

## INTERFERING RESONANCES IN THE $^{19}\text{F}(\alpha, p)^{22}\text{Ne}$ REACTION

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### Synopsis

Energies and strengths are reported of twenty-seven  $^{19}\text{F}(\alpha, p)^{22}\text{Ne}$  resonances in the  $E_\alpha = 1.3\text{--}2.6$  MeV region. Transitions are observed to the  $^{22}\text{Ne}$  ground state and first excited state. A yield curve for the  $E_\alpha = 1.3\text{--}3.3$  MeV region is presented. Angular distributions of the ground-state protons are measured as a function of energy. The analysis in terms of the theory of interfering resonances yields the spins of fifteen resonances and the parity of five of these. Information is obtained about widths, partial widths and reduced widths of the observed levels.

1. *Introduction.* The investigation of the  $^{19}\text{F}(\alpha, p)^{22}\text{Ne}$  reaction, reported in this article, is the last one in a series of investigations of  $(\alpha, p)$  reactions<sup>1)2)3)</sup>.

With alpha-particle energies up to  $E_\alpha = 3.3$  MeV, information can be obtained about excited states of  $^{23}\text{Na}$  in the region  $E_x = 11$  MeV to  $E_x = 13.2$  MeV. The yield curve shows that above  $E_\alpha \approx 2.6$  MeV the average level spacing compared to the average level width is too small to discuss properties of individual levels. For this reason the investigation is limited to levels below this energy.

Twenty-seven levels are observed in this region, corresponding to levels in  $^{23}\text{Na}$  between  $E_x = 11.5$  and 12.6 MeV. Most of these resonances can not be considered as isolated levels and therefore interference terms must be taken into account in the discussion of angular distributions. For the ground-state protons the channel spin for both the incoming and the outgoing channels equals one half. This considerably simplifies the analysis of the angular distribution measurements. At a given alpha-particle energy, the angular distribution is completely determined by the widths, the strengths and the energies of all contributing resonances as continuous parameters, and by the spin and parity of these resonances as discrete variables.

In the present paper it is shown that the angular distributions, which can be expected in such a reaction, profitably can be represented in terms of certain linear combinations of Legendre polynomials of which the properties are discussed.

To determine the spin and parity of the observed levels, angular distribution measurements are performed as a function of energy in eleven energy regions each containing one or two resonance levels. From the analysis the spins of fifteen resonances could be determined. In five of these cases the parity was also determined.

The experimental methods are briefly described in section 2. The angular distribution theory as applied to the  $^{19}\text{F}(\alpha, p)^{22}\text{Ne}$  reaction is discussed in section 3. Sections 4 and 5, respectively, present methods of analysis and the results.

2. *Experimental.* Targets of  $\text{SrF}_2$ , evaporated on a solid copper backing, are bombarded with alpha particles from the Utrecht 3 MV Van de Graaff generator. Although this fluorine compound can withstand the beam better than, say,  $\text{LiF}$  or  $\text{CaF}_2$ , a continuous target deterioration is observed during the bombardment, necessitating a frequent change of the target spot.

Angular distribution measurements are performed with eight surface barrier detectors, situated at laboratory angles of  $\theta = 40, 55, 70, 87, 105, 120, 140$  and  $172$  degrees with respect to the incoming beam. Detector pulses are amplified, selected and fed into a BORER scaler system, recording the data on punch tape suitable for analysis with the Utrecht University X1 computer.

A more extensive description of the experimental arrangement is given in ref. 3.

3. *Theory.* The application of the general angular distribution theory to a resonance reaction, involving overlapping levels, is relatively simple for the case that the channel spin ( $S$ ) for both the incoming and the outgoing channel equals one half, as in the  $^{19}\text{F}(\alpha, p)^{22}\text{Ne}$  reaction. For a given value of the spin and parity  $J^\pi$  of the resonance level, only one value is then allowed for the orbital momentum  $l$  of the incoming and the outgoing particles. With this restriction on orbital momentum and channel-spin values, the number of terms in the summation, appearing in the general angular distribution formula as given in ref. 4, is reduced considerably. The intensity of the outgoing particles at a given center-of-mass angle  $\theta$  and at an energy  $E$  of the incoming particles is then proportional to

$$W(\theta, E) \sim \sum_{JJ'} \text{Re}[\langle lS J | R | lS J \rangle \langle l'S' J' | R | l'S' J' \rangle^*] \times \\ \times \sum_k Z'(lJl'J'; Sk)^2 P_k(\cos \theta).$$

The summations over  $J$  and  $J'$  are extended to all resonance levels with different spin and/or parity, which are contributing to the angular distribution at the energy considered. The matrix elements of the operator  $R$  include the energy dependent part of the expression, whereas the angular

dependence occurs in the sum over  $k$ , as a combination of  $Z'$  coefficients (the usual  $Z$  coefficients with an extra phase correction<sup>4</sup>), and the Legendre polynomials  $P_k(\cos \theta)$ .

