Zijlstra, R. J. J.
Alkemade, C. Th. J.
1965

Physica 31
1486–1488

LETTER TO THE EDITOR

Statistics of recombination radiation in solids

Fluctuations in the generation and recombination rates of electrons and holes in solids have extensively been studied in connection with the resulting fluctuations in the number of free charge carriers[1]. The fluctuations in the recombination rate, however, may also directly show up as fluctuations in the radiation emitted because of the (radiative) recombination process. Here we shall consider theoretically the possible deviation of the statistics of emitted photons from Poisson statistics. The terms generation and recombination in solids are commonly used in connection with the generation of carriers from a lower energy level to a higher level, and with the recombination of electrons and holes, respectively. Since we want also to include in our analysis the case of $p-n$ luminescence, where injection of carriers plays a predominant role, we prefer to use the more general terms excitation and de-excitation instead.

In calculating carrier density fluctuations one can adopt the Langevin approach[2] and extend the macroscopic kinetic equations for the (de-)excitation rates with microscopic driving "forces" with suitable stochastic properties which formally describe the noise sources. Because of their shot noise character these forces are assumed to depend on the corresponding average (de-)excitation rates only. Their auto-correlation functions in the time and space domain are usually assumed to have the shape of a δ-function. They do not depend explicitly on the occupancy of the states concerned. The resulting fluctuations in the observable (de-)excitation rates, however, will depend on the fluctuations in the occupancy of the relevant levels, when these occupancies enter explicitly into the underlying kinetic equations. The situation can be compared with the analogous case of current fluctuations in the loaded external circuit of a thermionic diode, where the fluctuations in the random emission of electrons from the cathode are the "driving noise forces". Owing to the (finite) internal resistance of the tube the fluctuating voltage developed across the external load enters explicitly into the circuit equation governing the current fluctuations.

In the following an attempt will be made to prove theoretically for a simplified model, under which conditions deviations from Poisson statistics may occur for a stream of photons emitted in a radiative recombination process. We will assume that the driving noise force connected with the de-excitation process has a pure shot noise character, whereas the stochastic properties of the driving force connected with the excitation process remain as yet unspecified. In our model the occupancy n of the excited state only, occurs as an independent variable in the kinetic equations. Accordingly, the fluctuations in the de-excitation rate r and the excitation rate g around their steady state values are determined by

$$\Delta r(t) = r' \cdot \Delta n + R(t) \tag{1a}$$

$$\Delta g(t) = g' \cdot \Delta n + G(t) \tag{1b}$$

where

$$r' \equiv \left(\frac{\mathrm{d}r}{\mathrm{d}n}\right)_{\Delta n=0}, \quad g' \equiv \left(\frac{\mathrm{d}g}{\mathrm{d}n}\right)_{\Delta n=0}.$$

The driving noise forces are represented by $R(t)$ and $G(t)$, respectively. Making a Fourier analysis of equation (1*a*) in the frequency domain and denoting complex Fourier coefficients by a, one obtains

$$a_{\Delta r} = r' a_{\Delta n} + a_R \tag{2}$$

and

$$a^*_{\Delta r} = r' a^*_{\Delta n} + a^*_R \tag{3}$$

where the asterisk denotes complex conjugate quantities and the subscripts refer to the fluctuating quantities considered. Multiplying the corresponding sides of eqs 2 and 3, and averaging yields

$$S_{\Delta r} = r'^2 S_{\Delta n} + S_R + 2r' \langle \mathrm{Re}(a_{\Delta n} a^*_R) \rangle \tag{4}$$

where S stands for spectral noise intensity and Re( ) denotes the real part of the expression between brackets. Here use is made of the well-known relation between the Fourier coefficient and the corresponding spectral noise intensity [3].

Since we consider fluctuations Δn in the occupancy around a steady state value, we have in addition

$$\frac{\mathrm{d}\Delta n}{\mathrm{d}t} = \Delta g - \Delta r. \tag{5}$$

Substituting eq. 1 into eq. 5 gives the familiar relaxation equation

$$\frac{\mathrm{d}\Delta n}{\mathrm{d}t} + \frac{\Delta n}{\tau} = G - R \tag{6}$$

where $\tau^{-1} \equiv r' - g'$.

Making a Fourier analysis of eq. (6) we find

$$(\mathrm{j}\omega + 1/\tau)\, a_{\Delta n} = a_G - a_R \tag{7}$$

and

$$(-\mathrm{j}\omega + 1/\tau)\, a^*_{\Delta n} = a^*_G - a^*_R \tag{8}$$

where ω is angular frequency and j the imaginary unit.

