation. For example, for a discrimination experiment, the function may describe the change of perceiving a test stimulus as more intense than a particular reference stimulus, as a function of the test-stimulus intensity. In general, this function has a sigmoid shape. A number of different functions have been suggested, such as the cumulative Weibull distribution, the cumulative logistic distribution, and the cumulative normal (Gaussian) distribution (Treutwein 1995; Klein 2001; Wichmann and Hill 2001). In general, psychophysical data are not accurate enough to make a distinction between these very similarly shaped functions. For this reason, the choice which function to use is often made out of convenience. Usually, the function itself is not the subject of interest, but what is, are the parameters that can be derived from fitting such a function to psychophysical data, such as the point of subjective equality (PSE), and the just noticeable difference (JND) or discrimination threshold.

For a two-alternative discrimination experiment, chance level is at 50%. So, a psychometric function for such an experiment should cross the 50% line at the location where the test and reference stimuli are of equal intensity. Let's call this the point of objective equality (POE). On either side of this point lies an area with a width of the discrimination threshold in which discrimination is not reliably possible. The definition of 'reliably' differs, but is often set at 75% or 84% probability that the physically more intense stimulus is perceived as such. This means the function should pass through the 25% (or 16%) line at a point one discrimination threshold to the left of the POE, and through the 75% (or 84%) line at a point one discrimination threshold to the right of the POE. Often (but by far not always), the discrimination threshold is a more-or-less constant fraction of the stimulus intensity (Weber fraction). This means that, if you place points on the stimulus intensity line one discrimination threshold apart, subsequent points are separated by a constant factor. In other words, they are equidistant on a logarithmic scale. A psychometric function describing discrimination of stimuli on such a continuum would have to be antisymmetric around the (POE, 50%) point on a logarithmic scale. That is, the value of such a function as xPOE would have to be equal to 100% minus the value at POE$/x$. An infinite number of sigmoid functions satisfy this condition, but a particularly simple one is:

$$f(x) = 50\% + 50\% \operatorname{erf}\left[\frac{\log(x/p)}{\sqrt{2}\log(w+1)}\right].$$

Here, erf is the cumulative Gaussian distribution, p is the intensity of the POE or PSE, and w is the Weber fraction. A plot of the function is shown in figure 1. This function, based on the cumulative normal distribution, crosses the 16% line a factor

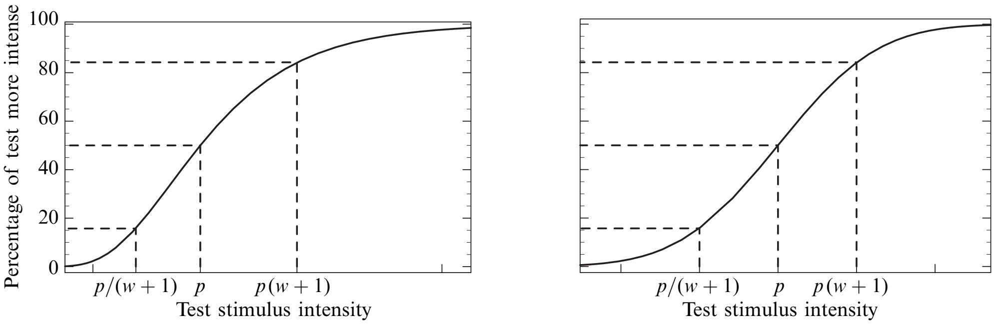

Figure 1. A psychometric function that is antisymmetric on a logarithmic scale plotted on a linear scale (left) and on a logarithmic scale (right). The tick marks on the horizontal axis are a factor of $w+1$ apart.

of one discrimination threshold to the left of the POE [$x = p/(w+1)$] and the 84% line a factor of one discrimination threshold to the right of the POE [$x = p(w+1)$]. This function is equivalent to a cumulative log-normal distribution with $\mu = \log p$ and $\sigma = \log(w+1)$. It has the advantage that the Weber fraction is present as a unique parameter (w) which follows directly from a fitting procedure, without having to transform the data. Compared to a function with the same properties based on the cumulative logistic distribution, this function is expressed in simpler terms.(1) A function based on the cumulative Weibull distribution is out of the question because this distribution is not antisymmetric.

This function has been successfully used to describe data from a haptic viscosity discrimination experiment (Bergmann Tiest et al 2010). We can recommend this function for any psychophysical data that display Weber-like behaviour.

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(1) A function with the same properties based on the cumulative logistic distribution would be

$$g(x) = \frac{100}{1 + (x/p)^{-[\log(w+1)]^{-1}}}\,\% \,.$$

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