

**GAUGING CONFORMAL ALGEBRAS WITH RELATIONS BETWEEN  
THE GENERATORS** <sup>1</sup>K. THIELEMANS <sup>2</sup>*Theoretical Physics Group, Imperial College  
London SW7 2BZ, UK*S. VANDOREN<sup>3</sup>*Instituut voor Theoretische Fysica, K.U.Leuven  
Celestijnenlaan 200D, B-3001 Leuven, Belgium***ABSTRACT**

We investigate the gauging of conformal algebras with relations between the generators. We treat the  $W_{5/2}$ -algebra as a specific example. We show that the gauge-algebra is in general reducible with an infinite number of stages. We show how to construct the BV-extended action, and hence the classical BRST charge. An important conclusion is that this can always be done in terms of the generators of the  $W$ -algebra only, that is, independent of the realisation.

The present treatment is still purely classical, but already enables us to learn more about reducible gauge algebras and the BV-formalism.

**1 Introduction**

It cannot be stressed enough that gauge symmetries play an extremely important role in our understanding of particle physics. Therefore it is very important to study the quantisation of models possessing a number of gauge invariances. By now, we know there is a large variety of these models, from electromagnetism, Yang–Mills and gravity theories to supergravity,  $W$ -gravities, superparticles and superstrings. All these models can, apart from their field content, be characterised by their *algebra of gauge transformations*. In Yang–Mills, this algebra is a Lie-algebra with structure constants satisfying Jacobi identities. In supergravity theories one has to extend this to more general gauge algebras, where one can have structure functions and where the algebra only closes modulo (graded) antisymmetric combinations

---

<sup>1</sup>This paper is based on proceedings for the Sixth Seminar on Quantum Gravity, Moscow, 12-17 June '95 and the Conference on Gauge theories, Applied Supersymmetry and Quantum Gravity, Leuven, 10-14 July '95.

<sup>2</sup>E-mail: k.thielemans@ic.ac.uk

<sup>3</sup>E-mail: stefan.vandoren@fys.kuleuven.ac.be

of the field equations [1, 2]. For these so called open gauge algebras, one cannot simply apply the same BRST quantisation method as for Yang–Mills theory, and an appropriate extension of the BRST formalism was given in [2]. In the case where the gauge symmetries are not independent, the level of reducibility is another important characterisation. As an example of a first level reducible theory, one can think about the antisymmetric tensor [3] where one has to introduce, on top of the ordinary ghosts, a ghost for ghosts. Other more complicated examples are the superparticle and the Green–Schwarz superstring, which are infinite stage reducible theories. In these cases, one has to work with an infinite tower of ghost for ghosts, see *e.g.* [4].

In this paper, we will even go one step further. We will start with an action  $S_0[\phi^i]$  with a number of global symmetries with generators  $T_a$ , which are then gauged by introducing gauge fields  $\mu^a$  :

$$S = S_0[\phi] + \int \mu^a T_a(\phi) , \quad (1)$$

where we call  $\phi^i$  the matter fields. We will concentrate on a specific example of a two dimensional conformal field theory based on the (nonlinear)  $W_{5/2}$ -algebra with the Virasoro spin 2 current  $T_1 = T$  and a spin 5/2 fermionic current  $T_2 = G$ . The new thing in this model is that the gauge algebra *does not close* on the two gauge symmetries, even when using antisymmetric combinations of field equations ! Instead, it generates 2 new unexpected symmetries (on shell zero) that act only on the gauge fields. It turns out that we have to include these 2 symmetries to find a gauge algebra that closes up to trivial symmetries (antisymmetric combinations of field equations). However, as we will show, adding these new symmetries to the original ones will make the complete set of symmetry generators dependent, so that we are dealing with a reducible theory. After introducing the necessary zero modes and their corresponding ghosts for ghosts, we will even see that the complete set of zero modes is reducible itself. This is a never ending story : the reducibility has an infinite number of stages, and there is an infinite tower of ghosts.

