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MODE INTERFERENCE PATTERNS IN RETINAL RECEPTOR OUTER SEGMENTS

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Abstract—Some calculated interference patterns of electromagnetic modes are given for a dielectric waveguide representative for the receptor outer segment. The results are qualitatively in accordance with observations of Enoch.

Retinal receptors are lightguiding structures of relatively small dia, in which the energy is guided in patterns called modes (Enoch, 1963). These modes have a definite phasevelocity. However, generally some modes with different phasevelocities occur inside the receptor outer segment. In this case the modes interfere with a different phase at different levels along the outer segment.

For some cases interference patterns of guided modes were calculated. Results qualitatively in accordance with some observations of Enoch (1963) were obtained.

THEORY

The receptor outer segment was approximated by a homogeneous isotropic circular cylindrical dielectric rod (Fig. 1). Apart from guided modes there are unguided modes which are not bound to the receptor. These unguided modes will not be considered here. The characteristics of the guided modes are determined by the normalized frequency $v = (2\pi\rho n_1/\lambda_{vac})\sqrt{\delta}$ and δ. Here $\delta = 1 - (n_2^2/n_1^2)$; n_1 and n_2 are the refractive indices of the inside and outside medium respectively, ρ is the radius of the rod and λ_{vac} is the vacuum wavelength of the incident light. For retinal receptors the parameter δ is of the order of 0·1, therefore the approx results for $\delta \to 0$ may be used for the field of the modes (Snyder, 1969a). The amplitude with which a mode is excited may be calculated neglecting the reflected light (Snyder, 1969b). In this approximation when the light incident on the outer segment is plane polarized, plane polarized modes and mode-combinations are excited (Wijngaard, 1971). Interference effects can only be obtained when the phasevelocity of the modes is different, therefore the approximation $\delta \to 0$ is not applicable to calculations of the phasevelocity. However, the difference in phasevelocity between the modes of a plane polarized mode-combination is small enough to conserve the plane polarization when the modes are guided along the outer segment.

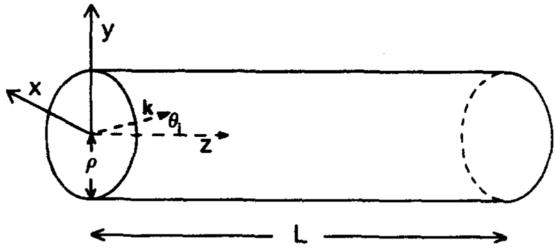

Fig. 1. Schematic drawing of a receptor outer segment.

The author has discussed minor deviations from this result for frog rods (Wijngaard, 1971). The approximate conservance of the plane and the degree of polarization is characteristic for the relatively short fibres of small δ in the retina. For the much longer fibres of, for example, Snitzer and Osterberg (1961) this may not be the case. These remarks should be considered in the designation of modes to patterns observed in retinal receptor outer segments.

To obtain physiologically significant interference patterns the amplitudes of the modes should be calculated from the field incident on the retina. However, even for an incident plane wave this problem is complex owing to the influence of the inner segment. Therefore the field at the entrance of the outer segment was assumed to be a plane wave restricted to the outer segment with the electric field vector perpendicular to the plane of incidence. The propagation vector **k** of the plane wave was assumed to lie in the X-Z plane (Fig. 1). This implies that the calculated interference patterns are symmetric about the plane through the axis of the outer segment and the X-axis. In this case only those modes and mode-combinations which have their transverse electric field vector directed along the Y-axis are excited. These modes and mode-combinations are, when the expressions for the field components given by Snyder (1969a) are used: $HE_{1,m}$, $TE_{0,m} + HE_{2,m}$ and $EH_{n-1,m} + HE_{n+1,m}$ $(n > 1)$.

NUMERICAL RESULTS

The interference patterns were calculated using the approximation $\delta \to 0$ for the field of the modes. The difference in phase $\Delta\Phi$, expressed in radians, between two modes due to the propagation over a length z along the rod is approximately given by

$$\Delta\Phi = (u_1^2 - u_2^2)\frac{\sqrt{\delta}}{2v}\cdot\frac{z}{\rho}. \tag{1}$$

Here u_1 and u_2 are the values of the parameters u of the modes, with $u = \rho(k_1^2 - h^2)^{1/2}$ (Snyder, 1969a), k_1

is the propagation constant of plane waves in a medium with refractive index n_1 (the refractive index of the inside medium) and h is the propagation constant of the guided mode. For guided modes $0 < u < v$. In this paper the value of u from the approximation $\delta \rightarrow 0$ will be used.

The interference patterns were given as a pattern of spots on an oscilloscope screen. To this end the intensity was calculated for the points of a grid. A number of spots proportional to the intensity are displayed in a region centred on each point of the grid. The maximum number of spots was 25. The inner circle indicates the circumference of the guide, the outer circle indicates the circumference of the grid of calculation points.

