VACUUM POLARIZATION EFFECTS IN ELASTIC SCATTERING OF PROTONS BY NUCLEI

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Sensitivity of spin observables in elastic proton scattering by nuclei to the vacuum polarization correction $\delta\rho_{\mathrm{VAC}}$ is explored. The Dirac optical potential is calculated from a complete set of Lorentz invariant amplitudes, Dirac–Hartree densities and the existing estimate of $\delta\rho_{\mathrm{VAC}}$ based on quantum hadrodynamics.

Spin observables in elastic scattering of protons by nuclei have been interpreted successfully in a relativistic approach based on the Dirac equation. Large scalar and vector potentials in the Dirac equation yield a simple description of the analyzing power, A_Y, and the spin rotation function, Q, over a wide energy range. Calculations of the optical potential in the Dirac equation are based on the nucleon–nucleon (NN) interaction and the nuclear density [1–5]. Recently this parameter-free approach has been generalized by developing a complete set of Lorentz invariant NN amplitudes from a relativistic meson exchange description of NN scattering [6–8]. The point of this generalization is to use a dynamical model which specifies the coupling between positive energy and negative energy states, the + to − coupling. In the original form of the impulse approximation, the + to − coupling was not tied to dynamics. Fermi covariants were assumed to provide a useful extension of the NN data to the full Dirac space of two nucleons [4].

Another immediate consequence of a relativistic meson theoretical description of the nuclear interaction is the existence of vacuum polarization contributions to the optical potential. Since they in principle may affect significantly the scattering observables, it is clearly of interest to study their effect. In this letter we examine for various nuclei the sensitivity of elastic proton nucleus scattering to these vacuum polarization corrections.

The Dirac optical potential is determined in a factorized form by the NN amplitude and the nuclear density according to the relation [9]

$$\hat{U}(p',p) = -\tfrac{1}{4} Tr_2\{\hat{M}(p, -\tfrac{1}{2}q \to p', +\tfrac{1}{2}q)\hat{\rho}(q)\}\,, \tag{1}$$

where $\hat{M}$ is the Feynman amplitude for NN scattering and $\hat{\rho}$ is the relativistic nuclear density. The latter is characterized by scalar, vector and tensor terms as follows.

$$\rho(q) = \rho_{\mathrm{S}}(q) + \gamma_2^0 \rho_{\mathrm{V}}(q) - \frac{\alpha_2 \cdot q}{2m}\rho_{\mathrm{T}}(q)\,. \tag{2}$$

Relativistic Hartree wave functions of the nucleus [10] are used to calculate the densities $\rho_{\mathrm{S}}(r)$, $\rho_{\mathrm{V}}(r)$ and $\rho_{\mathrm{T}}(r)$ in coordinate space and these are Fourier transformed to obtain the densities in (2) as functions of the momentum transfer, q. Neutron–proton

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differences are incorporated by evaluating eq. (1) once with the proton–proton amplitude, $\hat{M}_{pp}$, times the proton density, $\hat{\rho}_p$, and once with the proton–neutron amplitude, $\hat{M}_{pn}$, times the neutron density, $\hat{\rho}_n$, and then adding the two results.

Several papers have noted that spin observables in proton scattering are quite sensitive to the difference between scalar and vector terms in the Dirac optical potential [11,12]. Owing to the large scalar and vector components of the NN amplitudes, which have opposite signs, there is sensitivity to the difference, $\delta\rho(r) \equiv \rho_V(r) - \rho_S(r)$, between scalar and vector densities. For Dirac–Hartree wave functions, $\delta\rho(r)$ is determined by the lower components of the single particle wave functions, $F_\alpha(r)$, as follows,

$$\delta\rho_{LC}(r) = 2 \sum_\alpha \frac{2j+1}{4\pi} F_\alpha(r)^2 , \tag{3}$$

where α is summed over the occupied states. The proton vector density is constrained by electron scattering data and is relatively well known. Although there are uncertainties in neutron densities, the total vector density, ρ_V, is fairly well known. On the other hand, the scalar density is not constrained by electron scattering but one expects $\delta\rho_{LC}(r)$ to be small since it vanishes for nonrelativistic wave functions. Employing scattering data to constrain $\rho_V(r)$ and using Dirac–Hartree wave functions to determine $\delta\rho_{LC}(r)$, one may calculate the scalar density, $\rho_S(r) = \rho_V(r) - \delta\rho_{LC}(r)$, which is needed for the Dirac optical potential of (1). However, there is another contribution to the scalar density due to vacuum polarization.

