

# ERRORS IN THE METHOD OF LINE-REVERSAL

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It is shown that with a favourable experimental design the *random* error in the temperature measurement of hot gases with the method of line-reversal is of the order of 1 deg.K at about 2500°K. A number of *systematic* errors, each of the order of 10 deg.K, are considered. Some applications of the method are discussed quantitatively and a procedure is indicated for the reversal measurement of gas temperatures which are higher than the brightness temperature of the available background light source.

## Introduction

THE method of line-reversal is widely used for measuring gas temperatures, mostly between 1500°K and 3000°K. Since it is a null method the reliability of the results depends in the first place upon the calibration of the reference light source. However, owing to the apparatus used, some systematic errors still remain which concern the reversal measurement itself. With a normal experimental set-up the extent of these errors does not exceed one or two per cent in the temperature measurement. But, in view of the very small random error that is left in these measurements, as will be shown below, it is worthwhile to investigate also the systematic errors in detail.

Only those errors are dealt with which are due to the measuring method. The question whether the measured (excitation) temperature differs from the kinetic temperature (or any other 'temperature') of the gas because of deviations from a state of equilibrium is not discussed.

## Description of the Method

In a gas the population ratio  $N_n/N_m$  of two energy levels  $\epsilon_n$  and  $\epsilon_m$  ( $\epsilon_n > \epsilon_m$ ) with statistical weights  $g_n$  and  $g_m$  of any type of atom can be described by a (formal) temperature  $T_E$  according to

$$\frac{N_n}{N_m} = \frac{g_n}{g_m} \exp[-(\epsilon_n - \epsilon_m)/kT_E] \quad [1]$$

$T_E$  is called the excitation temperature of the pair of levels  $\epsilon_n$  and  $\epsilon_m$ . The intensity of the radiation, emitted in the transition  $n \rightarrow m$  is given by

$$U = A_{nm}N_n h\nu_{nm} \quad [2]$$

where  $A_{nm}$  is the transition probability for spontaneous emission and  $\nu_{nm}$  the frequency of the radiation. When near the gas a reference light source is installed which emits radiation with an intensity  $U_R$  in and around the wavelengths of the spectral line considered, light will be absorbed from the reference light source according to

$$U' = B_{mn}N_m U_R \quad [3]$$

where  $B_{mn}$  is the transition probability for absorption. To the radiation  $U_R$  a (formal) temperature  $T_R$  (in the restricted wavelength range) can be assigned according to Planck's radiation formula. Stimulated emission is negligible at the temperatures and wavelengths considered. From the relation between the transition probabilities  $A_{nm}$  and  $B_{mn}$  which contains the radiation formula, it follows that the contribution of the emission and the absorption in the spectral line to the continuous spectrum behind the gas are equal ( $U = U'$ ), when  $T_E = T_R$ . This equality of absorption and emission is independent of the number of atoms  $N_0$  and of the value of the transition probabilities. It is also independent of the occurrence of self-absorption in the gas; the line profiles of

the emission line and the absorption line remain identical. However, the values of  $N_0$  and  $A_{nm}$  and the occurrence of self-absorption do influence the accuracy of the measurement (see below).

The possibility of equalizing  $T_E$  and  $T_R$  experimentally (normally by varying  $T_R$ ) is the basis of the method of line-reversal: the spectrum of the light from the reference light source, as seen through the hot gas, should not show the spectral line under consideration [Figure 1(b)]. When the radiation temperature of the reference light source is lower than the excitation temperature an emission line appears in the spectrum [Figure 1(a)]; when the radiation temperature is higher an absorption line appears [Figure 1(c)]. The foregoing holds, of course, only if the areas and the solid angles of emitted and absorbed radiation contributing to the comparison are equal.

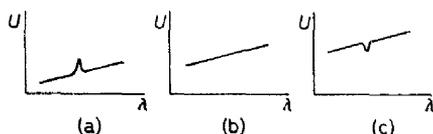


FIGURE 1. The intensity  $U$  of the spectrum when a reference light source is seen through a hot gas, at three different brightness temperatures  $T_R$  of the reference light source. The intensity in the spectral line of the hot gas is governed by an excitation temperature  $T_E$ . The relation between the two temperatures is: (a)  $T_R < T_E$ ; (b)  $T_R = T_E$ ; (c)  $T_R > T_E$

The method for detecting the radiation depends upon the specific problem in hand. The spectral range containing the transition(s) considered may be selected by a non-dispersive filter and the gas may be brought intermittently into the optical path, or one can use a double beam system. Then the variation of the intensity in the spectral line with time is observed. When the spectral line under consideration lies rather isolated in the spectrum of the gas one may use a prism or grating spectrograph and compare the intensity in the spectral line with the intensity of the continuous spectrum at either side of the line. The last method is the most common one and can be employed in visual,

photographic and photo-electric modifications when the gas system is stationary during the comparison.