To obtain patterns qualitatively comparable to the patterns observed by Enoch (1963) at the terminations of retinal receptors v was chosen to be 4·5 for λ_{vac} = 400 nm. A representative value of δ for cone outer segments is 0·072; this implies a value for ρ of 0·768 μm. The relative length L/ρ of the outer segment was assumed to be 30. The values of δ and L/ρ are only necessary to compute the difference in phase between the modes, therefore the effect of a smaller value of δ may be compensated by a larger value of L/ρ and reversely. A relative angle of incidence θ_r is defined with $\theta_r = \theta_i/\theta_c$. Here θ_i is the angle of incidence of the plane wave (Fig. 1) and θ_c is the angle of incidence of a meridional ray which strikes the wall of the outer segment at the critical angle.

To investigate the influence of the wavelength of the incident light the interference patterns are given for v equal to 3·0, 3·5, 4·0 and 4·5 with θ_r = 1, L/ρ = 30 and δ = 0·072 in Fig. 2. In Fig. 3 the effect of a variation of the angle of incidence is demonstrated for v = 4·5, δ = 0·072 and L/ρ = 30. The relative power with which the modes and mode-combinations are excited is given in Table 1. A comparison of Figs. 2 and 3 demonstrates that an increase of the angle of incidence θ_r from 2/3 to 1 and an increase of v from 3·5 to 4·5 (decrease of λ_{vac}) have qualitatively the same influence on the interference pattern.

The energy may beat between one side of the receptor and the other. This effect will be demonstrated for v = 4·5, δ = 0·072 and θ_r = 1/3 in Fig. 4. In this figure the patterns are given for z/ρ equal to 10, 20 and 30.

When v is larger than 4·5 the interference pattern is more complicated for the same θ_r. This fact is demonstrated for parameters representative for frog red rod 4·5, θ_r = 2/3 and δ = 0·072 and with only the ampli-

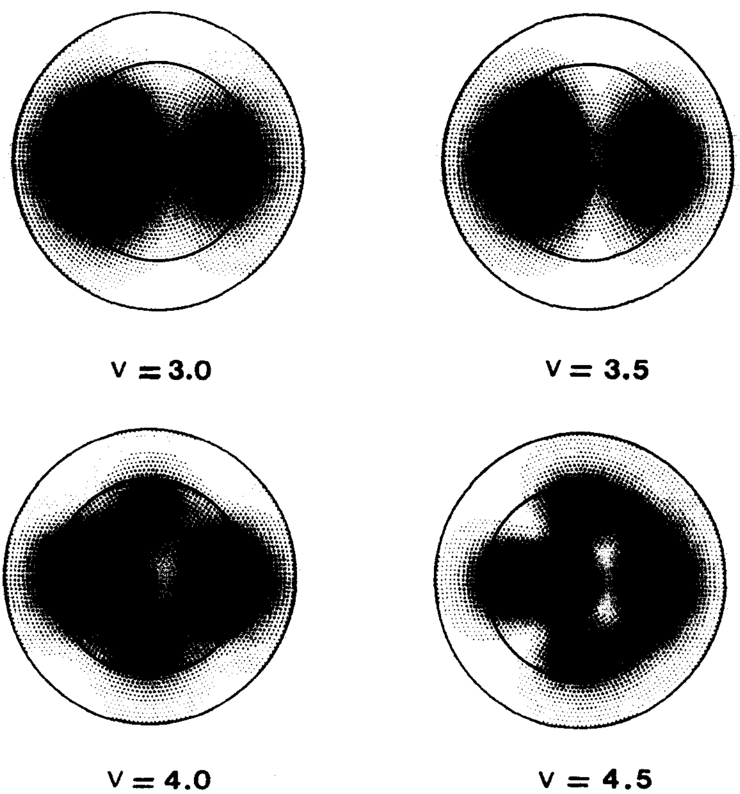

Fig. 2. Calculated interference patterns as a function of wavelength. L/ρ = 30; θ_r = 1; δ = 0·072 (v is proportional to $1/\lambda$).

