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SOME REMARKS ON THE OPTICAL MODEL FOR NUCLEON-NUCLEUS ELASTIC SCATTERING

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Synopsis

A review is given of the published work on the analysis of high energy nucleon scattering from complex nuclei on the basis of the quasi-optical model originally introduced by Fernbach, Serber and Taylor, with inclusion of polarization effects.

1. *Introduction.* In view of the large number of calculations carried out recently to interpret high energy elastic scattering of nucleons by nuclei in terms of the so-called optical model, in view also of the variable amount of success of such calculations and of the uncertainties they leave in the choice of the optical potential, we have chosen to discuss a number of general indications and trends revealed by a comparison of the various published calculations rather than to report on them individually. For that reason we shall quote the papers relevant to our general discussion and we make no claim of completeness regarding our references to work on the subject.

The two ingredients of the optical model of the nucleus, aimed at describing in simple phenomenological terms the nuclear interaction of a nucleon (proton or neutron) with a not too light nucleus, are a complex-valued central potential $V(r)$ for the spin-independent part of the interaction, and a spin-orbit coupling $U(r)\ \boldsymbol{s}\cdot\boldsymbol{l}$ of the Mayer-Jensen type ($\boldsymbol{s}$ and $\boldsymbol{l}$ are the spin and orbital angular momenta of the nucleon in units of $\hbar$) for the part of the interaction depending on the spin of the nucleon. The optical model pretends to describe in detail the elastic scattering of the nucleon by the nucleus (including polarization effects), while for all other processes (inelastic scattering and all reactions) it gives one single total cross-section (usually called absorption cross section σ_a) **).

The use of a complex-valued central potential $V(r)$ was first proposed by Fernbach, Serber and Taylor [1]) for the case of nucleons of high incident

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**) To be more exact, that part of the elastic scattering which occurs through formation of a compound system is also included in σ_a. This type of elastic scattering is quite small at high energies. (See reference 2).

energy (energy $E \gtrsim 100$ MeV). It was later shown by Feshbach, Porter and Weisskopf [2]) that the same type of potential with suitably modified parameters (see below) could be used with considerable success at low energies ($E \gtrsim 0{,}1$ MeV). Also at intermediate energies a fair amount of agreement has been obtained, in this case mainly for elastic scattering of protons [3]). Furthermore it is well known that the shell model of the nucleus is based on a similar type of potential, with vanishing imaginary part, and this case can indeed be considered as the extreme low energy limit of scattering ($E \sim -$ binding energy of the last nucleon).

It is in the latter case (shell model) that the occurrence of a strong spin-orbit coupling was first recognized [4]) and it was later shown by Fermi that this coupling is of the right order of magnitude to account for the very large polarizations observed in high energy elastic scattering of protons by nuclei [5]). As shown by Adair, Darden and Fields, and also by Sternheimer, the same spin-orbit coupling is probably consistent with the much smaller polarizations observed at lower energies ($E \lesssim 50$ MeV) [6]). Also the sign of the polarization could be shown to agree with the sign of the spin-orbit coupling as required by the shell model [7]).

When interpreting a given set of experimental data by means of the optical model, one has to fit two total cross-sections, the total cross section σ_e for elastic scattering and the total cross-section σ_a for all other processes, and two functions of the scattering angle θ, the angular distribution $d\sigma/d\Omega$ of elastically scattered particles and the polarization P as a function of θ [8]). The values of σ_e and σ_a simply determine two parameters of the optical potential, which are usually taken to be the depths of the real and imaginary parts of $V(r)$, and the values obtained are quite reasonable. One has further at one's disposal the strength of the spin-orbit force (depth of U(r)) and the radial dependence of $V(r)$ and $U(r)$ in order to fit $d\sigma/d\Omega$ and P as functions of θ, but the available choice of radial dependence is severely restricted (see below) by the well established uniformity of nuclear matter inside the nucleus and the fairly precise estimates we have for the nuclear radius as well as for the diffuseness of the nuclear surface. Thus the model inescapably implies certain pronounced qualitative features of the $d\sigma/d\Omega$ and P versus θ variation, of the type of diffraction maxima and minima. Such diffraction features have clearly been observed in the angular distribution $d\sigma/d\Omega$ at moderate energies [9]) and are probably also present at higher energies, at least for not too light nuclei [10]). They have also been observed to some extent in the θ-dependence of the polarization [11]).

