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ESTIMATING FOREIGN TRADE FUNCTIONS

A Comment and a Correction

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In this note a more flexible restricted error covariance matrix is applied than Winters (1984) uses in order to estimate import demand functions. We show that the alternative specification performs better in the sense of yielding a considerably larger value of the log-likelihood function. Moreover, some slight errors in Winters' tables 2 and 3 are corrected.

1. Introduction

In a recent issue of the *Journal of International Economics*, Winters (1984) uses the AIDS model, introduced by Deaton and Muellbauer (1980), in order to specify import demand functions. For the estimation of the parameters of the model he uses either an unrestricted error covariance matrix Σ or an error covariance matrix restricted according to

$$\Sigma=\sigma^2(I-ll'), \tag{1}$$

where I denotes the unit matrix and $l=n^{-1/2}\ \iota$ with n the number of countries and ι a vector with all elements equal to 1 (i.e. the summation vector). He states that (1) is 'the only permissible restricted form of Σ that treats all flows symmetrically' [Winters (1984, p. 248)].

The disadvantage of specification (1) is that all variances σ_{ii} are assumed to be equal, i.e. $\sigma_{ii}=(1-n^{-1})\sigma^2$.

In the appendix Winters states: 'It may also be shown that the attempt to allow σ_{ii} to vary but to have common correlations between all flows reduces

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to (A.4)' [i.e. to (1)]. This statement is true. However, if one drops the requirement that there are common correlations between all flows, a more flexible restricted form than (1) has been proposed by De Boer and Harkema (1983). It treats all flows symmetrically as well and is as demanding with respect to the number of observations as (1). The purpose of this note is to apply that specification, briefly reviewed in section 2, to the problem at hand and to compare its performance with specification (1). This will be done in section 3. Unfortunately, Winters' tables 2 and 3 appear to contain some slight errors. In order to obtain a fair comparison, we therefore present in this section also the right figures, which have kindly been put at our disposal by the author.

2. An alternative covariance matrix and its estimation

The restricted covariance matrix proposed by De Boer and Harkema (1983) reads:

$$\Sigma = D - \frac{1}{d}\delta\delta', \tag{2}$$

where

$$D = \begin{bmatrix} d_1 & \dots & 0 \\ \vdots & & \vdots \\ 0 & \dots & d_n \end{bmatrix}, \quad d = \textstyle\sum_{i=1}^{n} d_i \quad \text{and} \quad \delta' = [d_1 \dots d_n].$$

It is obvious that specification (2) reduces to (1) when

$$d_i = \sigma^2, \quad i = 1, \dots, n. \tag{3}$$

The variances, as well as the covariances, are allowed to differ from each other, as can easily be seen from the scalar form of (2):

$$\sigma_{ii} = d_i - \frac{d_i^2}{d}, \qquad i = 1, \dots, n,$$

$$\sigma_{ij} = -\frac{d_i d_j}{d}, \qquad i \neq j.$$

De Boer and Harkema show that maximum likelihood estimates of the parameters d_i can be obtained from the following system of equations:

$$\hat{d}_i - \frac{\hat{d}_i^2}{\hat{d}} = \frac{1}{T}\hat{u}_i'\hat{u}_i, \quad i = 1, \dots, n, \tag{4}$$

where $\hat{u}_i$ denotes the vector of residuals of the import demand equation pertaining to the ith country and T denotes the length of the sample period. They also show that, apart from a special case which occurs with probability zero, there is a unique solution to (4) that can be obtained by means of an algorithm that uses a one-dimensional search procedure and that works very quickly. The algorithm is presented in De Boer and Harkema (1986); for details and proofs we refer to the original publication De Boer and Harkema (1983) which may be obtained from the authors upon request.

3. Application to import demand functions

From the various models that Winters estimates we have selected the 'simplified' AIDS applied to 10 import sources of the United Kingdom for the period 1952–1979. For details on the data we refer to Winters (1984). The 'simplified' AIDS reads as follows:

$$w_{it} = \alpha_i + \sum_j \gamma_{ij} \log p_{jt} + \beta_i \log(M_t / P_t^*),$$

where w_{it} denotes the share of imports from country i among total imports, p_{jt} the import price pertaining to country j, M_t total (current price) expenditure on imports and $\log P_t^* = \sum_k w_{kt} \log p_{kt}$ represents the Stone index.

Economic theory imposes the following constraints with respect to the parameters:

(i) additivity: $\sum_{i=1}^{n} \alpha_i = 1$, $\sum_{i=1}^{n} \gamma_{ij} = 0$ and $\sum_{i=1}^{n} \beta_i = 0$,

(ii) homogeneity: $\sum_{j=1}^{n} \gamma_{ij} = 0$,

and

(iii) symmetry: $\gamma_{ij} = \gamma_{ji}$.

As suggested by Winters we have included a dummy which takes on the value zero until 1972 and unity thereafter in order to measure accession to the EEC. In table 1 we present the values of the log-likelihood function

Table 1

Values of the log-likelihood function and the likelihood ratio test statistic.

	Specification (1)	Specification (2)	Likelihood ratio test statistic
Additivity	897.96	971.32	146.73
Homogeneity	874.35	939.58	130.47
Symmetry	821.15	888.32	134.35

evaluated at the optimum for specifications (1) and (2) of the covariance matrix and for the three versions of the 'simplified' AIDS. Since specification (1) is nested into (2), [see (3)], we can apply the likelihood ratio test for comparing the performance of the two specifications. In the third column of table 1 we present the value of the test statistic.

At a size of 0.1 percent the critical value of a $\chi^2(9)$ is 27.9. Hence, whenever there is a relatively small number of observations and one wishes to distinguish some import sources, as is usually the case for foreign trade functions, it seems worthwhile to apply the specification proposed by De Boer and Harkema.

The values of the log-likelihood function according to specification (1) for the models under additivity and under homogeneity substantially differ from the two log-likelihood values reported by Winters because of some errors in his table 2. The corrected figures[1] for tables 2 and 3, which have been put at our disposal by Dr. Winters, are shown in the tables 2 and 3. The errors do not change any of the fundamental results of Winters' paper, but only do affect his incidental comments about the forms of the covariance matrix. His principal interest was in the symmetry forms, which are unaffected by the errors.

Table 2

Adding domestic prices to the import allocation model.

Model	2 × log-likelihood	χ^2-statistic (9 d.f.)	
		p^d	p^d/P^*
N_1	1782.7	131.9	129.0
H_1	1762.0	27.5	103.8

Table 3

A non-separable allocation model.

Model	2 × log-likelihood
N_1	3072.4
H_1	2591.1
S_1	2258.2
N_Ω	3661.6
H_Ω	3348.2

[1]There are still slight differences between the figures of tables 1 and 2, but it seems likely that these must be attributed to rounding errors.

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