### Systematic Errors

One of the advantages of the method of line-reversal lies in it being a null method; the detecting apparatus need only indicate the presence or absence of a difference of intensities. Another advantage is that the compensation occurs at the beginning of the optical path: apart from the requirement that the aperture of the apparatus is completely filled by light from the reference light source, the optical path between the hot gas and the detector as well as the properties of the detecting apparatus have, in first order, no systematic influence on the result of the measurement.

However, in view of the accuracy that can be obtained in these measurements, it is important to discuss the minor systematic errors that still may occur.

### The reference light sources

The choice of the reference light source depends in the first place upon the temperature of the hot gas to be investigated. For temperatures up to about 2600°K, i.e. for most measurements in flames, tungsten strip lamps are widely used, and also tungsten arc lamps. For higher temperatures the anode of a carbon arc (up to about 3800°K) is used. For still higher temperatures one has applied the radiation of the sun and the continuum radiation from high pressure gas discharges in mercury and xenon (up to about 6000°K). We will mention the accuracies that are attributed to the calibration of these sources: strip lamp calibrations are furnished by standard laboratories with an accuracy<sup>1</sup> of about 5 deg.K up to about 2600°K. The temperature of the anode of a carbon arc is considered<sup>2</sup> to be correct within about 25 deg.K, and the accuracy of the temperature of the hottest sources, the sun<sup>3</sup> (through the atmosphere) and gas discharges, is not better than about 100 deg.K. However, it should be remembered that at a wavelength of 4000 Å a difference of 100 deg. at 6000°K is equivalent to a difference of 17 deg. at 2500°K.

*The optical path before the flame*

Normally an image of the reference light source is formed in the hot gas by a lens ( $L_1$  in Figure 2). With a single lens it mostly suffices to use the estimate that four per cent of the light is lost per surface by reflection. When a complex lens (often with coated surfaces) is used the transmission has to be measured with great care. Systematic errors in the transmission measurement easily creep in, since the lens not only weakens but also deforms the transmitted light beam. Figure 3 shows the transmission of a Leitz Summicron lens (eight coated surfaces).

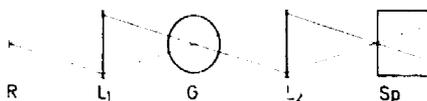


FIGURE 2. Normal arrangement for the observation of line-reversal. The radiation of a reference light source R is viewed with a spectroscope Sp through a hot gas G, which emits (and absorbs) radiation of one or more discrete wavelengths

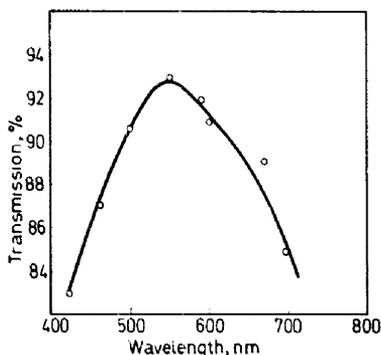


FIGURE 3. The transmission of a Leitz Summicron lens versus wavelength

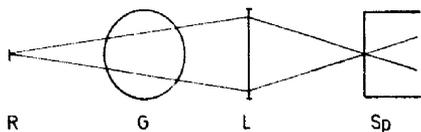


FIGURE 4. Modified arrangement for the observation of line-reversal (without a lens between the light source and the hot gas)

To avoid the problem of the transmission of the lens it sometimes is possible not to use the lens at all, namely when the temperature of either the reference light source or the hot gas or of both is homogeneous over a sufficiently large area (Figure 4). One may of course decrease the area used by decreasing the aperture of the light beam (by a diaphragm behind L), but this also increases the random error of the measurement (see below).

*Reflection of radiation*

A systematic error may often occur by reflection of radiation which is emitted by the hot gas in the direction of the reference light source. The radiation reflected, for instance, from the first lens, or, when a strip lamp is used, from the window of the lamp and from the strip itself, is (partly) added to the radiation from the reference source into the dispersing apparatus. The adjustment to the reversal point then requires a higher level of background radiation than without this reflected radiation. In Table 1 some examples of this effect are given. These values were found in an experiment in which the spectral line showed self-absorption, as is usual in reversal measurements. The increase is therefore mainly due to the flanks of the line. Without self-absorption the effect would have been greater. The different values obtained with the strip lamp refer to different orientations of the lamp. The maximum value was found when the strip was in a normal position, i.e. when its surface was perpendicular to the optical axis.

TABLE 1. Examples of the effect of reflected radiation: the relative increase of the detected radiation ( $\lambda = 5890 \text{ \AA}$ ) from a flame, coloured with sodium, caused by reflection from (an) object(s) before the flame (see Figure 2)

Object at the place of the reference light source	Object at the place of the first lens	Increase of line intensity %
Flat glass plate	—	4
Aluminium mirror	—	25
—	Coated lens	1
Strip lamp	Coated lens	1-5
—	Strip lamp (Figure 4)	0-7

No reflection was found when the strip was turned  $\frac{1}{4}$  radian (or more) around a vertical axis. This position is of course to be preferred, but the change of brightness temperature with viewing angle<sup>4</sup> (about 3°) should be allowed for. Errors of the order of 10° (four per cent) may occur because of this effect of reflected radiation.