In order to facilitate the discussion of the angular distributions, this expression can be reduced to

$$W(\theta, E) \sim \sum_{JJ'} G(J\pi, J'\pi', E) W(J, J', \pi \times \pi', \theta)$$

where the energy depending coefficients are given by

$$G(J\pi, J'\pi', E) = \sqrt{(2J+1)(2J'+1)} \times \text{Re}[\langle lS J | R | lS J \rangle \langle l'S J' | R | l'S J' \rangle^*],$$

and the angular distribution components by

$$W(J, J', \pi \times \pi', \theta) = \sum_k \{(2J+1)(2J'+1)\}^{-\frac{1}{2}} Z'(lJl'J'; Sk)^2 P_k(\cos \theta).$$

These linear combinations of Legendre polynomials can easily be computed for all possible combinations of  $J\pi$  and  $J'\pi'$ . An inversion of both parities does not affect the coefficients  $Z'(lJl'J'; Sk)^2$ . It is thus possible to label the  $W$  polynomials by the product of the parities  $\pi \times \pi'$  instead of giving both parities. They can be divided into three different types:

a)  $J' = J; \pi' = \pi.$

This component appears only once in the summation. In cases where only one resonance contributes, it is the only term, thus presenting the angular distribution at an isolated resonance with spin and parity  $J\pi$ . The coefficients are tabulated in ref. 1. When more resonances are contributing, such terms can be considered as being the pure level components. Because  $l' = l$ , all odd order coefficients disappear, resulting in symmetry around the  $\theta = 90^\circ$  plane.

b)  $J' \neq J; \pi' = \pi.$

Each of the interference terms with different spins and equal parities appears twice in the summation. Odd  $k$  coefficients still vanish because of the even difference between  $l$  and  $l'$ . The  $k = 0$  coefficient also equals zero; this is in agreement with the general behaviour of an interference term, which does not contribute to the total yield.

c)  $\pi' \neq \pi.$

Because  $(l - l')$  is odd, the even  $k$  coefficients vanish. Only the odd  $k$  terms remain, giving rise to a distortion of the angular distribution symmetry around the  $\theta = 90^\circ$  plane.

Table I presents the orders of Legendre polynomials contributing to the angular distribution components for spin values up to  $J = 11/2$ , which corresponds to orbital momentum values up to  $l = 5$ . The three parts of

this table, pure level terms, equal parity, and opposite parity interference terms correspond to the three polynomial types discussed above.

If for a given spin and parity combination not more than one level (resonance energy  $E_0$ , width  $\Gamma$ ) contributes, the reaction matrix elements are proportional to

$$\exp(i\xi_l) \times i \frac{g_1 g_2}{(E_0 - E) - i\Gamma/2}.$$

The angle  $\xi_l$  is the sum of the phase shifts for nuclear and potential scattering, for incoming and outgoing particles.

The partial widths of the resonance are directly correlated to the  $g$  factors by  $g_1 = \pm\sqrt{\Gamma_\alpha}$  and  $g_2 = \pm\sqrt{\Gamma_p}$ . The matrix element can thus be written as

$$\pm \exp\{i(\xi_l + \pi + \phi)\} \times \sqrt{\frac{\Gamma_\alpha \Gamma_p}{(E - E_0)^2 + (\Gamma/2)^2}}.$$

The phase angle

$$\phi = \text{arctg} \left( \frac{E - E_0}{\Gamma/2} \right)$$

varies from  $-\frac{1}{2}\pi$  to  $+\frac{1}{2}\pi$  as the energy  $E$  increases.

If only two resonances with different spin and/or parity  $J_1\pi_1$  and  $J_2\pi_2$  contribute, the angular distribution is given by

$$W(\theta, E) \sim \mu_1^2 W(J_1, J_1, +, \theta) + \mu_2^2 W(J_2, J_2, +, \theta) + 2\mu_1\mu_2 \cos \varphi W(J_1, J_2, \pi_1 \times \pi_2, \theta),$$

where

$$\mu_i = \sqrt{\frac{(2J_i + 1) \Gamma_{i\alpha} \Gamma_{ip}}{(E - E_{i0})^2 + (\Gamma_i/2)^2}} \quad \text{and} \quad \varphi = (\phi_1 - \phi_2 + \xi_{l_1} - \xi_{l_2} + \frac{1}{2}(\pi \pm \pi)).$$

The  $G$  coefficients are correlated to these  $\mu$  factors by

$$G(J_1\pi_1, J_1\pi_1, E) = \mu_1^2, \quad G(J_2\pi_2, J_2\pi_2, E) = \mu_2^2, \\ G(J_1\pi_1, J_2\pi_2, E) = \mu_1\mu_2 \cos \varphi;$$

and thus one has

$$\cos \varphi = G(J_1\pi_1, J_2\pi_2, E) / \sqrt{G(J_1\pi_1, J_1\pi_1, E) \times G(J_2\pi_2, J_2\pi_2, E)}.$$