All this can be better understood in terms of “nonfreely generated” conformal algebras. These are algebras where the Jacobi identities are only satisfied if certain combinations of the generators are considered to be zero (“null fields”). The simplest example, which we also consider in this paper, is the  $W_{5/2}$ -algebra, discovered in the quantum case in [6]. It is then clear that, because there are relations between the currents  $T_a$ , there will be extra symmetries in the theory [13]. They are precisely those needed to close the gauge algebra. Together with the original ones, they will form a reducible system [13, 4]. It is the gauge theory of this conformal algebra that we want to treat here.

To handle such a complicated system, we resort to the antifield formalism of Batalin–Vilkovisky (BV) [5]. We sketch how to deal with further zero modes that vanish on shell, a point that is not well discussed in the literature so far. To do this, we make use of the acyclicity of the Koszul–Tate differential, the basic ingredient of the (BV) formalism. Details are given in [11].

The main motivation for this work lies in the further study of gauge algebras. The gauging of nonfreely generated  $W$ -algebras is however interesting in its own right, as this could provide a new class of  $W$ -string theories. Indeed, up to now, all  $W$ -string theories are constructed by gauging a  $W$ -algebra where all generators are linearly independent. Furthermore, a particular class of nonfreely generated quantum  $W$ -algebras have been

studied lately. They provide “unifying”  $W$ -algebras for the more familiar algebras in the Drinfeld–Sokolov series [7]. The study of  $W$ -strings based on the unifying algebras will however be complicated by the fact that the classical versions of these  $W$ -algebras have an infinite number of generators. Clearly, we first have to understand the case of nonfreely, but finitely generated  $W$ -algebras.

So, in the next section we study the  $W_{5/2}$  current algebra, and discuss how the relations between the currents follow from the Jacobi identities. Then, in section 3 we show how the extra symmetries are generated starting from the (open) gauge algebra based on the gauged (super)conformal symmetries. In section 4 we show that the model is infinitely reducible. In a last section, we discuss briefly the gauge fixing procedure in the BV formalism and determine the structure of the BRST charge. We end with some conclusions.

## 2 The current algebra

The  $W_{5/2}$ -algebra was one of the first  $W$ -algebras constructed, see [6] where it is presented in the quantum case with Operator Product Expansions. We need it here as a classical  $W$ -algebra, *i.e.* using single contractions. The algebra consists of two currents :  $T$  the Virasoro generator and a primary dimension  $\frac{5}{2}$  current  $G$ . They satisfy the brackets :

$$\begin{aligned} \{T(z), T(w)\} &= -2T(w)\partial\delta(z-w) + \partial T(w)\delta(z-w) \\ \{T(z), G(w)\} &= -\frac{5}{2}G(w)\partial\delta(z-w) + \partial G(w)\delta(z-w) \\ \{G(z), G(w)\} &= T^2(w)\delta(z-w) . \end{aligned} \quad (2)$$

The last bracket leads us to call  $G$  a (generalised) supersymmetry generator. In the quantum case, the Jacobi identities are only satisfied for a specific value of the central charge  $c = -\frac{13}{14}$  and even then only modulo a “null field”. In this context, we call “null fields” all the combinations of  $T$  and  $G$  which should be put to zero such that the Jacobi identities are satisfied. Similarly, we find in the classical case that the algebra does not admit a central extension and there is a classical null field :

$$N_1 \equiv 4T\partial G - 5\partial T G . \quad (3)$$

We can check by repeatedly computing brackets with  $N_1$  that the null fields are generated by  $N_1$  and

$$N_2 \equiv 2T^3 - 15\partial G G . \quad (4)$$

More precisely, all other null fields are of the form :

$$f_n(T, G) \partial^n N_1 + g_m(T, G) \partial^m N_2 \quad (5)$$

where  $f_n, g_m$  are differential polynomials in  $T$  and  $G$ .

