On an extension theorem by Sikorski

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It is a classical result of Sikorski (see [17]) that the injective Boolean algebras are precisely the complete Boolean algebras. Sikorski actually proves a bit more: if B is a Boolean algebra with card $B \leq \alpha$, if $A \subset B$ is a Boolean subalgebra, if C is an α-complete Boolean algebra and if $\varphi : A \to C$ is a Boolean algebra homomorphism, then there exists a Boolean algebra homomorphism $B \to C$ which extends φ. Sikorski's theorem on injective Boolean algebras has been generalized in several directions, the most surprising perhaps being a characterization of majorizing-injective [2] Riesz spaces. By now many proofs exist of the fact that Dedekind complete Riesz spaces are majorizing injective (see [12], [10] and [2]), but from none of these is it clear what cardinal version of Sikorski's theorem holds in the setting of Riesz spaces. In this paper we prove two main theorems. One of them will be the exact analogue of the cardinal version of Sikorski's theorem in Riesz spaces, thus giving a sufficient condition for a Riesz space to be majorizing α-injective (Main Theorem II). Our second result characterizes the Riesz spaces of the form $C(K)$ which are majorizing α-injective in the class of $C(K)$-spaces. We present some additional results on extension of Riesz homomorphisms and give some applications of the main theorems. All the Riesz spaces that occur are Archimedean, though this is not a serious restriction (see [2]). For the sake of convenience we work with real, rather than complex Riesz spaces, though some of the results easily extend to the complex setting. Our terminology will follow the standard text books [13], [18] and [19].

1. INTRODUCTION

Suppose A is a partially ordered set. For $U, V \subset A$ we write $U \leq V$ if for all $u \in U$ and all $v \in V$ we have $u \leq v$. Throughout α denotes a cardinal number bigger than or equal to ω_0. A is said to have the α-interpolation property if for every two nonempty subsets U, V of A, such that $U \leq V$ and both card U and card V are less than or equal to α, there exists $a \in A$ such that $U \leq \{a\} \leq V$. Clearly, if A has the α-interpolation property for every α, then A is Dedekind complete. A map φ between two partially ordered sets A and B is said to be order preserving if for all $a \leq a'$ we have $\varphi(a) \leq \varphi(a')$.

As a matter of fact, if they have a smallest and a largest element, partially ordered sets C with the α-interpolation property are characterized by the property that for every partially ordered set B with card $B \leq \alpha$, every subset $A \subset B$ and every order preserving map $A \to C$ there exists an order preserving extension $B \to C$. Though we will be interested in sets and mappings with more structure, the above remark shows how the α-interpolation property comes in rather naturally.

Checking the proof of Sikorski's theorem in [17] we see the following.

THEOREM 1. *Suppose $\mathcal{M}$, $\mathcal{B}$ and $\mathcal{C}$ are Boolean algebras, $\mathcal{M} \subset \mathcal{B}$ is a subalgebra, $\mathcal{C}$ has the α-interpolation property, card $\mathcal{B} \leq \alpha$ and $\varphi : \mathcal{M} \to \mathcal{C}$ is a Boolean algebra homomorphism. Then there exists a Boolean algebra homomorphism $\mathcal{B} \to \mathcal{C}$ which extends φ.*

2. MAIN THEOREM I

Though apparently other ways of approach are available to prove the generalizations of theorem 1 which we have in mind, it does seem rather convenient and even economical to take theorem 1 as a starting point.

We will then take instead of $\mathcal{M}$, $\mathcal{B}$ and $\mathcal{C}$ in the above, spaces of continuous functions $C(K)$ and use a technique which was suggested by a paper of Negrepontis [14]. Instead of the condition card $\mathcal{B} \leq \alpha$ we need the so called density character of $C(K)$. The density character of $C(K)$ is defined to be the infimum over all those cardinal numbers for which there exists a dense subset of $C(K)$ with that cardinality. Here K is assumed to be compact all the time and dense means dense in the supremum norm. For technical reasons we need to relate this density character with a cardinal number attached to K. The weight of K is defined to be the infimum over all those cardinal numbers for which there exists a base for K of this cardinality (Concerning both cardinal functions we will commonly refer to either [7] or [16]). The following proposition gives the obvious relation between the density character of $C(K)$ denoted $\delta(C(K))$ on the one hand and the weight of K, denoted $w(K)$ on the other.

PROPOSITION 2. $\delta(C(K)) = w(K)$.

We now have all the machinery to state the following formal generalization of theorem 1.