The plus-minus sign in these formulae arises from the sign uncertainty of the  $g$  products. When the  $g$  product  $g_1 g_2$  has equal signs for both resonances, the plus sign is correct; the minus sign is valid in the opposite case. It is seen from the expressions above that the energy dependence of the pure level coefficients  $G(J_1\pi_1, J_1\pi_1, E)$  and  $G(J_2\pi_2, J_2\pi_2, E)$  is given by a Breit-Wigner curve.

4. *Analysis.* a. Yields and widths. The thick target yield of a resonance for  $p_0$  or  $p_1$  emission is proportional to the resonance strength

$S_0$  or  $S_1$ , defined by  $S_0 = (2J + 1) \Gamma_\alpha \Gamma_{p_0} / \Gamma$  and  $S_1 = (2J + 1) \Gamma_\alpha \Gamma_{p_1} / \Gamma$ . When  $\Gamma$  can be determined from the yield curve and when  $J$  is known from angular distribution measurements, two solutions for the partial widths for  $\alpha$ ,  $p_0$ , and  $p_1$  emission, can be found, supposing that all other channels are closed.

In practical cases where  $S_{0,1} \ll \Gamma$ , the first solution is approximated by

$$\Gamma_\alpha = (S_0 + S_1) / (2J + 1), \text{ and}$$

$$\Gamma_{p_0, p_1} = \frac{S_{0,1}}{S_0 + S_1} \Gamma.$$

The second solution, having  $\Gamma_\alpha \approx \Gamma$  is mostly rejected by Wigner limit considerations. If  $\Gamma$  is not known and/or if other channels are open, the value  $(S_0 + S_1) / (2J + 1)$  can still be used as a lower limit for  $\Gamma_\alpha$ .

The conversion to dimensionless reduced widths is described in ref. 5. For the calculation of Coulomb functions in the formation channel the method of Feshbach *et al.*<sup>6)</sup> is used. For the Coulomb functions in the decay channel a power series expansion computer program<sup>7)</sup> (written by B. Bošnjaković) is applied.

*b. Angular distributions.* All observed angular distributions were expanded in terms of Legendre polynomials  $\sum_0^n B_i P_i$ . The order  $N$  of the highest polynomial contributing in an energy region around one or two resonances is determined from the value of  $\chi^2$  as a function of  $n$ . From the knowledge of  $N$ , one can find, with the aid of table I, the highest spin value contributing to the angular distributions in the energy region considered. If no odd order coefficients appear, significantly different from zero, it can be concluded that all contributing resonances have the same parity.

Using the information obtained above, an attempt is now made to interpret the angular distributions as a linear combination of  $W$  polynomials with angular momenta up to the maximum value, determined from the  $B$  coefficients.

When only two levels with spin and parity  $J_1^{\pi_1}$  and  $J_2^{\pi_2}$  interfere in a given energy region, four possibilities have to be considered:

I)  $J_2 = J_1; \pi_2 = \pi_1$ . The angular distribution, which is the same as if only one single level  $J_1^{\pi_1}$  exists, is proportional to  $W(J_1, J_1, +, \theta)$ . The least squares procedure yields the energy dependent coefficient in front. This coefficient can be compared with the theoretical two level expression<sup>8)</sup>.

II)  $J_2 = J_1; \pi_2 \neq \pi_1$ . Pure level components coincide. The fitting procedure determines the coefficients  $(G(J_1 \pi_1, J_1 \pi_1, E) + G(J_1 \pi_2, J_1 \pi_2, E))$  and  $2G(J_1 \pi_1, J_1 \pi_2, E)$  of the polynomials  $W(J_1, J_1, +, \theta)$  and  $W(J_1, J_1, -, \theta)$ , respectively.

III)  $J_2 = J_1 \pm 1; \pi_2 = \pi_1$ . The interference term  $W(J_1, J_1 \pm 1, +, \theta)$  appears to be a linear combination with known coefficients of the two pure

level terms. The angular distributions near such a doublet are then determined by the two independent polynomials  $W(J_1, J_1, +, \theta)$  and  $W(J_2, J_2, +, \theta)$ . The coefficients of these polynomials obtained in a least squares procedure are linear combinations of the three  $G$  coefficients  $G(J_1\pi_1, J_1\pi_1, E)$ ,  $G(J_2\pi_2, J_2\pi_2, E)$ , and  $G(J_1\pi_1, J_2\pi_2, E)$ .