A realisation for the algebra (2) was found in [8] :

$$\begin{aligned} T &= -\frac{1}{2}\psi\partial\bar{\psi} + \frac{1}{2}\partial\psi\bar{\psi} , \\ G &= \frac{1}{2}(\psi + \bar{\psi})T , \end{aligned} \quad (6)$$

where  $\psi$  is a complex fermion satisfying the Dirac brackets  $\{\psi(z), \bar{\psi}(w)\} = \delta(z-w)$ . One can easily verify for this realisation that the null fields  $N_i$  vanish.

In fact, for any realisation in terms of fields of an underlying Poisson (or Dirac) algebra (e.g. free fields), the null fields will vanish identically. Indeed, they appear in the *rhs* of a Jacobi identity, which is of course satisfied for a Poisson algebra. This means that in any realisation, the generators  $T, G$  are not independent. They satisfy (at least) the relations  $N_i = 0$ . In the following section, we will see that these relations have important consequences for the gauge algebra.

### 3 The gauge algebra

In order to construct a gauge theory based on this algebra, one must be able to work in a certain realisation, *i.e.* one must specify an action  $S_0$  for matter fields  $\phi^i$ <sup>4</sup>. Using this action  $S_0$ , one can define light-cone Poisson (or Dirac) brackets between the fields and their momenta. With respect to these brackets, we assume that we can find conserved currents  $T(\phi), G(\phi)$  satisfying the algebra (2). The transformations of the fields are obtained by taking brackets with the generators :

$$\delta_{\epsilon^a} \phi = \int \epsilon^a \{T_a, \phi\} \quad (7)$$

where the index  $a$  runs over the number of generators, and there is no summation on the *rhs*. We will not make a choice for the realisation and use only the information contained in the algebra of the generators to construct a gauge theory.

The above assumption implies that the action  $S_0$  transforms under the conformal symmetry and supersymmetry with parameters  $\epsilon$  and  $\alpha$  respectively, as

$$\delta_{\epsilon} S_0 = - \int \bar{\partial} \epsilon T \quad \delta_{\alpha} S_0 = - \int \bar{\partial} \alpha G , \quad (8)$$

where the transformations of the Noether currents  $T, G$  follow from eq. (7) :

$$\begin{aligned} \delta_{\epsilon} T &= \epsilon \partial T + 2 \partial \epsilon T & \delta_{\alpha} T &= \frac{3}{2} \alpha \partial G + \frac{5}{2} \partial \alpha G \\ \delta_{\epsilon} G &= \epsilon \partial G + \frac{5}{2} \partial \epsilon G & \delta_{\alpha} G &= \alpha T^2 \end{aligned} \quad (9)$$

The commutators between two symmetries can be computed using the Jacobi identities:

$$\begin{aligned} [\delta_{\epsilon^a}, \delta_{\bar{\epsilon}^b}] \phi &= \int \epsilon^a \int \bar{\epsilon}^b \left( (-1)^{ab} \{T_a, \{T_b, \phi\}\} - \{T_b, \{T_a, \phi\}\} \right) \\ &= - \int \epsilon^a \int \bar{\epsilon}^b \{ \{T_b, T_a\}, \phi \} . \end{aligned}$$

We find :

$$\begin{aligned} [\delta_{\epsilon_1}, \delta_{\epsilon_2}] &= \delta_{\bar{\epsilon} = \epsilon_2 \partial \epsilon_1 - \epsilon_1 \partial \epsilon_2} \\ [\delta_{\epsilon}, \delta_{\alpha}] &= \delta_{\bar{\alpha} = -\epsilon \partial \alpha + 3/2 \alpha \partial \epsilon} \\ [\delta_{\alpha_1}, \delta_{\alpha_2}] &= \delta_{\bar{\epsilon} = 2\alpha_2 \alpha_1 T} . \end{aligned} \quad (10)$$

---

<sup>4</sup>For the complex fermion in (6), one has  $S_0 = \psi \bar{\partial} \bar{\psi}$ .