#### The optical path behind the hot gas

A stringent condition for the validity of the method of line-reversal is that light from the reference light source is (at least) available both in every part of the hot gas and in every solid angle contributing to the detected radiation from the hot gas. The hot gas emits light in every direction, but the solid angle of the competing light from the reference light source is restricted by the aperture of the first lens. A sufficient condition is that the aperture of the first lens is not restricting the solid angle of reference light used behind the first lens. If this condition is not fulfilled the measured temperature will be too high because of an excess of light from the hot gas.

#### Stray light

Since the image of the reference light source is mostly small as compared to the area covered by the hot gas, light from parts of the hot gas outside this image may interfere with the comparison of intensities behind the spectrograph when it is scattered from lens  $L_2$  into the solid angle actually used (Figure 5). An excess of light from the gas will be detected and the measured reversal temperature will be too high. By dusting  $L_2$  with some powder we could raise the measured reversal temperature of a

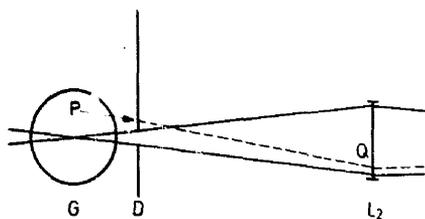


FIGURE 5. The avoidance of stray light, from the flame by means of a diaphragm D. Without D the light from P, scattered at Q, may reach the detector

flame by 35 degrees. Normal errors amount to 5 or 10 deg.K. To eliminate this effect the radiation from the hot gas can be intercepted by a diaphragm D (Figure 5), which should at most be as large as the image of the reference light source in the hot gas.

#### Polarization

The oblique surfaces of a prism partly polarize the transmitted light beam. This effect may introduce an error when the light from the reference light source is also partly polarized. A check can be made when the reversal temperature is determined with a rotatable polarizer in the light path behind the hot gas. We have not found an error caused by polarization, even with a strip lamp viewed from an angle of 15° from the normal.

The errors due to the apparatus, as discussed in the preceding sections, were checked separately. When care is taken to avoid these errors the remaining uncertainty in the comparison of the temperatures of the reference light source and the hot gas, due to these effects, is as follows:

Reflection losses	1 deg.K
Reflection of flame light	1 deg.K
Equality of solid angles	0 deg.K
Stray light	1 deg.K
Polarization	0 deg.K

We therefore estimate the total amount of these systematic errors at about 2 deg.K. The error in the absolute value of the measured temperature is, in these circumstances, mainly determined by the accuracy of the calibration of the reference light source, which is about 3 to 5 deg.K.

#### The Random Error

In reversal measurements the main (and unavoidable) noise source is due to the fluctuations in the number of detected quanta  $B$  from the line and from the reference light source in the transmitted spectral range. Normally the primary events of detection (e.g. with photoelectric detection: the photo-electrons) occur independently and the root-mean-square  $R$  of the number of detected quanta is given by

$$R(= B^{\frac{1}{2}}) = (\eta \bar{N} \tau)^{\frac{1}{2}} \quad [4]$$

where  $\eta$  is the efficiency of the detector,  $\bar{N}$  the

mean number of incident quanta per second (in the optical signal  $B$ ) and  $\tau$  the measuring time. The threshold signal  $\Delta B$ , defined as the extra signal in order that  $B + \Delta B$  can be 'just' distinguished from the main signal  $B$ , is then usually defined as  $\Delta B = K\sqrt{2R}$ , in which  $\Delta B = \eta\bar{n}\tau$ ,  $\bar{n}$  being the mean number of incident quanta per second in the optical signal  $\Delta B$ . The factor  $K$ , the signal-to-noise ratio at the threshold, is determined by the reliability required in the measurements of the difference of two optical signals, and is mostly taken as one or two. Inserting the expression for  $\Delta B$  in equation 4 yields

$$\bar{n} = K\sqrt{2\bar{N}}^{\frac{1}{2}}(\eta\tau)^{-\frac{1}{2}} \quad [5]$$

Additionally one often defines the contrast threshold  $C$  as

$$C \equiv \frac{\bar{n}}{\bar{N}} = \frac{\Delta B}{B} = K\sqrt{2(\eta\bar{N}\tau)^{-\frac{1}{2}}} \quad [6]$$

The relations 5 and 6 were derived under the assumption that the efficiency  $\eta$  does not change when the extra signal  $\Delta B$  is added to  $B$ . In later sections (the photographic plate and the eye) the validity of equations 5 and 6 is reconsidered.