IV) Other cases. The three polynomials  $W(J_1, J_1, +, \theta)$ ,  $W(J_2, J_2, +, \theta)$ , and  $W(J_1, J_2, \pi_1 \times \pi_2, \theta)$  are independent. The least squares method determines the three coefficients  $G(J_1\pi_1, J_1\pi_1, E)$ ,  $G(J_2\pi_2, J_2\pi_2, E)$ , and  $2G(J_1\pi_1, J_2\pi_2, E)$ . The value of  $\cos \varphi$  can be determined directly from these coefficients.

TABLE I

Orders of Legendre polynomials, contributing to the angular distributions of interfering resonances in a reaction with initial and final channel spins $S = 1/2$ , for spin values $J$ and $J'$ up to $11/2$									
$J \rightarrow$		1/2	3/2	5/2	7/2	9/2	11/2	$J$	
$J'$	$\downarrow$	0	0-2	0-4	0-6	0-8	0-10	pure level terms (even)	
1/2	0	1	1	3	3	5	5	opposite parity interference terms (odd)	
3/2	0-2	2	1-3	1-3	3-5	3-5	5-7		
5/2	0-4	2	2-4	1-5	1-5	3-7	3-7		
7/2	0-6	4	2-4	2-6	1-7	1-7	3-9		
9/2	0-8	4	4-6	2-6	2-8	1-9	1-9		
11/2	0-10	6	4-6	4-8	2-8	2-10	1-11		
$J'$	pure level terms (even)	equal parity interference terms (even)							

For all four types of interference it is possible, for a given set of the six resonance parameters, energies, widths, and strengths, to calculate the behaviour of the coefficients obtained. A least squares method will then give the best values for these quantities. The spins, parities, and relative  $g$ -product signs will enter as discrete parameters. The resonance strengths do not appear into the fitting procedure for  $\cos \varphi$ , which only can be applied in case IV.

If a two level analysis is not possible, some qualitative conclusions can still be made from the behaviour of the coefficients  $B_k$  in the Legendre polynomial expansion.

When only one resonance interferes with a low background to which several other resonances contribute, the total phase angle will increase by  $\pi$ , when the energy varies from far below to far above the resonance energy. This will in general cause a change of sign for all interference terms. The  $B$  coefficients corresponding to Legendre polynomials, appearing only in interference terms, thus also change sign. Pure level  $B$  coefficients are

expected to behave like a Breit-Wigner curve, superimposed on a low background. Such coefficients will always be positive. After a subtraction of the background, they will vary with  $B_0$ ; the proportionality factors are given in ref. 1 (table I).

5. *Results.* A yield curve for the  $^{19}\text{F}(\alpha, p)^{22}\text{Ne}$  reaction in the  $E_\alpha = 1.3 - 3.3$  MeV region, as observed with one detector at  $\theta = 120^\circ$ , is presented in fig. 1. The ground-state proton yield is shown for the whole energy region; the yield for protons, decaying to the first excited state in  $^{22}\text{Ne}$  is given above  $E_\alpha = 1.45$  MeV. For lower energies pulses from the latter proton group get obscured in the detector noise. No ground-state proton resonance was observed in the  $E_\alpha = 0.9 - 1.3$  MeV region; Wigner limit considerations exclude the existence of observable resonances below this region.

Above the neutron threshold ( $E_\alpha = 2.36$  MeV) the average level width is much higher than below this energy, which, together with the increasing level density, makes it difficult to consider properties of individual levels. For this reason the discussion is mainly limited to levels below  $E_\alpha \approx 2.6$  MeV. All information obtained about the twenty-seven resonances observed in this region is collected in table II. The first columns of this table present resonance energies, excitation energies computed with  $E_b = 10.465$  MeV<sup>9</sup>), thick  $\text{SrF}_2$  target yields and resonance strengths for both transitions. The target deterioration is the main reason of the uncertainty of about 40% in yields and strengths.

The angular distribution measurements performed around resonances indicated with an arrow in fig. 1 and the spin and parity assignments given will be discussed individually for each resonance. The original measurements are only presented for the 1879–1884 keV doublet, the only case which could fully be analysed with the two-level interference theory, and where contributions from other nearby resonances could be neglected.

The resonance at 1318 keV.

Angular distributions, measured at twenty-three alpha-particle energies in a 50 keV region are isotropic within the experimental error, indicating that no important interference effects contribute. A unique  $J = \frac{1}{2}$  assignment follows from  $\chi^2$  considerations.

The resonances at 1492 and 1507 keV.

