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DEGREE OF COHERENCE *)

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Abstract

A new simple definition of the degree of coherence is proposed from which a previously proved theorem, also used as a definition, can easily be derived.

In the thirties two totally different definitions of the degree of coherence were given and from these definitions some theorems concerning this concept were derived. One of these treatments started from the assumption of a monochromatic lightsource, consisting of a great number independently emitting elements, oscillating with different phases. Then in a plane, illuminated by this source, the existence of any correlation between the resulting vibrations in adjacent points was investigated. Indeed such a correlation was found. If the mean illumination in the plane is I_0, then the probability for the vibrations in adjacent points P and P_0 to have amplitudes A and A_0 and phases ϑ and ϑ_0 respectively was proved to be (van Cittert, [1])

$$W(\xi_0, \eta_0, \xi, \eta) = \frac{1}{\pi^2(1-c^2)I_0^2} \exp\left\{- \frac{(\xi_0^2 - 2c\xi_0\xi + \xi^2) + (\eta_0^2 - 2c\eta_0\eta + \eta^2)}{(1-c^2)I_0}\right\}.$$

Here

$$\xi_0 = A_0 \cos \vartheta_0;\ \eta_0 = A_0 \sin \vartheta_0;$$
$$\xi = A \cos \vartheta;\ \eta = A \sin \vartheta;$$

and

$$c = \frac{\sin 2\pi x\alpha_0/\lambda}{2\pi x\alpha_0/\lambda},$$

α_0 being half the angle subtended by the lightsource at P_0, the lightsource supposed to be rectangular and centered on the normal through P_0. From this the existence of a relation between the vibrations (ξ_0, η_0) in P_0 and (ξ, η) in P is evident and for this relation the fraction c is determinative. This number is called the *degree of coherence*, as was proposed by Zernike [2]). The maximum value of $|c|$ is 1 and the minimum value is 0.

*) Dedicated to professor F. Zernike on the occasion of his 70th birthday.

If $c = 1$, then $W = 0$, except for $\xi = \xi_0$ and $\eta = \eta_0$. In this case the vibrations in P_0 and P are perfectly coherent. If however $c = 0$, then

$$W = \frac{1}{\pi I_0} \exp\left(-\frac{\xi_0^2 + \eta_0^2}{I_0}\right) \cdot \frac{1}{\pi I_0} \exp\left(-\frac{\xi^2 + \eta^2}{I_0}\right)$$

and hence the vibrations in P_0 and P are completely incoherent.

Considering c as a function of x, this function is identical with the (amplitude) diffraction-function of a diaphragm of the same size as the lightsource. More generally we have the following theorem:

I. *The degree of coherence in a plane illuminated by a lightsource is identical with the diffraction-function of a diaphragm of the same shape and size as the lightsource.*

Furthermore the following theorems were proved:

II. *The visibility in the interference-pattern as obtained from the vibrations at* P_0 *and* P *is equal to the degree of coherence between these two points.*

III. *The distribution of the degree of coherence in a plane, illuminated by imaging an extended source on this plane by means of a lens is completely identical with the distribution obtained by direct illumination from a source of the same shape as the lens.*

In other words: *there is no difference whatsoever between direct illumination and illumination by means of a lens.*

Of these theorems some applications were discussed, e.g. with the help of theorem I a new interpretation of Michelson's wellknown experiment on the determination of stellar diameters could be given.

Regarding the rather intricate statistical calculations on which the above considerations were based, Zernike [2]) introduced a totally different definition of the degree of coherence. He reversed theorem II, now *defining the degree of coherence between the vibrations in* P_0 *and* P *as being equal to the visibility of the interferences obtained with these vibrations.* Then starting from this definition, he proved the theorems I and III once more. Zernike also gave some applications, among which there was again Michelson's experiment. Zernike's method undeniably has the advantage of avoiding the always complicated statistical calculations, but has the disadvantage of not being based directly on the cause of the difference in composition of the vibrations in the two points, taking a rather arbitrary, though obvious consequence as a starting point.

There is however a possibility for another and very simple definition of the degree of coherence. Let P_0 and P be illuminated by two similar beams of light of equal intensity I_0. Each of these beams is supposed to consist of two parts A and B; parts A with intensities cI_0 being *perfectly coherent* and parts B with intensities $(1 - c)I_0$ being *completely incoherent*, both mutually and with respect to A. The quantity c, thus introduced, is now defined as being the degree of coherence between the beams, (composed of parts A and B) illuminating P_0 and P. If the two beams are made to interfere, only parts A are effective, forming an interference pattern in which the intensity-distribution as a function of the phase-difference φ, is given by

$$I_1 = 4cI_0 \cos^2 \tfrac{1}{2}\varphi.$$

The maximum intensity therefore is $4cI_0$ and the minimum intensity is 0. On this pattern a uniform distribution with an intensity

$$I_2 = 2(1 - c)I_0$$

is superimposed. As a result the maximum intensity becomes

$$I_{max} = 4cI_0 + 2(1 - c)I_0 = 2(1 + c)I_0$$

and the minimum intensity

$$I_{min} = 2(1 - c)I_0.$$

Then the visibility V is immediately found to be equal to c, *viz.*

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2(1 + c)I_0 - 2(1 - c)I_0}{2(1 + c)I_0 + 2(1 - c)I_0} = c$$

So from the new definition theorem II follows immediately and since Zernike's considerations are based on this theorem, the other theorems evidently are most suitably proved in the way as indicated by Zernike.

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2) Zernike, F., Physica **5** (1938) 785–795.