PROPOSITION 3. *Suppose X and Y are compact Hausdorff spaces, Y is totally disconnected, $\delta(C(X)) \leq \alpha$, $C(Y)$ has the α-interpolation property, $H \subset C(X)$ is a Riesz subspace containing 1_X and $\varphi : H \to C(Y)$ is a Riesz homomorphism with $\varphi(1_X) = 1_Y$. Then there exists a Riesz homomorphism $C(X) \to C(Y)$ which extends φ.*

Our proof will make use of three lemmas (lemma 4 and lemma 5 are in [14] and lemma 6 is a combination of 6.2.11 and 3.22 in [7]), Sikorski's theorem 1 and Stone's duality theorem for Boolean algebras.

LEMMA 4. *Let f be a continuous surjection of the compact totally disconnected space Y onto the compact space Z. Then there exist a compact totally disconnected space Z' with $w(Z)$ open-and-closed sets and continuous surjections $g : Y \to Z'$ and $h : Z' \to Z$ such that $h \circ g = f$.*

LEMMA 5. *Given compact totally disconnected spaces Z_1, Z_2, Y and continuous surjections $h_1 : Z_1 \to Y$, $h_2 : Z_2 \to Y$ there exist a compact totally disconnected space E such that $w(E) \leq w(Z_1) + w(Z_2)$ and continuous surjections $k_1 : E \to Z_1$ and $k_2 : E \to Z_2$ such that $h_1 \circ k_1 = h_2 \circ k_2$.*

LEMMA 6. *Every compact space of weight α is a continuous image of a closed (hence totally disconnected) subspace of D^α (where D is the two point set $\{0, 1\}$).*

PROOF OF PROPOSITION 3

Suppose X and Y are compact Hausdorff spaces, Y is totally disconnected, $C(Y)$ has the α-interpolation property, $C(X)$ has density character α, $H \subset C(X)$ is a Riesz subspace containing 1_X and $\varphi : H \to C(Y)$ is a Riesz homomorphism with $\varphi(1_X) = 1_Y$. We may just as well assume that H is a normclosed Riesz subspace of $C(X)$. By Kakutani's theorem $H \cong C(Z)$ for a compact Hausdorff space Z. By 7.6.6. in [16] we have $w(Z) \leq w(X) \leq \alpha$.

Thus we have a continuous surjection $h_1 : X \twoheadrightarrow Z$ and a continuous map $h_2 : Y \to Z$, where h_1 corresponds to the natural injection $C(Z) \subset C(X)$ and where $\varphi(f) = f \circ h_2$ for all $f \in C(Z)$. We are looking for a continuous map $g : Y \to X$ such $h_1 \circ g = h_2$. We may assume that h_2 is surjective. All of this is summarized in the following diagram.
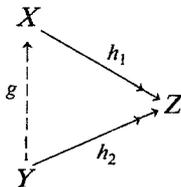
X, Y, Z are compact Hausdorff spaces; Y is totally disconnected, $C(Y)$ has the α-interpolation property; $w(Z) \leq w(X) \leq \alpha$.

Lemmas 4, 5 and 6 produce the following diagram.
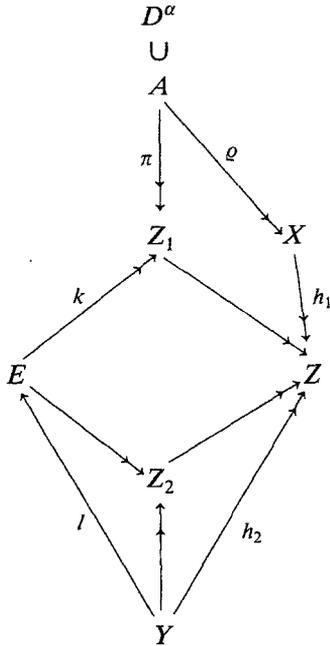
$$w(Z_1) \le w(Z) \le \alpha;$$
$$w(Z_2) \le w(Z) \le \alpha;$$
$$w(E) \le w(Z_1) + w(Z_2) \le \alpha;$$

E, Y, Z_1, Z_2, A are totally disconnected;
A is a closed subspace of D^α.

The arrows and spaces in the diagram are constructed in the following order. By lemma 6 there exists a closed subspace A of D^α which maps surjectively onto X via a map called ϱ. The composition of that map with h_1 can be factored via a totally disconnected space Z_1 with the aid of lemma 4. The same lemma provides a factoring via Z_2 of h_2. The weight of Z_1 and the weight of Z_2 are less than or equal to α. The remaining space E and the mappings from E onto Z_1 and Z_2 are constructed with lemma 5. Some of the mappings (see diagram) have been given a name.