Angular distributions measured at fifteen different alpha-particle energies in a 50 keV region, can be fitted well by assuming  $N = 2$ ; the  $B_0$ ,  $B_1$ , and  $B_2$  coefficients are then significantly different from zero. The  $B_0$  coefficient exhibits two peaks. It can be seen from table I, that such a behaviour can be expected only for a pair of resonances with spins  $\frac{1}{2}$  and  $3/2$  and opposite

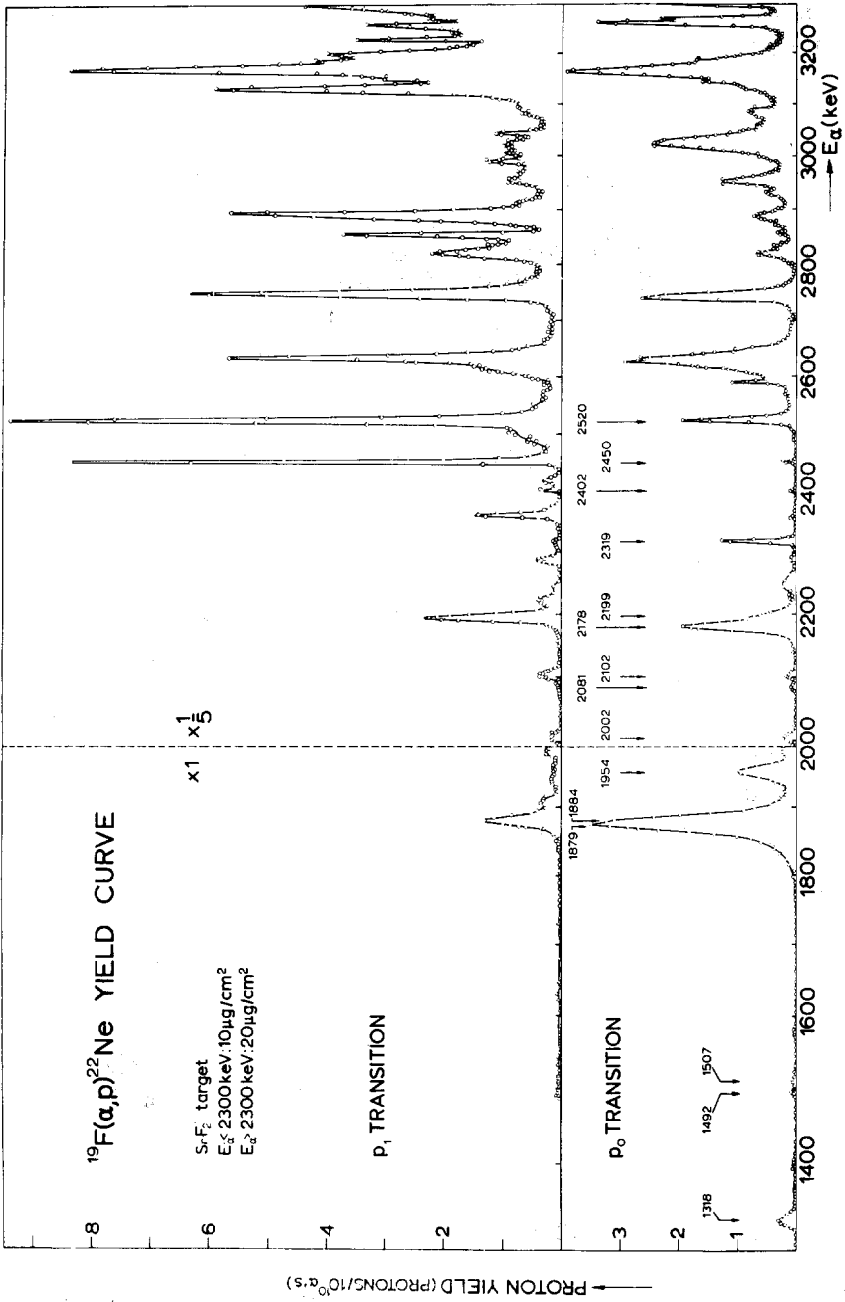


Fig. 1. Yield curve for the  $^{19}\text{F}(\alpha, p)^{22}\text{Ne}$  reaction, observed at a laboratory angle  $\theta = 120^\circ$ . Ground-state proton angular distributions are investigated at resonances, indicated with an arrow.