In the quantum hadrodynamics (QHD) model of Serot and Walecka [13], filled negative-energy states of the Dirac sea are shifted in energy due to the scalar and vector potentials. The resulting change of the total energy of the system, ΔE_{VAC}, due to this vacuum polarization is a function of the mean scalar field $\phi_0(r)$. A functional derivative of ΔE_{VAC} with respect to the mean scalar field determines the vacuum polarization correction to the scalar density, $\delta\rho_{VAC}(r) = \delta\Delta E_{VAC}/\delta\phi_0(r)$. The energy correction ΔE_{VAC} has been calculated in a mean field approximation to quantum hadrodynamics after introducing suitable counter terms for renormalization [14,15]. The result is that $\delta\rho_{VAC}$ is determined from the effective nucleon mass, M^*, as follows.

$$\delta\rho_{VAC}(r) = -(1/\pi^2)[M^*(r)^3 \ln(M^*(r)/M) + \tfrac{1}{3} M^3 - \tfrac{3}{2} M^2 M^*(r) + 3MM^*(r)^2 - \tfrac{11}{6} M^*(r)^3] , \tag{4}$$

where $M^*(r)$ is calculated from the total scalar density (proton plus neutron),

$$M^*(r) \cong M - (g_S^2/M_S^2)\rho_S(r) , \tag{5}$$

and M is the nucleon mass, g_S is the scalar meson coupling constant and m_S is the scalar meson mass. Eq. (4) is derived for infinite nuclear matter and then applied to finite nuclei by using the local density to determine $M^*(r)$ in (5). Finite range effects are neglected in eq. (5).

Regarding the vector density as fixed, the total scalar density is found by subtracting the Hartree lower component density, $\delta\rho_{LC}(r)$, of eq. (3), and then adding the vacuum polarization correction of eq. (4),

$$\rho_S(r) = \rho_V(r) - \delta\rho_{LC}(r) + \delta\rho_{VAC}(r) . \tag{6}$$

It is necessary to determine $\delta\rho_{VAC}$ self consistently from eqs. (4)–(6) due to the highly nonlinear dependence of $\delta\rho_{VAC}$ on M^*. A straightforward calculation shows that $\delta\rho_{VAC}$ is about 5% of the vector density. Perry [16] has considered the vacuum polarization for a finite nucleus within QHD. Extra terms are obtained in that case and the vacuum polarization effect causes changes in both the scalar and vector densities. However, the difference $\delta\rho = \rho_V - \rho_S$ is quite similar to the mean field result of eq. (4).

In this paper, the effect of the vacuum polarization on spin observables in proton scattering is considered. The vacuum correction $\delta\rho_{VAC}$ is estimated based on mean field theory using eqs. (4)–(6). Hartree vector and lower-component densities, ρ_V and $\delta\rho_{LC}$, are kept fixed and scalar meson parameters $g_S^2 = 54.289$ and $m_S = 458$ MeV are used [15]. Fig. 1 shows $\delta\rho_{VAC}$ for Ca and Pb nuclei. Somewhat more consistent values of $\delta\rho_{VAC}$ can be obtained by refitting the Hartree density to obtain an improved estimate of $\delta\rho_{LC}(r)$ including the vacuum polarization corrections to the nuclear binding energy. However, the result is quite similar to that given by eq. (4). We do not consider such refinements of the calculation because dynamical uncertainties exist in the deter-

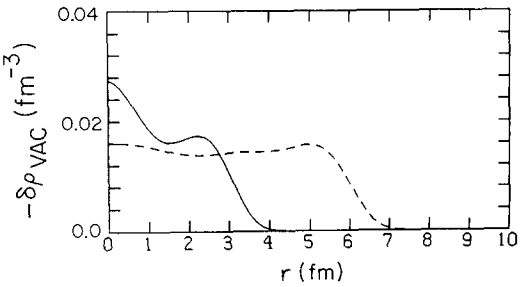

Fig. 1. Negative of vacuum corrections, $\delta\rho_{VAC}$, for ^{40}Ca (solid line) and ^{208}Pb (dashed line).

mination of realistic vacuum polarization corrections. For example, the mean field theory does not include pion exchange dynamics. The values in fig. 1 represent the simplest model. It is conceivable that mean field theory based on scalar and vector mesons overestimates the effect because it yields a rather low value of M^* in the nuclear interior. The optical potential based on complete sets of NN amplitudes as in eq. (1) predicts smaller scalar and vector potentials. This implies a larger value for M^* and therefore a smaller vacuum correction than mean field theory.