Now, we can gauge these symmetries by introducing gauge fields  $\mu$  (bosonic) and  $\nu$  (fermionic) for the conformal and susy symmetries. The action is then

$$S = S_0 + \int \mu T + \int \nu G . \quad (11)$$

The transformation rules for the gauge fields such that the action is invariant, are

$$\begin{aligned} \delta_\epsilon \mu &= \nabla^{-1} \epsilon & \delta_\alpha \mu &= \alpha \nu T \\ \delta_\epsilon \nu &= \epsilon \partial \nu - \frac{3}{2} \nu \partial \epsilon & \delta_\alpha \nu &= \nabla^{-(\frac{3}{2})} \alpha , \end{aligned} \quad (12)$$

with the notation  $\nabla^j = \bar{\partial} - \mu \partial - j \partial \mu$ . These rules enable us to study the gauge algebra. Computing the commutators of the gauge symmetries on the gauge fields, we see that they close only after using equations of motion.

In the usual case for open algebras [2] one has the following structure :

$$[\delta_{\epsilon^a}, \delta_{\epsilon^b}] \phi^A = R_c^A T_{ab}^c \epsilon^b \epsilon^a - y_B E_{ba}^{BA} \epsilon^a \epsilon^b , \quad (13)$$

where  $\phi^A$  now stands for all the classical fields (matter fields  $\phi^i$  and gauge fields  $\mu^a$ ), and the symmetries are written in the form  $\delta_\epsilon \phi^A = R_a^A \epsilon^a$ . The structure functions  $T_{bc}^a$  are graded antisymmetric in  $(bc)$ . The first term in the *rhs* can be rewritten as  $\delta_{\tilde{\epsilon}} \phi^A$  for  $\tilde{\epsilon}^c = T_{ab}^c \epsilon^a \epsilon^b$ , so this is again a symmetry of the action. The second term is proportional to field equations  $y_A$ . As it arises in the commutator of two symmetries, it leaves the action invariant too. This is trivially the case when the matrix  $E^{AB} = E_{ab}^{AB} \epsilon^a \epsilon^b$  is graded antisymmetric in  $(AB)$ , because then it generates trivial field equation symmetries of the form  $\delta \phi^A = E^{AB} y_B$ , and one does not need to take these into account for quantising the theory. All previously known examples of gauged algebras are of the type (13), with a graded antisymmetric  $E^{AB}$ -matrix and  $R_a^A$  on shell nonzero.

In the case of  $W_{5/2}$  however, the commutator of two supersymmetries gives us something unexpected. Computing the commutator (10) on the gauge fields, we find :

$$\begin{aligned} \left( [\delta_{\alpha_1}, \delta_{\alpha_2}] - \delta_{\tilde{\epsilon}=2\alpha_2\alpha_1 T} \right) \mu &= -[\nabla^{-3}(\alpha_2\alpha_1)T + 2\alpha_2\alpha_1 \nabla^2 T] \\ &\quad - \left[ \frac{5}{2} \partial(\alpha_2\alpha_1) \nu G + 3\alpha_2\alpha_1 \nu \partial G \right] \\ \left( [\delta_{\alpha_1}, \delta_{\alpha_2}] - \delta_{\tilde{\epsilon}=2\alpha_2\alpha_1 T} \right) \nu &= -[3\alpha_2\alpha_1 \partial(T\nu) + \frac{1}{2} \partial(\alpha_2\alpha_1) \nu T] \\ &\quad + 9\alpha_2\alpha_1 \nu \partial T + \partial(\alpha_2\alpha_1 \nu) T , \end{aligned} \quad (14)$$

upon using (9,12). All the terms in square brackets can be absorbed by trivial field equation symmetries using the field equations  $y_\mu = T$  and  $y_\nu = -G$ .<sup>5</sup> However, the two terms on the last line of the *rhs* for  $\nu$  remain. So, they come from a nontrivial symmetry, which is zero on shell :

$$\delta_n \phi^i = 0 \quad \delta_n \mu = 0 \quad \delta_n \nu = 9n \partial T + 4 \partial n T . \quad (15)$$

---

<sup>5</sup>The field equations are defined as the right derivative of the action w.r.t. the field.