TABLE II

Resonances in the $^{19}\text{F}(\alpha, p)^{22}\text{Ne}$ reaction below $E_\alpha = 2.6$ MeV														
$E_\alpha$ (keV)	$E_x$ (MeV)	$p_0$ transition		$p_1$ transition		$N$	$J^\pi$	$\Gamma$ (keV)	$\Gamma_\alpha$ (eV)	$\Gamma_{p_0}$ (keV)	$\Gamma_{p_1}$ (keV)	$\theta_\alpha^2 \times 10^2$	$\theta_{p_0}^2 \times 10^2$	$\theta_{p_1}^2 \times 10^2$
		yield a)	strength (eV) b)	yield a)	strength (eV) b)									
1318 ± 4	11.552	0.29	5.0			0	1/2	<15	> 2.5			> 13		
1364 ± 5	11.590	0.027	0.47					<15						
1395 ± 5	11.615	0.051	0.90					<15						
1492 ± 5	11.696	0.092	1.7	0.069	1.3	2	1/2	<15	> 0.84			> 0.72		
1507 ± 5	11.707	0.074	1.4	0.014	0.26	2	3/2	<15	> 0.34			> 0.53		
1574 ± 7	11.764	0.041	0.77	<0.04	< 0.7			<15						
1879 ± 5	12.015	4.9	100	<0.2	< 4	4	1/2 <sup>+</sup>	30 ± 5	63	30	<1	3.6	0.93	> 0.81
1884 ± 5	12.019	1.1	24	1.3	27	4	5/2 <sup>+</sup>	20 ± 5	11	9	11	3.4	1.10	
1901 ± 9	12.033	< 0.2	< 4	0.21	4.5			<15						
1954 ± 5	12.077	2.0	42	<0.3	< 6	2	1/2 <sup>-</sup>	15 ± 5	22	15	<2	1.4	0.67	
1982 ± 5	12.100	0.29	6.2	0.16	3.5			<14						
2002 ± 4	12.117	1.1	23	0.78	17	4	1/2	< 8	> 20			> 0.51		
2081 ± 4	12.182	0.70	15	0.24	5.2	4	5/2	< 8	> 3.4			> 0.31		
2102 ± 4	12.199	0.92	21	1.9	43	4	5/2	7 ± 5	11	2.2	4.8	> 1.0	> 0.23	> 0.29
2178 ± 4	12.262	10	230	0.62	14	2	1/2 <sup>+</sup>	13 ± 3	220	12	0.76	2.7	0.34	> 0.33
2199 ± 5	12.279	11	260	4.4	100	2	1/2 <sup>-</sup>	24 ± 5	180	17	6.8	3.7	0.69	> 0.73
2216 ± 4	12.293	< 0.5	< 10	0.69	16			<15						
2243 ± 6	12.315	32	740	<3	< 10			25 ± 10						
2284 ± 4	12.349	0.77	18	0.99	23	3	3/2	<13	> 66			> 0.69		
2319 ± 2	12.378	9.6	230	1.5	36			< 5						
2354 ± 4	12.407	0.72	17	2.3	54			< 6						
2360 ± 4	12.412	< 0.3	< 7	9.6	230	6	7/2	< 6	> 6.7			> 0.57		
2402 ± 2	12.447	0.62	15	1.6	39			< 5						
2420 ± 4	12.462	< 0.2	< 5	2.8	69			10 ± 3						
2450 ± 2	12.486	1.5	36	11	250	3	3/2	< 5	> 73			> 0.46		
2490 ± 10	12.519	2.1	52	2.6	64			15 ± 8						
2520 ± 3	12.544	19	470	15	370	3	3/2	7 ± 3	> 210			> 1.0		

a) Yield of a thick SrF<sub>2</sub> target in protons/10<sup>10</sup> alpha particles. The error may amount to 40%.b) Resonance strength  $(2J + 1)\Gamma_\alpha\Gamma_p/\Gamma$ .c) The order  $N$  of the highest Legendre polynomial, contributing to the angular distribution of the ground state protons.

parities. The measurements of  $W(E, \theta)$ , as analysed in three terms  $W(\frac{1}{2}, \frac{1}{2}, +, \theta)$ ,  $W(3/2, 3/2, +, \theta)$ , and  $W(\frac{1}{2}, 3/2, -, \theta)$ , show that  $J = \frac{1}{2}$  should be assigned to the lower, and  $J = 3/2$  to the upper resonance. The relatively bad statistics obtained at these weak resonances, together with the smallness of their widths, compared to the target thickness, precludes a parity assignment.

The resonances at 1879 and 1884 keV.

Angular distribution measurements, performed at seventeen different alpha-particle energies, covering a 90 keV region, are fitted well by Legendre polynomial analysis with  $N = 4$ . The  $B$  coefficients obtained are plotted in fig. 2a. The smooth curves in this figure are derived from the analysis below.

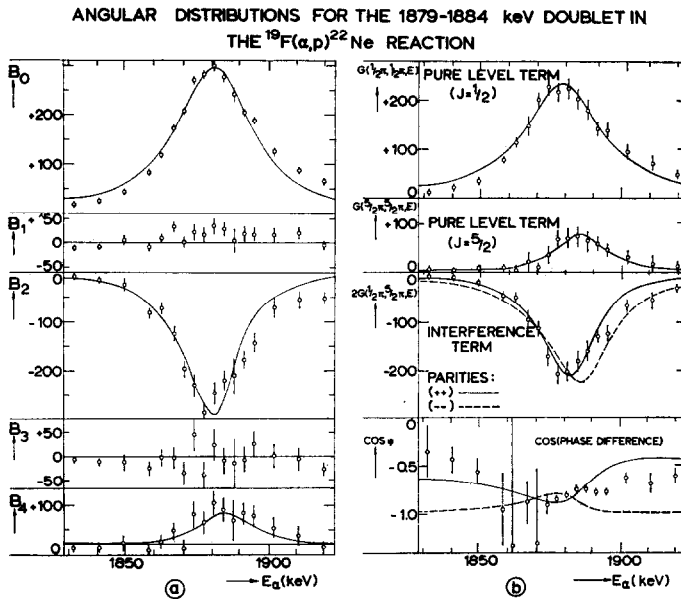


Fig. 2. Angular distributions, observed at the 1879-1884 keV doublet analysed in terms of