Although qualitatively good enough to be quite significant, the agreement between experiment and predictions of the optical model is rarely accurate and is often quantitatively poor, mainly at large scattering angles [12]). Still by combining the results obtained by various authors with what one expects on general grounds concerning the shape of the optical potentials one can reach some definite qualitative conclusions which are in themselves quite instructive and which will be useful guides in attempts at understanding the optical model in terms of the actual nuclear structure. These conclusions will now be discussed for the central potential and the spin-orbit coupling separately. One remark has however still to be made. There seems to be empirical evidence that the optical model is more successful for medium or heavy nuclei than for very light ones [13]). This should not be surprising. The optical potentials, which are supposed to give a global account of the interaction between nucleon and nucleus, are indeed more likely to have the simple shapes adopted for them when they result from the collective effect of many nucleons than in the case of a nucleus composed of a very few particles. One is therefore entitled to limit the discussion to nuclei heavy enough to contain homogeneous nuclear matter in a considerable portion of their volume. This limitation is made in the following.

2. *The central potential.* In view of the short range of nuclear forces, both the real part $V_1(r)$ and the imaginary part $V_2(r)$ of the central potential $V(r)$ should be constant throughout that region of the nuclear volume where the density of nuclear matter is essentially uniform. We denote these constant values by $-v_1$ and $-v_2$ respectively (v_1 and v_2 are positive energies). They should be characteristics of nuclear matter in bulk and thus independent of the atomic weight A. The whole radial variation of $V_1(r)$ and $V_2(r)$ is concentrated in the diffuse boundary layer of the nucleus, both quantities vanishing of course outside [14]). A few words will be said about this variation at the end of the present section.

While independent of A, the quantities v_1 and v_2 may of course depend upon the energy E of the scattered nucleon (E is defined as the incident kinetic energy). Enough empirical evidence has been gathered, not only to show that this variation indeed exists, but also to determine its qualitative nature over a wide energy range *). There is a fairly fast and regular decrease of v_1 in the region $0 \lesssim E \lesssim 150$ MeV, from a value of 40 to 50 MeV at $E \sim 0$ to a value of the order of 15MeV at $E \sim 150$ MeV [15]) followed by a marked flattering off of the v_1 versus E curve at higher energies, up to at least $E \sim 400$ MeV [16]). Although Jastrow has presented long ago a qualitative argument to explain this remarkable energy variation of the depth v_1, it will undoubtedly provide a very interesting test of any structure theory of nuclear matter [17]).

The depth v_2 of the imaginary part of the central potential has also a marked variation with the nucleon energy E. On the basis of the empirical evidence v_2 increases smoothly from the vanishing value required by the shell model for nucleons bound inside a nucleus to a value of 10 or 15 MeV

*) The figures hereafter are based on the ordinary mass value of the free nucleon. They could of course be interpreted otherwise by introduction of an effective mass.

for E around 40 MeV [18]); at higher energies v_2 seems to go on increasing, but much more slowly and may have a value around 20 MeV for $E \sim 400$ MeV [19]). As a consequence of this behaviour the mean free path of a nucleon in nuclear matter, considered for increasing nucleon energy, must first rapidly decrease, than increase more slowly. (At high nucleon energy E, assuming v_1 practically constant, one gets a $E^{1/2}$-dependence of the mean free path). The minimum seems to lie in the $E \sim 30$ MeV region and may have an order of magnitude of 2×10^{-13} cm [20]).

In contrast to the case of v_1, it has been possible to make surprisingly successful theoretical estimates of v_2 on the basis of a very crude picture of the "absorption" process, the probability of which v_2 is supposed to describe. Following an idea first used by Goldberger for high energies [21]) one treats the nucleus as an ideal Fermi gas of nucleons in a (real) potential well and one supposes that each collision between the incident nucleon and a nucleon of the target nucleus produces "absorption". A rough estimate of the nucleon-nucleon scattering cross sections is sufficient for such a calculation, which makes essential use of the Pauli exclusion principle in selecting the possible modes of nucleon-nucleon collision inside the nucleus. It is very remarkable that such rough calculations give for v_2 results in excellent qualitative agreement with the empirical values, at least over the energy range for which they have been carried out so far, $0 \lesssim E \lesssim 100$ MeV, where v_2 varies from a value of the order of 3 MeV [2]) to a value of the order of 15 MeV [22]). The small value obtained for low E is entirely due to the effect of the Pauli-principle. According to Lane and Wandel [22]) neglection of this principle would give $v_2 \sim 30$ MeV at $E \sim 0$. Still such crude calculations are of course not able to account quite convincingly for the further decrease of v_2 down to zero when E takes negative values (bound nucleon), as required by the shell model. Furthermore, despite their success, they require a well-founded theoretical justification which will perhaps be provided by the Brueckner theory of nuclear structure [17]).