By Sikorski's theorem 1 and Stone's duality theorem for Boolean algebras we produce the map $l: Y \to E$ as in the diagram. Now $k \circ l(Y) =: B$ is a totally disconnected subset of Z_1 and $\pi^{-1}(B)$ is a totally disconnected subset of A. Certainly we keep track of the weight of new spaces that arise, i.e. $w(B) \le \alpha$. These observations result into the following diagram.
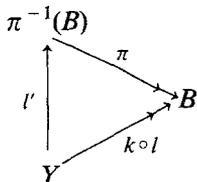
B, $\pi^{-1}(B)$, Y compact and totally disconnected Hausdorff spaces;
$w(B) \le \alpha$;
$C(Y)$ has the α-interpolation property.

Again, by Sikorski's theorem 1 and Stone's duality theorem, a mapping is produced which we have called l'. The desired function g is defined by $g = \varrho \circ l'$.

$\square$

The conditions $1_X \in H$ and $\varphi(1_X) = 1_Y$, natural as they are in the context of Boolean algebras, are not desirable in the present set up. It is easy to replace $1_X \in H$ and $\varphi(1_X) = 1_Y$ merely by the condition that H is majorizing. A little bit more surprising is the fact that we can allow any Riesz subspace H if we assume φ to be continuous. The heart of the fact lies in the following lemma for which we refer to [15] or [9] (However a straightforward proof not using the Krein-Milman theorem can be given).

LEMMA 7. *Suppose X is a compact Hausdorff space and $H \subset C(X)$ is a Riesz subspace. If $\varphi : H \to R$ is a continuous Riesz homomorphism, then there exists a Riesz homomorphism $\Phi : C(X) \to R$ with $\Phi|_H = \varphi$ and $\|\Phi\| = \|\varphi\|$.*

Interestingly, lemma 7 actually characterizes AM-spaces and can be used in many circumstances. For us, it will be the main step in the proof of proposition 8.

PROPOSITION 8. *Suppose X is a compact space of weight less than or equal to α. Y is compact and totally disconnected, $C(Y)$ has the α-interpolation property, $H \subset C(X)$ is a Riesz subspace and $\varphi : H \to C(Y)$ is a continuous Riesz homomorphism. Then there exists a norm preserving Riesz homomorphic extension $C(X) \to C(Y)$.*

PROOF. We may assume that H is a closed Riesz subspace. Let G be the closed Riesz subspace generated by $\varphi(H)$ and 1_Y. It follows that the density character of G is less than or equal to α. Let $\{g_w | w \leq \delta(G)\}$ be a dense subset of G^+ and choose for each $w \leq \delta(G)$ an element y_w in Y such that $g_w(y_w) = \|g_w\|$. Define $\psi : G \to l^\infty(\delta(G))$ by $\psi(g)(w) = g(y_w)$ ($w \leq \delta(G)$). It follows that ψ is an isometric Riesz isomorphism from G into $l^\infty(\delta(G))$. By applying lemma 7 pointwise it follows that $\psi \circ \varphi$ can be extended to a norm preserving Riesz homomorphism $\varphi_1 : C(X) \to l^\infty(\delta(G))$. Let E be the closed Riesz subspace of $l^\infty(\delta(G))$ generated by $\varphi_1(C(X))$ and $1_{\delta(G)}$. Again, $\delta(E) \leq \alpha$ and of course E is Riesz isomorphic to $C(X')$ for some compact Hausdorff space X' with $w(X') \leq \alpha$. Let $\varphi_2 : \psi(G) \to C(Y)$ be the isometric Riesz homomorphism which is the inverse of ψ. By theorem 3 there exists a norm one Riesz homomorphic extension of φ_2, say $\varphi_0 : E \to C(Y)$. It follows that $\varphi_0 \circ \varphi_1$ is a norm preserving Riesz homomorphic extension $C(X) \to C(Y)$ of φ. $\square$

It is a well-known fact that in the category of Riesz spaces and Riesz homomorphisms injective objects do not exist. A way out of that valley has been to restrict the class of subspaces (e.g. majorizing-injectivity in [12], [10] and [2] or ideal-injectivity in [3], [4] and [5]), another way out is to restrict the class of spaces (e.g. AM-spaces). The latter was the route we have followed up to now.