a) Legendre polynomials;

b) the polynomials  $W(\frac{1}{2}, \frac{1}{2}, +, \theta) = 1$ ,  $W(5/2, 5/2, +, \theta) = 1 + 1.14 P_2(\cos \theta) + 0.86 P_4(\cos \theta)$ , and  $W(\frac{1}{2}, 5/2, +, \theta) = 1.73 P_2(\cos \theta)$ .

In the latter case the value of  $\cos \varphi$  is also plotted.

No odd order coefficients exist, significantly different from zero. The behaviour of the  $B$  coefficients indicates that at least two resonances contribute. If one assumes a two level interference, table I gives  $1/2-5/2$  or  $3/2-5/2$  spin possibilities with equal parities. For a  $3/2-5/2$  combination it should be possible to write the observed angular distributions as a linear combination of the two polynomials  $W(3/2, 3/2, +, \theta)$  and  $W(5/2, 5/2, +, \theta)$  (case III).

Using  $\chi^2$  considerations, this possibility can be excluded. For the discussion of the  $1/2-5/2$  assumption, the angular distributions are written as a linear combination of the polynomials  $W(\frac{1}{2}, \frac{1}{2}, +, \theta) = 1$ ,  $W(5/2, 5/2, +, \theta) = 1 + 1.14 P_2(\cos \theta) + 0.86 P_4(\cos \theta)$ , and  $W(1/2, 5/2, +, \theta) = 1.73 P_2(\cos \theta)$ . Application of the least squares procedure, determines the corresponding coefficients  $G(\frac{1}{2}\pi_1, \frac{1}{2}\pi_1, E)$ ,  $G(5/2\pi_1, 5/2\pi_1, E)$ , and  $2G(\frac{1}{2}\pi_1, 5/2\pi_1, E)$ . These coefficients are plotted in figure 2b, together with the value of  $\cos \varphi$ . A least squares fit of the two pure level coefficients to Breit-Wigner distributions, yields the curves shown in this figure. For the interference coefficient and for  $\cos \varphi$  four possibilities exist, of which only the possibilities with equal  $g$ -product signs are given in the figure. The other two, opposite to these curves, are in distinct disagreement with the observed data. The agreement of the curves shown in figure 2 is much better for a double positive parity assumption than for a double negative assignment. The difference between the double positive curve and the data points is expected to be produced by target thickness effects.

The resonance at 1954 keV.

Angular distributions, determined at twelve energies in a 45 keV region can be fitted well for  $N = 2$ . The  $B_0$  coefficient exhibits one resonance at 1954 keV, whereas with increasing alpha particle energy  $B_2$  changes slowly from small negative values to about zero. The  $B_1$  coefficient increases from negative to positive values. This behaviour can be understood by supposing a  $J = \frac{1}{2}^-$  resonance to be interfering with the high energy tail of the strong and broad 1879 keV resonance. The fact that the  $B_1$  coefficient is significantly different from zero suggests parity difference. The  $B_2$  coefficient is probably produced by the 1879–1884 keV interference term, whereas the 1884–1954 keV interference  $B_3$  coefficient can be expected to be too small for observation.

The resonance at 2002 keV.

Angular distributions are measured at twenty alpha-particle energies, covering a 35 keV region. Analysis can be performed with  $N = 4$ . The  $B_0$  coefficient shows one resonance at 2002 keV, whereas all other coefficients are small compared to  $B_0$  and vary slowly with energy. For a spin assignment higher than  $J = \frac{1}{2}$  one would expect a  $B_2$  coefficient of the same order as  $B_0$ .

The resonances at 2081 and 2102 keV.

No  $B_k$  coefficients significantly different from zero exist for  $k > 4$  in a 50 keV region, investigated at twenty-five energies. The  $B_0$ ,  $B_2$ , and  $B_4$  coefficients, having about equal strengths, exhibit two peaks. Small  $B_1$  and  $B_3$  terms change sign several times. The behaviour of the even coefficients

indicates  $J = 5/2$  as the spin value for both levels. The absence of a  $B_5$  coefficient implies equal parities. The behaviour of the odd coefficients, and small deviations from the pure ratios for the even coefficients prove that at least two other resonances in this energy region contribute.