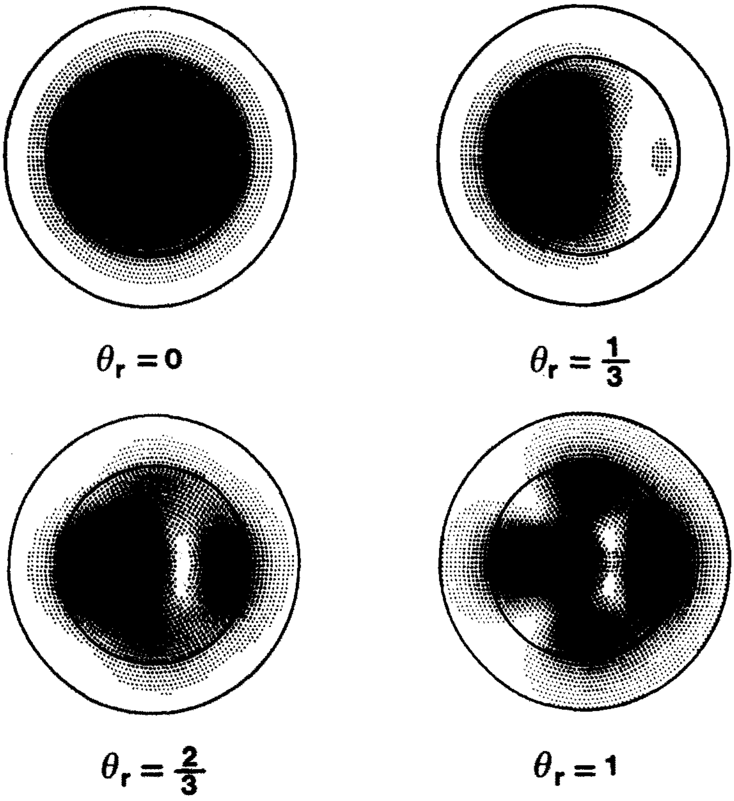

Fig. 3. Calculated interference patterns as a function of the angle of incidence of the incident wave. $L/\rho = 30$; $v = 4{\cdot}5$; $\delta = 0{\cdot}072$.

outer segments with $v = 15{\cdot}8$, $\delta = 0{\cdot}097$ and $L/\rho = 16$. In Fig. 5 the intensity patterns are given for $\theta_r = 0{\cdot}5$ and for $\theta_r = 1$. In this case 35 modes and mode-combinations are excited. These calculated patterns are qualitatively comparable to the patterns observed by Enoch and Tobey (1973).

As was mentioned in the section on theory, plane polarized incident light excites plane polarized modes and mode-combinations. This implies that a $TE_{0,1}$ mode, for example, will not be excited alone even when the incident light is unpolarized. The single ring pattern found by Enoch (1963) is not necessarily a $TE_{0,1}$ or a $TM_{0,1}$ or a $HE_{2,1}$ mode, but the pattern may even be a combination of the $HE_{1,1}$ and the $HE_{1,2}$ mode. This combination is plane polarized with a plane of polarization determined by the incident field. Interference patterns of the $HE_{1,1}$ and the $HE_{1,2}$ modes are given in Fig. 6. These patterns were obtained with $v =$

Table 1. The power with which the Y-polarized modes and mode-combinations are excited by a Y-polarized plane wave restricted to the outer segment. The propagation vector of the plane wave was assumed to lie in the X-Z plane (Fig. 1). The incident power was assumed to be 1. v is the normalized frequency and θ_r is the relative angle of incidence

v	θ_r	$HE_{1,1}$	$TE_{0,1} + HE_{2,1}$	$EH_{1,1} + HE_{3,1}$	$HE_{1,2}$
3·0	1·0	0·089	0·314	—	—
3·5	1·0	0·034	0·285	—	—
4·0	1·0	0·008	0·200	0·200	0·030
4·5	1·0	0·0004	0·111	0·217	0·096
4·5	2/3	0·109	0·430	0·190	0·056
4·5	1/3	0·537	0·308	0·026	0·0001
4·5	0	0·849	0	0	0·022

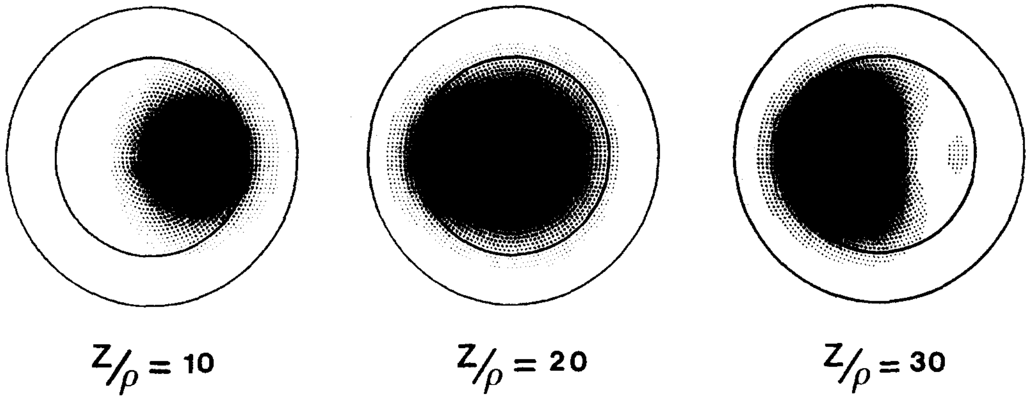

Fig. 4. Calculated interference patterns for different levels inside the outer segment. $v = 4{\cdot}5$; $\theta_r = 1/3$; $\delta = 0{\cdot}072$.