A few remarks must still be made on the variation of the central potential $V(r) = V_1(r) + iV_2(r)$ at the nuclear surface. Considering first the case of the real part $V_1(r)$ the most natural assumption, given the present evidence, is to let $V_1(r)$ vary smoothly from $-v_1$ to zero across the nuclear boundary, the width of which is known to be of the order of 2×10^{-13} cm. A representation of this variation which seems quite satisfactory is provided by the potential proposed by Woods and Saxon [3])

$$V_1(r) = -\{1 + \exp[(r - R)a^{-1}]\}^{-1} v_1$$

with $a = 0{,}5 \times 10^{-13}$ cm and $R = 1.33\, A^{1/3} \times 10^{-13}$ cm. A more detailed study of the best choice of $V_1(r)$ in the boundary region, and the closely related problem of the best definition of the nuclear radius R, does not seem warranted until the experimental data are more accurate and our theoretical understanding of the optical potential is more satisfactory.

Although it seems reasonable enough to apply the same considerations to the imaginary part of the potential – most authors simply take $V_2(r)$ proportional to $V_1(r)$ – a different argument can be brought forward in this case. As mentioned above, for rather low nucleon energy the exclusion principle reduces radically the imaginary potential by forbidding many modes of two-body collisions. It has often been argued that this effect of the Pauli principle will be weaker in the less dense boundary layer of the nucleus than in its central region, so that $|V_2(r)| = - V_2(r)$ may reach a maximum in this region and go down to zero outside and to $- v_2$ inside the nucleus. One may have however some doubts on the validity of this argument, because it is by no means clear why two-body collisions occurring at the boundary would preferentially involve nucleon states entirely localized in the boundary region, as is supposed by such type of reasoning *).

Our notation $V(r)$ (and $U(r)$ for that matter) has suggested that we assume spherical symmetry for the optical potentials. This has been done quite generally until now in all optical model calculations, but, as mentioned e.g. by Köhler [11]), non-spherical deformation corrections are of course in principle to be included for all actual nuclei except those with closed shells.

3. *The spin-orbit coupling.* Starting our discussion in the same way as for the central potential, we consider first the motion of a nucleon in that part of a nucleus where nuclear matter is homogeneous, *i.e.* essentially inside nuclear matter in bulk. As seen immediately by imagining a nuclear medium of infinite extension, symmetry considerations forbid any spin-orbit force to act on a nucleon moving through this medium. We are thus led to surmise that the spin-orbit coupling between a nucleon and an actual nucleus will be essentially localized in the boundary region of the nucleus, *i.e.* that the function $U(r)$ will have non vanishing values in the boundary layer only. According to Wilson there exists empirical support for this conclusion: whereas a spin-orbit coupling uniformly distributed over the whole nucleus should have a strength depending on atomic weight A in order to fit the experimental data, a A-independent strength can be adopted when assuming the spin-orbit force to be concentrated in the boundary layer [23]).

This boundary nature of the spin-orbit force once accepted, it is rather natural to adopt for $U(r)$ a mathematical form suggested by the Thomas

*) *Note added in proof*: In a private discussion Prof. V. F. Weisskopf drew the attention to another form of the argument leading to an increased absorption in the boundary layer. It is based on the fact that the nucleon states of the continuum have (except at resonance) a smaller amplitude inside the nucleus than outside, so that their overlap with bound nucleon states is more concentrated at the boundary.