After multiplying both sides of eqs. (7) and (8) by a^*_R and a_R, respectively, and after subsequently adding and subtracting the resulting equations one finds with the help of some algebra

$$\langle 2\mathrm{Re}(a_{\Delta n} \cdot a^*_R) \rangle = \frac{-2\tau S_R}{1 + \omega^2 \tau^2}\,. \tag{9}$$

In addition, the familiar result [4] for the spectral noise intensity $S_{\Delta n}$ is obtained by multiplying the corresponding sides of eqs. (7) and (8) with each other and averaging

$$S_{\Delta n} = \frac{\tau^2 (S_G + S_R)}{1 + \omega^2 \tau^2}\,. \tag{10}$$

Here it is assumed that $R(t)$ and $G(t)$ are uncorrelated. Inserting the expressions (9) and (10) into eq. (4) one obtains for the noise quantity under discussion

$$S_{\Delta r} = \frac{g'^2 S_R + r'^2 S_G + \omega^2 S_R}{(r' - g')^2 + \omega^2}\,. \tag{11}$$

Inspection of eq. (11) learns that in the limit of $\omega \to \infty$ $S_{\Delta r}$ is, indeed, equal to $S_R = 2\langle r \rangle$. Under this condition the value of S_G is immaterial as long as $S_G/\omega^2 \to 0$ in the limit of $\omega \to \infty$. In the low frequency limit $\omega \to 0$, however, we have

$$S_{\Delta r}(0) > 2\,\langle r \rangle \quad \text{for} \quad r'g' > 0$$
$$S_{\Delta r}(0) < 2\,\langle r \rangle \quad \text{for} \quad r'g' < 0$$
$$S_{\Delta r}(0) = 2\,\langle r \rangle \quad \text{for} \quad r'g' = 0$$

if the usual assumption is now made that $S_G = 2\langle g \rangle = 2\langle r \rangle = S_R$. Thus, even when the driving noise forces show full shot effect the fluctuations in the net de-excitation rate can be either larger or smaller than full shot noise at low frequencies, whereas they equal full shot noise at high frequencies. Since in stable situations r' must be positive, the interesting case that $S_{\Delta r}(0)$ is below the full shot noise level could occur only for $g' < 0$. In this case the minimum value of $S_{\Delta r}(0) = \langle r \rangle$ is obtained when $-g' = r'$.

Considering $p-n$ luminescence we deal with a $p-n$ junction diode biased in the forward direction. Holes and electrons are injected into the n-type and p-type region, respectively, where they subsequently recombine with charge carriers of opposite sign under emission of radiation. If the recombination occurs directly from band to band, we can apply our simple model. In this case the excitation rate in fact equals the injection rate and we have $g' = 0$. Then eq. (11) reveals clearly how $S_{\Delta r}$ and by that the noise in the recombination radiation depends on the statistical behaviour of the injection rate through S_G. If S_G would be smaller than $2\langle r \rangle$ and $g' = 0$, then $S_{\Delta R}$ would be suppressed below its full shot noise value for $\omega \ll r'$. So, if the luminescence efficiency would be close to unity and if no partition noise effects would occur in the detection of the emitted photons, the fluctuations in the number of photons counted would be below the value predicted by Poisson statistics. Partition noise, however, is likely to occur due to collection losses or to a quantum efficiency of the photon detector below unity.

The noise suppression below the full shot noise level in the outgoing recombination radiation is here in fact achieved by means of a kind of internal feed-back in the model considered. One might argue that the same effect could be obtained by a trivial external feed-back system, where e.g. part of the radiation from an electrical light source is detected by a phototube the output of which is fed back into the lamp circuit. With an arrangement like this a similar suppression of the noise in the available radiation (as distinguished from the radiation used in the feed-back circuit) cannot be obtained, however, since the shot noise in divided beams is essentially uncorrelated.

The above noise considerations might be of some value in considering the statistics of so-called radistors [5]). These are phototransistors which combine efficiently an electroluminescent $p-n$ junction as a radiation source and a photosensitive $p-n$ junction as a radiation detector.

R. J. J. ZIJLSTRA
C. TH. J. ALKEMADE

Fysisch Laboratorium der Rijksuniversiteit
Utrecht, Nederland

Received 23-6-65

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