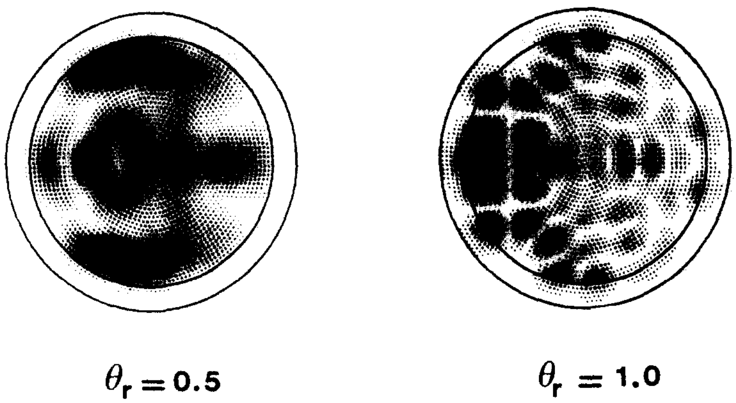

Fig. 5. Calculated interference patterns for parameters representative for frog red rods with λ approx 525 nm. $v = 15{\cdot}8$; $L/\rho = 16$; $\delta = 0{\cdot}097$.

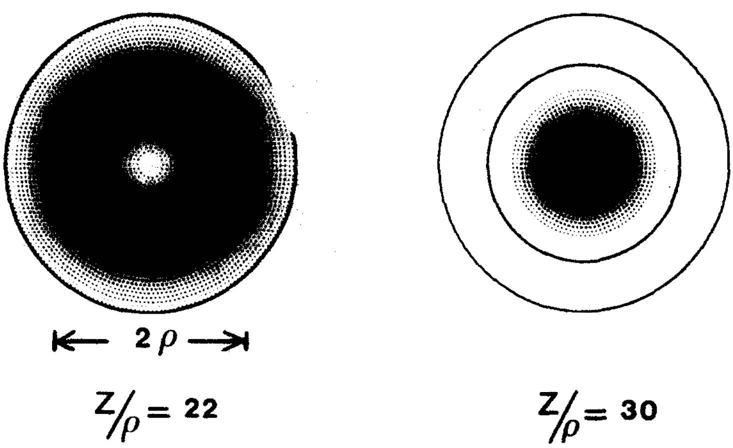

Fig. 6. Calculated interference patterns of the $HE_{1,1}$ and the $HE_{1,2}$ modes. For $z/\rho = 22$ these modes are approximately in antiphase, for $z/\rho = 30$ they are approximately in phase. $v = 4{\cdot}5$; $\theta_r = 2/3$; $\delta = 0{\cdot}072$.

tudes of the $HE_{1,1}$ and the $HE_{1,2}$ modes nonzero. For $z/\rho = 22$ the $HE_{1,1}$ and the $HE_{1,2}$ modes are approximately in antiphase, for $z/\rho = 30$ these modes are approximately in phase.

DISCUSSION

The results given here may be valid for a complete cone model with for example a conical inner segment and a cylindrical outer segment when the normalized frequency v is constant throughout the receptor. In this case when it is assumed that the modes do not couple to each other or to the radiation field the situation is equivalent to the situation discussed above except that formula (1) for the phase difference is not valid.

The amplitudes have been calculated by the use of the assumption that the plane wave is truncated to the area of the receptor. For a complete cone model this assumption will be approximately valid for touching cone inner segments when coupling between the modes of different cones is neglected.

The results given above have been obtained neglecting for example the birefringence of the outer segment. This approximation is valid due to the fact that for $\delta \ll 1$ for the guided modes the component of the E-field in the direction of the axis is small compared with the transverse component. The small refractive index difference between the receptor and the matrix medium justifies neglection of reflected light. The results given here are for receptors without absorption. However, the effects of absorption may be included in the way indicated by Snyder (1972).

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Résumé—On calcule quelques figures d'interférence des modes électromagnétiques pour un guide d'ondes diélectrique, représentant le segment externe d'un récepteur. Les résultats sont en accord qualitatif avec les observations d'Enoch.

Zusammenfassung—Für einen dielektrischen Wellenleiter, der das äussere Segment eines Rezeptors beschreibt, werden einige Interferenz-Muster von elektromagnetischen Moden berechnet. Die Ergebnisse stimmen qualitativ mit den Beobachtungen von Enoch überein.

Резюме—Рассчитаны некоторые интерференционные паттерны электромагнитного поля электрического волновода, представляющего наружный сегмент рецептора. Результаты качественно соответствуют наблюдениям.