It may sound unnatural to the reader at first that we assumed Y to be totally disconnected. That the condition, however, cannot be removed follows from

the following observation. If $C(Y)$ has the α-interpolation property for $\alpha \geq \omega_0$, then in particular $C(Y)$ has the σ-interpolation property and Y is what is called an F-space. Suppose $y \in Y$ and the component of $y \in Y$ contains $y' \neq y$. Choose a continuous function $f \in C(Y)^+$ with $f(y) = 0$ and $f(y') = 1$ and $\|f\| = 1$. It then follows that $f : Y \rightarrow [0, 1]$ is surjective. Choose a continuous surjection $h : D^{\omega_0} = \mathbb{D} \rightarrow [0, 1]$. If in the diagram below the continuous function g can be constructed such that $h \circ g = f$, we have found a contradiction, using the total disconnectedness of $\mathbb{D}$.
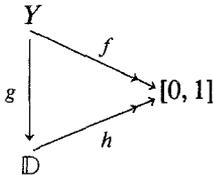
We are now in the position that we have proved most of the following.

MAIN THEOREM I

For a compact Hausdorff space Y the following are equivalent.

(1) *Y is totally disconnected and $C(Y)$ has the α-interpolation property.*

(2) *For every compact space X such that $\delta(C(X)) \leq \alpha$ and every Riesz subspace $H \subset C(X)$ such that $1_X \in H$, every Riesz homomorphism $\varphi : H \rightarrow C(Y)$ with $\varphi(1_X) = 1_Y$ extends to a Riesz homomorphism $C(X) \rightarrow C(Y)$.*

(3) *For every compact space X such that $\delta(C(X)) \leq \alpha$ and every majorizing Riesz subspace $H \subset C(X)$, every Riesz homomorphism $\varphi : H \rightarrow C(Y)$ extends to a Riesz homomorphism $C(X) \rightarrow C(Y)$.*

(4) *For every compact space X such that $\delta(C(X)) \leq \alpha$ and every Riesz subspace H of $C(X)$, every continuous Riesz homomorphism $\varphi : H \rightarrow C(Y)$ extends to a Riesz homomorphism in a norm preserving way.*

Furthermore if $\alpha = \omega_0$ then each of (1) to (4) is equivalent to

(5) *For each majorizing Riesz subspace $H \subset C(\mathbb{D})$ and each Riesz homomorphism there exists a (norm preserving) Riesz homomorphic extension $C(\mathbb{D}) \rightarrow C(Y)$, where $\mathbb{D} = D^{\omega_0}$.*

Some remarks before giving a proof. Of course (1) ⇔ (2) generalizes Gleason's theorem on projective topological spaces (see [8]), while the equivalence of (1) and (4) generalizes Peck's characterization of extremally disconnected spaces (see [15]). We have included (3) for its resemblance with the theorem by Luxemburg-Schep-Lipecki (see [12], [10] and [2]). (5) merely reflects the universality of $\mathbb{D}$ within the class of compact metric spaces. It is not without interest to observe the difference between theorem I and the results in [6] and [5].

PROOF OF THE MAIN THEOREM I

(1) ⇒ (4) is the content of proposition 8. (4) ⇒ (3) and (3) ⇒ (2) are trivial. A

proof of (5)$\Rightarrow$(1) if $\alpha = \omega_0$ is left to the reader. This leaves us with (2)$\Rightarrow$(1). Suppose (2) holds. From the discussion just before Main Theorem I we see that Y is totally disconnected. To prove that $C(Y)$ has the α-interpolation property, assume $A, B \subset C(Y)^+$, $A \le B$ and card $A \le \alpha$ and card $B \le \alpha$ and assume $1_Y \in A \cup B$. Let E be the closed Riesz subspace of $C(Y)$ generated by the elements of $A \cup B$. By Kakutani's theorem $E \cong C(X)$ for a compact Hausdorff space X of weight less than or equal to α. There exists $g \in C(X)^{**}$ such that $A \le \{g\} \le B$ in $C(X)^{**}$. Let F be the closed Riesz subspace of $C(X)^{**}$ generated by $A \cup B \cup \{g\}$. By Kakutani's theorem $F \cong C(\tilde{X})$ where $\tilde{X}$ is a compact Hausdorff space of weight less than or equal to α. By (2) there exists a Riesz homomorphism $\Phi : C(\tilde{X}) \to C(Y)$ which extends the natural embedding $C(X) \subset C(Y)$. It follows that $A \le \{\Phi(g)\} \le B$. $\qquad\square$

Among the uses for Main Theorem I there is a wide generalization of lemma 7. It is well known and proved by Ando in [1] that for an AM-space F and a closed Riesz subspace $E \subset F$ there exists a simultaneous extension operator $E^* \to F^*$, i.e. a contractive linear operator T such that for all $x^* \in E^*$ and all $y \in E$ $T(x^*)(y) = x^*(y)$. (See also exercise 29, chapter III in [18]). A bit more can be said about T.