In order to explore the sensitivity of proton scattering observables in a phenomenological way, calculations are given for three cases: (1) not including $\delta\rho_{VAC}$(dashed lines); (2) including 50% of $\delta\rho_{VAC}$(solid lines); and (3) including 100% of $\delta\rho_{VAC}$(dotted lines). Figs. 2 and 3 show the proton scattering observables for ^{40}Ca and ^{208}Pb at 500 MeV. Without the vacuum polarization correction, the spin observables are in reasonable agreement with experimental data for ^{40}Ca but the agreement is less good for ^{208}Pb. Including 50% or 100% of the vacuum correction enhances the agreement with experimental data on spin observables at 500 MeV. The results for 500 MeV protons scattering from ^{208}Pb are notably better when vacuum polarization effects are included. Inclusion of $\delta\rho_{VAC}$ yields a similar beneficial effect at 800 MeV. However, at 200 MeV, the results for spin observables are excellent when no vacuum polarization corrections are included. Including 50% or 100% of $\delta\rho_{VAC}$ worsens the agreement. Medium corrections not included in the calculations have been shown by Murdock and Horowitz [17] to be important at 200 MeV. When both medium corrections and vacuum polarization corrections are included, there

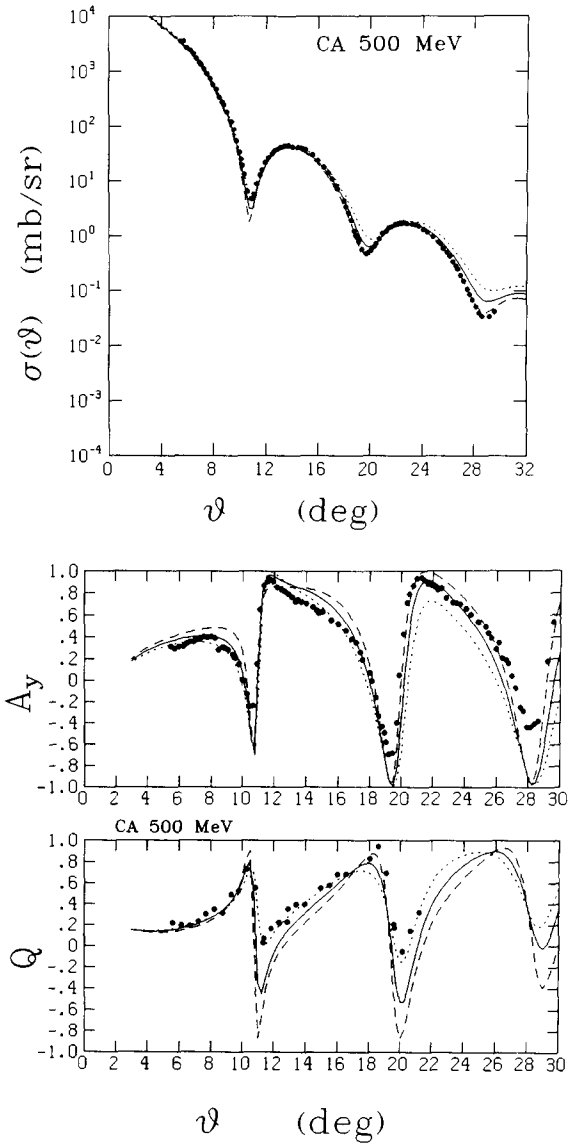

Fig. 2. Analyzing power (A_Y) and spin rotation function (Q) for 500 MeV protons scatteriong from ^{40}Ca.

is agreement with experimental data at 200 MeV. A detailed exposition of these results will be given elsewhere. Since the vacuum corrections are not reliably known, the main point of these comparisons is that there is potentially significant sensitivity to them. Existing estimates of $\delta\rho_{VAC}$ [13–15] are compatible with proton scattering data at intermediate energy.