Note that it acts only on the gauge fields, and hence leaves  $S_0$  invariant. The variation of the action (11) under these transformation rules is proportional to the relation  $N_1$  (3). Of course, in hindsight it is obvious there is a corresponding symmetry associated with such a relation. However, if one tries to quantise the action without knowing the algebra of the previous section, one is surprised that the gauge transformations (12) do not form a closed algebra, even after using trivial equation of motion symmetries.

Completely analogously, one can find a second new symmetry. This symmetry will appear in the commutator  $[\delta_\alpha, \delta_n]$ , again acting on the gauge fields. The second symmetry, with bosonic parameter  $m$  can be written as

$$\delta_m \phi^i = 0 \quad \delta_m \mu = 2mT^2 \quad \delta_m \nu = -15m\partial G, \quad (16)$$

which indeed leaves the action (11) invariant when using  $N_2 = 0$  (4).

The two new symmetries are proportional to field equations. The way they are written down is not unique. For instance, one could change (15) to  $\delta_n \mu = -4n\partial G; \delta_n \nu = 5n\partial T$ . This choice is however equivalent, since it corresponds to (15) by adding a trivial field equation symmetry, and this does not change the theory and its quantisation.

## 4 Reducibility

Having found all the gauge symmetries, we should see if they are all independent, *i.e.* is the gauge algebra irreducible? We have to investigate if we can find zero modes  $Z$  of the matrix  $R$  of gauge transformations :

$$R_a^A Z_{a_1}^a \epsilon^{a_1} = y_B f^{BA}, \quad (17)$$

where the index  $a$  runs over all symmetries (1...4). Remark that the  $Z_{a_1}^a$  are only zero modes on the stationary surface, and the  $f^{BA}$  are graded antisymmetric, see e.g. [9]. We expect that these zero modes will be related to the relations  $N_i = 0$  [13]. Let us first look at the transformations of the matter fields  $\phi^i$ . Consider the transformations generated by taking Poisson brackets with the  $N_i$ . Because the relations contain only the generators  $T, G$ , we can use the Leibniz rule and some partial integrations to rewrite the transformation as a combination of those generated by  $T, G$ . For example :

$$\int \zeta^1 \{N_1, \phi^i\} = \left( \delta_{\epsilon=9\zeta^1 \partial G + 5\partial \zeta^1 G} - \delta_{\alpha=9\zeta^1 \partial T + 4\partial \zeta^1 T} \right) \phi^i. \quad (18)$$

However, because  $N_1 = 0$ , the previous equation gives us a relation between the transformations of the matter fields (valid for every realisation). Similarly, via  $N_2$  we can find another relation between the gauge transformations acting on the matter fields.

These two relations satisfy eq. (17) for the  $A$ -index running over the matter fields (with  $f^{Bi} = 0$ ). However, for the gauge fields  $\mu, \nu$ , we find that we need the extra symmetries eqs. (15,16) to make  $f$  graded antisymmetric. Of course, we can include these terms in eq. (18) as the extra symmetries do not act on the matter fields. Summarising, we find two

zero modes (one for every relation  $N_i$ ) giving the following entries in the matrix  $Z_{a_1}^a$  :

$$\begin{array}{c|c} -9\partial G - 5G\partial & -6T^2 \\ -9\partial T - 4T\partial & 30\partial G + 15G\partial \\ \nabla^{-\frac{9}{2}} & -\frac{1}{2}\nu T \\ \frac{3}{2}\partial\nu - \frac{1}{2}\nu\partial & \nabla^{-5} \end{array} \quad (19)$$

where the rows correspond to the conformal symmetry, the supersymmetry and the two extra symmetries (15,16), and each column corresponds to a zero mode.