COROLLARY 9. *T can be chosen to map Riesz homomorphisms on E to Riesz homomorphisms on F, i.e., all Riesz homomorphisms on E can be extended simultaneously.*

PROOF. Remark that E^{**} is a closed Riesz subspace of F^{**} and that E^{**} as well as F^{**} contains a unit and again is an AM-space. By Main Theorem I the identity $I : E^{**} \to E^{**}$ can be extended norm preservingly to a Riesz homomorphism $Q : F^{**} \to E^{**}$. Write $J : F \to F^{**}$ for the natural embedding of F into F^{**} and define $T : E^* \to F^*$ by $T(x^*)(x) = QJ(x)(x^*)$ for all $x^* \in E^*$, $x \in F$. Leaving the other details to the reader we will show that for a Riesz homomorphism $x^* \in E^*$ the operator $T(x^*)$ is a Riesz homomorphism in F^*. This follows from $T(x^*)(|x|) = QJ(|x|) = |QJ(x)|(x^*) = \sup \{QJ(x)(y) \mid 0 \le |y| \le x^*\} = \sup \{QJ(x)(\lambda x^*) \mid -1 \le \lambda \le 1\} = \sup \{\lambda QJ(x)(x^*) \mid -1 \le \lambda \le 1\} = |T(x^*)(x)|$, where we have used the characterization of Riesz homomorphisms as extreme rays. $\qquad\square$

3. MAIN THEOREM II

In this section we will give the promised cardinal version of the Luxemburg-Schep-Lipecki result. Our way of approach is essentially that of [2] and therefore we will not dwell on details.

Let G be any Riesz space and $f \in G^+$. By $K_{f,G}$ we denote the compact Hausdorff space of the maximal ideals of the principal ideal G_f generated by f in G under the hull-kernel topology. $D(K_{f,G})$ will denote the lattice of all continuous functions $K_{f,G} \to \bar{\mathbb{R}}^+$ where $\bar{\mathbb{R}}^+$ are the positive extended real numbers. There is a natural lattice homomorphism $\varphi_{f,G} : G \to Q(K_{f,G})$, where

$Q(K_{f,G})$ is the maximal ring of quotients of $C(K_{f,G})$ (see page 464, notes 26.2.11 and page 459 in [16]). If $U \subset G^+$ is a maximal disjoint subset, then G^+ naturally embeds as a lattice into $D(\sum_{u \in U} K_{u,G})$ and G embeds as a vector lattice into $Q(\sum_{u \in U} K_{u,G})$. The following two lemmas are well known.

LEMMA 10. *If G is σ-Dedekind complete then $K_{u,G}$ is basically disconnected for every $u \in G^+$. More generally, $K_{u,G}$ is α-extremally disconnected (see [16]) if G is α-Dedekind complete.*

LEMMA 11. *For every F-space Y, $C^\infty(Y)$ (see [13]) is vector lattice isomorphic to $Q(Y)$.*

For the present we will call a Riesz space G α-*small* if it satisfies both of the following conditions.
(1) Every disjoint subset of G^+ has cardinality $\leq \alpha$.
(2) For every $f \in G^+$, $w(K_{f,G}) \leq \alpha$.
Lemma 12 is straightforward.

LEMMA 12. *Suppose $I \subset G$ is an ideal, $f \in G^+$ and $\bar{f}$ is the image of f under the natural map $G \to G/I$. Then $w(K_{\bar{f},G/I}) \leq w(K_{f,G})$.*

We can now state and prove a cardinal version of the Luxemburg-Schep-Lipecki result.

THEOREM 13. (*Main Theorem II*). *Suppose E is an α-small Riesz space, F is α-Dedekind complete and $H \subset E$ is a majorizing Riesz subspace. Then every Riesz homomorphism $\varphi : H \to F$ extends to a Riesz homomorphism $E \to F$.*

Two corollaries read as follows.