However, there are circumstances in which other noise sources than the one due to the shot noise from the background light source dominate and the contrast threshold is consequently increased. For instance, when a photocell is used as a detector, the Johnson noise of the anode resistance will be the largest noise source, unless a very high value of  $B$  can be used. Another example is the haze in a photographic plate which, for small values of  $B$ , is the main noise source. In photo-electric detection methods also flicker noise may occur (for instance in the dark current), in addition to the photocurrent shot noise or to the dark current noise<sup>2</sup>. Since the spectral power of the flicker noise is approximately inversely proportional to the noise frequency it can be suppressed by using a.c. methods of detection at sufficiently high modulation frequencies.

Still, when the contrast threshold is governed by the normal shot noise in the number of detected quanta, in some detectors there remains the complication that the quantum efficiency

depends upon  $\bar{N}$  (e.g. the human eye) or upon  $\bar{N}\tau$  (e.g. the photographic plate): the adaptation of the eye and the a-linearity of response of the photographic plate partly cancel the gain in the contrast threshold when  $\bar{N}$  (and  $\bar{N}\tau$ , respectively) is increased, and finally lead to an increase of the contrast threshold in the region of over-exposure.

In Figure 6 the contrast thresholds of a number of detectors are plotted as a function of the intensity of the optical signal, using reasonable assumptions of the quantum efficiencies and the spectral noise powers.

### Threshold of a Spectral Line in a Continuous Spectrum

#### Thresholds in a continuous spectrum

The method of line-reversal implies the determination of a contrast threshold evoked by a spectral line in a part of a continuous spectrum. We therefore consider the energy in a continuous spectrum as emitted, for instance, by a tungsten strip, of which an image is formed on the slit of a spectrograph. Behind the spectrograph the energy in a spectral range  $\Delta\lambda$  is given by

$$U = U(\lambda, T)h_s b_s \Omega \alpha \Delta\lambda \quad [7]$$

In this expression  $U(\lambda, T)$  is the energy per  $\text{cm}^2$ , per nm of wavelength, per second and steradian according to Planck's radiation formula at the wavelength  $\lambda$  and at the radiation temperature  $T$  of the image of the strip,  $h_s$  and  $b_s$  the slit height and the slit width,  $\Omega$  the aperture (in sr) before the prism in the spectrograph and  $\alpha$  the transmission factor of the apparatus. Inserting, for example, the data for a medium glass prism spectrograph and the energy emitted by a black-body at 2500°K and 600 nm in a spectral interval of 0.1 nm one finds  $U = 4 \times 10^{-9}$  watt ( $= 1.2 \times 10^{10}$  photons per second).

Assuming that shot noise in the number of detected quanta is the main noise source, equation 6 yields the contrast threshold in this continuous spectrum, i.e. the minimum increase in the intensity of the transmitted spectral range of the continuum that can be 'just' detected.

$$C = K\sqrt{2[\eta U(\lambda, T)h_s b_s \Omega \alpha \Delta\lambda \tau]^{-\frac{1}{2}}} \quad [8]$$