Surprisingly, this is not the end of the story. When trying to construct a BRST charge in the BV formalism, or an extended action in BV, no solution can be found with the above symmetries and zero modes. Indeed, many other zero modes exist. They all have zero entries in the first two rows of  $Z$ , that is, they are relations between the extra symmetries. Furthermore, the remaining two entries are differential polynomials in  $T, G$ , which means that they are zero on shell. We give as an example the zero mode  $Z = (0, 0, T, 0)^t$ , for which there indeed exists a graded antisymmetric  $f^{AB}$  such that eq. (17) is satisfied.

However, most of these zero modes do not solve the problems mentioned above. In fact, we have to look more closely at the existence proof of the relevant object (BRST-charge or extended action). Both proofs involve the computation of the cohomology of the so-called Koszul–Tate differential  $\delta_{KT}$  [10]. We do not wish to go into details here (see [11]), but give only the gist of the argument.

Eq. (17) corresponds to the existence of a KT-invariant (or cocycle). However, only those invariants which are not exact — *i.e.* not the  $\delta_{KT}$  of something else — determine the non trivial zero modes. In the BV language, this can be stated as follows. If the BV master equation cannot be satisfied at a certain antifield level, it is because a KT-nontrivial cocycle  $A$  exists. One then introduces a cochain  $a$  by hand, such that  $\delta_{KT}a = A$ , making  $A$  exact. In our case, the cochain  $a$  would be the antifield of a ghost for ghost.

All this means we have to compute the cohomology of  $\delta_{KT}$  (at antifield level 2). One can do this for cocycles organised by engineering<sup>6</sup> and conformal dimension. The result of this calculation in our present case is that  $(0, 0, T, 0)^t$  corresponds to a trivial zero mode, *i.e.* it is  $\delta_{KT}$  exact without introducing a new ghost for ghost. In fact, we need the two following zero modes for which we give only the two bottom rows of  $Z$  (the first two contain zeroes) :

$$\begin{array}{c|c} 0 & T^2 \\ \partial T & 0 \end{array} \quad (20)$$

So, we find that the acyclicity of the KT-differential implies the introduction of zero modes which vanish on shell. To our knowledge, this is the first algebra where this has been observed. See [11] for more details. It is not clear to us how the zero modes (20) relate to the Poisson algebra of  $T, G$ .

With the information in (19,20), we can continue the computation of the extended action one step further. At the next level, we have to look for zero modes of the  $Z$  matrix :

$$Z_{a_1}^a Z_{a_2}^{a_1} \epsilon^{a_2} = y_B f^{Ba} . \quad (21)$$

---

<sup>6</sup>The engineering dimension is minus the dimension in meters.  $\partial$  and  $\bar{\partial}$  increase it by one.

Again, this is only a necessary condition for the elements of  $Z_{a_2}^{a_1}$ , and we have to compute the cohomology of  $\delta_{KT}$  (now at antifield level 3) to see what the nontrivial zero modes are. This calculation was done in Mathematica [14]. We get the following table for  $Z_{a_2}^{a_1}$  dropping the first two lines which contain only zeroes :

$$\begin{array}{c|c|c|c|c|c} \nabla^{-8} & 0 & 0 & T^2 & 0 & 0 \\ 4\partial\nu - \frac{3}{4}\nu\partial & \nabla^{-\frac{17}{2}} & \partial T & 0 & T^2 & TG \end{array} \quad (22)$$

It is now clear that we will find zero modes for  $Z_{a_2}^{a_1}$  and so on. This means that a gauged  $W_{5/2}$  system is reducible with an infinite number of stages.

## 5 Gauge fixing

In this section, we show briefly how the gauge fixing can be performed and how the resulting BRST charge looks. In general, we need to introduce ghosts  $c^a$  for every symmetry  $R_a^A$ , ghosts for ghosts  $c^{a_1}$  for every zero mode  $Z_{a_1}^a$  and so on. We will split the ghosts in two classes. Ghosts for which there appears a  $\nabla$  in  $Z_{a_i}^{a_{i-1}}$ , we denote by  $c^{\{i\}}$ , the remaining ones by  $\tilde{c}^{\{i\}}$ . The number of  $\tilde{c}^{\{i\}}$  is equal to the number of  $c^{\{i+1\}}$ .