The resonances at 2178 and 2199 keV.

Measurements performed at twenty different alpha-particle energies in a 50 keV region exhibit a highest Legendre polynomial order  $N = 2$ . The  $B_0$  coefficient shows that two levels contribute; a large  $B_1$  component indicates opposite parities for the two resonances. The  $B_2$  coefficient is only significantly different from zero around the upper resonance and changes sign at the actual resonance energy. The fact that this coefficient sometimes has negative values proves that it can not result only from a pure level term. This means that other resonances, probably situated outside the energy region considered, contribute. The smallness of the  $P_2$  term establishes that both resonances observed have  $J = \frac{1}{2}$  spin. A weak background produced by a remote  $J = 3/2$  resonance probably causes the  $B_2$  component being different from zero. A least squares fit of the  $B_0$  coefficient as the sum of two Breit-Wigner curves determines the parameters of the  $J = 1/2$  resonances. The curve for the  $B_1$  coefficient, evaluated from these values, fits well around the high energy resonance for the plus-minus parity possibility and equal  $g$  product signs. Deviations for the low energy resonance can be expected from the  $3/2-1/2$  interference. Other parity (minus-plus) and/or opposite  $g$ -product solutions give very poor agreement.

The resonance at 2319 keV.

Angular distribution measurements at sixteen energy settings in a 30 keV region show equally large  $B_0$  and  $B_2$  coefficients, both exhibiting one peak. Small  $B_1$  and  $B_3$  terms, the ratio of which is energy dependent, prove that contributions from at least two other resonances should be considered. The equal magnitude of the  $B_0$  and  $B_2$  coefficients and the absence of higher orders establish a  $J = 3/2$  assignment.

The resonance at 2402 keV.

Angular distributions at twelve energies in a 20 keV region exhibit one weak resonance in the  $B_0$ ,  $B_2$ ,  $B_4$ , and  $B_6$  coefficients. The odd order coefficients are all small and vary slowly with energy. Performing the analysis with the assumption that interference effects do not contribute significantly to the even order terms, one obtains a unique  $J = 7/2$  assignment from the relative magnitude of these terms.

The resonance at 2450 keV.

A 30 keV region, investigated at sixteen different energies, shows Legendre-

polynomial coefficients up to the third order. The  $B_0$  and  $B_2$  coefficients show one equally strong resonance peak. Smaller odd terms prove interference with several other resonances. A  $J = 3/2$  assignment is made under the assumption that the  $B_2$  coefficient is mainly a pure level effect.

The resonance at 2520 keV.

Angular distributions, determined at seventeen energies in a 40 keV region, exhibit the same behaviour as the measurements at the 2450 keV resonance, thus also leading to a  $J = 3/2$  assignment.

Other resonances below  $E_\alpha \approx 2.6$  MeV are too weak for angular distribution measurements.

Ground state proton yields are corrected for anisotropy in the angular distribution at the resonances discussed above. Other yields are determined at  $\theta = 120^\circ$  with respect to the incoming beam.

The last columns of table II present the data obtained about widths, partial widths, and dimensionless reduced widths. Lower limits for the dimensionless reduced widths are given if the total width or the parity of the resonance is unknown. In the latter case, as well as for the  $p_1$  decay, the lowest possible value for the orbital momentum is used.

Resonance energies and yields are in agreement with older results from the  $^{19}\text{F}(\alpha, p\gamma)^{22}\text{Ne}$  reaction<sup>10) 11) 12)</sup>. Several levels reported there are resolved now into doublets. The correspondence of levels established in the present experiment with levels given in a preliminary report of a  $^{22}\text{Ne}(p, p'\gamma)^{22}\text{Ne}$  experiment<sup>9)</sup> is definitely not very good.

The number of levels, expected in the energy region investigated, calculated following the procedure of ref. 3, is about ten times the observed number of levels, suggesting thus that many weak resonances exist, which are not observed. This may partly explain that hardly any correspondence could be found between the  $^{23}\text{Na}$  levels observed from the reactions  $^{19}\text{F}(\alpha, p)^{22}\text{Ne}$  and  $^{22}\text{Ne}(p, p'\gamma)^{22}\text{Ne}$ .

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## ERRATUM

**Flux pumps and superconducting solenoids.**[*Physica* *31* (1965) 413]

by H. VAN BEELEN, MISS A. J. P. T. ARNOLD, H. A. SYPKENS, J. P. VAN BRAAM HOUCKGEEST, R. DE BRUYN OUBOTER, J. J. M. BEENAKKER and K. W. TACONIS

The correct time it takes to generate a persistent current of 280 A in a 1.8 mH Nb-Zr coil with the device of Buchholz is 80 seconds, instead of the 160 seconds mentioned on page 416, 2*b* of the paper.