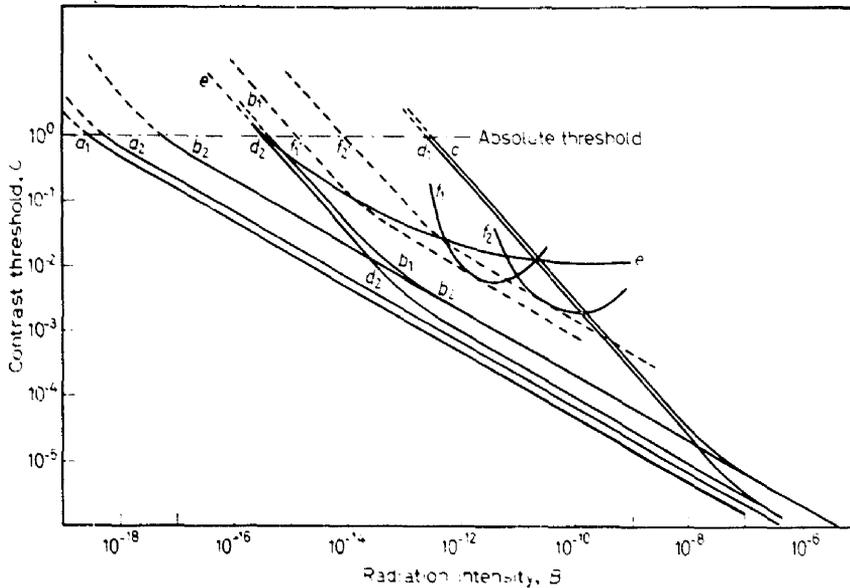


FIGURE 6. Contrast thresholds  $C$  versus the radiation intensity  $B$  in watts for various photodetectors. For all detectors the measuring time is one second.  $a_1$ , ideal detector,  $\lambda = 400$  nm;  $a_2$ , ideal detector,  $\lambda = 800$  nm;  $b_1$ , photomultiplier,  $\lambda = 400$  nm,  $\eta = 0.1$  (ref. 5);  $b_2$ , cooled photomultiplier,  $\lambda = 400$  nm,  $\eta = 0.1$  (ref. 5);  $c$ , photocell,  $\lambda = 400$  nm,  $\eta = 0.1$ , anode resistance  $10^8 \Omega$  (ref. 6);  $d_1$ , Ge p-n diode,  $\lambda = 400$  nm,  $\eta = 0.6$  (ref. 6);  $d_2$ , cooled Ge p-n diode,  $\lambda = 400$  nm,  $\eta = 0.6$  (ref. 6);  $e$ , human eye, adapted to intensity  $B$ ,  $\lambda = 550$  nm (ref. 7);  $f_1$ , photographic plate, Kodak Royal X,  $\lambda = 400$  nm,  $A = 1 \text{ mm}^2$  (ref. 7);  $f_2$ , Kodak Pan X,  $\lambda = 400$  nm,  $A = 1 \text{ mm}^2$  (ref. 7). Here:  $\lambda$  is the wavelength of the radiation,  $\eta$  is the efficiency of the detector at this wavelength, and  $A$  is the area on the photographic plate. Logarithmic curves with a slope of  $-\frac{1}{2}$  denote that the contrast threshold is governed by shot noise from the reference light source; curves with a slope of  $-1$  denote that another noise source is dominant. The graphs also show the link between the absolute thresholds (i.e. the magnitude of the signal  $\Delta B$  that is 'just' detectable when no background signal  $B$  is present) and the contrast thresholds  $C$ . When the slope  $-\frac{1}{2}$  is maintained up to  $C = 1$ , there is no source of fluctuations but the shot noise of the detected photons. Above  $C = 1$  these curves have a slope of  $-1$  since then the shot noise in  $\Delta B$  itself is the main noise source

Using the relations

$$b_s = Df\Delta\lambda \quad [9]$$

and

$$\Omega = S/f^2 \quad [10]$$

equation 8 can be written as

$$C = K\sqrt{2}(\Delta\lambda)^{-1}[\eta U(\lambda, T)(h_s/f)DS\alpha\tau]^{-\frac{1}{2}} \quad [11]$$

Here  $D$  is the angular dispersion,  $S$  the cross section of the light beam in the dispersing element and  $f$  the focal length of the collimator of the spectrograph. That we take the spectral interval  $\Delta\lambda$  of equation 9 (pertaining to the

entrance slit) identical to the interval  $\Delta\lambda$  of equations 7 and 8 (pertaining to the exit slit) implies that we employ the normal slit setting, in which a (geometrical) image of the entrance slit is equal in width to the exit slit. In photographic detection one may use a smaller slit width (of the densitometer) to be followed by an averaging procedure in the densitogram over the interval  $\Delta\lambda$ . Still, the entrance slit width should not be smaller than the value given by equation 9 when the minimum resolvable wavelength difference  $\Delta\lambda_s$  is inserted, since the limit of the validity of equation 9 is reached for this value of  $\Delta\lambda$ .

When the contrast threshold, given by equation 11, is taken to exist between two black-bodies at slightly different temperatures, the minimum observable temperature difference can be obtained from Planck's formula. For a detector with an efficiency  $\eta = 0.01$ , a measuring time of one second and the energy of the example above equation 11 yields

$$C = 1.2 \times 10^{-4}$$

which is equivalent to a temperature difference of 0.03 deg.K at 2500°K. This figure would pertain, for example, to the accuracy obtainable with a photo-electric pyrometer, which has the spectral and geometrical properties as assumed above.

#### Thresholds for a spectral line in a continuous spectrum

In the method of line-reversal the relevant threshold exists between a spectral line in a continuum and the mean of the adjacent parts of the continuum at either side of the line. There is no difference in the definition of or the expression for the contrast threshold in this case as compared to the previous section, but a difference does exist when we want to convert the contrast threshold into a minimum detectable temperature difference. The reasons are that the flame may not be optically thick at the wavelength of the spectral line (not even in the centre

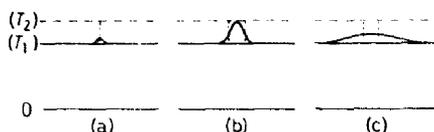


FIGURE 7. Equivalent widths of spectral lines, demonstrated in the situation of reversal measurements. The solid and interrupted horizontal lines denote the intensity of black-body radiation at two different temperatures  $T_1$  and  $T_2$ .  $T_2$  is the gas temperature to be measured,  $T_1$  is the (radiation) temperature of the background light source. The solid curves show the line profiles, the dotted lines show the equivalent widths of the spectral lines. Figures 7(a) and 7(b) pertain to the same spectral apparatus, 7(a) for a weak line and 7(b) for a strong line. Figures 7(b) and 7(c) pertain to the same line (with equal intensity = equal area under the profile). In Figure 7(c) the line is smoothed by insufficient resolving power, but the equivalent width is the same as in Figure 7(b). The figures show a case in which  $T_2 > T_1$ , but an analogous discussion holds when  $T_2 < T_1$ .