Vacuum polarization corrections are not well determined for several reasons but they in principle

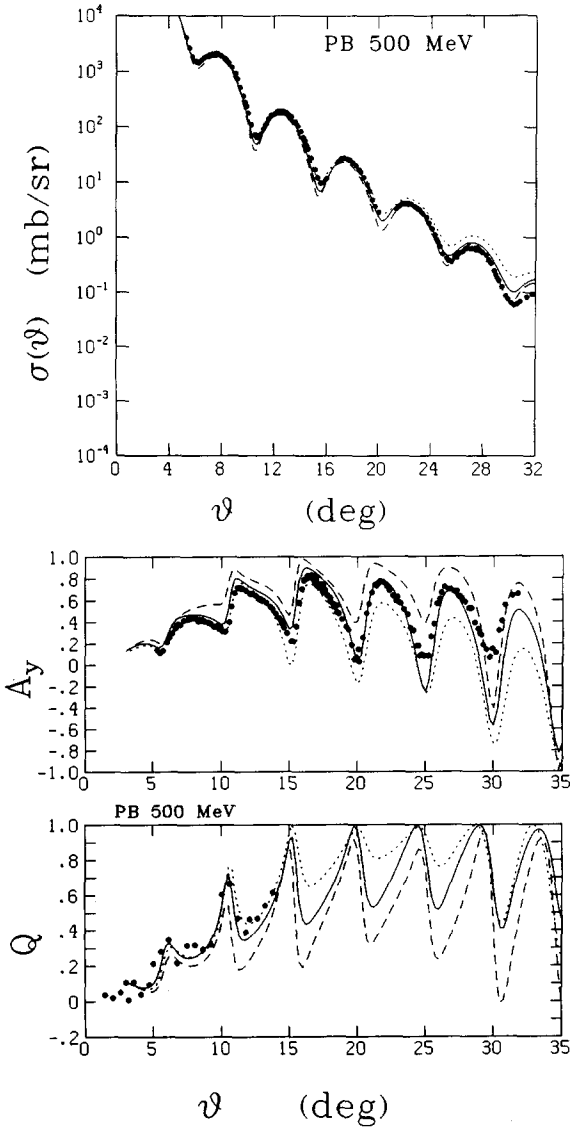

Fig. 3. Analyzing power (A_Y) and spin rotation function (Q) for 500 MeV protons scattering from ^{208}Pb.

should be present in relativistic models of nuclear structure and scattering. From the above study we find that existing estimates of $\delta\rho_{VAC}$ based on a mean field approximation to the quantum hadrodynamics model can have a significant effect on the prediction of spin observables.

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References

[1] J.A. McNeil, J.R. Shepard and S.J. Wallace, Phys. Rev. Lett. 50 (1983) 1439.
[2] J.R. Shepard, J.A. McNeil and S.J. Wallace, Phys. Rev. Lett. 50 (1983) 1443.
[3] B.C. Clark, S. Hama, R.L. Mercer, L. Ray and B.D. Serot, Phys. Rev. Lett. 50 (1983) 1644.
[4] J.A. McNeil, L. Ray and S.J. Wallace, Phys. Rev. C 27 (1983) 2123.
[5] B.C. Clark, S. Hama, R.L. Mercer, L. Ray, G.W. Hoffman and B.D. Serot, Phys. Rev. C 28 (1983) 1421.
[6] J.A. Tjon and S.J. Wallace, Phys. Rev. Lett. 54 (1985) 1357.
[7] J.A. Tjon and S.J. Wallace, Phys. Rev. C 32 (1985) 267.
[8] J.A. Tjon and S.J. Wallace, Phys. Rev. C 35 (1987) 280; C 32 (1985) 1667.
[9] J.A. Tjon and S.J. Wallace, submitted to Phys. Rev. C.
[10] C.J. Horowitz and B.D. Serot, Nucl. Phys. A 368 (1981) 503.
[11] L. Ray and G.W. Hoffman, Phys. Rev. C 31 (1985) 538.
[12] M.V. Hynes, A. Picklesimer, P.C. Tandy and R.M. Thaler, Phys. Rev. Lett. 52 (1984) 978; Phys. Rev. C 31 (1985) 1438.
[13] B.D. Serot and J.D. Walecka, in: Advances in nuclear physics, Vol. 16, eds. J. Negele and E. Vogt (Plenum, New York, 1986) pp. 116–126.
[14] S.A. Chin, Ann. Phys. (NY) 108 (1977) 301.
[15] C.J. Horowitz and B.D. Serot, Phys. Lett. B 140 (1984) 181.
[16] R.J. Perry, Phys. Lett. B 182 (1986) 269.
[17] D.P. Murdock and C.J. Horowitz, Phys. Rev. C 35 (1987) 1442.