The gauge fixing is most easily done in the BV formalism [5], or its Hamiltonian counterpart. For details, see [9]. One introduces antifields for every field (including the ghosts) and forms the extended action :

$$S_{BV} = S + \phi_A^* R_a^A c^a + c_{a_i}^* Z_{a_{i+1}}^{a_i} c^{a_{i+1}} + \dots \quad (23)$$

where the ellipsis denotes terms at least quadratic in antifields or ghosts. They are determined by the (classical) master equation.

The gauge choice we take consists of putting the gauge fields  $\mu, \nu$  and all the ghosts  $\tilde{c}^{\{i\}}$  to zero. This can be done using the symmetries associated with columns with  $\nabla$  in  $R$  and the  $Z$ 's. In BV, this can be accomplished by performing a canonical transformation transforming the antifields  $\mu^*, \nu^*$  into the antighosts  $b_{\{1\}}$  and vice versa. Similarly, we transform  $\tilde{c}_{\{i\}}^*$  into  $b_{\{i+1\}}$ . The gauge fixed action  $S_{gf}$  is then obtained by putting the new antifields equal to zero. It is nearly a free field action (due to the presence of the  $\nabla$ 's), but there are additional terms like  $b^2 \bar{\partial} c^1 c^1$ . An additional canonical transformation (similar to the one used for  $W_3$  in [12]) gives us a free field action for the ghosts :

$$S_{gf} = S_0 + b_{\{i\}} \bar{\partial} c^{\{i\}} . \quad (24)$$

Moreover, the extended action is linear in the new antifields<sup>7</sup>. This means that the BRST transformations in this gauge choice are nilpotent off shell. The corresponding BRST charge is :

$$Q = \oint cT + \gamma G + (5b\partial G - 4\partial bG)r^1 + (15bT^2 - 2\beta\partial G)r^2 + \dots , \quad (25)$$

where we called the ghost of the conformal symmetry  $c$  (antighost  $b$ ), of the supersymmetry  $\gamma$  (antighost  $\beta$ ), and the ghosts for ghosts  $r_1, r_2$ .

<sup>7</sup>This follows from dimensional arguments. In a conformal field theory, we can associate two dimensions with every field: the conformal dimension  $d$  and  $\bar{d} = D - d$ , where  $D$  is the engineering dimension. All terms in the extended action need to have  $d = 1, \bar{d} = 1$ . In our gauge choice, all fields have  $\bar{d} = 0$  and antifields have  $\bar{d} = 1$ .

## 6 Discussion

From the example of the  $W_{5/2}$ -algebra, we can draw some general conclusions for systems with relations between the generators and where minimal coupling is sufficient for gauging.

In general, two kinds of relations are possible. The ones we studied in this paper arise from the algebra, in particular from Jacobi identities. These relations have to be satisfied for any realisation having that particular symmetry algebra. On the other hand, accidental relations (valid in a particular realisation) are also possible. An example is given by the realisation (6) of the  $W_{5/2}$  algebra. There we have additional relations like :

$$\psi\bar{\psi}T = 0 \quad G - \frac{1}{2}(\psi + \bar{\psi})T = 0 . \quad (26)$$

These relations explicitly involves matter fields.

We find an extra symmetry for every relation between the generators [13, 4]. These symmetries act only on the gauge fields. If the relations involve only the generators the symmetries will be zero on shell. If the relations arise because of Jacobi identities, the extra symmetries have to be included in the algebra (13) to make the matrix  $E^{AB}$  graded antisymmetric. If there are only accidental relations, the gauge symmetries that correspond to the global symmetries form a subalgebra. However, the extra symmetries do show up in the cohomology of  $\delta_{KT}$ .

The presence of the relations (and the extra symmetries) makes the gauge algebra reducible. There is a zero mode for every relation [13]. By studying the cohomology of the Koszul–Tate differential, we can find other zero modes. If the relations consist of generators only, these extra zero modes vanish on shell. The zero modes then turn out to be dependent themselves.