of the line) and that the real spectral width of the line often is smaller than the transmitted spectral range. To avoid the explicit expressions for the profiles—only the areas under the profiles are of interest here—we describe in the following the spectral line by an 'equivalent width'  $\Delta\lambda_{eq.}$ , defined as the width of a line with a rectangular profile, the height of which corresponds to black-body radiation (at the temperature of the hot gas) and the area of which equals the area under the real line profile (Figure 7).

One may distinguish two cases. The resolving power of the spectral apparatus may be sufficient to transmit the spectral line profile virtually unsmoothed. In that case equation 11 can be applied without change. Still, it is of advantage to increase the slit widths until  $\Delta\lambda = \Delta\lambda_{eq.}$ . Then the minimum detectable temperature difference can be obtained as described previously.

Often, however, the resolving power will not be sufficient and the spectral line is smoothed ( $\Delta\lambda > \Delta\lambda_{eq.}$ , see Figure 7). In that case the contrast between the situations with and without the spectral line in the range  $\Delta\lambda$  is furnished within the smaller range  $\Delta\lambda_{eq.}$ . In the measurement the contrast is observed within the range  $\Delta\lambda$ , but the temperature difference is correlated to  $\Delta\lambda_{eq.}$ . The threshold contrast to be used to obtain the minimum detectable temperature difference should therefore be increased by a factor  $\Delta\lambda/\Delta\lambda_{eq.}$ . Equation 11 then becomes

$$C = K\sqrt{2(\Delta\lambda_{eq.})^{-1}[\eta U(\lambda, T)(h_s/f)DS\alpha\tau]^{-\frac{1}{2}}} \quad [12]$$

Taking a reasonable value of the equivalent width of 0.005 nm (measured with quite a high sodium concentration in a flame at 1 atm) the contrast threshold according to equation 12 becomes

$$C = 2.4 \times 10^{-3}$$

which is equivalent to a temperature difference of 0.6 deg.K.

It should be noted that, since the equivalent width of a spectral line is used in equation 12, the actual slit width, and therefore the resolving power of the spectrograph, does not influence the threshold, provided the slit width is not too

small (see above) and provided that the other factors involved remain constant (see, however, below as to the variation of the efficiency).

#### *The equivalent width*

In the equivalent width of the spectral line are implicit the factors that determine the intensity of the line, which is of paramount importance for the accuracy of a reversal measurement. We mention the most important factors.

(1) The population of the energy levels of the transition. Because of the Boltzmann distribution, resonance lines are strongly favoured but other lines are not excluded in principle.

(2) The transition probability. Large values are of course favoured.

(3) The product of the concentration  $P$  of the particles involved and the length  $l$  of the optical path in the hot gas. For small values of  $Pl$  the integrated line intensity is proportional to  $Pl$ . However, from a certain value of  $Pl$  upward the integrated line intensity, and therefore  $\Delta\lambda_{\text{eq}}$ , becomes roughly proportional to  $(Pl)^2$ , yielding a slower increase of  $\Delta\lambda_{\text{eq}}$ . (The influence of the wavelength of the line and the temperature of the hot gas is implicit in the use of Planck's radiation formula.)

#### **Application**

##### *Photo-electric detection in steady state measurements*

The simplest case to deal with quantitatively is the photo-electric measurement of line-reversal with the aid of a photomultiplier. The almost noiseless amplification makes the Johnson noise in the anode resistance and possible other noise sources in the amplifier or the indicator negligible, while the quantum efficiency of the detector is constant up to high values of  $\bar{N}$ . As is shown in Figure 6 the obtainable contrast thresholds are quite low.

We performed experiments with a prism apparatus using a modified scanning technique<sup>9</sup>. The part of the spectrum that contained the spectral line to be reversed was made to vibrate across a fixed exit slit by means of a mirror, which was mounted on the axis of a milliammeter. An a.c. current of 25 c/s and about 2 mA through the coil of the meter caused a vibration of the mirror and consequently of the spectrum.

The spectral line crossed the exit slit 50 times per second. This 50 c/s signal was detected by a photomultiplier behind the slit. The electronics consisted of a phase-sensitive narrow-band (around 50 c/s) amplifier. The measurement concerned the temperature of an acetylene-air flame (about 2500 K), in which a sodium solution was sprayed. The background light source, a tungsten strip lamp, was adjusted until no 50 c/s signal was detected. We obtained an accuracy, limited by the shot noise of the background spectrum, of 0.8 deg. K in a measuring time of one second using one component of the yellow Na-D doublet, in agreement with the result above.