correction of atomic theory

$$U(r) = \frac{\alpha\hbar^2}{2M^2c^2}\frac{1}{r}\frac{d}{dr}V_1(r) \qquad (1)$$

where M is the nucleon mass and $V_1(r)$ is, as before, the real part of the central potential. On the basis of nuclear ground state properties (shell model) the low energy value of the phenomenological coefficient α was determined [24]) to be of the order of -30 (α is unity for a Dirac electron in a central potential). Of course there is at present no reason to attach any deeper significance to this Thomas-type of spin-orbit coupling than to any other simple formula giving a $U(r)$-function concentrated at the nuclear surface. The form (1) may even be slightly misleading when considering the variation of $U(r)$ with the energy E of the incident nucleon, because it suggests that $U(r)$ might decrease more or less proportionally to $V_1(r)$ for increasing E. This assumes the phenomenological coefficient α to be independent of E, an assumption which lacks at present all theoretical foundation.

The empirical evidence regarding the E-dependence of the spin-orbit force is unfortunately still uncertain. While Eriksson [16]) and Köhler [11]) base their optical model analyses of polarization data on the assumption of a spin-orbit force (1) with α independent of E, Sternheimer [6]) makes the (on simplicity grounds as plausible) assumption of an energy independent spin-orbit coupling, *i.e.* of a coefficient α varying with E in inverse ratio of the depth of V_1. In neither case is the agreement with experiment so good as to be quite conclusive. One might however find an indication in favour of Sternheimer's assumption in the fact that this author studies the energy range 50 MeV $\lesssim E \lesssim$ 130 MeV where the polarization varies rapidly (for low energies, up to $E \sim 50$ MeV, it is usually quite small, of the order of 10 or 20%; it then increases to values of the order of 70% at 130 MeV, see Adair *et al.* [6]) and Dickson *et al.*[10])) and shows that this marked variation can be entirely accounted for by the decrease of $V_1(r)$ for increasing E, keeping $U(r)$ energy-independent. Köhler and Eriksson work at somewhat higher energies, where the polarization, while being quite large, does not show such a pronounced variation. Still, as remarked by Eriksson [25]), at the lower end of the energy range he investigates ($E \sim 90$ MeV) the fast increase of the polarization with energy seems difficult to explain with constant α. It is a priori clear why Sternheimer's assumption is more successful in this respect: instead of taking the spin-orbit force to decrease with increasing E in the same proportion as $V_1(r)$, it keeps it constant and allows only V_1 to decrease.

Very little can be said at present about a theoretical understanding of the spin-orbit coupling and even less about its calculation in terms of nucleon-nucleon (or nucleon-meson) interactions. We shall content ourselves with mentioning two often quoted papers, one by Tamor [26]) showing in crude terms that the large polarization effects observed in elastic nucleon-nucleus collisions are not in conflict with the smaller polarizations occurring in nucleon-nucleon scattering, the second due to Fernbach, Heckrotte and Lepore [27]) establishing that a spin-orbit force of the form (1) can be obtained rather naturally on the basis of a number of approximations, among which the impulse approximation [28]), valid at very high energy. Still, such an approach to the problem of understanding the spin-orbit force in terms of basic nuclear structure does not seem very satisfactory because it relies in an essential way on high energy approximations, whereas perhaps the most striking property of the observed spin-orbit interaction is its large degree of independence of the nucleon energy. To this criticism one might object that the high energy method of calculating the imaginary potential $V_2(r)$ turned out to be surprisingly good at medium and low energies. V_2 is however a measure of the total cross-section for all processes where the target nucleus changes its quantum state, while $U(r)$ as well as $V_1(r)$, dealing with processes elastic in the target nucleus, concern the coherent motion of the colliding nucleon accross nuclear matter and probably require a fairly deep-going theory of nuclear structure [17]) [29]).

Short communications directly following this paper were read by Meyer, p. 1173, Baz, p. 1180, Shapiro, p. 1168, Skyrme, p. 1179 and Clementel. p. 1167 of this volume.

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12) It should be said that most of the theoretical calculations until now are based on approximation methods (Born or WKB approximations), the accuracy of which is not certain for this kind of problem. Woods and Saxon, ref. 3, have done a complete numerical calculation.
13) See e.g. Köhler, ref. 11.
14) Whenever a qualitatively different r-dependence of the potential is used as a parabolic well (Köhler, H. S., Nuovo Cimento **2** (1955) 911 and ref. 11), it should be considered as an approximation to the more realistic dependence described in the text and supported by all available evidence.
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