It is proven in [10] that for a class of theories, called “regular”, no on shell vanishing symmetries or zero modes can occur. If we avoid using a realisation, the  $W_{5/2}$  –theory is an example of a nonregular theory. Nevertheless, we showed that the BV formalism can still be applied by studying the cohomology of the Koszul–Tate differential, see also [11]. The realisation (6) is regular however. The apparent contradiction with the theorem of [10] is resolved by noting that now there are more extra symmetries (which do not vanish on shell) corresponding to the relations (26). The symmetries (15,16) are then  $\delta_{KT}$  trivial, and do not have to be included.

In contrast to the superparticle and superstrings, gauge fixing does not present any problems, at least for  $W$ -algebras.

It is surprising that we can start from the symmetries arising from only the “necessary” relations, construct an extended action (or BRST charge), and perform a valid gauge fixing. For example, in the case of the realisation (6), we did not include the accidental symmetries (26). Still, the resulting gauge–fixed action (24) does not have any remaining gauge symmetries. This is related to the reducibility of the system. It would be interesting to investigate whether or not the theories constructed using only necessary symmetries are related to those where all symmetries are gauge fixed.

An important question that remains is of course what happens when quantising these systems. Are the extra symmetries anomalous ? We leave this for further study.

### Acknowledgements

It is a pleasure to thank José Figueroa-O’Farrill, Antoine Van Proeyen, Ruud Siebelink, Jordi París, Kelly Stelle and Roya Mohayaee for helpful discussions, and Lucy Wenham for numerous improvements of the manuscript.

This work was partly carried out in the framework of the project “Gauge theories, applied supersymmetry and quantum gravity”, contract SC1-CT92-0789 of the European Economic Community.

## References

- [1] R. Kallosh, Zh. Eksp. Teor. Fiz. Pis’ma **26** (1977) 575; Nucl. Phys. **B141** (1978) 141.
- [2] B. de Wit, J.W. van Holten, Phys. Lett. **B79** (1978) 389.
- [3] W. Siegel, Phys. Lett. **B93** (1980) 170.
- [4] E.A. Bergshoeff, R. Kallosh, A. Van Proeyen, Class. Quan. Grav. **9** (1992) 321.
- [5] I.A. Batalin, G.A. Vilkovisky, Phys. Lett. **B102** (1981) 27; Phys. Rev. **D28** (1983) 2567 (E:**D30** (1984) 508).
- [6] A.B. Zamolodchikov, Theor. Math. Phys. **63** (1985) 1205.
- [7] R. Blumenhagen, W. Eholzer, A. Honecker, K. Hornfeck, R. Hubel, Phys. Lett. **B** 332 (1994) 51;  
K. Hornfeck, Phys. Lett. **B343** (1995) 94.
- [8] H. Lu, C.N. Pope, K. Thielemans, X.J. Wang, K.W. Xu, Int. J. Mod. Phys. **A10** (1995) 2123.
- [9] W. Troost, A. Van Proeyen, *An introduction to Batalin–Vilkovisky Lagrangian quantization*, Leuven Notes in Math. Theor. Phys., in preparation.
- [10] J. Fisch, M. Henneaux, J. Stasheff, C. Teitelboim, Commun. Math. Phys. **120** (1989) 379;  
J. Fisch, M. Henneaux, Commun. Math. Phys. **128** (1990) 627;  
M. Henneaux, Commun. Math. Phys. **140** (1991) 1.
- [11] K. Thielemans, S. Vandoren, in preparation
- [12] S. Vandoren, A. Van Proeyen, Nucl. Phys. **B411** (1994) 257.
- [13] I.A. Batalin, R.E. Kallosh, A. Van Proeyen, in Proc. of the Fourth Seminar on *Quantum Gravity*, May 25-29, Moscow, 1987, eds. M.A. Markov et al., (World Scientific, 1987), pp. 423.
- [14] K. Thielemans, J. Vervoort, *A Mathematica package for computing antibrackets in the BV formalism*, in preparation.