##### *Photographic detection*

The use of a photographic plate for reversal measurements has the well-known advantage that a number of spectral lines can be investigated simultaneously. For this purpose one has to make a series of spectra with slightly different temperatures of the background light source. Since the efficiency of a plate, defined as the number of developed grains per photon, decreases (after an initial increase) when the number of photons per unit area increases, there is for a fixed slit width (see above) an optimum exposure time for which the smallest contrast threshold is obtained (whereas with photo-electric detectors an increase of the measuring time was, in principle, always of advantage).

It is, however, still possible to increase the exposed area of the photographic plate by optical enlargement of the spectrum and, by doing so, to increase the exposure time while keeping the efficiency of the plate at its maximum value (interrupted curves in Figure 6). In Figure 6 the practical upper and lower limits of the useful area of a plate were taken to be 1 cm<sup>2</sup> and 10<sup>-4</sup> cm<sup>2</sup> respectively. Another lower limit is set by the practicable optical aperture. When the area of the plate used is not adjusted to maximum efficiency, a (slow) plate with high contrast is to be preferred. However, when the areas can be adjusted the gain in speed of a fast plate overcompensates for the low contrast and the larger graininess of such plates and the contrast threshold is smaller than with slow

plates. The scanning area of the densitometer should include the width (see above) and, if possible, the total height of the image of the spectral line.

In equations 5 and 6 it was assumed that the efficiency is independent of  $\bar{N}$  or of  $\bar{N}\tau$ . In the photographic plate the efficiency decreases (after an initial increase) with  $\bar{N}\tau$ . This dependence tends to reduce the effect of an exposure to  $\Delta B$ . This is very clearly so in the region of over-exposure, when an extra signal does not increase the density on the plate any more and the contrast threshold has lost its meaning. In the curves of Figure 6 this effect has been allowed for, since the curves were deduced from measured values of just detectable energy differences<sup>7</sup>.

With a fine grain plate (Ilford R 50) we measured a contrast threshold of one per cent in the density at a density of 1.8, while measuring the reversal temperature of the CN bands at 388 nm in an arc discharge of 1 atm. The random error of the measurement amounted to about 20 deg.K at 6000°K.

#### Visual detection

In the human eye efficiency decreases with the background intensity in such a way that the contrast threshold is fairly constant (at about one per cent) over a range of background intensities. However, there is no change in efficiency when the extra signal  $\Delta B$  is added to the signal  $B$ , since such changes require times of at least seconds whereas the duration of  $\Delta B$  is mostly smaller than one second. Furthermore, the measuring time is limited to about 0.2 second per measurement. Also the solid angle outside the eye, from which the light is integrated in the eye, is limited. Because of the last phenomenon an adjustment of the geometrical width of the line (by optical magnification) to the spatial integrating properties of the eye does increase the accuracy. A further increase of the magnification again decreases the accuracy, because then the intensity gradient on the retina is decreased and because from a certain magnification the pupil is no longer completely filled with light.

Assuming a contrast threshold of one per cent an optimal accuracy of about 30 deg.K

(around 2500°K) is calculated from Planck's formula, if the line is smoothed a factor of ten by the spectrograph. This is in reasonable agreement with visual measurements we performed<sup>8</sup>. With an apparatus of high resolving power more accurate measurements can be made visually.

#### Time-resolved measurements

The studies of transient temperature phenomena with the method of line-reversal have yielded valuable results. In principle, the accuracy of the measurements is less than for stationary systems since the time available for the measurement is very limited. As an example we calculate the accuracy of the measurements of reversal temperatures in shock waves, as performed by Clouston, Gaydon and Glass<sup>9</sup> with a spectrograph, and by Clouston, Gaydon and Hurle<sup>10</sup> who used interference filters.

Applying equation 12 we insert the values given by the authors:  $\eta = 0.01$  (EMI 6097 F at 589 nm),  $U_\lambda$  (2000°K) =  $3 \times 10^{15}$  photons per sec. sr. cm<sup>2</sup>. nm,  $b_s = 1.8 \times 10^{-2}$  cm and  $\Omega = 2.5 \times 10^{-3}$  sr, and values estimated by us:  $\Delta\lambda_{\text{eq.}} = 0.16$  nm,  $K = 1$ ,  $h_s = 0.2$  cm,  $\alpha = 0.25$  and  $\tau = 10^{-5}$  sec; we find for the measurements first mentioned

$$C = 0.14$$

which is equivalent to a temperature difference of 35 deg at 2000°K.