COROLLARY 14. *Suppose F is α-Dedekind complete, E is a Riesz space with cardinality less than or equal to α and $H \subset E$ is a majorizing Riesz subspace. Then every Riesz homomorphism $\varphi : H \to F$ extends to a Riesz homomorphism $E \to F$.*

COROLLARY 15. (*Luxemburg-Schep-Lipecki*). *Suppose F is Dedekind complete, E is any Riesz space, $H \subset E$ is a majorizing Riesz subspace. Then every Riesz homomorphism $\varphi : H \to F$ extends to a Riesz homomorphism $E \to F$.*

PROOF OF THEOREM 13 (Main Theorem II).

Let U be a maximal disjoint subset of H^+. It follows that card $U \leq \alpha$. We consider the following diagram.

$$H \overset{\mu}{\hookrightarrow} E \overset{q}{\longrightarrow} E/\mu(U)^{\perp}$$

$$\varphi \downarrow$$

$$F \longleftarrow \varphi(H)^{\perp\perp}$$
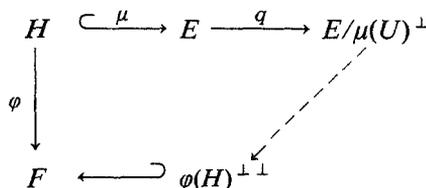
Here μ is the natural embedding $H \subset E$ and q is the quotient map $E \rightarrow E/\mu(U)^\perp$. Remark that it suffices to produce the dotted arrow in the above diagram. Because $q \circ \mu(U)$ is a maximal disjoint subset in $E/\mu(U)^\perp$ and by lemma 12 for every $f \in E/\mu(U)^\perp$ we have $w(K_{f, E/\mu(U)^\perp}) \le \alpha$ and $\varphi(H)^{\perp\perp}$ is an α-Dedekind complete space, we may assume that there exists a maximal disjoint subset U of H^+ which is maximal disjoint in E^+ as well and such that $\varphi(U)$ is a maximal disjoint subset of F^+.

For every $u \in U$ we get the following diagram.

$$
\begin{array}{ccc}
K_{u,E} & & \\
\uparrow & \searrow & \\
\varphi_u^{\#} \vdots & & \rightarrow K_{u,H} \\
\vdots & \nearrow & \\
K_{\varphi(u),F} & &
\end{array}
$$
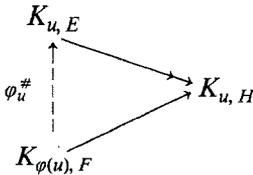
The two non-dotted arrows are continuous mappings which naturally belong to the embedding $H \subset E$, respectively the Riesz homomorphism φ. They are produced as in the paper by Aron, Hager and Madden [2]. The dotted arrow is produced by proposition 8. In a natural way this gives us a continuous map

$$ \varphi^{\#} : \sum_{u \in U} K_{\varphi(u),F} \rightarrow \sum_{u \in U} K_{u,E}. $$

For every $f \in E^+$ define $\Phi(f) = f \circ \varphi^{\#} \in D(\sum_{u \in U} K_{\varphi(u),F})$. Remark that

$$ \Phi(f) \in C^\infty(\sum_{u \in U} K_{\varphi(u),F}) = Q(\sum_{u \in U} K_{\varphi(u),F}) $$

because it is dominated by an element of F. (It is here that we use that $H \subset E$ is majorizing). Also $\Phi(f) \wedge n\varphi(u) \in F$ for all $f \in E^+$, $n \in \mathbb{N}$ and $u \in U$. By α-Dedekind completeness of F, hence σ-Dedekind completeness of F, $\sup_{n \in \mathbb{N}} \Phi(f) \wedge n\varphi(u) = \tilde{f}_{\varphi(u)} \in F$. Furthermore $\sup_{u \in U} \tilde{f}_{\varphi(u)} = \Phi(f) \in F$ by α-Dedekind completeness of F. The rest of the proof is left to the reader. $\square$

In many circumstances both, the condition that card $E \le \alpha$ and the condition that F is α-complete in corollary 14 are a bit too restrictive as can be seen from the main theorem I. To give one more example of such a theorem we observe the following

THEOREM 16. *Any of the conditions* (1) *to* (4) *in Main Theorem I is equivalent to*
(α) *For every pair of Banach lattices $B_1 \supset B_2$ such that $\delta(B_1) \le \alpha$ and B_2 is majorizing in B_1 and for every Riesz homomorphism $\varphi : B_2 \rightarrow C(Y)$ there exists a Riesz homomorphic extension $B_1 \rightarrow C(Y)$.*

A proof of Theorem 16 can be given following the lines of proposition 8, except for the use of lemma 7 which should be replaced by corollary 15. Remark that the extension may possibly not preserve the norm.

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