For the measurements with an interference filter the authors give  $\eta = 0.01$  (EMI 6095 F) and  $U_\lambda$  (2000°K) =  $3 \times 10^{15}$ . The other values were estimated as:  $h_s \times b_s = 2 \times 10^{-2}$  cm<sup>2</sup>,  $\Omega = 10^{-2}$  sr,  $\alpha = 0.25$  and  $\tau = 10^{-5}$  sec. Because there is no angular dispersion in this case we have to use equation 8, but still have to multiply the result by the ratio of the transmitted spectral range ( $\Delta\lambda = 6$  nm) and the equivalent width of the line, again estimated at 0.16 nm. Equation 8 then yields a value of

$$C = 0.18$$

which is equivalent to a temperature difference of 45 deg.K in  $10^{-5}$  sec. At 2500°K, where the background intensity is about ten times higher than at 2000°K, this value is about 15 deg.K.

The values found are in fair agreement with the results of the measurements mentioned. The authors give values of 30 deg.K at 2000 K in the first example and of 20 deg. (in one measurement 5 deg.) at 2500 K in the second example. However, two parameters estimated by us need some comment. The estimation of the equivalent width of the spectral line of 0.16 nm may seem rather large. We have used this value because (a) the value is not improbably high in view of the high sodium content used and pressure broadening effects and (b) it is the value that corresponds to the slit widths employed in the measurements. The estimation of the measuring time is also a doubtful item. We have estimated a value of  $10^{-5}$  sec from the pictures in the papers, but a revaluation can easily be made when the mean of the photomultiplier signal is taken over a longer or a shorter period.

#### Measurements above the reversal point

When the temperature of the reference light source cannot be matched to the temperature of the hot gas (for instance because of technical limitations) it is often possible to measure separately the emission of the hot gas, and the absorption in the gas (preferably with an a.c. method). It is also possible to measure the

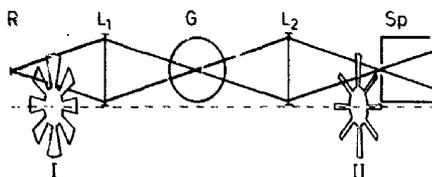


FIGURE 8. Modified set-up for reversal measurements of stationary gas temperatures which are higher than the temperature of the reference light source (cf. Figure 2). The two synchronous choppers I and II differ in that the first has a tooth-opening ratio of 1:1, whereas the second has a ratio of say 1:4

emission of the hot gas, the emission of the reference light source and the 'combined emission' alternately<sup>11</sup>. From these data the temperature of the hot gas can still be determined with

the aid of Planck's formula, but the essential advantage of the reversal method, the optical compensation of the radiation, is lost and with it the high accuracy of the method. However, we are able to decrease by a known factor the signal due to the emission of the gas as compared to the signal due to absorption of background radiation by the gas and thus to realize line-reversal, by making use of an extra modulation.

For this purpose a sectored disc I (see Figure 8) was placed between the background source and the hot gas. The teeth of this disc were as wide as the openings, as usual. Between the hot gas and the monochromator another disc II was placed with an equal number of teeth, which was fixed with disc I on to a common axis. The teeth of disc II were narrower than the openings. The resulting time course of the various light signals is as shown in Figure 9. The amplitude of the 600 c/s signal is (a) decreased by absorption in the flame and (b) increased by emission from the flame, but, considering only the 600 c/s component, with a smaller weight factor. Therefore the emission of the atoms in the flame must be

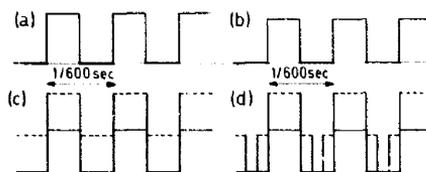


FIGURE 9. The light modulations as employed in the reversal measurement of gas temperatures which are higher than the temperature of the reference light source used; (a) the intensity of the reference light behind disc I; (b) the intensity of the reference light decreased by absorption in the hot gas; (c) the intensity of the light signal, decreased by absorption (full curve), and increased by continuous emission (interrupted curve) of the hot gas; (d) the intensity of the light signal behind disc II. At the 'reversal point' the 600 c/s components of curves (a) and (d) are equal

larger than the absorption by these atoms to restore the original amplitude of the 600 c/s signal from the background source alone. The reversal point is obtained when the (excitation) temperature of the spectral line is higher than the (radiation) temperature of the background

source. To detect this reversal point the part of the spectrum around the spectral line was again scanned periodically with the aid of a vibrating mirror, as described above, with a frequency of 25 c/s. The electric signal is then quite complicated: a 100 c/s signal modulated with 50 c/s, with another 50 c/s signal superposed. Filtering, demodulating and rectifying this signal yields the information wanted. Using a width ratio of 1:4 between tooth and opening in the second disc we obtained reversal in reality at a difference of about 300 deg.K between the brightness temperature of the background source and the flame temperature. It seems possible to extend the method to differences up to about 800°K, i.e. to use strip lamps to measure flame temperatures up to about 3500 K. Because of the modulations implied the method is only applicable to stationary systems and only with electronic detection.

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